

# Hadronic Top-quark Polarimetry with ParticleNet

LHCTopWG meeting, Nov 12, 2024

Alberto Navarro



Based on arXiv:2407.01663 and arXiv:2407.07147

with Z. Dong, D. Gonçalves, K. Kong, and A. J. Larkoski

# Motivation

- Spin correlation studies in top quark physics plays an important role in new physics searches (e.g., resonant or non-resonant searches), observation of entanglement, etc.
- Polarization is a valuable tool when studying top-quark physics.



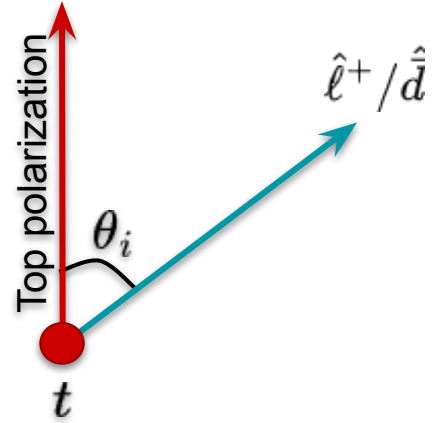
$$\left\{ \begin{array}{l} \tau_{\text{top}} \approx 5 \times 10^{-25} \text{ s} \\ \tau_{\text{had}} \approx 2 \times 10^{-24} \text{ s} \\ \tau_{\text{flip}} \approx 10^{-24} \text{ s} \end{array} \right.$$

- The top polarization can be probed from the kinematics of its decay products.

# Top polarization

- The direction of the top decay products correlates with the top polarization axis

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_i} = \frac{1}{2} (1 + p\beta_i \cos \theta_i)$$



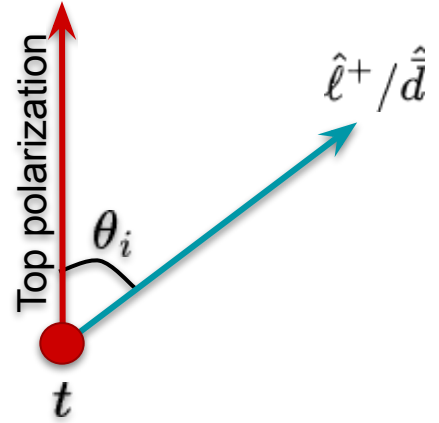
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Spin analyzing power

$\ell^+/\bar{d}$	$\nu/u$	$b$
1	-0.32	-0.41



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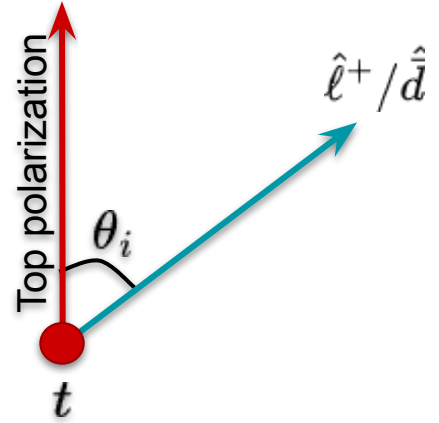
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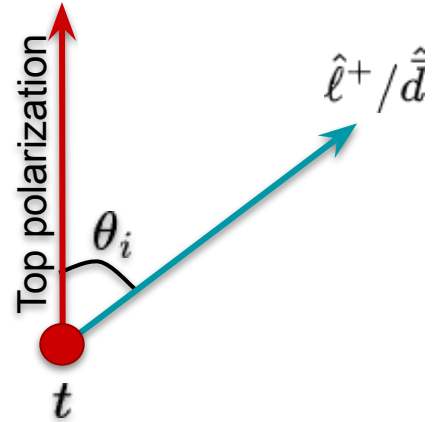
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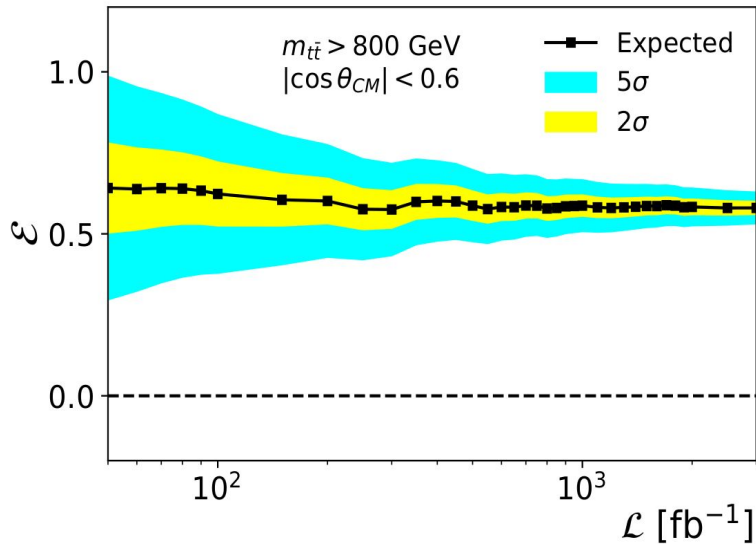
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- The charged lepton and down quark have the largest spin analyzing power.
- Hadronic decaying top offers higher statistics so fully recovering its polarization is crucial.

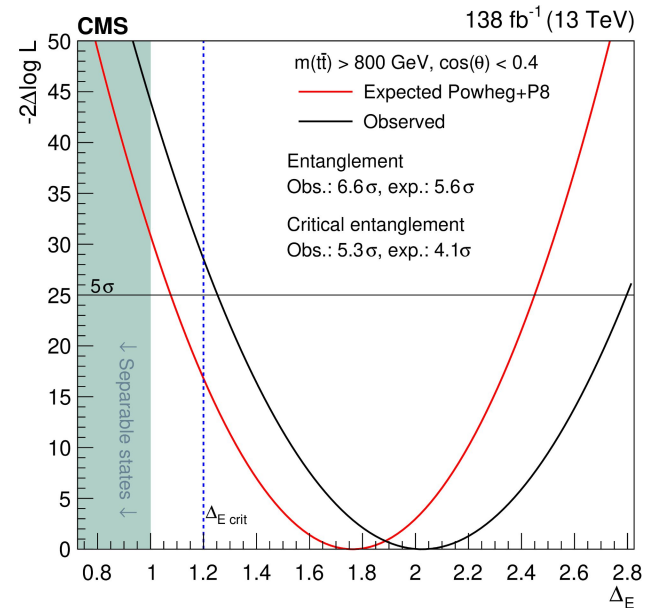
# Entanglement with semileptonic tops

- very recent application of hadronic top polarization in measuring entanglement with semileptonic top quarks



Dong, Gonçalves, Kong and **AN** (2023)

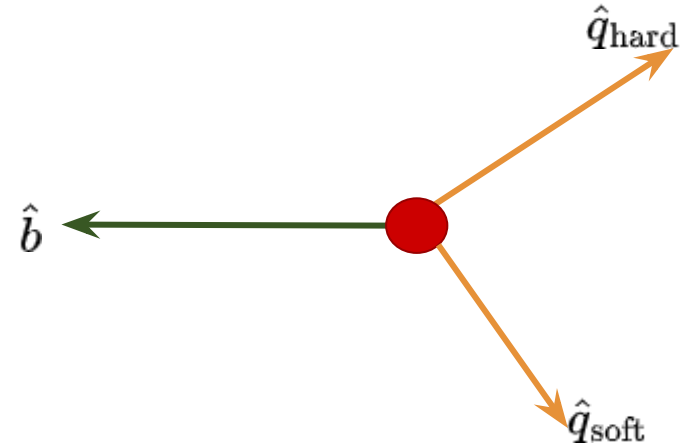
See also Han, Low, Wu (2023)



**CMS** (2024)

# Hadronic top polarimetry

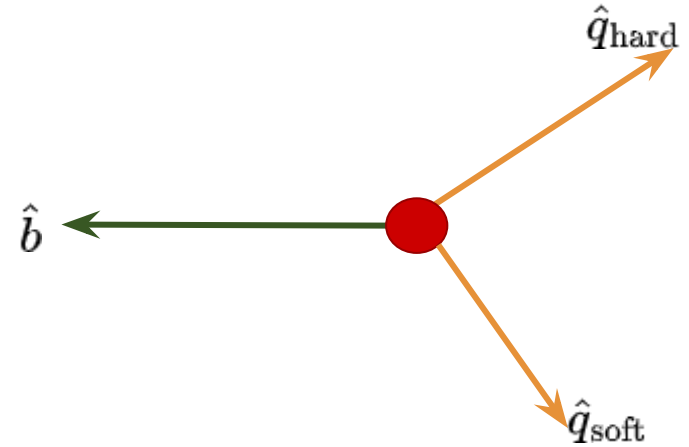
- Identifying a down-quark in a collider environment is challenging.





