

Looking for axions in exotic, highly magnetised White Dwarf stars

Ruben Zatini

IAC - Instituto de Astrofísica de Canarias
LAPTh/CNRS - Laboratoire d'Annecy-le-Vieux de Physique Théorique

October 8, 2024



Education

Università degli Studi di Padova, MSc in Physics of the Fundamental Interactions

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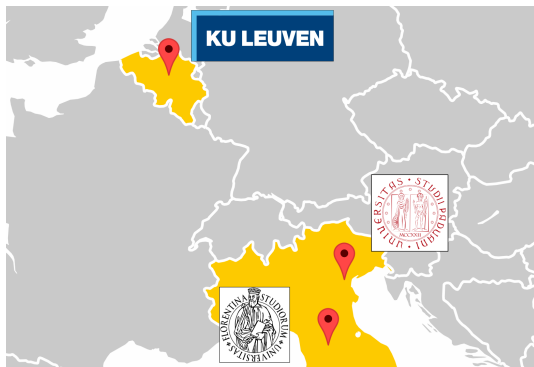
Supervised by Luca Di Luzio, Sebastian Hoof

KU Leuven, Study Abroad: Erasmus+ student

Università degli Studi di Firenze, BSc in Physics and Astrophysics

On the gravitational interaction of two massless particles

Supervised by Dimitri Colferai



Joint PhD project

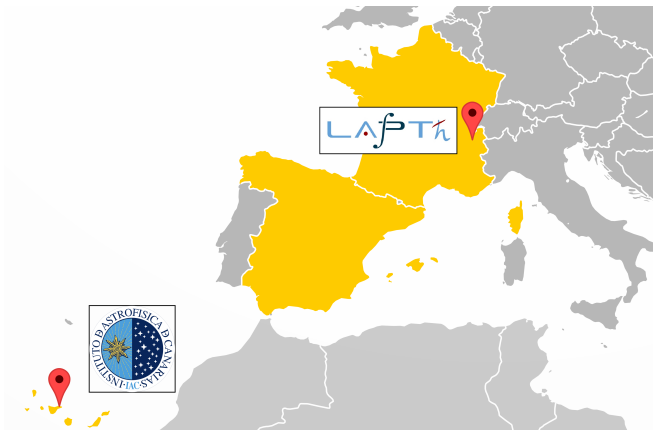
Instituto de Astrofísica de Canarias - IAC

Supervised by Jorge Martín Camalich

Laboratoire d'Annecy-le-Vieux de Physique Théorique - LAPTh/CNRS

Supervised by Francesca Calore

Probing light dark sectors in extreme astrophysical objects



Theoretical motivations and phenomenology

- The strong CP problem: absence of CP violation in strong interactions
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Axion mass and interactions with SM particles: $m_a, g_a^{\text{SM}} \propto 1/f_a$

Experiments imply a **large value** of f_a ⇒ axion is **light** and **weakly-interacting**
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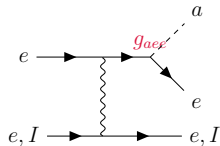
Thesis phenomenology: axion production inside white dwarfs and subsequent conversion into X-ray photons inside their magnetosphere

⇒ g_{aee} and $g_{a\gamma\gamma}$ are the most important couplings

Axion phenomenology in WDs

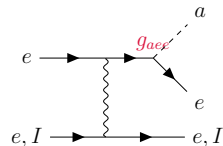
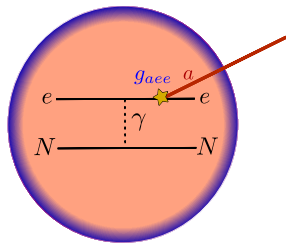
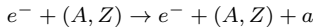
g_{aee} enables axion emission through *bremstrahlung*:

$$e^- + (A, Z) \rightarrow e^- + (A, Z) + a$$



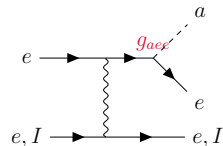
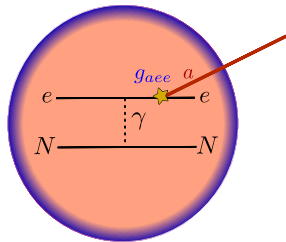
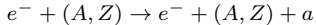
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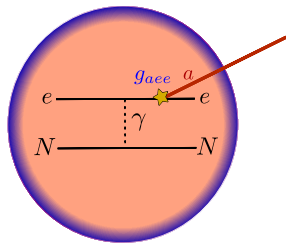
Phenomenological consequence:

