Combined effective field theory interpretation of Higgs boson, electroweak vector boson, top quark, and multi-jet measurements

LHC TOP Working Group Meeting

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Documentation

- Physics Analysis Summary: CMS-PAS-SMP-24-003; CDS: https://cds.cern.ch/record/2911229

Available on the CERN CDS information server **CMS PAS SMP-24-003**

CMS Physics Analysis Summary

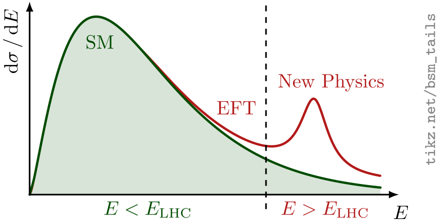
Contact: cms-pag-conveners-smp@cern.ch 2024/09/23

Combined effective field theory interpretation of Higgs boson, electroweak vector boson, top quark, and multi-jet measurements

The CMS Collaboration

Indirect Search for BSM physics

- Looking for BSM physics in the LHC data
  - → Direct searches (resonances, ...)
  - → But what if BSM particles are too heavy to be produced on-shell at the LHC?
  - → In addition, we need to look for indirect evidence of BSM physics via deviations in known SM processes



- One approach to indirect searches: The Standard Model Effective Field Theory (SMEFT)

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)}$$

- → Consistent and model independent way to parametrise deviations in all SM processes
- → Set constraints on Wilson coefficients c_i (and let theorists match to any UV model)
- → In this work we consider only operators with $d = 6$

Motivation for a combined SMEFT interpretation

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \cancel{\frac{1}{\Lambda}\sum_i c_i \mathcal{Q}_i^{(5)}} + \underbrace{\frac{1}{\Lambda^2}\sum_j c_j \mathcal{Q}_j^{(6)}}_{\text{leading deviation from SM}} + \ldots$$

(violates lepton number)

- 129 operators considered (dim-6, CP-even, `topU3l` flavour symmetry): Typically, each operator impacts multiple processes, and each process is sensitive to multiple operators
  - → Setting constraints within a single measurement requires assumption that all other Wilson coefficients are fixed to SM value of 0
  - → To constrain multiple Wilson coefficients simultaneously, we need to combine a global set of measurements
- **This analysis is the first combined SMEFT interpretation of data from four sectors of the SM (Higgs, Top, Electroweak, QCD) done by CMS**

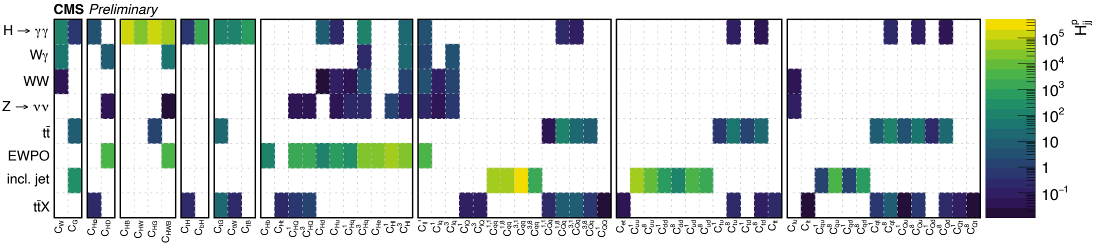
Analyses included in the combination

- Combination of seven sets of CMS measurements, and electroweak precision observables measured at LEP and SLC
- Inputs chosen to provide sensitivity to broad set of SMEFT operators (64 in total), negligible overlap in event selections, small backgrounds (or estimated from data)

Analysis	Type of measurement	Observables used	Experimental likelihood
$\mathrm{H} \to \gamma\gamma$	Diff. cross sections	STXS bins [41]	✓
$\mathrm{W}\gamma$	Fid. diff. cross sections	$p_{\mathrm{T}}^{\gamma} \times \lvert\phi_f\rvert$	✓
WW	Fid. diff. cross sections	$m_{\ell\ell}$	✓
$\mathrm{Z} \to \nu\nu$	Fid. diff. cross sections	$p_{\mathrm{T}}^{\mathrm{Z}}$	✓
$\mathrm{t}\bar{\mathrm{t}}$	Fid. diff. cross sections	$M_{\mathrm{t}\bar{\mathrm{t}}}$	$\times$
EWPO	Pseudo-observables	Γ_Z, σ^0_{had}, R_ℓ, R_c, R_b, $A^{0,\ell}_{FB}$, $A^{0,\mathrm{c}}_{FB}$, $A^{0,\mathrm{b}}_{FB}$	$\times$
Inclusive jet	Fid. diff. cross sections	$p_{\mathrm{T}}^{\mathrm{jet}} \times \lvert y^{\mathrm{jet}}\rvert$	$\times$
$\mathrm{t}\bar{\mathrm{t}}\mathrm{X}$	Direct EFT	Yields in regions of interest	✓

Analyses included in the combination

- Combination of seven sets of CMS measurements, and electroweak precision observables measured at LEP and SLC
- Inputs chosen to provide sensitivity to broad set of SMEFT operators (64 in total), negligible overlap in event selections, small backgrounds (or estimated from data)
- Which input channel is sensitive to which operators?

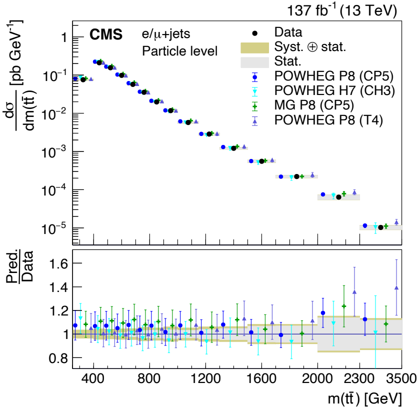


- plot shows diagonal entries of the Hessian matrix, $H_{jk} = \frac{\partial^2 \ln \mathcal{L}}{\partial c_j \partial c_k}$

  $\rightarrow$ $(H^p_{jj})^{-1/2}$: estimate of half the expected 68% confidence interval on Wilson coefficient c_j/Λ^2, evaluated with input channel p)

