(Basic) Introduction to toponium physics

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Thanks for the input to:

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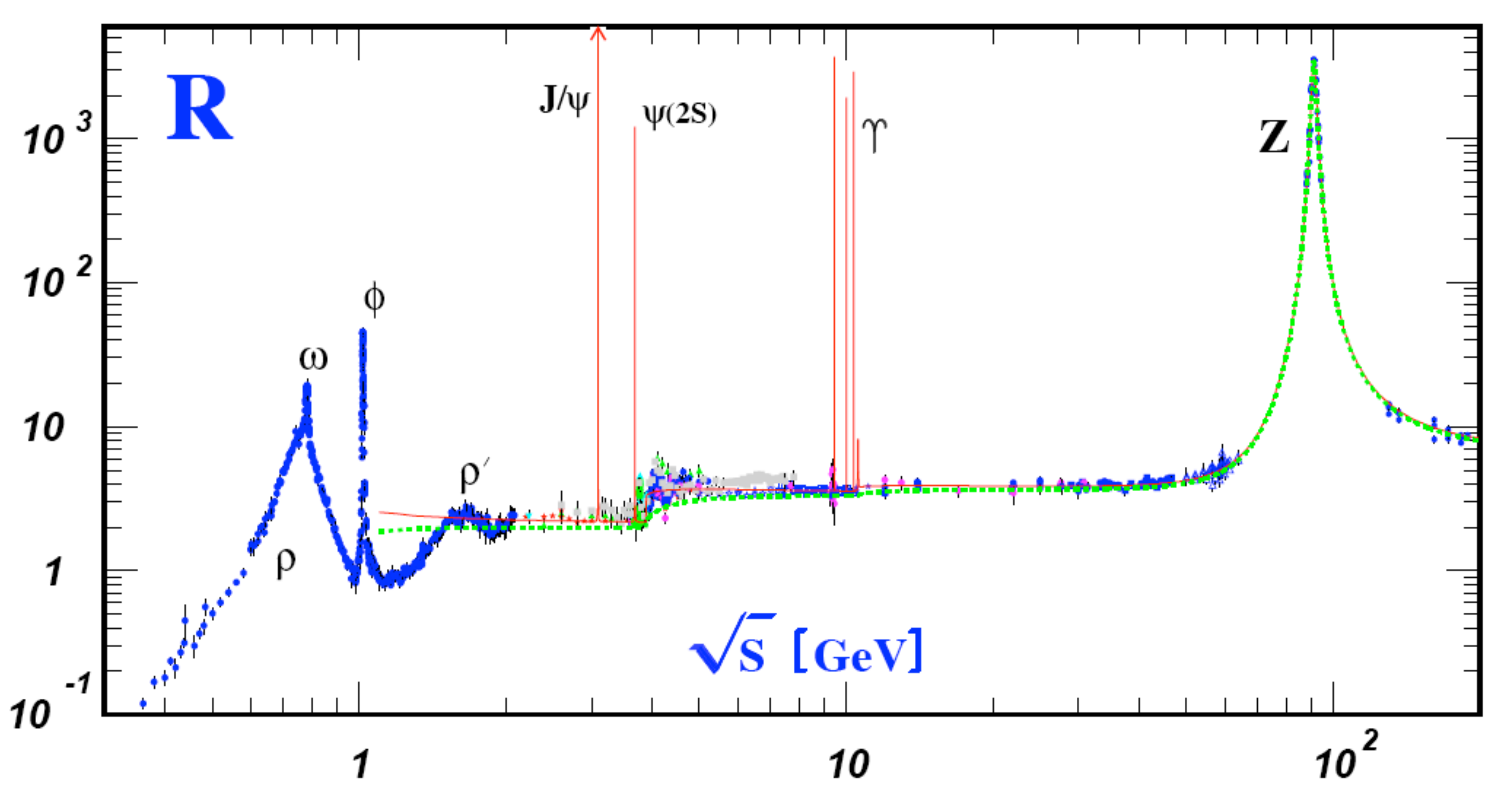
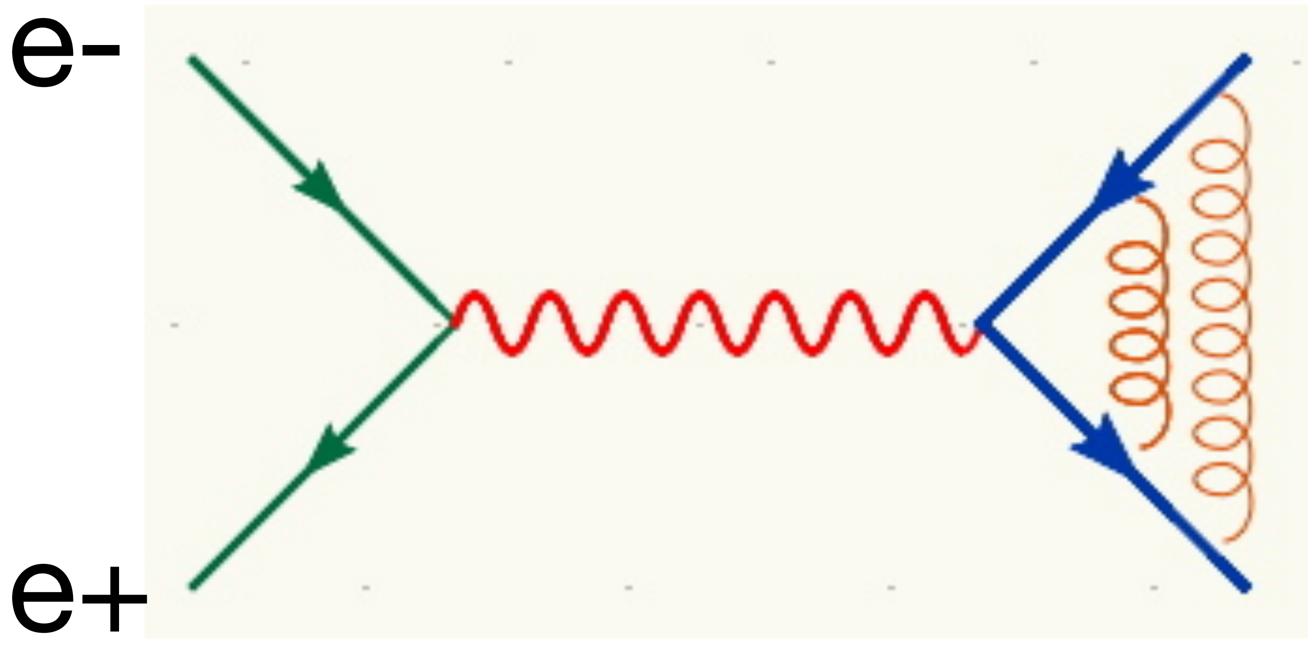
Heavy-quark bound states

