Precision top- threshold physics

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arXiv:2409.05969[hep-ph]

with M. Beneke (Technische Universität Müchen)

arXiv:1605.03010[hep-ph]

MB, YK, A. Maier(Desy Zeuthen),
J. Piclum(Universität Siegen)

TUM-HEP-919/13
TTK-13-26
SFB/CPP-13-110
arXiv:1312.4791 [hep-ph]
September 07, 2024

Third-order correction to top-quark pair production near threshold I. Effective theory set-up and matching coefficients

M.Beneke, YK, K. Schuller 2013 (updated in 2024)

TUM-HEP-1525/24
September 08, 2024

Third-order correction to top-quark pair production near threshold II. Potential contributions

MB, YK , 2024

We described detailed NNNLO QCD computation of threshold top quark production in Part I and Part II papers.

Near-threshold production of heavy quarks with `QQbar_threshold`

M. Beneke[a], Y. Kiyo[b], A. Maier[c], J. Piclum[d,e]

QQbar_threshold

QQbar_threshold computes the top-quark pair production cross section near threshold in electron-positron annihilation at NNNLO in resummed non-relativistic perturbation theory [1, 2]. It includes Higgs, QED, electroweak and non-resonant corrections at various accuracies and a consistent implementation of initial-state radiation. Details can be found in

- M. Beneke, Y. Kiyo, A. Maier, and J. Piclum
  Near-threshold production of heavy quarks with QQbar_threshold
  Comput. Phys. Commun. 209 (2016) 96-115, arXiv:1605.03010 [hep-ph]
- M. Beneke, A. Maier, T. Rauh, and P. Ruiz-Femenía
  Non-resonant and electroweak NNLO correction to the $e^+ e^-$ top anti-top threshold
  arXiv:1711.10429 [hep-ph]

C++ QQbar_threshold(2016),
Can be downloaded at
hepforge.org

We expect to update soon based on Part II.

Top in the SM

$$\delta\mathcal{L}_t = -\frac{g}{2\sqrt{2}}\,\bar{t}\,\not{W}(1-\gamma_5)\,b - \frac{y_t}{\sqrt{2}}\left[\langle h^0\rangle_{vac} + h^0\right]\bar{t}t + \cdots$$

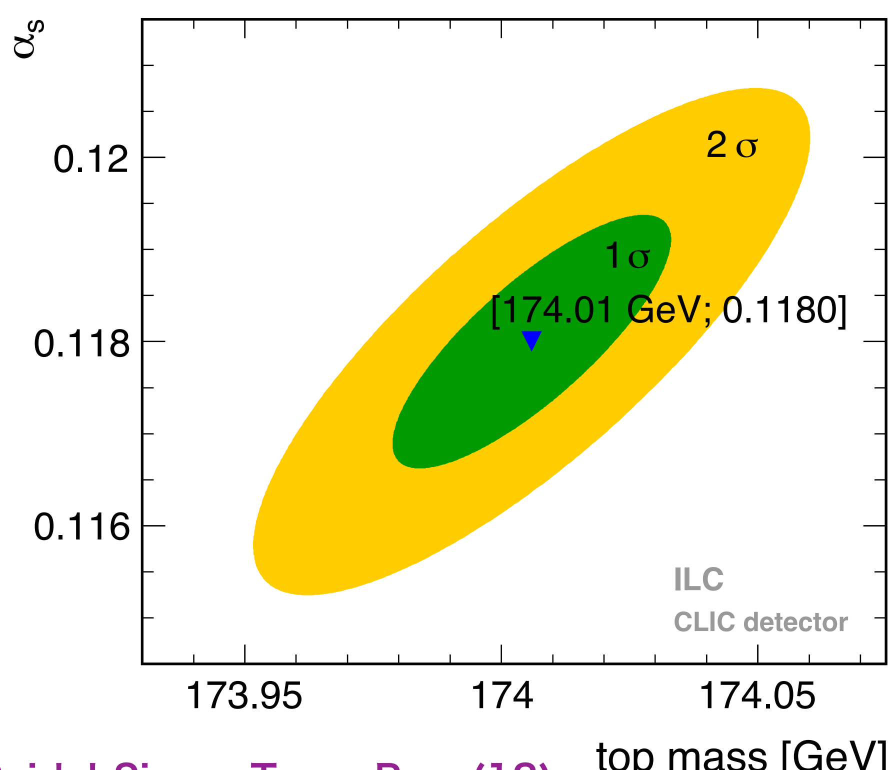
Eaten NG

$$\Gamma_{t\to bW^+} = \frac{G_F m_t^3 |V_{tb}|^2}{8\pi\sqrt{2}}\left[1 + F\left(\frac{M_W^2}{m_t^2}\right)\right]$$

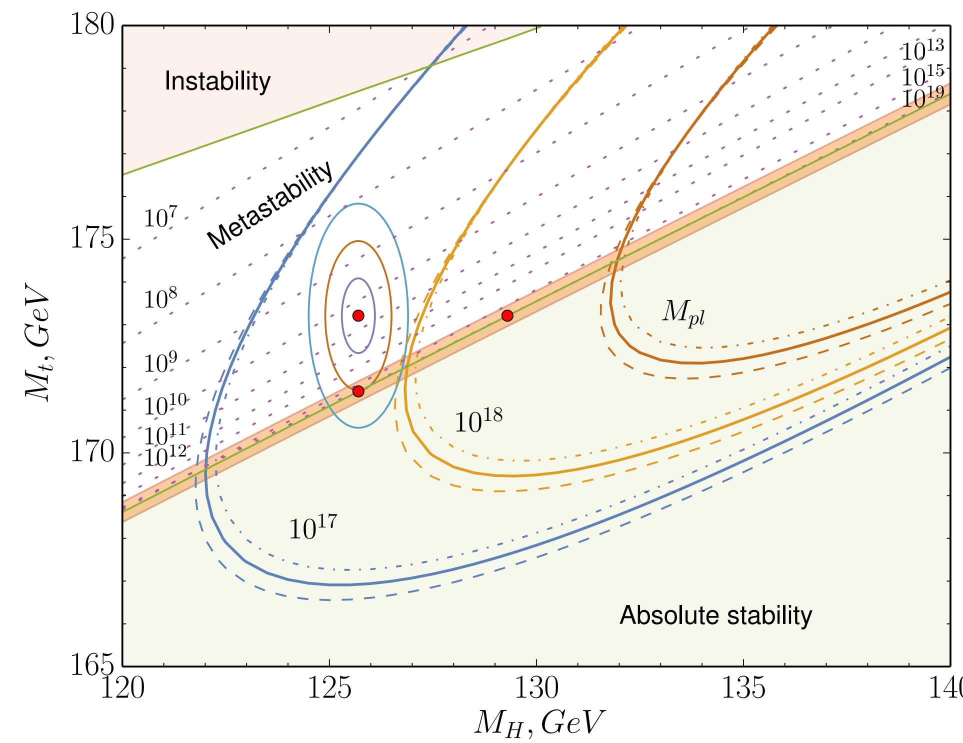
BEH mechanism

$$\frac{y_t}{\sqrt{2}} = \frac{m_t}{v} = \frac{m_t}{(\sqrt{2}G_F)^{\frac{1}{2}}}$$

- Top quark mass is an important SM parameter. Its precise determination will give a hint for new physics



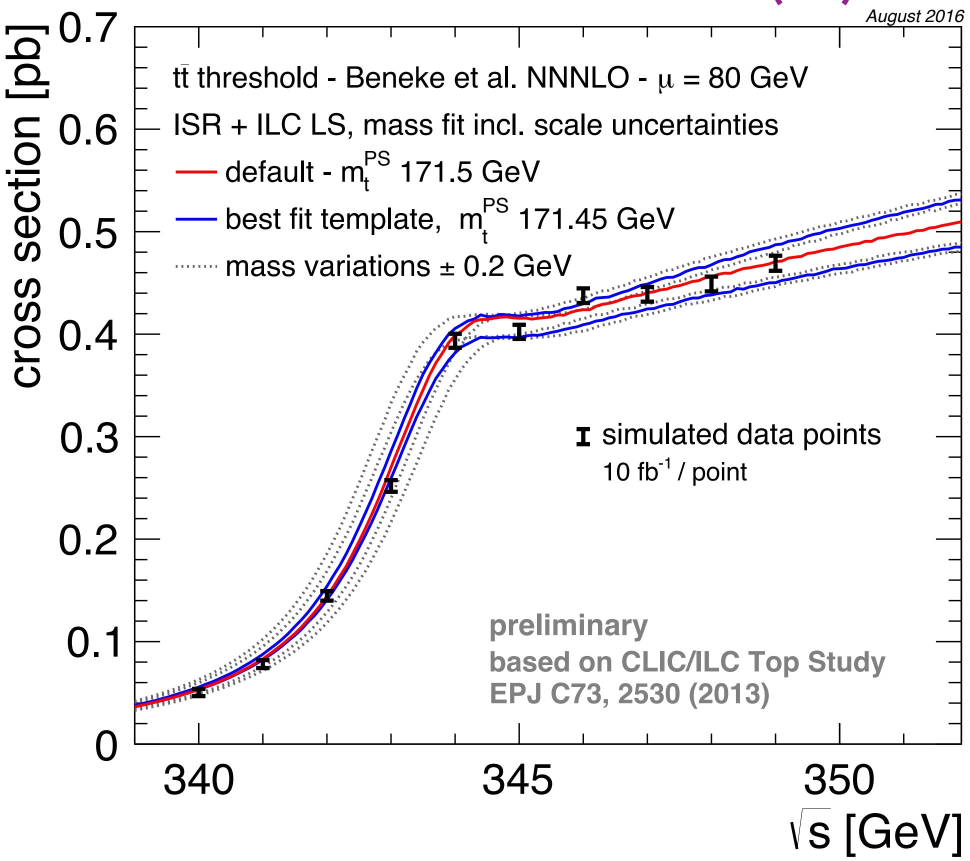
Seidel-Simon-Tesar-Poss(13)



Bednyakov-Kniehl-Pikelner-Veretin (15)

Top threshold in e+e-

Seidel-Simon-Tesar-Poss(13)



- Experimentally clean threshold scan
- Theoretically clean

(1) small α_s → perturbation

(2) Short life time : $m_t \gg \Gamma_t \gg \Lambda_{QCD}$

→ cut off hadronization effect

(3) color singlet observable (tt-bound state)_→ renormalon free

Decoupling of QCD infrared effect ↔ theoretically well defined short distance masses

Precision m_t determination with short distance masses!

Top mass is already known quite well:

$$m_t = 172.52 \pm 0.14(\text{stat.}) \pm 0.33(\text{syst.})$$ (ATLAS+CMS)

Measurement of a toponium resonance at LHC is still interesting (color singlet, theoretically clear mass def.)

Hagiwara-Sumino-Yokoya(2008), YK-Kuehn-Moch-Steinhauser-Uwer(2009)

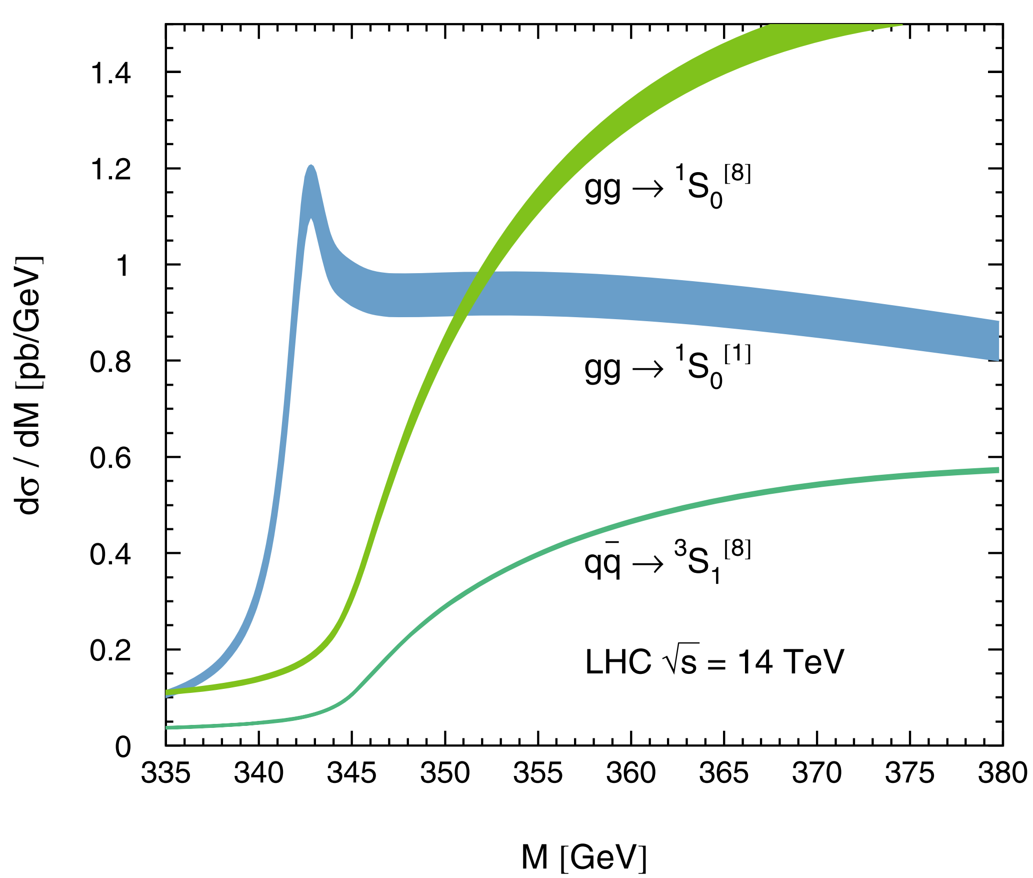


Fig. 6.1 (Color online) Invariant-mass distributions for leading sub-processes: $gg \to {}^1S_0^{[1,8]}$ (*blue* and *light green*, respectively) and $q\bar{q} \to {}^3S_1^{[8]}$ (*green*). For each process the bands take into account scale variation of the hard cross sections

