$t(\bar{t})\ldots$ bound states & and contact interactions

Felipe J. Llanes-Estrada

Collaborators: Alejandro Alonso, Daniel Berzal, Mario Pardo and Clara Peset
Universidad Complutense de Madrid, Dept. de Física Teórica and IPARCOS
(on leave at CERN-TH)

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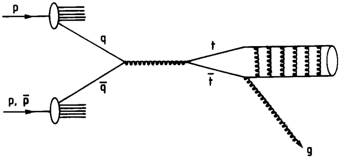
Motivation: toponium (dedicate time tomorrow) & topball

Long standing prediction of $t\bar{t}$ states near threshold



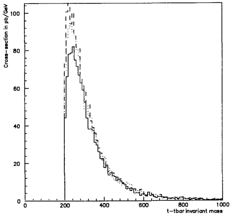


Figure 8: Invariant mass distribution of $t\bar{t}$ pairs for the LHC collider at 15 TeV, with m_t = 100 GeV. Notation as in Fig. 6.

Fadin, Khoze and Sjöstrand,
Z.Phys.C 48 (1990) 613-622

Motivation: toponium (dedicate time tomorrow) & topball

Very speculative: $6t6\bar{t}$ Tball

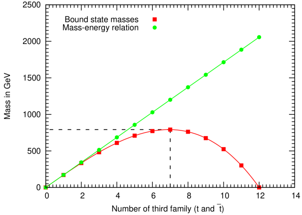


Fig. 4: The dependence of the T-ball's mass of the number $\mathbf{N_{cost.}}$ of the NBS constituents.

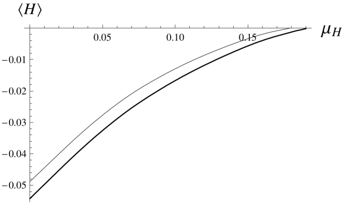


FIG. 2: The total energy $\langle H \rangle$ of the multi-top versus the mass of the Higgs boson $m_{\rm H}$, energy is measured in units of $E_0 = N(N-1)^2\, \alpha_{\rm H}^2\, m_t c^2$ (compare Eq.(5)), mass of the Higgs

Froggatt and Nielsen, Surveys HEP **18**, 55-75 (2003); Froggatt et al. (2008)
Kuchiev, Flambaum and Shuryak, PRD **78**, 077502 (2008)

Outline

1. Multi-c, b states
2. Multi $t\bar{t}$ states
3. A closer look at toponium

Toponium: Schrödinger equation

$$E\psi(r) = -\frac{1}{2(m_t/2)}\nabla^2\psi(r) + V(r)\psi(r)$$

yields, at LO,

$$E = -\frac{4}{9}m_t\alpha_s^2 \qquad \text{or} \qquad M = 2m_t\left(1 - \frac{2}{9}\alpha_s^2\right)$$

For higher orders see M. Beneke et al. PRL **115** (2015) 19, 192001

Many-body states: Hartree-Fock method

Combine Hartree-Fock equation:

$$\varepsilon\phi_k(q) = -\frac{1}{2m}\nabla^2\phi_k(q) + \kappa_c \sum_{l=1}^{N} \int dq' \left|\phi_l(q')\right|^2 \frac{\alpha_s}{|\mathrm{r}-\mathrm{r}'|}\phi_k(q) -$$

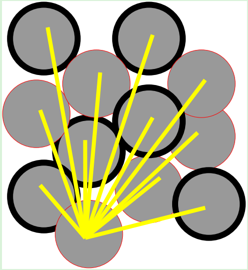
$$-\kappa_c \sum_{l=1}^{N} \delta(\chi_l, \chi_k)\delta(\sigma_l, \sigma_k) \int dq' \phi_l^*(q') \frac{\alpha_s}{|\mathrm{r}-\mathrm{r}'|}\phi_k(q')\phi_l(q)$$

with Koopman's theorem:

$$E = \sum_i \varepsilon_i - \alpha \sum_{i<j} \int \frac{|\phi_i(\mathrm{r},\sigma)|^2|\phi_j(\mathrm{r}',\sigma')|^2}{|\mathrm{r}-\mathrm{r}'|} d^3\mathrm{r}\, d^3\mathrm{r}'$$

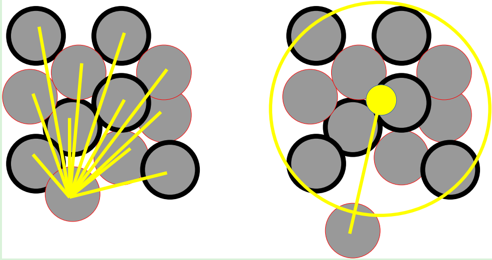
$$+\alpha \sum_{i<j} \int \frac{\phi_i^*(\mathrm{r},\sigma)\phi_j(\mathrm{r},\sigma)\phi_j^*(\mathrm{r}',\sigma')\phi_i(\mathrm{r}',\sigma')}{|\mathrm{r}-\mathrm{r}'|} d^3\mathrm{r}\, d^3\mathrm{r}'$$