Charmonium and Bottomonium

Consider how the charm and the bottom quarks were discovered:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$





$$^{2S+1}L_J^{[C]} = \, ^3S_1^{[1]}$$

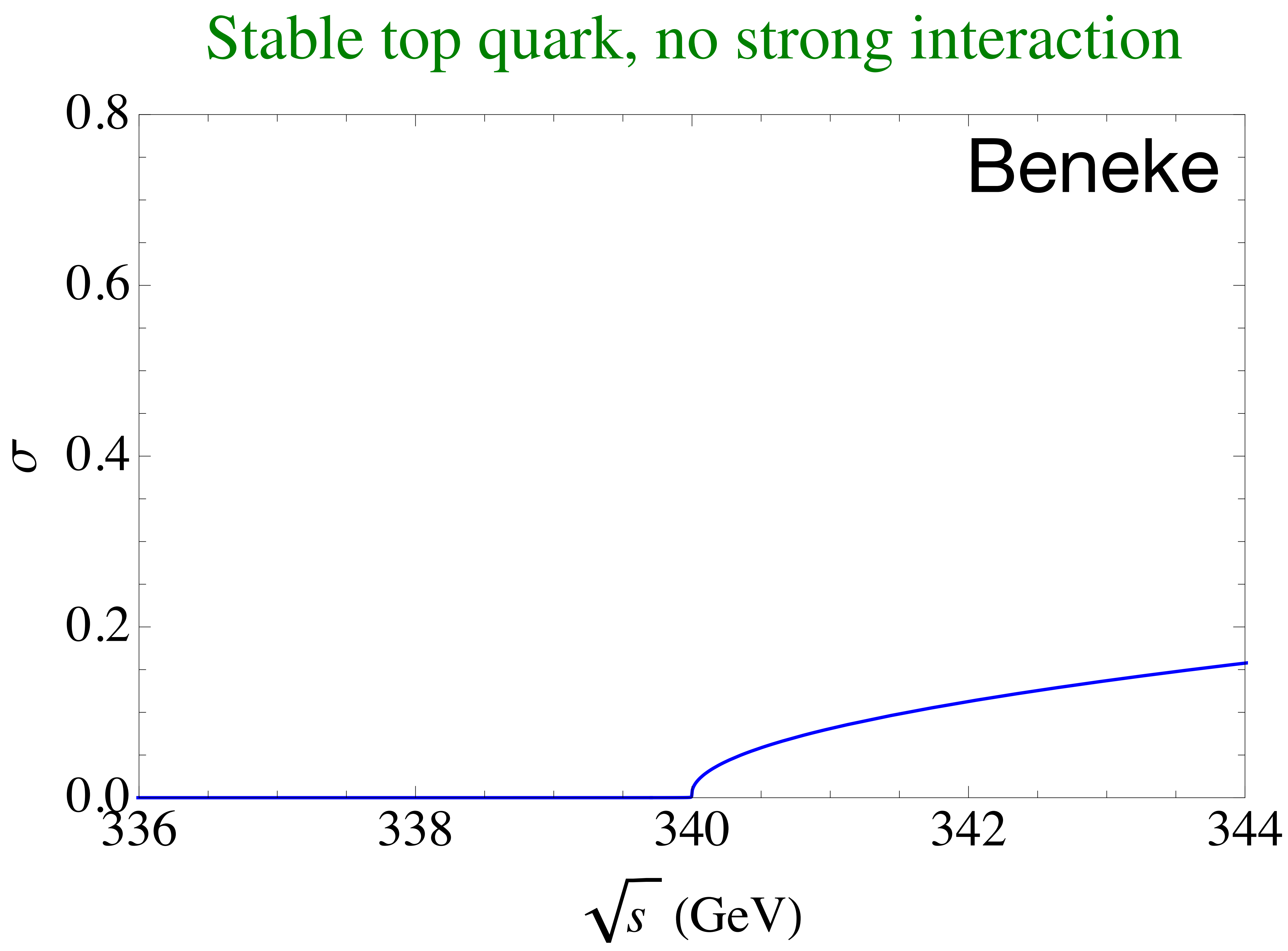
Very sharp peaks => small widths (~ 100 KeV) compared to hadronic resonances (100 MeV) => very long lived states.
QCD is "weak" at scales >> ΛQCD (asymptotic freedom), non-relativistic bound states are formed like positronium!

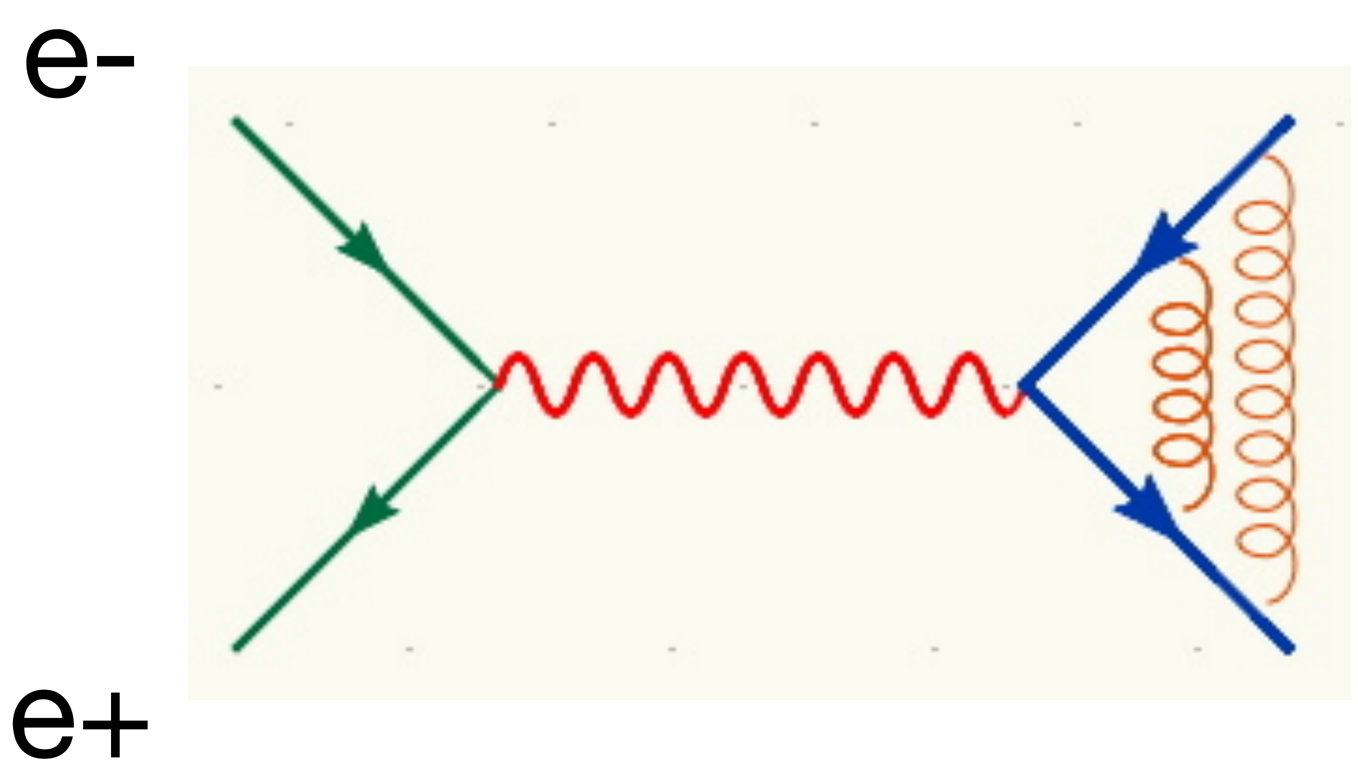
The QCD potential is like

$$V(r) = -C_F \frac{\alpha_S(1/r)}{r} + \sigma r \qquad C_F = 4/3 \quad \sigma = 0.18\,\mathrm{GeV}^2$$