Axion conversion into detectable photons in the B-field of the WD \Rightarrow axion indirect detection

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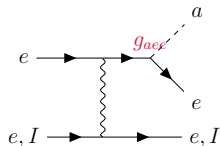
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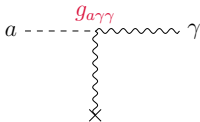


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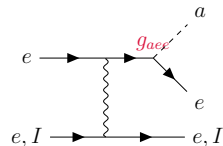
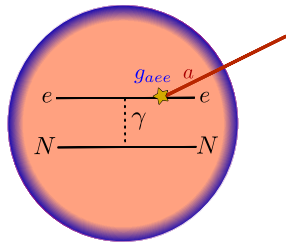
$g_{a\gamma\gamma}$ mediates axion-photon oscillation through *Inverse Primakoff* process \Rightarrow X-ray emission



Axion phenomenology in WDs

g_{aee} enables axion emission through *bremsstrahlung*:

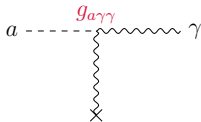
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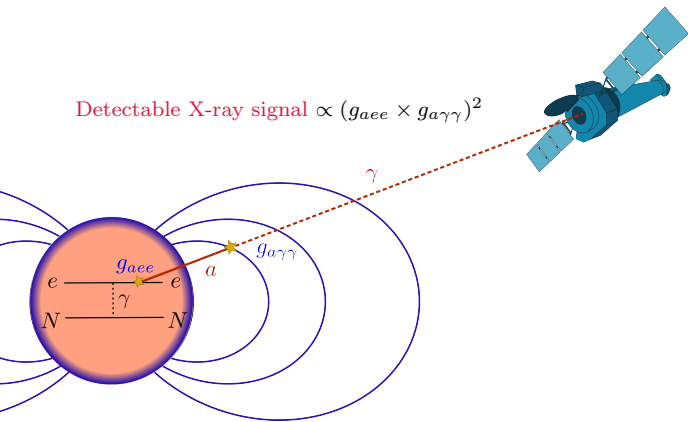
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WDs are not expected to emit X-ray photons \Rightarrow clean environment for detection
[Bilikova et al. (2010)]

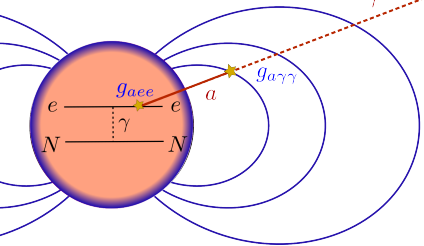
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Detectable X-ray signal $\propto (g_{aee} \times g_{a\gamma\gamma})^2$



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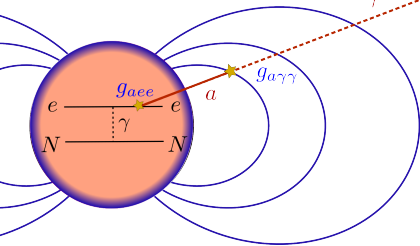


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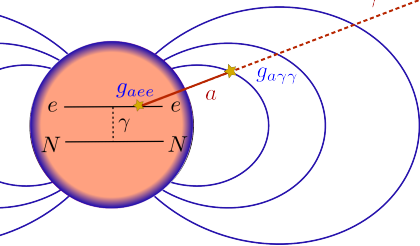


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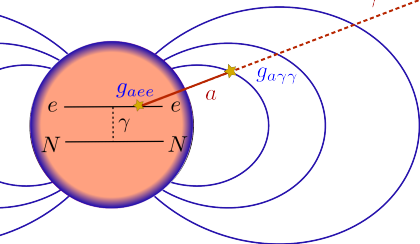