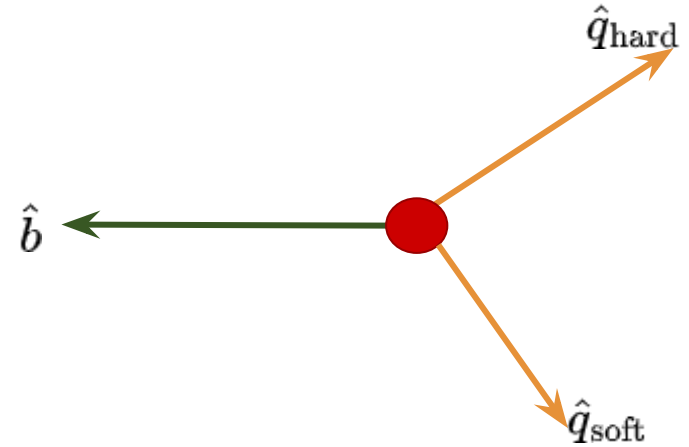
# Hadronic top polarimetry

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- An alternative approach is to use the light jet that is softest in the top rest frame.
- This choice gives a spin analyzing power equals to **0.5** Ježabek (1994)



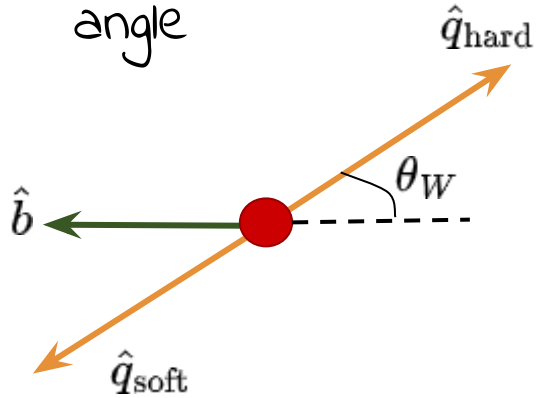
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- An alternative approach is to use the light jet that is softest in the top rest frame.
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- A better polarimeter can be constructed using the kinematic information of the top decay.



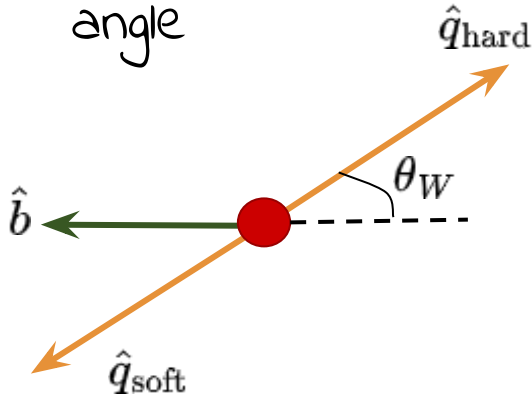
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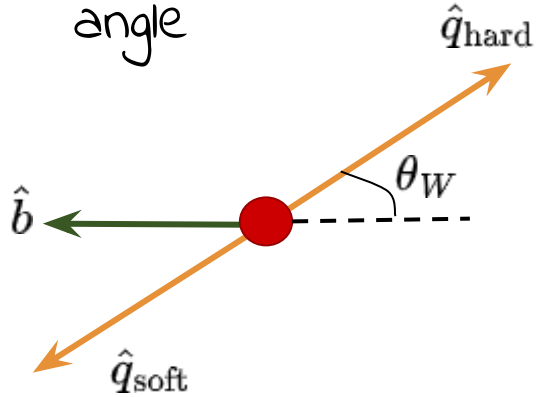


$$p(d \rightarrow q_{\text{hard}} | c_W) = \frac{p(|c_W|)}{p(|c_W|) + p(-|c_W|)}$$

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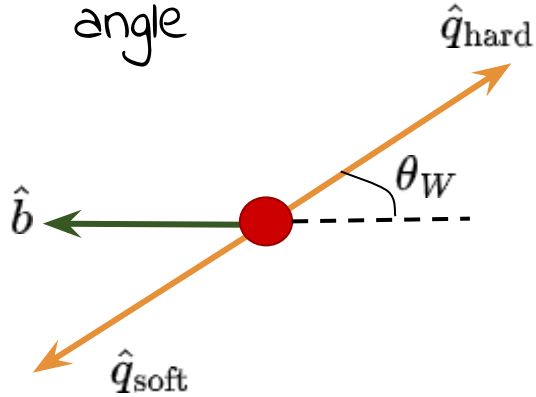


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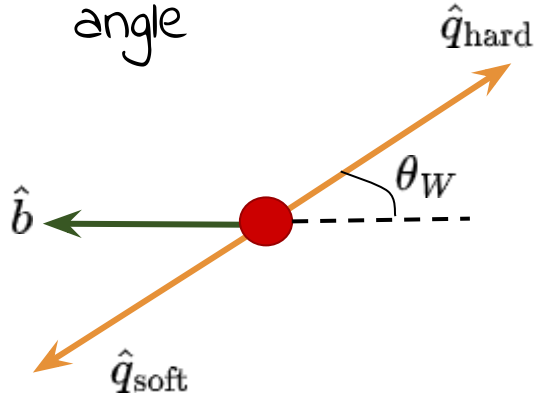
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- Define the optimal hadronic direction through kinematics as

$$\vec{q}_{\text{opt}}^{\text{kin}} = p(d \rightarrow q_{\text{hard}} | c_W) \hat{q}_{\text{hard}} + p(d \rightarrow q_{\text{soft}} | c_W) \hat{q}_{\text{soft}}$$

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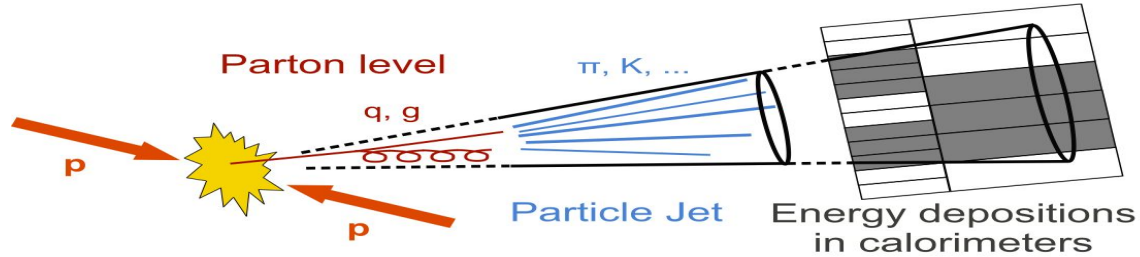
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- The length of this vector is the spin analyzing power and equals 0.64.

Tweedie (2014)

# Going beyond kinematics

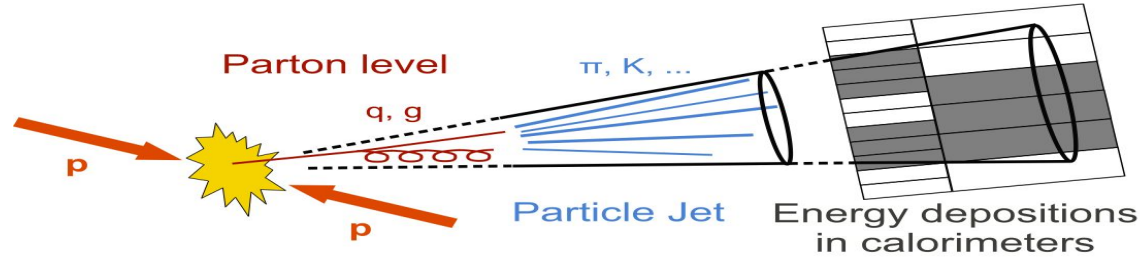
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# Going beyond kinematics

- At particle level, there's more information available from the jets.