Heavy-quark bound states

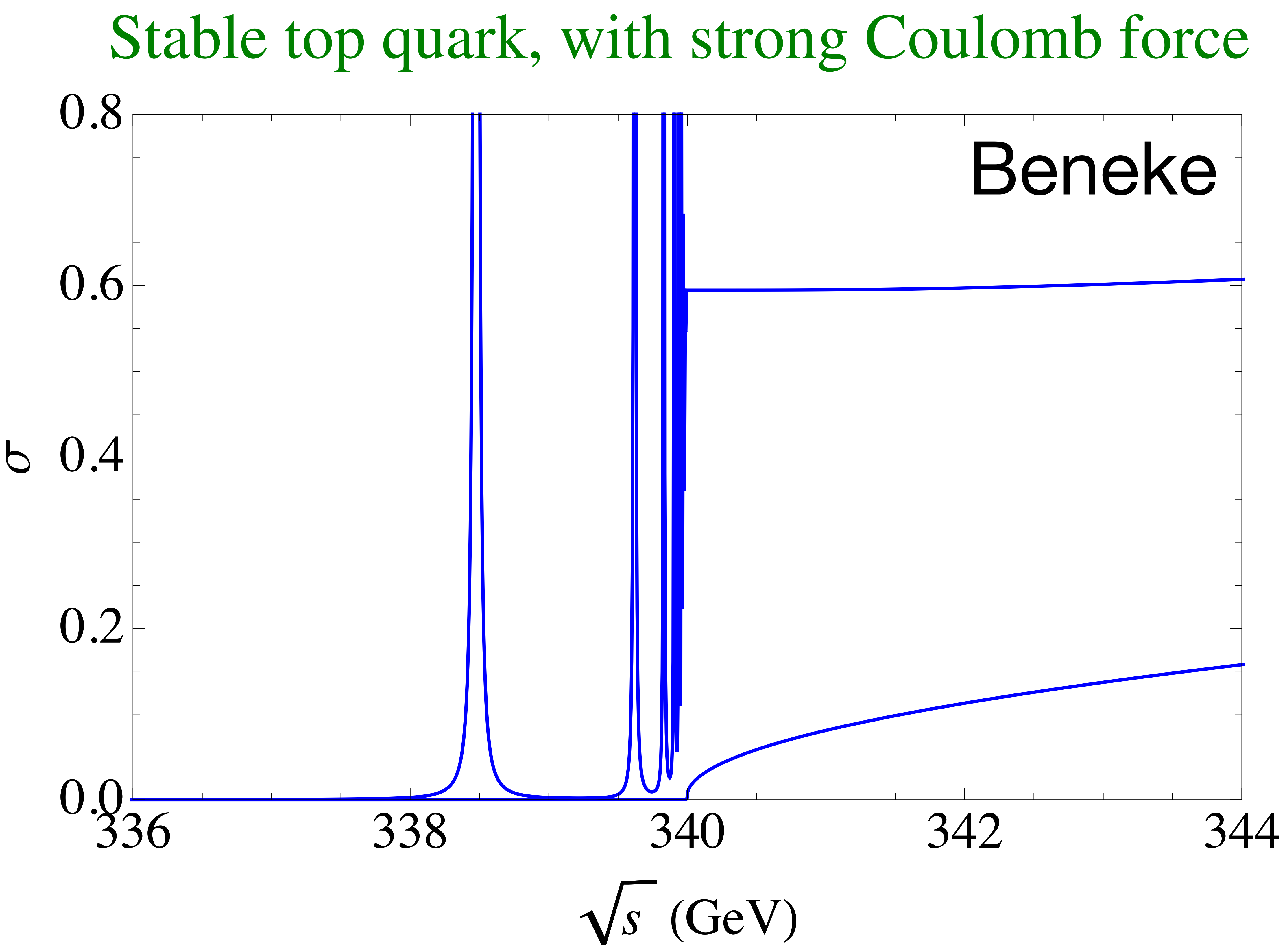
Toponium

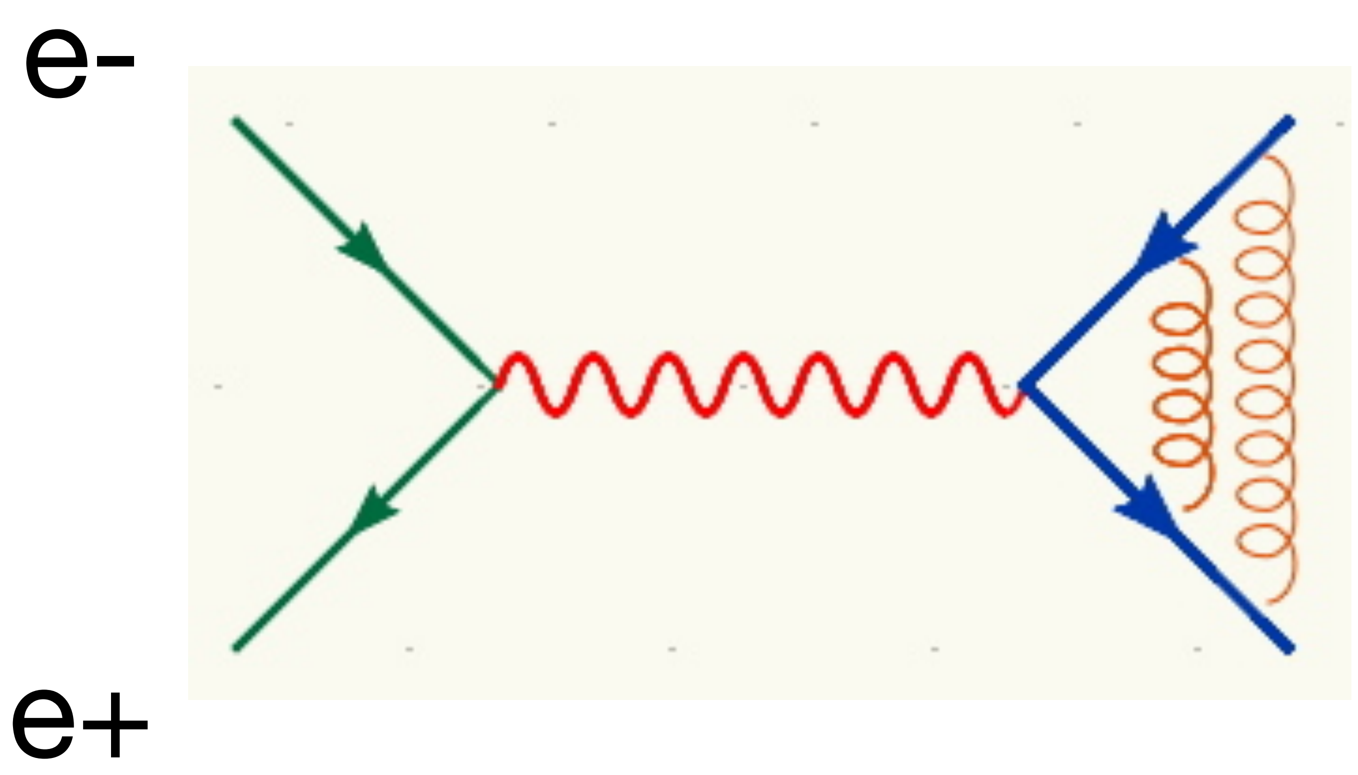




Heavy-quark bound states

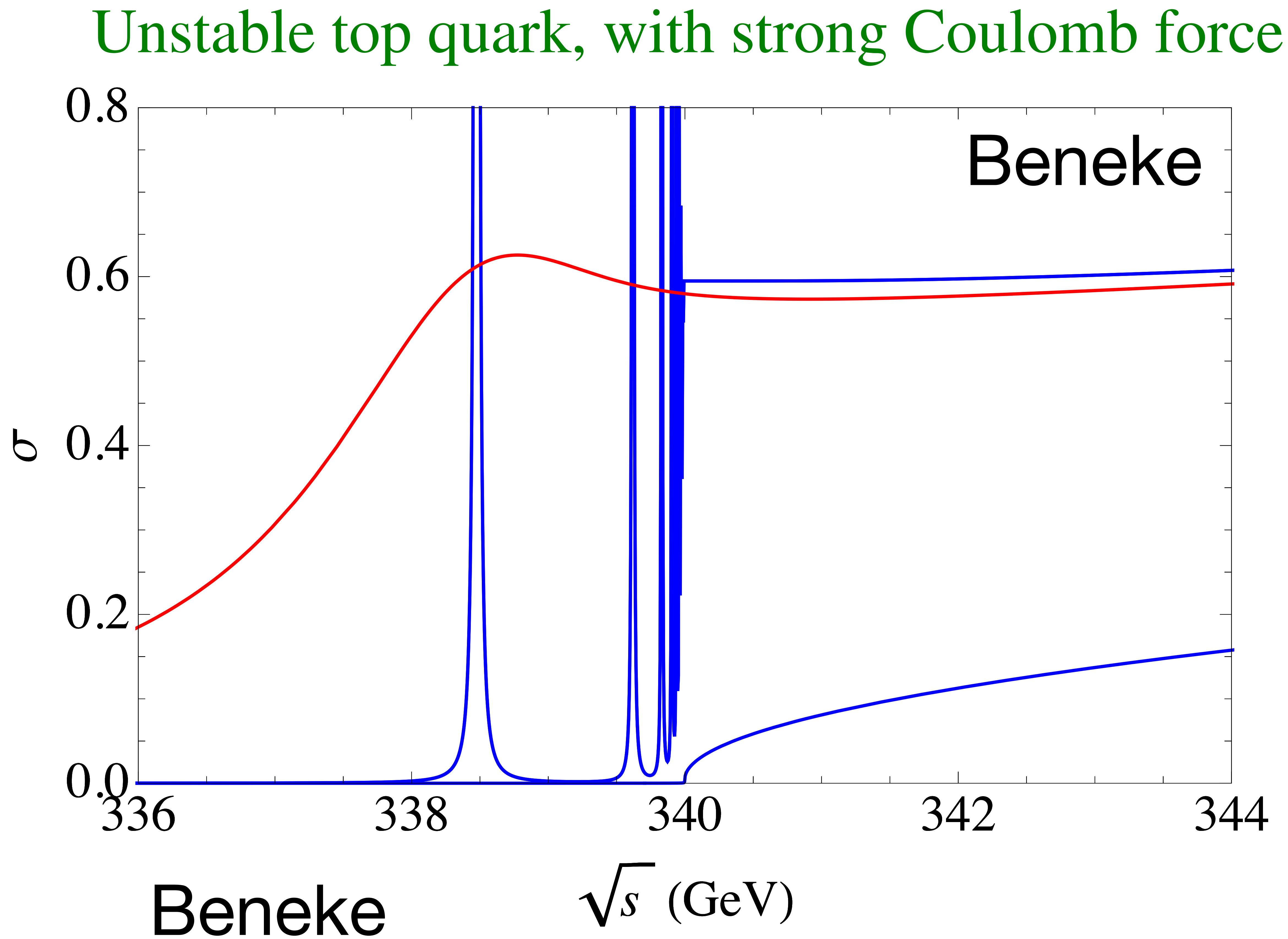
Toponium





Heavy-quark bound states

