

# Potential influence from stochastic GW on Spring-8 and other storage rings

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# Outline

- Introduction
- Time domain analysis
- Frequency domain analysis
- Comparison to LIGO
- Conclusion

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SRGW2021



# SRGW2021 - ARIES WP6 Workshop: Storage Rings and Gravitational Waves

2 February 2021 to 31 March 2021

Europe/Zurich timezone

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## Storage Rings and Gravitational Waves:

Summary and Outlook [arXiv:2105.00992](https://arxiv.org/abs/2105.00992)

A. Berlin<sup>1</sup>, M. Brüggen<sup>2</sup>, O. Buchmueller<sup>3</sup>, P. Chen<sup>4</sup>, R. T. D'Agnolo<sup>5</sup>, R. Deng<sup>6</sup>,  
J. R. Ellis<sup>7,\*,\*</sup>, S. Ellis<sup>5</sup>, G. Franchetti<sup>8</sup>, A. Ivanov<sup>9</sup>, J. M. Jowett<sup>8</sup>,  
A. P. Kobushkin<sup>10</sup>, S. Y. Lee<sup>11</sup>, J. Liske<sup>2</sup>, K. Oide<sup>12</sup>, S. Rao<sup>2</sup>, J. Wenninger<sup>13</sup>,  
M. Wellenzohn<sup>9</sup>, M. Zanetti<sup>14</sup>, F. Zimmermann<sup>13,\*,†</sup>

***We succeeded A. N. Ivanov  
and A. P. Kobushkin***

## Storage rings as detectors for relic gravitational-wave background ?

A. N. Ivanov,<sup>1,\*</sup> [A. P. Kobushkin](#),<sup>2,†</sup> and M. Wellenzohn<sup>1,3,‡</sup>

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*Kiev and Physical and Technical National University KPI, Prospect Pobedy 37, 03056, Kiev, Ukraine*

<sup>3</sup>*FH Campus Wien, University of Applied Sciences, Favoritenstraße 226, 1100 Wien, Austria*

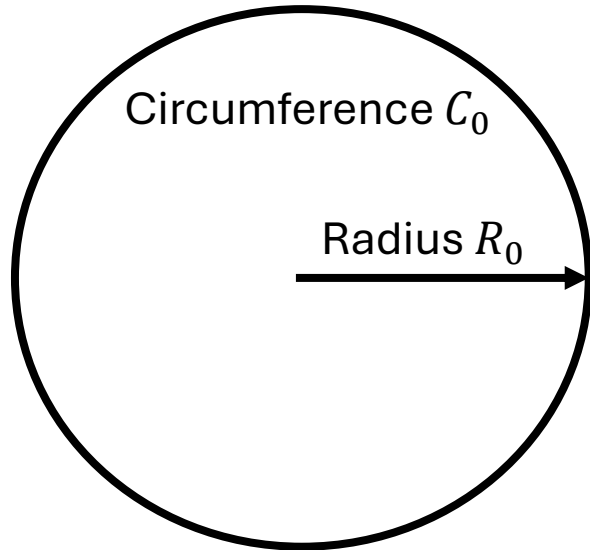
(Dated: March 31, 2021)

arXiv:gr-qc/0210091

# Main idea

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}\eta^{\mu\nu}(h_{\nu\alpha,\beta} + h_{\nu\beta,\alpha} - h_{\alpha\beta,\nu}) - \frac{1}{2}h^{\mu\nu}(h_{\nu\alpha,\beta} + h_{\nu\beta,\alpha} - h_{\alpha\beta,\nu}) + O(h^3)$$

→ Laser interferometer



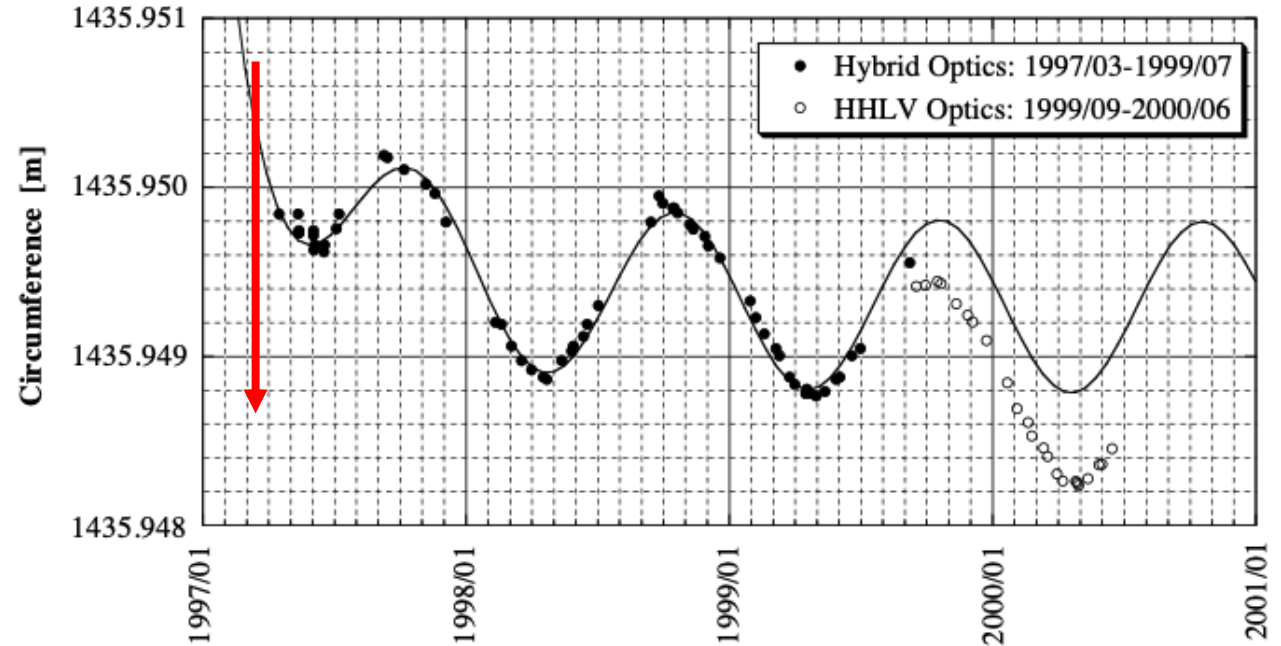
$$\frac{1}{r} \frac{d^2 r}{dt^2} = -\frac{1}{4} \left[ \left( \frac{\partial h_{xx}}{\partial t} \right)^2 + \left( \frac{\partial h_{xy}}{\partial t} \right)^2 \right]$$

$$\rightarrow \frac{1}{C_0} \frac{dC(t)}{dt} \sim \frac{1}{16} h_0^2 \omega \sin(2\omega t + 2\delta)$$

Different from S. Rao et al PRD 102 122006 (2020) and arXiv:2301.08331 where

$$\Delta T_{GW}(T) = -\frac{1}{2} \left( 1 - \frac{v_i^2}{2c^2} \right) \int_{t_0}^{t_0+T} (F_+ \mathbf{h}_+ + F_\times \mathbf{h}_\times) dt$$

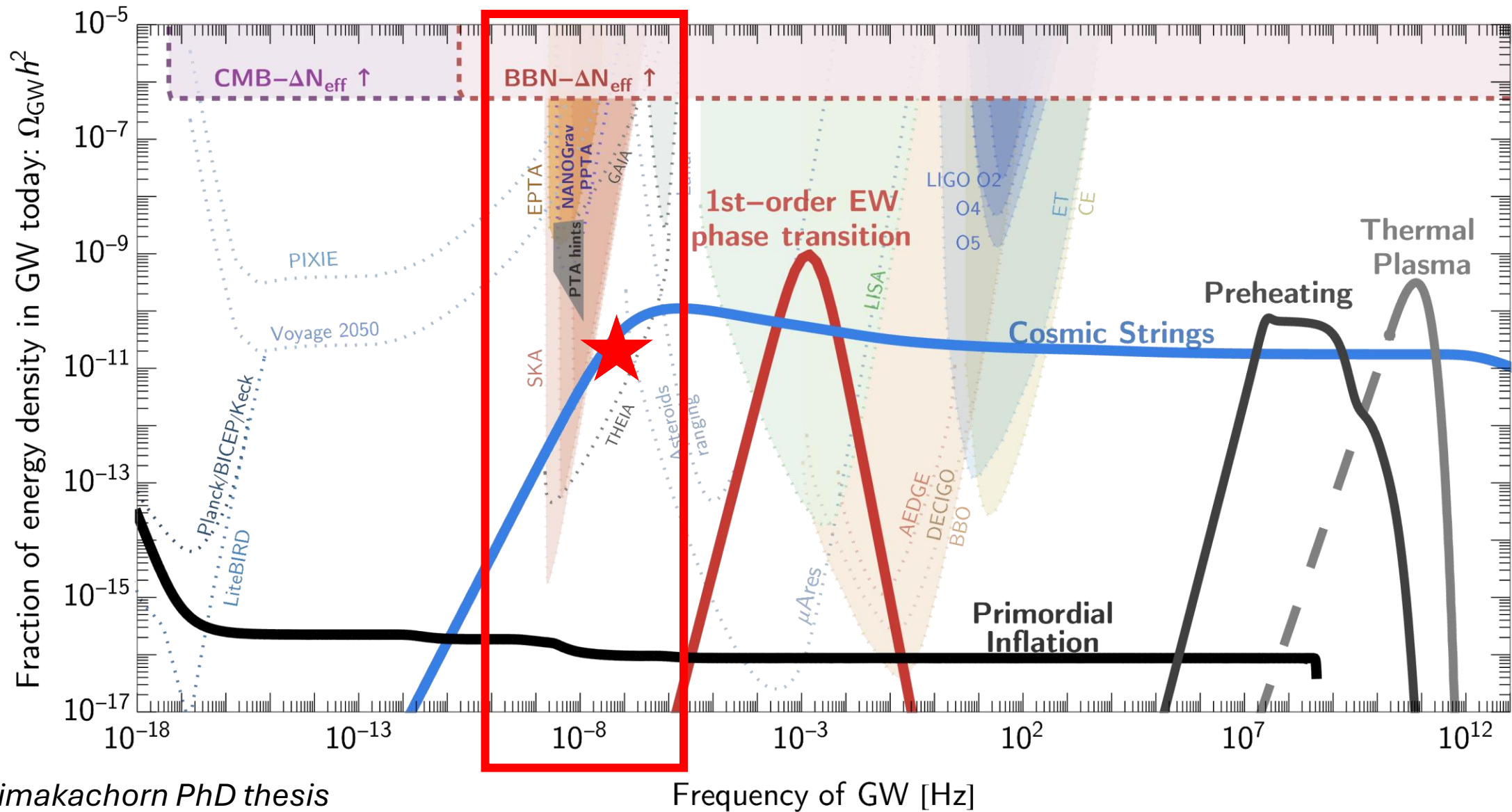
## 1st 4-years data of SPring-8



M. Takao and T. Shimada, <http://accelconf.web.cern.ch/accelconf/e00/PAPERS/MOP5A04.pdf>

GW perturbation of the metric at the SR  $h_0 \sim 7 \times 10^{-4}$   
 Relic GW perturbation in FLWR metric  $h_0^{GW} \sim 5 \times 10^{-16}$   
 Relic GW frequency  $\omega \sim 10^{-7}$  Hz