- Assume some set of observables  $\{\mathcal{O}\}$  is measured and generalized the optimal hadronic direction to

$$\vec{q}_{\text{opt}} = p(d \rightarrow q_{\text{hard}} | \{\mathcal{O}\}) \hat{q}_{\text{hard}} + p(d \rightarrow q_{\text{soft}} | \{\mathcal{O}\}) \hat{q}_{\text{soft}}$$

Dong, Gonçalves, Kong, Larkoski and AN (2024)

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# The analytical approach

- Start by considering "global jet observables" such as kinematics, jet charge and particle multiplicity.

$$p(\bar{d} \rightarrow q_{\text{hard}} | c_W, \mathcal{Q}_{\kappa,h}, N_h, \mathcal{Q}_{\kappa,s}, N_s) = \frac{1}{p(\bar{d} \rightarrow q_{\text{hard}} | c_W) + \frac{p(\mathcal{Q}_{\kappa,h} | u \rightarrow q_{\text{hard}}, N_h)}{p(\mathcal{Q}_{\kappa,h} | \bar{d} \rightarrow q_{\text{hard}}, N_h)} p(u \rightarrow q_{\text{hard}} | c_W)}$$
$$\times \frac{1}{\frac{p(\mathcal{Q}_{\kappa,s} | \bar{d} \rightarrow q_{\text{soft}}, N_s)}{p(\mathcal{Q}_{\kappa,s} | u \rightarrow q_{\text{soft}}, N_s)} p(\bar{d} \rightarrow q_{\text{soft}} | c_W) + p(u \rightarrow q_{\text{soft}} | c_W)}$$
$$\times p(\bar{d} \rightarrow q_{\text{hard}} | c_W).$$

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Likelihood ratio of the up and down being the hard jet  $\rightarrow$

$\left( \frac{p(\mathcal{Q}_{\kappa,h} | u \rightarrow q_{\text{hard}}, N_h)}{p(\mathcal{Q}_{\kappa,h} | \bar{d} \rightarrow q_{\text{hard}}, N_h)} \right)$  Likelihood ratio of the up and down being the hard jet

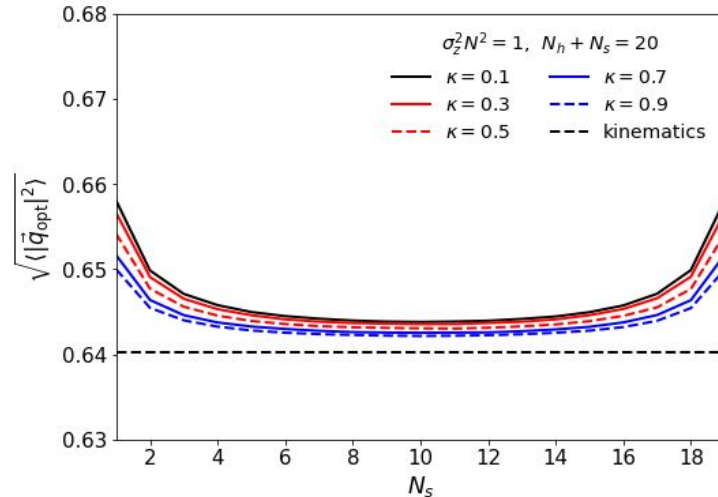
Likelihood ratio of the down and up being the soft jet  $\leftarrow$

Dong, Gonçalves, Kong, Larkoski and AN (2024)

# Improvements with analytical approach

- Jet charge distributions conditioned on multiplicity are Gaussians. Kang, Larkoski and Yang (2023)
- Compute the spin analyzing power as the square root of the mean squared-vector length

$$\langle |\vec{q}_{\text{opt}}|^2 \rangle = \int dc_W p(c_W) \int d\mathcal{Q}_{\kappa,h} d\mathcal{Q}_{\kappa,s} p(\mathcal{Q}_{\kappa,h}, \mathcal{Q}_{\kappa,s} | c_W, N_h, N_s) |\vec{q}_{\text{opt}}|^2$$

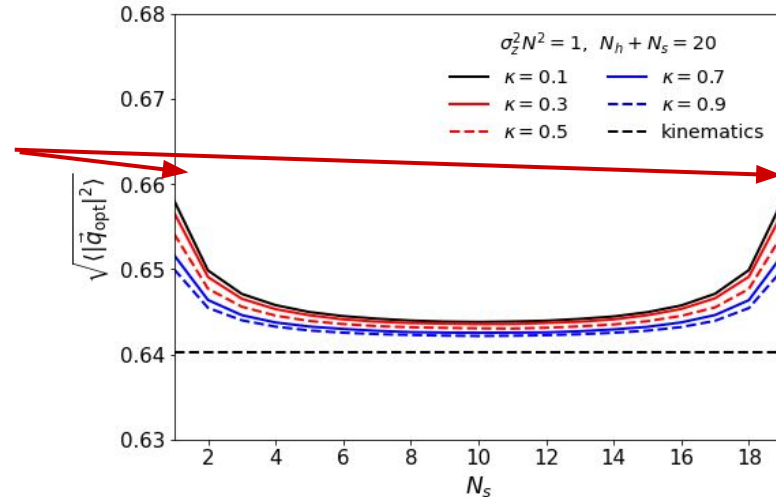


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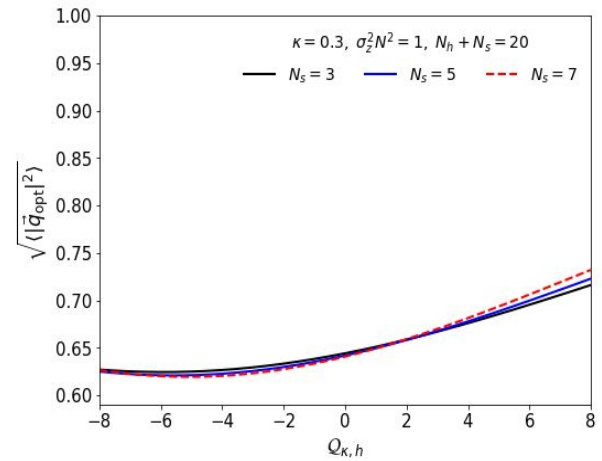
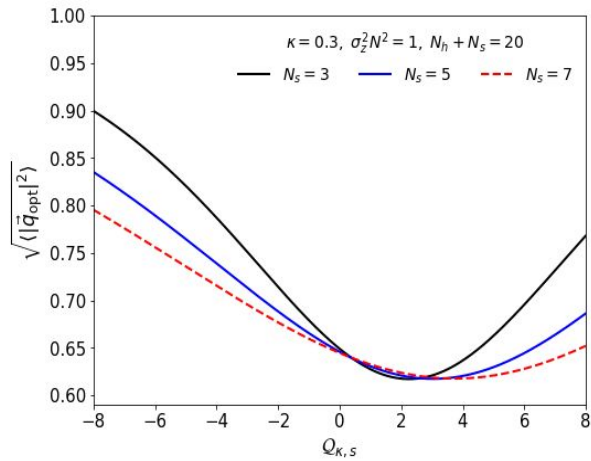
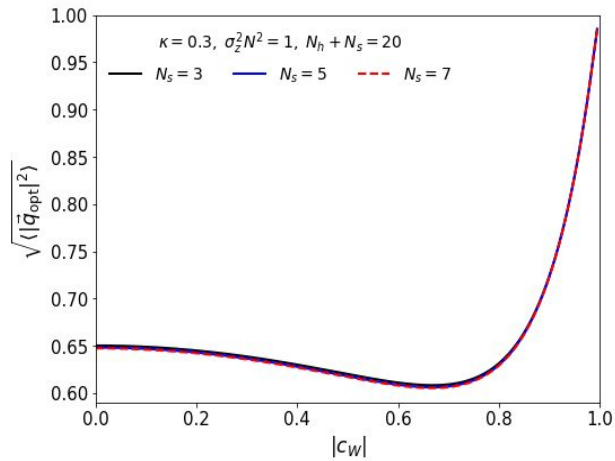
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Better  
discrimination for  
low  $N_s$  and  $N_h$



