Gravitational lensing time-delays and implications for dark matter in galaxies

Kick-off meeting UNDARK

Teodori Luca

IAC, October 2024

WEIZMANN INSTITUTE OF SCIENCE

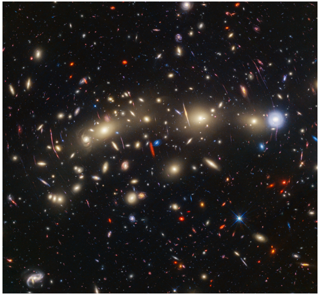
Based on:
K. Blum, LT [2105.10873]
LT, K. Blum, E. Castorina, M. Simonović, Y. Soreq (2022) [2201.05111]
LT, K. Blum [2305.19151]
K. Blum, LT [2409.04134]

UNDARK
Funded by the European Union

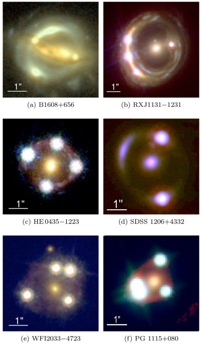
Gravitational lensing systems as DM tracers

$m_{\rm dm}$[eV] wave particle macro $\sim 10^{-9} M_\odot$

10^{-20} 10^{-18} 10^{-16} ≀≀ 10^{6} 10^{8} 10^{10} 10^{12} 10^{14} ≀≀ 10^{53} 10^{55} 10^{57}



MACS 0416 (Nasa, Esa)



K.C. Wong et al 2019 [1907.04869]

- Spacetime curvature feels everything, even “nightmare scenario” dark matter
- Light deflection features connect to galaxy/clusters density models
- New data incoming! LSST forecast: $\sim$ 1000 quasar lenses with measurable time delay

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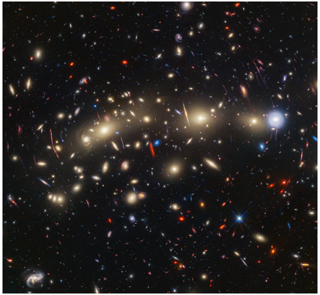
$m_{\rm dm}$[eV] — wave: 10^{-20}, 10^{-18}, 10^{-16} — particle: 10^{6}, 10^{8}, 10^{10}, 10^{12}, 10^{14} — macro: 10^{53}, 10^{55}, 10^{57} — $\sim 10^{-9} M_{\odot}$



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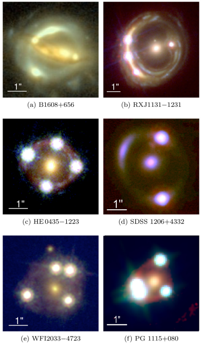

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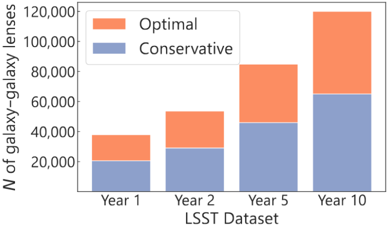
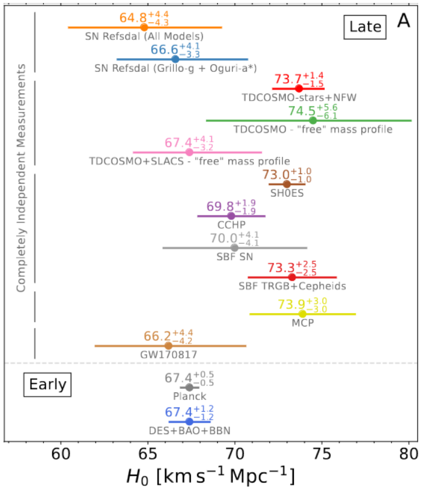


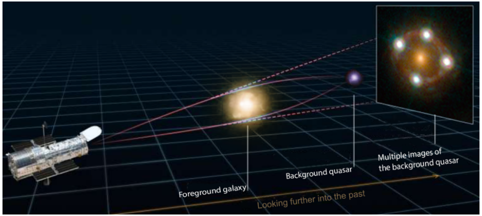
A. J. Shajib et al 2024 [2406.08919]

Time-delay cosmography



P.L. Kelly et al 2023 [2305.06367]

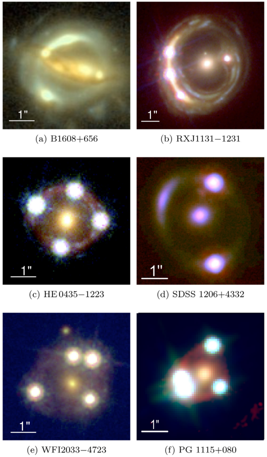
- Dimension-full observable, as of now primarily used for cosmology (H_0), but large error bars $\Longrightarrow$ degeneracies
- Can they be used to constrain DM profile of galaxies/clusters?



Martin Millon, Image from Hubble Space Telescope

Degeneracies (or opportunities?)

