

SRF for GW detection: MAGO

Sebastian A. R. Ellis
University of Geneva

Conclusion This Morning

Identify regime of GW by hierarchy with respect to size of detector:

- Resonant regime: $\omega_g \sim 1/L \gg c_s/L$ — use PDF and account for current & boundary changes

e.g. *axion cavity experiments*

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (2021)

- High-frequency regime: $\omega_g \gg 1/L \gg c_s/L$ — use TT gauge

e.g. *MADMAX*

Domcke, Ellis, Kopp (2024)

- Low-frequency regime: $1/L \gg \omega_g$ — use PDF and account for current & boundary changes

e.g. *Magneto-quasistatic experiments*

Domcke, Garcia-Cely, Rodd (2022)

e.g. *Heterodyne experiments*

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel, Wentzel (2023)

Domcke, Garcia-Cely, Lee, Rodd (2023)

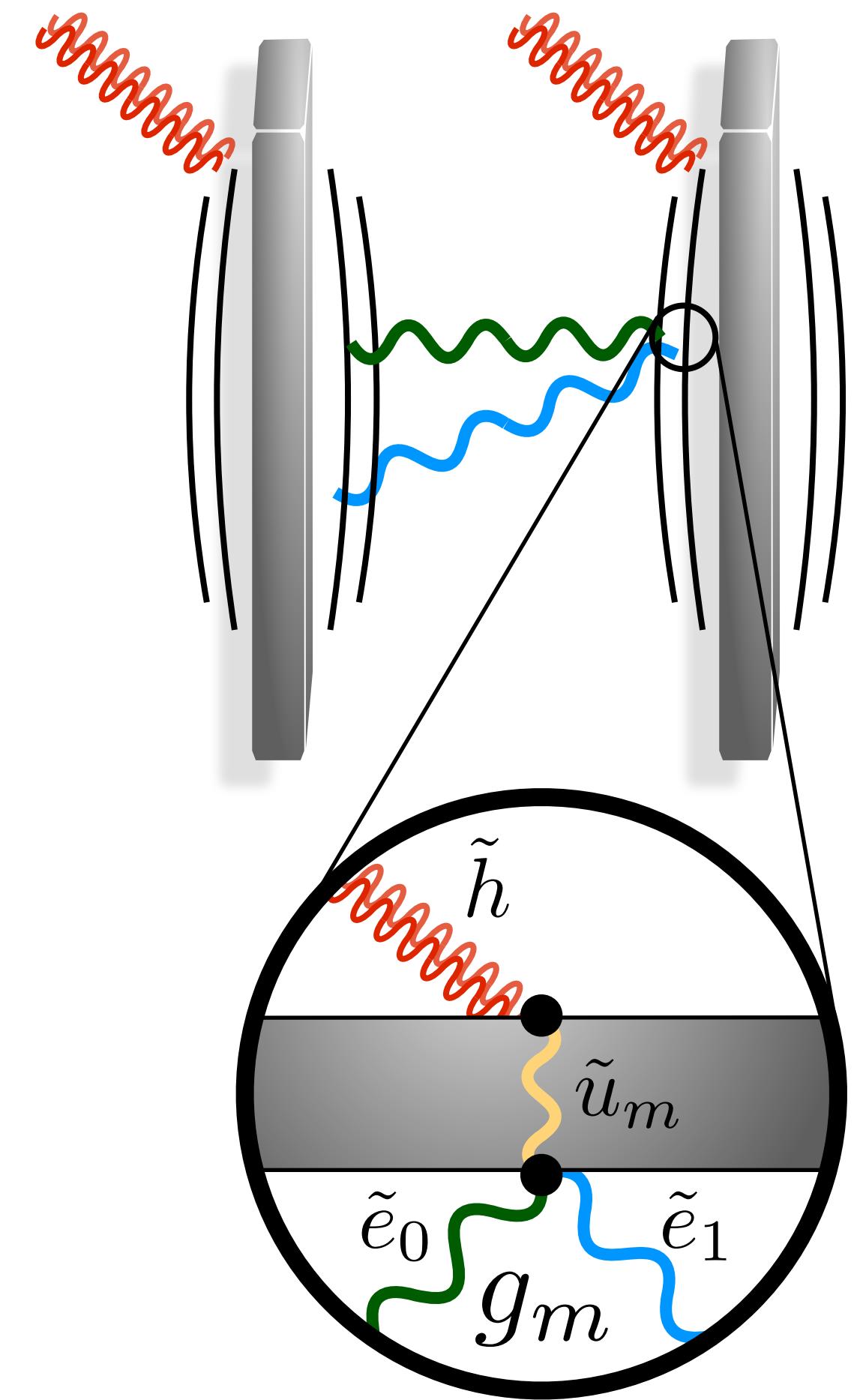
Domcke, Ellis, Rodd (2024)

Transfer function for mechanical transduction

$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{\omega_g^2 L}{2}\tilde{h}^{\text{TT}}(\omega)$$
$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{e}_1(\omega) \simeq \int d\omega' \tilde{e}_0(\omega - \omega')g_m\tilde{u}_m(\omega')$$

$$g_m \equiv -\frac{2\omega_1^2}{L}$$

$$\mathcal{T}_{\text{mech}}^2(\omega) = \frac{\omega_g^4 \omega_1^4}{\left((\omega_1^2 - \omega^2)^2 + \frac{\omega^2 \omega_1^2}{Q^2}\right) \left((\omega_m^2 - \omega_g^2)^2 + \frac{\omega_g^2 \omega_m^2}{Q_m^2}\right)}$$