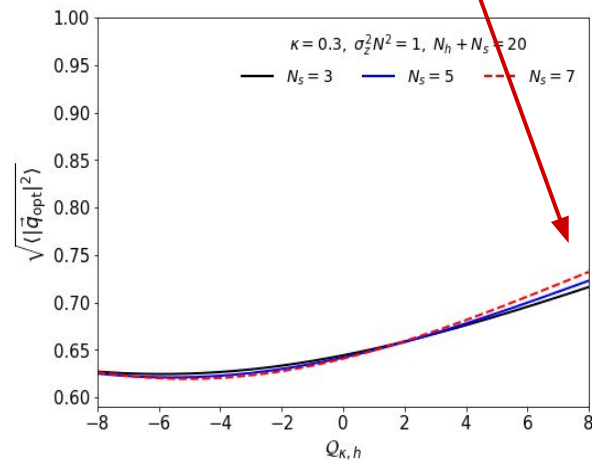
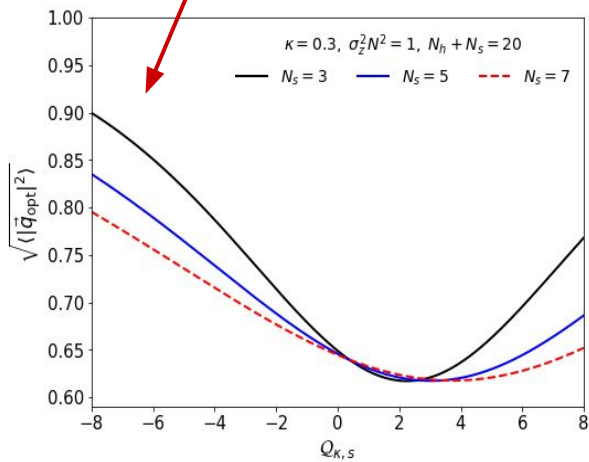
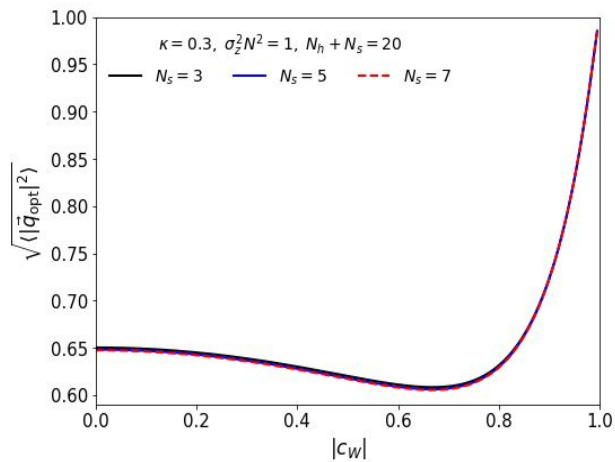
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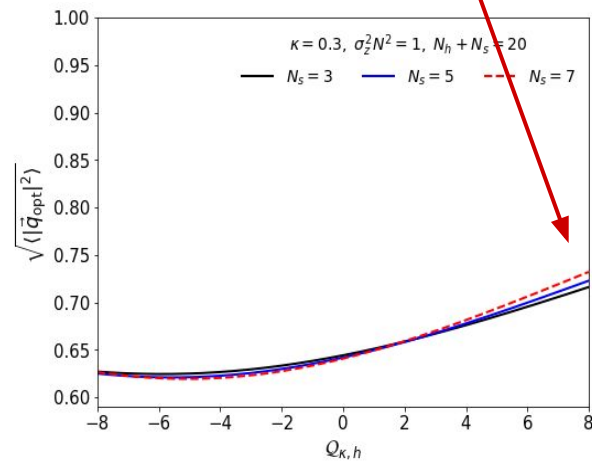
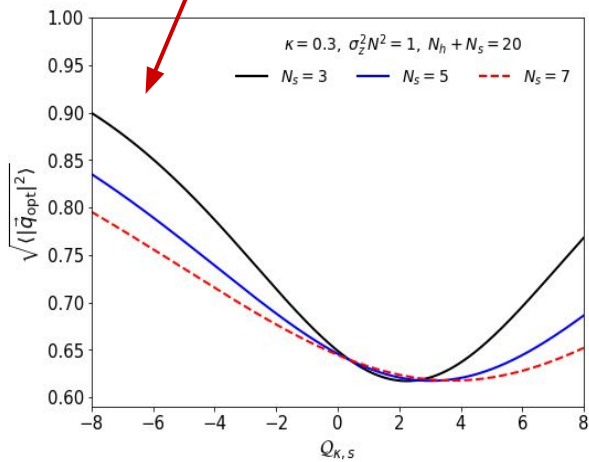
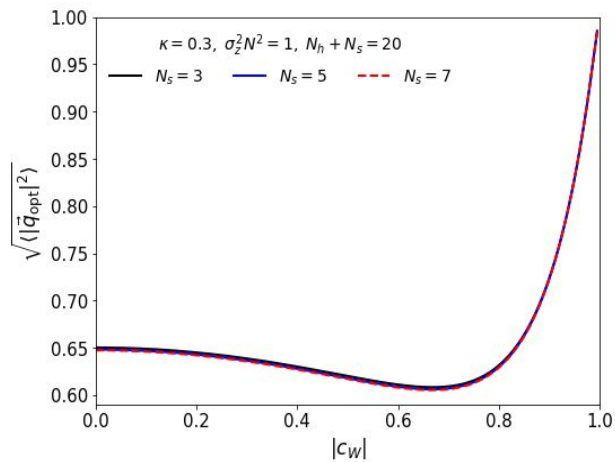


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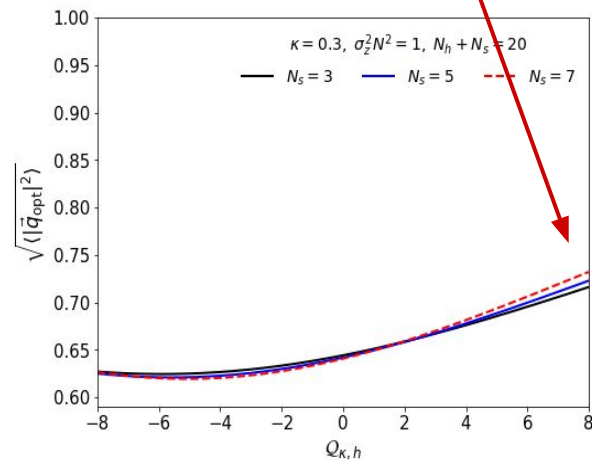
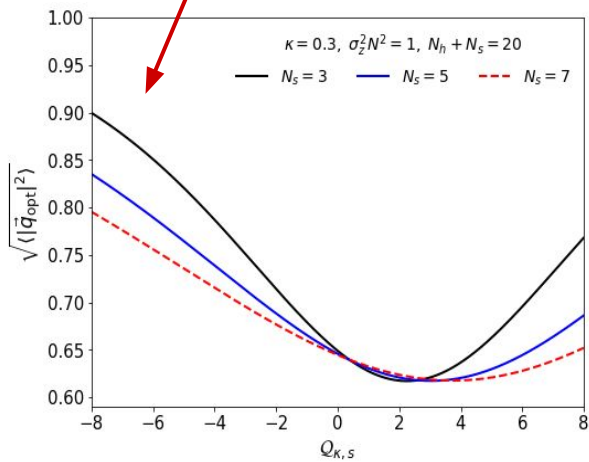
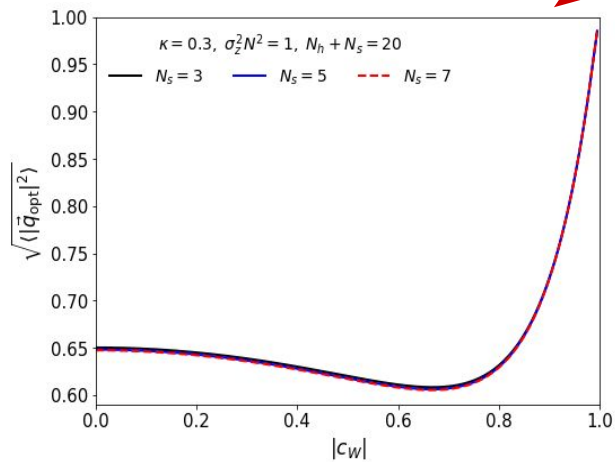