Many-body states: Hartree-fock method



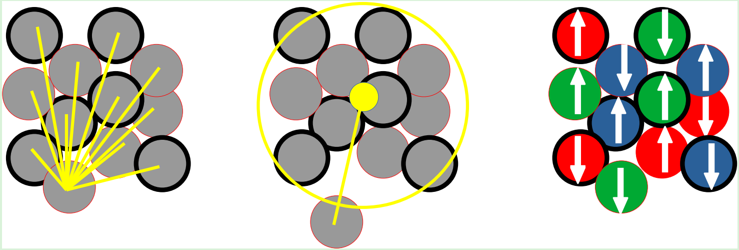
All-to-all interactions

Many-body states: Hartree-fock method



Mean-field simplification

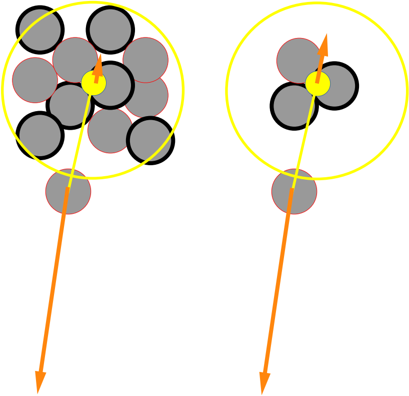
Many-body states: Hartree-fock method



Fill the 1S orbital:

12= 2(spin) $\times$ 3 (color) $\times$ 2 (particle/antiparticle)

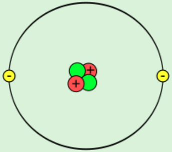
Need to subtract cm energy



Correct for recoil at the one-body level:

$$T_i = \frac{p_i^2}{2m_t} \to \frac{p_i^2}{2m_t} \times \left(1 - \frac{1}{N}\right) \text{ ($\pm$ uncertainty 50% of correction)}$$

Validation: Helium and positronium



Ground state energy (total electron binding energy) of the neutral Helium atom, in eV.

This work, HF1	This work, HF2	Thijssen, HF	Experimental value
-77.9	-78.57	-77.69	-79.01

Positronium e^-e^+: BE$= -\frac{1}{2}\frac{m_e}{2}\alpha_s^2 \simeq -6.8$ eV

HF: -5.9 ± 1.6 eV

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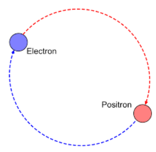
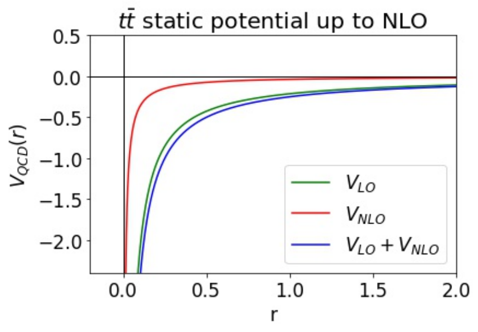


Positronium $e^- e^+$: BE$= -\frac{1}{2}\frac{m_e}{2}\alpha_s^2 \simeq -6.8$ eV

HF: -5.9 ± 1.6 eV

Validation: multi-c and -b systems

With V_{NLO} potential

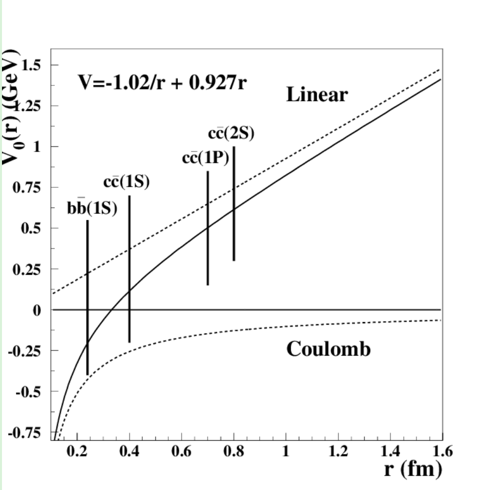


All-heavy baryons: color part of nuclear physics

	LO	NLO	NLO+Spin
$\Omega_{ccc}\left(\frac{3}{2}\right)^+$	4650(120)	4660(120)	4660(120)
$\Omega_{ccb}\left(\frac{1}{2}\right)^+$	7920(110)	7930(110)	7920(110)
$\Omega_{ccb}\left(\frac{3}{2}\right)^+$	7920(110)	7920(110)	7920(110)
$\Omega_{bbc}\left(\frac{1}{2}\right)^+$	11100(100)	11080(100)	11100(100)
$\Omega_{bbc}\left(\frac{3}{2}\right)^+$	11100(100)	11090(100)	11100(100)
$\Omega_{bbb}\left(\frac{3}{2}\right)^+$	14280(90)	14270(90)	14280(90)

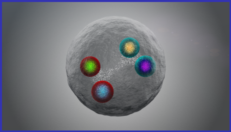
(Remember that LHCb has reported a doubly-heavy one)