- Unaccounted galactic density profile features (core? Nearby group?): internal mass sheet degeneracy
- Large scale structure "on the way": external mass sheet degeneracy (weak lensing)



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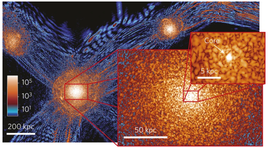

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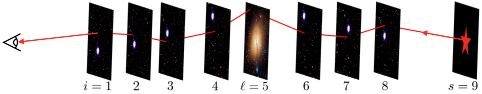
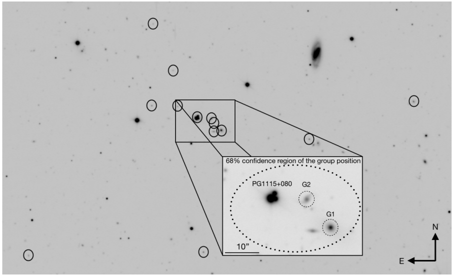


C. McCully et al 2014 [1401.0197]

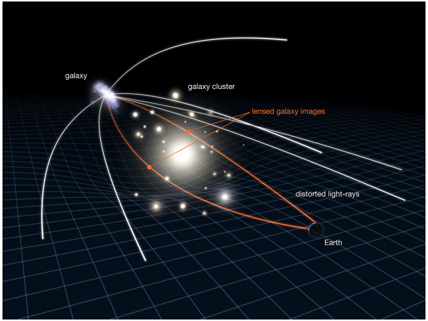


G.C.-F. Chen et al 2019 [1907.02533]

Outline

- Small recap: time delay cosmography for H_0
- Large scale structures effects subtleties
- Internal Mass sheets, unaccounted cores (ULDM?)
- Can multiple source systems break degeneracies?

Strong Gravitational Lensing in Elliptical Galaxies



ESA/Hubble

- Lens model $\rightarrow$ convergence (dimensionless) $\kappa \sim \int \mathrm{d}\eta\, q(\eta)\rho(\eta)$.
- Lens model + time delay measurement

$$\Delta t_{ij} \propto \frac{1}{H_0}(\text{Geometric} + \text{Shapiro}) \,.$$

- Degeneracies (Mass sheet degeneracy): source position and mass of galaxy unknown

$$\kappa \rightarrow \lambda\kappa + (1-\lambda) \implies H_0 \rightarrow \lambda H_0$$

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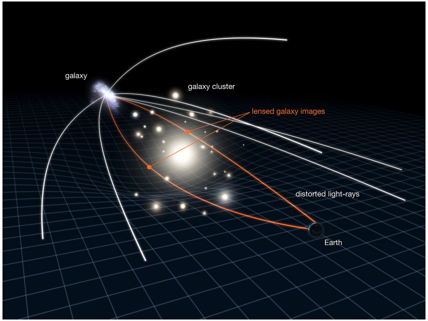

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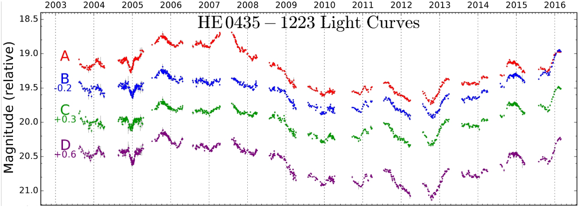

V. Bonvin et al 2016 [1607.01790]

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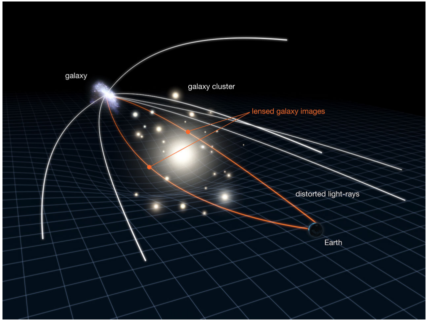

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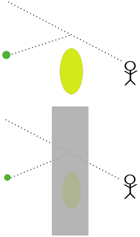

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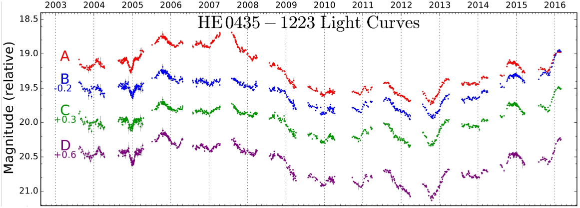

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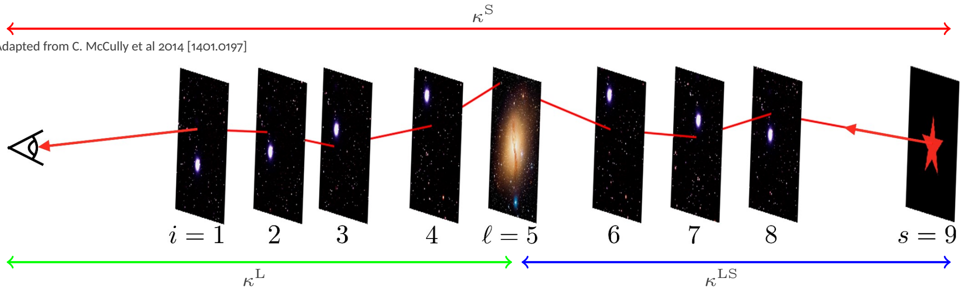
Effect of Large Scale Structures



Adapted from C. McCully et al 2014 [1401.0197]

- External convergences can be removed from imaging modelling, unobservable
- Time delays do change!

$$\Delta t \rightarrow \frac{(1-\kappa_{\mathrm{S}})(1-\kappa_{\mathrm{L}})}{1-\kappa_{\mathrm{LS}}}\Delta t$$

$$H_0 \rightarrow \frac{(1-\kappa_{\mathrm{S}})(1-\kappa_{\mathrm{L}})}{1-\kappa_{\mathrm{LS}}}H_0$$

- External convergence maps from simulations only for κ^{S}.