Input Measurements: CMS-TOP-20-001, $t\bar{t}$ semileptonic

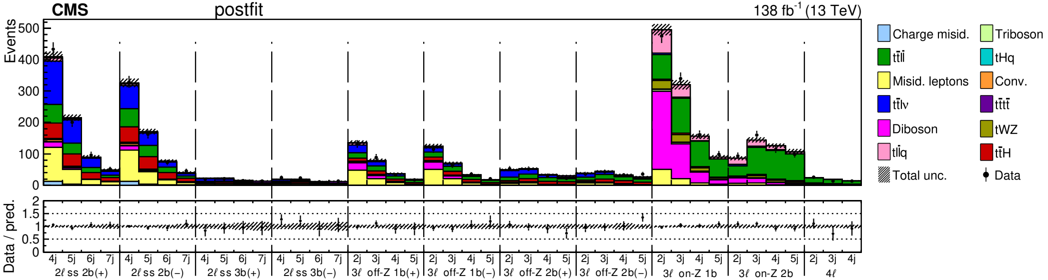
- $t\bar{t}$: CMS-TOP-20-001
  (Phys. Rev. D 104 (2021) 092013)



- Observable: $M_{t\bar{t}}$ (based on sensitivity studies)
- Full Run 2 data, 138 fb^{-1}
- Experimental likelihood model not available
  → Build simplified likelihood model based on:
  - Diff. cross section measurements with experimental covariance matrix from HEPData
  - SM predictions (Powheg+Pythia8, CP5 tune) from the authors
  - Theory uncertainty covariance matrix derived using MC samples and Rivet plugin
- SMEFT parameterizations: SMEFTsim3 (pp → $t\bar{t}$ events with up to one extra jet, SMEFT effects in top quark decays not considered)

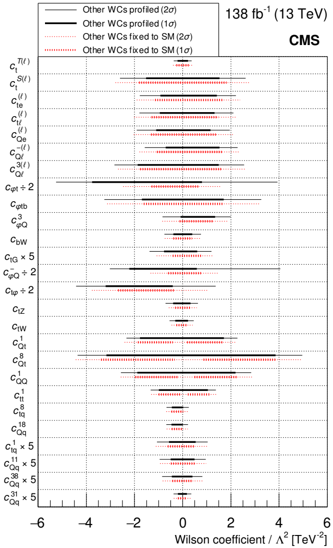
Input Measurements: CMS-TOP-22-006, $t\bar{t}X$ EFT

- CMS-TOP-22-006 (JHEP 12 (2023) 068):
  EFT in associated top quark production ($t\bar{t}H$, $t\bar{t}W$, $t\bar{t}Z$, tZq, tHq, and $t\bar{t}t\bar{t}$ processes)
- Each process can be studied individually, but they are irreducible backgrounds to each other
- Event selection based on number of leptons, jets, and b-tagged jets
  → 43 categories, binning in a kinematical variable within each category
- Total predicted event yield in each observable bin parameterized as a quadratic function of 26 WCs
  → Detector-level predictions accounting for all relevant EFT effects on each of the signal processes simultaneously



Input Measurements: CMS-TOP-22-006, $t\bar{t}X$ EFT

- Original analysis sets constraints on 26 Wilson coefficients in the `dim6top` basis
- In the combination we use `topU3l` SMEFT basis as implemented in SMEFTsim3, and consider additional operators that may affect the $t\bar{t}X$ processes
- To include $t\bar{t}X$ analysis in the combination in a consistent way, we therefore had to
  - $\rightarrow$ rotate from `dim6top` to SMEFT `topU3l`
  - $\rightarrow$ study the effect of missing operators on $t\bar{t}X$ processes
- $\mathcal{Q}_{H\Box}$ uniformly scales all SM Higgs boson couplings
  - $\rightarrow$ added to $t\bar{t}X$ analysis as **rescaling** of $t\bar{t}H$ and tHq signals
- **2-heavy-2-light quark operators** enhance $t\bar{t}Z$ and $t\bar{t}H$ production rates (effect on shape and normalization)
  - $\rightarrow$ added to $t\bar{t}X$ analysis by **reweighting the signal samples** using standalone reweighting modules produced by MadGraph («post-mortem reweighting»)
- Effect of other operators found to be negligible



SMEFT parameterization

- Scattering cross section is proportional to matrix element squared

$$\sigma = |\mathcal{M}_{\mathrm{SM}} + \sum_j \frac{c_j}{\Lambda^2}\mathcal{M}_j|^2 = |\mathcal{M}_{\mathrm{SM}}|^2 + 2\sum_j \frac{c_j}{\Lambda^2}\mathrm{Re}\left(\mathcal{M}_j\mathcal{M}^*_{\mathrm{SM}}\right) + \sum_{j,k}\frac{c_j c_k}{\Lambda^4}\mathrm{Re}\left(\mathcal{M}_j\mathcal{M}^*_k\right)$$

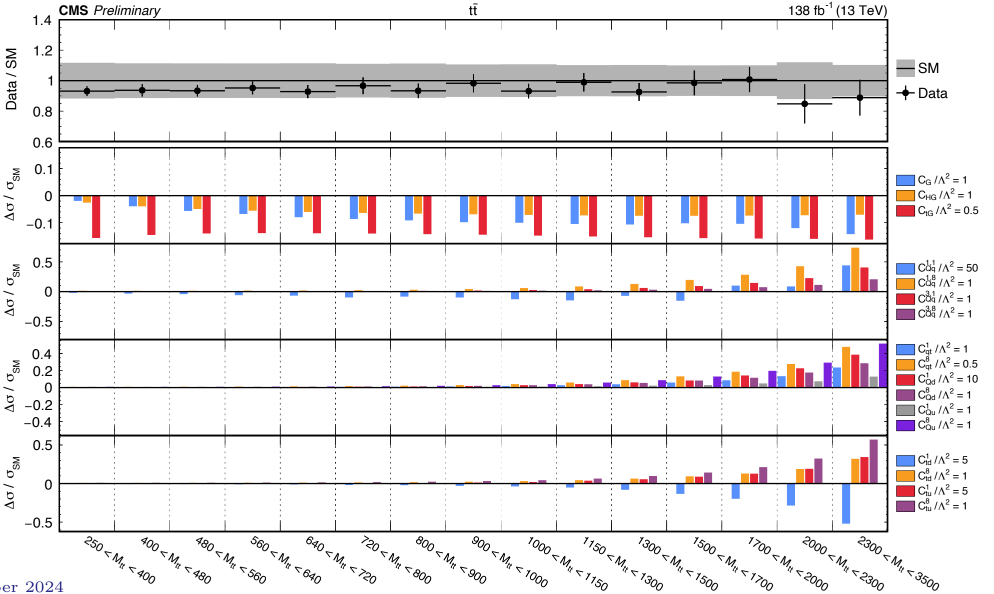
- This means that cross section of process p in kinematic bin i can be written as

$$\sigma^{i,p}_{\mathrm{SMEFT}} = \sigma^{i,p}_{\mathrm{SM}} + \sigma^{i,p}_{\mathrm{int.}}(\vec{c}) + \sigma^{i,p}_{\mathrm{BSM}}(\vec{c}) = \sigma^{i,p}_{\mathrm{SM}}\left(1 + \sum_j A^{i,p}_j \frac{c_j}{\Lambda^2} + \sum_{j,k} B^{i,p}_{jk}\frac{c_j c_k}{\Lambda^4}\right)$$