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- Assess the impact of different background models

Axion emission in WDs

Axion bremsstrahlung emissivity and luminosity:

$$\frac{d\varepsilon_a}{d\omega} = \frac{\alpha_{EM}^2 g_{aee}^2}{4\pi^3 m_e^2} \frac{\omega^3}{e^{\omega/T} - 1} \sum_s \frac{Z_s^2 \rho_s F_s}{A_s u} \Rightarrow \frac{dL_a}{d\omega}(\omega) = 4\pi \int_0^{R_{WD}} dr r^2 \frac{d\varepsilon_a}{d\omega}(r)$$

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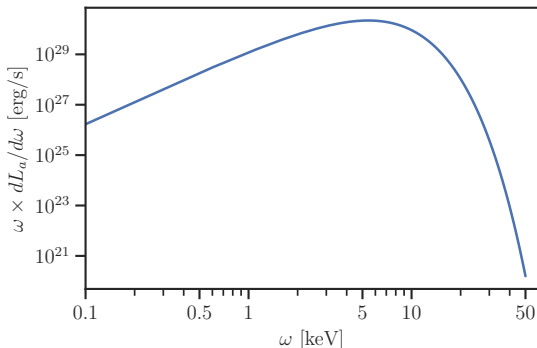
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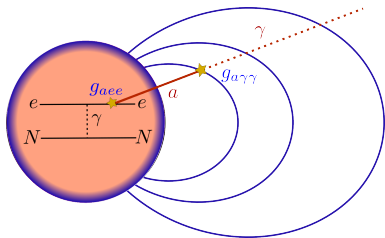
Application to **RE J0317-853**



Axion-photon conversion

Axion electrodynamics: system of equations that mixes axions and photons

[Raffelt & Stodolsky (1988); Millar et al. (2017)]

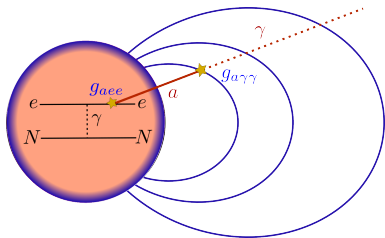


$$\left[i\partial_r + \omega + \begin{pmatrix} \Delta_{\parallel} & \Delta_B \\ \Delta_B & \Delta_a \end{pmatrix} \right] \begin{pmatrix} A_{\parallel} \\ a \end{pmatrix} = 0$$

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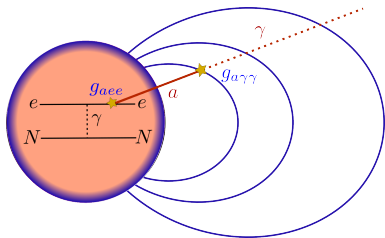
In **weak-mixing** limit:

$$p_{a \rightarrow \gamma} = \left| \int_{R_{\text{WD}}}^{\infty} dr' \Delta_B(r') e^{i\Delta_a r' - i \int_{R_{\text{WD}}}^{r'} dr'' \Delta_{\parallel}(r'')} \right|^2$$

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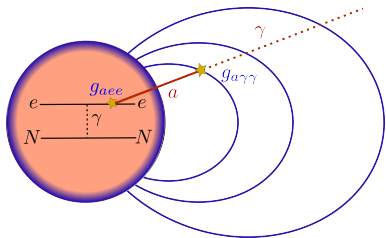
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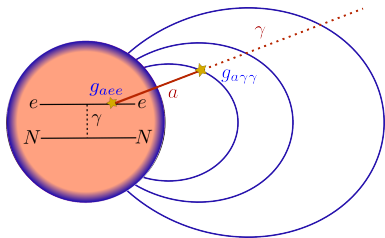
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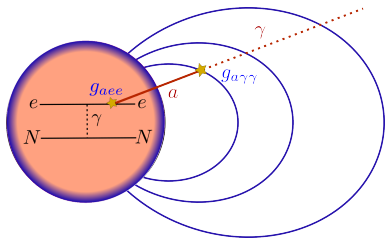
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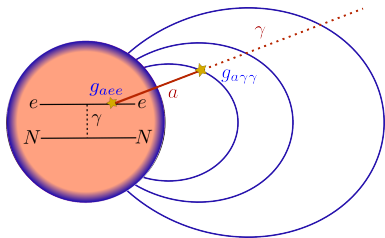
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Application to **RE J0317-853**: $p_{a \rightarrow \gamma} \sim \mathcal{O}(10^{-4}) \times \left(\frac{g_{a\gamma\gamma}}{10^{-11} \text{ GeV}^{-1}} \right)^2$