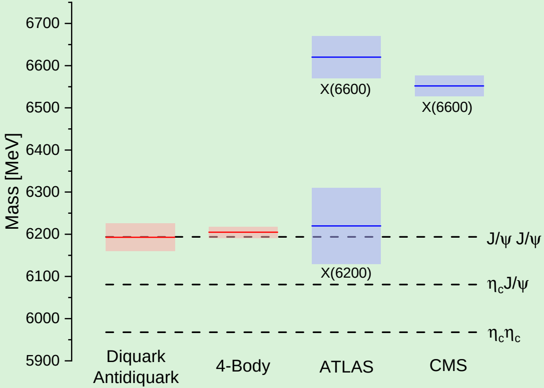
In the Cornell-potential



K. Seth, EPJ Web of Conferences 3 07006 (2010)

Tetraquarks





Pentaquarks

	LO	NLO	Spin
$(ccc)(c\bar{c})$	7860(200)	7850(200)	7860(200)
$(ccb)(c\bar{c})$	11040(190)	11020(190)	11040(190)
$(ccc)(c\bar{b})$	11060(190)	11050(190)	11060(190)
$(ccb)(b\bar{c})$	14240(180)	14210(180)	14240(180)
$(bbc)(c\bar{c})$	14240(180)	14240(180)	14250(180)
$(ccb)(b\bar{b})$	17440(170)	17430(170)	17440(170)
$(bbc)(c\bar{b})$	17450(170)	17450(170)	17450(170)
$(bbb)(c\bar{b})$	20650(160)	20640(160)	20640(160)
$(bbc)(b\bar{b})$	20650(160)	20660(160)	20650(160)
$(bbb)(b\bar{b})$	23830(150)	23830(150)	23830(150)

Dibaryons

- $6 \times c$ and $6 \times b$ just too heavy;
- But Deuteron-like $ccb - bbc$: $M = 18860(50)$ MeV $< M(\Omega_{bbb}) + M(\Omega_{ccc}) \simeq 18940(210)$ MeV

Order of magnitude cross sections (in μbarn)

c	$\sim 10^2$
cc	$\sim 10^{-1}$
ccc	$\sim 10^{-4}$

b	$\sim 10^2$
bb	$\sim 5 \cdot 10^{-2}$
bbb	$\sim 10^{-5}$

Empirical rule: $+c$-quark $\implies \sigma \to \sigma/10^3$, (larger for the b).

Pentaquarks $(ccc)(c\bar{c})$ $\to$0.1 fbarn (HL-LHC?)

$(bbb)(b\bar{b}) \to 10^{-4}$ fb (ooof)

Outline

The t sometimes forms bound states

- $2\pi a_0 \simeq 0.25$ fm for a Bohr orbit
- $N = N_0 e^{-15t/\mathrm{fm}}$ pairs survive a time t
- CMS $t\bar{t}$ pair sample: 87k (e+jets) + 140k (μ+jets) =240k
- 360 pairs survive long enough to form a bound state

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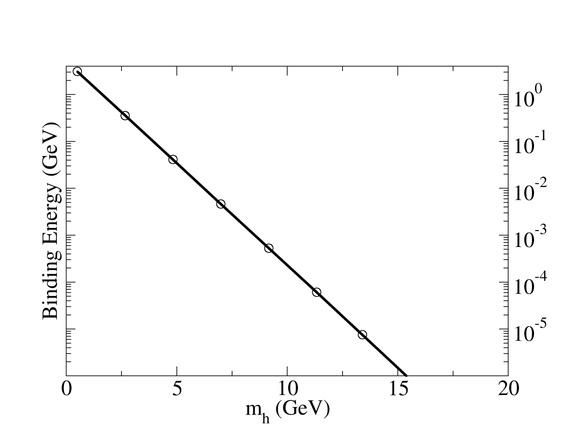
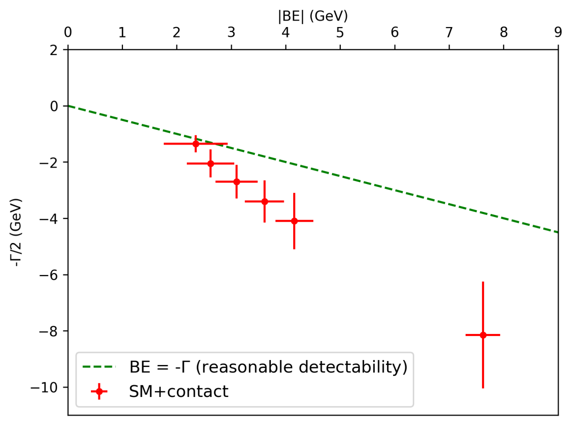
Confirm: NO T-ball ($6t6\bar{t}$) from Higgs binding



Qualitative agreeement with Kuchiev, Flambaum and Shuryak PRD **78** (2008) 077502

Onto QCD

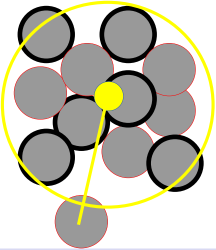
- Bound states of (multi) $t\bar{t}$ are about QCD



$$N = 2, 3, 4, 5, 6, 12$$

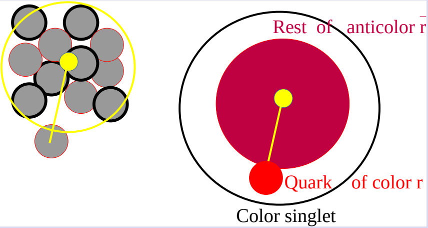
Soft scale $\mu = m_t \alpha_s \implies \alpha_s = 0.16$

Binding energy grows linearly with N



Remember the Hartree-Fock philosophy?