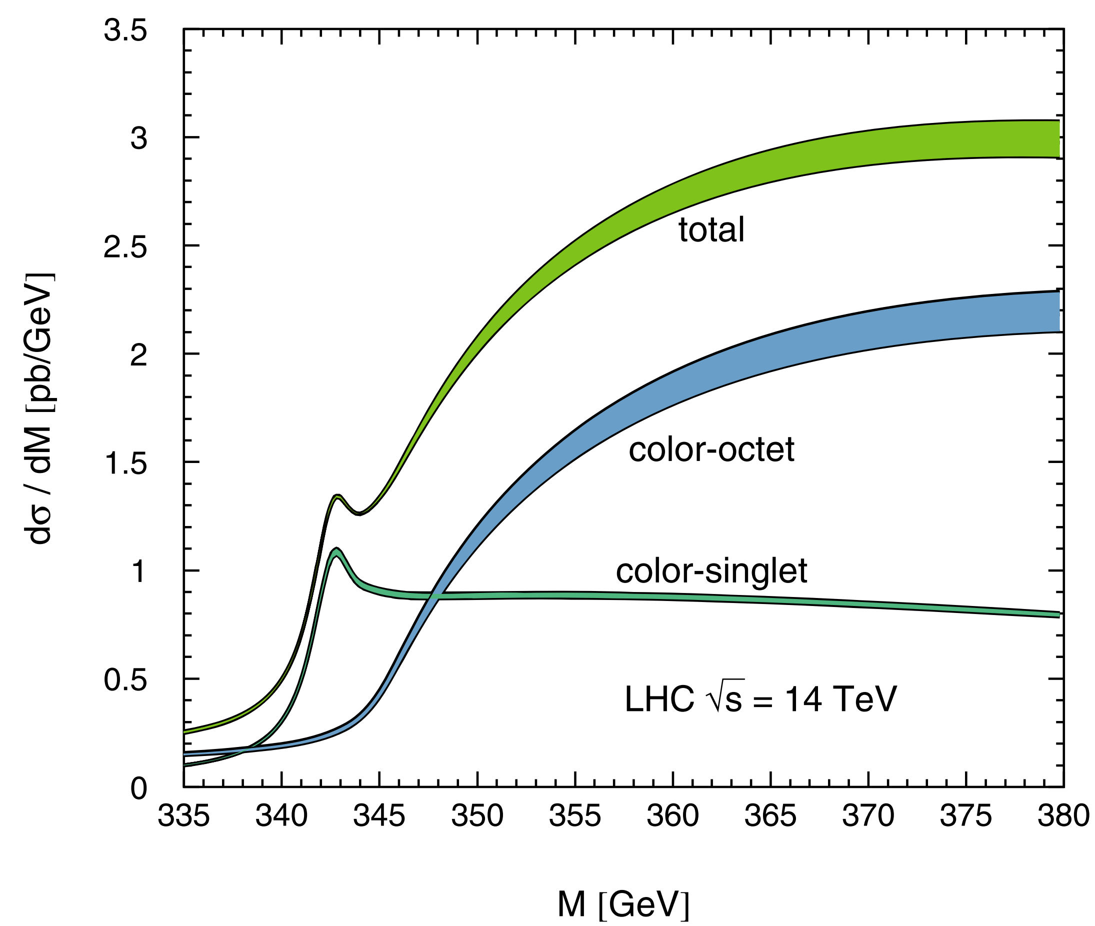
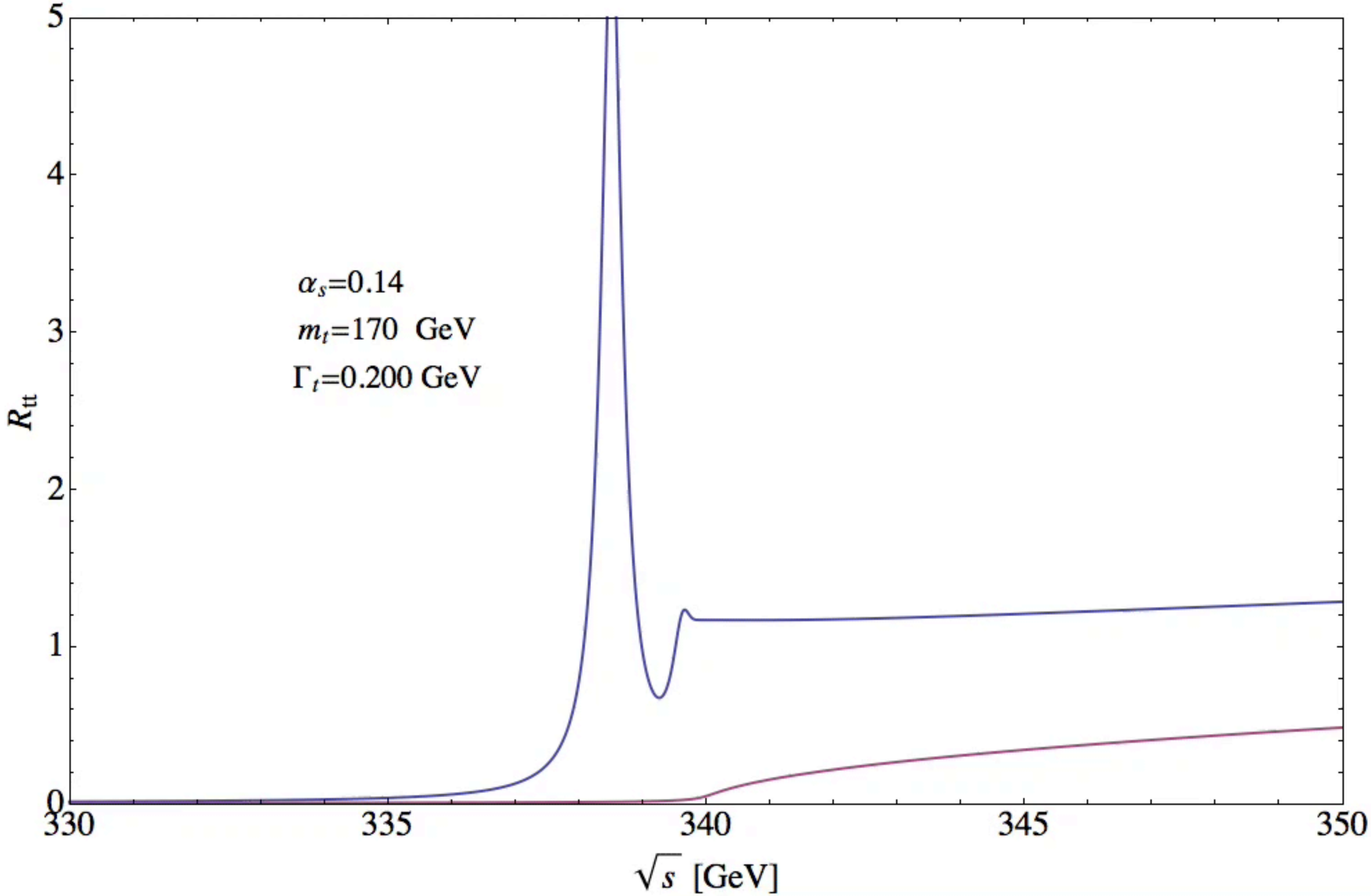


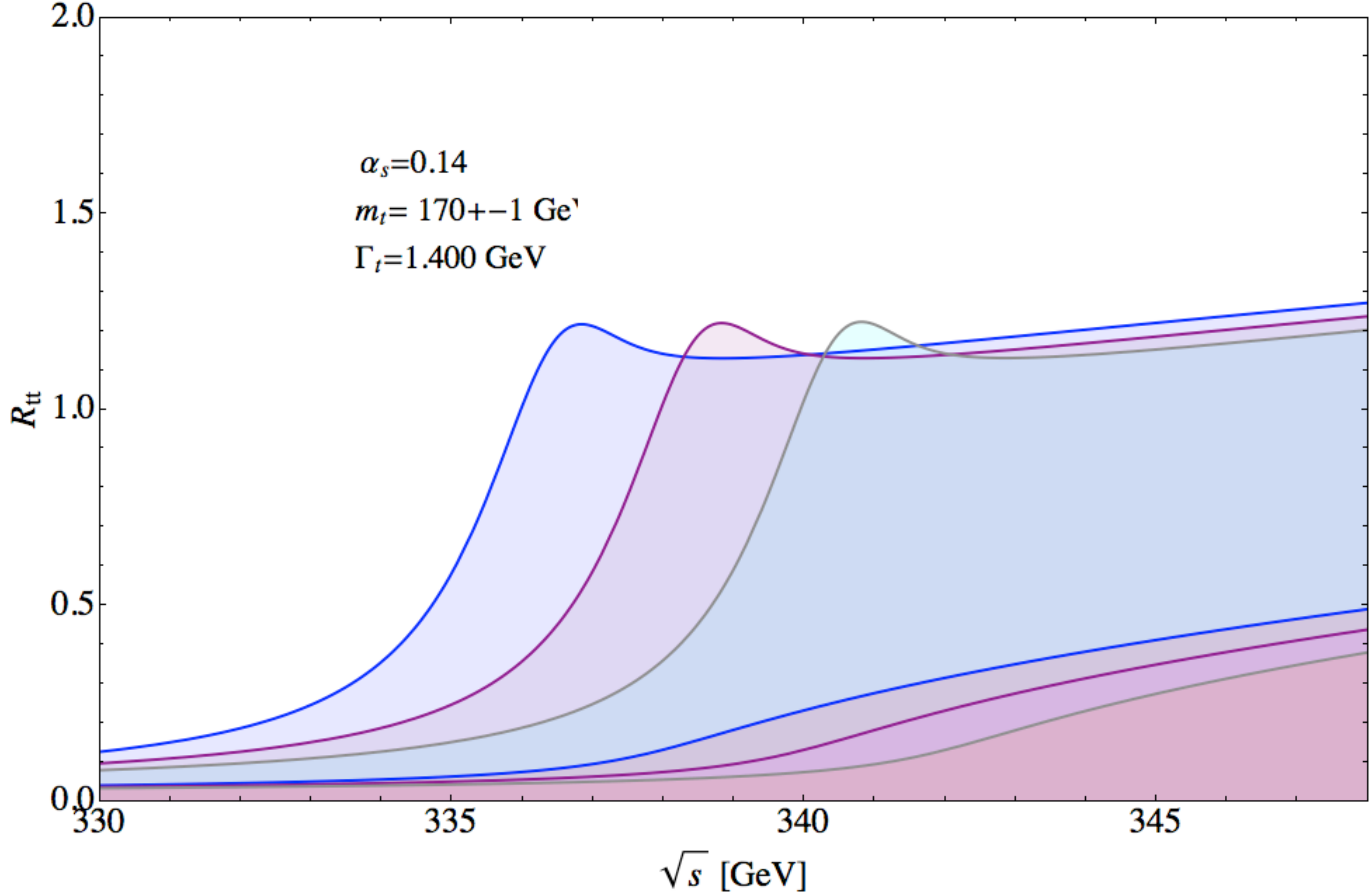
Fig. 6.2 Invariant-mass distribution including all production channels shown in Table 3.2. The width of the bands reflect the scale dependence of the hard scattering parts

Threshold cross section $\sigma(e^+e^- \to t\bar{t})$ as a function of $(s; m_t, \Gamma_t, \alpha_s)$

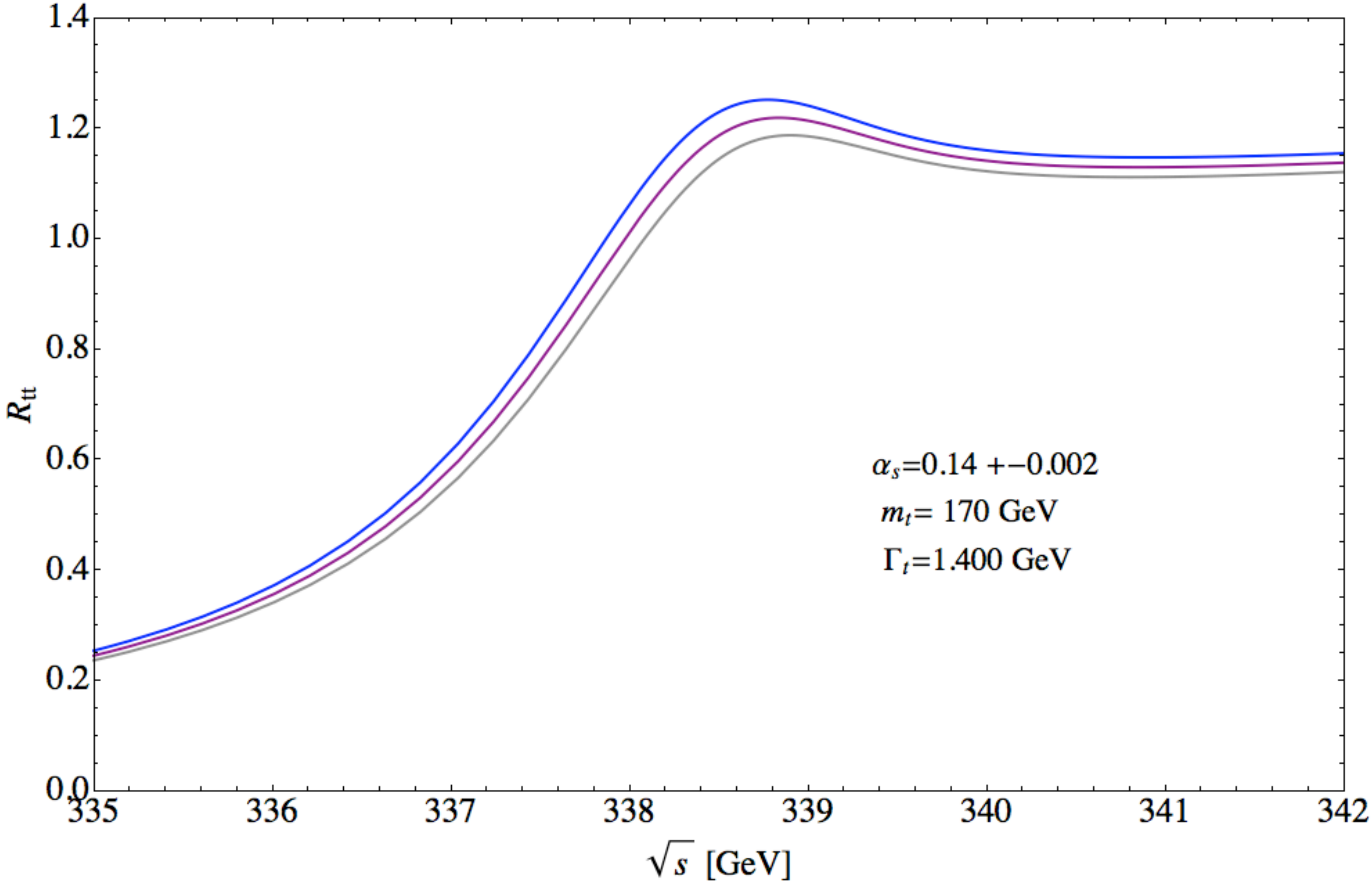
Γ_t dependence (LO)