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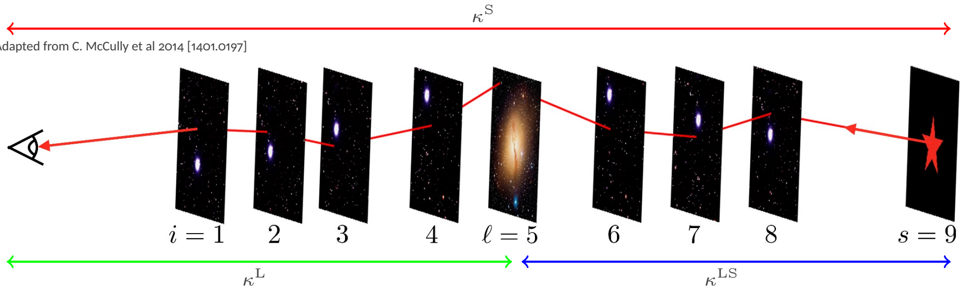

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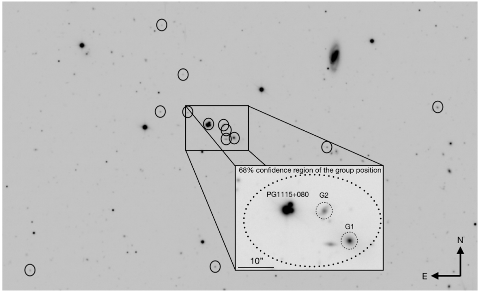
Internal mass sheet degeneracy: lens residing in a group



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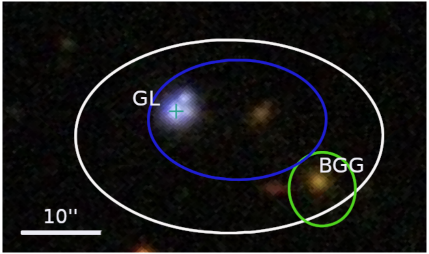
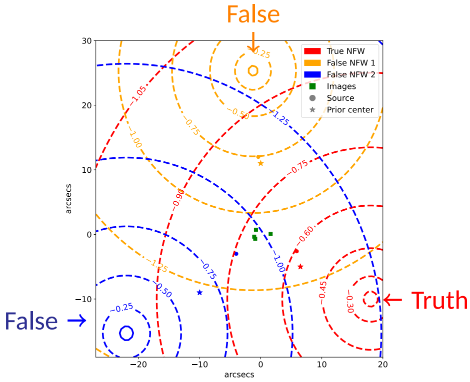


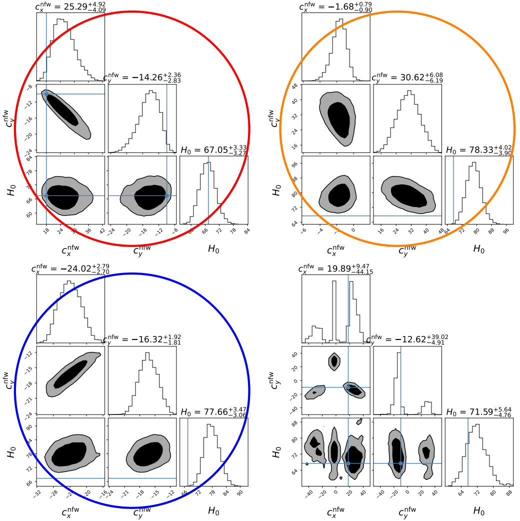
LT, K. Blum 2023 [2305.19151]

- The sheet comes from mass components internal to the lens
- Lens galaxies often reside in groups
- PG1115 example: use kinematics to constrain mass and center of the halo position
- X-ray? Dependence on the quasar emission subtraction, not reliable

Discrete flexion degeneracy from a NFW group

- First order α expansion $\sim$ convergence; second order: flexion F, G.
- $F \sim \mathrm{e}^{\mathrm{i}\phi} F_0(h)$, $G \sim \mathrm{e}^{3\mathrm{i}\phi} G_0(h)$
- Falling into the false center yields a bias in H_0 (due to the different convergence inferred)





Internal Mass sheet degeneracy: lens core component

- Possible cores from non-trivial DM models (Not related to core-cusp debate!). A generic core-MSD (approximate degeneracy) is what inflated the error bars on the TDCOSMO collaboration measurement.
- Ultralight dark matter, cores

$$\lambda_{\rm deBroglie} = \frac{1}{mv} \sim 20\,\mathrm{kpc}\frac{1\times 10^{-24}\,\mathrm{eV}}{m}$$

- Subdominant component (order 10%) compatible with constraints, can form cores affecting time delay measurements

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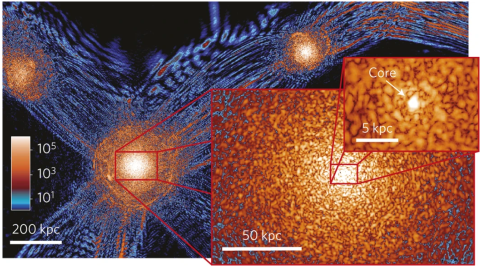

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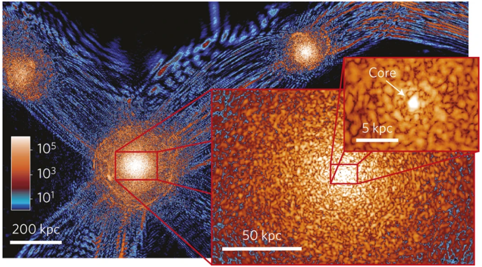