Binding energy grows linearly with N



- Each quark as in a meson (from color point of view)
- N bodies $\implies$ N interactions

Onto new physics

- Bound states of (multi) $t\bar{t}$ are about QCD
- and perhaps new physics?

Contact interactions in HEFT (SMEFT) Lagrangian

$$\mathcal{L}_{4t}^{\text{HEFT}} = \frac{16\pi^2}{\Lambda^2}\Big[(r_1+r_3+r_4)\bar{Q}_L Q_R \bar{Q}_L Q_R + (r_9+r_{10}+r_{11})\,\bar{Q}_L\gamma^\mu Q_L \bar{Q}_L \gamma_\mu Q_L + (r_{13}+r_{14}+r_{15})\bar{Q}_R\gamma^\mu Q_R \bar{Q}_R\gamma_\mu Q_R + (r_{17}+r_{18}+r_{19}+r_{20})\bar{Q}_L\gamma^\mu Q_L \bar{Q}_R\gamma_\mu Q_R + (r_5+r_7+r_8)\bar{Q}_L\vec{\lambda}Q_R\bar{Q}_L\vec{\lambda}Q_R + (r_{22}+r_{23}+r_{24}+r_{25})\bar{Q}_L\gamma^\mu\vec{\lambda}Q_L\bar{Q}_R\gamma_\mu\vec{\lambda}Q_R + h.c.\Big]$$

(For this work, group them and respect P, C)

$$\mathcal{L}_{4t}^{\text{BSM}} = K_1\left(\bar{Q}Q\right)\left(\bar{Q}Q\right) + K_2\left(\bar{Q}\gamma_\mu Q\right)\left(\bar{Q}\gamma^\mu Q\right) + K_5\left(\bar{Q}\lambda^A Q\right)\left(\bar{Q}\lambda_A Q\right) + K_6\left(\bar{Q}\gamma_\mu\lambda^A Q\right)\left(\bar{Q}\lambda_A\gamma^\mu Q\right).$$

I. Brivio *et al.* Eur. Phys. J. C 76, 416 (2016).

Contact interactions in HEFT (SMEFT) Lagrangian

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$$(r_{13}+r_{14}+r_{15})\bar{Q}_R\gamma^\mu Q_R \bar{Q}_R\gamma_\mu Q_R + (r_{17}+r_{18}+r_{19}+r_{20})\bar{Q}_L\gamma^\mu Q_L \bar{Q}_R\gamma_\mu Q_R +$$
$$(r_5+r_7+r_8)\bar{Q}_L\vec{\lambda}Q_R\bar{Q}_L\vec{\lambda}Q_R + (r_{22}+r_{23}+r_{24}+r_{25})\bar{Q}_L\gamma^\mu\vec{\lambda}Q_L\bar{Q}_R\gamma_\mu\vec{\lambda}Q_R + h.c.\Big]$$

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$$+\; K_5\left(\bar{Q}\lambda^A Q\right)(\bar{Q}\lambda_A Q) + K_6\left(\bar{Q}\gamma_\mu\lambda^A Q\right)(\bar{Q}\lambda_A\gamma^\mu Q)\;.$$

I. Brivio *et al.* Eur. Phys. J. C **76**, 416 (2016).

Contact interactions in HEFT (SMEFT) Lagrangian

95% constraints with scale $\Lambda = 1$ TeV. from CMS data

Type	Operator	Composition	Coefficient	Allowed range (TeV^{-2})
Scalar	$\mathcal{O}_{\mathcal{S}}$	$\bar{Q}Q\bar{Q}Q$	K_1, -K_4	[-1.8,1.9]
Vectorial	$\mathcal{O}_{\mathcal{V}}$	$\bar{Q}\gamma^{\mu}Q\bar{Q}\gamma_{\mu}Q$	K_2, K_3, $K_4/4$	[-0.5,0.4]
Color⊗ scalar	$\mathcal{O}_{\mathcal{SC}}$	$\bar{Q}\lambda^{A}Q\bar{Q}\lambda_{A}Q$	K_5	[-6.8,8.0]
Color⊗ vector	$\mathcal{O}_{\mathcal{VC}}$	$(\bar{Q}\gamma^{\mu}\lambda^{A}Q)^2$	K_5, -K_6	[-6.8,8.0]

Eventual bound-state constraints:
could use $N_{\text{constituents}}$ to disentangle

Contact interactions for bound states

$$V(\mathrm{r}) = \frac{g^2}{M^2}\delta^{(3)}(\mathrm{r}) \ .$$

Wilson coefficients $C_i \leftrightarrow g^2$ coupling
EFT scale $\Lambda \leftrightarrow M$ "BSM particle"

Regulated for numerical work:

$$V^{\mathrm{BSM}} = \frac{\alpha_{\mathrm{contact}}}{|\mathbf{r}' - \mathbf{r}|} e^{-\Lambda|\mathbf{r}' - \mathbf{r}|}$$