Axion-induced X-ray flux prediction

$$\frac{dF_{\gamma a}}{d\omega} \propto \frac{dL_a}{d\omega}(\omega) \times p_{a \rightarrow \gamma}(\omega) \propto g_{aee}^2 \times g_{a\gamma\gamma}^2$$

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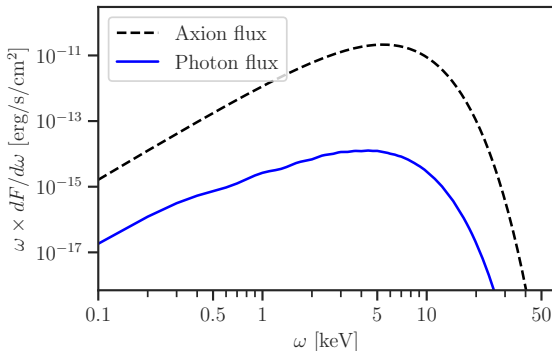
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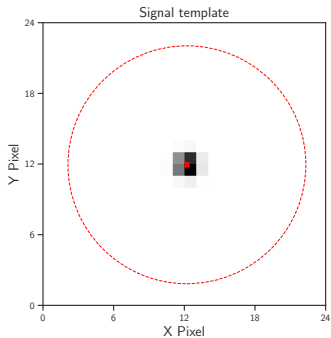
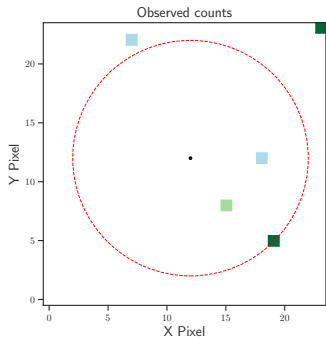


⇒ The X-ray signal peaks in the keV range, with its intensity modulated by a signal parameter $\theta_s \propto (g_{aee} \times g_{a\gamma\gamma})^2$

Observation of RE J0317-853 and analysis

Chandra observation: 37.42 ks

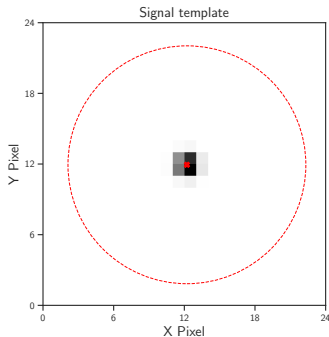
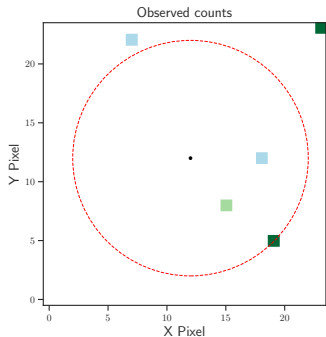
Implement *Chandra* response functions