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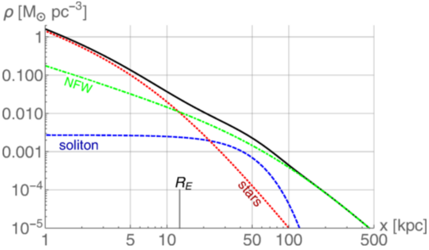

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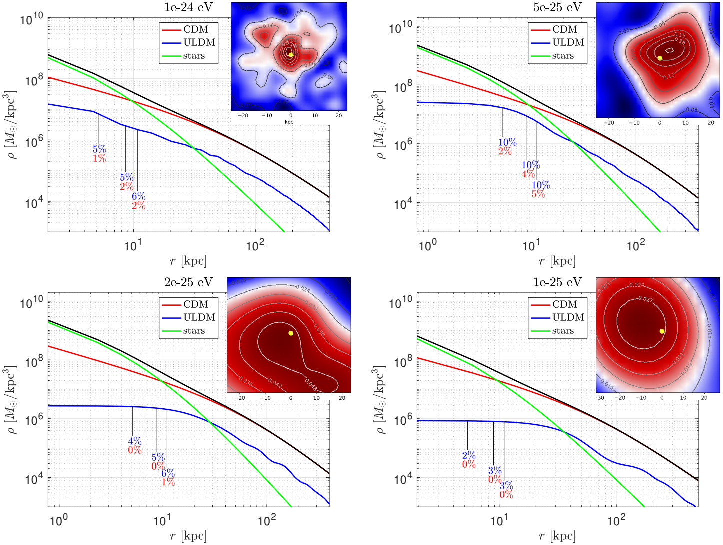
AxionHOgraphy?

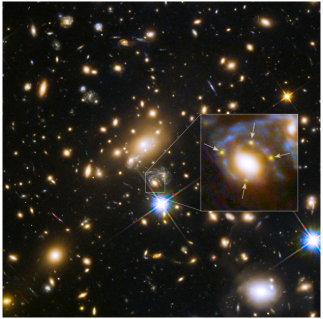


K. Blum and LT [2409.04134]

- Can you look for subdominant (order 10 %) component ULDM with time delay bias?
- (Non cosmological) simulations (we acknowledge E. Hardy and M. Gorghetto) of galactic environment suggests formation of core structure which can possibly be detected
- Need for more refined simulations to sharpen the theoretical prediction

Multi-source systems and MSD

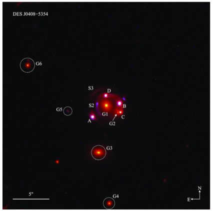
Cluster MACS J1149.5+2223. Credit: NASA



- Multiple sources do not break the mathematical MSD! See e.g. P. Schneider 2014 [1409.0015]
- MSD factors can be fully reabsorbed for a single source only. Observable:

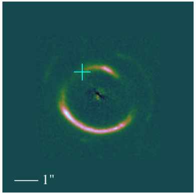
$$C_i \frac{\left|\vec{\theta}_1^{\mathrm{E}}\right|}{\left|\vec{\theta}_i^{\mathrm{E}}\right|} \approx 1 + \gamma_{1i}\,, \gamma_{1i} := \delta\kappa_{1i}^{\mathrm{S}} - \delta\kappa_{1i}^{\mathrm{LS}} + \kappa_{c1}\left(1 - C_i\right)$$

A.J. Shajib et al 2019 [1910.06306]



D.J. Ballard et al [2309.04535]

HST I-band



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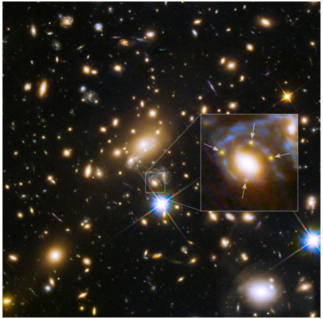

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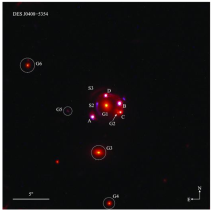

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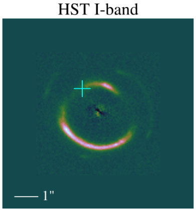

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Summary

What can we use time delay cosmography for?

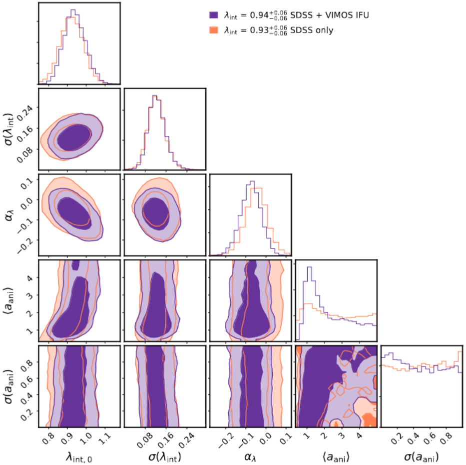
- As of now, primarily used as cosmology tool (most notably H_0)
- Challenge: modelling degeneracies; important to point out all the shortcomings to lensing collaborations, in other to achieve an *accurate* H_0 measurement.
- Promising! New data arriving, including precise stellar kinematics, new lensed systems etc.
- Effects from observer-lens and lens-source sightlines *must* be taken into account for a non-biased inference of time delays and non-biased stellar kinematics constraint.
- The presence of the group can yield additional degeneracies to the lensing reconstruction problem
- ULDM as a possible internal mass sheet (AxionHography?)
- With multi-source systems, you might spot internal profile mismodeling (e.g. cores).
- With an H_0 prior, we can measure galactic features like large cores, difficult to spot otherwise, and better characterization of host groups and weak lensing field effects.