→ $A^{i,p}_j \frac{c_j}{\Lambda^2}$: *linear terms* or *interference terms*, from interference of SM and BSM
→ $B^{i,p}_{jk}\frac{c_j c_k}{\Lambda^4}$: *quadratic terms* or *cross terms* (when $j \neq k$)
→ Note that majority of results we report use parameterizations truncated at $\mathcal{O}(c)$

- $A^{i,p}_j$, $B^{i,p}_{jk}$ computed using `MG5_aMC@NLO`+`Pythia`+`SMEFTsim3` (`SMEFT@NLO` for gg → H, ZH)
- SMEFT modifications of masses and decay widths also taken into account
- For H → $\gamma\gamma$ decay and EWPO, analytic calculations are used

SMEFT parameterization: $t\bar{t}$ measurement



Modifications to input analyses and likelihood model

Likelihood model:

- Combined likelihood model:

$$\mathcal{L}\left(\text{data}\,;\,\vec{c},\vec{\nu}\right)=\mathcal{L}^{\text{expt}}\left(\vec{c},\vec{\nu}\right)\mathcal{L}^{\text{simpl}}\left(\vec{c}\right)$$

- «experimental likelihood»:

$$\mathcal{L}^{\text{expt}}\left(\vec{c},\vec{\nu}\right)=\prod_i \text{Poisson}\Big(n_i \,\Big|\, \sum_j \mu'^{j}(\vec{c})s_i^j(\vec{\nu})+b_i(\vec{\nu})\Big)\prod_k p_k\left(y_k \,\middle|\, \nu_k\right)$$

  → covers the $\mathrm{H}\to\gamma\gamma$, $\mathrm{W}\gamma$, WW, and $\mathrm{Z}\to\nu\nu$ measurements
  → in the $\mathrm{t\bar{t}X}$ measurement, instead of $\sum_j \mu'^{j}(\vec{c})s_i^j(\vec{\nu})$, signal yield takes the form $\sum_j s_i^j(\vec{\nu},\vec{c})$

- «simplified likelihood»:

$$\mathcal{L}^{\text{simpl}}\left(\vec{c}\right)=\frac{\exp\Big(-\frac{1}{2}\left(\vec{\mu}(\vec{c})-\hat{\vec{\mu}}\right)^T V^{-1}\left(\vec{\mu}(\vec{c})-\hat{\vec{\mu}}\right)\Big)}{\sqrt{(2\pi)^m \det(V)}}$$

  → covers the EWPO, $\mathrm{t\bar{t}}$, and inclusive jet measurements

Modifications to input analyses and likelihood model

Likelihood model:

- Combined likelihood model: $\mathcal{L}\left(\text{data}\,;\,\vec{c},\vec{\nu}\right) = \mathcal{L}^{\text{expt}}\left(\vec{c},\vec{\nu}\right)\mathcal{L}^{\text{simpl}}\left(\vec{c}\right)$
- $\mathcal{L}^{\text{expt}}\left(\vec{c},\vec{\nu}\right) = \prod_i \text{Poisson}\Big(n_i \,\Big|\, \sum_j \mu'^j(\vec{c}) s_i^j(\vec{\nu}) + b_i(\vec{\nu})\Big) \prod_k p_k\left(y_k \,\middle|\, \nu_k\right)$ (H, Wγ, WW, Z, t$\bar{\text{t}}$X)
- $\mathcal{L}^{\text{simpl}}\left(\vec{c}\right) = \exp\Big(-\frac{1}{2}\big(\vec{\mu}(\vec{c}) - \hat{\vec{\mu}}\big)^T V^{-1}\big(\vec{\mu}(\vec{c}) - \hat{\vec{\mu}}\big)\Big) \Big/ \sqrt{(2\pi)^m \det(V)}$ (t$\bar{\text{t}}$, inclusive jet, EWPO)

Modifications to input measurements:

- SM predictions for inclusive jet measurement rederived with up-to-date PDF set
- For all (CMS) measurements, derived theoretical uncertainties on SM predictions (PDF, factorization and renormalization scale uncertainties)
- PDF and luminosity uncertainties (partially) correlated between the input measurements
- SMEFT parameterizations derived up to $\mathcal{O}(c^2/\Lambda^4)$ in a consistent way for all input measurements

Combination Procedure: Principal Component Analysis

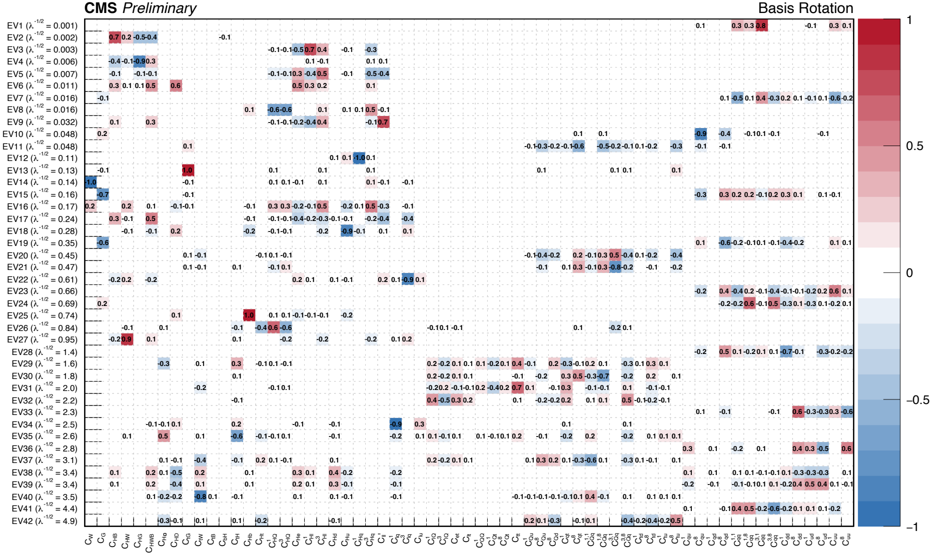
- Can not constrain all Wilson coefficients simultaneously, as some of them have nearly degenerate effects on the input measurements
- Principal component analysis (PCA) is a useful tool to identify linear combinations of WCs that can be constrained:
  - Obtain the Hessian matrix H of the combined measurement, parameterized in terms of the Wilson coefficients ($H_{jk} = \frac{\partial^2 \ln \mathcal{L}}{\partial c_j \partial c_k}$)
  - Eigendecomposition of H yields the eigenvectors x_i (which are used to write linear combinations of Wilson coefficients, «EVi») and the corresponding eigenvalues λ_i
- EV$i = \sum_k x_i^k c_k$ are uncorrelated linear combinations of Wilson coefficients

