Toponium simulations at the LHC

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In collaboration with Kaoru Hagiwara, Kai Ma & Ya-Juan Zhang:

- PRD 104 (2021) 034023
- arXiv:2411.NNNNN [hep-ph]

LHC Top WG meeting

CERN – 13 November 2024

Toponium production - a tale of scales…

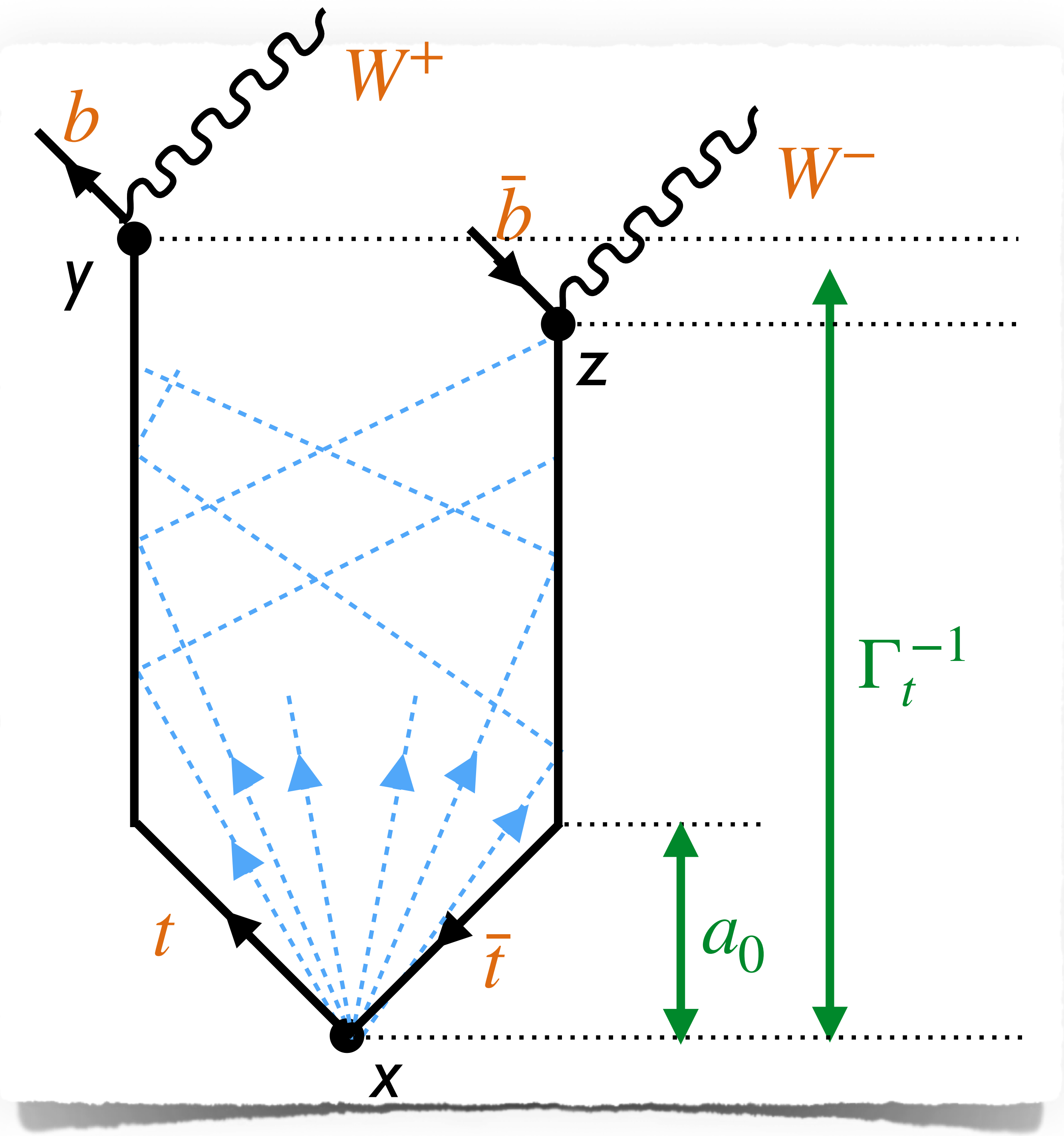
Three important scales at play

- No hadronisation, no top mesons
  ➙ Top decay before hadronisation ⟷ $\Lambda_{\rm QCD} \ll \Gamma_t$
- QCD interactions of a $t\bar{t}$ bar before it decays
  ➙ As a two-particle state
  ➙ Gluon exchanges
- Comparison with the top Bohr radius $a_0 = \dfrac{1}{C_F\,\alpha_s\,m_t/2}$
- Possible gluon exchanges before the top decay
  ➙ Toponium bound-state effects

$$a_0 \ll \frac{1}{\Gamma_t} \ll 1\ \text{fm}$$

(20 GeV) $^{-1}$ (1.5 GeV) $^{-1}$ (0.2 GeV) $^{-1}$

Top-antitop production near threshold



[Fadin & Khoze (JETP Lett`87)]
[Fadin, Khoze & Sjöstrand (Z.Phys.C`90)]
[Sumino, Fujii, Hagiwara, Murayama & Ng (PRD`93)]

A position-space picture

- A $t\bar{t}$ pair created at $x = (t_0, \vec{x})$ ➙ two-particle state
  - ≃ wave packet propagating until the QCD potential barrier
    - ➙ Typical scale: the Bohr radius a_0
- Oscillations within the potential barrier until the system decays
  - ➙ Top $[y = (t_1, \vec{y})]$ or antitop $[z = (t_2, \vec{z})]$ decay
  - ➙ Typical scale $\simeq \Gamma_t^{-1}$
- Probe of the QCD potential
  - ➙ toponium effects

Three-point correlation function in the non-relativistic limit

$$K_{abcd}(x,y,z) = \left\langle 0 \middle| T\left\{ t_c(y)\bar{t}_d(z) : \bar{t}_a(x) t_b(x) : \right\} \middle| 0 \right\rangle$$

$$= \frac{(1+\gamma^0)_{ca}}{2}\,\frac{(1-\gamma^0)_{bd}}{2} \int \mathrm{d}^3 r \Big[K_1\big(y; (z^0, \vec{r})\big)\, K_2(z^0, \vec{r}, \vec{z}; x^0, \vec{x}, \vec{x}) + K_1\big(z; (y^0, \vec{r})\big)\, K_2(y^0, \vec{y}, \vec{r}; x^0, \vec{x}, \vec{x}) \Big]$$