# Latest *chart* of GW theory and experiments





# Pulsar timing array

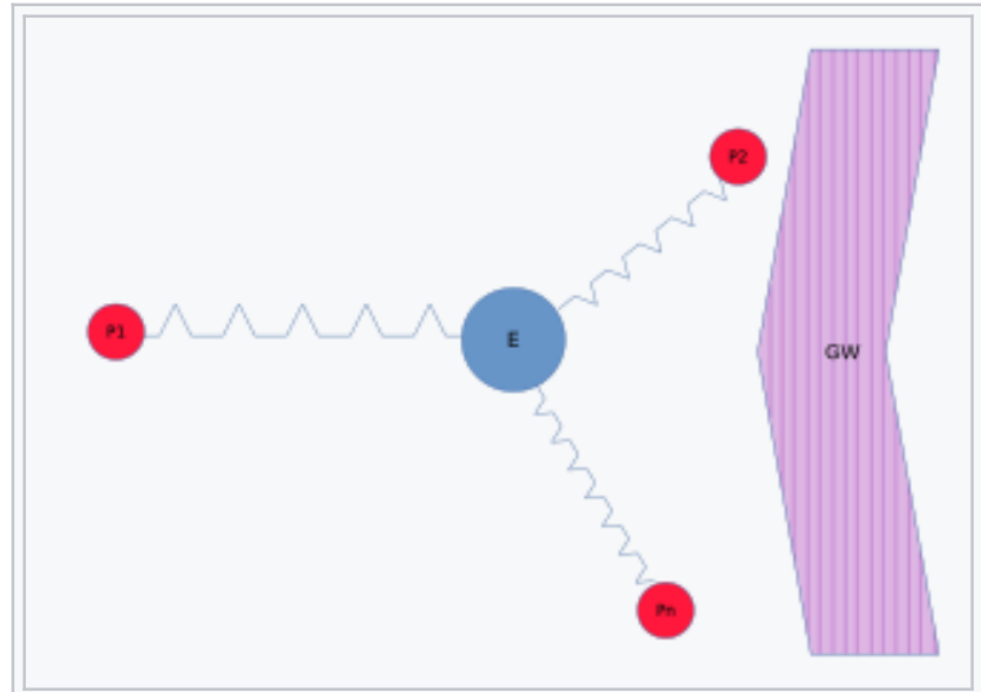
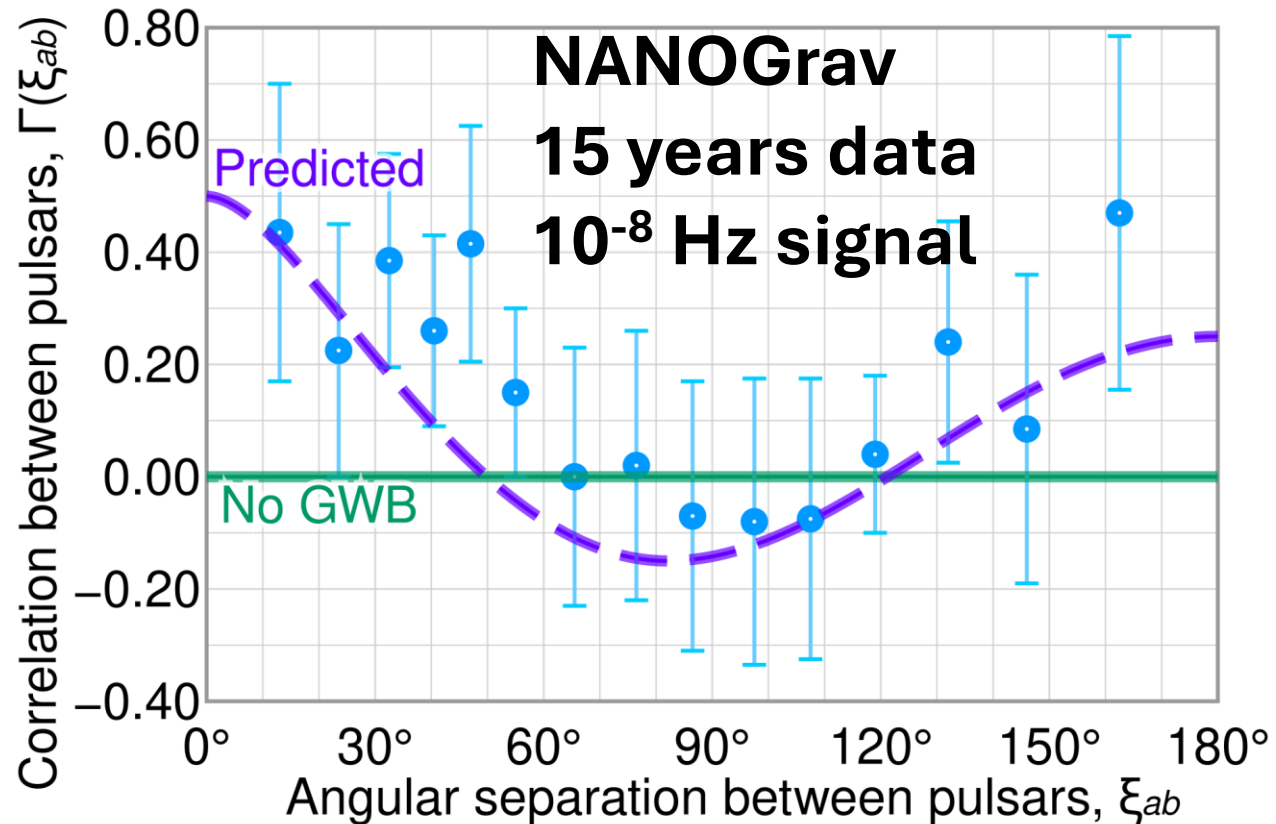
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From Wikipedia, the free encyclopedia

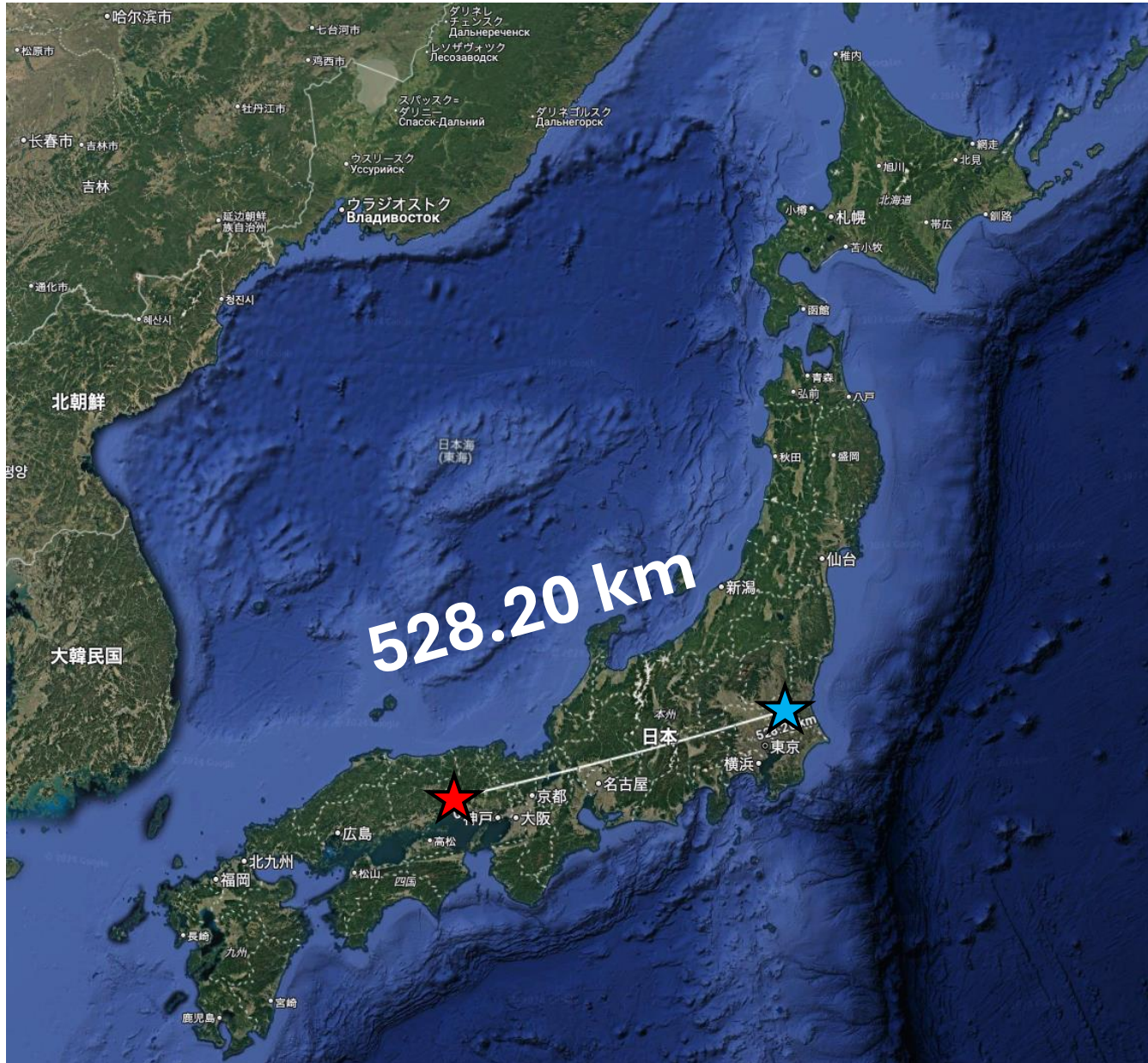
A **pulsar timing array (PTA)** is a set of [galactic pulsars](#) that is monitored and analyzed to search for correlated signatures in the pulse arrival times on Earth. As such, they are galactic-sized detectors. Although there are many applications for pulsar timing arrays, the best known is the use of an array of [millisecond pulsars](#) to detect and analyse long-wavelength (i.e., low-frequency) [gravitational wave background](#). Such a detection would entail a detailed measurement of a [gravitational wave](#) (GW) signature, like the GW-induced [quadrupolar correlation](#)<sup>[1]</sup> between arrival times of pulses emitted by different millisecond pulsar pairings that depends only on the pairings' angular separations in the sky. Larger arrays may be better for GW detection because the quadrupolar spatial correlations induced by GWs can be better sampled by many more pulsar pairings. With such a GW detection, millisecond pulsar timing arrays would open a new low-frequency window in [gravitational-wave astronomy](#) to peer into potential ancient [astrophysical](#) sources and [early Universe](#) processes, inaccessible by any other means.<sup>[2][3]</sup>



The pulsars P1 ... Pn are sending signals periodically, which are received on Earth. A gravitational wave (GW) perturbs spacetime in between the pulsar and Earth (E) and changes the time of arrival of the pulses. By measuring the spatial correlation of the changes in the pulse parameters of many different pulsar pairings, a GW can be detected.

We need correlation among multiple sources / detectors to reveal stochastic GW background

# SPring-8 vs KEKB circular accelerators



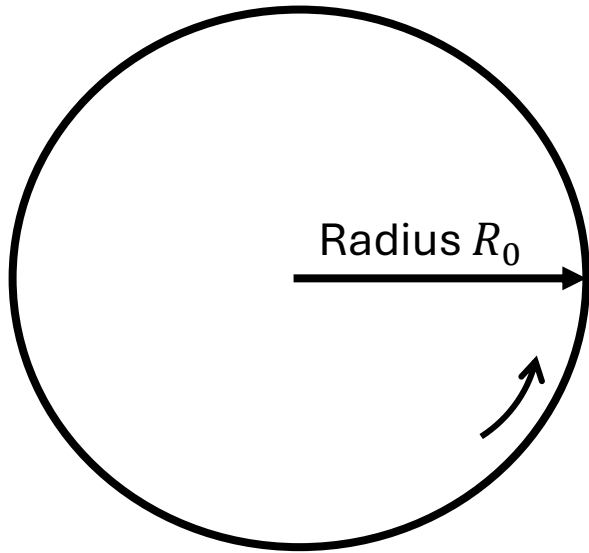
★ SPring-8 accelerator **Today's talk**  
34° 56' 41" N; 134° 25' 38" E  
Circumference 1435 m  
8 GeV e<sup>-</sup>  
RF frequency: 506.756 MHz  
Harmonic number: 2436  
Momentum compaction: 1.46e-4  
Operation: 1997- **25-years data**

★ KEKB accelerator  
36° 9' 17" N; 140° 4' 19" E  
Circumference 3016 m  
8 GeV e<sup>-</sup> / 3.4 GeV e<sup>+</sup>  
RF frequency: 508.887 MHz  
Harmonic number: 5120  
Momentum compaction: 3.4e-4  
Operation: 1998- data prepared

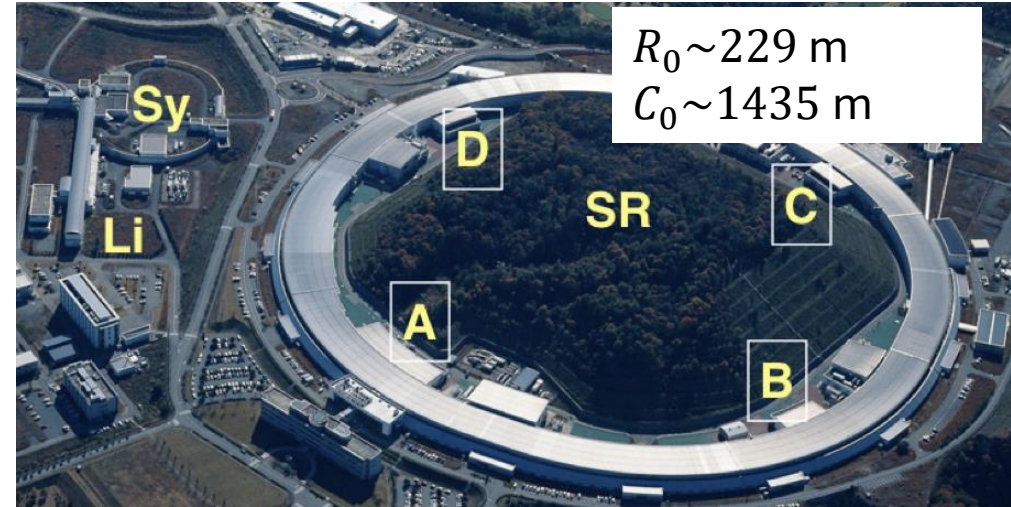


# RF frequency and circumference of Spring-8

Circumference  $C_0$



- 1 turn:  $T_0$
- Speed of electrons  $v$
- Beam energy 8 GeV
- $v \sim c$
- Harmonic number  $n = 2436$
- RF frequency  $f_{RF} = 506.756$  MHz