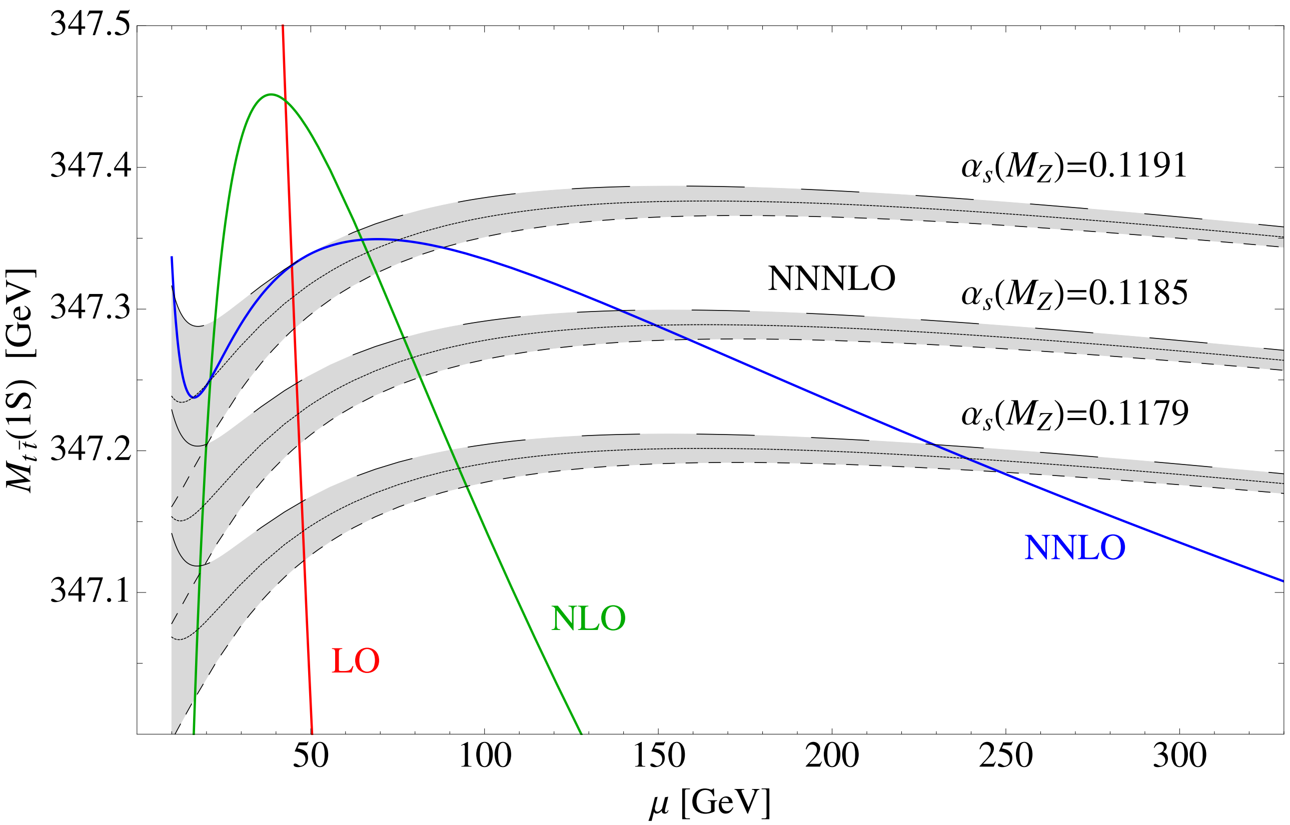
m_t dependence (LO)

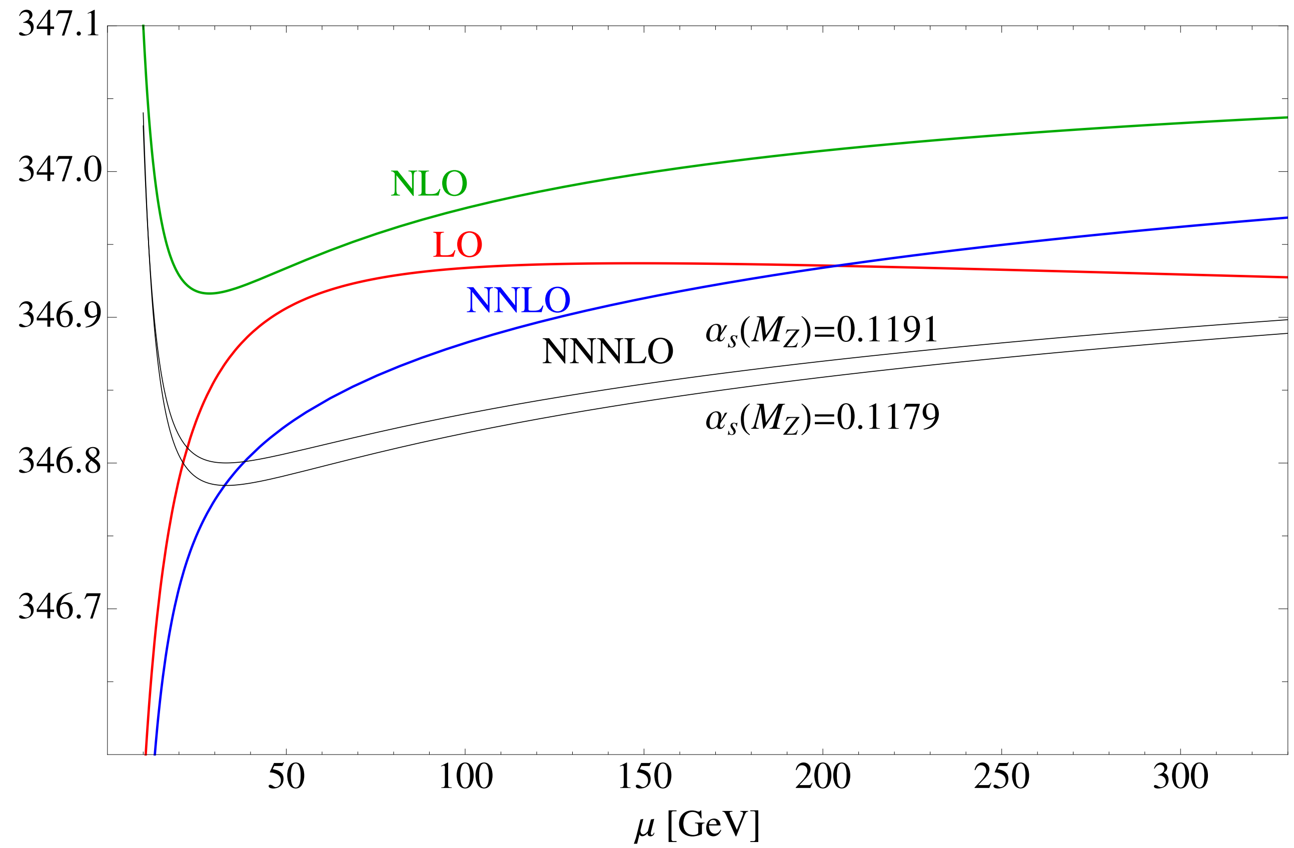


α_s dependence (LO)



Precision of top mass is determined by the relation between $M_{t\bar{t}}$ and m_t





- black bands due to numerical error of d3(exact)
- three lines for NNNLO for $\alpha_s(M_z) = 0.1185 \pm 0.0006$

➡ $\delta M_{1S} = 2\delta m_{\overline{\text{MS}}} = (40_{\mu} + 10_{d_3} + 90_{\alpha_s})$ MeV

YK. Mishima. Sumino(15)

The precision of N3LO top MSbar-mass determination is $\delta m_t \simeq 70\,\text{MeV}$.
Uncertainty in α_s is the largest source of the error (2015).

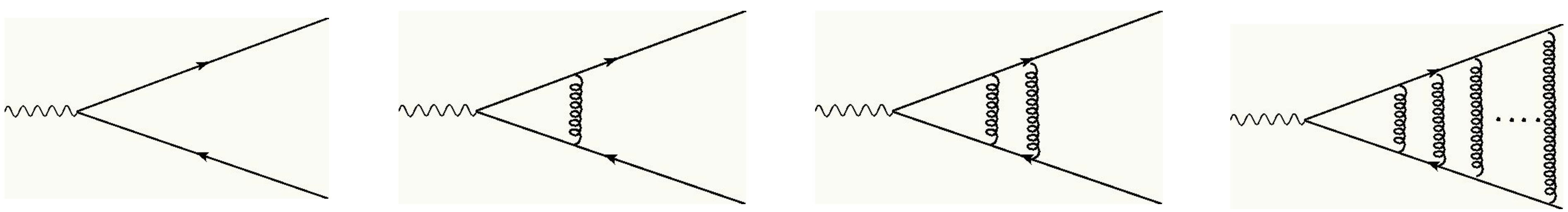
N^3LO Top Cross section

- Coulomb singularity α_s/v near threshold

√s～350GeV

$$v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s$$

- Resummation scheme(NRQCD)



$$\mathrm{N^0LO} = v \times \sum_n \left(\frac{\alpha_s}{v}\right)^n \times 1$$

$$\mathrm{N^1LO} = v \times \sum_n \left(\frac{\alpha_s}{v}\right)^n \times \{v, \alpha_s\}$$

$$\mathrm{N^2LO} = v \times \sum_n \left(\frac{\alpha_s}{v}\right)^n \times \{v^2, v\alpha_s, \alpha_s^2\}$$

$$\mathrm{N^3LO} = v \times \sum_n \left(\frac{\alpha_s}{v}\right)^n \times \{v^3, v^2\alpha_s, v\alpha_s^2, \alpha_s^3\}$$

Perturbative/exact

Analytical calculations are only possible through the perturbative expansion (insertion): dimensional regularization is adopted, and UV-divergences are renormalized in MSbar scheme.
(renormalizable order by order)

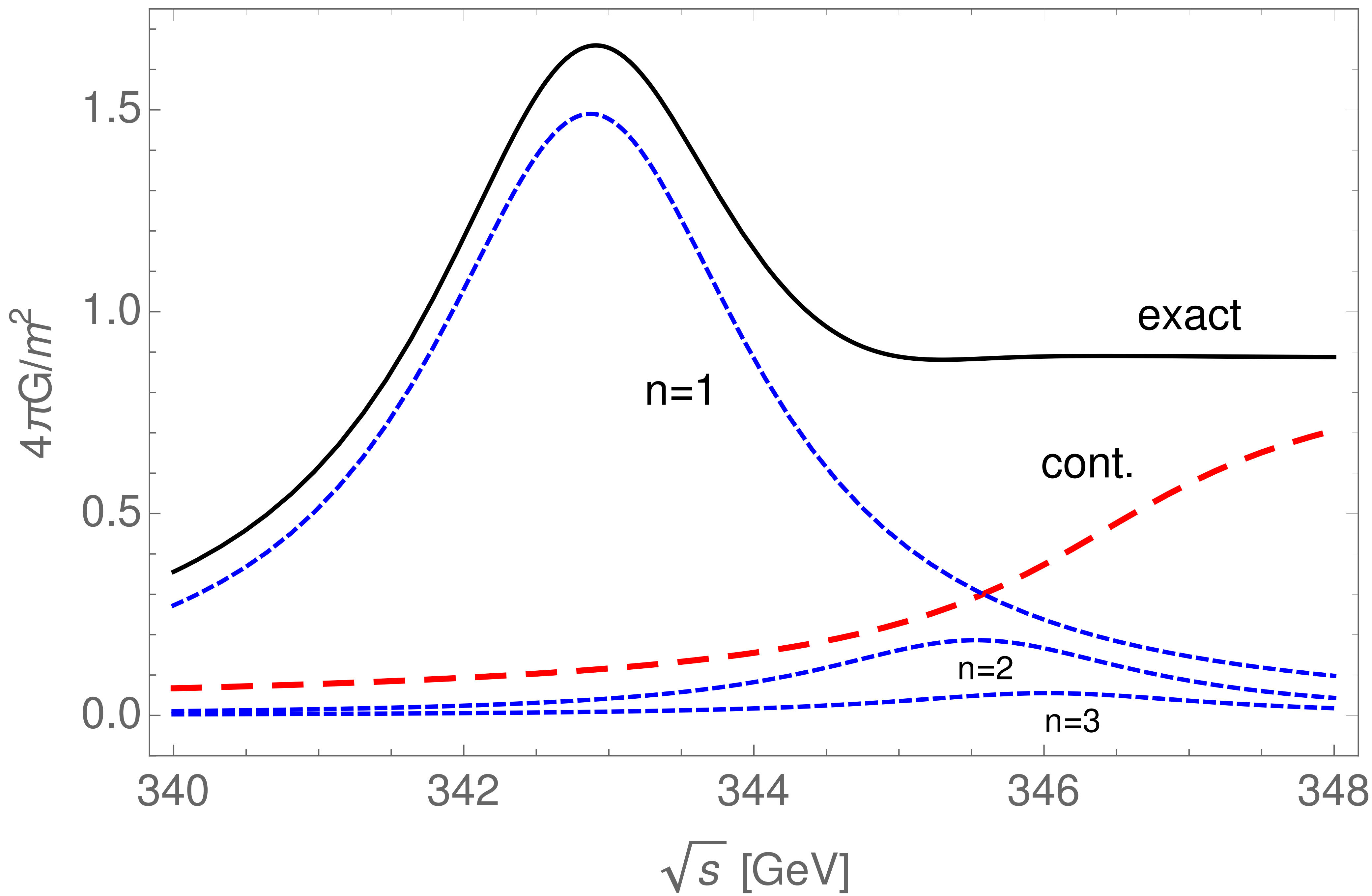
$$\sigma_{t\bar{t}} = \mathrm{Im}\left\langle \frac{1}{\frac{p^2}{m} - \frac{C_F\alpha_s}{r} - (E + i\Gamma_t) + \delta V} \right\rangle$$

$$= \mathrm{Im}\left\langle G_c \right\rangle - \left\langle G_c \delta V G_c \right\rangle + \left\langle G_c \delta V G_c \delta V G_c \right\rangle + \cdots$$