Antitop-decay first

Top-decay first

Non-relativistic spin projection operators

Momentum space Green's functions (I)

The three-point correlation function in momentum space

$$\widetilde{K}_{abcd}(p_t, p_{\bar{t}}) = \underbrace{\frac{(1-\gamma^0)_{bd}}{2}\,\frac{(1+\gamma^0)_{ca}}{2}}_{\text{Spin structure}}\;\underbrace{\widetilde{G}(E;p^*)}_{\text{Toponium Green's function}}\;\underbrace{\left[D(p_t)+D(p_{\bar{t}})\right]}_{\text{Top \& antitop propagators}}$$

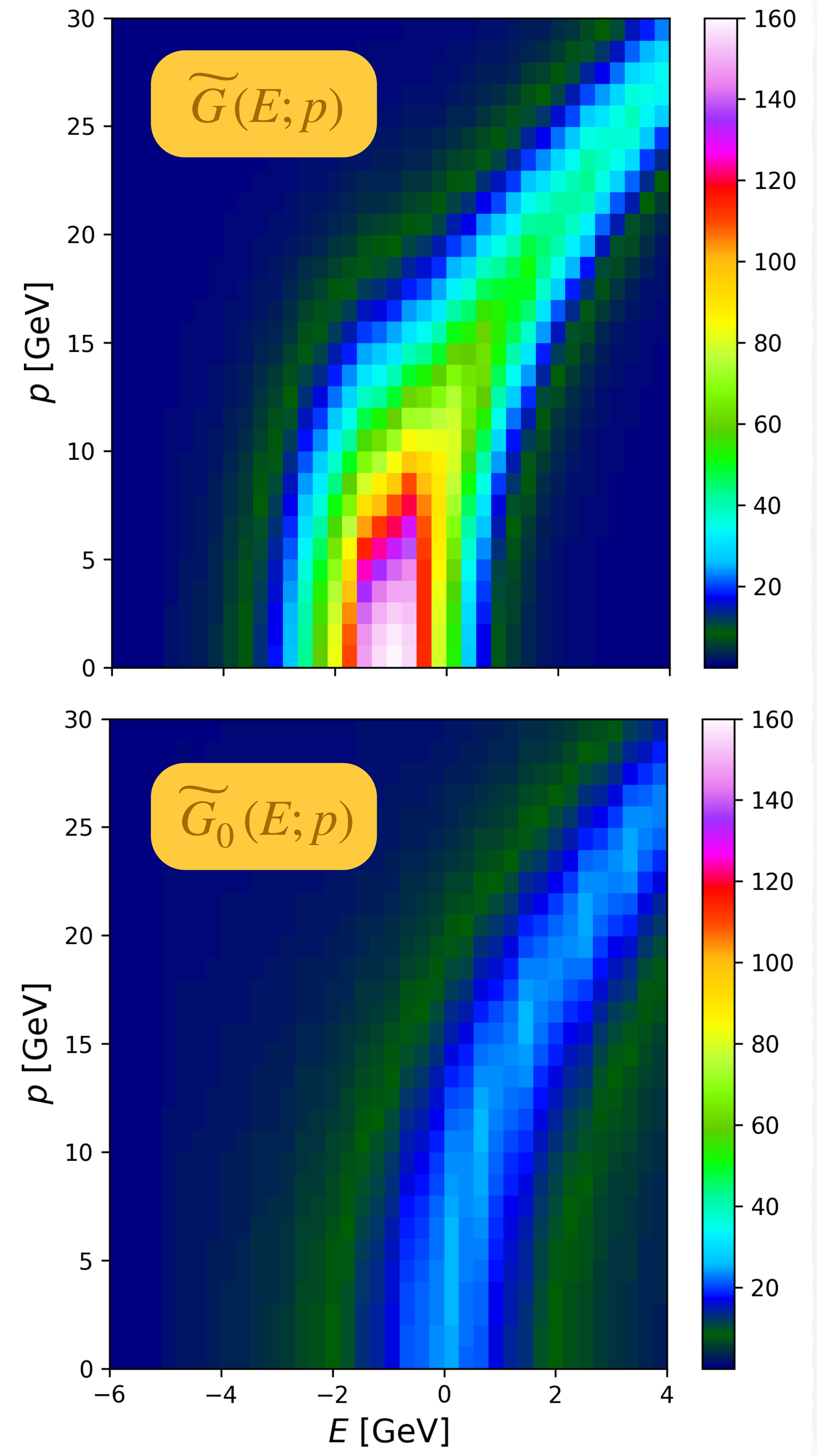
- $D(p)$: $t/\bar{t}$ non-relativistic propagator
- $\widetilde{G}(E;p^*)$: toponium Green's function
  - ➞ dependance on the binding energy E
  - ➞ dependance on the top recoil momentum p^*

The toponium Green's function

- Solution to the Lippmann-Schwinger equation
  - ➞ Fourier transform of the QCD potential

$$\widetilde{G}(E;p) = \underbrace{\widetilde{G}_0(E;p)}_{\text{Free Green's function}} + \int \frac{\mathrm{d}^3q}{(2\pi)^3}\,\widetilde{V}_{\mathrm{QCD}}(\vec{p}-\vec{q})\,\widetilde{G}(E;q)$$

- To be solved numerically **[Jezabek, Kuhn & Teubner (Z.Phys.C`92)]**
  - ➞ choice: Coulomb potential



Momentum space Green's functions (2)

Strong impact on the *(E,p)* distribution

- $\neq$ normalisation (factor of a few)
- $\neq$ shapes above and below threshold!
- Breit-Wigner behaviour

  ➙ $\widetilde{G}_0 \equiv \forall E$, peak at $p_{\text{peak}} \simeq \dfrac{2m_t + E}{2}\sqrt{1 - \dfrac{4m_t^2}{(2m_t+E)^2}}$

  ➙ $\widetilde{G} \equiv \forall E \geq 2$ GeV (different normalisation persists at low E)

- Significant toponium effects between −4 GeV and 0 GeV

Green's function ratio as a seed for toponium modelling

- Valid near threshold

  ➙ Non-relativistic matrix element (relativistic effects negligible)