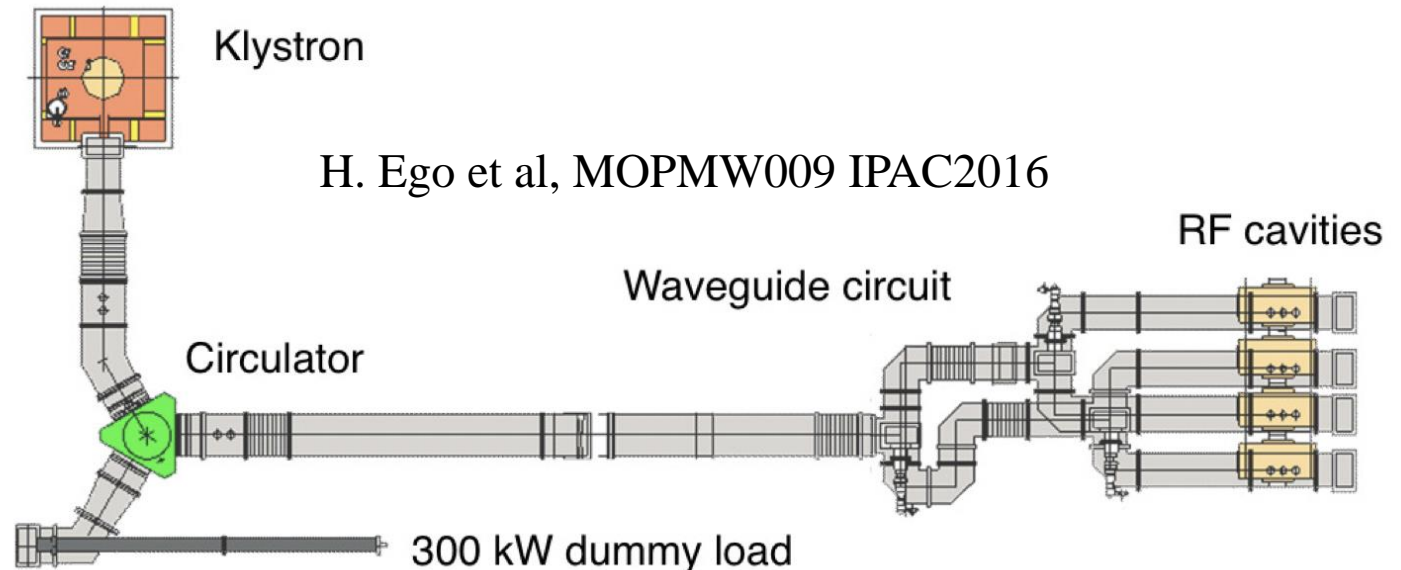


ABCD: RF stations

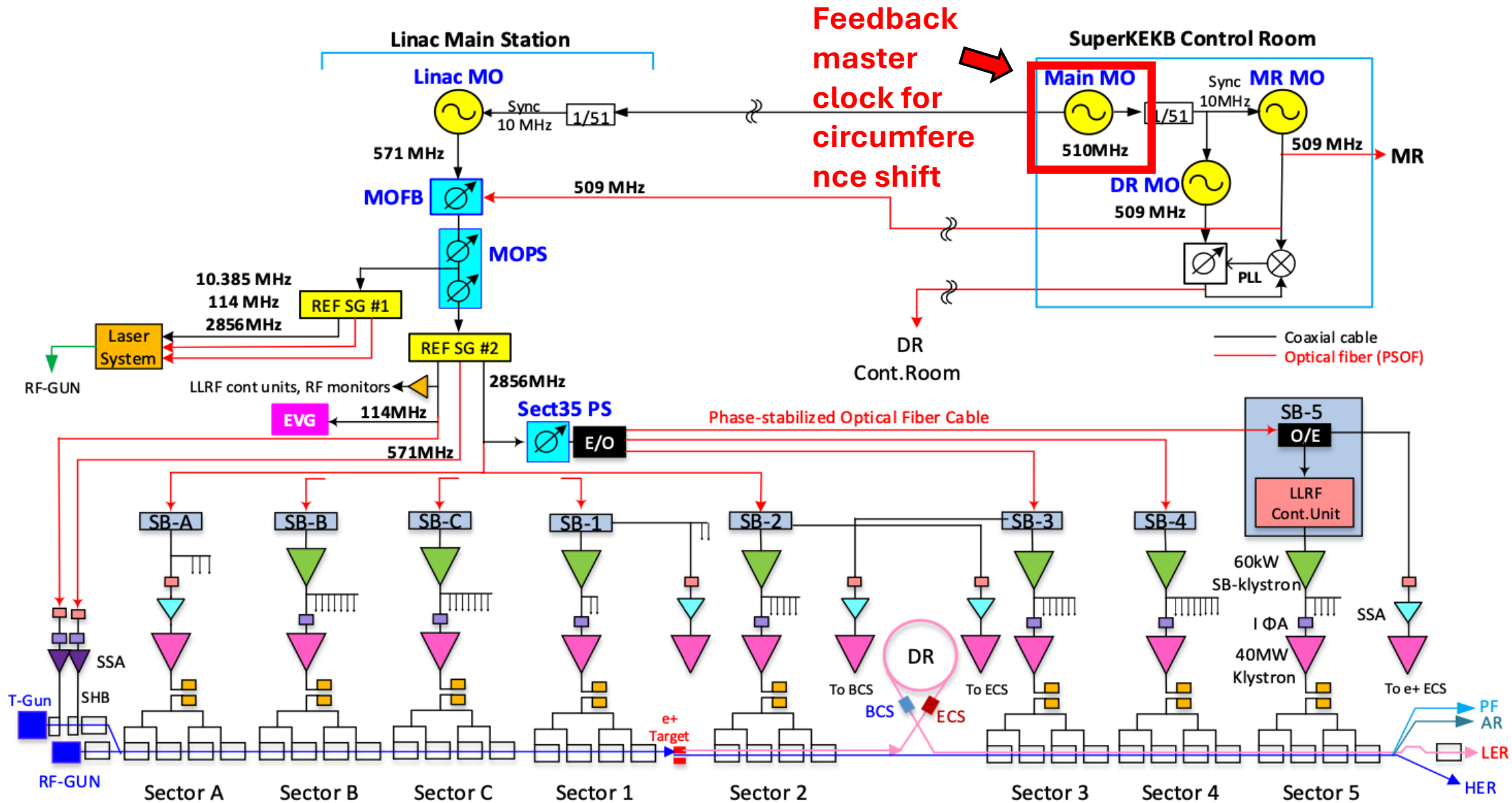
$$C_0 \sim T_0 v = \frac{c}{f_{RF}/n}$$

$C_0 = 1435.4512$  m for the official specification

→ We have real data



# RF infrastructure of KEKB presented in LINAC2024

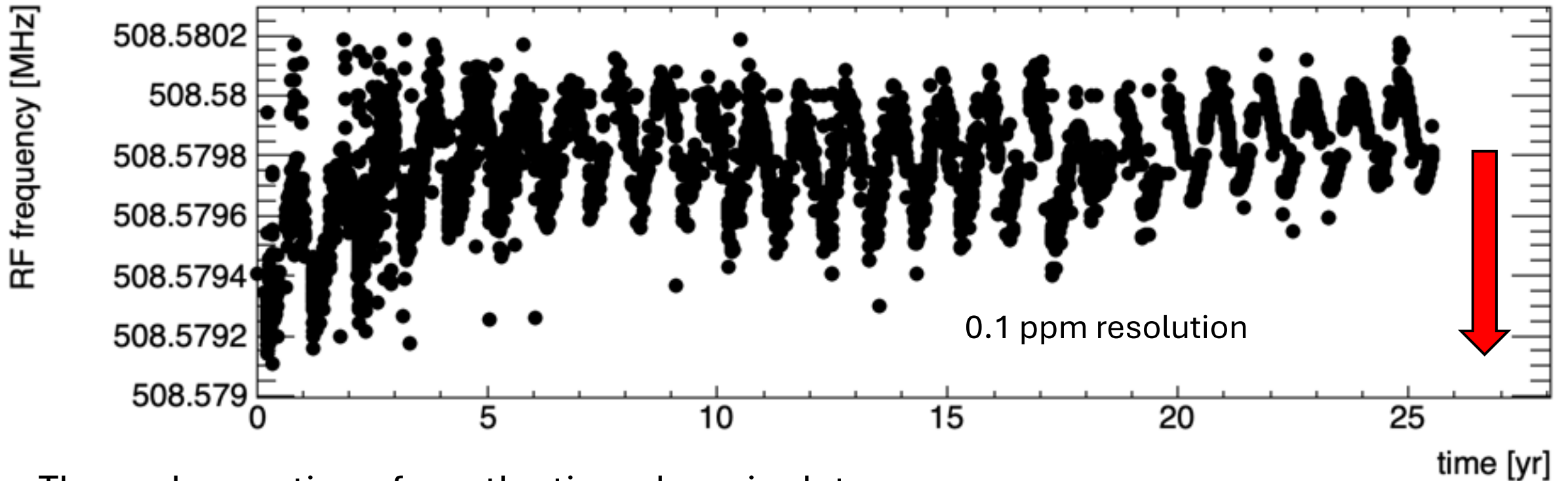


➔ KEK will give us data (not ready today)

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# SPring-8 data over 25 years

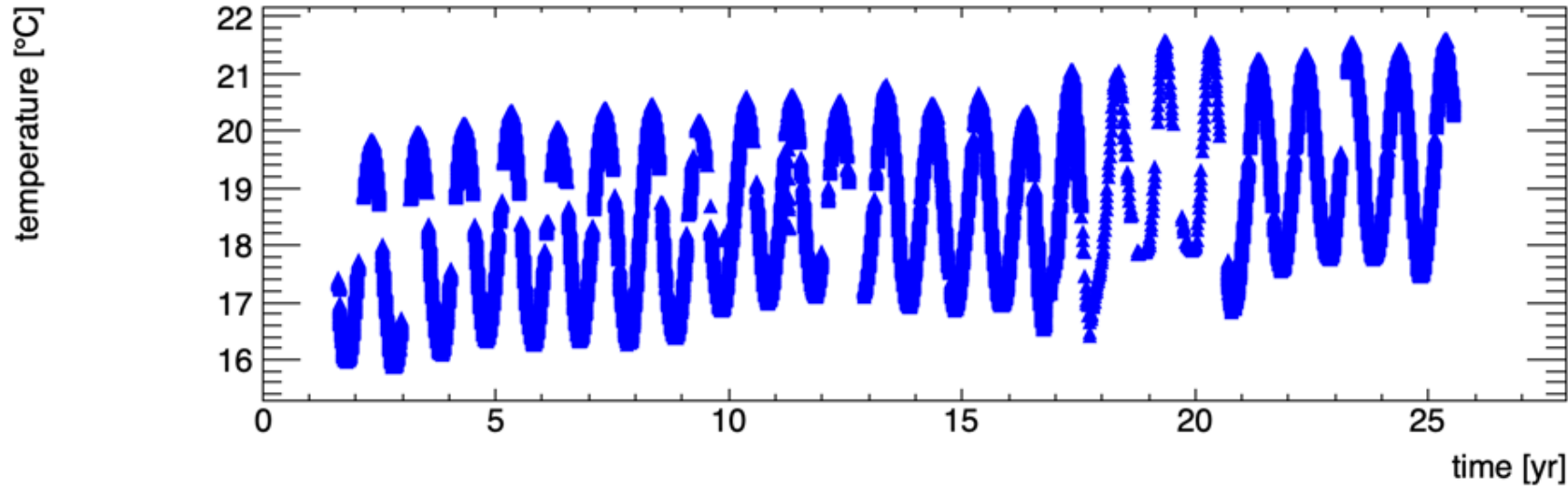


## Three observations from the time domain data

- Annual modulation (**0.32 ppm**)
  - Thermal shrinkage / expansion due to temperature modulation?
  - Seasonal force from the gravitational force between the Sun and the Earth?
- Significant shift of RF in the first 2 years cannot be explained by temperature  
→ Inspired A. N. Ivanov and A. P. Kobushkin
- Slower (~ 5-10 years) oscillation seems also visible → nHz physics