Toy example: power law

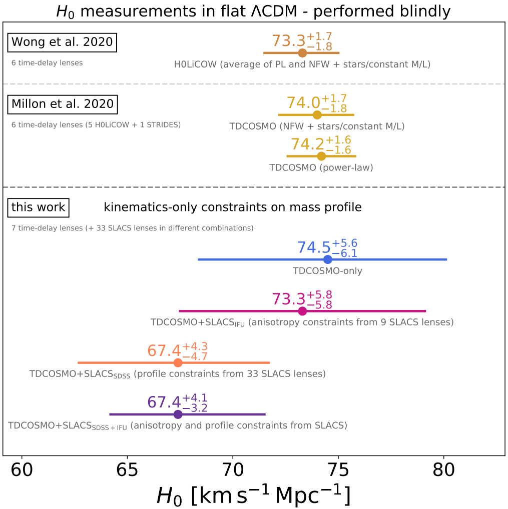
Absorb in source coord. Absorb in external shear
Absorb in MSD

- $\alpha(\theta) \sim \theta^0 + (\kappa + \text{shear})\theta + (F+G)\theta^2$, $F = F_0(h)e^{i\phi}$, $G = G_0(h)e^{3i\phi}$
- Inference constrains $\frac{F}{1-\kappa} = F_{\rm eff}$, $\frac{G}{1-\kappa} = G_{\rm eff}$
- In a false minimum, $G_{\rm eff}^{\rm inference} = G_{\rm eff}^{\rm truth}$, $F_{\rm eff}^{\rm inference} \to 0$.
- In power law, $\rho \sim \theta^{-\gamma}$, $\frac{F_{\rm eff}}{G_{\rm eff}} = \frac{\gamma-3}{\gamma+1}$, $G_{\rm eff} = \frac{-\gamma^2+1}{3-\gamma}\frac{\kappa}{h(1-\kappa)}$
- Decrease $\frac{F_{\rm eff}}{G_{\rm eff}} \implies$ increase γ, keep constant $G_{\rm eff} \implies$ decrease κ/h.
- Decrease $\kappa \implies$ bias up H_0.

TDCOSMO IV

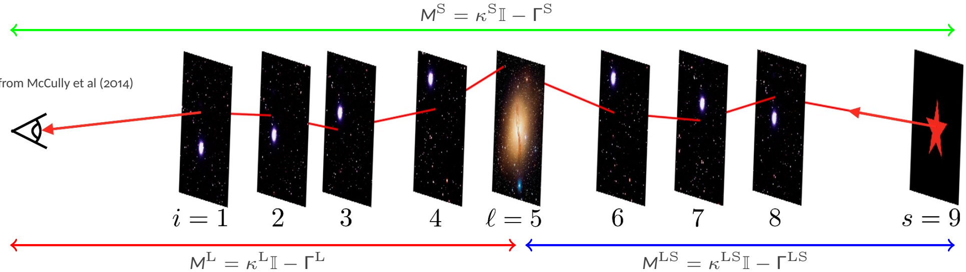


S. Birrer et al 2020 (TDCOSMO IV) [2007.02941]



The multilens equation

The tidal approximation



from McCully et al (2014)

Lens equation in tidal approximation

$$\vec{\beta} = (\mathbb{I} - M^S)\vec{\theta} - (\mathbb{I} - M^{LS})\vec{\alpha}((\mathbb{I} - M^L)\vec{\theta})$$

Degeneracy ("revised" MSD)

$$1 - M^R \longmapsto \lambda_R (1 - M^R),$$
$$\vec{\beta} \longmapsto \lambda_S \vec{\beta},$$
$$\vec{\alpha}(\vec{\theta}) \longmapsto \lambda_S\, \lambda_{LS}^{-1}\, \vec{\alpha}(\lambda_L^{-1}\, \vec{\theta}),$$
$$\Psi(\vec{\theta}) \longmapsto \lambda_S\, \lambda_{LS}^{-1}\, \lambda_L\, \Psi(\lambda_L^{-1}\, \vec{\theta}).$$

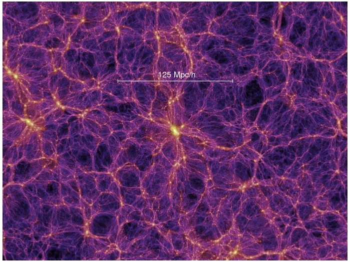
Choose

$$\lambda_S = \frac{1}{1 - \kappa^S}, \quad \lambda_{LS} = \frac{1}{1 - \kappa^{LS}}, \quad \lambda_L = \frac{1}{1 - \kappa^L},$$

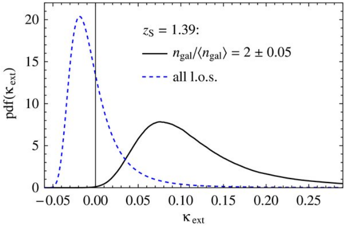
external convergence removed from the modeling