  ➙ Start from standard matrix element

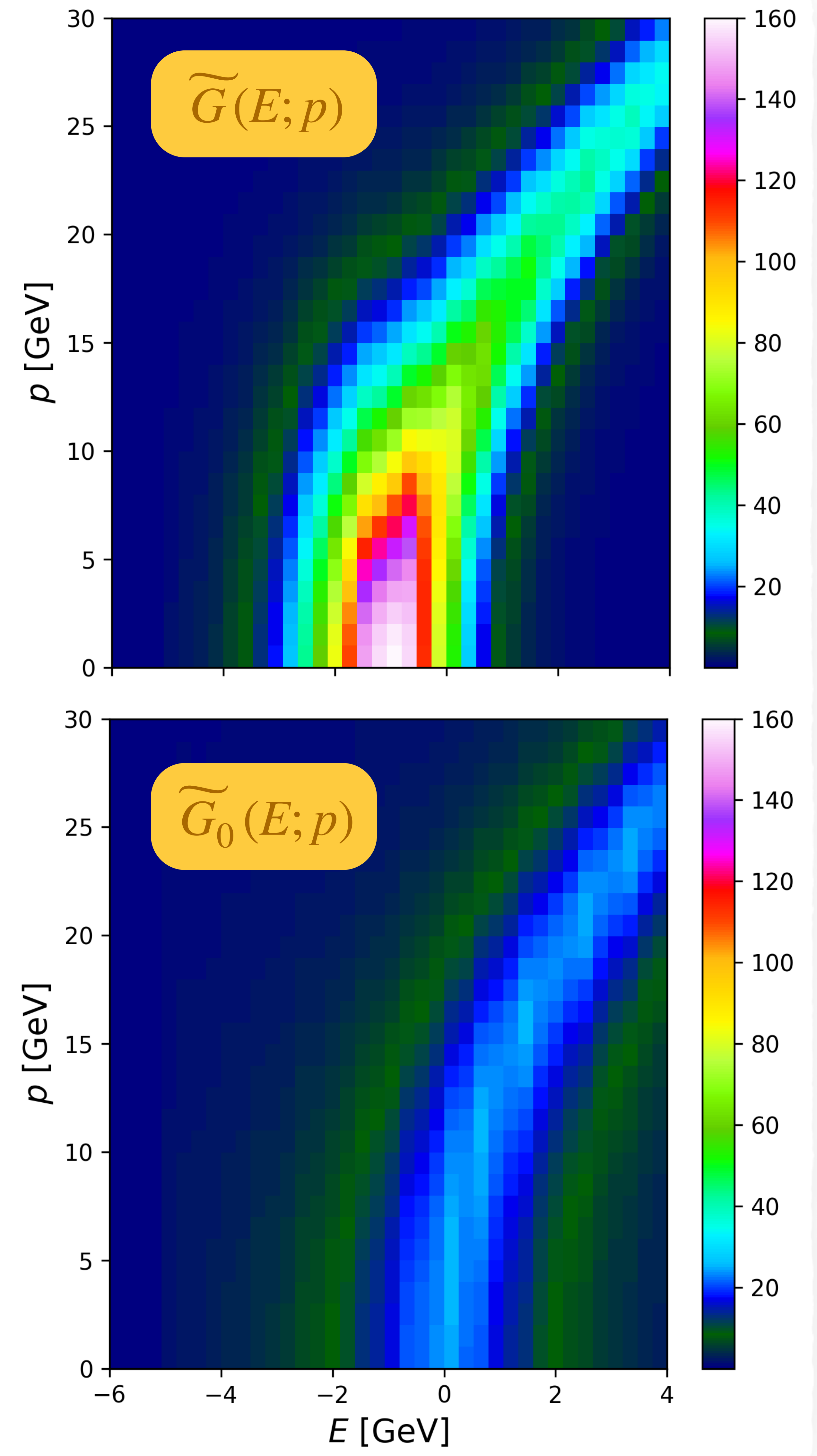
- Matrix-element re-weighting

  ➙ Ratios of non-relativistic Green's functions

  ➙ For $E < 4$GeV and $p^* < 50$ GeV

  ➙ Use within custom event generators

$$iM^{(c)} \to iM^{(c)} \times \frac{\widetilde{G}(E;p^*)}{\widetilde{G}_0(E;p^*)}$$