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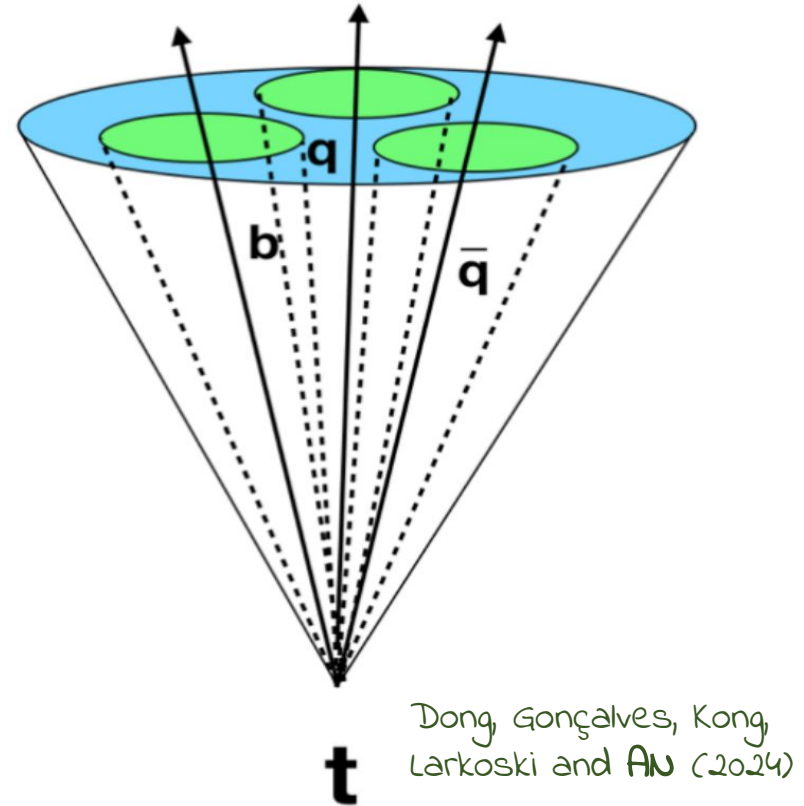
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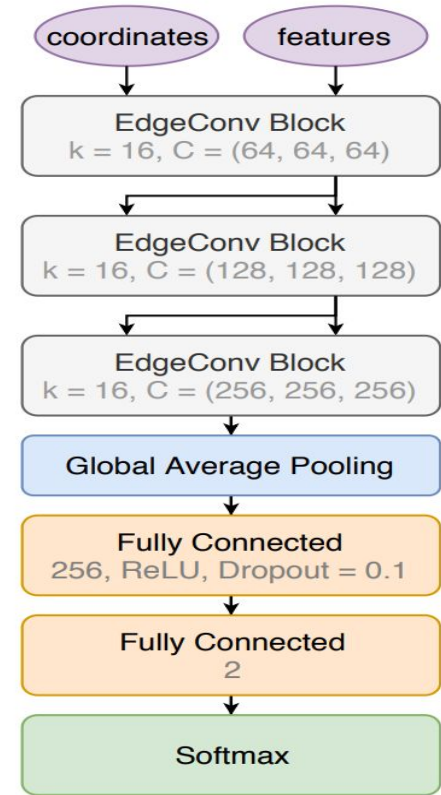
# The ML approach

- Could improve result further by incorporating information of jet constituents such as charge, momenta and ID.
- Train network to identify the down jet within the top jet.
- Using a simple DNN may not be ideal due to complexity of data and number of features.



# ParticleNet

- Idea is to represent a jet as an unordered, permutation invariant set of particles (a particle cloud).
- Analogous to the point cloud representation of 3D shapes used in computer vision.
- Has been successfully used for top and quark-gluon tagging.
- Need to modify for our purpose.

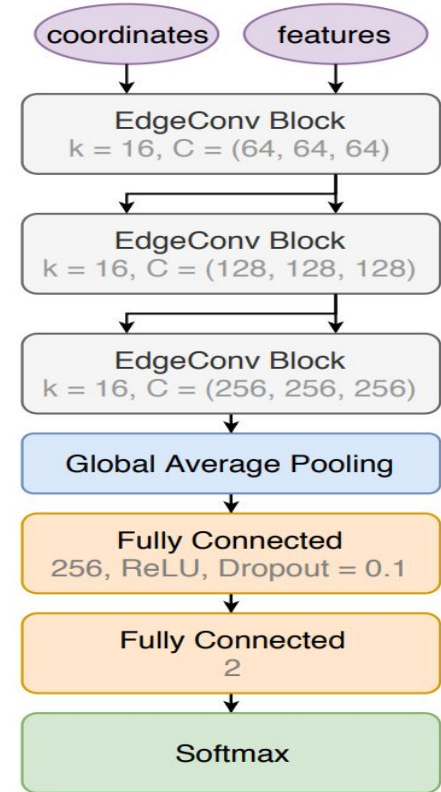
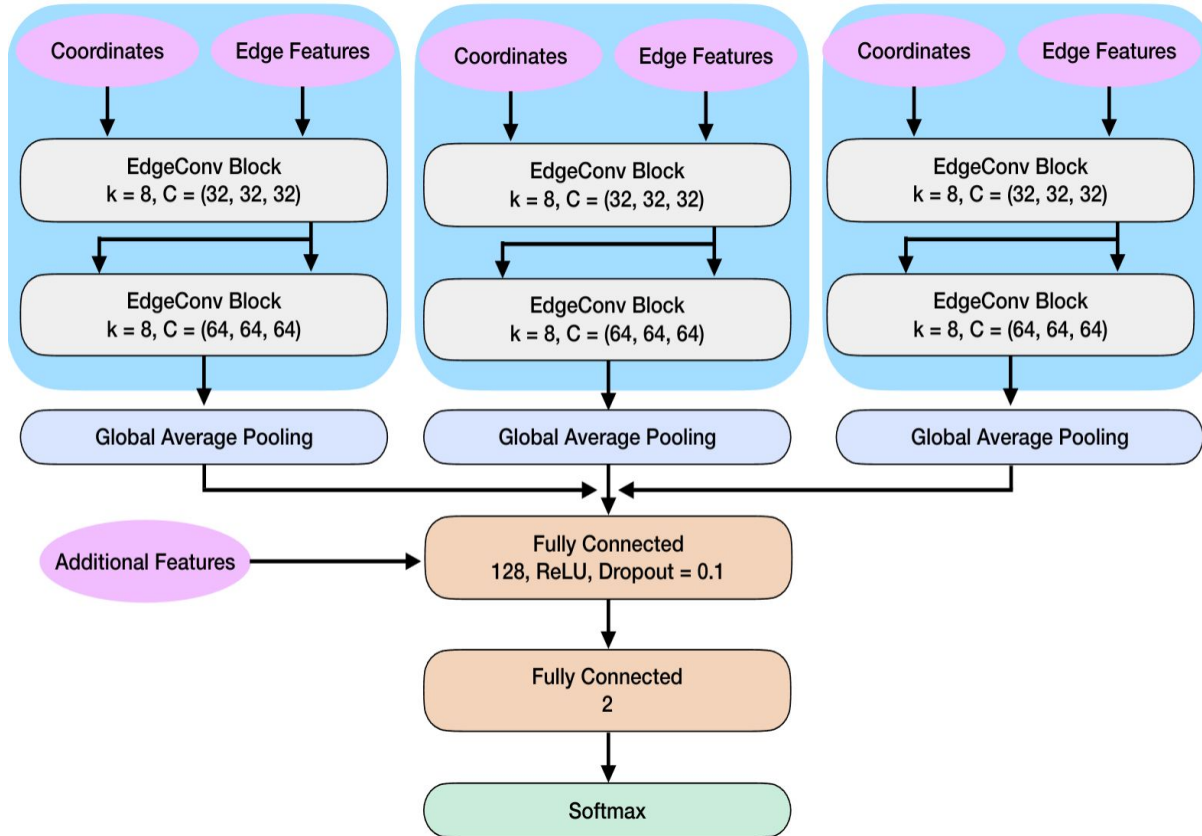