Interpreting the Mass Sheet Degeneracy

Springer et al 2005



Suyu et al 2010



- Time delays do change!

$$\Delta t \to \lambda_S \lambda_{LS}^{-1} \lambda_L \Delta t$$

$$H_0 \to \lambda_S \lambda_{LS}^{-1} \lambda_L H_0$$

- Bias in time delays:

$$\frac{H_0^{\text{inferred}}}{H_0^{\text{true}}} = \frac{1 - \kappa^{LS}}{(1 - \kappa^{L})(1 - \kappa^{S})}$$

- Estimate κ^S via ray-tracing through Millennium Simulation and characterization of the lens field
- Stellar kinematics can constrain mass in the galaxy $\sigma^2 \sim GM/R$, sensitive to normalization of $\vec{\alpha}$, in particular it is sensitive to

$$1 - \kappa^{\text{ext}} := \frac{1 - \kappa^S}{1 - \kappa^{LS}} \implies \frac{H_0^{\text{inferred}}}{H_0^{\text{true}}} \approx 1 + \kappa^L$$

- Direction of bias? Need estimates of κ^L, κ^{LS} variances and correlations, but order few percent.

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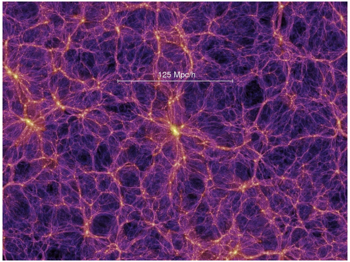

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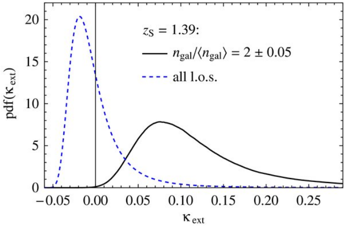

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$$\frac{H_0^{\rm inferred}}{H_0^{\rm true}} = \frac{1-\kappa^{\rm LS}}{(1-\kappa^{\rm L})(1-\kappa^{\rm S})}$$

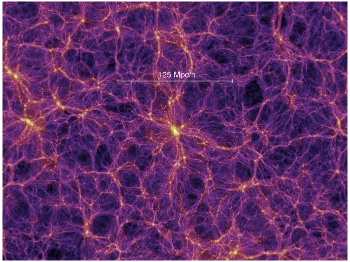
- Estimate $\kappa^{\rm S}$ via ray-tracing through Millennium Simulation and characterization of the lens field
- Stellar kinematics can constrain mass in the galaxy $\sigma^2 \sim GM/R$, sensitive to normalization of $\vec{\alpha}$, in particular it is sensitive to

$$1-\kappa^{\rm ext} := \frac{1-\kappa^{\rm S}}{1-\kappa^{\rm LS}} \implies \frac{H_0^{\rm inferred}}{H_0^{\rm true}} \approx 1+\kappa^{\rm L}$$

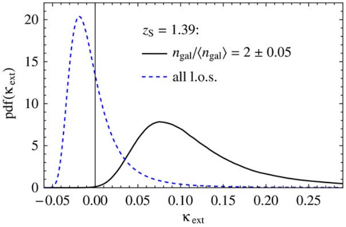
- Direction of bias? Need estimates of $\kappa^{\rm L}$, $\kappa^{\rm LS}$ variances and correlations, but order few percent.

Interpreting the Mass Sheet Degeneracy

Springer et al 2005



Suyu et al 2010



- Time delays do change!

$$\Delta t \to \lambda_{\rm S}\lambda_{\rm LS}^{-1}\lambda_{\rm L}\Delta t$$
$$H_0 \to \lambda_{\rm S}\lambda_{\rm LS}^{-1}\lambda_{\rm L}H_0$$

- Bias in time delays:

$$\frac{H_0^{\rm inferred}}{H_0^{\rm true}} = \frac{1-\kappa^{\rm LS}}{(1-\kappa^{\rm L})(1-\kappa^{\rm S})}$$

- Estimate $\kappa^{\rm S}$ via ray-tracing through Millennium Simulation and characterization of the lens field
- Stellar kinematics can constrain mass in the galaxy $\sigma^2 \sim GM/R$, sensitive to normalization of $\vec{\alpha}$, in particular it is sensitive to

$$1-\kappa^{\rm ext} := \frac{1-\kappa^{\rm S}}{1-\kappa^{\rm LS}} \implies \frac{H_0^{\rm inferred}}{H_0^{\rm true}} \approx 1+\kappa^{\rm L}$$

- Direction of bias? Need estimates of $\kappa^{\rm L}$, $\kappa^{\rm LS}$ variances and correlations, but order few percent.