$t\bar{t}$ production near threshold with MG5aMC

Process: $gg \to t\bar{t} \to b\ell^+\nu_\ell \bar{b}\ell'^-\bar{\nu}'_\ell$

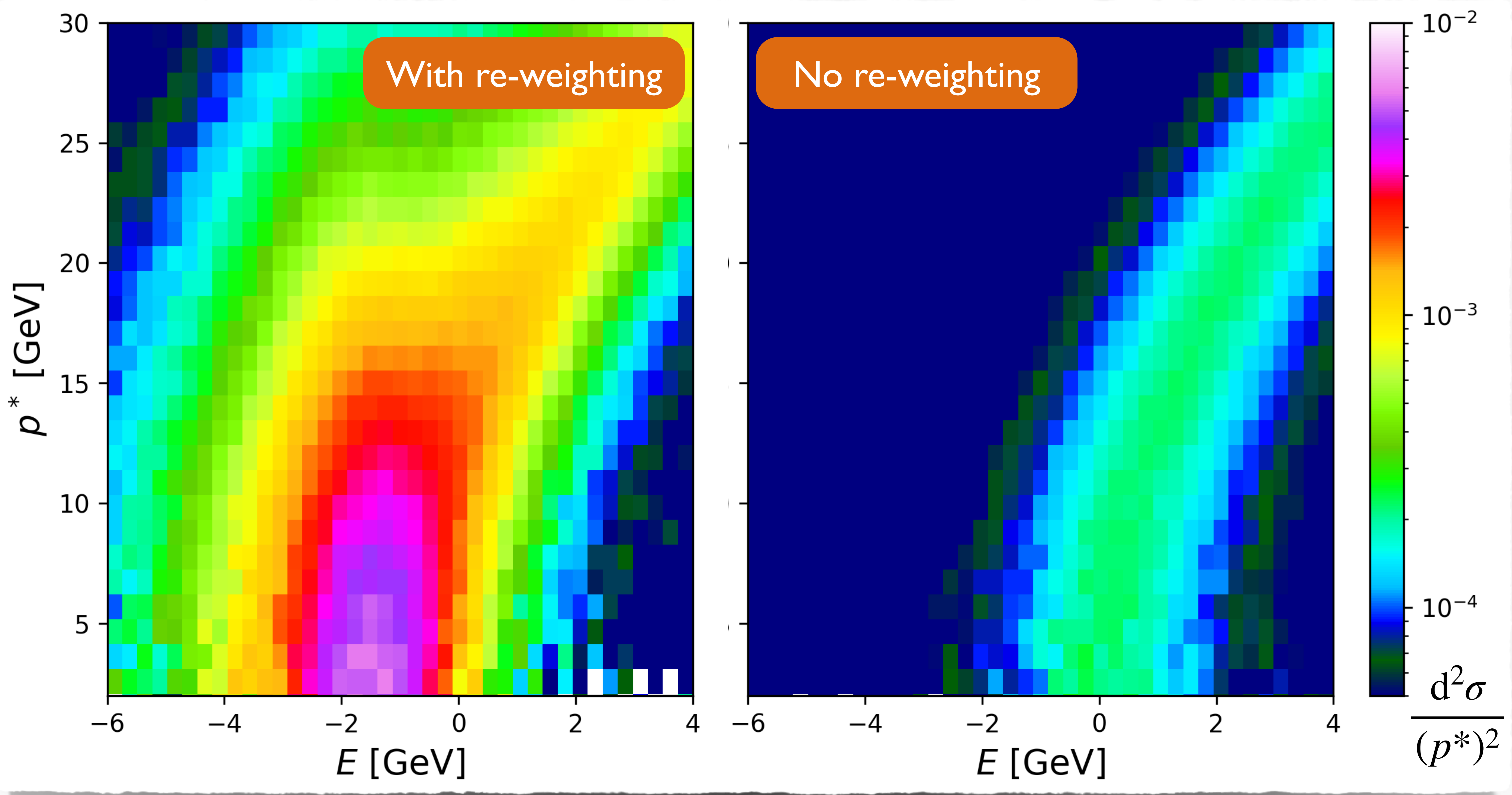
- Six-body final state: spin correlations included
- Projection on the colour singlet
- Matrix-elements with and without re-weighting
- No matching with parton showers

Without re-weighting

- Rates ≡ phase space x Breit-Wigner
- Similar heat map as for $\widetilde{G}_0$

With re-weighting

- Normalisation and shape affected
- Access to the QCD Green's function
  ➙ ratio of the re-weighted/non-re-weighted predictions



Typical top momentum in the toponium rest frame

$$\langle p(E)\rangle = \frac{\int p^3\,\mathrm{d}p\,\frac{\mathrm{d}\sigma}{p^2\mathrm{d}p\mathrm{d}E}}{\int p^2\,\mathrm{d}p\,\frac{\mathrm{d}\sigma}{p^2\mathrm{d}p\mathrm{d}E}}$$

➡ for $E \simeq -2$ GeV: 20 GeV (the Bohr radius!)

[Hagiwara, BF, Ma & Zhang (2411.NNNNN)]

Invariant masses: tops, toponium

$m_{t\bar{t}}$ invariant mass distribution

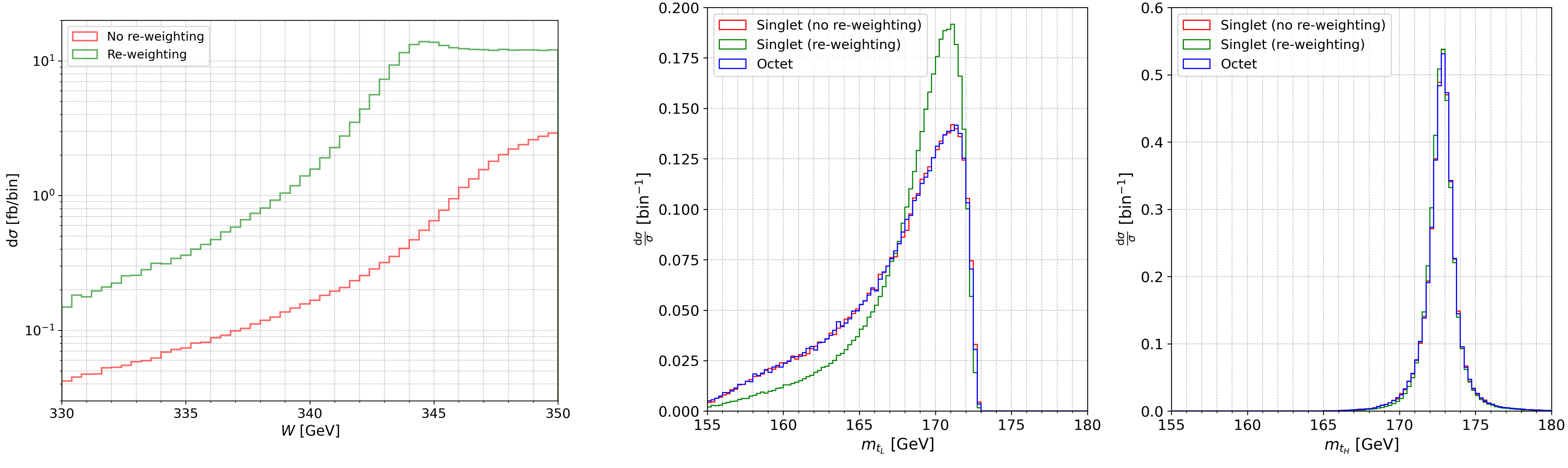
- Peak at $E \simeq -2$ GeV $\oplus$ extended bound state effects
- Shape in agreement with pNRQCD
- Normalisation 20% lower (*cf.* octet contributions)

⚠️ Comparison (relativistic pieces, octet, α_s, PDFs, *etc.*)

[Sumino & Yokoya (JHEP`10)]

Top mass distributions

- Heavy top ↔ Breit-Wigner
  - ➞ Effectively stable until t_L decays
  - ➞ No QCD potential form the other top when it decays
- Light top ↔ invariant mass shifted to lower values
  - ➞ Governed by the QCD Green's function
  - ➞ Bound state within the Coulomb potential generated by t_H



[Hagiwara, BF, Ma & Zhang (2411.NNNNN)]

Validity of pseudo-scalar toy models?