Qu, Gouskos (2019)

See also Gong et al. (2022)

Bogatskiy, Hoffman, Miller, offermann (2022)

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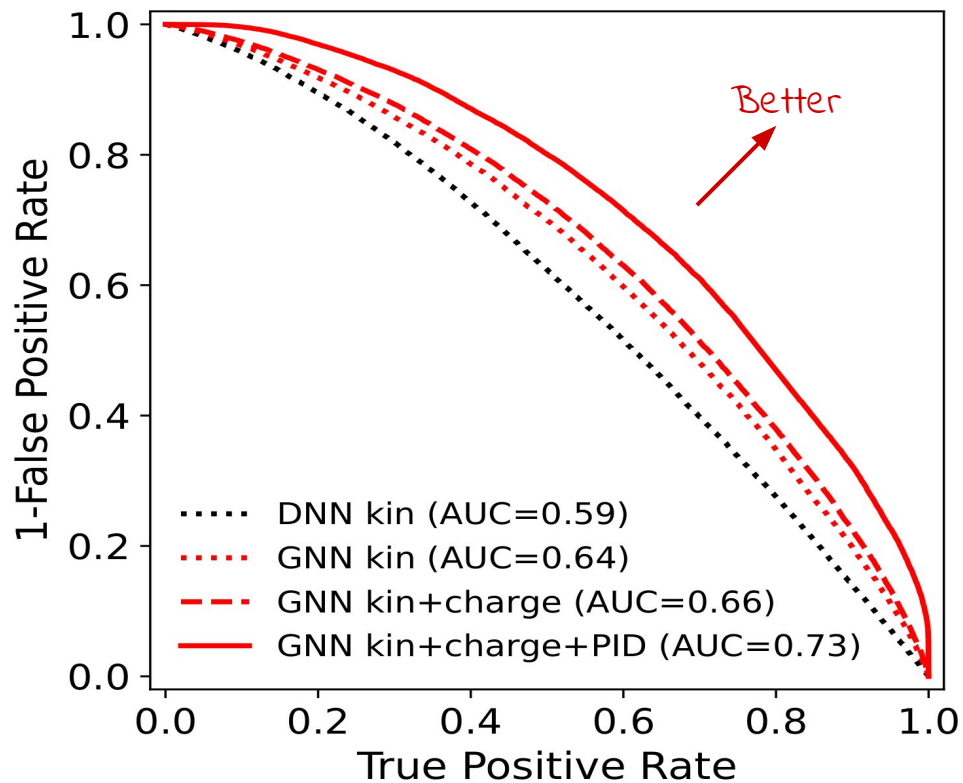
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# GNN performance