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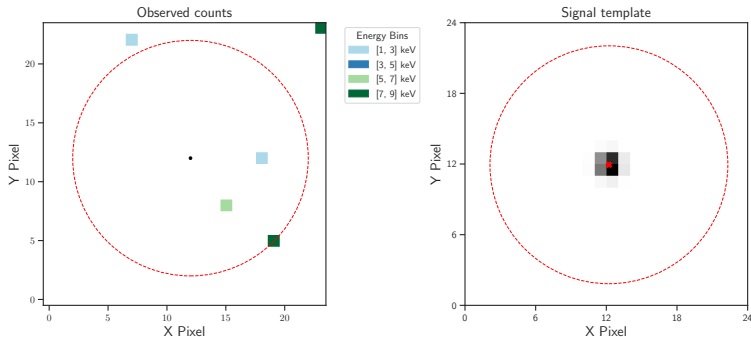


$$L(\boldsymbol{\theta}) = \prod_{i=1}^4 \prod_{j=1}^{N_{\text{pix}}} \frac{\mu_{i,j}(\boldsymbol{\theta})^{n_{i,j}} e^{-\mu_{i,j}(\boldsymbol{\theta})}}{n_{i,j}!} \quad \text{with} \quad \mu_{i,j}(\boldsymbol{\theta}) = s_{i,j}(\boldsymbol{\theta}_s) + b_{i,j}(\boldsymbol{\theta}_b)$$

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Employing different background models, compute profile likelihood for $\theta_s \Rightarrow$ determine an upper limit on the product of the couplings $g_{aee} \times g_{a\gamma\gamma}$

Statistical analysis

Implement different background models

- **Free** background: Four parameters, one in each energy bin, rescaling the background spatial template
- **Constant** background: Single parameter for all energy bins and pixels
- **Linear** background: Background described by a linear spectrum
- **Power-law** background: Background described by a power law spectrum

$$\frac{dN_{\text{bkg}}}{d\omega} = K \cdot \left(\frac{\omega}{\omega_0} \right)^{-\alpha}$$

physically well-motivated, possible astrophysical background due to accretion or binary companion \Rightarrow X-ray emission [Kluzniak et al. (1989)]

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Little impact on the analysis, free background for consistency with [Dessert et al. 2022]

$$g_{aee} \times g_{a\gamma\gamma} < 2.3 \times 10^{-25} \text{ GeV}^{-1} \quad (2\sigma) \quad \text{This thesis}$$

$$g_{aee} \times g_{a\gamma\gamma} < 1.3 \times 10^{-25} \text{ GeV}^{-1} \quad (2\sigma) \quad [\text{Dessert et al. 2022}]$$

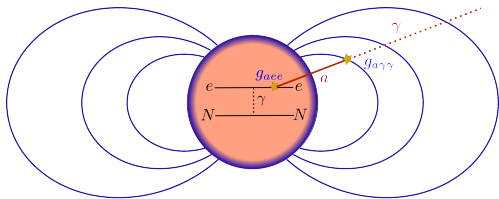
\Rightarrow Dessert et al. obtained a stronger bound by a factor ~ 1.7

Summary

Axion emission from white dwarfs
will induce a hard X-ray signature.

[Dessert, Long, & Safdi (2019)]

[Dessert, Long, & Safdi (2022)]

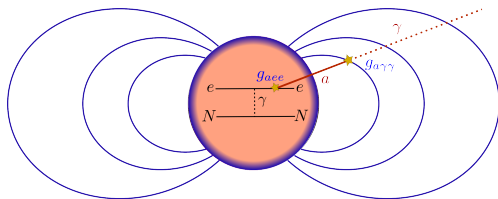


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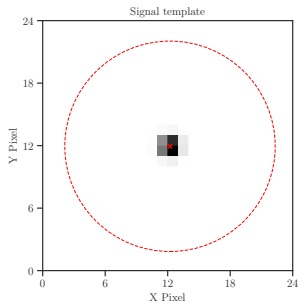
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Development of an analysis pipeline for this type of
signal: **Construct of a signal template and
provide an application for RE J0317-853**

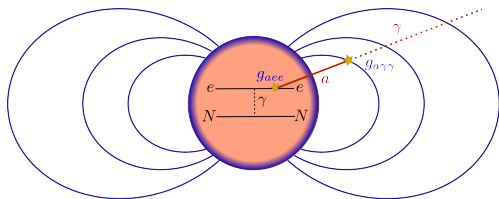


Summary

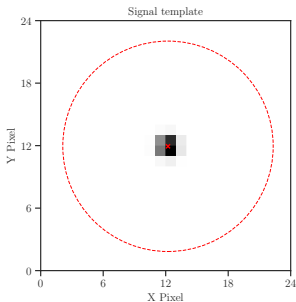
Axion emission from white dwarfs will induce a hard X-ray signature.