# Annual modulation: temperature variation

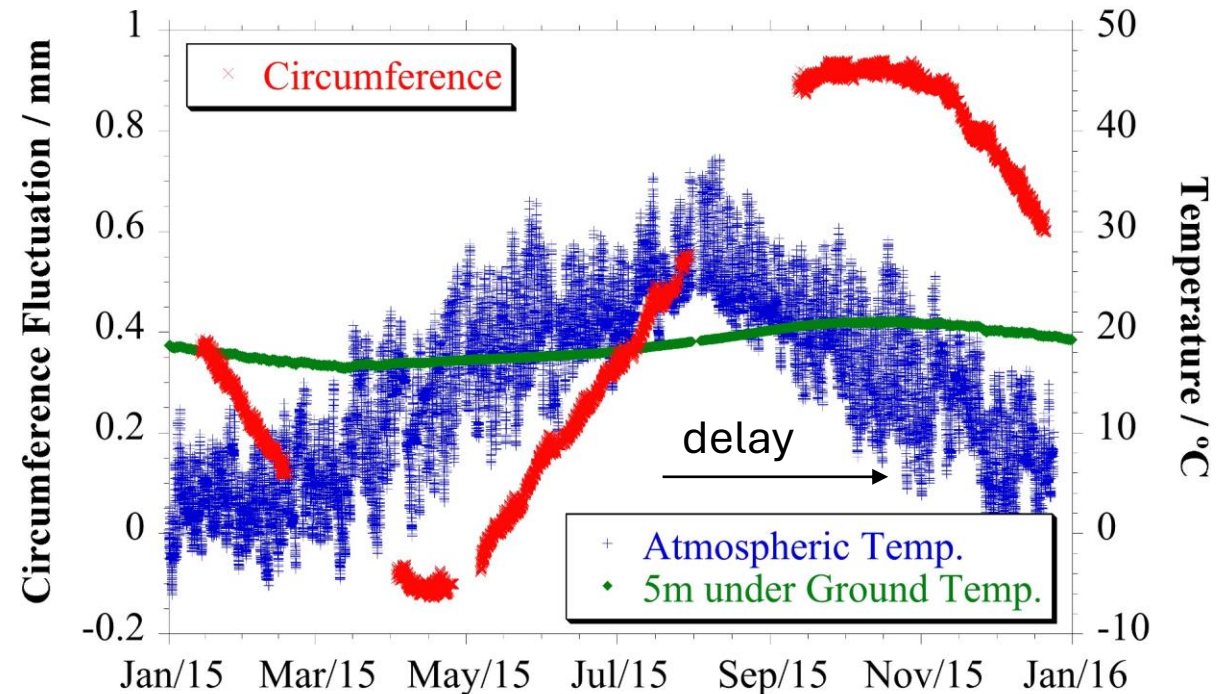


## Gravitational origin?

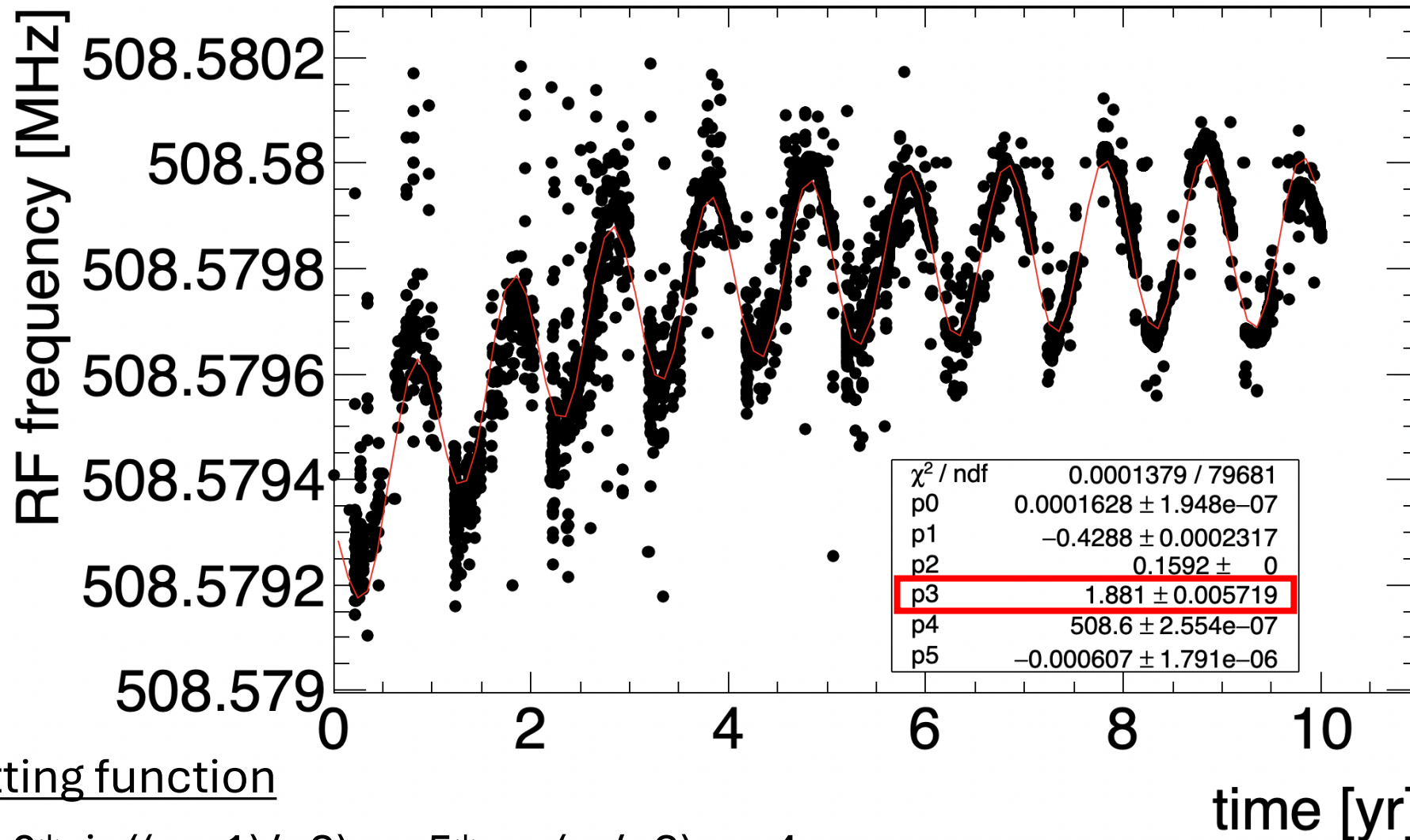
$$U_s(\mathbf{r}) = G_N M_{\odot} \left( \frac{1}{|\mathbf{R}_s - \mathbf{r}|} - \frac{1}{R_s} - \frac{\mathbf{r} \cdot \mathbf{R}_s}{R_s^3} \right)$$

## No! → correlation to temperature

- The data is in-phase to the temperature underground **delayed** from the atmospheric temperature
- Earth-Sun orientation → atmospheric T



# Heuristic fitting of the initial RF shift



The ring shrunk by around 1.2 mm (**0.86 ppm**) with a time constant of 1.88 years

**Very good resolution  
0.1 ppm to monitor strain**

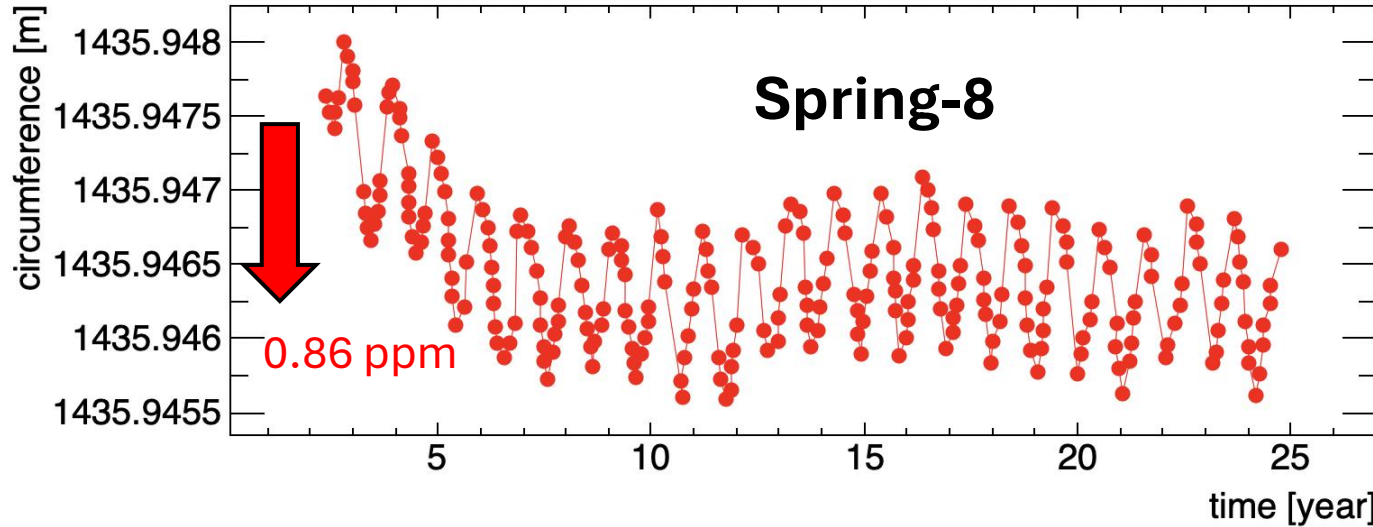
Fitting function

$$p0 \cdot \sin((x-p1)/p2) + p5 \cdot \exp(-x/p3) + p4$$

p2 was fixed at  $2\pi \times 1$  yr

(no fundamental reason to select this function)

# Hypothesis: Shrinkage of drying concrete?



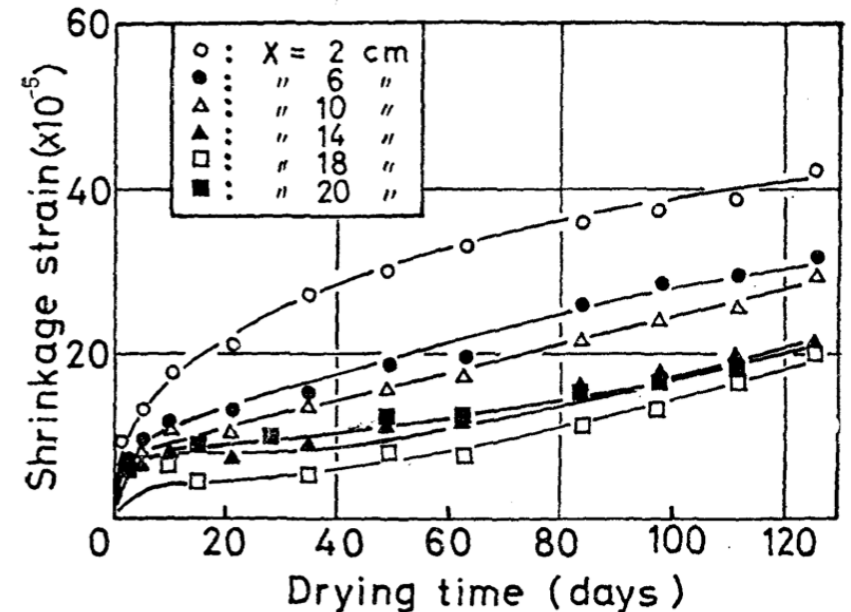
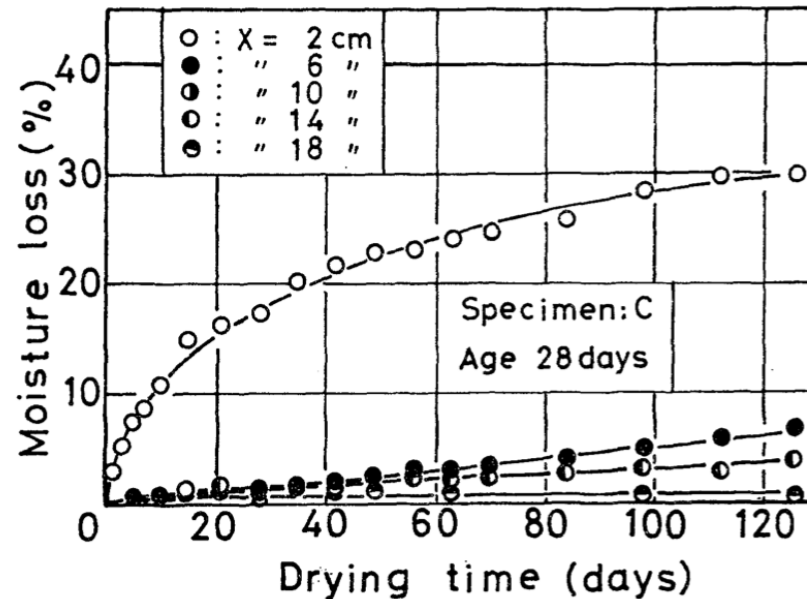
→ Compare to a dedicated study of concrete (Japanese paper)

K. Sataka and K. Osamu, A study of the water diffusion and shrinkage in concrete by drying, 土木学会論文報告集 第316号 1981年 12月

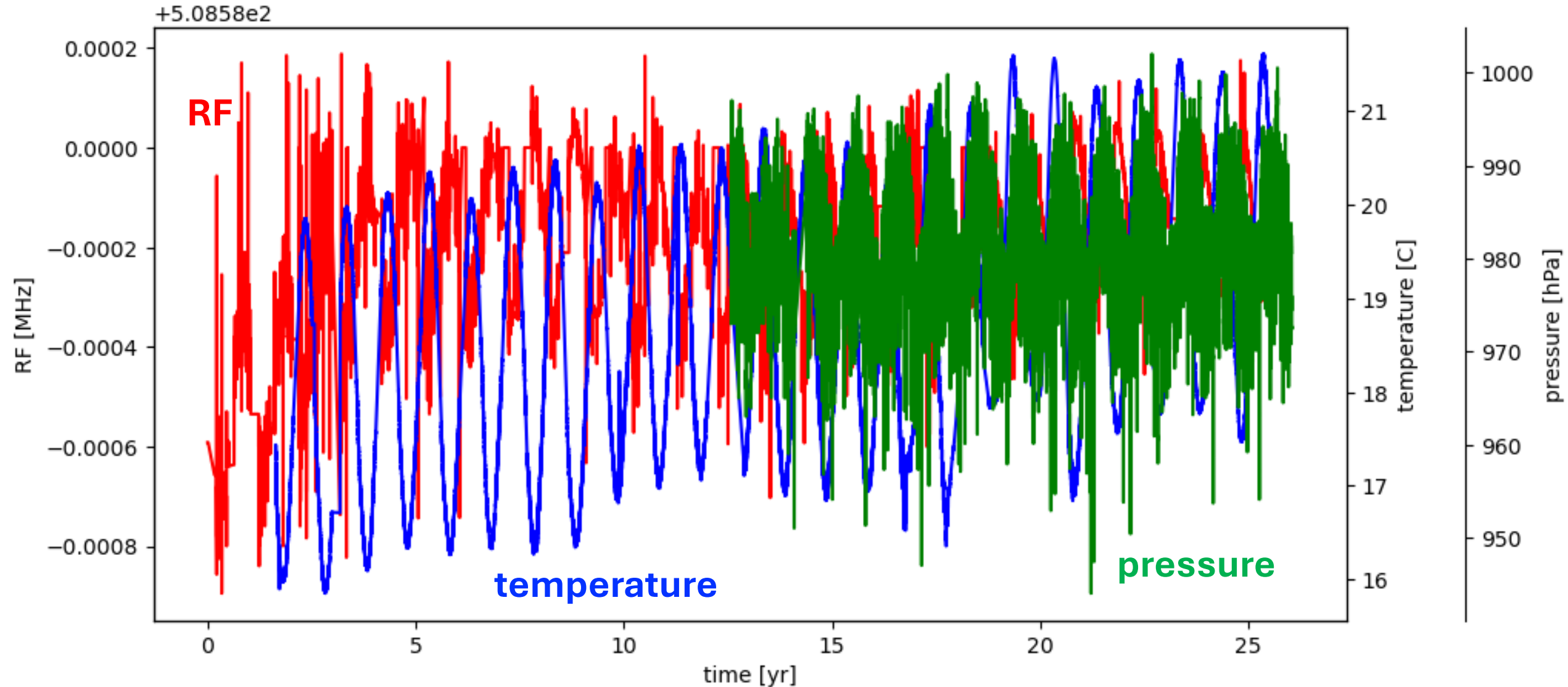
Constant temperature (20C) and humidity (60%) room