Variable	Definition
$\Delta\eta_t$	difference in pseudorapidity between the particle and the top jet axis
$\Delta\phi_t$	difference in azimuthal angle between the particle and the top jet axis
$\Delta\eta_j$	difference in pseudorapidity between the particle and the subjet axis
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$\log p_T$	logarithm of the particle's $p_T$
$\log E$	logarithm of the particle's Energy
$q$	electric charge of the particle
isElectron	if the particle is an electron
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isChargedHadron	if the particle is a charged hadron
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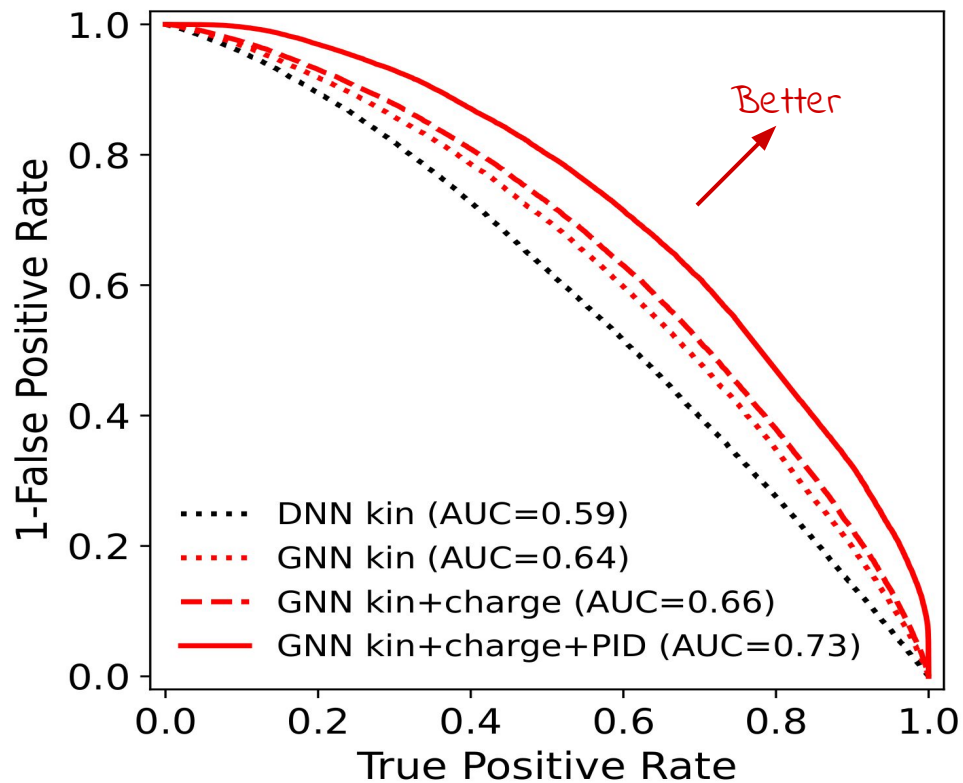
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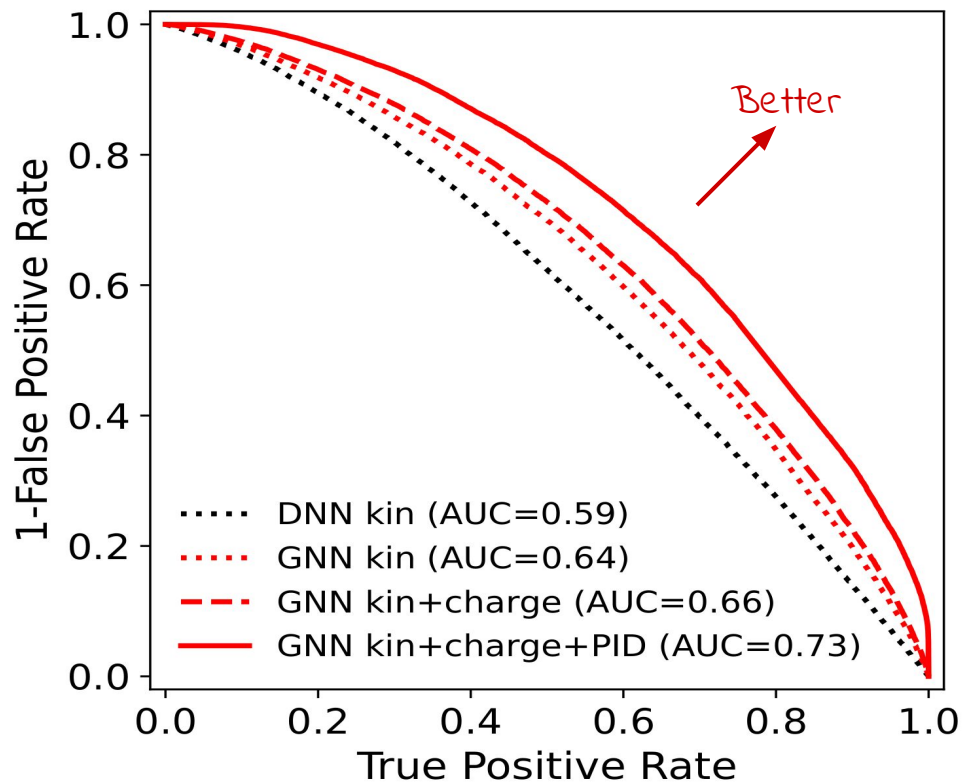
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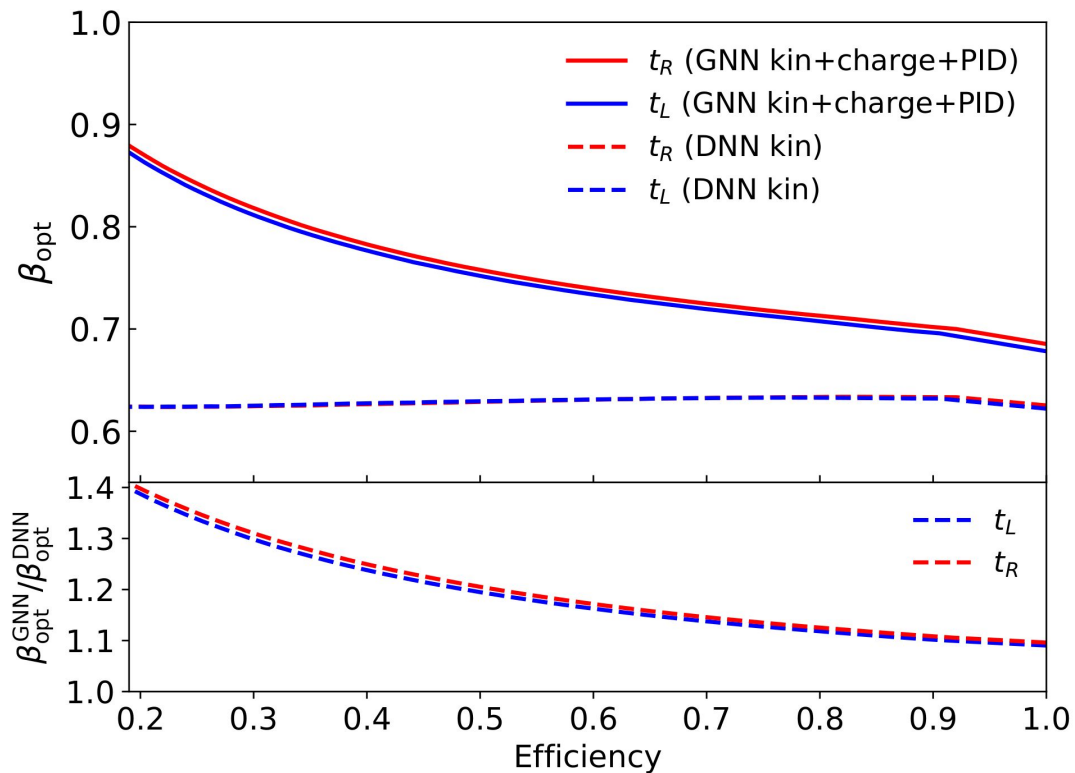
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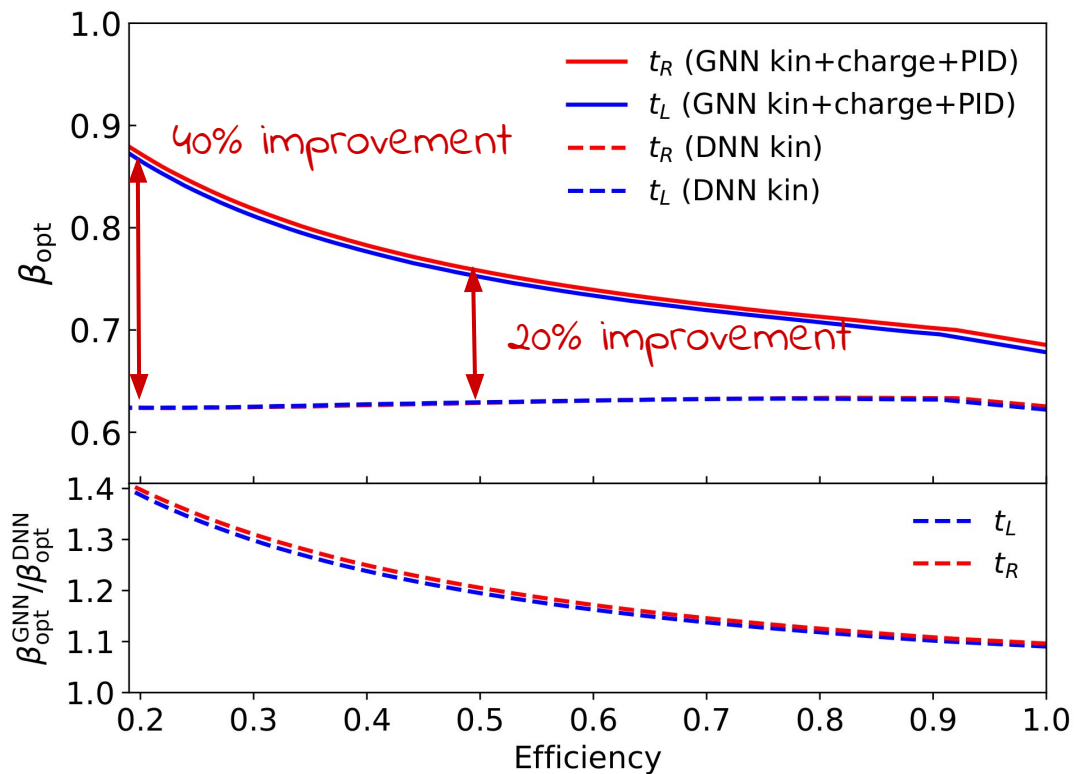
# Improvements with ML approach

- Use network score as the probability of the down-type jet being the soft jet.
- Apply cuts on the network score to enhanced the results as long as total rate is still larger than in dileptonic case.



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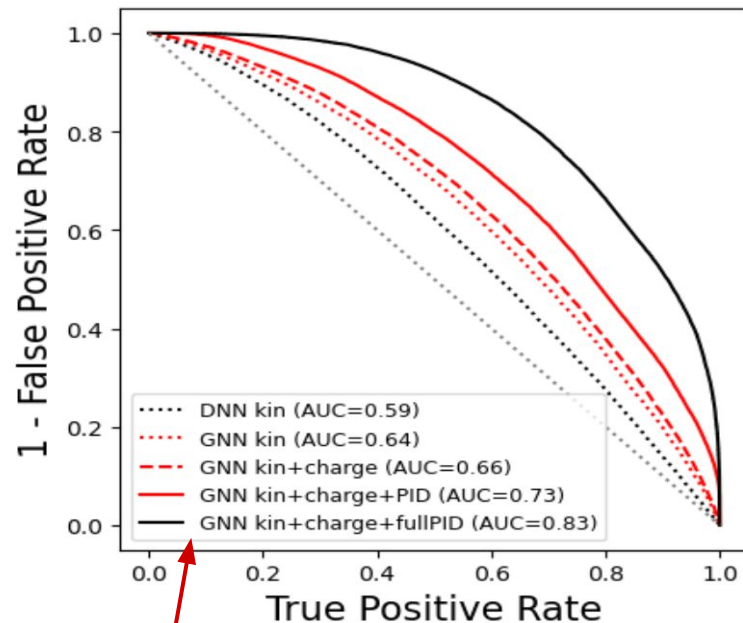
# Summary

- Hadronic top-quark polarization can be improved by incorporating information of global jet dynamics and constituents.
- Owing to the larger rate (factor of for semileptonic w.r.t dileptonic case), the spin analyzing power in hadronic decays can improve by approximately 20% (40%) compared to the kinematic approach, assuming an efficiency of 0.5 (0.2) for the GNN.
- This results could be used to further improve studies where the top polarization is relevant such as new physics searches, spin correlations, etc.