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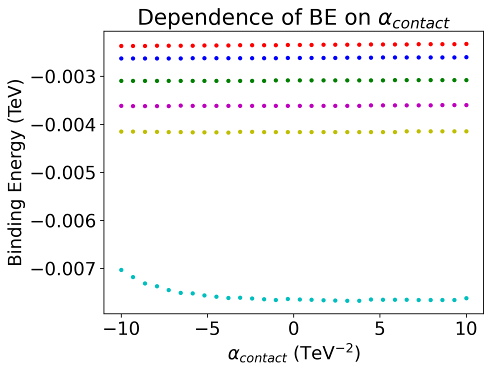
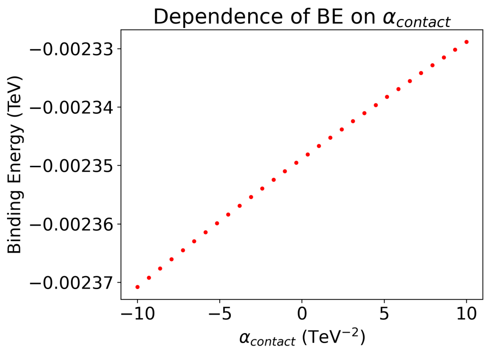
Variation with α_{contact}



Variation with $\alpha_{\mathrm{contact}}$ (closer look at $N = 2$)



- To have an $O(1)$ effect on the binding energies, $K_i \sim O(50-100)$
- (Remember other constraints are already $O(1-10)$
- Need GeV-level expt. and theory precision to be helpful

Outline

1. Multi-c, b states
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3. A closer look at toponium

Spanish saying: "Blanco y en botella"

If it is white and comes in a bottle,
it must be milk

Much simpler 2-body Coulombic system

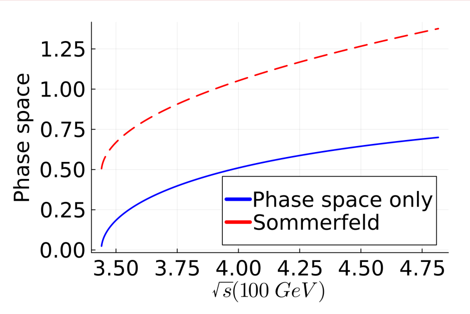
- Mass $M = m_t(2 - \frac{4}{9}\alpha_s^2) \implies$ 4 GeV binding energy
- Width $\Gamma = 2\Gamma_t \simeq 3$ GeV
- $\sigma \sim 6 - 7$ pbarn. . . (will show next)

Then it must be the η_t

Threshold $t\bar{t}$ production: phase space factor

$$\beta_t = \sqrt{1 - \frac{4m_t^2}{s_{t\bar{t}}}} \rightarrow$$

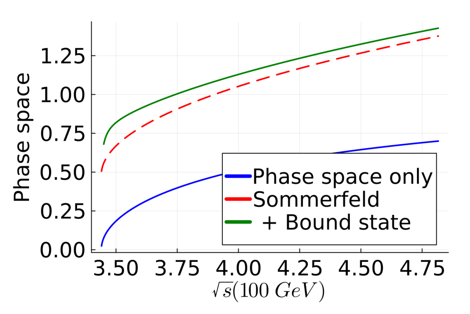
$$\frac{p_+}{m_t} + 2\frac{p_0}{m_t}\,\mathrm{atan}\frac{p_+}{p_-}$$



(definitions at the back: following Fadin, Khoze and Sjöstrand *op.cit.*)

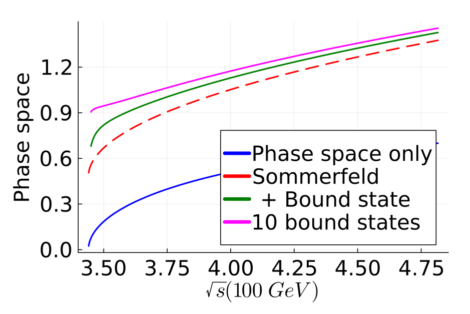
Above threshold effect of toponium ($\Gamma_t = 0$)

$$+\frac{2p_0^2}{m_t^2}\frac{1}{n^4}\frac{p_+(n^2E+p_0^2/m_t)}{(E+p_0^2/(m_t n^2))^2}\bigg|_{n=1}$$



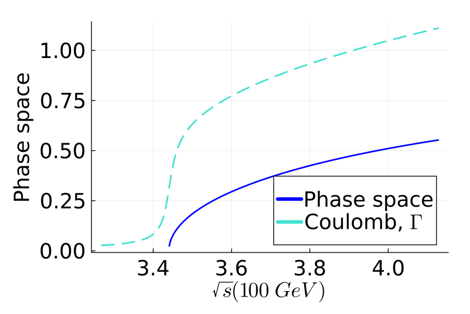
Add nine more bound states of the series

$$+\sum_n \frac{2p_0^2}{m_t^2}\frac{1}{n^4}\frac{p_+(n^2E+p_0^2/m_t)}{(E+p_0^2/(m_tn^2))^2}$$