  $\rightarrow$ Half the width of the expected 68% confidence interval for EVi is equal to $1/\sqrt{\lambda_i}$

  $\rightarrow$ Linear combinations with small eigenvalue $\lambda_i \rightarrow 0$ correspond to flat directions and are fixed to their SM value of 0 (cutoff $\lambda_i < 0.04$)

Basis rotation

- Perform PCA to identify 42 linear combinations of Wilson coefficients that can be constrained simultaneously
- 22 «flat directions» $(1/\sqrt{\lambda} > 5)$ fixed to zero



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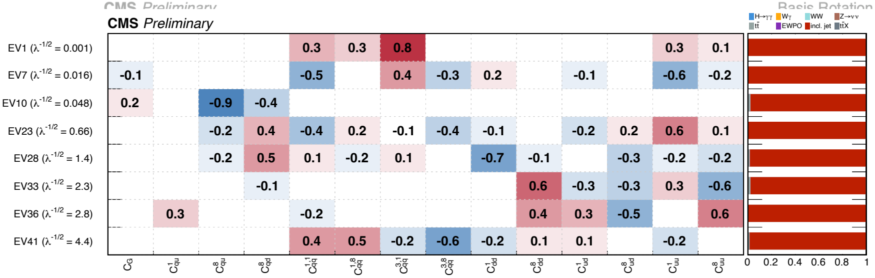
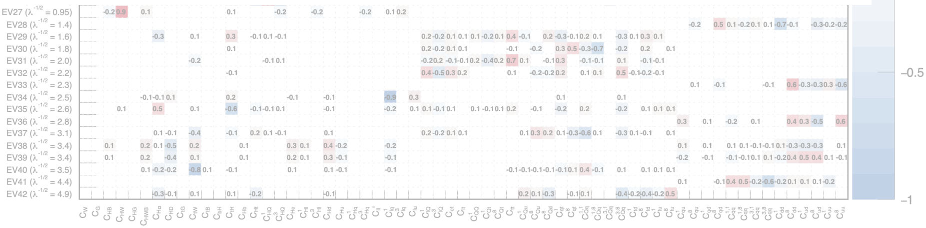


- 8 linear combinations constrained by inclusive jet measurement ($\mathcal{Q}_G$, ψ^4 (4-light-quark) operators)



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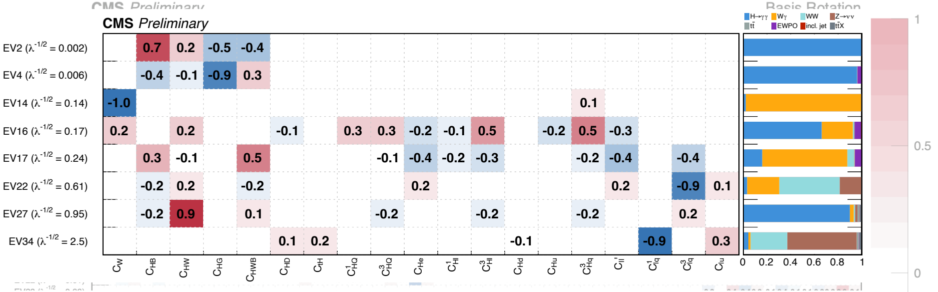
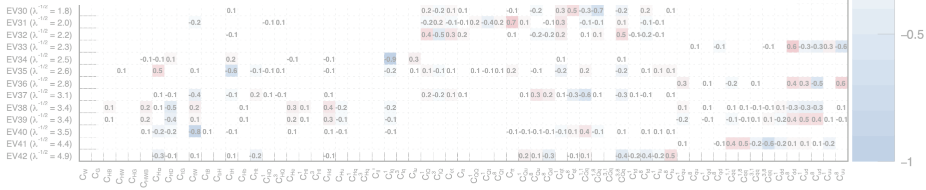


- 8 linear combinations constrained by inclusive jet measurement ($\mathcal{Q}_{\rm G}$, ψ^4 (4-light-quark) operators)
- 8 by Higgs and electroweak measurements ($\mathcal{Q}_{\rm W}$, $\mathcal{Q}_{\rm HD}$, H+V, H+ψ, ψ^4 (2ℓ or 4ℓ) operators)



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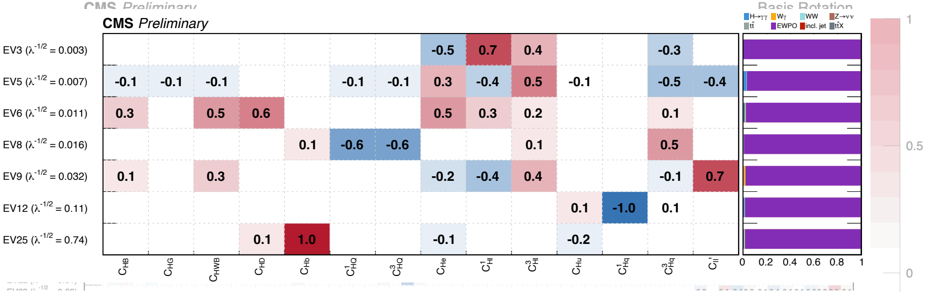

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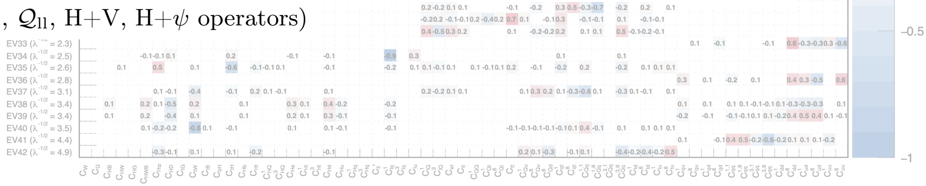
- 8 by Higgs and electroweak measurements ($\mathcal{Q}_W$, $\mathcal{Q}_{HD}$, H+V, H+ψ, ψ^4 (2ℓ or 4ℓ) operators)
- 7 by EWPO ($\mathcal{Q}_{HD}$, $\mathcal{Q}_{ll}$, H+V, H+ψ operators)