Backup

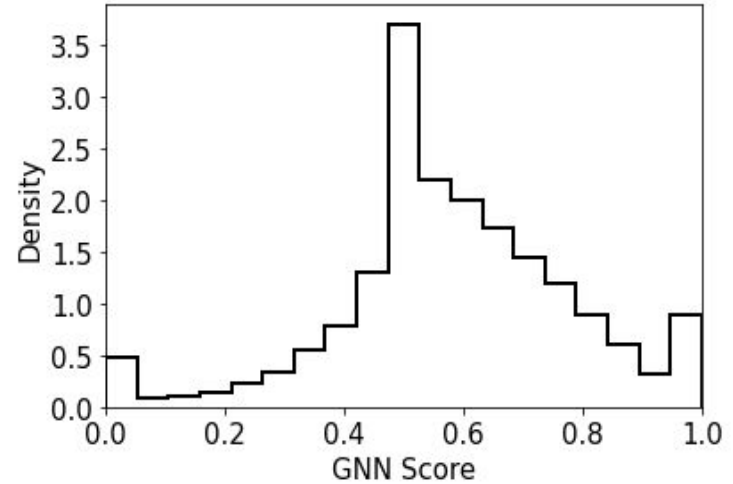
# Path for further improvements

- Performance can be improved with pion/kaon discrimination.
- They are not well distinguished at the LHC. This may change with future detectors, though.
- Charm tagging can also be incorporated.
- Any extra information should further improve our current results.



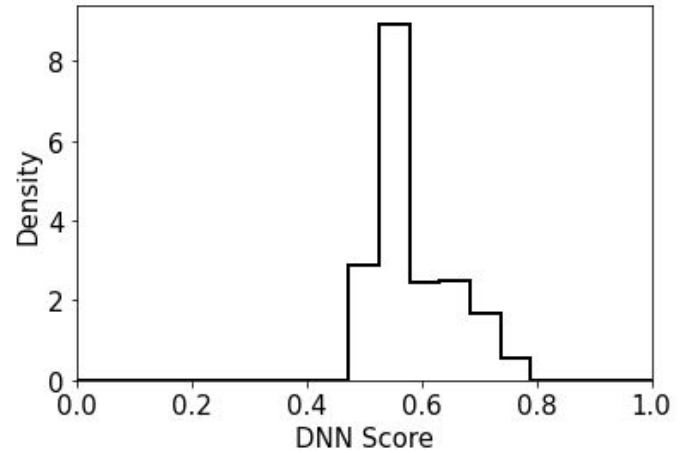
# GNN score

- Total of 77k trainable parameters.
- Trained on 200k unpolarized sample.
- Tested on 50k right-hand and left-hand polarized samples.



# DNN score

- Using jets momenta as input.
- Total of 20k trainable parameters.
- Trained on 200k unpolarized sample.
- Tested on 50k right-hand and left-hand polarized samples.



# Jet charge

➤ A definition of jet charge is given by

$$Q_\kappa \equiv \sum_{i \in J} z_i^\kappa Q_i$$

$$z_i^\kappa$$

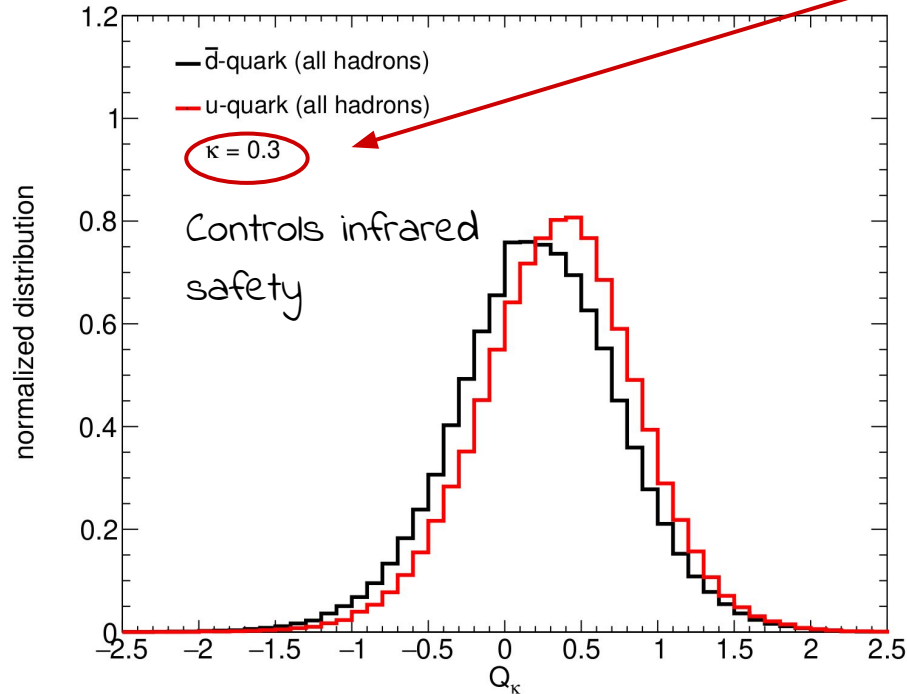
$$Q_i$$

Field and Feynman (1978)  
Kang, Larkoski and Yang  
(2023)

Charge of jet  
constituent

$$z^\kappa = \left( \frac{E_i}{E_J} \right)^\kappa$$

Energy fraction of  
jet constituent





# Parametric form of jet charge distributions

Kang, Larkoski and Yang (2023)

- Assume hadrons in the jet are produced through identical independent processes and the multiplicity is large

$$p(Q_\kappa|u, N) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(Q_\kappa - \mu_u)^2}{2\sigma^2}} \quad p(Q_\kappa|\bar{d}, N) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(Q_\kappa - \mu_{\bar{d}})^2}{2\sigma^2}}$$

- Assume particles in the jets are pions and the SU(2) isospin of the pions is exact

$$\mu_u = \frac{2}{3} N^{-\kappa} \left(1 + \frac{\kappa}{2} (\kappa - 1) \sigma_z^2 N^2 + \dots\right) \quad \mu_{\bar{d}} = \frac{1}{3} N^{-\kappa} \left(1 + \frac{\kappa}{2} (\kappa - 1) \sigma_z^2 N^2 + \dots\right) \quad \sigma^2 = \frac{2}{3} N^{1-2\kappa} \left(1 + \kappa(2\kappa - 1) \sigma_z^2 N^2 + \dots\right)$$

- The discrimination power

$$\eta = \frac{(\langle Q_\kappa \rangle_u - \langle Q_\kappa \rangle_{\bar{d}})^2}{\sigma^2} \sim \frac{1}{N} (1 - \kappa^2 \sigma_z^2 N^2 + \dots)$$

# Data preparation

- we generate 14 TeV  $pp \rightarrow t\bar{t} \rightarrow \ell^\pm \nu 2b2j$  events using MG5, with no cuts except for  $p_T(t) > 200$  GeV.
- Three sets of samples where the top quark is unpolarized, left hand polarized and right hand polarized in the  $t\bar{t}$  rest frame.
- Parton shower and hadronization are done with PYTHIA8 without MPI.
- Identify the top jet using CA algorithm with  $R = 1.5$ , and  $p_T > 250$  GeV. And decluster to find the subjects with  $m < 30$  GeV.
- Keep the three subjects that satisfy  $165 \text{ GeV} < m(\text{top}) < 195 \text{ GeV}$ .
- Match the hadron level jets with true parton level momenta, by using the smallest  $\Delta R$  between the two.

# EdgeConv block

- Represents a point cloud as a graph, whose vertices are the points themselves, and the edges are constructed as connections between each point to its  $k$  nearest neighboring points
- A local patch needed for convolution is defined for each point as the  $k$  nearest neighboring points connected to it.
- Can be viewed as a mapping from a point cloud to another point cloud with the same number of points.