X: depth from surface

- Shrinkage  $10^{-4}$  / 100 days if the depth is  $>20$  cm
- Difficult to quantitatively prove



# Time domain data with T and P over 25 years



Power spectrum with Fourier transform is of great interest

But the data was **not recorded periodically** → FFT is not applicable



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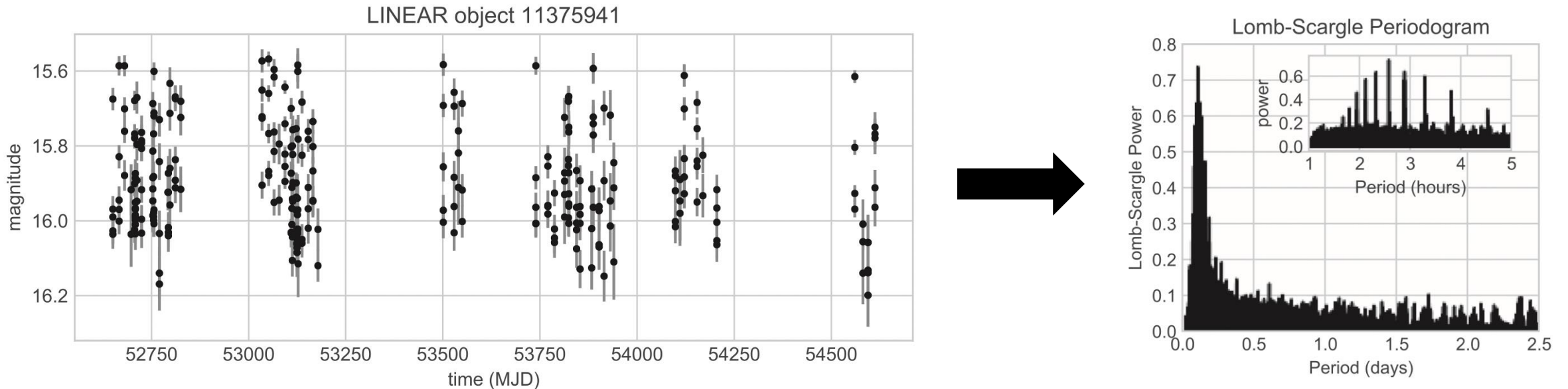
## Understanding the Lomb–Scargle Periodogram

Jacob T. VanderPlas 

University of Washington, eScience Institute, 3910 15th Ave NE, Seattle WA 98195

Received 2017 August 26; revised 2017 December 6; accepted 2017 December 7; published 2018 May 11

Thanks to  
Yuto Minami



- Commonly used algorithm in astronomy
- Modified from classical Fourier transform of unevenly spaced data
- The y-axis is not the classical spectral power density but chi2 of fitting time domain data by a sum of sinusoidal functions → Power Spectral Density can also be plotted

# Lomb Scargle Power

## The Lomb-Scargle Model

The Lomb-Scargle periodogram fits a sinusoidal model to the data at each frequency, with a larger power reflecting a better fit. With this in mind, it is often helpful to plot the best-fit sinusoid over the phased data.

$$P(f) = \frac{1}{N} \left| \sum_{n=1}^N g_n e^{-2\pi i f t_n} \right|^2$$
$$= \frac{1}{N} \left[ \left( \sum_n g_n \cos(2\pi f t_n) \right)^2 + \left( \sum_n g_n \sin(2\pi f t_n) \right)^2 \right]$$

$$y(t; f) = A_f \sin(2\pi f(t - \phi_f))$$

$$\chi^2(f) \equiv \sum_n (y_n - y(t_n; f))^2$$

$$P(f) = \frac{1}{2} [\hat{\chi}_0^2 - \hat{\chi}^2(f)]$$

Equally spacing  
data set  $y_n$

$$P(f) = \frac{1}{N} |FFT(y_n)|^2$$

Lomb Scargle periodogram is one of the possible generalization of FFT

# LombScargle



```
class astropy.timeseries.LombScargle(t, y, dy=None, fit_mean=True,  
center_data=True, nterms=1, normalization='standard') \[source\]
```

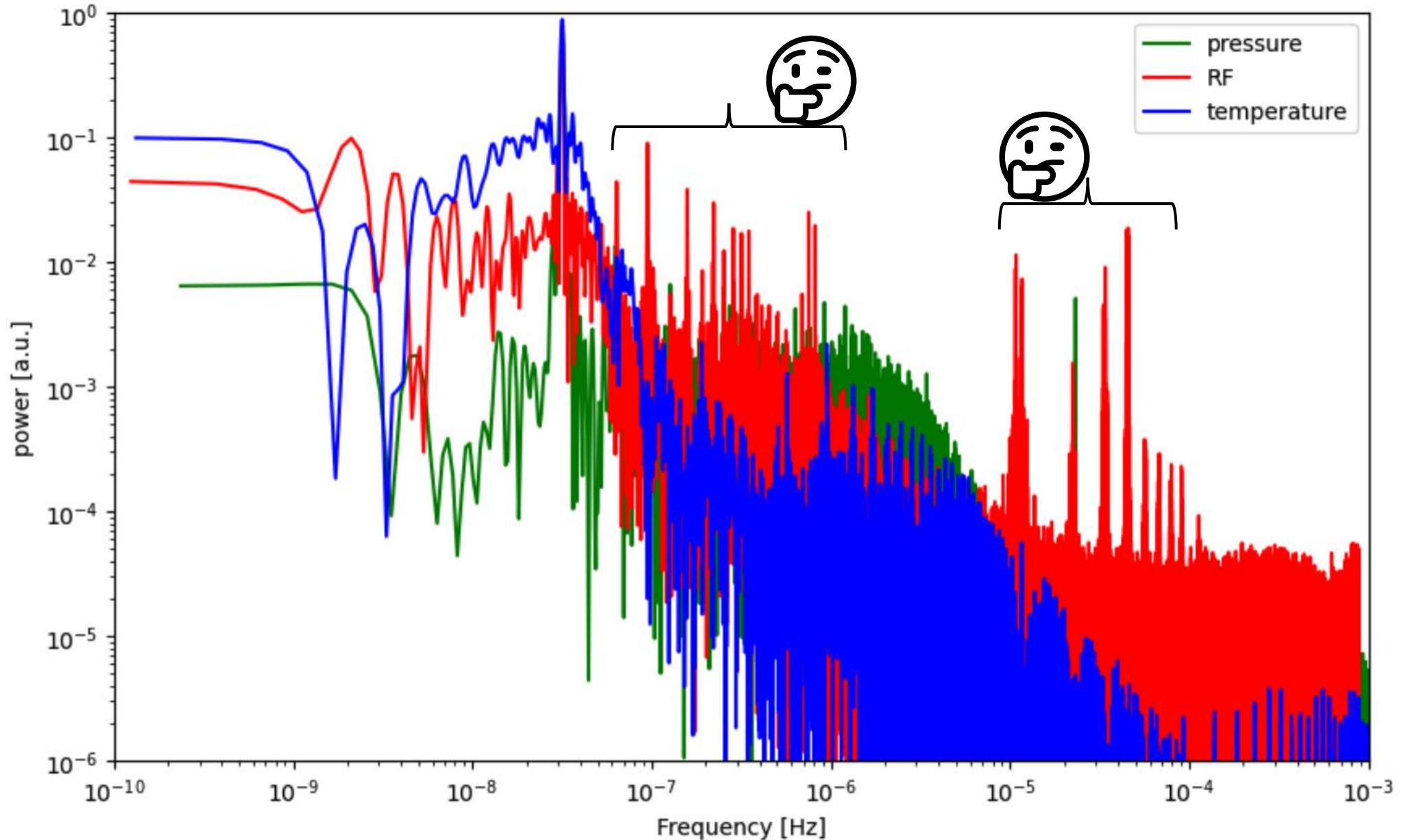
Bases: [BasePeriodogram](#)

```
[11]: from astropy.timeseries import LombScargle  
frequency, power = LombScargle(time_raw, delta_raw, normalization='psd').autopower()
```

- Extremely simple to perform the analysis...10 minutes to prepare working environment in Jupyter notebook if you wish
  - Installation through conda or pip
- Default normalization: Lomb-Scargle power  $\chi^2$ 
  - Useful to compare different data (RF, temperature, pressure)
- Normalization = 'psd' for the Power Spectral Density
  - Useful to see noise level