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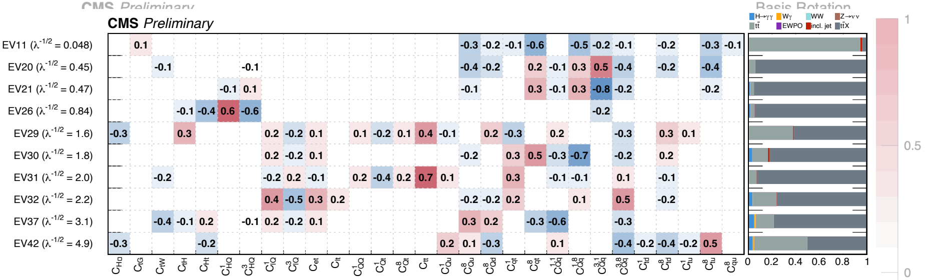

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- 7 by EWPO ($\mathcal{Q}_{\mathrm{HD}}$, $\mathcal{Q}_{\mathrm{ll}}$, H+V, H+$\psi$ operators)
- 10 by top quark measurements ($\mathcal{Q}_{\mathrm{H}\Box}$, V+$\psi$, H+$\psi$, ψ^4 (2- or 4-heavy-quark) operators)

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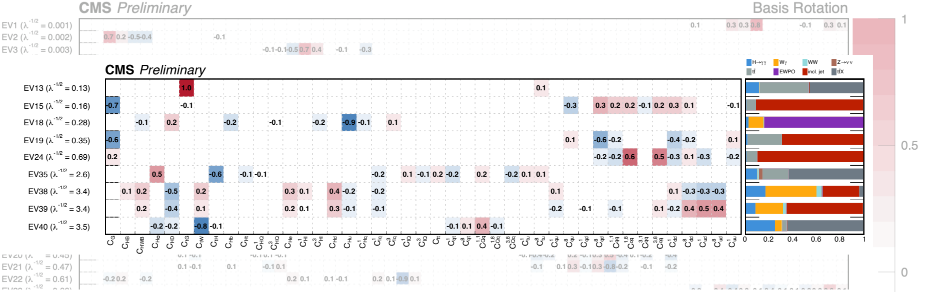

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- 7 by EWPO ($\mathcal{Q}_{\mathrm{HD}}$, $\mathcal{Q}_{\mathrm{ll}}$, H+V, H+$\psi$ operators)
- 10 by top quark measurements ($\mathcal{Q}_{\mathrm{H}\Box}$, V+$\psi$, H+$\psi$, ψ^4 (2- or 4-heavy-quark) operators)
- 9 by a mixture
  - → Higgs+Top (EV13, EV35, EV40), multi-jet+Top (EV15, EV19, EV24)
  - → CMS EWK+EWPO (EV18), Higgs+EWK+multi-jet (EV38, EV39)

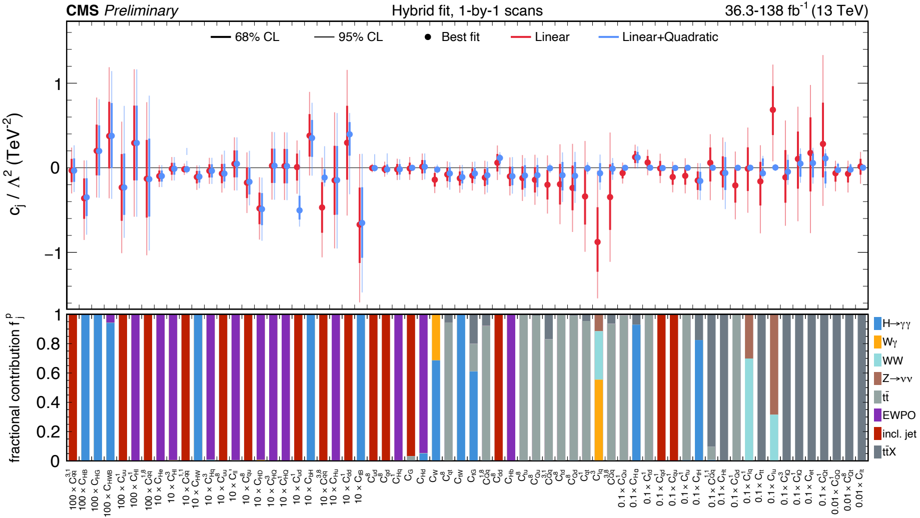
Results

Individual constraints on Wilson coefficients

- Setting constraints on 64 individual Wilson coefficients (each obtained by fixing all others to 0)
- 95% confidence intervals range from around ± 20 to ± 0.003
- Breakdown of contributions of each measurement p, calculated as

$$f_j^p = \frac{H_{jj}^p}{H_{jj}^{\text{comb.}}}$$

- Several operators constrained by multiple analyses

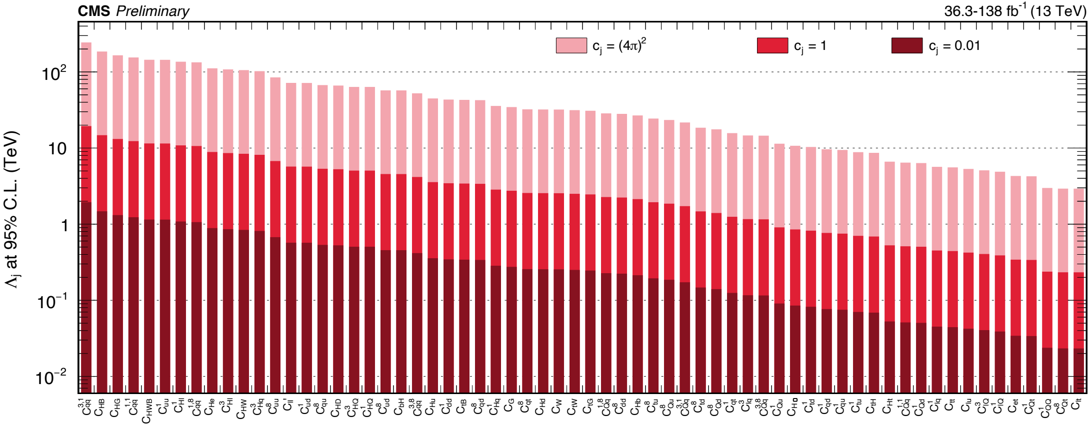


Individual constraints on Wilson coefficients

- Several Wilson coefficients receive significant constraints from multiple measurements, e.g.
  - → Through combination of **$\mathbf{t\bar{t}}$ cross section measurements and the dedicated $\mathbf{t\bar{t}X}$ EFT analysis**, we obtain stronger constraints on the **2-heavy-2-light quark operators**
  - → Combination of **$\mathbf{W\gamma}$ and $\mathbf{H \to \gamma\gamma}$** yields an improved constraint on $\boldsymbol{c_W}$ with respect to any single-analysis result (linear-only sensitivity ∼45% higher than CMS Wγ result)
  - → **2-(light)quark-2-lepton operators** constrained by combination of **EW vector boson** measurements
  - → Interplay of Higgs- and Top-sectors (c_{tG}, c_{tH}, $c_{H\Box}$)
- Comparison of results with linear-only ($\mathcal{O}(c/\Lambda^2)$) and linear+quadratic parameterization ($\mathcal{O}(c^2/\Lambda^4)$) gives an indication of how much the inclusion of orders $1/\Lambda^4$ could change the sensitivity of the results