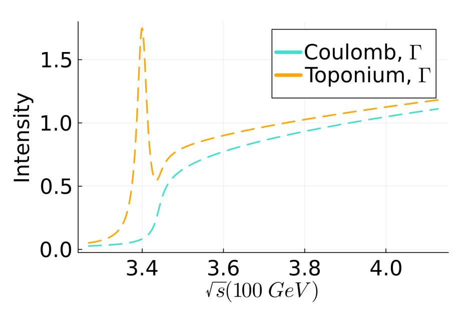


Start over, but now with the t width Γ_t

Phase-space×Coulomb factor
now spread below threshold



Bound state visible under threshold



Note: contact interactions can also produce one bound state

$t\bar{t}$ bound by contact interaction instead

$$H = -\frac{1}{2m_t/2}\nabla^2 + v\delta^{(3)}(r)$$

Needs renormalization

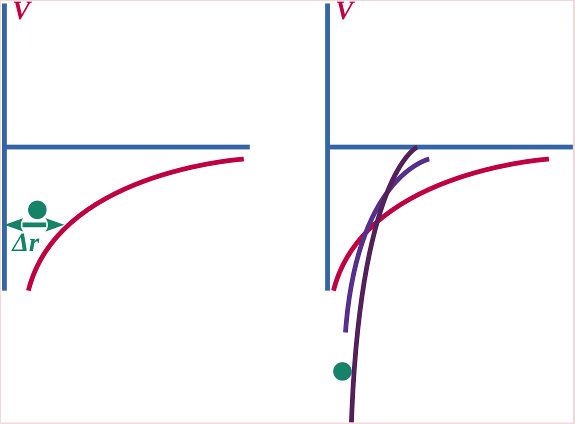
$$g = \frac{v}{1 + \frac{m_t \Lambda v}{\pi^2}}$$

Binding energy of the one bound state:

$$\mathrm{BE} = \frac{1}{m_t^3}\frac{\pi^2}{2g^2}$$

R. Jackiw, entry 313843 in `inspirehep.net`

Why renormalization?

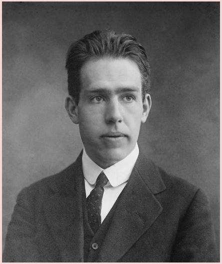


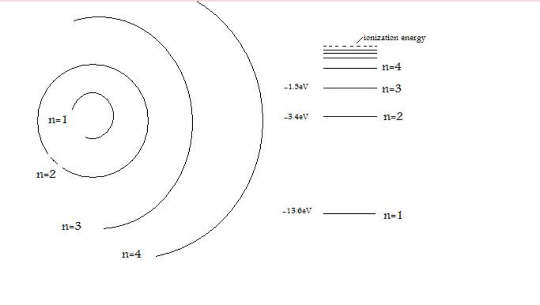
In three dimensions: $T \propto \frac{1}{(\Delta r)^2}$ (positive)

$V \propto -\frac{1}{r^\alpha}$ (negative, if $\alpha > 2$, dominant)

A. Galindo, P. Pascual, Quantum Mechanics, Springer Verlag (1990)

How to distinguish them? Excited Bohr levels...



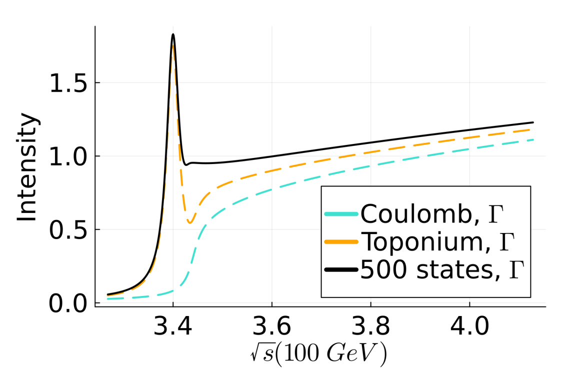


$$E_n = -\frac{E_{\mathrm{Bohr}}}{n^2}$$

Chemistry libretext

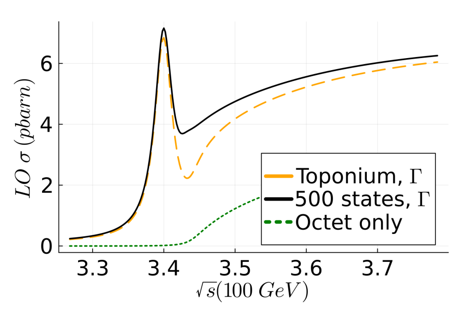
Add those excited states

$$\frac{p_+}{m_t} + 2\frac{p_0}{m_t}\,\mathrm{atan}\frac{p_+}{p_-}$$
$$+\sum_n \frac{2p_0^2}{m_t^2}\frac{1}{n^4}\frac{\Gamma_t p_0 n + p_+(n^2\sqrt{E^2+\Gamma_t^2} + p_0^2/m_t)}{(E + p_0^2/(m_t n^2))^2 + \Gamma_t^2}$$

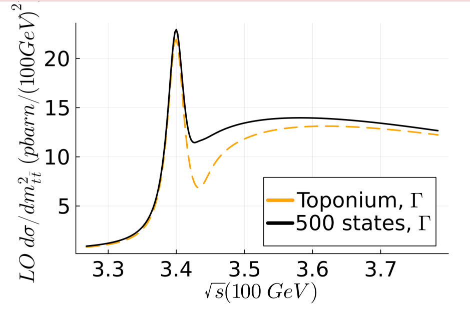


Add the color-octet $t\bar{t}$ production

(No physical states there)