Multi-lens MSD with lens coupling

- "Real model" with sheets ($i = 0$ is first plane, or lens plane)

$$\beta_i = (1 - \kappa(z_i, 0))\theta - \sum_{j=0}^{i-1}(1 - \kappa(z_i, z_j))C_{ij}(\alpha_j(\beta_j) + \kappa_{cj}\beta_j)\ ,\ C_{ij} := \frac{D_{\mathrm{S_{j+1}}} D_{\mathrm{LS_i}}}{D_{\mathrm{LS_{j+1}}} D_{\mathrm{S_i}}}$$

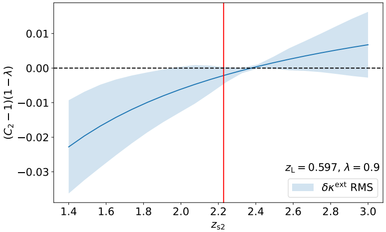
- Target model (we want to remove κ_{c0})

$$\tilde{\beta}_i = \theta - \sum_{j=0}^{i-1} \tilde{C}_{ji}(\tilde{\alpha}_j(\tilde{\beta}_j) + \tilde{\kappa}_{cj}\tilde{\beta}_j) \implies \begin{cases} \tilde{\beta} & = \lambda_i \beta\ ,\ \lambda_i = (1 - \kappa(z_i, 0) - C_{i0}\kappa_{c0}(1 - \kappa(z_i, z_0))(1 - \kappa(z_0, 0)))^{-1} \\ \tilde{C}_{ij} & = \lambda_{ij} C_{ij}\ ,\ \lambda_{ij} = (1 - \kappa(z_i, z_j))\lambda_i \\ \tilde{\alpha}_i(\beta) & = \alpha_i(\beta/\lambda_i) \\ \tilde{\kappa}_{c0} & = 0\ ,\ \tilde{\kappa}_{ci} = \kappa_{c0}/\lambda_i \text{ if } i > 0 \end{cases}$$

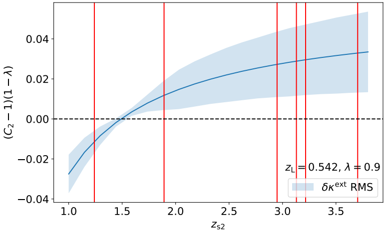
- Differential convergence effect encoded in C_{ij} rescaling, but κ_{ci}, $i > 0$, will not be rescaled away (but still it becomes a differential convergence factor)
- Schneider [1409.0015]: use careful κ_{ci} determination to mask away differential convergences as well, relevant if other planes need explicit modelling.

External convergence reinterpretation?

DESJ0408-5354

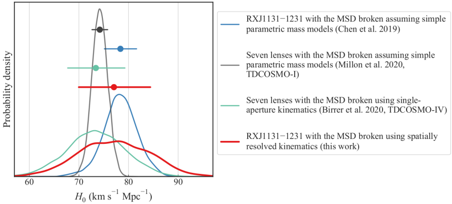


Cluster MACS J1149.5+2223



- If γ_{1i} is too large to be explained by external convergences, it might be an indication of a non-zero, unaccounted κ_c!
- $\gamma_{1i} = \delta\kappa^{S}_{1i} - \delta\kappa^{LS}_{1i} + (1 - \lambda)(1 - C_i)$
- Cluster here only as proof of concept, in the actual cluster model you should have λ terms for any substructure, and not just a single one

MSD: internal and external (TDCOSMO XII)



The real internal convergence profile, to be tested against kinematics

$$\kappa_{\rm gal}(\theta) = (1-\kappa^{\rm ext})(\lambda\kappa_{\rm model}(\theta) + (1-\lambda)\kappa_{\rm s}(\theta))$$

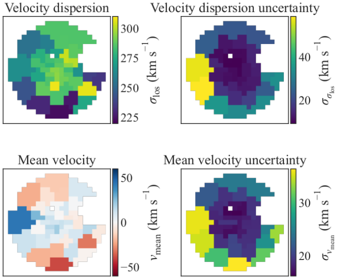
$$\Delta\tau = (1-\kappa^{\rm ext})\lambda\Delta\tau^{\rm model} \implies H_0 = (1-\kappa^{\rm ext})\lambda H_0^{\rm model}$$

$$\kappa_{\rm s}(\theta) = 1 + \text{finite core corrections}\ ,\ 1-\kappa^{\rm ext} := \frac{1-\kappa^{\rm S}}{1-\kappa^{\rm LS}}$$

$$\theta = \frac{\tilde{\beta}}{1-\kappa^{\rm S}-\kappa_{\rm c}} + \frac{(1-\kappa^{\rm LS})\tilde{\alpha}}{1-\kappa^{\rm S}-\kappa_{\rm c}(1-\kappa^{\rm LS})}$$

$$\kappa_{\rm c} =: (1-\lambda)(1-\kappa^{\rm ext})$$

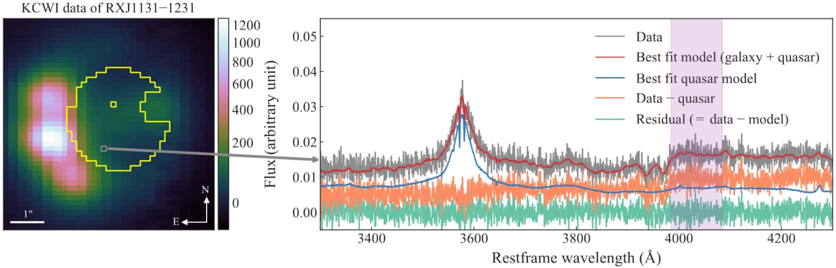
TDCOSMO XII



- Resolved kinematics with Keck Cosmic Web Imager (KCWI), to mitigate systematic effects
- New result for a single system:

$$H_0 = 77.1^{+7.3}_{-7.1}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- Goal for the future: Spatially resolved velocity dispersion measurements for around 40 time-delay lens galaxies will yield an independent 2% H_0 measurement without any mass profile assumption



Steps for the kinematic measurement

- For each pixel, compute the spectroscopic S/N ratio, to perform Voronoi binning (have the best spatial resolution given a S/N treshold).
- Fit the spectra in each Voronoi bin using pPXF and the X-shooter Spectral Library (XSL)
- Change template setup to estimate variance-covariance matrix (hopefully catching possible systematics)
- Solve the axisymmetric Jeans equation given an anisotropy profile and a gravitational potential. The gravitional potential is obtained from deprojecting $\kappa_{\rm gal}$ (they consider oblate and prolate case), which includes the finite core term.
- With this, infer a constraint on the possible mass of the lens $\implies$ constraint on the mass sheet degeneracy, hence on the error bars on the inferred H_0.