[Dessert, Long, & Safdi (2019)]

[Dessert, Long, & Safdi (2022)]



Development of an analysis pipeline for this type of signal: **Construct of a signal template and provide an application for RE J0317-853**



No evidence of X-ray emission from *Chandra* observation of RE J0317-853: **Strong limit on axion couplings and quantify the impact of different background models**

$$\Rightarrow g_{aee} \times g_{a\gamma\gamma} < 2.3 \times 10^{-25} \text{ GeV}^{-1} (2\sigma)$$

in line with [Dessert et al. (2022)]

Outlook and improvements

- Apply the analysis to various axion models
 - ⇒ which ones can be constrained more than others with WDs observations?
- Extend the analysis to processes mediated by other couplings
 - ⇒ ^{57}Fe de-excitation via axion-nucleon coupling
- Refine the computation of F factors
 - ⇒ Numerical parametrization available in literature is not totally adequate

Backup slide: medium factors F_s

Axion bremsstrahlung emissivity spectrum:

$$\frac{d\varepsilon_a}{d\omega} = \frac{\alpha_{\text{EM}}^2 g_{aee}^2}{4\pi^3 m_e^2} \frac{\omega^3}{e^{\omega/T} - 1} \sum_s \frac{Z_s^2 \rho_s F_s}{A_s m_u}$$

Backup slide: medium factors F_s

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Formal expression for F from axion emissivity calculation

$$F = \int \frac{d\Omega_2}{4\pi} \int \frac{d\Omega_a}{4\pi} \frac{(1 - \beta_F^2)[2(1 - c_{12}) - (c_{1a} - c_{2a})^2]}{(1 - c_{1a}\beta_F)(1 - c_{2a}\beta_F)(1 - c_{12})(1 - c_{12} + \kappa_s)}$$

in weakly
coupled plasma

Backup slide: medium factors F_s

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WD plasma is strongly coupled \Rightarrow Numerical parametrization is needed

AXION BREMSSTRAHLUNG IN DENSE STARS. II. PHONON CONTRIBUTIONS
MASAYUKI NAKAGAWA, TOMOO ADACHI, YASU HARU KOHYAMA, AND NAOKI ITOH
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Received 1987 June 22; accepted 1987 August 21

Only for WD composed by a single element
 \Rightarrow compute different F_s and then sum up

Backup slide: medium factors F_s

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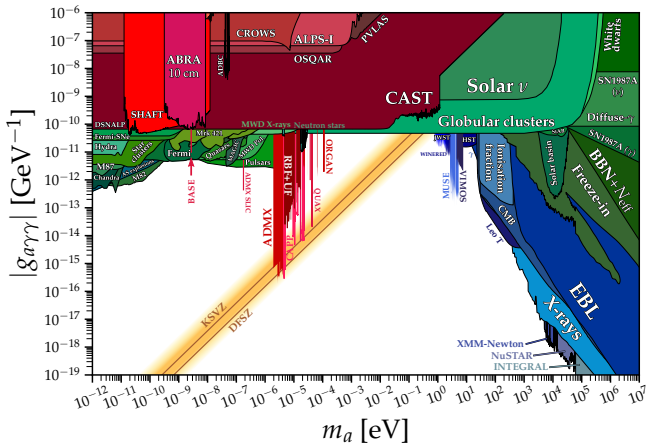
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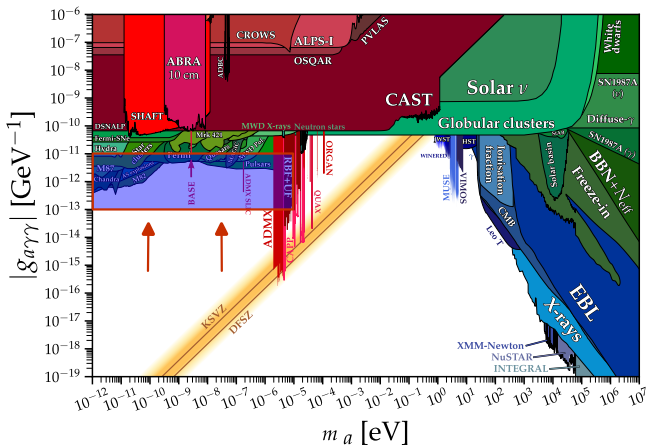
Only for WD composed by a **single element**
 \Rightarrow compute different F_s and then sum up