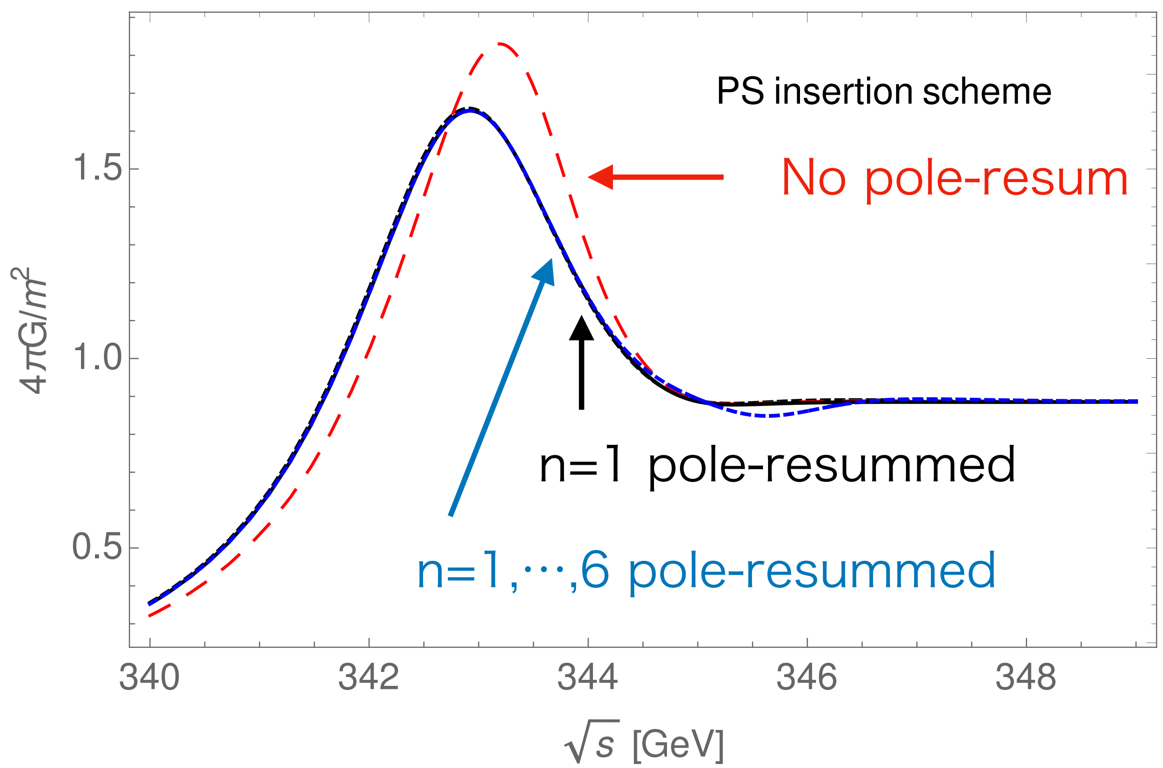
but this approach disrupts the single-pole structure of bound-state Green function → pole-resummation to recover single-pole structure

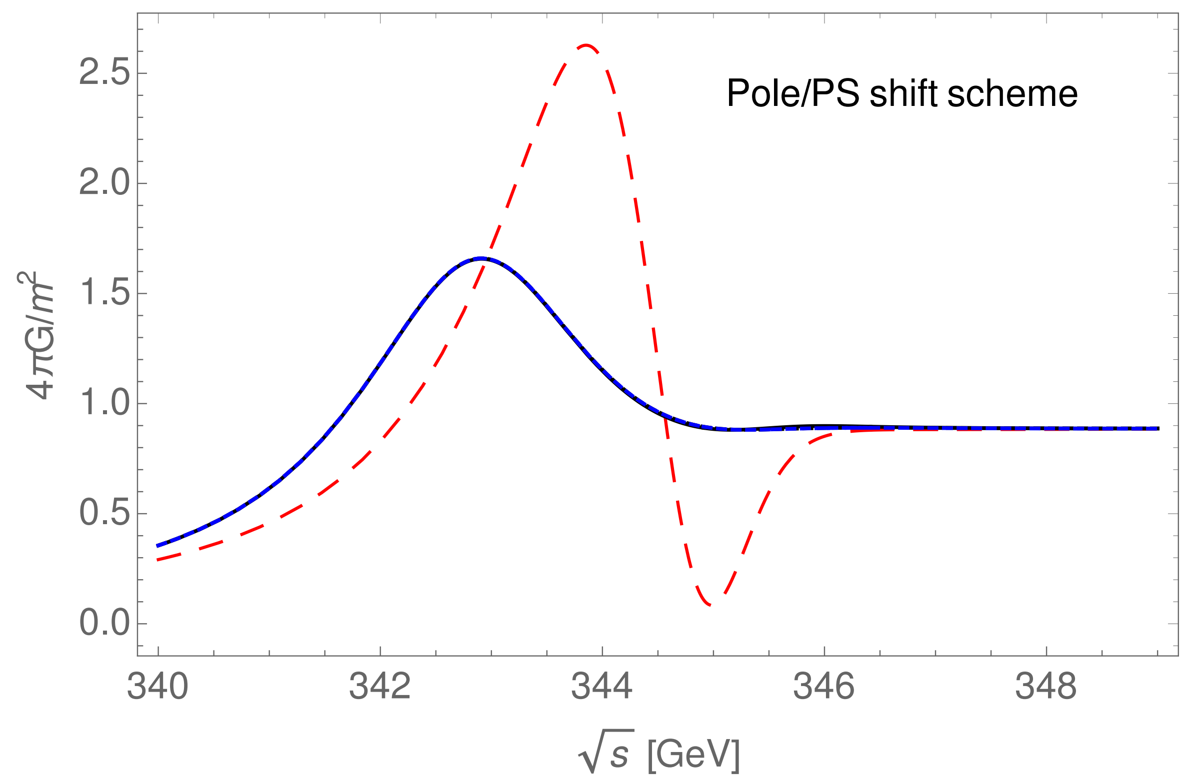
continuum/pole contributions

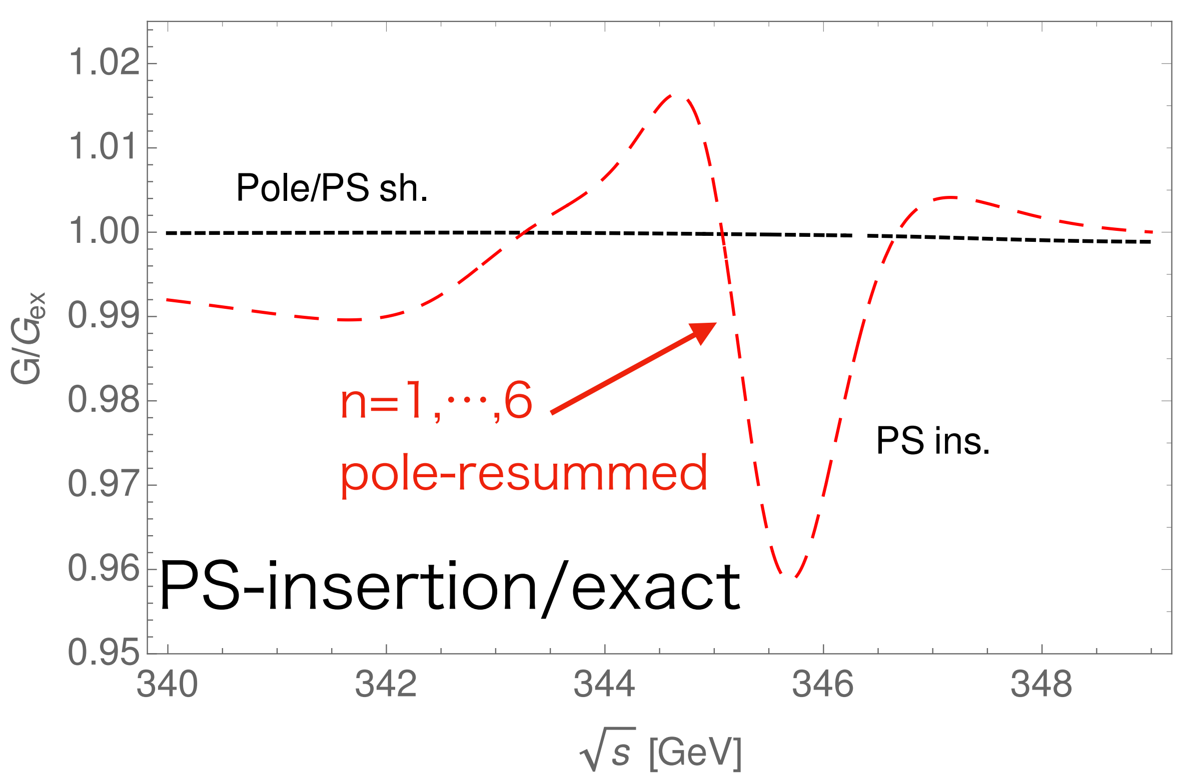
N-th resonance and continuum contributions are compared
For Coulomb potential