A toponium toy Lagrangian with a pseudo-scalar state

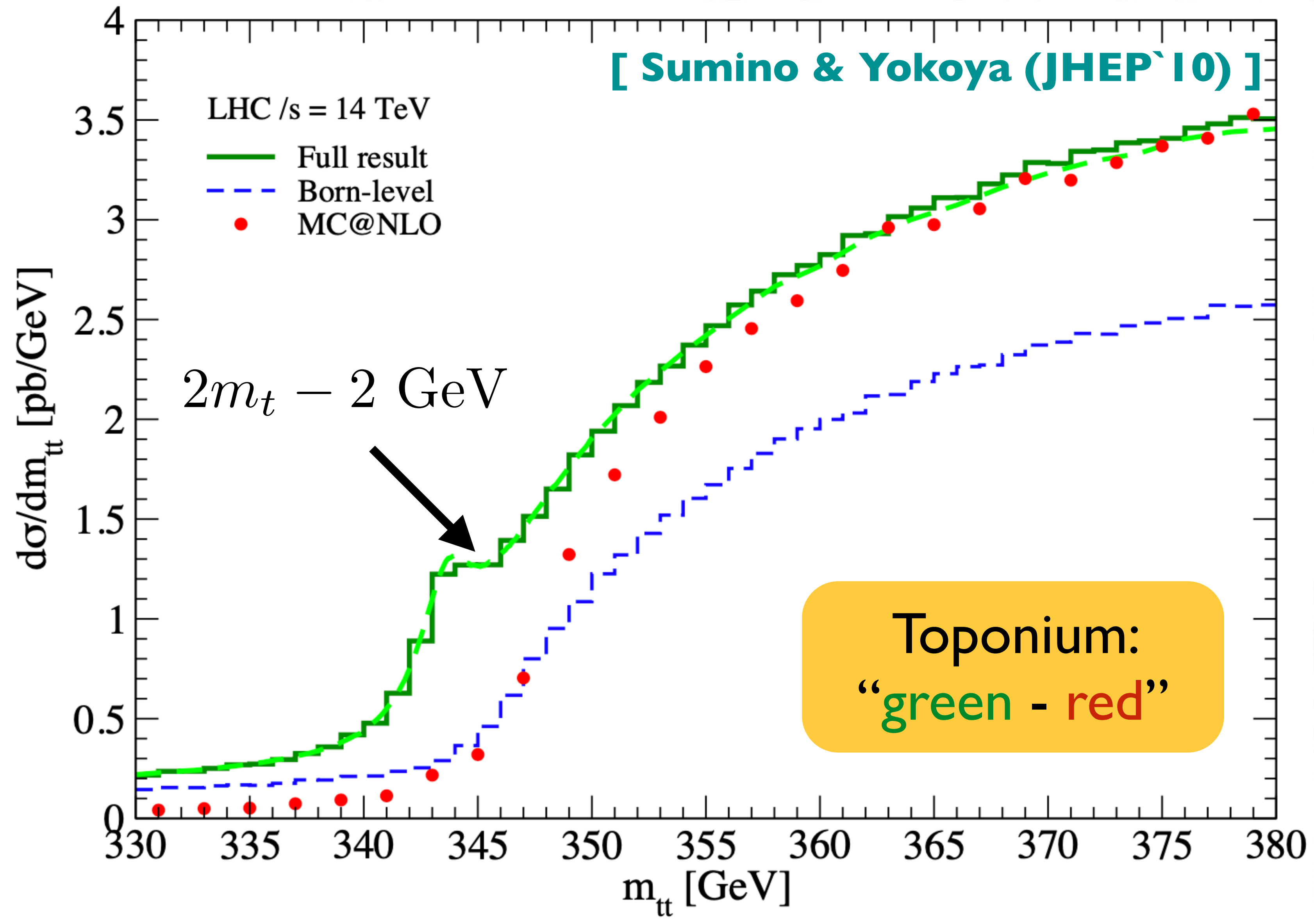
$$\mathcal{L}_{\eta_t} = \frac{1}{2}\partial_\mu\eta_t\partial^\mu\eta_t - \frac{1}{2}m_{\eta_t}\eta_t^2 - \frac{1}{4}g_{gg}\eta_t G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - ig_{tt}\eta_t\bar{t}\gamma_5 t$$

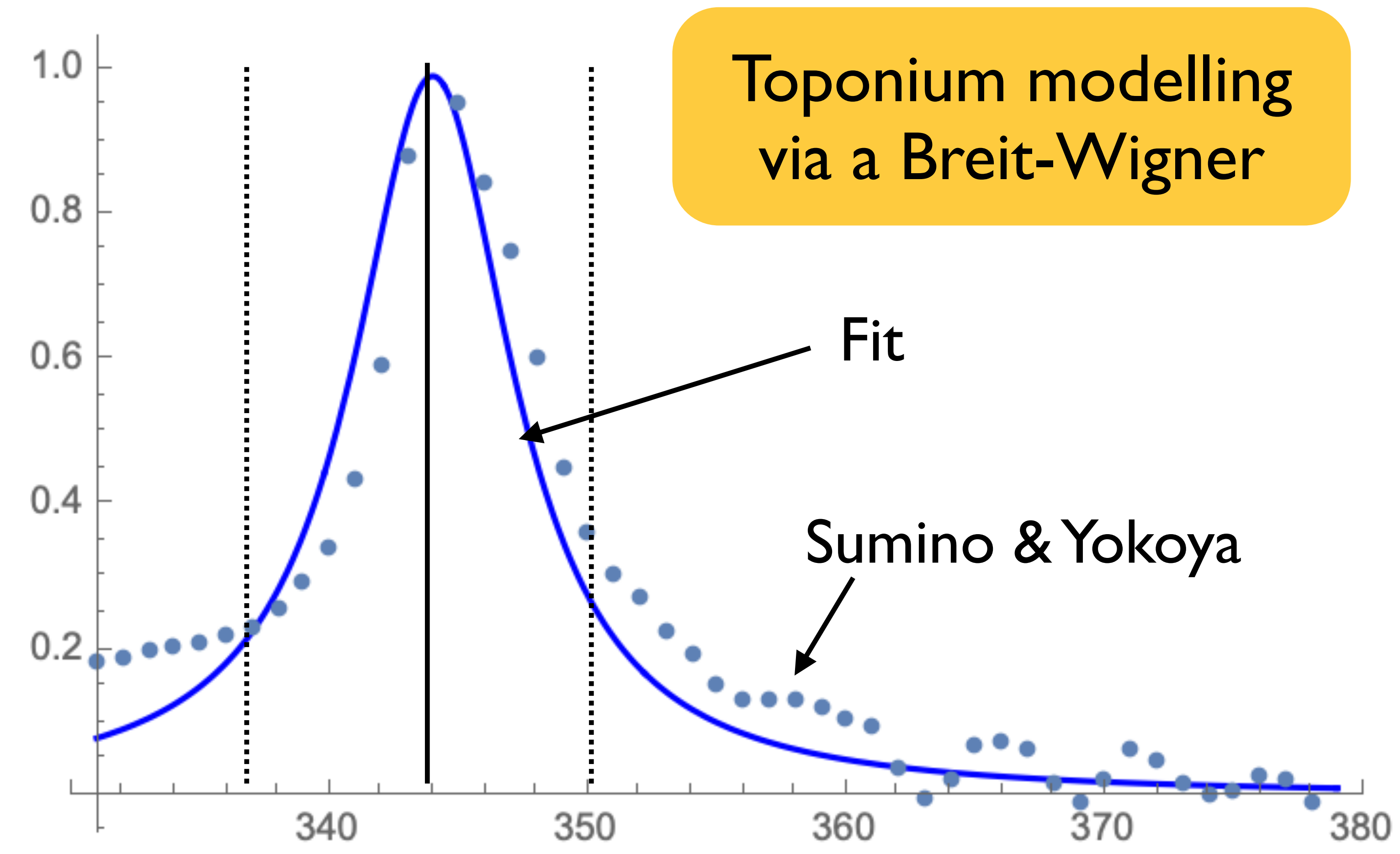
$m_{\eta_t} = 344$ GeV $\quad \Gamma_{\eta_t} \approx 7$ GeV $\quad \sigma(13\ \text{TeV}) \sim 6.5$ pb