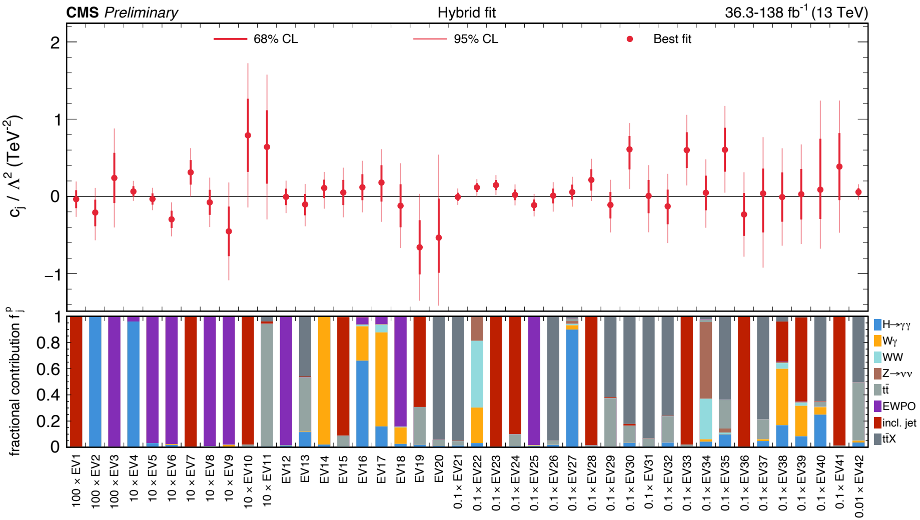
Lower limits on energy scales of new physics

- Translate constraints on c_j/Λ^2 into 95% CL lower limits on the scale of new physics Λ_j, by setting c_j to specific values (of 0.01, 1 and $(4\pi)^2$)



Constraints on linear combinations of Wilson coefficients

- Setting constraints on 42 linear combinations of Wilson coefficients (each varied simultanoeusly)
- Majority of POI receives significant contributions from multiple channels
- 8 linear combinations constrained by inclusive jet measurement
- 8 by Higgs and electroweak measurements
- 7 by EWPO
- 10 by top quark measurements
- 9 by a mixture



Constraints on linear combinations of Wilson coefficients

- Setting constraints on 42 linear combinations of Wilson coefficients (varied simultanoeusly)
- 95% confidence intervals range from around ± 10 to ± 0.002
- Majority of POI receives significant contributions from multiple channels
  - $\rightarrow$ 8 linear combinations constrained by inclusive jet measurement
  - $\rightarrow$ 8 by Higgs and electroweak measurements
  - $\rightarrow$ 7 by EWPO
  - $\rightarrow$ 10 by top quark measurements
  - $\rightarrow$ 9 by a mixture
- The p-value for the compatibility with the SM is 1.7%
  - $\rightarrow$ Deviation from SM is mostly driven by inclusive jet measurement
  - $\rightarrow$ When excluding it from the combination, the p-value is found to be 26%

Summary

- Indirect searches for evidence of BSM physics via deviations in known SM processes are an important complementary strategy to direct searches
  - $\rightarrow$ SMEFT provides a framework for such indirect searches
  - $\rightarrow$ We present the first combined SMEFT interpretation of CMS measurements from four sectors of the SM (Higgs, Top, Electroweak, QCD)
- Technical and conceptual developments enable us to combine and interpret different types of measurements (differential cross sections and direct EFT measurements) in a consistent way
- Through a combined interpretation of seven sets of CMS measurements and electroweak precision observables from LEP & SLC, we constrain 64 Wilson coefficients individually and 42 linear combinations of Wilson coefficients simultaneously
- The p-value for the compatibility with the SM (all $c_i = 0$) is 1.7% (deviation mostly driven by inclusive jet)

BACKUP

$\mathrm{H} \to \gamma\gamma$ and $\mathrm{W}\gamma$ measurements

HIG-19-015: Simplified template cross sections (STXS) stage 1.2, ggH, VBF, VH, ttH, and tH production modes, $\gamma\gamma$ decay channel

- Full Run 2 data, 138 fb^{-1}
- Experimental likelihood model available ✓
- Theoretical uncertainties on SM predictions taken from model in original analysis
- SMEFT parameterization taken from the Higgs combination (common parameterization of CMS/ATLAS/theorists)
  - $\to$ SMEFTsim3 (SMEFT@NLO for loop-induced processes gg $\to$ H and gg $\to$ ZH)
  - $\to$ analytic calculation for $\mathrm{H} \to \gamma\gamma$ decay (from arXiv:1807.11504)

SMP-20-005: Double differential cross section measurements of $\mathrm{W}\gamma$ production ($p_{\mathrm{T}} \times |\phi_f|$), $\ell\nu$ decay channel

- Full Run 2 data, 138 fb^{-1}
- Experimental likelihood model available ✓
- Theoretical uncertainties on SM predictions taken from model in original analysis
- SMEFT parameterization: SMEFTsim3

WW, Z $\to \nu\nu$, and inclusive jet measurements

SMP-18-004: Differential cross section measurements of WW production ($m_{\ell\ell}$), $\ell\ell\nu\nu$ decay channel

- 2016 data, 36.3 fb^{-1}
- Experimental likelihood model available ✓
- Theoretical uncertainties on SM predictions rederived
- SMEFT parameterization: SMEFTsim3

SMP-18-003: Differential cross section measurements of Z production (p_{T}^{Z}), $\nu\nu$ decay channel

- 2016 data, 36.3 fb^{-1}
- Experimental likelihood model available ✓
- Theoretical uncertainties on SM predictions rederived
- SMEFT parameterization: SMEFTsim3