Computing differential convergences expectations

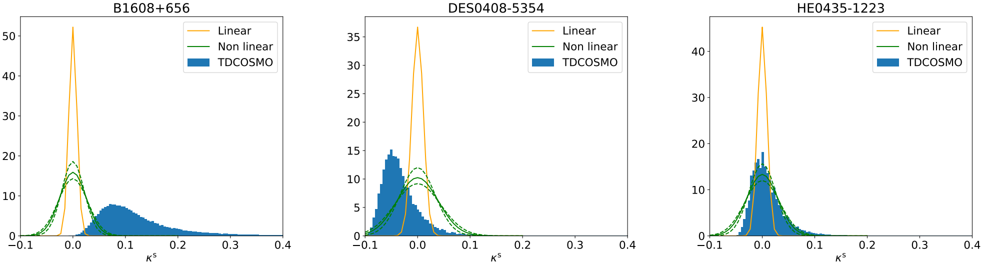
Lensing kernel | Unequal time power spectrum

- $\kappa(\eta_i, \eta_j) = \int \mathrm{d}\eta\, q_{ij}(\eta)\delta(\eta)$
- $\langle(\kappa(\eta_i, \eta_j) - \kappa(\eta_k, \eta_l))^2\rangle = \int \mathrm{d}\eta \int \mathrm{d}\eta'\, q_{ijkl}(\eta, \eta') \int \mathrm{d}k\, k^2 j_0(k(\eta - \eta'))P_\delta(k, \eta, \eta')$
- In Zel'dovich approx.,

Linear growth factor

$$P_\mathrm{Z}(k, \eta, \eta') = \mathrm{e}^{-k^2\Sigma^2(D(\eta)-D(\eta'))^2/2}P_\mathrm{Z}(k, \bar{\eta})\,,\ D(\eta)D(\eta') = D^2(\bar{\eta})\,,\ \Sigma^2 = \frac{1}{6\pi^2}\int \mathrm{d}k\, P_\mathrm{lin}(k)$$

- Ansatz $P(k, \eta, \eta') = \mathrm{e}^{-k^2\Sigma^2(D(\eta)-D(\eta'))^2/2}P(k, \bar{\eta})$



κ^{S} comparisons

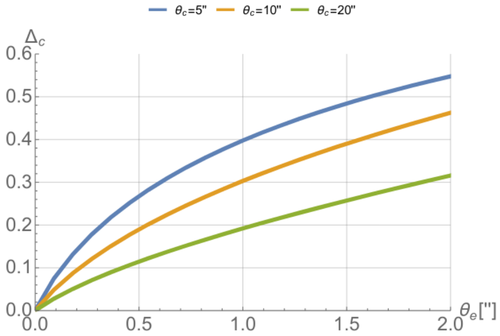
System	σ_{lin}	$\sigma_{\mathrm{halofit}}$	$\sigma^{\mathrm{TDCOSMO}}$	$\kappa^{\mathrm{S}}_{\mathrm{TDCOSMO}}$
DES0408-5354	0.0109	0.0390	0.0380	$-0.0397^{+0.0421}_{-0.0242}$
HE0435-1223	0.0088	0.0299	0.0342	$0.0040^{+0.0363}_{-0.0215}$
PG1115+080	0.0089	0.0303	0.0330	$-0.0054^{+0.0358}_{-0.0209}$
SDSS1206+4332	0.0092	0.0313	0.0410	$-0.0037^{+0.0402}_{-0.0215}$
B1608+656	0.0076	0.0251	0.0903	$0.1026^{+0.0949}_{-0.0451}$
RXJ1131-1231	0.0037	0.0110	0.0433	$0.0695^{+0.0480}_{-0.0260}$
WFI2033-4723	0.0087	0.0295	0.0660	$0.0591^{+0.0863}_{-0.0442}$

Using stellar kinematics

$$\sigma^2_{\rm los}(\theta) = \frac{2G}{I(\theta)}\int_1^\infty \frac{{\rm d}y}{y} K(y) I(y D_{\rm L}\theta) M(y D_{\rm L}\theta)$$

In internal MSD ($\lambda =: 1 - \kappa_{\rm c}$),

$$M = (1-\kappa_{\rm c}) M^{\rm model} + M_{\rm core}\ , \ M_{\rm core}(r) \propto \kappa_{\rm c}\frac{r^3}{r_{\rm c}}\ , \ \kappa_{\rm c} \sim \rho_0 r_{\rm c}$$



- A perfect MSD limit is not conservative! Used in TDCOSMO IV

$$\left(\frac{\sigma_{\rm los}}{\sigma^{\rm model}_{\rm los}}\right)^2 = 1 - \kappa_{\rm c}(1-\Delta_{\rm c})$$

- Mass-anisotropy degeneracy: stellar kinematics and lensing probe two different profiles!

K. Blum, LT 2021 [2105.10873]