- No free parameters [$m/\Gamma/\sigma$ known]
- Non-perfect Breit-Wigner fit ➙ impact relative to pNRQCD

Disclaimer: the toy model should not be used anymore!

- Fitting a non-Breit-Wigner contribution by a Breit-Wigner
- Used in ATLAS/CMS searches & varied pheno works
  ➙ With or without Green's functions re-weighting
  ➙ Including ours [Hagiwara, BF, Ma & Zhang (PRD`21)]
- Differences with pNRQCD predictions
  ➙ Octet/singlet contributions, non-relativistic, matched,…





The toy model *versus* pNRQCD

Predictions with the toy model

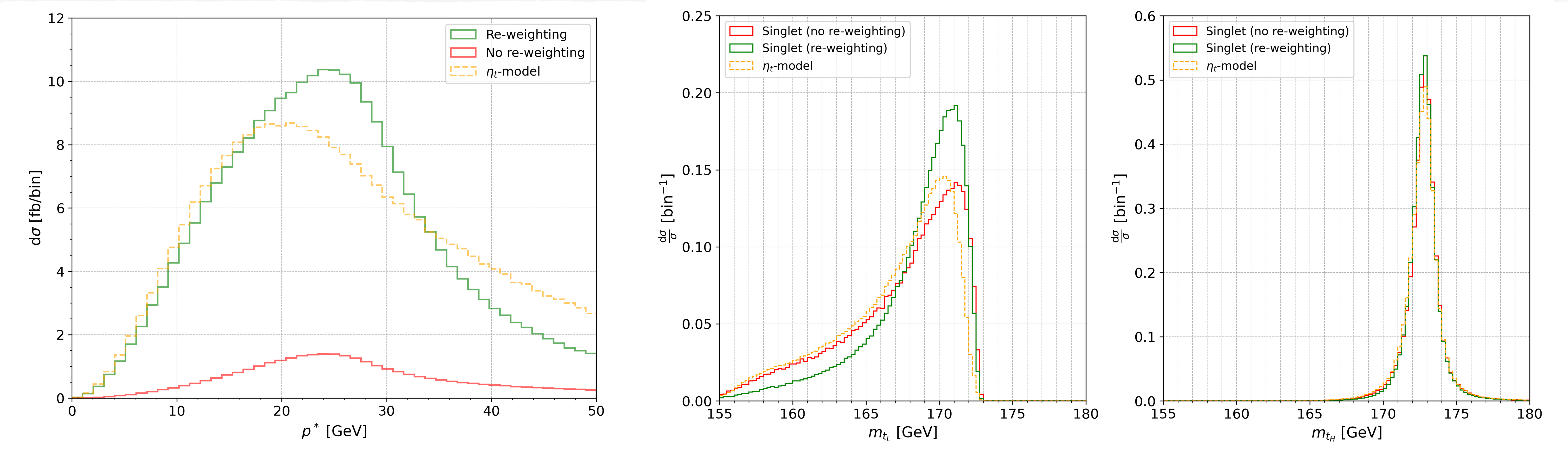
- Without re-weighting (dashed orange)
- As used presently [except in Hagiwara, BF, Ma & Zhang (PRD`21)]

pNRQCD predictions

- With (solid green) and without (solid red) re-weighting

Imperfections of the toy model visible in key distributions

- Small/large effects … easily avoidable
  - ➙ p^* peak at 12-13 GeV
  - ➙ t_L mass mis-modelled around the peak
  - ➙ *etc.*
- Potential of m_{t_L} in single-leptonic $t\bar{t}$ decays
  - ➙ Up to *b*-tagging performance and MET resolution
  - ➙ Direct access to toponium effects!



Seeking toponium at the LHC

Basic cuts targeting a di-leptonic final state:

- 2 leptons / 2 b-jets

Bulk of toponium events (cf. spin density matrices)

- Small $\Delta\varphi_{\ell\ell}$ / $m_{\ell\ell}$

Insights from the toponium system (proxy for $m_{t\bar{t}}$)