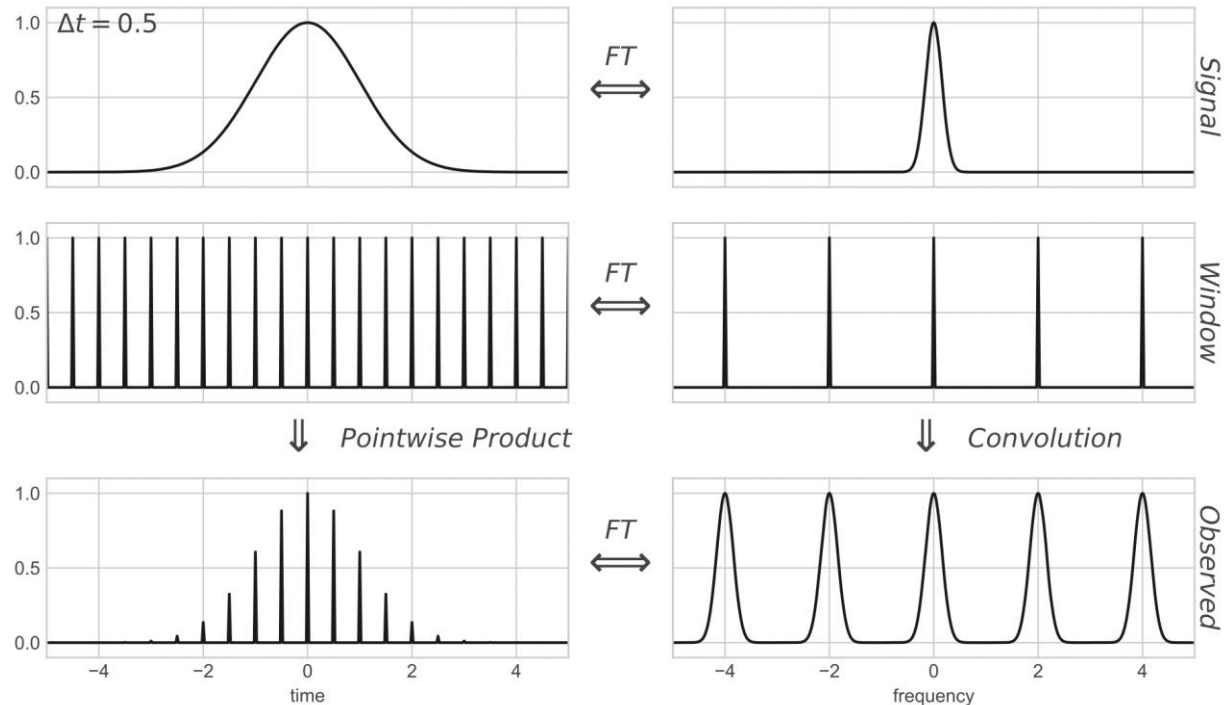


# Frequency domain spectrum: y-axis in $\chi^2$

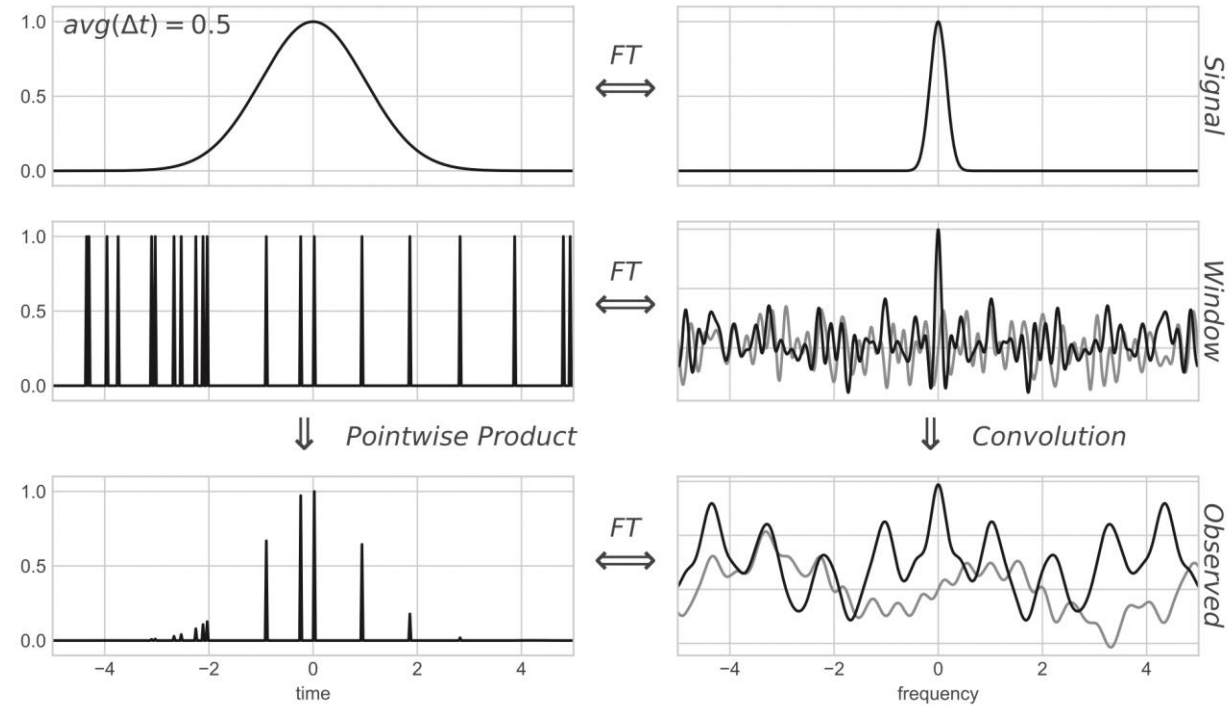


# Uneven Fourier analysis: uneven Dirac comb

## Uniform data acquisition (Dirac comb)



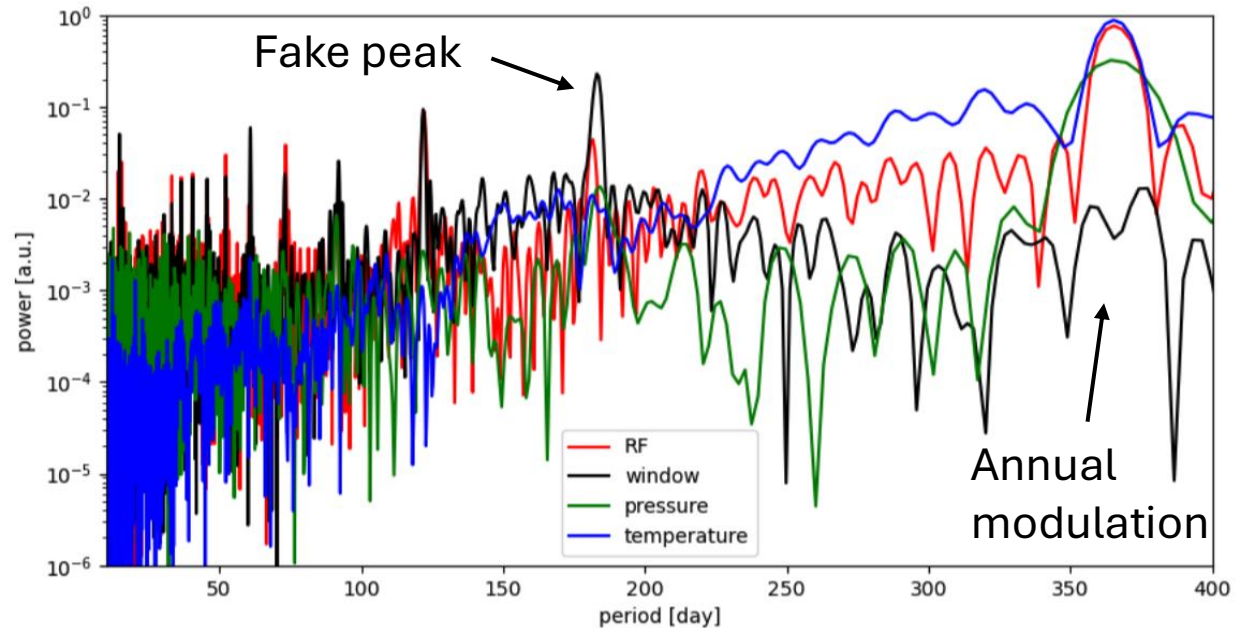
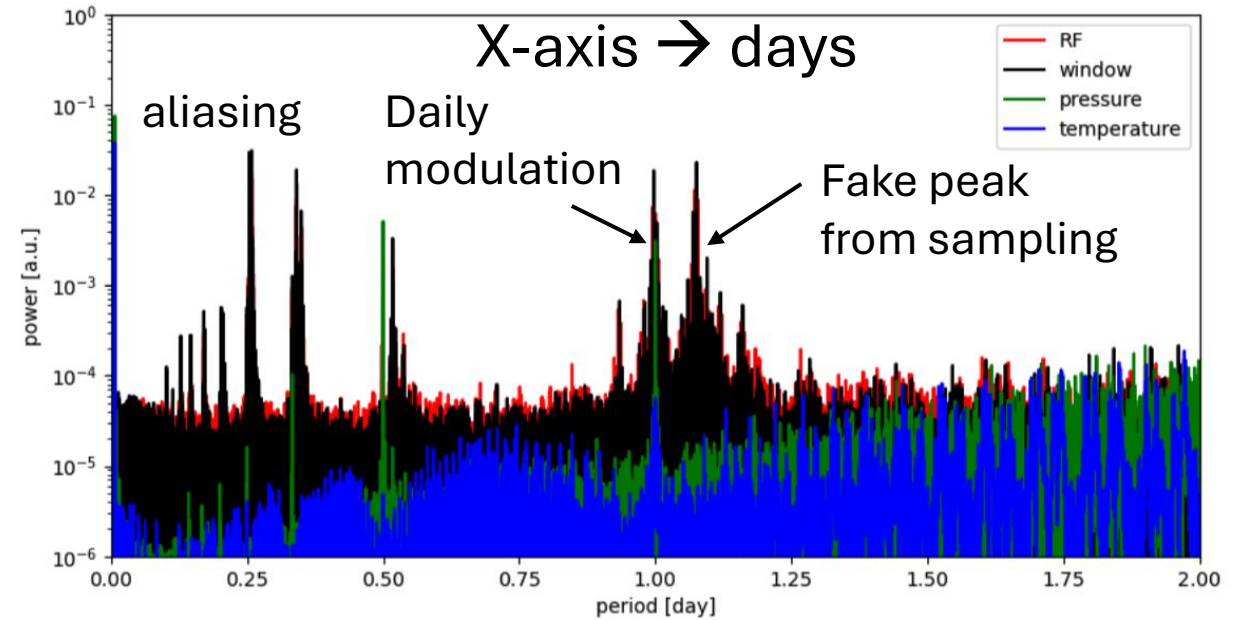
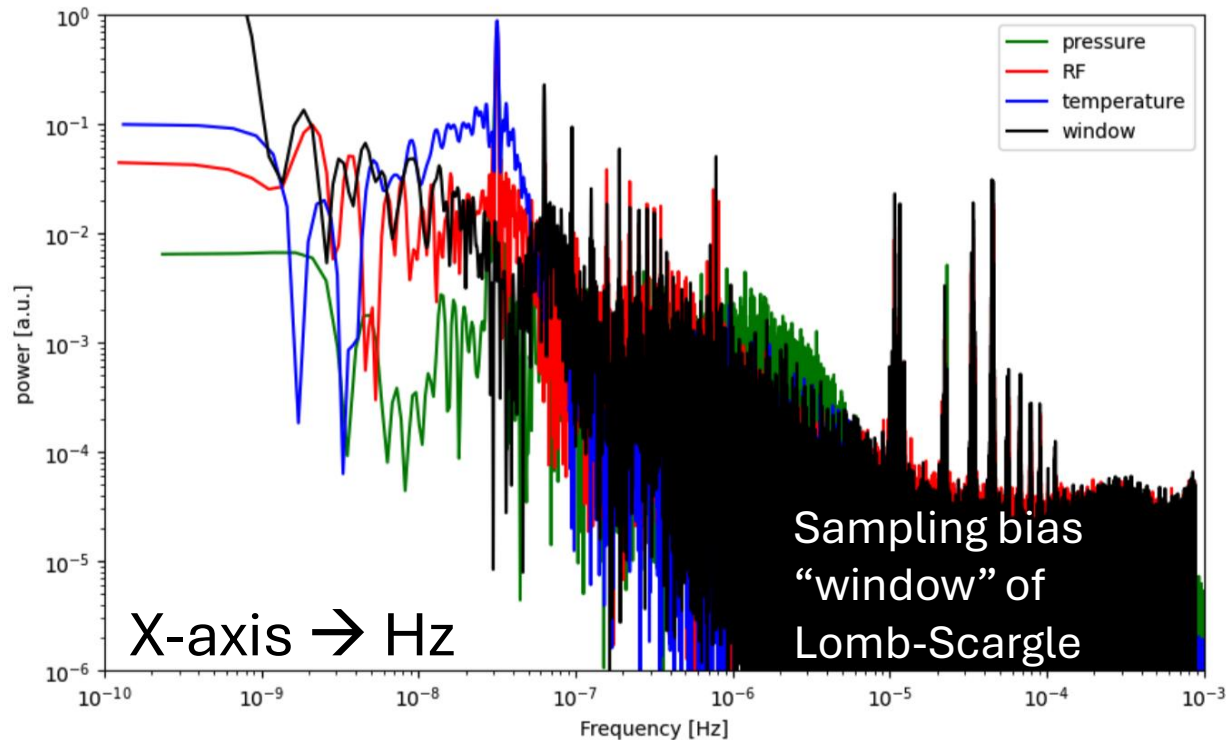
## Nonuniform data acquisition



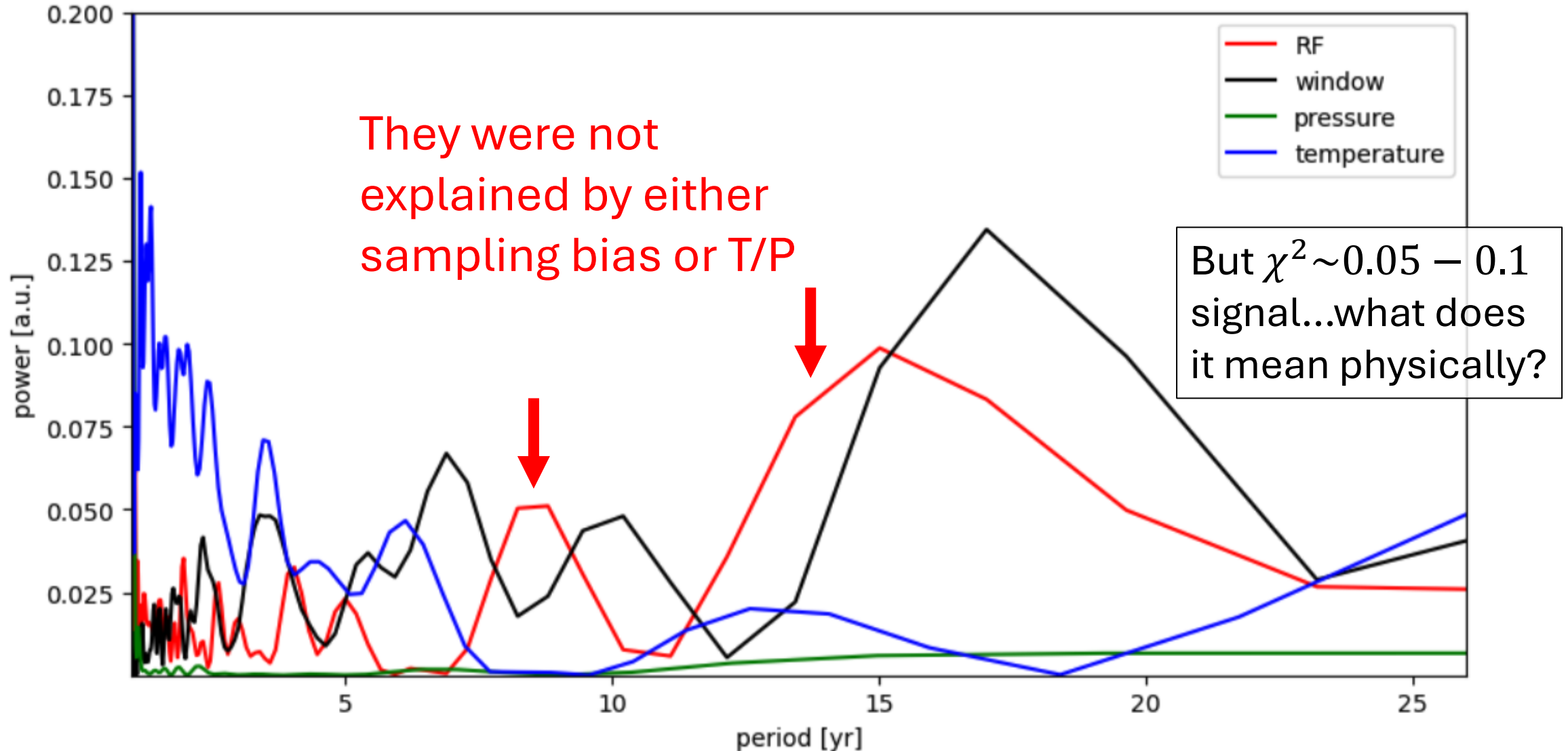
The Fourier spectrum is a convolution of the signal and the measuring (sampling) window  $\rightarrow$  fake peaks even within Nyquist range