Toponium



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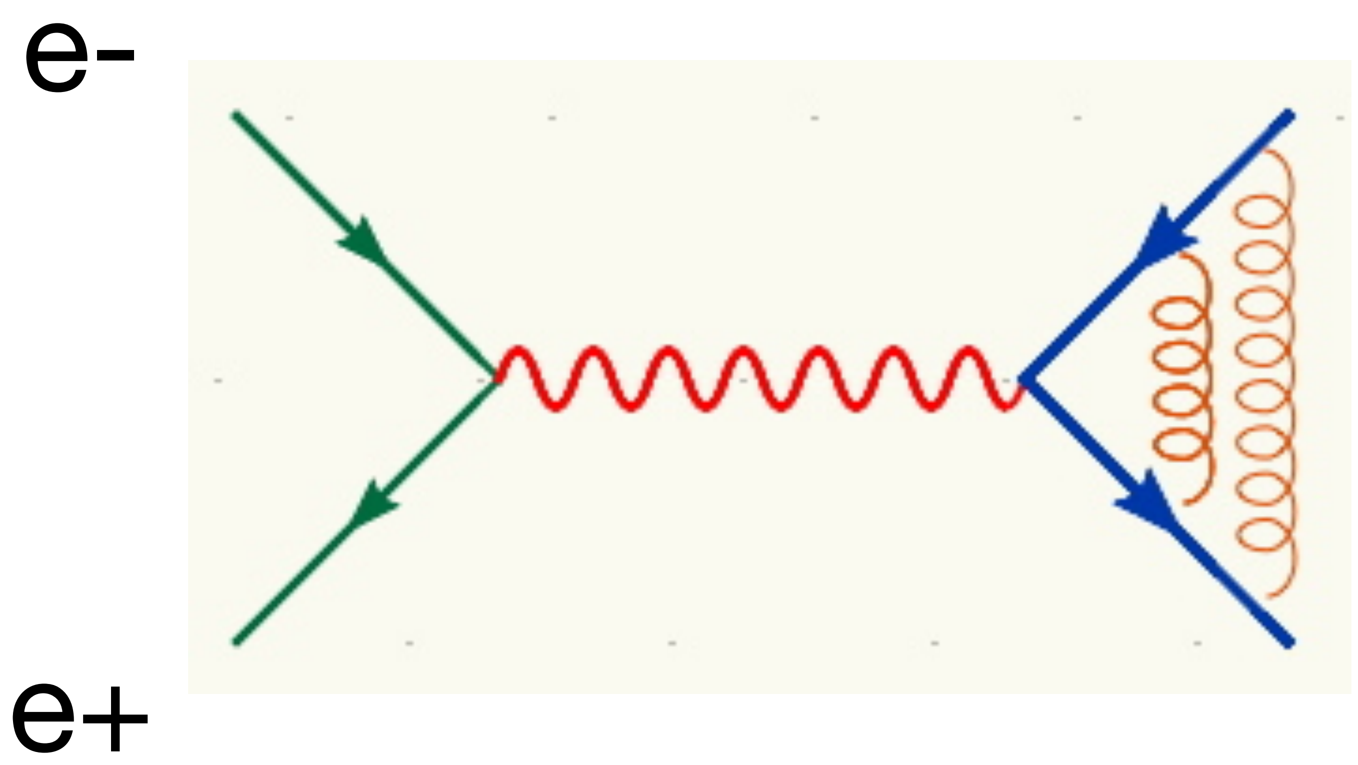
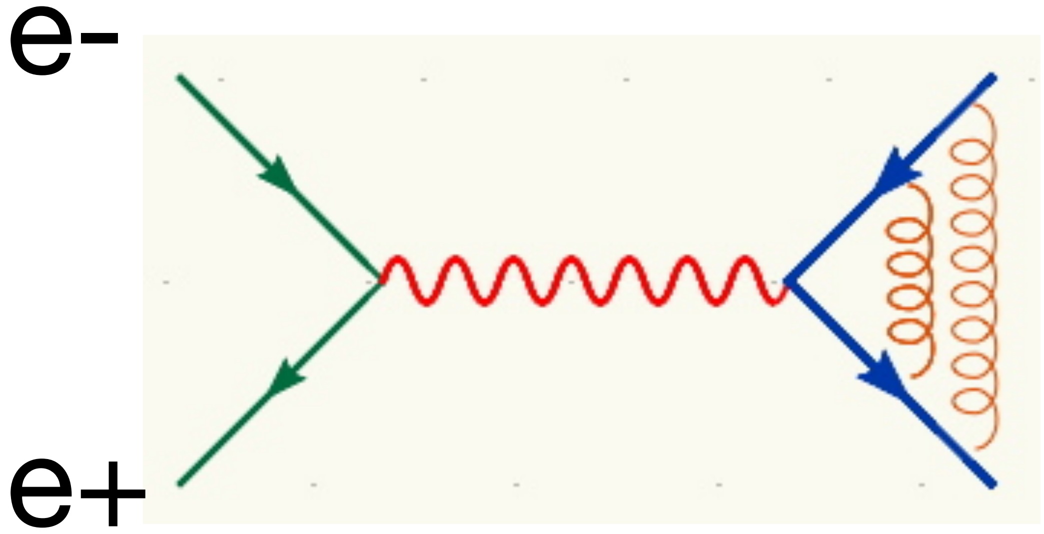
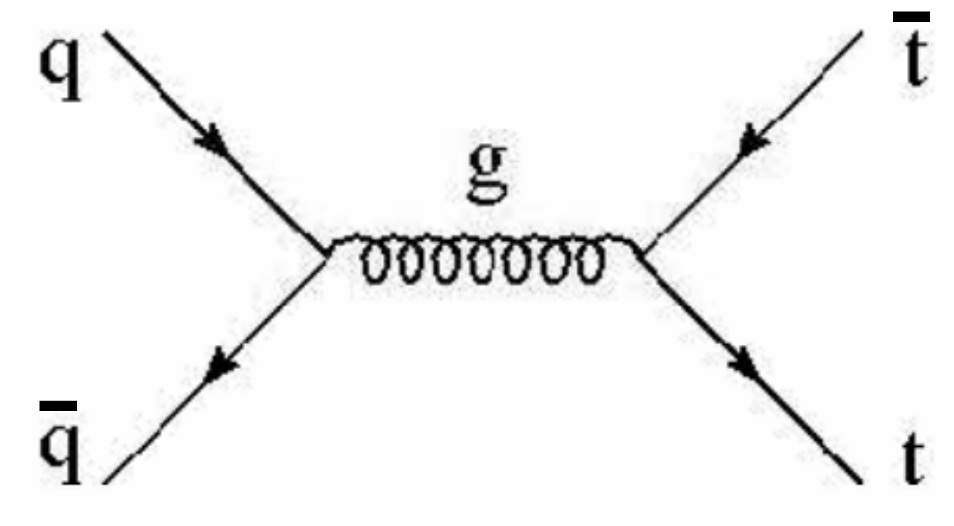
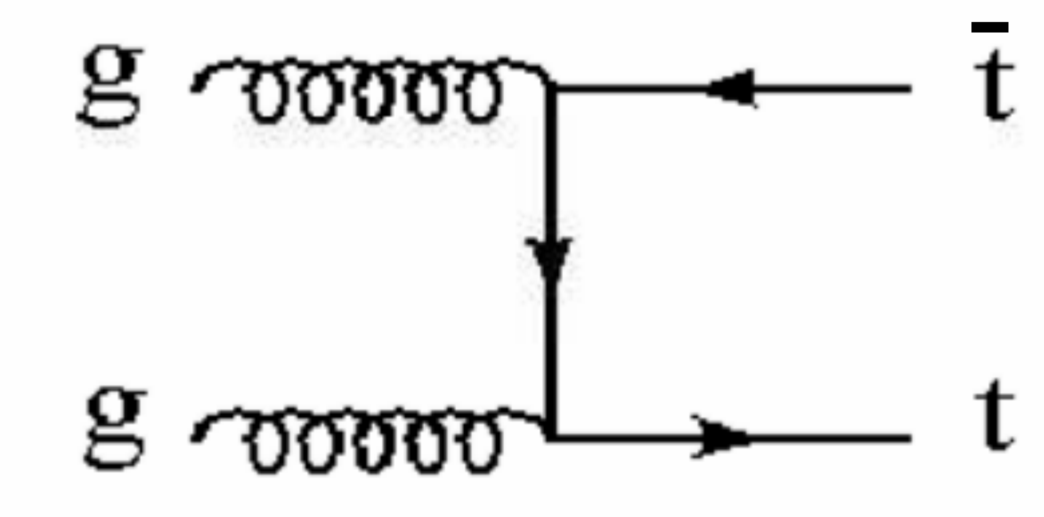