SMP-20-011: Double-differential inclusive jet cross section measurements ($p_{\mathrm{T}}^{\mathrm{jet}} \times |y^{\mathrm{jet}}|$)

- 2016 data, 36.3 fb^{-1}
- Experimental likelihood model not available ×
- Nominal SM predictions and theoretical uncertainties rederived
- SMEFT parameterization: SMEFTsim3

$t\bar{t}$ and $t\bar{t}X$ measurements

TOP-20-001: Differential cross section measurements of $t\bar{t}$ production ($M_{t\bar{t}}$), single lepton channel

- Full Run 2 data, 138 fb^{-1}
- Experimental likelihood model not available ×
- Theoretical uncertainties on SM predictions rederived
- SMEFT parameterization: SMEFTsim3

TOP-22-006: Search for BSM physics in top quark production with additional leptons

- Full Run 2 data, 138 fb^{-1}
- Experimental likelihood model available ✓
- Theoretical uncertainties on SM predictions taken from model in original analysis
- SMEFT parameterization translated from `dim6top` to SMEFT basis and more operators added

Electroweak precision observables from LEP & SLC

The measurement of eight pseudo-observables measured at LEP and SLC [35, 36] is incorporated in the interpretation presented here. These observables are the Z boson total width, Γ_Z; the hadronic pole cross section, σ^0_{had}, defined as

$$\sigma^0_{\text{had}} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{\text{ee}}\Gamma_{\text{had}}}{\Gamma_Z^2}, \tag{2}$$

where Γ_{ee} is the partial decay width of the Z boson to an electron-positron pair and $\Gamma_{\text{had}} = \Gamma_{\text{uu}} + \Gamma_{\text{dd}} + \Gamma_{\text{cc}} + \Gamma_{\text{ss}} + \Gamma_{\text{bb}}$ is the hadronic Z boson decay width; three ratios of partial decay widths, R_ℓ, R_c, R_b, defined as

$$R_\ell = \frac{\Gamma_{\text{had}}}{\Gamma_{\ell\ell}}, \quad R_{c,b} = \frac{\Gamma_{\text{cc,bb}}}{\Gamma_{\text{had}}}; \tag{3}$$

and three forward-backward asymmetries, $A^{0,\ell}_{FB}$, $A^{0,\text{c}}_{FB}$, $A^{0,\text{b}}_{FB}$, defined as

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B}. \tag{4}$$

In this expression, N_F (N_B) is the number of events where the charged lepton, c quark, or b quark is produced in the direction of the electron beam (anti-electron beam). SMEFT corrections on these pseudo-observables were evaluated analytically in Ref. [36].

SMEFT parameterization (details on event generation)

$$\sigma^i_{p,\mathrm{SMEFT}} = \sigma^i_{p,\mathrm{SM}} \left(1 + \sum_j A^i_{p,j} \frac{c_j}{\Lambda^2} + \sum_{j,k} B^i_{p,jk} \frac{c_j c_k}{\Lambda^4} \right)$$

$\rightarrow$ $A^i_{p,j} \frac{c_j}{\Lambda^2}$: *linear terms* or *interference terms*, from interference of SM and BSM
$\rightarrow$ $B^i_{p,jk} \frac{c_j c_k}{\Lambda^4}$: *quadratic terms* or *cross terms* (when $j \neq k$)

- Constants $A^i_{p,j}$, $B^i_{p,jk}$ are computed by generating events at LO with `MADGRAPH5_AMC@NLO` + `PYTHIA 8.3`. SMEFT effects are modelled using `SMEFTsim3` (and `SMEFT@NLO` for loop-induced processes gg $\rightarrow$ H and gg $\rightarrow$ ZH)
- Phase space selections for each kinematic bin are reproduced using `RIVET 3.1.9`
- NNPDF 3.1 is used when computing the parameterizations
- All parameterizations use $\{m_W, m_Z, G_\mathrm{F}\}$ input scheme and `topU3l` flavour symmetry

SMEFT operators (1/2)

X^3		
$\mathcal{Q}_{\mathrm{G}} = f^{abc} G_\mu^{a\nu} G_\nu^{b\rho} G_\rho^{c\mu}$	$\mathcal{Q}_{\mathrm{W}} = \varepsilon^{ijk} W_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}$	
$H^4 D^2$		
$\mathcal{Q}_{\mathrm{H}\Box} = (H^\dagger H)\Box(H^\dagger H)$	$\mathcal{Q}_{\mathrm{HD}} = (D^\mu H^\dagger H)(H^\dagger D_\mu H)$	
$X^2 H^2$		
$\mathcal{Q}_{\mathrm{HG}} = H^\dagger H G^a_{\mu\nu} G^{a\mu\nu}$ $\mathcal{Q}_{\mathrm{HWB}} = H^\dagger H W^i_{\mu\nu} B^{\mu\nu}$	$\mathcal{Q}_{\mathrm{HW}} = H^\dagger H W^i_{\mu\nu} W^{i\mu\nu}$	$\mathcal{Q}_{\mathrm{HB}} = H^\dagger H B_{\mu\nu} B^{\mu\nu}$
$\psi^2 H^3$		
$\mathcal{Q}_{\mathrm{tH}} = (H^\dagger H)(\bar{Q}\tilde{H} t)$	$\mathcal{Q}_{\mathrm{bH}} = (H^\dagger H)(\bar{Q} H b)$	
$\psi^2 X H$		
$\mathcal{Q}_{\mathrm{tW}} = (\bar{Q}\sigma^{\mu\nu} t)\sigma^i \tilde{H} W^i_{\mu\nu}$	$\mathcal{Q}_{\mathrm{tB}} = (\bar{Q}\sigma^{\mu\nu} t)\tilde{H} B_{\mu\nu}$	$\mathcal{Q}_{\mathrm{tG}} = (\bar{Q}\sigma^{\mu\nu} T^a t)\tilde{H} G^a_{\mu\nu}$
$\psi^2 H^2 D$		
$\mathcal{Q}^{(1)}_{\mathrm{Hl}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$ $\mathcal{Q}^{(1)}_{\mathrm{Hq}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$ $\mathcal{Q}_{\mathrm{Hd}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$ $\mathcal{Q}_{\mathrm{Ht}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{t}\gamma^\mu t)$	$\mathcal{Q}^{(3)}_{\mathrm{Hl}} = (H^\dagger i\overleftrightarrow{D}^i_\mu H)(\bar{l}_p \sigma^i \gamma^\mu l_r)$ $\mathcal{Q}^{(3)}_{\mathrm{Hq}} = (H^\dagger i\overleftrightarrow{D}^i_\mu H)(\bar{q}\sigma^i\gamma^\mu q)$ $\mathcal{Q}^{(1)}_{\mathrm{HQ}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{Q}\gamma^\mu Q)$ $\mathcal{Q}_{\mathrm{Hb}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{b}\gamma^\mu b)$	$\mathcal{Q}_{\mathrm{He}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$ $\mathcal{Q}_{\mathrm{Hu}} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$ $\mathcal{Q}^{(3)}_{\mathrm{HQ}} = (H^\dagger i\overleftrightarrow{D}^i_\mu H)(\bar{Q}\sigma^i\gamma^\mu Q)$