# Fake peaks from uneven sampling (+aliasing)

- A lot of peaks were identified as unphysical
- Sampling bias (tunnel access, shutdown, maintenance, etc) makes fake peaks in frequency domain
- Some peaks are physical and correlated to pressure and/or temperature



# Too early to be excited 😊



→ We need to understand noise level



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# Power Spectral Density (PSD)

$$w_T(t) = \begin{cases} 1 & : t \in [-T/2, T/2] \\ 0 & : t \notin [-T/2, T/2] \end{cases}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |n(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} |n(t)w_T(t)|^2 dt \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} |n_T(t)|^2 dt$$

Applying Parseval's theorem (unitarity of Fourier transform) for  $\forall a: \mathbb{R} \rightarrow \mathbb{C}$

$$\int_{-\infty}^{\infty} |a(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\tilde{a}(\omega)|^2 d\omega$$

Then

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} |\tilde{n}_T(\omega)|^2 \frac{d\omega}{2\pi}$$

1<sup>st</sup> fundamental axiom of physics: limit and integral are always exchangeable

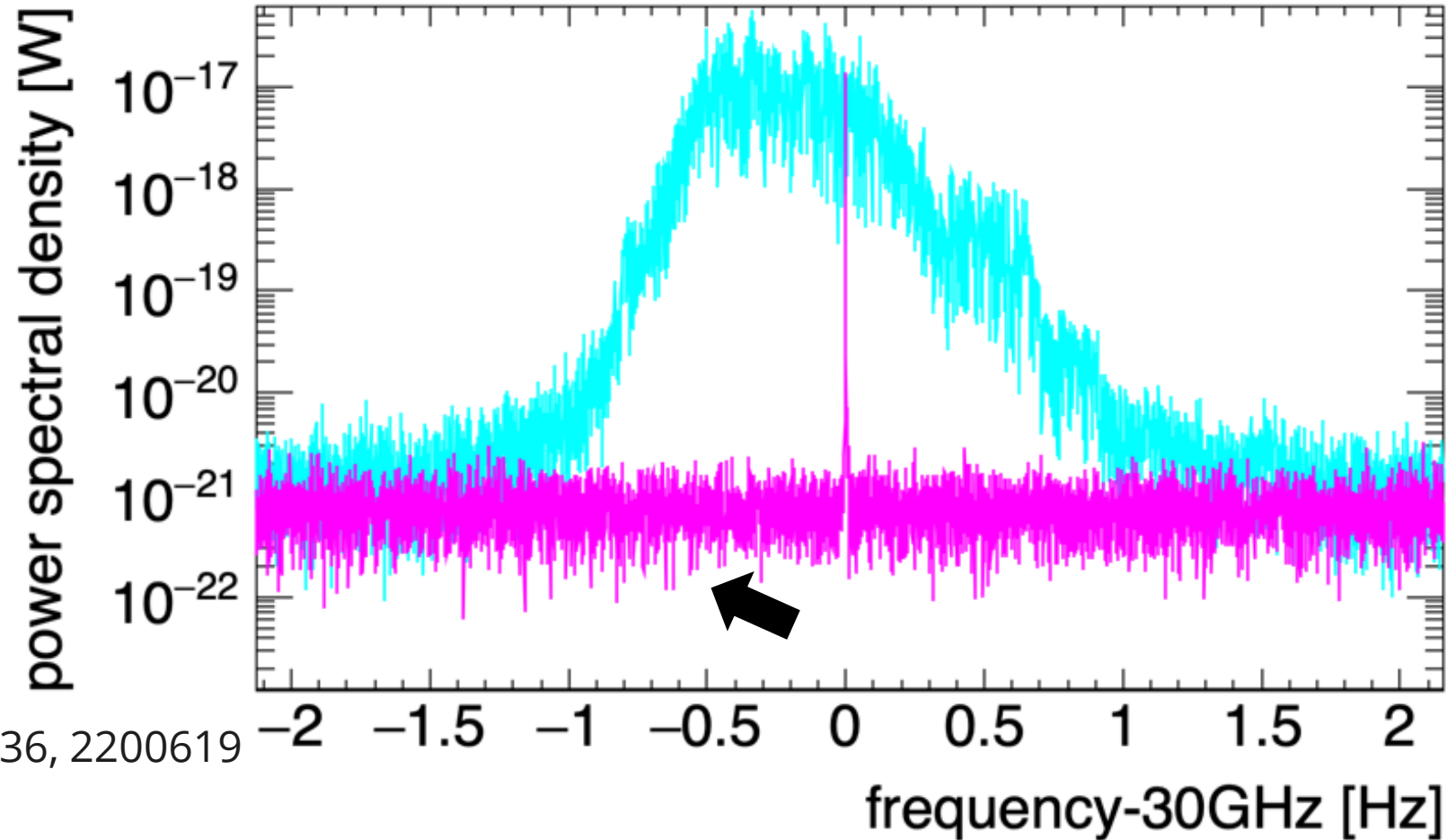
$$P = \int_{-\infty}^{\infty} \boxed{\lim_{T \rightarrow \infty} \frac{1}{T} |\tilde{n}_T(\omega)|^2} \frac{d\omega}{2\pi}$$

$\equiv S_n(f)$ : (noise) power spectral density

← supported by astropy's LombScargle

# PSD is a noise level in classical wave data

Eg) RF noise in  
spectrum analyzer

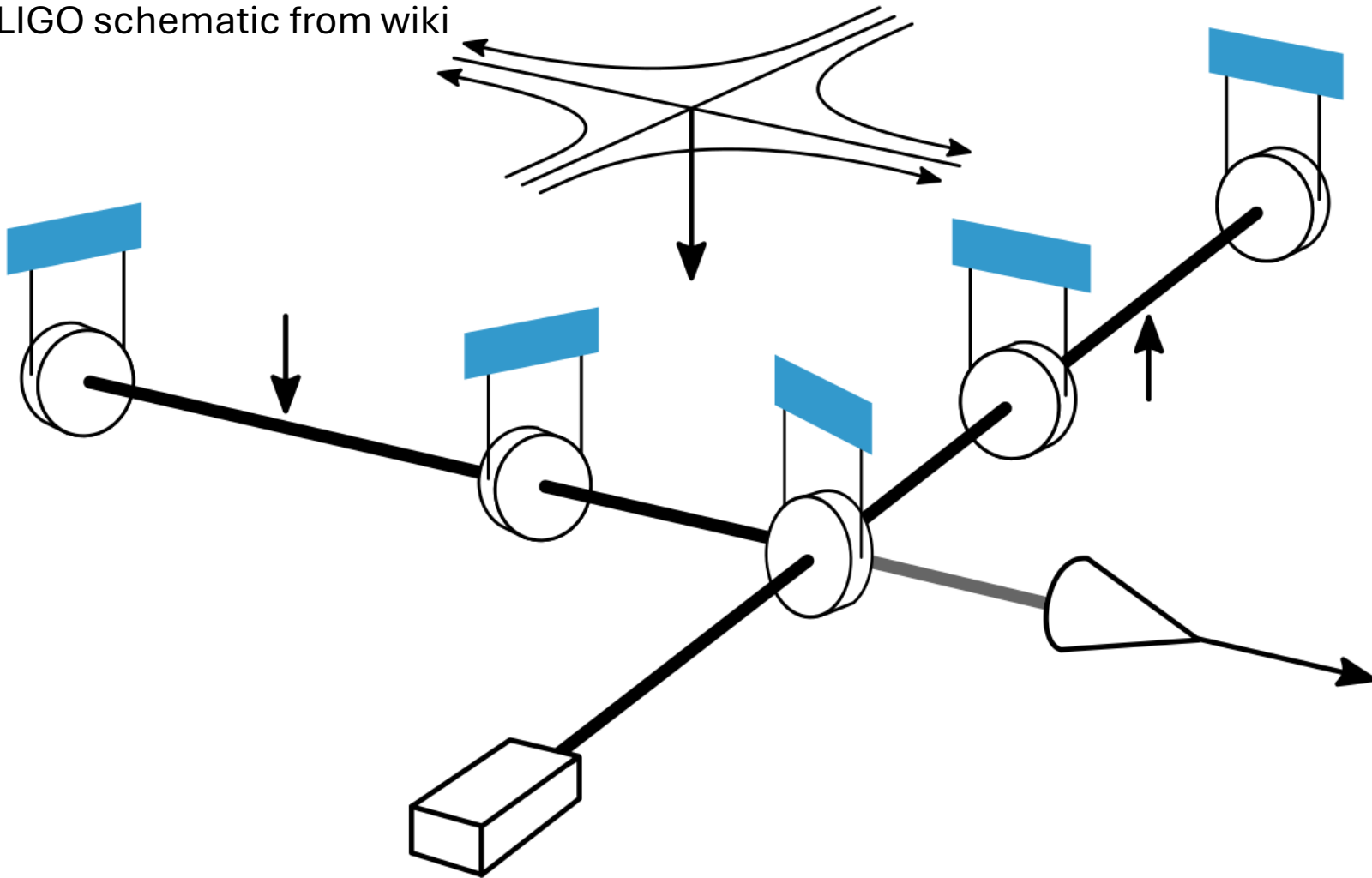


*AM et al, ANNALEN DER PHYSIK 2023, 536, 2200619*

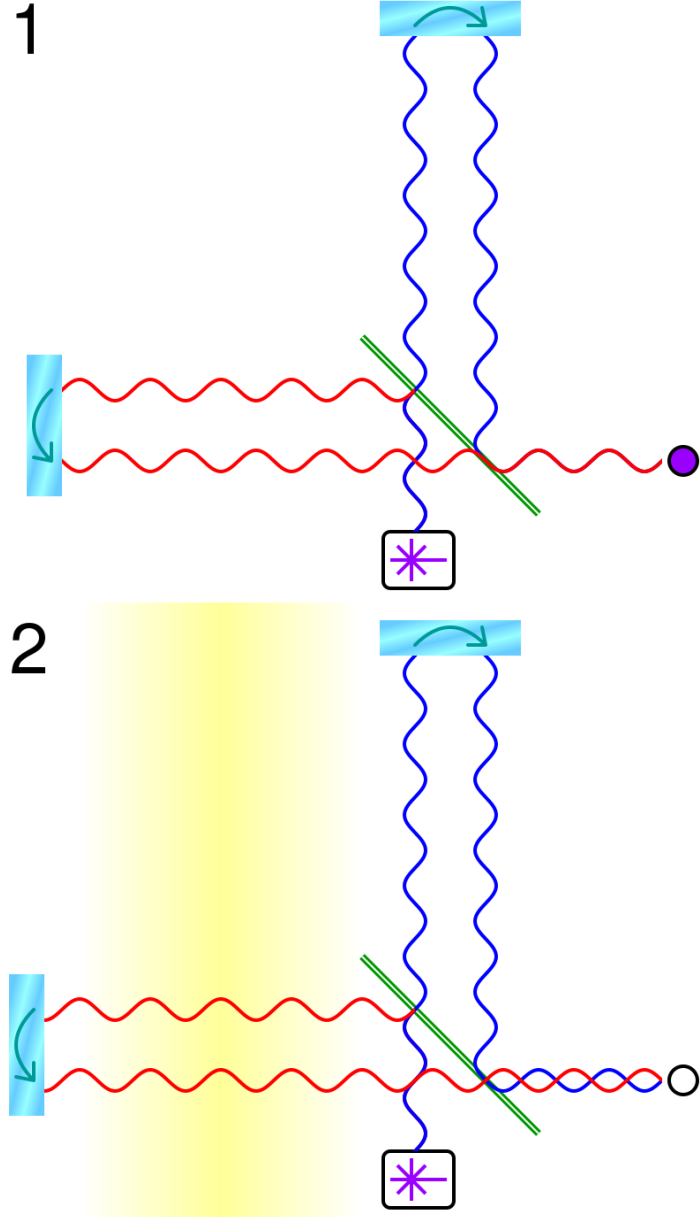
## Background of typical axion dark matter experiments

# Compare PSD to LIGO

LIGO schematic from wiki



Cmglee, CC BY-SA 3.0  
<<https://creativecommons.org/licenses/by-sa/3.0/>>, via Wikimedia Commons



# Free Python code in Jupyter notebook & LIGO data

```
import numpy as np
from matplotlib import pyplot as plt
from scipy import fftpack
from matplotlib import mlab
from astroML.datasets import fetch_LIGO_large
```

```
from astroML.plotting import setup_text_plots
setup_text_plots(fontsize=8, usetex=True)
```

```
# Fetch the LIGO hanford data
data, dt = fetch_LIGO_large()
```

```
# subset of the data to plot
```

```
t0 = 646
```

```
T = 2
```

```
tplot = dt * np.arange(T * 4096)
```

```
dplot = data[4096 * t0: 4096 * (t0 + T)]
```

```
tplot = tplot[::10]
```

```
dplot = dplot[::10]
```