Heavy-quark bound states

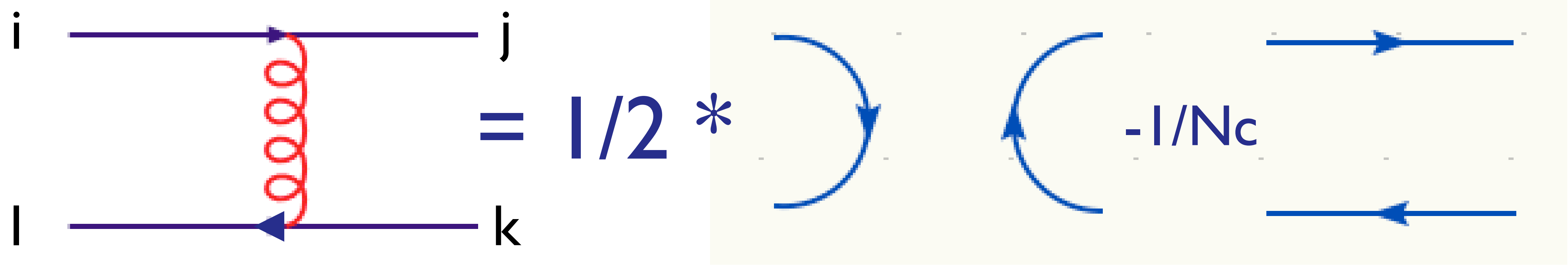
Production at colliders

	3S1[1]	100% SINGLET, attractive
	3S1[8]	100% OCTET, repulsive
	1S0[1] 1S0[8]	10% SINGLET, attractive 90% OCTET, repulsive