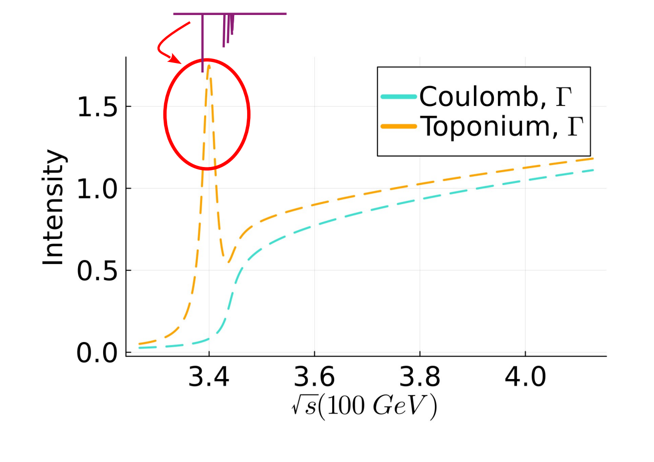
Multiply by parton luminosity



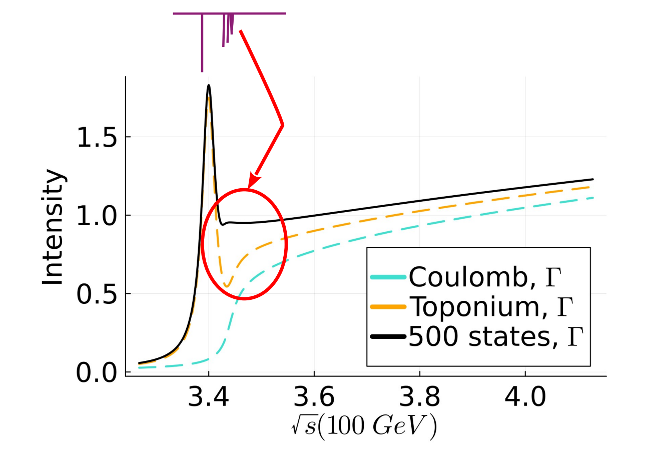
$$\frac{d\sigma}{dm^2_{t\bar{t}}} = \sigma_{g\hat{g}\to tt} \underbrace{\frac{d\mathcal{L}}{dm^2_{t\bar{t}}}}$$

$$\int_{m^2_{t\bar{t}}}^{1} \frac{dx}{x\ s_{\rm LHC}} f(x) f\left(\frac{m^2_{t\bar{t}}}{x\ s_{\rm LHC}}\right)$$

The peak is close to the η_t

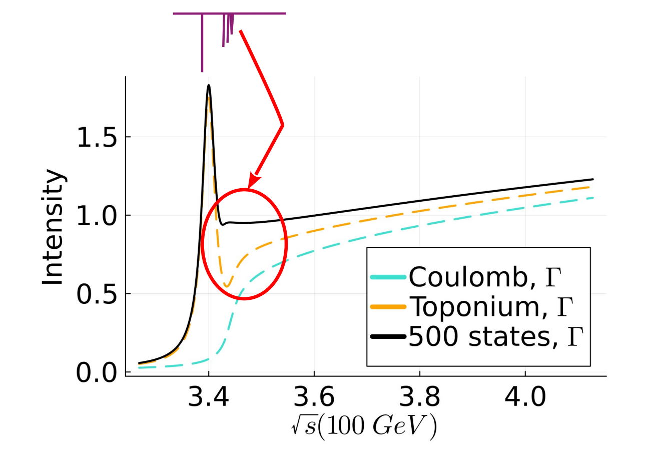


But excited toponia fill the dip:



This dip has a width $O(5-10)$ GeV: need better experimental energy resolution

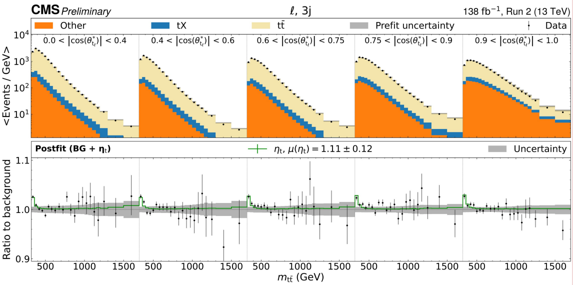
But excited toponia fill the dip:



This dip has a width $O(5-10)$ GeV: need better experimental energy resolution

Now current reality...

- CMS spectrum: large background
- Only after subtraction a threshold excess appears



- & 50 GeV energy bins vs. 2 GeV, better resolution needed

CMS PAS-HIG-22-013

Summary

- Hartree-Fock approximation to systems with several heavy quarks/antiquarks (c,b)
- Extended to several $t\bar{t}$ systems
- Confirm that the 12-body T-ball is only lightly bound (by QCD gluons)
- BSM HEFT/SMEFT coefficients need to be quite large to shift binding energies with near/mid-term precisions
- Whether QCD toponium or new-physics: dip at threshold? (Bohr excitations)

$t(\bar{t})\ldots$ bound states & and contact interactions

Felipe J. Llanes-Estrada

Collaborators: Alejandro Alonso, Daniel Berzal, Mario Pardo and Clara Peset
Universidad Complutense de Madrid, Dept. de Física Teórica and IPARCOS
(on leave at CERN-TH)