But F is **not linear** on the species, a **unified procedure** to treat multiple elements is **needed!**

Backup slide: Parameter space



Backup slide: Parameter space



⇒ ALPs: no restriction, they can lie everywhere in the parameter space

Backup slide: $p_{a \rightarrow \gamma}$ computation

Axion-photon conversion probability:

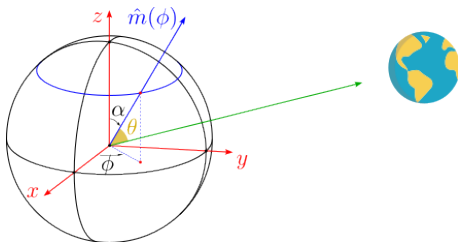
$$p_{a \rightarrow \gamma} = \left| \int_{R_{\text{WD}}}^{\infty} dr' \Delta_B(r') e^{i\Delta_a r' - i \int_{R_{\text{WD}}}^{r'} dr'' \Delta_{\parallel}(r'')} \right|^2.$$

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$p_{a \rightarrow \gamma}(\theta)$, with θ the angle between axion radial trajectory \hat{r} and magnetic axis \hat{m} .
Dipole field + viewing angle and \hat{m} misalignment: [Burleigh et al. (1999)]

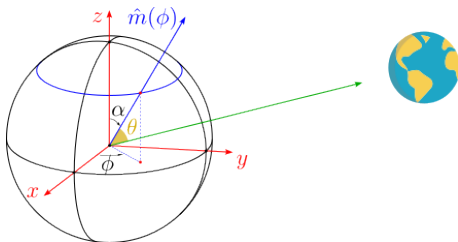


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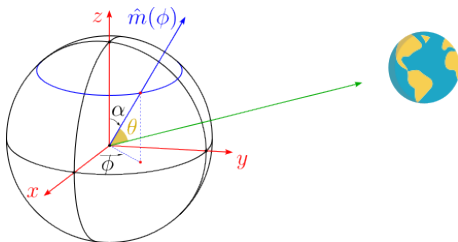
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What value of θ should we take?

\Rightarrow Observation time ~ 40 ks $\gg \sim 700$ s rotational period of RE J0317-853

Backup slide: $p_{a \rightarrow \gamma}$ computation

Generate trajectory with uniformly distributed ϕ

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Generate trajectory with **uniformly distributed** ϕ + parametrization of $\theta = \theta(\phi)$

Backup slide: $p_{a \rightarrow \gamma}$ computation

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 \Rightarrow compute $p_{a \rightarrow \gamma}(\phi) = p_{a \rightarrow \gamma}(\theta(\phi))$ and then **average** over ϕ :

$$\langle p_{a \rightarrow \gamma} \rangle_{\phi} = \frac{1}{2\pi} \int_0^{2\pi} p_{a \rightarrow \gamma}(\phi) d\phi \quad \text{using } \text{np.trapz}$$

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Repeat the numerical integration for a range of energies $\omega \in [0, 50]$ keV

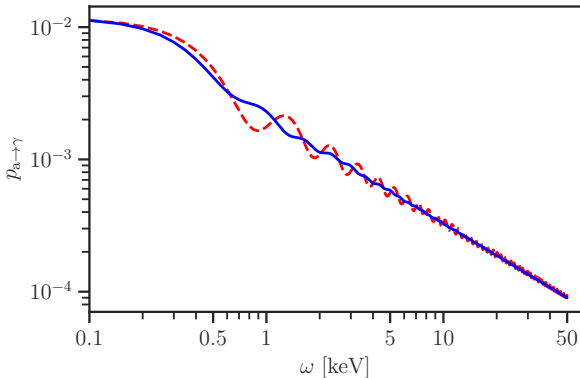
⇒ Construct energy profile for $p_{a \rightarrow \gamma}(\omega)$