Mass insertion scheme, and pole resummation

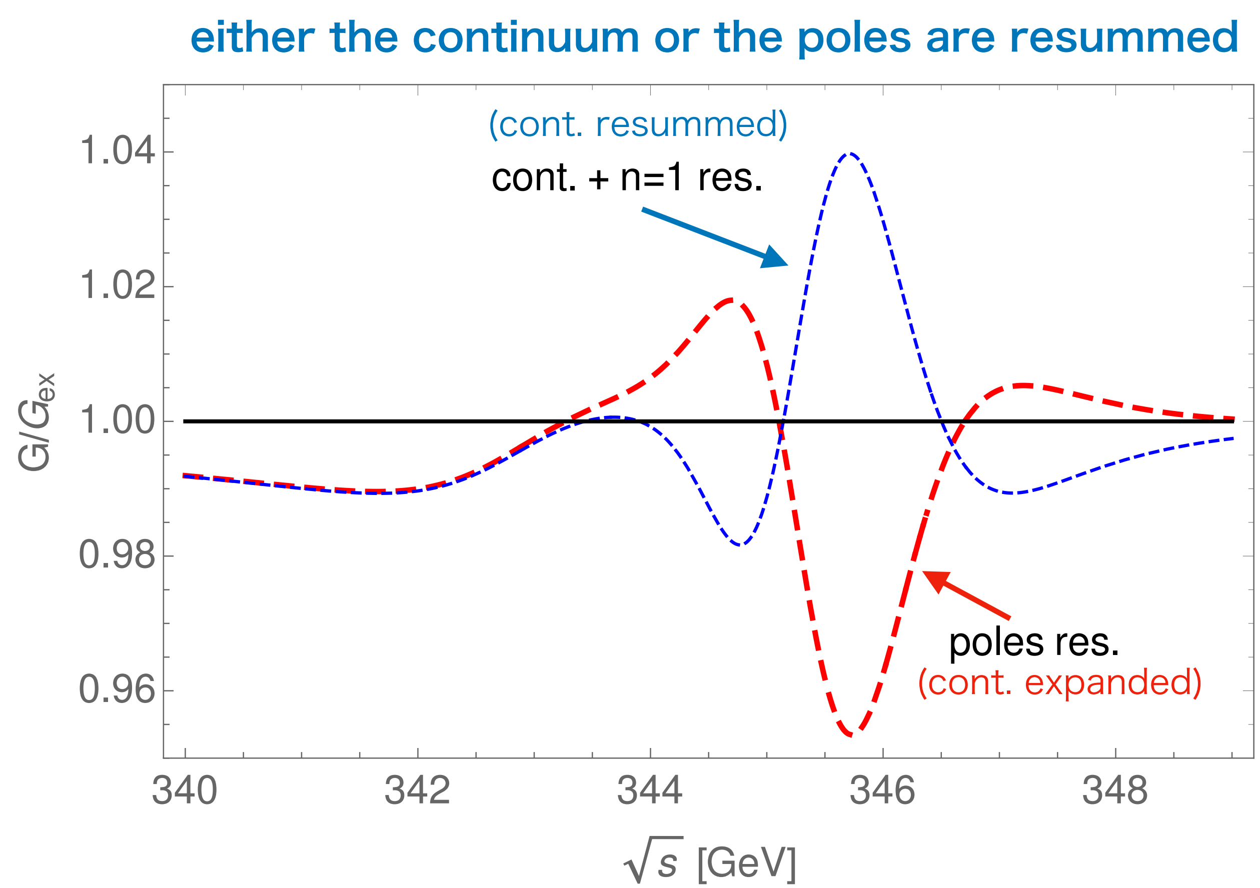


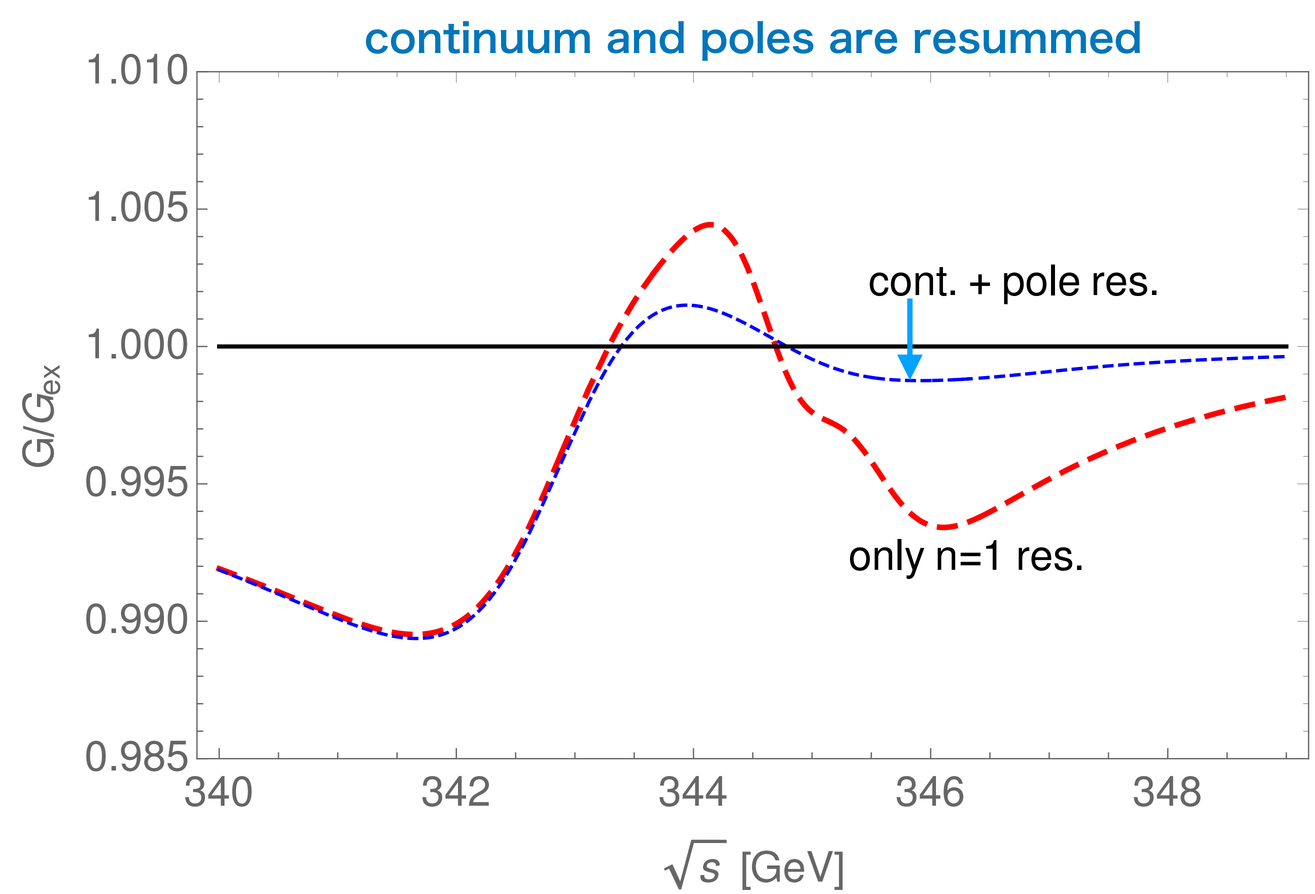




To achieve renormalon cancelation in short distance masses, mass corrections must be treated in insertion scheme, and they have to be resummed parameter

continuum/pole resume





The pole and continuum are treated coherently, a much improved approximation is obtained: Accuracy is few ‰ above threshold), and 1% below threshold

⇨ **PS-shift mass (our standard scheme) is a good scheme for the threshold cross section**

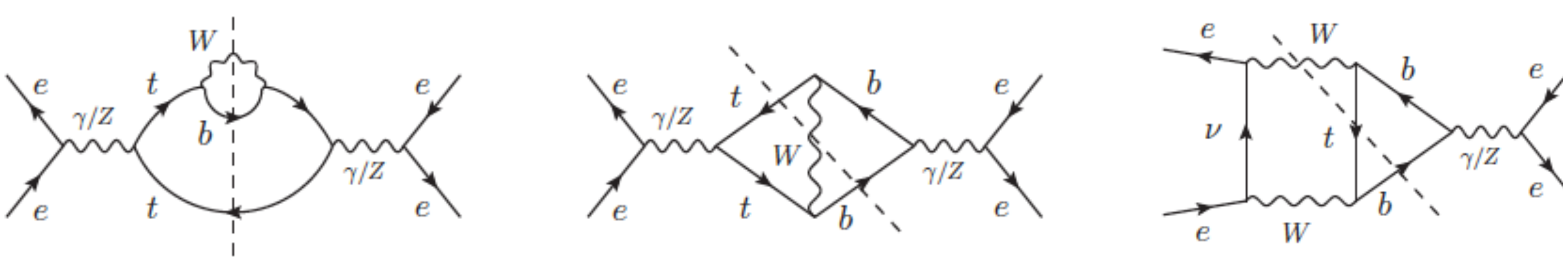
X section formula

Unstable top and (bW) can not be distinguished. Include non-resonant (bW) productions! Hoang-Reisser(04)

$$\sigma_{e^+e^-\to W^+W^-b\bar{b}} = \sigma_{t\bar{t},\mathrm{resonant}}(s,\mu,\mu_w) + \sigma_{\mathrm{non-resonant}}(s,\mu,\mu_w)$$

$$\sigma_{\mathrm{resonant}} = \sigma(e^+e^- \to t\bar{t})$$

$$\sigma_{\mathrm{non-resonant}} = \sigma(e^+e^- \to (bW^+)\bar{t}) + \cdots$$

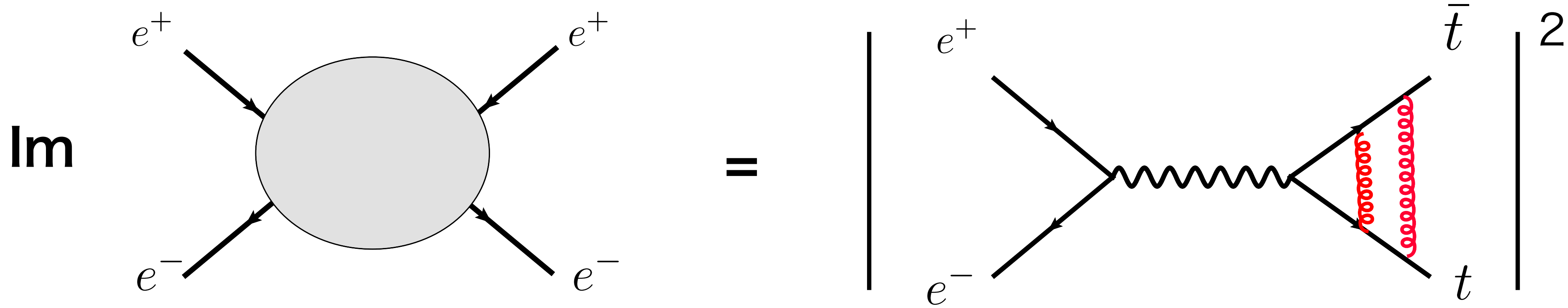


Nonresonant N2LO Beneke-Jantzen-Femenia(2010)

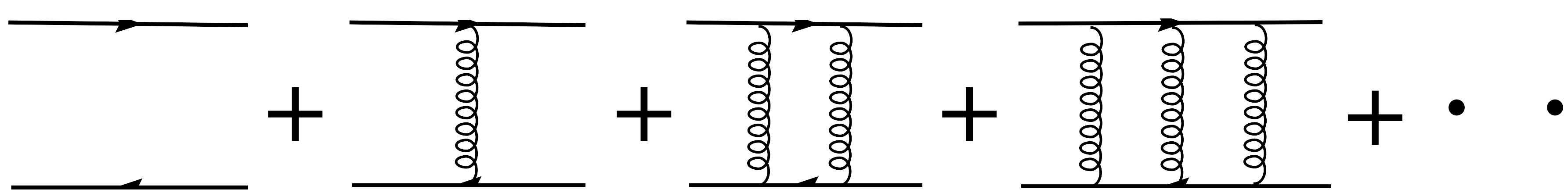
$$\frac{d\sigma^{(\mathrm{N^3LO})}_{\mathrm{res}}(\mu,\mu_w)}{d\ln\mu} = \mathcal{O}(v\alpha_s^4) \quad (\mathrm{N^4LO}), \quad \frac{d\sigma^{(\mathrm{N^3LO})}_{\mathrm{res}}(\mu,\mu_w)}{d\ln\mu_w} = \mathcal{O}(\alpha_s\Gamma) \quad (\mathrm{N^2LO})$$

μ_w scale independence (cancelation) requires non-resonant contribution!

Resonant $t\bar{t}$ production



$$\sigma_{\rm res} = {\rm Im}\left[C^{(i)}C^{(j)}\, i\int d^4x\, e^{iqx}\langle 0|{\rm T}[\psi^\dagger\sigma^i\chi]^\dagger(x)[\psi^\dagger\sigma^i\chi](0)|0\rangle\right.$$



$$G = \left\langle \mathbf{r}\left|\frac{1}{\frac{p^2}{m} + V(r) - (E + i\Gamma_t)}\right|\mathbf{r}'\right\rangle$$

$$V_s(r) = -\frac{4\alpha_s}{3r} + \cdots$$

$$V_o(r) = \frac{\alpha_s}{6r} + \cdots$$

Resonant part correponds to QM Green-function(Fadin-Khoze),
described by EFT (NRQCD/pNRQCD) Brambilla-Pineda-Soto-Vairo(99)

Top width, Resonant, non-resonant,

Resonant cross section is divergent:

$$\mathrm{Im}\Pi_{VV}(q) = \mathrm{Im}\, C_v^2 \times i \int d^d x e^{iEt} \langle 0 | \mathrm{T} j_V(x) j_V^\dagger(0) | 0 \rangle \quad + \quad \mathrm{Im}\Pi_{VV,\mathrm{non-res}}$$

Cancelation of μ-dependent

$$\mathrm{Im}\left[\delta_2 G\right]_{\mathrm{div}} = \frac{\alpha_s^2 C_F}{6\epsilon}\left(C_F + \frac{3}{2} C_A\right) \mathrm{Im}\left[G_0(E)\right] + \frac{m\alpha_s C_F \Gamma_t}{8\pi\epsilon}$$

UV divergence

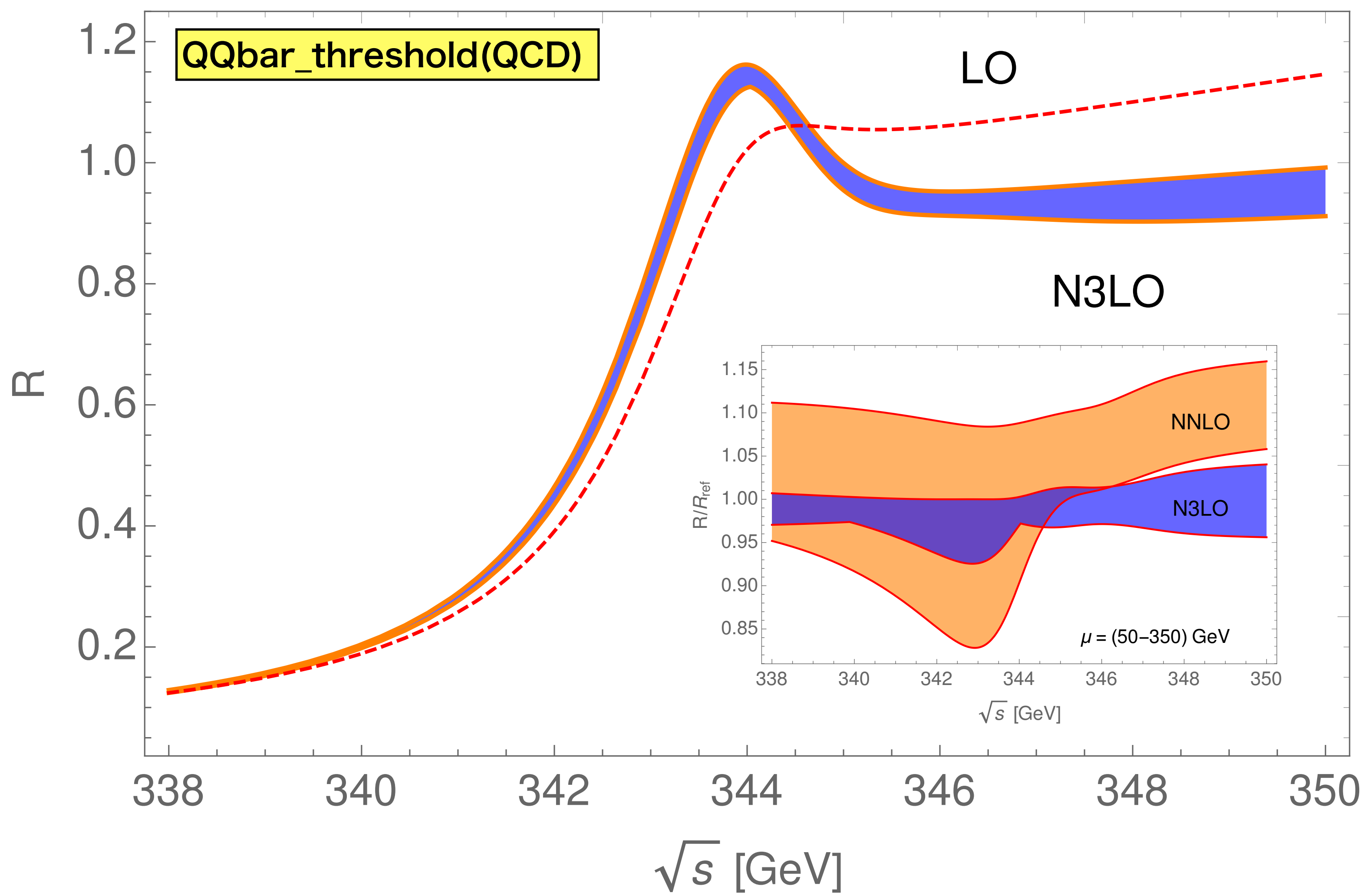
Cancelation of μ_w-dependence

In QCD, Π_{VV} is physical quantity and μ, μ_w independent

＊non-resonant N[3]LO computation is not known
＊ numerically μ_w-dependence is smaller than μ-dependence