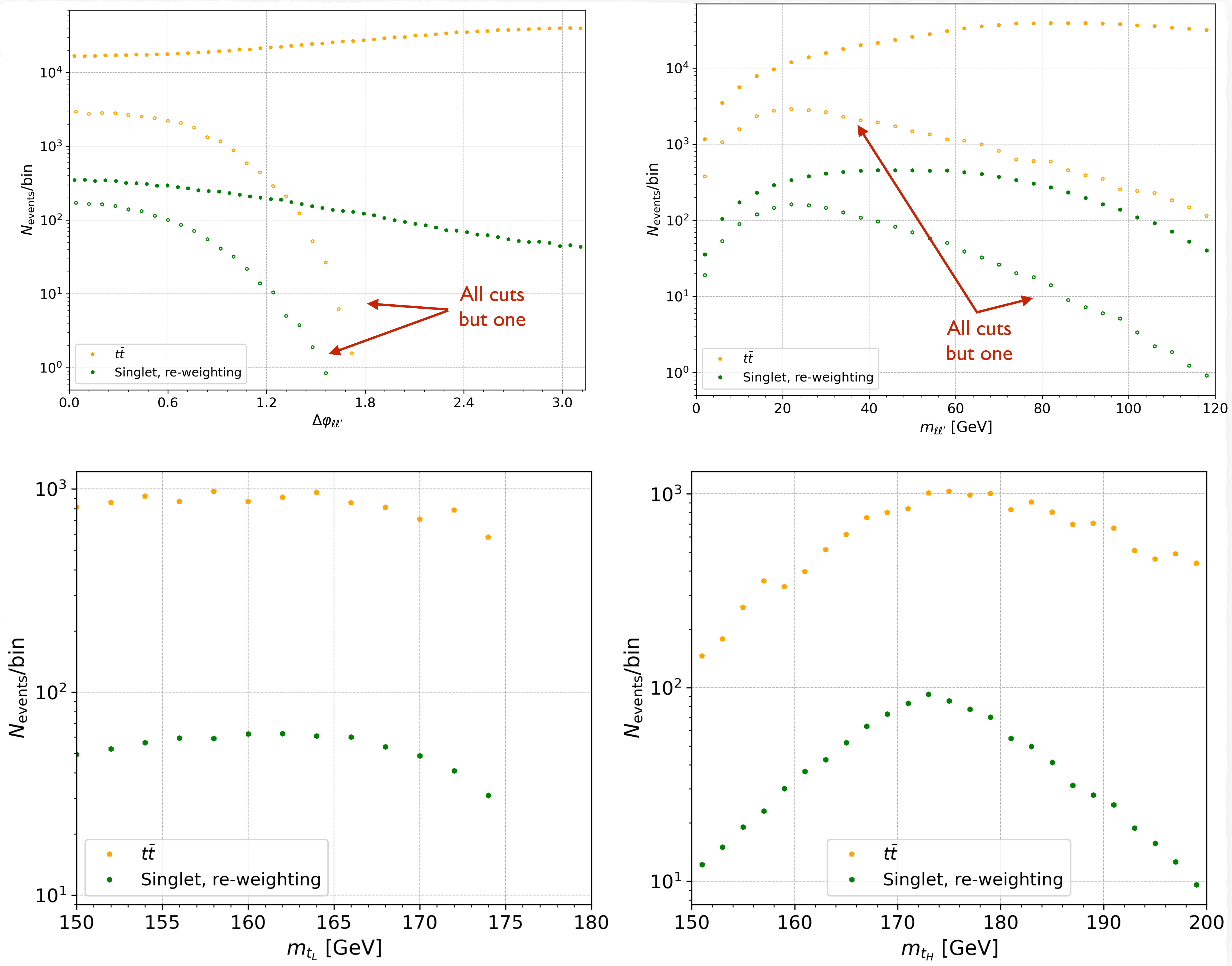
- The $\ell^+\ell^- b\bar{b} + E_T^{\text{miss}}$ system at low transverse mass

Kinematical reconstruction of the t_L/t_H system

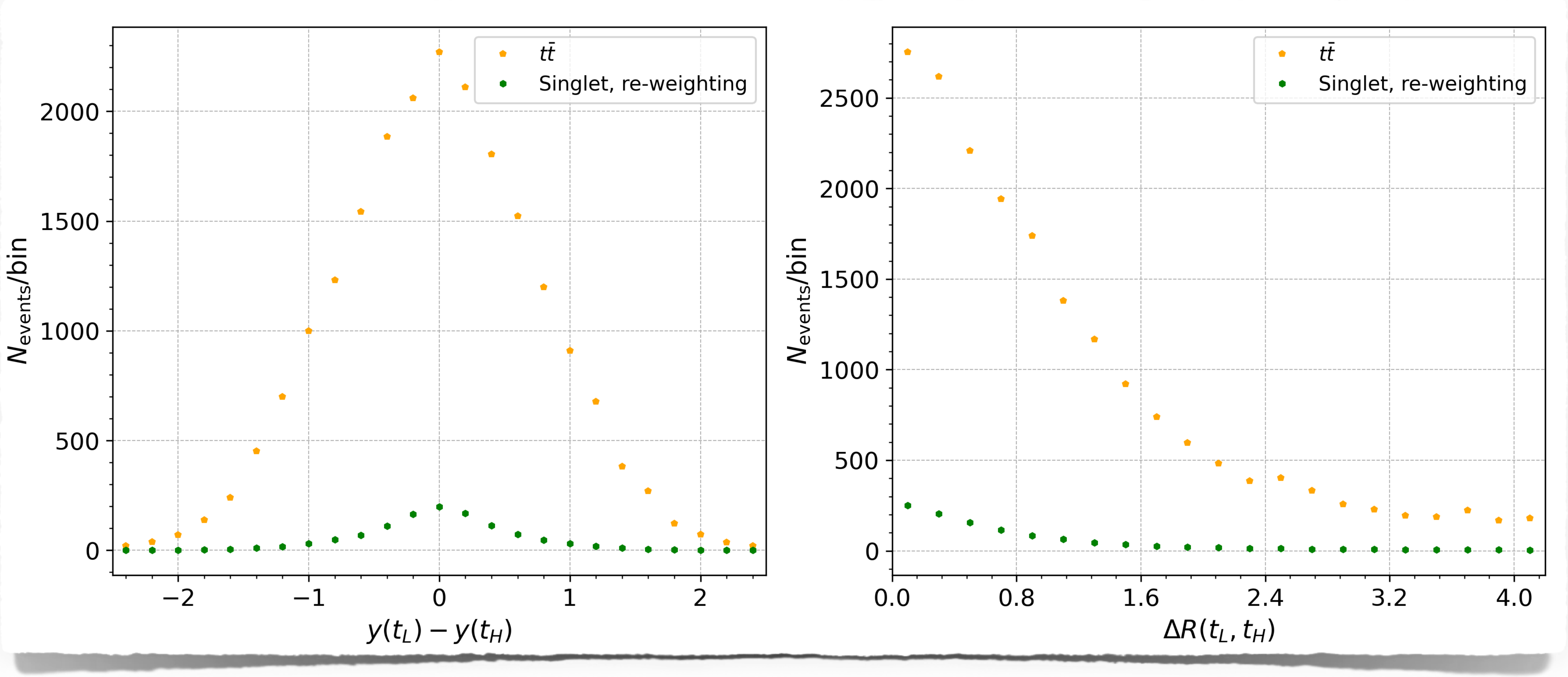
- Assumption: $\vec{p}_T(t_H) = \vec{p}_T(t_L)$
- Identification of all final-state objects
  - ➙ ℓ_1 is the leading lepton, ℓ_2 the sub-leading one
  - ➙ b-jet pairing: enforcing $m(\ell_1, b_1) > m(\ell_2, b_2)$
- Neutrino momenta ➙ '*top*' reconstruction