Backup slide: $p_{a \rightarrow \gamma}$ computation

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Backup slide: Statistical analysis

First type of analysis: utilize only spectral information

$$L(\boldsymbol{\theta}) = \prod_{i=1}^4 \frac{\mu_i(\boldsymbol{\theta})^{n_i} e^{-\mu_i(\boldsymbol{\theta})}}{n_i!} \quad \text{with} \quad \mu_i(\boldsymbol{\theta}) = s_i(\boldsymbol{\theta}_s) + b_i(\boldsymbol{\theta}_b),$$

and **signal parameter** $\boldsymbol{\theta}_s \propto (g_{aee} \times g_{a\gamma\gamma})^2$

Backup slide: Statistical analysis

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2. **Construct $LLR(\theta_s)$ profile for a range of hypothesized θ_s :**

$$LLR(\theta_s) = -2 \ln \left(\frac{L(\theta_s, \hat{\boldsymbol{\theta}}_b)}{L(\hat{\theta}_s, \hat{\boldsymbol{\theta}}_b)} \right)$$

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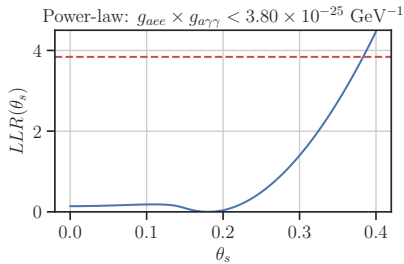
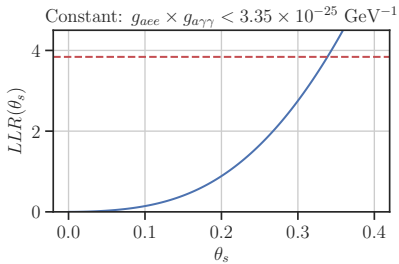
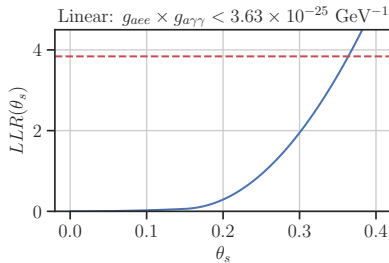
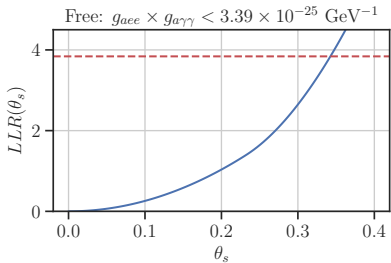
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3. **Determine the upper limit:** asymptotic formulae to constrain θ_s : we identify the value of θ_s at which the $LLR(\theta_s)$ **crosses the 95% threshold**

Backup slide: Statistical analysis



Backup slide: PQ symmetry

Realization of PQ symmetry in the UV is **not** unique; it requires colored PQ-charged fermions (quarks) for the color anomalous shift, replacing the $\bar{\theta}$ -term with a dynamical field (the axion).

PQ-charged fermions can also be EM charged, leading to an EM anomalous term.
⇒ After SSB, the axion couples with photons and the PQ current:

$$\mathcal{L}_a \supset \frac{a}{v_{\text{PQ}}} \frac{g_s^2 N}{16\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \frac{a}{v_{\text{PQ}}} \frac{e^2 E}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial^\mu a}{v_{\text{PQ}}} J_\mu^{\text{PQ}}$$

Anomaly coefficients E, N and the fermionic PQ current vary by model:

- **Minimal KSVZ model:** PQ-charged fermions are heavy quarks, EM neutral ⇒ axion interacts only with gluons (*hadronic model*)
- **DFSZ model:** PQ-charged fermions include SM quarks and leptons ⇒ interaction with photons and electrons; $g_{a\gamma\gamma}$ and g_{aee} emerge already in the UV