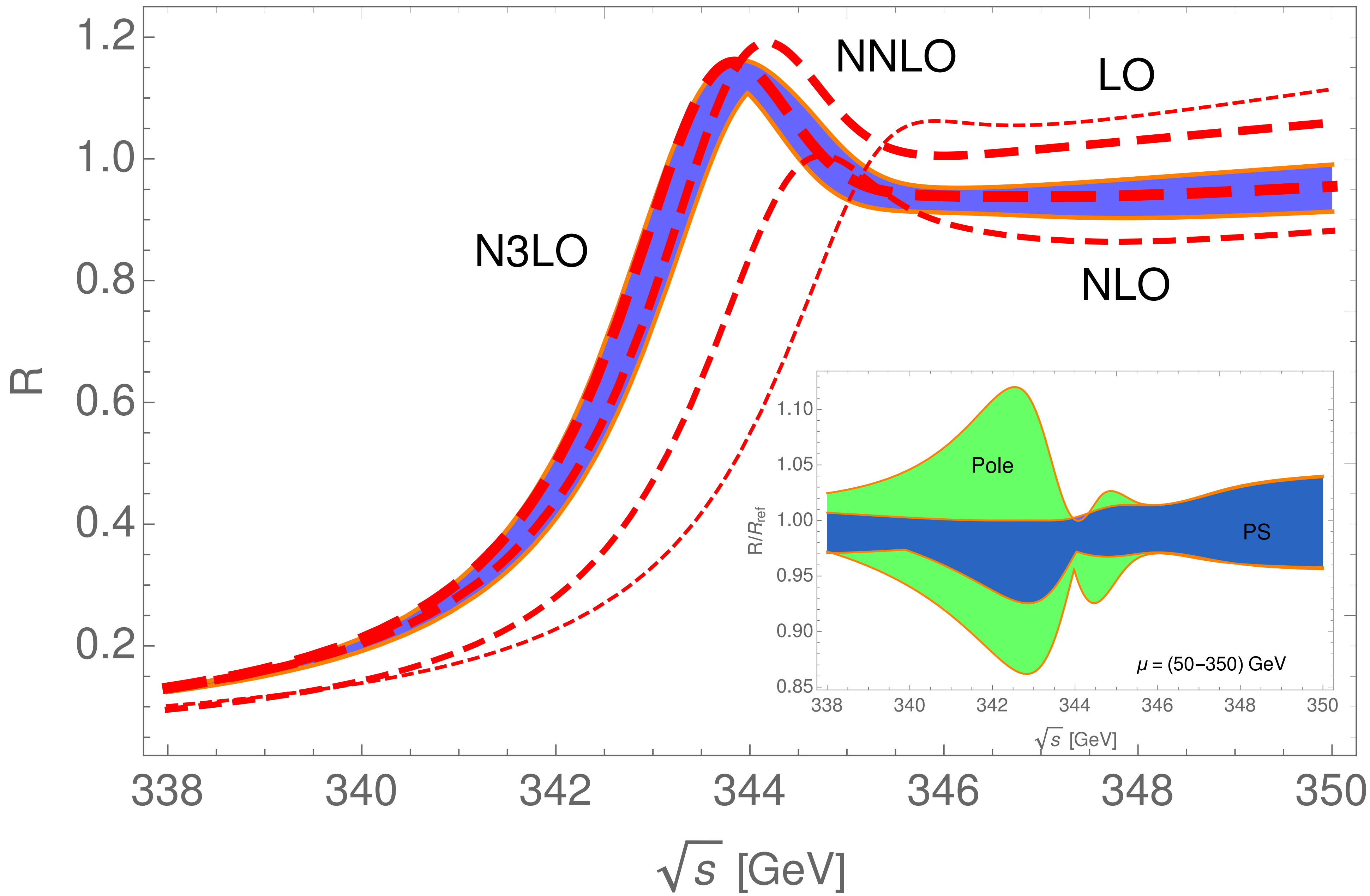
X section at N3LO

X section order and uncertainty



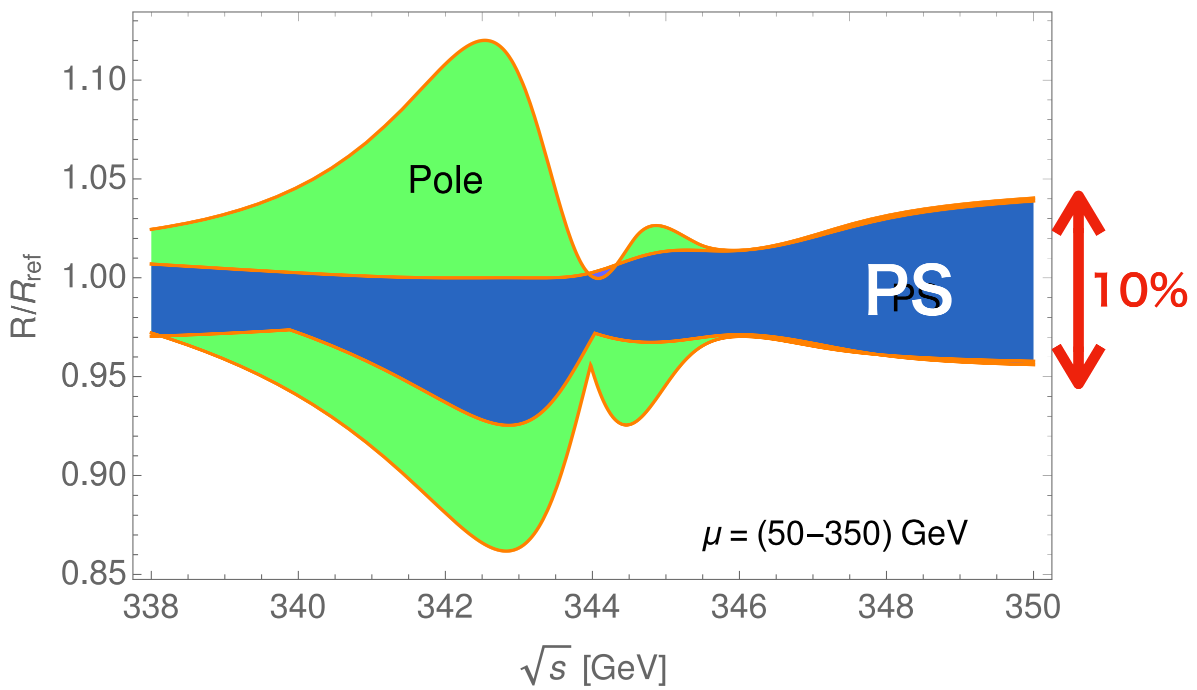
- $\mu = 80$ GeV ($\mu \in [50,350]$GeV)
- $\mu_w = 350$ GeV
- $m_t^{\mathrm{PS}}(\mu_f = 20\mathrm{GeV}) = 171.5$ GeV
- $\Gamma_t = 1.36$ GeV
- $\alpha_s(m_Z) = 0.1180$
- $m_Z = 91.1876$ GeV
- $\sin^2\theta_w = 0.222897$

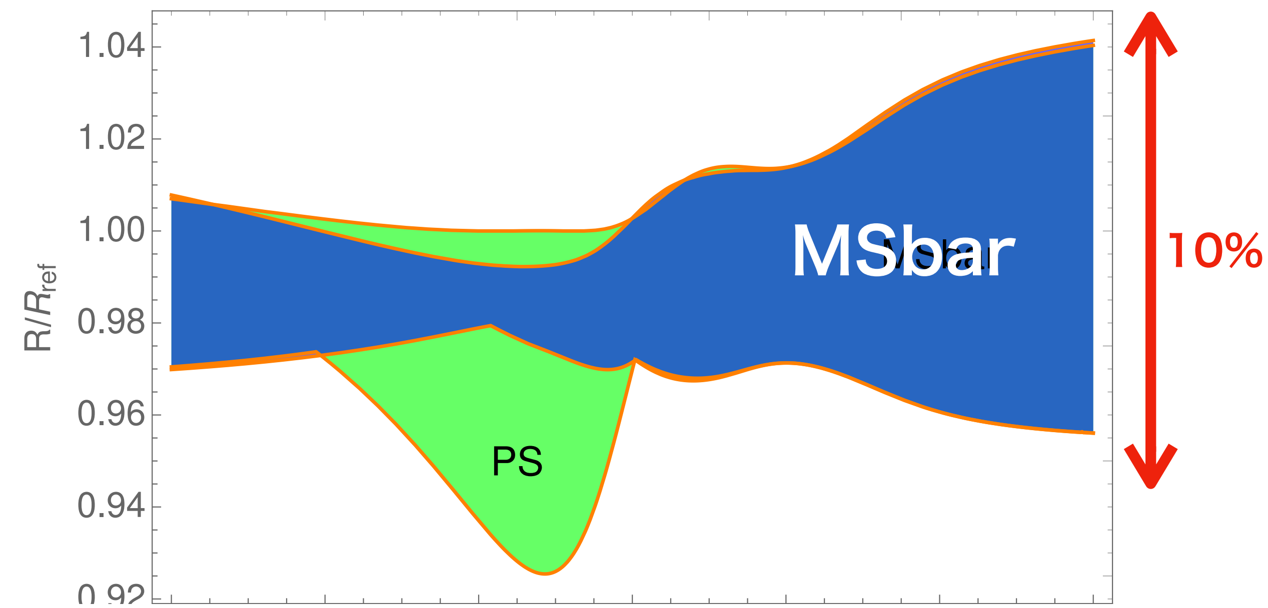
X section order and uncertainty (Pole mass scheme)

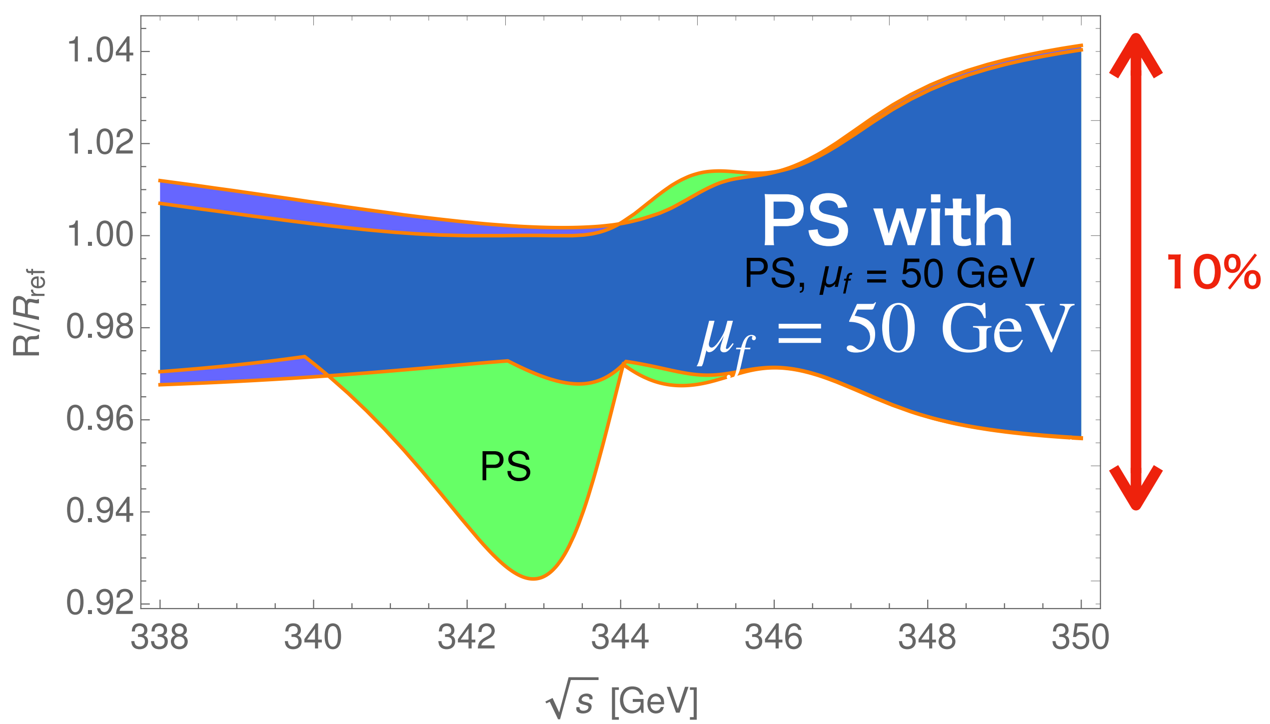


Bad convergence for peak location/height in pole scheme.
(renormalon ambiguity)