13 TeV, 140/fb

Cut	$t\bar{t}$	toponium
2ℓ, 2 b-jets	1,330,000	8,400
$\Delta\varphi_{\ell\ell'}$	171,000	3,250
$m_{\ell\ell'}$	77,100	2,200
$m_T(\ell\ell' bb; \vec{p}_T^{\text{miss}})$	42,800	1,970
$t\bar{t}$ kinematical fit	20,800	1,130



Potential key observables: angles

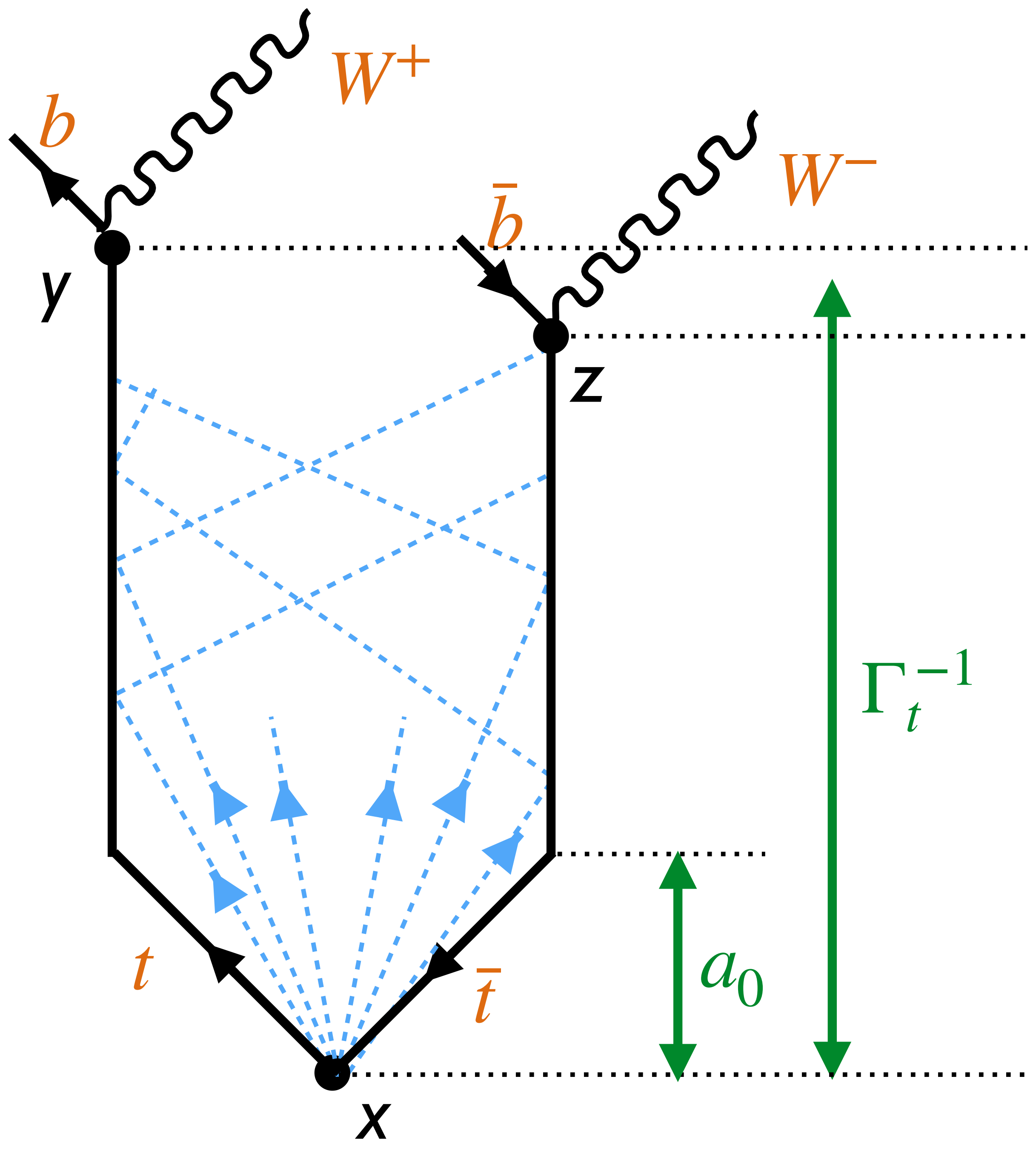


The Δy or ΔR distributions

- Signal: peak at the origin
  ➙ small and similar t_L/t_H momentum in the toponium rest frame
- Relatively insensitive to Green's function re-weighting
- Toponium effects ≡ 10% enhancement

10% effect…
Impossible to miss?

A few last words



Top-antitop production near the threshold

- Emergence of a toponium system at a time scale of 0.05 GeV^{-1}
- Decay at a time scale of $1/\Gamma_t$ ~ 0.6 GeV^{-1}
- Occurs well before hadronisation at 5 GeV^{-1}

Possibility to experimentally probe the toponium wave function

- 'The smallest hadron in the SM'

Simulating toponium formation signals at the LHC

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November 12, 2024

Abstract We present a method to simulate toponium formation events at the LHC using the Green's function of non-relativistic QCD in the Coulomb gauge, which governs the momentum distribution of top quarks in the presence of the QCD potential. This Green's function can be employed to re-weight any matrix elements relevant for $t\bar{t}$ production and decay processes where a colour-singlet top-antitop pair is produced in the S-wave at threshold. As an example, we study the formation of η_t toponium states in the gluon fusion channel at the LHC, combining the re-weighted matrix elements with parton showering.