Heavy-quark bound states

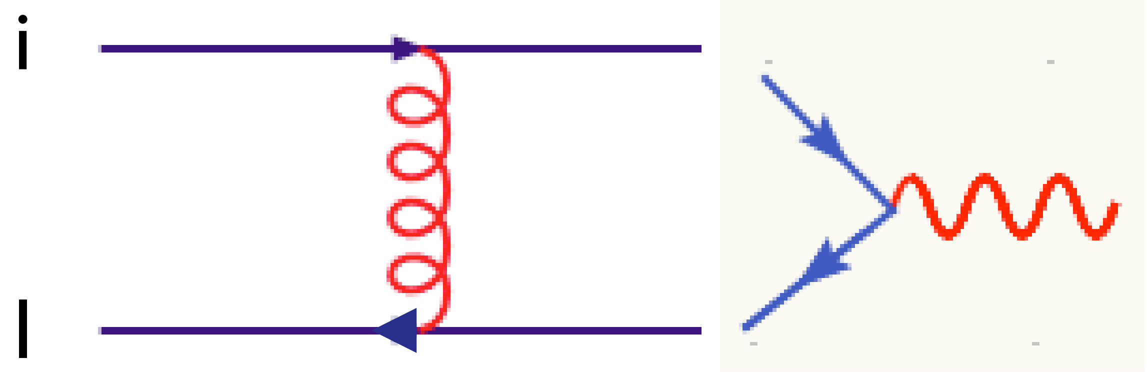
Singlets versus octets

$$t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$



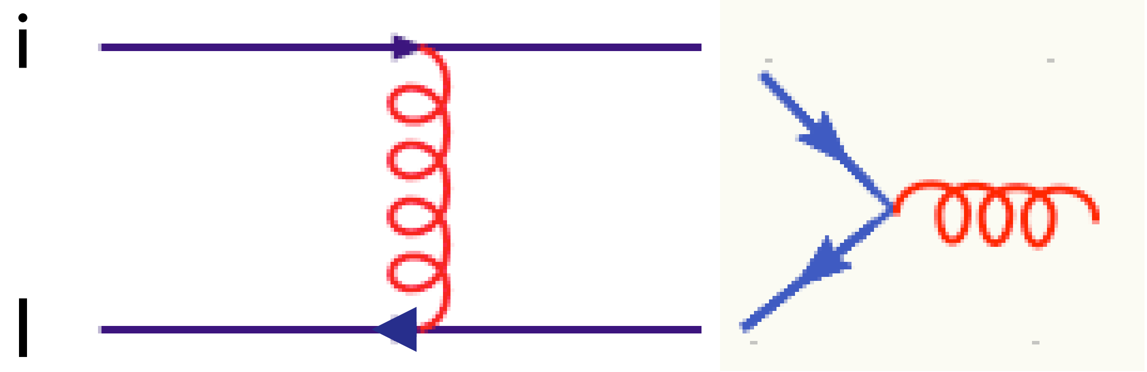
Problem: Show that the one-gluon exchange between quark-antiquark pair can be attractive or repulsive. Calculate the relative strength.

Solution: a q qb pair can be in a singlet state (photon) or in octet (gluon) : $\bar{3} \otimes 3 = 1 \oplus 8$



$$\frac{1}{2}\left(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk}\right)\delta_{ki} = \frac{1}{2}\delta_{lj}\left(N_c - \frac{1}{N_c}\right) = C_F\delta_{lj}$$

SINGLET:
>0, attractive



$$\frac{1}{2}\left(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk}\right)t^a_{ki} = -\frac{1}{2N_c}t^a_{lj}$$

OCTET:
<0, repulsive

Heavy-quark bound states

Toponium

For very small distances, the interaction of a single gluon can be approximated by a Coulomb interaction. Let analyse the scales which characterise the lower state of a Coulombic potential.

$$V(r) \simeq -C_F \frac{\alpha_S(1/r)}{r}$$

The scales can be found using the energy of the ground state and the Bohr radius: $m_R = m_t/2$ $\psi(x) = \frac{1}{\sqrt{\pi R_0^3}} e^{-r/R_0}$

$$E_0 = -\frac{1}{2}\frac{m_t}{2}(C_F \alpha_S)^2 \qquad R_0 = 1/(C_F \alpha_S m_t/2)$$

Using the Virial theorem: $\langle T \rangle = -\frac{1}{2}\langle V \rangle$ one gets the self consistent relation

$$v_0 \simeq C_F \alpha_S(m_R v_0) = C_F \alpha_S(1/R_0) = 0.21$$

Valid for the ground state only

The equation can be solved iteratively and gives scales that are all perturbative and well separated.

What about toponium?

Scales

		v=0.0073	v=0.21
Scale	Quantity	e+e-	toponium
m_R	annihilation time	0.5/2 MeV	172/2 GeV
$m_R v_0$	size p~1/R	1.9 KeV	18 GeV
$m_R v_0^2$	Formation time	14 eV	4 GeV

“Unfortunately” the formation time for the bound state is longer than the toponium lifetime.

$$\tau_{\text{form}} \approx 2\pi R_0/v_0 \simeq \pi/E_0 = 2\pi/(m_R v_0^2) \simeq 1.6\,\text{GeV}^{-1}$$

$$\tau_{\text{life}} \approx \tau_t/2 \simeq 1/3\,\text{GeV}^{-1} < \tau_{\text{form}}$$

$$\tau_{\text{form}}/\tau_{\text{life}} \approx 5$$

Toponium spectrum

States

In a Coulombic potential , E~ 1/n^2 , R~n^2, v~1/n:

$$\frac{N(\tau_{\text{form}})}{N_0} = e^{-\frac{n^3 9\pi\Gamma_t}{4\alpha_s^2\mu_{Q\bar{Q}}}} \simeq (0.7\%)^{n^3}$$