relative scale dependence of X section



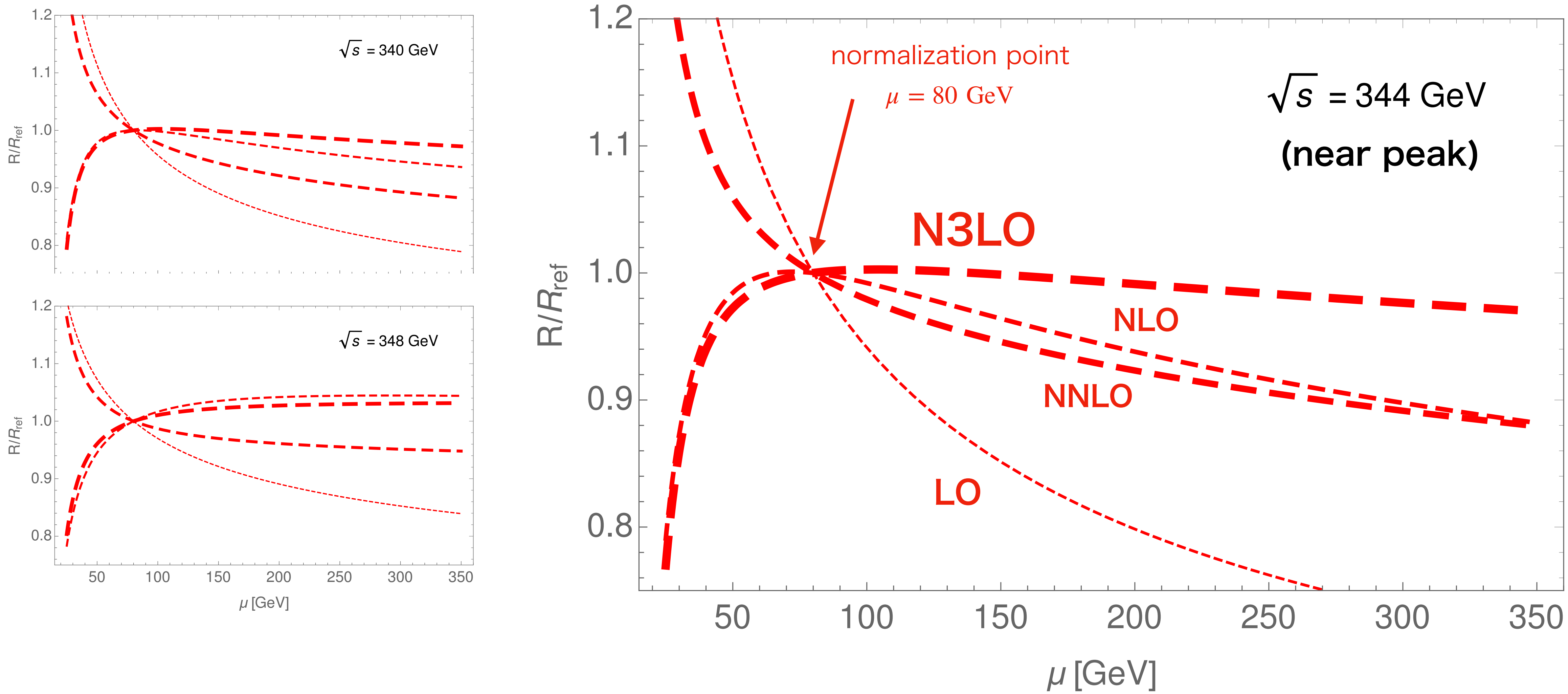




i) scale dependence in short distance mass scheme is 5%(on peak) and 10% above treshold

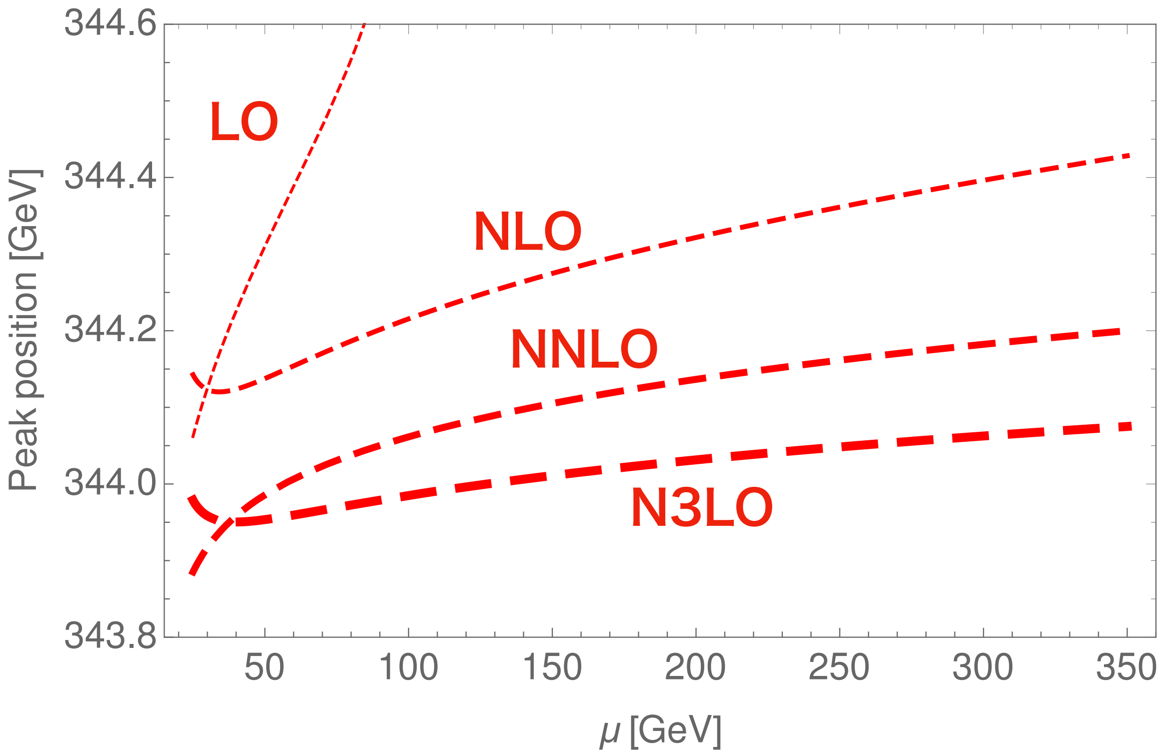
ii) One can observe that the PS mass approaches the MSbar mass as the scale becomes larger ($\mu_f = 50$ GeV)

scale dependence of height



- Convergence and and reduction of μ dependence(〜5% N3LO)

scale dependence of peak location and magnitude



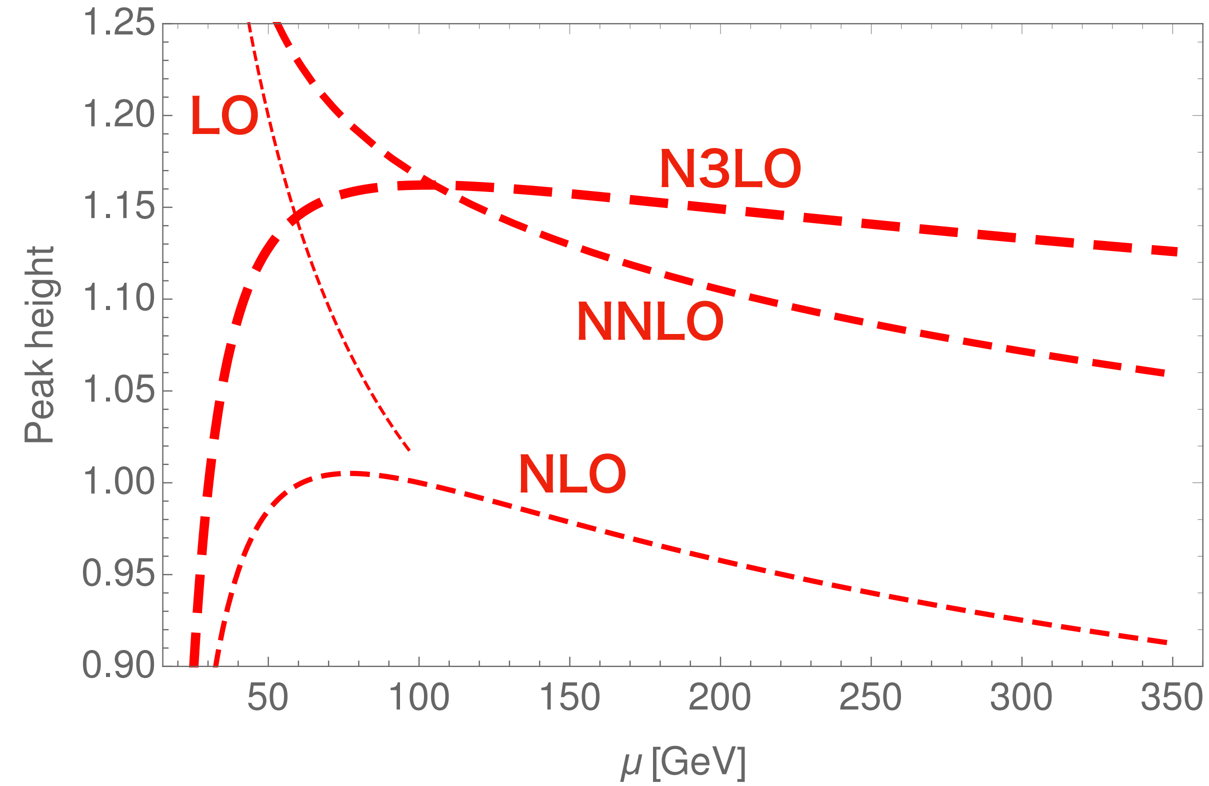
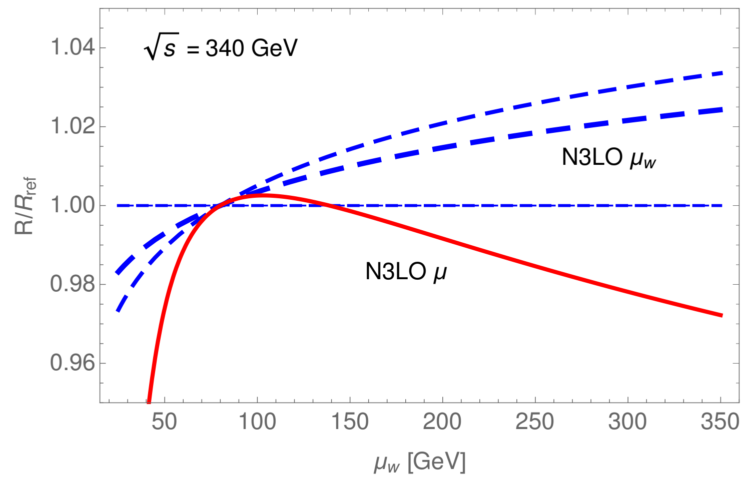
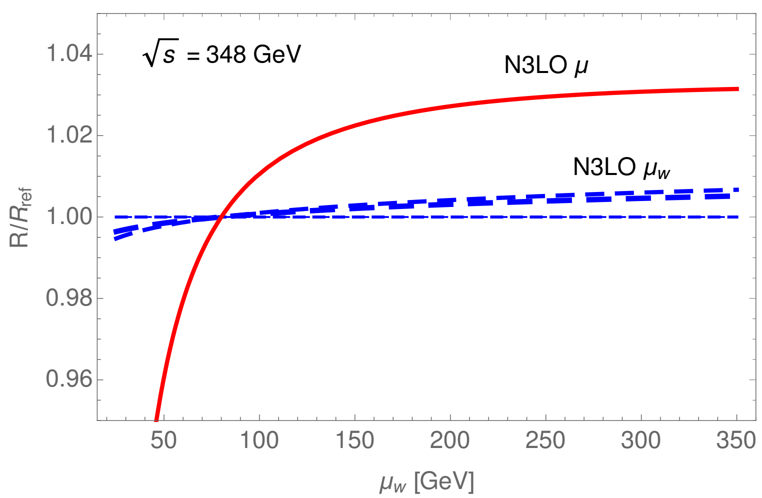


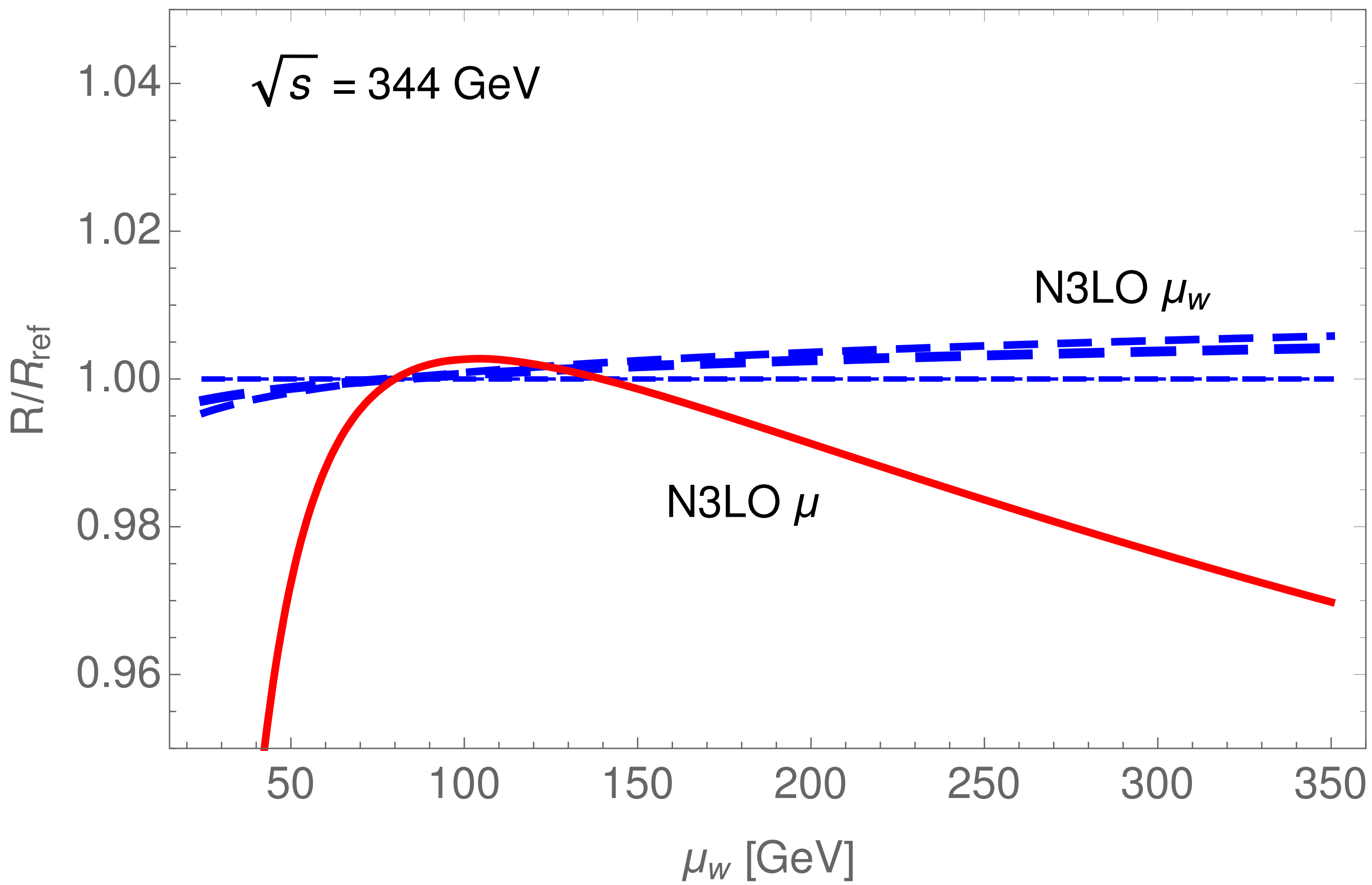
Table 2: Shift of the peak position with respect to the previous order / ± scale variation of the peak position in the four mass schemes discussed in this section. All numbers in units of MeV. At NNNLO, the peak is located at $\sqrt{s}$ = 343.972 (PS shift) / 343.841 (pole) / 343.985 (MSshift) / 343.972 (PS shift, μ_f = 50 GeV) GeV in the four schemes.