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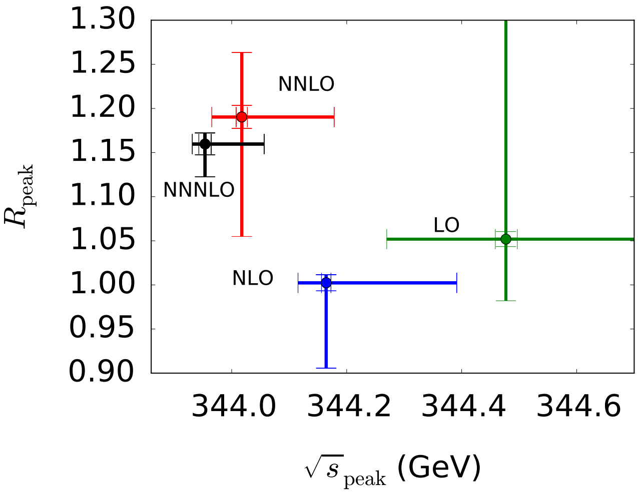
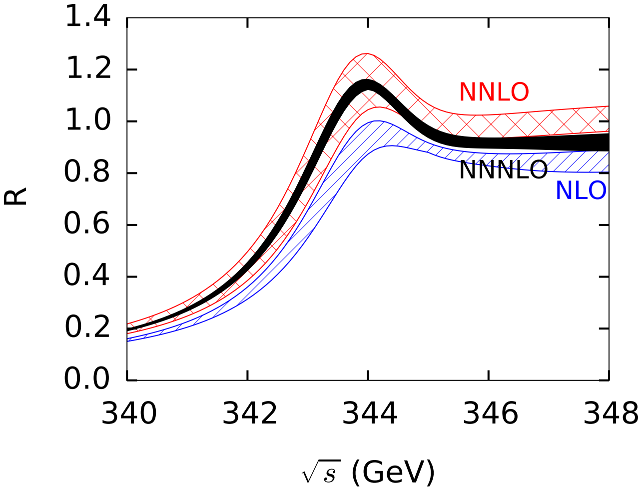
Peak position (here in e^-e^+) with QCD corrections



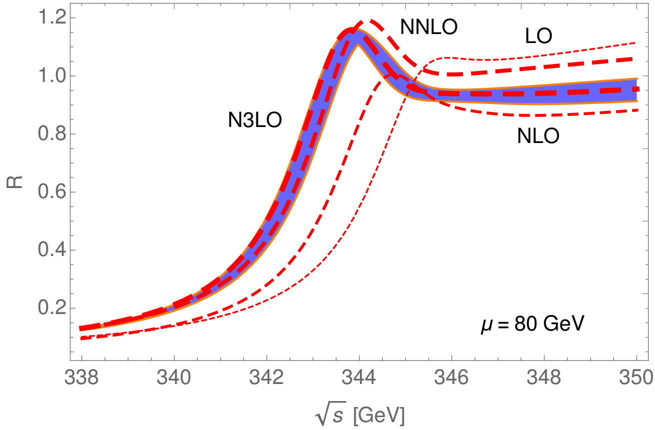
M. Beneke *et al.* PRL **115** (2015) 19, 192001

Cross section (here in e^-e^+) with QCD corrections



M. Beneke *et al.* PRL **115** (2015) 19, 192001

Higher order computations achieve scale independence



(Note: LO can capture line shape if assigning ultrasoft scale to binding energy but hard scale to production)

M. Beneke and Y. Kiyo, arXiv:2409.05960

Fix parameters: quarkonium spin averages

$c\bar{c}$, $b\bar{b}$ and mixed-flavor B_c and experimental reference values.

	$c\bar{c}$ ($\pm 80\mathrm{MeV}$)		$b\bar{b}$ ($\pm 60\mathrm{MeV}$)		B_c ($\pm 70\mathrm{MeV}$)	
	S	P	S	P	S	P
LO	3100	3160	9470	9540	6310	6370
NLO	3030	3140	9410	9520	6270	6350
PDG spin average	3069	3526	9445	9889	6274	6714

Final parameter set for all-c, b multiquark hadrons.

m_c (MeV)	$\alpha_s(m_c^2)$	m_b (MeV)	$\alpha_s(m_b^2)$
1575	0.34	4768	0.21

Fix parameters: quarkonium spin averages

Sizeable uncertainty sources to be propagated.

	Difference to employed average				CM
	p-wave	s-wave	LO	NLO	recoil
m_c (MeV)	180	10	20	20	20
$\alpha_s(m_c^2)$	0.013	0.005	0.001	0.001	0.005
m_b (MeV)	180	10	10	10	20
$\alpha_s(m_b^2)$	0.001	0.001	0.001	0.001	0.003

Uncertainty
40
0.006
30
0.004

Momenta from Fadin & Khoze

- $p_0 \equiv \frac{2}{3} m_t \alpha_s$ is Bohr's momentum
- $p_\pm = \sqrt{\frac{m_t}{2}\left(\sqrt{E^2 + \Gamma_t^2} \pm E\right)}$ are spread by t-width

NLO spectrum with ultrasoft scale