SMEFT operators (2/2)

$\psi^4, (\bar{L}L)(\bar{L}L)$

$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p\gamma_\mu l_r)(\bar{q}\gamma^\mu q)$	$\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p\sigma^i\gamma_\mu l_r)(\bar{q}\sigma^i\gamma^\mu q)$	$\mathcal{Q}_{lQ}^{(1)} = (\bar{l}_p\gamma_\mu l_r)(\bar{Q}\gamma^\mu Q)$
$\mathcal{Q}_{lQ}^{(3)} = (\bar{l}_p\sigma^i\gamma_\mu l_r)(\bar{Q}\sigma^i\gamma^\mu Q)$	$\mathcal{Q}_{QQ}^{(1)} = (\bar{Q}\gamma_\mu Q)(\bar{Q}\gamma^\mu Q)$	$\mathcal{Q}_{ll} = (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$\mathcal{Q}_{qq}^{(1,1)} = (\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$	$\mathcal{Q}_{qq}^{(1,8)} = (\bar{q}T^a\gamma_\mu q)(\bar{q}T^a\gamma^\mu q)$	$\mathcal{Q}_{qq}^{(3,1)} = (\bar{q}\sigma^i\gamma_\mu q)(\bar{q}\sigma^i\gamma^\mu q)$
$\mathcal{Q}_{qq}^{(3,8)} = (\bar{q}\sigma^i T^a\gamma_\mu q)(\bar{q}\sigma^i T^a\gamma^\mu q)$	$\mathcal{Q}_{Qq}^{(1,1)} = (\bar{Q}\gamma_\mu Q)(\bar{q}\gamma^\mu q)$	$\mathcal{Q}_{Qq}^{(1,8)} = (\bar{Q}T^a\gamma_\mu Q)(\bar{q}T^a\gamma^\mu q)$
$\mathcal{Q}_{Qq}^{(3,1)} = (\bar{Q}\sigma^i\gamma_\mu Q)(\bar{q}\sigma^i\gamma^\mu q)$	$\mathcal{Q}_{Qq}^{(3,8)} = (\bar{Q}\sigma^i T^a\gamma_\mu Q)(\bar{q}\sigma^i T^a\gamma^\mu q)$	

$\psi^4, (\bar{R}R)(\bar{R}R)$

$\mathcal{Q}_{et} = (\bar{e}_p\gamma_\mu e_r)(\bar{t}\gamma^\mu t)$	$\mathcal{Q}_{tt} = (\bar{t}\gamma_\mu t)(\bar{t}\gamma^\mu t)$	$\mathcal{Q}_{uu}^{(1)} = (\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$
$\mathcal{Q}_{uu}^{(8)} = (\bar{u}T^a\gamma_\mu u)(\bar{u}T^a\gamma^\mu u)$	$\mathcal{Q}_{tu}^{(1)} = (\bar{t}\gamma_\mu t)(\bar{u}\gamma^\mu u)$	$\mathcal{Q}_{tu}^{(8)} = (\bar{t}T^a\gamma_\mu t)(\bar{u}T^a\gamma^\mu u)$
$\mathcal{Q}_{dd}^{(1)} = (\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$	$\mathcal{Q}_{dd}^{(8)} = (\bar{d}T^a\gamma_\mu d)(\bar{d}T^a\gamma^\mu d)$	$\mathcal{Q}_{ud}^{(1)} = (\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$
$\mathcal{Q}_{ud}^{(8)} = (\bar{u}T^a\gamma_\mu u)(\bar{d}T^a\gamma^\mu d)$	$\mathcal{Q}_{td}^{(1)} = (\bar{t}\gamma_\mu t)(\bar{d}\gamma^\mu d)$	$\mathcal{Q}_{td}^{(8)} = (\bar{t}T^a\gamma_\mu t)(\bar{d}T^a\gamma^\mu d)$

$\psi^4, (\bar{L}L)(\bar{R}R)$

$\mathcal{Q}_{lu} = (\bar{l}_p\gamma_\mu l_r)(\bar{u}\gamma^\mu u)$	$\mathcal{Q}_{lt} = (\bar{l}_p\gamma_\mu l_r)(\bar{t}\gamma^\mu t)$	$\mathcal{Q}_{qu}^{(1)} = (\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u)$
$\mathcal{Q}_{qu}^{(8)} = (\bar{q}T^a\gamma_\mu q)(\bar{u}T^a\gamma^\mu u)$	$\mathcal{Q}_{Qu}^{(1)} = (\bar{Q}\gamma_\mu Q)(\bar{u}\gamma^\mu u)$	$\mathcal{Q}_{Qu}^{(8)} = (\bar{Q}T^a\gamma_\mu Q)(\bar{u}T^a\gamma^\mu u)$
$\mathcal{Q}_{qt}^{(1)} = (\bar{q}\gamma_\mu q)(\bar{t}\gamma^\mu t)$	$\mathcal{Q}_{qt}^{(8)} = (\bar{q}T^a\gamma_\mu q)(\bar{t}T^a\gamma^\mu t)$	$\mathcal{Q}_{Qt}^{(1)} = (\bar{Q}\gamma_\mu Q)(\bar{t}\gamma^\mu t)$
$\mathcal{Q}_{Qt}^{(8)} = (\bar{Q}T^a\gamma_\mu Q)(\bar{t}T^a\gamma^\mu t)$	$\mathcal{Q}_{qd}^{(1)} = (\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d)$	$\mathcal{Q}_{qd}^{(8)} = (\bar{q}T^a\gamma_\mu q)(\bar{d}T^a\gamma^\mu d)$
$\mathcal{Q}_{Qd}^{(1)} = (\bar{Q}\gamma_\mu Q)(\bar{d}\gamma^\mu d)$	$\mathcal{Q}_{Qd}^{(8)} = (\bar{Q}T^a\gamma_\mu Q)(\bar{d}T^a\gamma^\mu d)$	