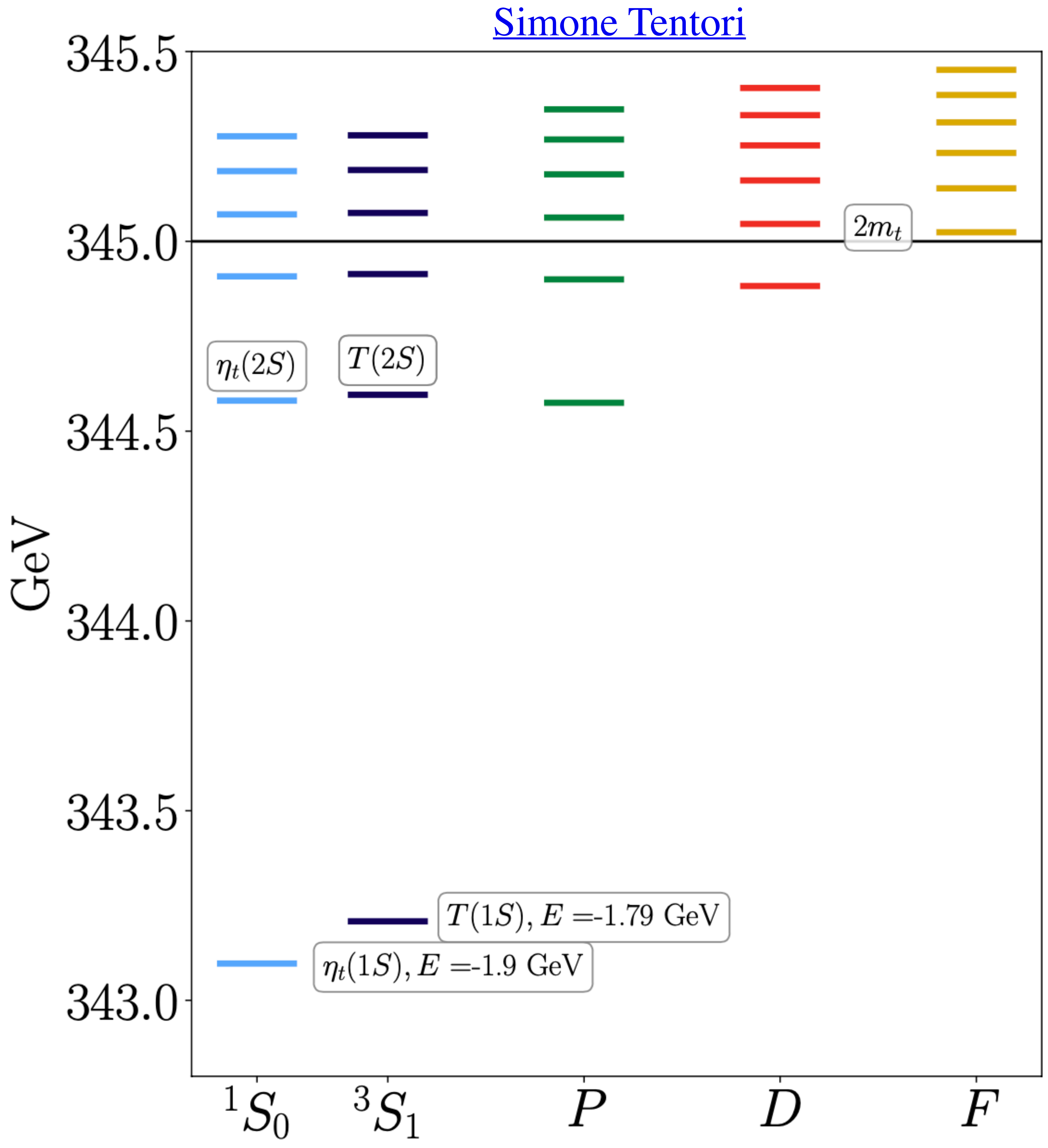
Only the n=1 states are Coulombic and could be actually accessible.

The energy of the ground states is about 2 GeV below the threshold.

The energy difference between n=1 states is

$$V_{ss} = \frac{8\pi}{9}\alpha_s \frac{f(j)}{m_q m_{\bar{q}}}\delta(\vec{x}) \quad \Rightarrow \quad \Delta E_{ss} = \frac{8\pi}{9}\frac{4\alpha_s}{m_q m_{\bar{q}}}|\psi(0)|^2$$

The difference is around 100 MeV.



0812.0919 [hep-ph]
0804.1014 [hep-ph]

Toponium production

QCD

So now, how to calculate the cross section for producing a toponium?

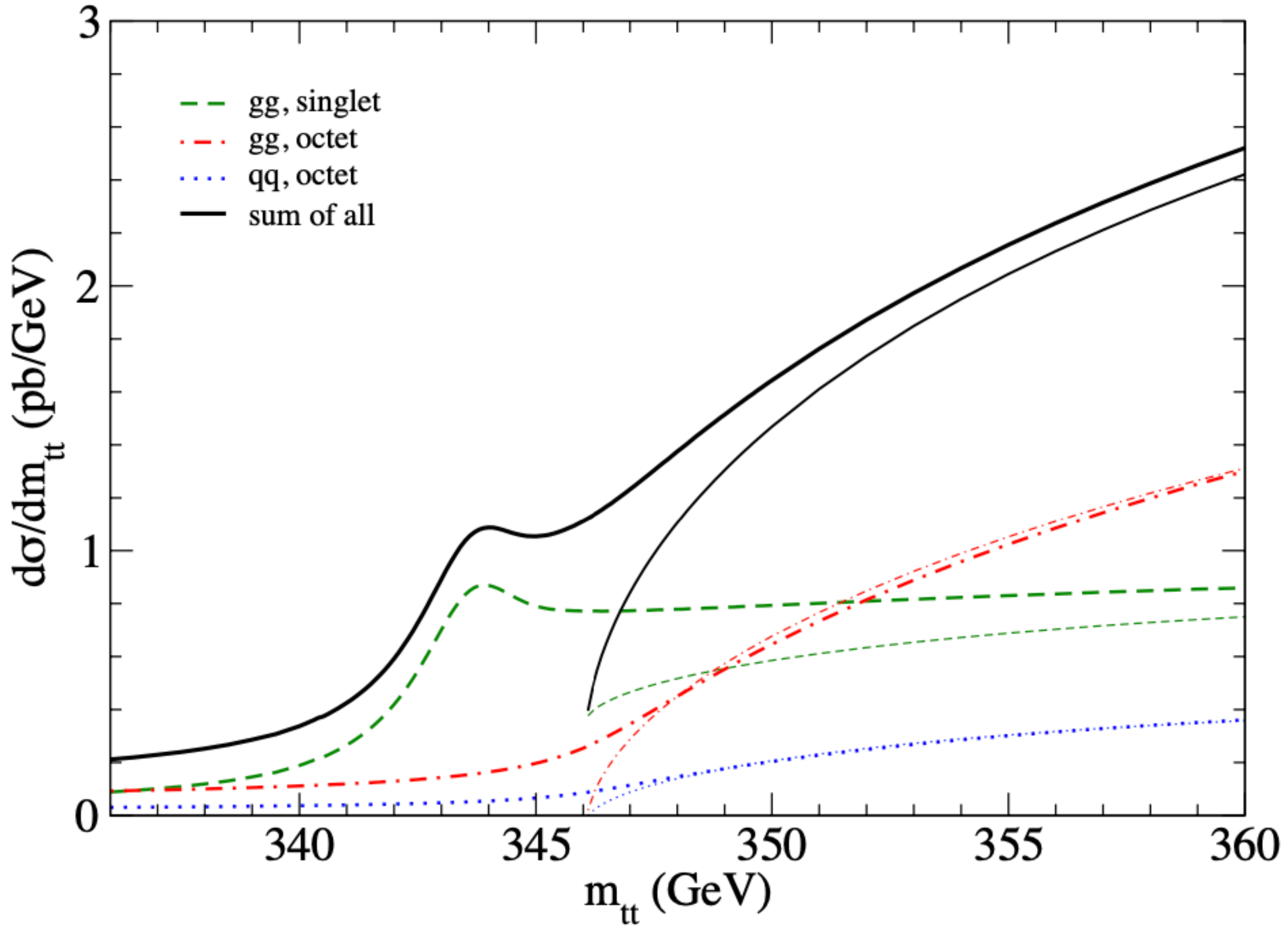
$$\hat{\sigma}(s'; i \to f) = \left[\hat{\sigma}(s'; i \to f)\right]_{\text{tree}} \times \frac{\text{Im}[G^{(c)}(\vec{0}; E)]}{\text{Im}[G_0(\vec{0}; E)]}$$