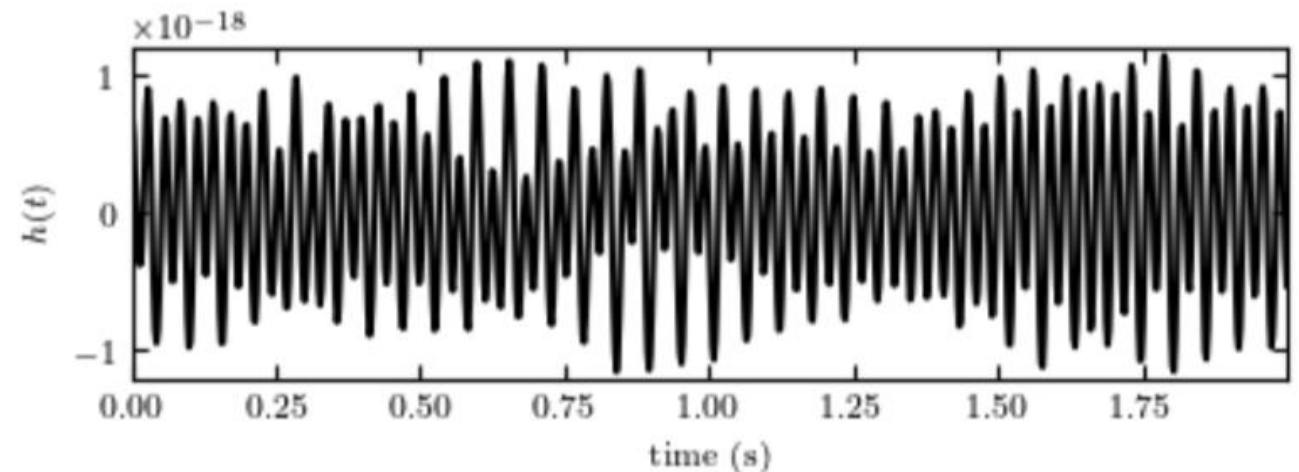
```
fmin = 40
```

```
fmax = 2060
```

```
# Plot the data
fig = plt.figure(figsize=(5, 5))
fig.subplots_adjust(bottom=0.1, top=0.9, hspace=0.3)

# top panel: time series
ax = fig.add_subplot(311)
ax.plot(tplot, dplot, '-k')
ax.set_xlabel('time (s)')
ax.set_ylabel('$h(t)$')

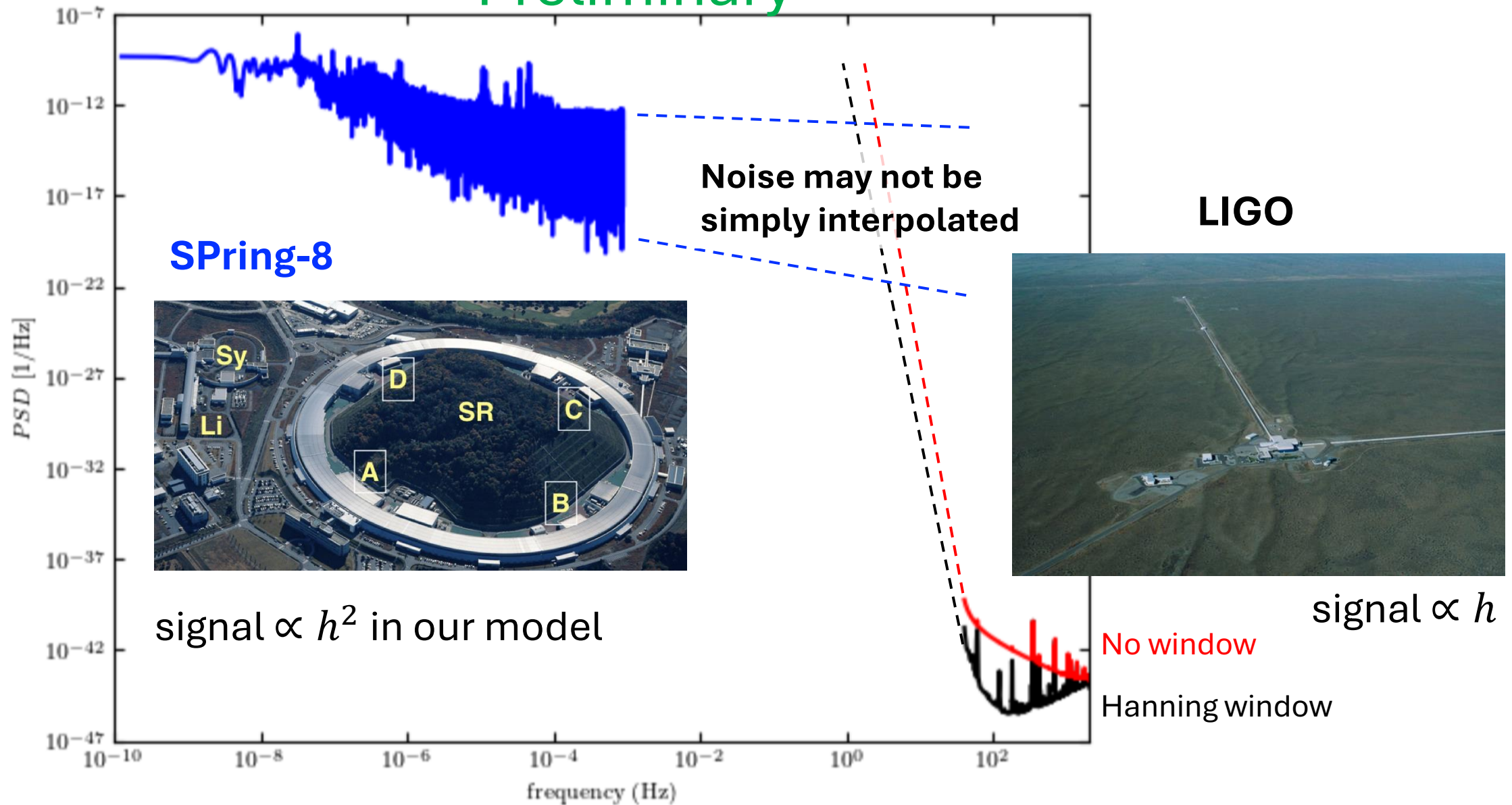
ax.set_ylim(-1.2E-18, 1.2E-18)
plt.show()
```





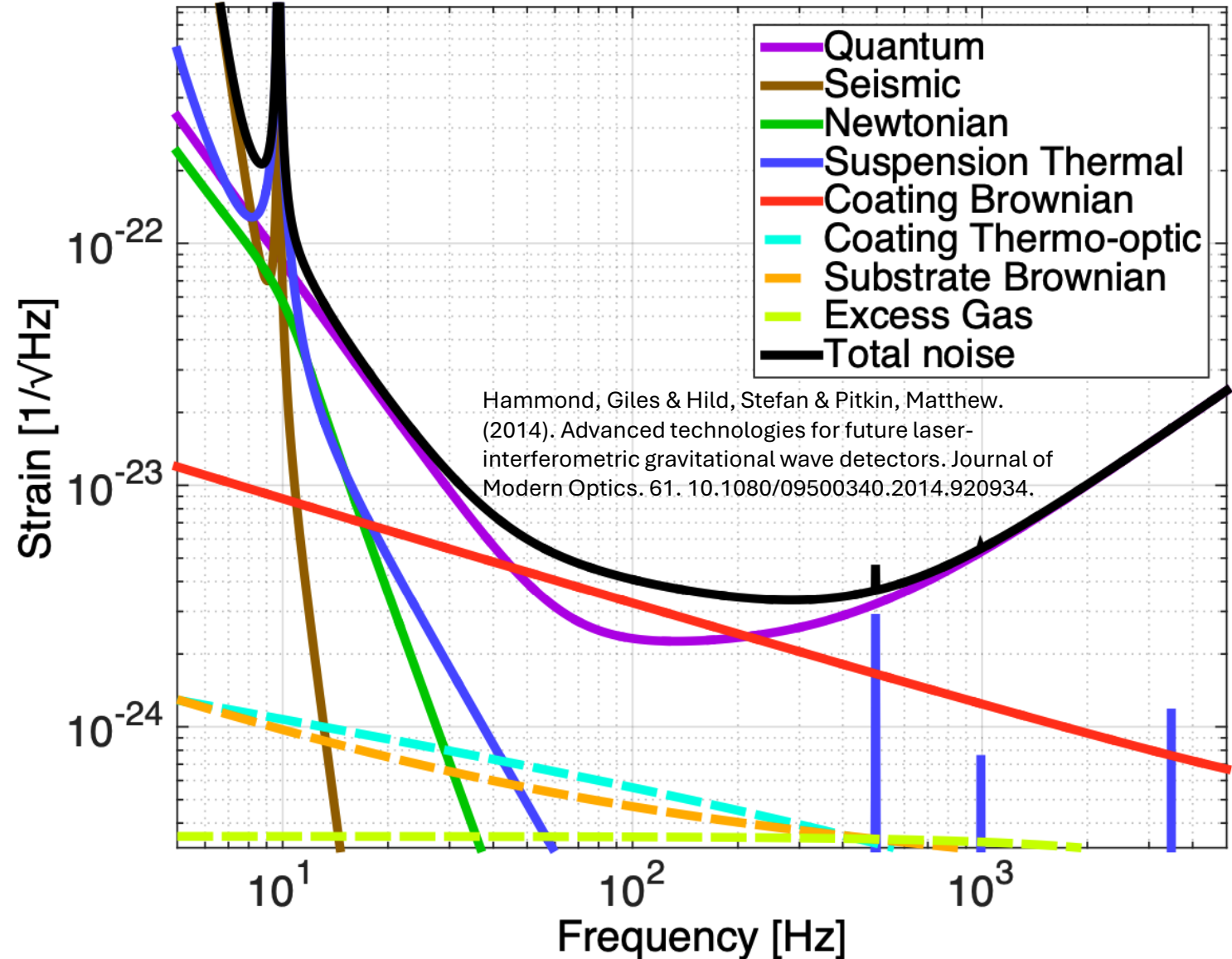
# Comparison: noise PSD of SPring-8 vs LIGO

Preliminary



# Advanced LIGO noise source models

- Most of the noise sources are specific to the LIGO setup (eg laser system)
- **Noise model of Storage Ring is of great importance**
- Another aspect: normalization of y-axis before FFT...relative length (?)  $\delta L/L$



# Noise estimation by Rao et al + momentum compaction

name	cause	$\Delta T$ [s]
Quantum noise	Uncertainty in pos / mom	1e-20
Gravity gradient noise	Tidal force	1e-16
Seismic noise	Mechanical vibration	1e-17
<b>RF phase noise</b>	<b>????</b>	<b>1e-12</b>
Detector noise	Timing jitter in longitudinal monitor	5e-11
Photon shot noise	Single photon detector	1e-17

Slow  
frequency  
(nHz-mHz)  
drift ?

$$-\frac{\Delta L}{L} \sim \frac{\Delta f}{f} = \eta_c \frac{\Delta p}{p}$$

$$\eta_c = \frac{1}{\gamma^2} - \alpha_c$$

SPring-8

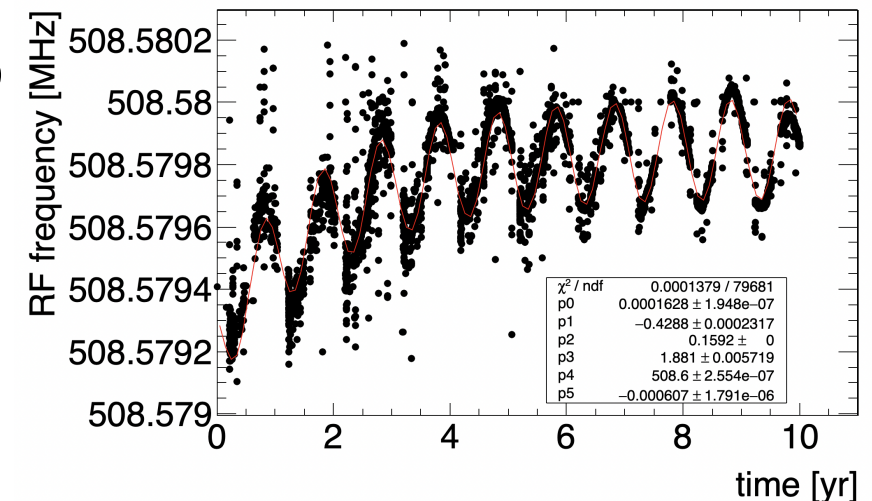
$$8 \text{ GeV} \rightarrow 1/\gamma^2 \sim 4 \times 10^{-9}$$

$$\alpha_c \sim 1.46 \times 10^{-4}$$

$$\Delta p/p \sim 1.08 \times 10^{-3}$$

$$\rightarrow \Delta L/L \sim 10^{-7} = 0.1 \text{ ppm}$$

Consistent with data 0.1 ppm resolution



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# Conclusion

- We succeeded the work by A. N. Ivanov and A. P. Kobushkin
  - GW could vary circumference of storage rings with  $h^2$
- RF data of storage ring records variation of circumference over years
  - SPring-8 data (and soon KEKB data) to see 25 years → access to nHz
- Time domain analysis
  - Annual modulation was explained by thermal shrinkage
  - Initial shift may be due to drying concrete
  - There remain other frequency components
- Frequency domain analysis: simple FFT does not work
  - Lomb-Scargle to process unevenly taken data set
  - Sampling bias generates fake peaks
  - There remain some peak not explained by sampling bias or environmental data
- Noise PSD analysis compared with LIGO data (preliminary!)
  - No clear noise model in storage ring data
  - $h^2$  vs  $h$  differences
- Goal: (probably very weak but 1<sup>st</sup>) constraint on  $\Omega_{GW}$  below mHz by using real accelerator data
  - We are also working on mathematics of  $h$ ,  $\Omega_{GW}$ ,  $S_h$  in general relativity
- This is a result from a “1<sup>st</sup> generation GW detector” (like storage ring for synchrotron radiation)
  - How to improve it toward a storage ring dedicated to GW detection?