Order	PS shift	Pole	MS shift	PS shift, $\mu_f = 50$ GeV
NLO	$-367/\pm 145$	$-1167/\pm 385$	$+1008/\pm 482$	$+154/\pm 44$
NNLO	$-149/\pm 107$	$-571/\pm 304$	$+46/\pm 103$	$+2/\pm 16$
NNNLO	$-65/\pm 61$	$-333/\pm 228$	$-81/\pm 26$	$-9/\pm 13$

μ_w dependence





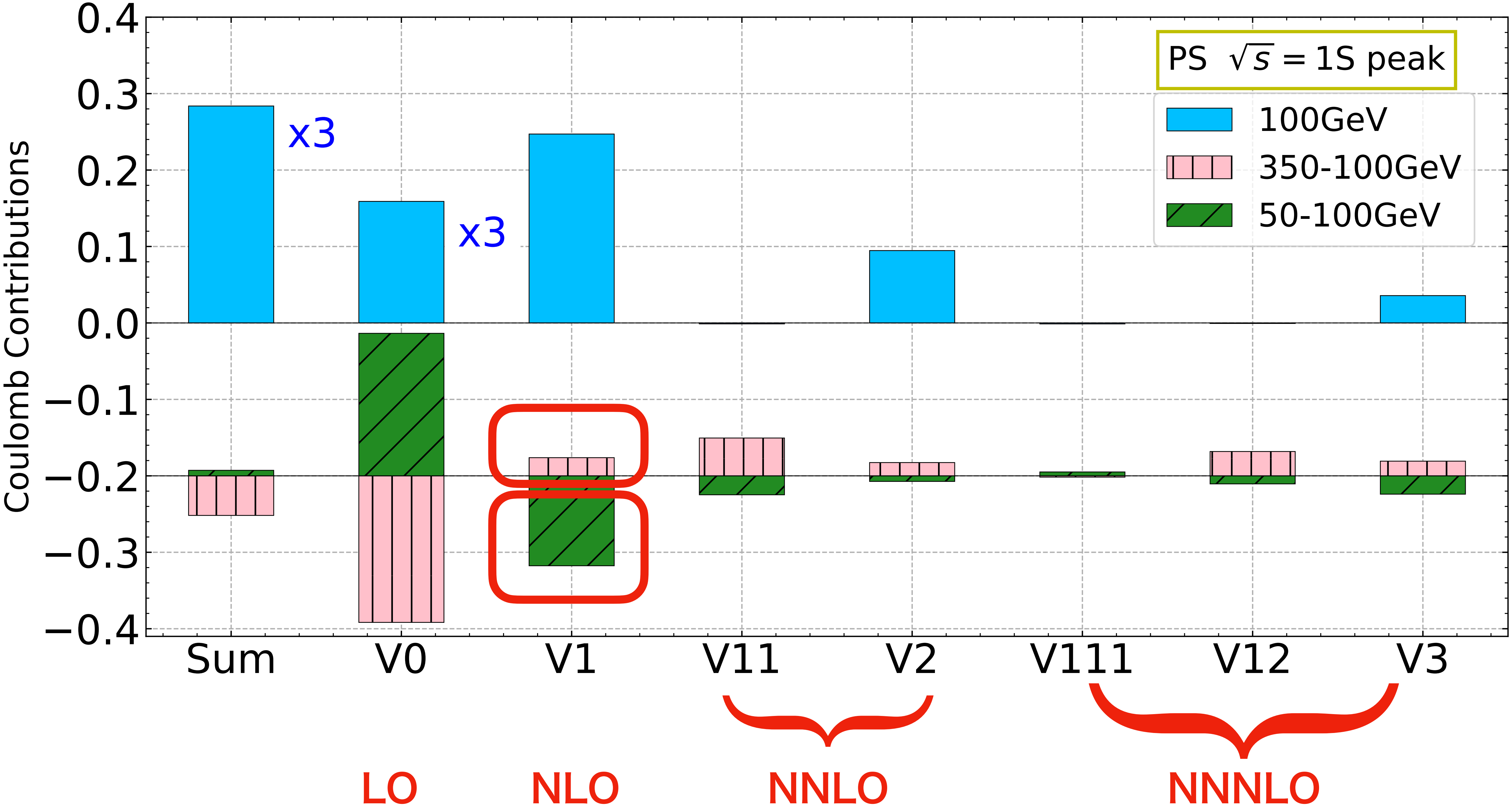


Finite-width(μ_w) scale dependence of the cross section($\mu = 80$ GeV)

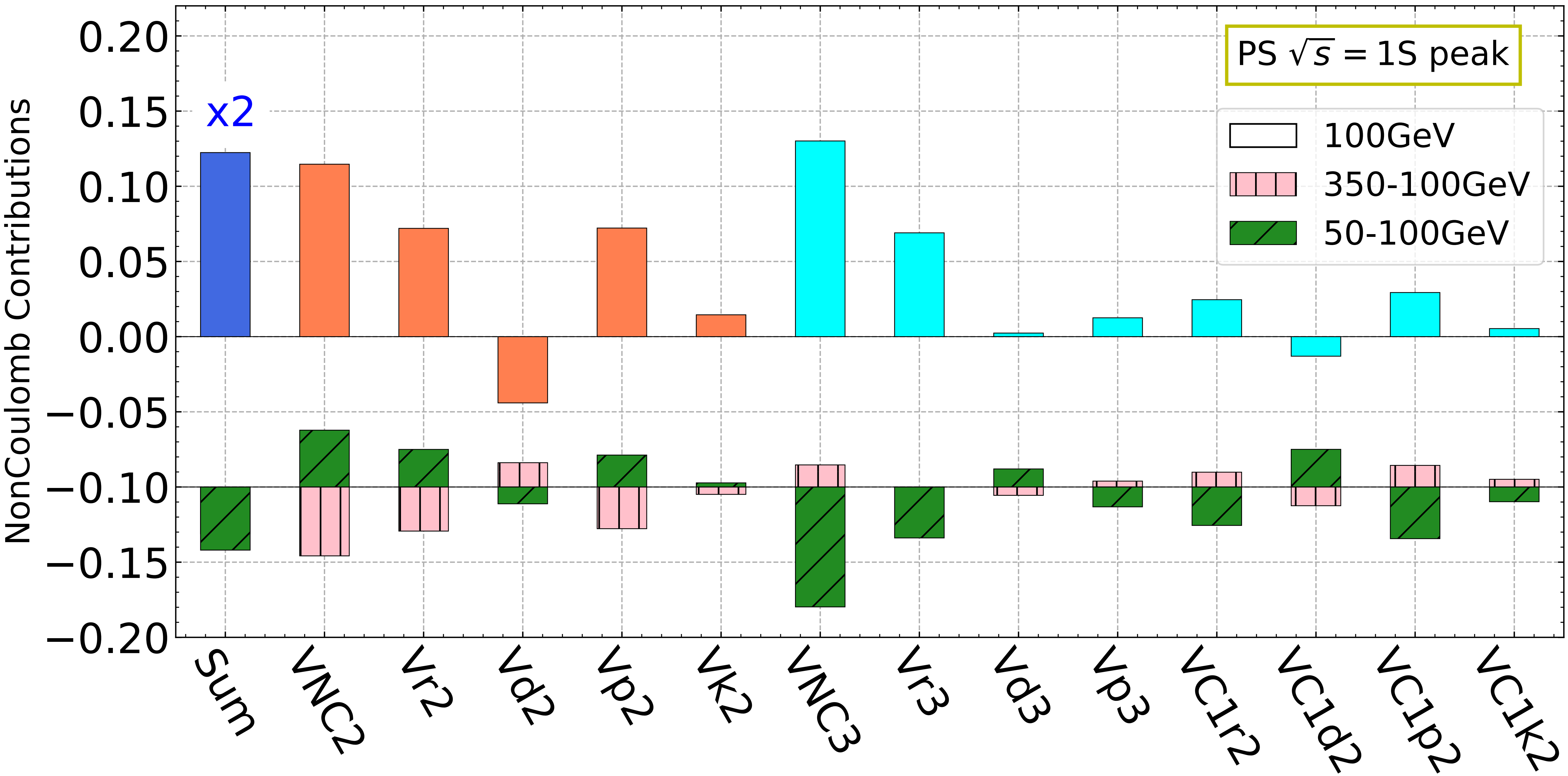
Our QCD computation contains uncanceled μ_w dependence at N3LO, but it is numerically smaller than μ-dependence.

Contributions

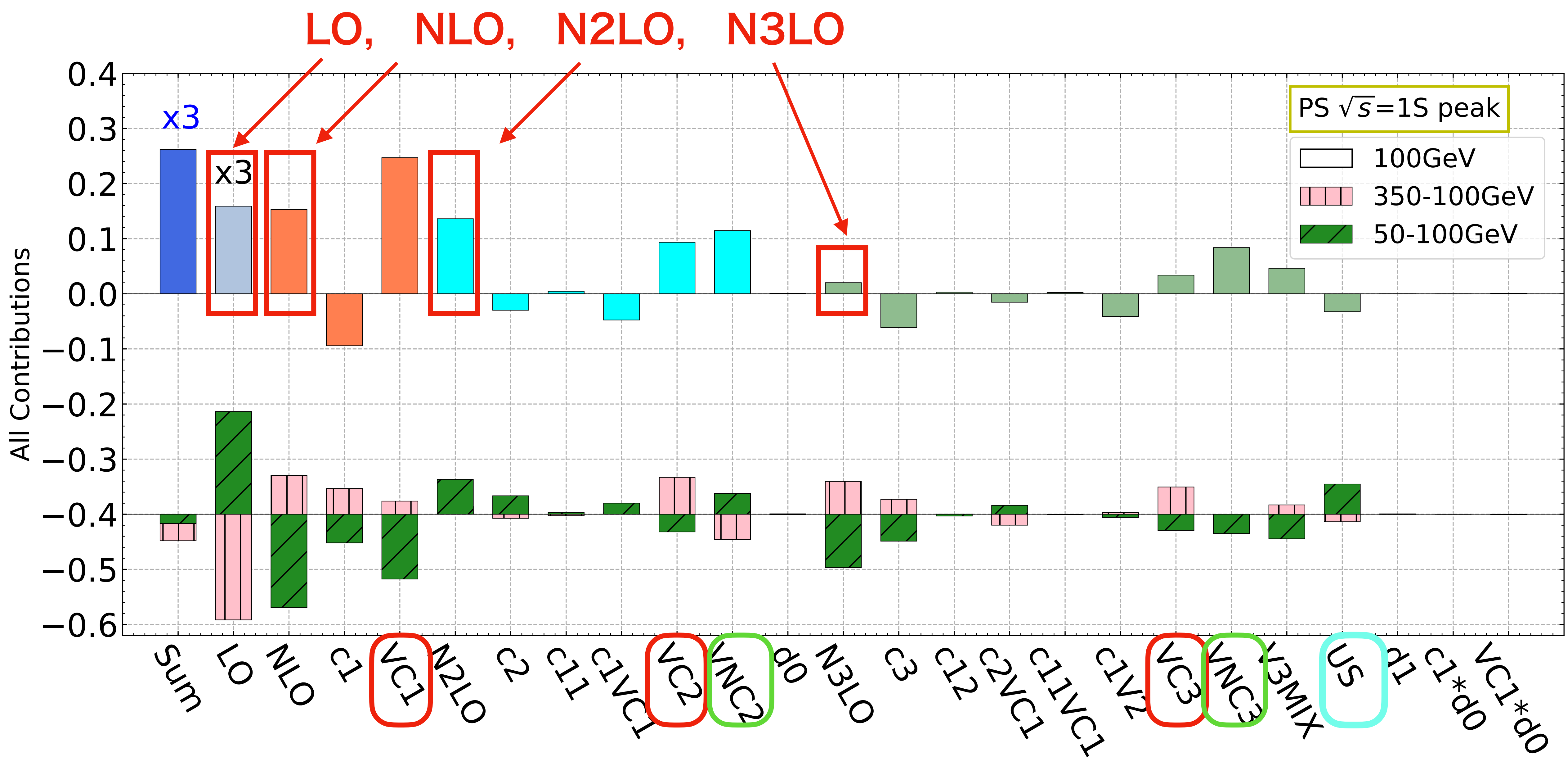
Coulomb contributions



NonC contributions



all contributions



Summary

- **$e^+e^- \to t\bar{t}X$ cross section at N3LO is complete in framework of pNRQCD approach**
  - scale(μ)-cancellation is checked order by order to N3LO (partially numerically)
  - finite-width scale(μ_w)-dependence is computed (for non-resonant counter term)
- **C++ code for simulation study is available in public**
  - QQbar_threshold(2016), placed at hepforge.org
  - Phenomenological analysis, update for newly computed corrections (ongoing)