The Green function is obtained by solving the Schrodinger equation

$$\left[(E + i\Gamma_t) - \left\{-\frac{\nabla^2}{m_t} + V^{(c)}_{\text{QCD}}(r)\right\}\right] G^{(c)}(\vec{x}; E) = \delta^3(\vec{x})$$

With a potential

$$V^{(c)}_{\text{QCD}}(r; \mu_{\text{B}}) = C^{(c)} \frac{\alpha_s(\mu_{\text{B}})}{r} \left[1 + \frac{\alpha_s(\mu_{\text{B}})}{4\pi} \left\{2\beta_0 \left[\ln(\mu_{\text{B}} r) + \gamma_E\right] + a_1^{(c)}\right\}\right]$$



$$C^{(1)} = -C_F, \qquad C^{(8)} = \frac{C_A}{2} - C_F,$$

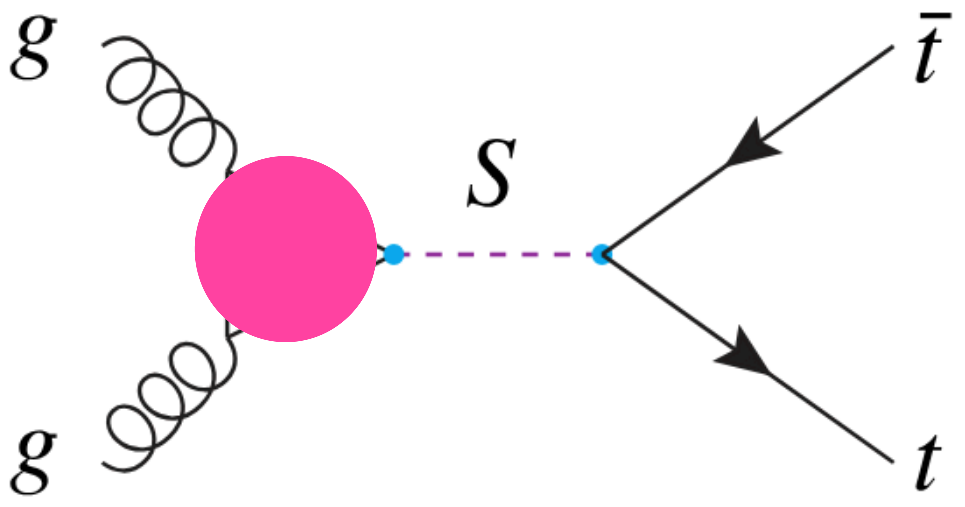
$$a_1^{(1)} = a_1^{(8)} = \frac{31}{9} C_A - \frac{10}{9} n_q,$$

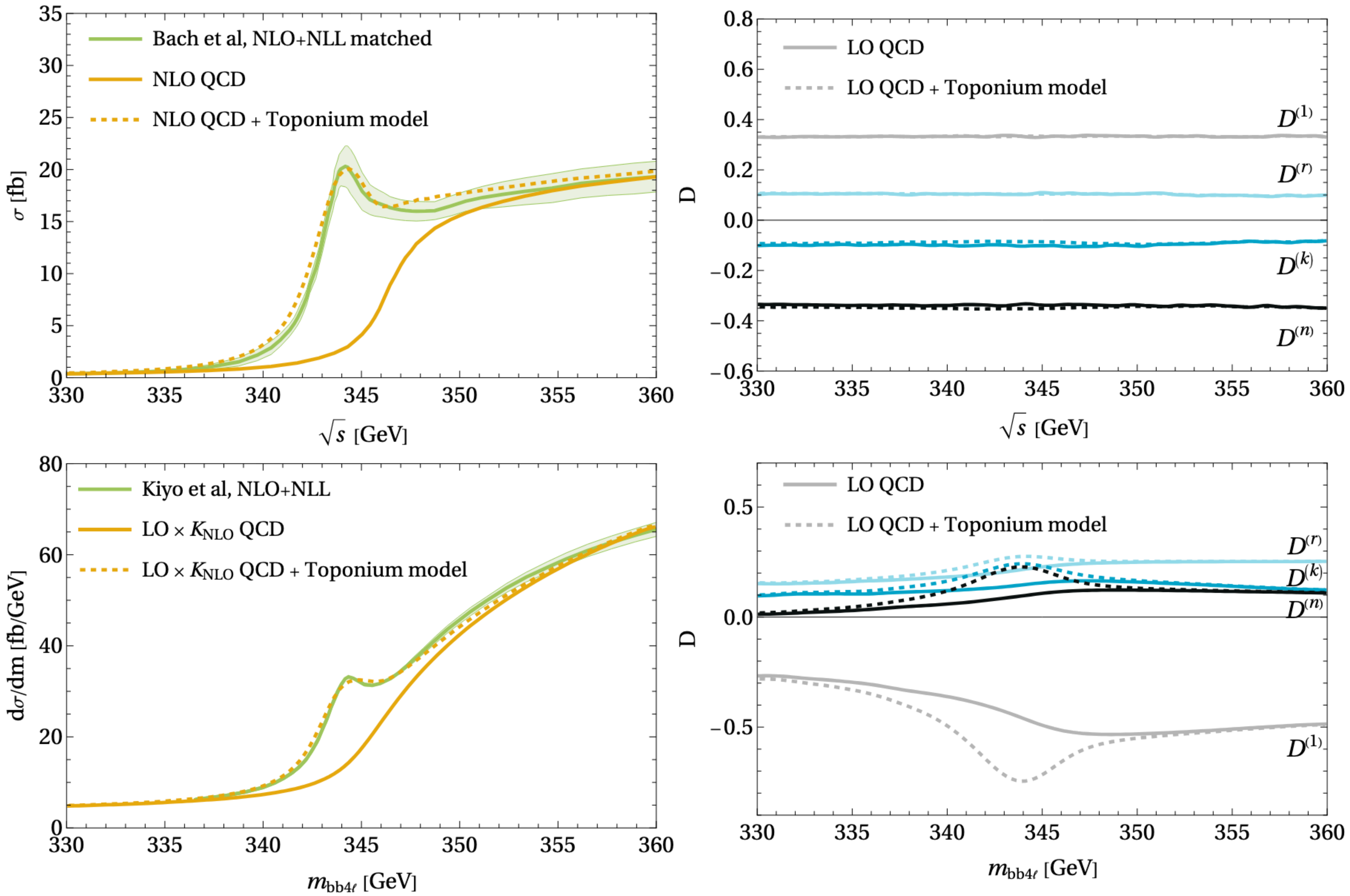
2404.08049 [hep-ph]

Toponium production

Modelling

A simple way go account for toponium is to add to the MC a pseudo-scalar state produced from gg through a contact interaction and tune the mass, the width and the cross section to the QCD prediction.





This can be also tested at e+e- with a vector 3S1[1] where the predictions are much more precise.

Spin correlations can be easily kept.

Conclusions

1. Toponium is not a bound state as Charmonium or Bottomonium. The top quarks don’t have the time to hadronise and they in fact decay before a propagating bound state is formed.

2. Toponium does not decay because the quark and anti-quark annihilate (as for J/psi or Y) but because one of the tops decay.

3. Only the color singlet states enhance locally the cross section at threshold. Octets do not attract each other.

4. At the end the toponium is seen as an enhancement of the cross section at threshold when the partonic channel is in a color and spin singlet.

1508.06864 [hep-ph]

e+e- predictions NLO,NNLO,N3LO