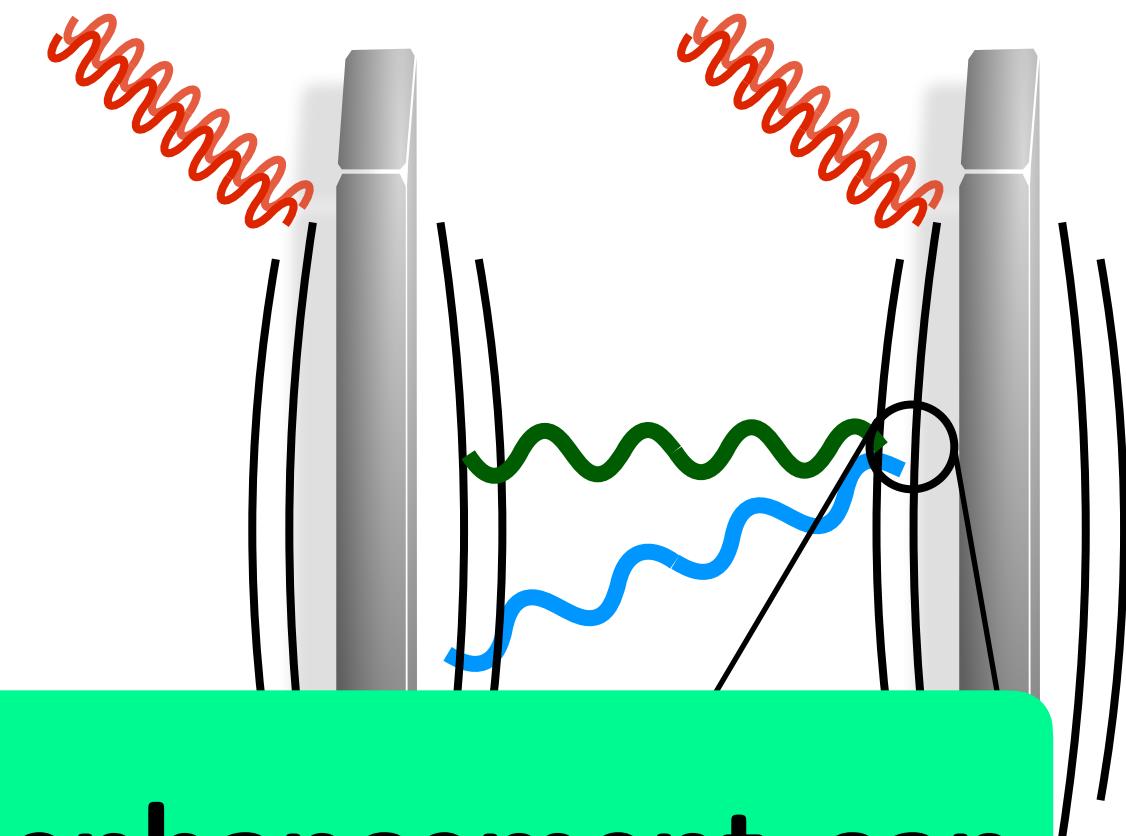


D'Agnolo, SARE (gr-qc/2412.17897)

Transfer function for mechanical transduction

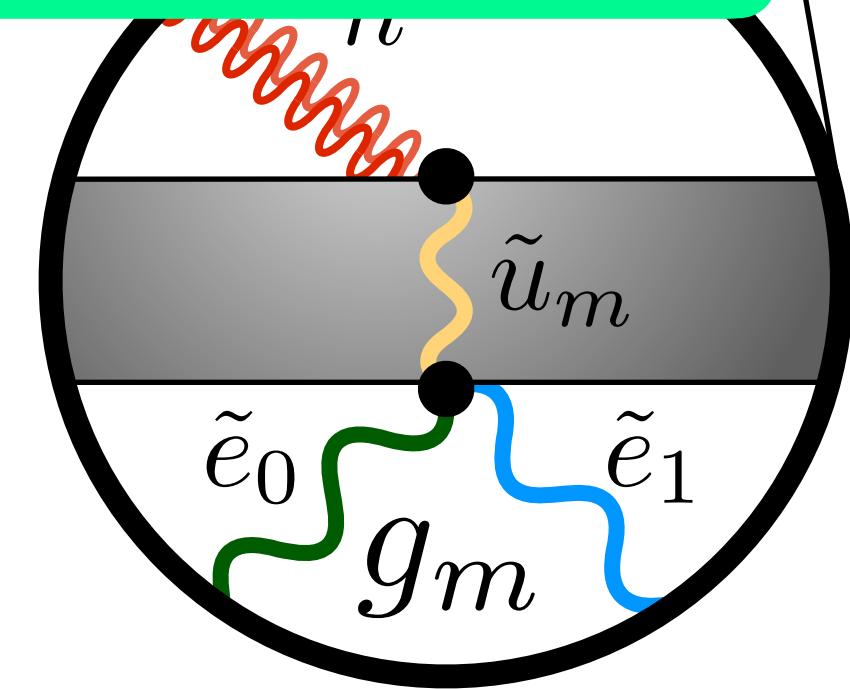
$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{\omega_g^2 L}{2}\tilde{h}^{\text{TT}}(\omega)$$

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{e}_1(\omega) \simeq \int d\omega' \tilde{e}_0(\omega - \omega') g_m \tilde{u}_m(\omega')$$



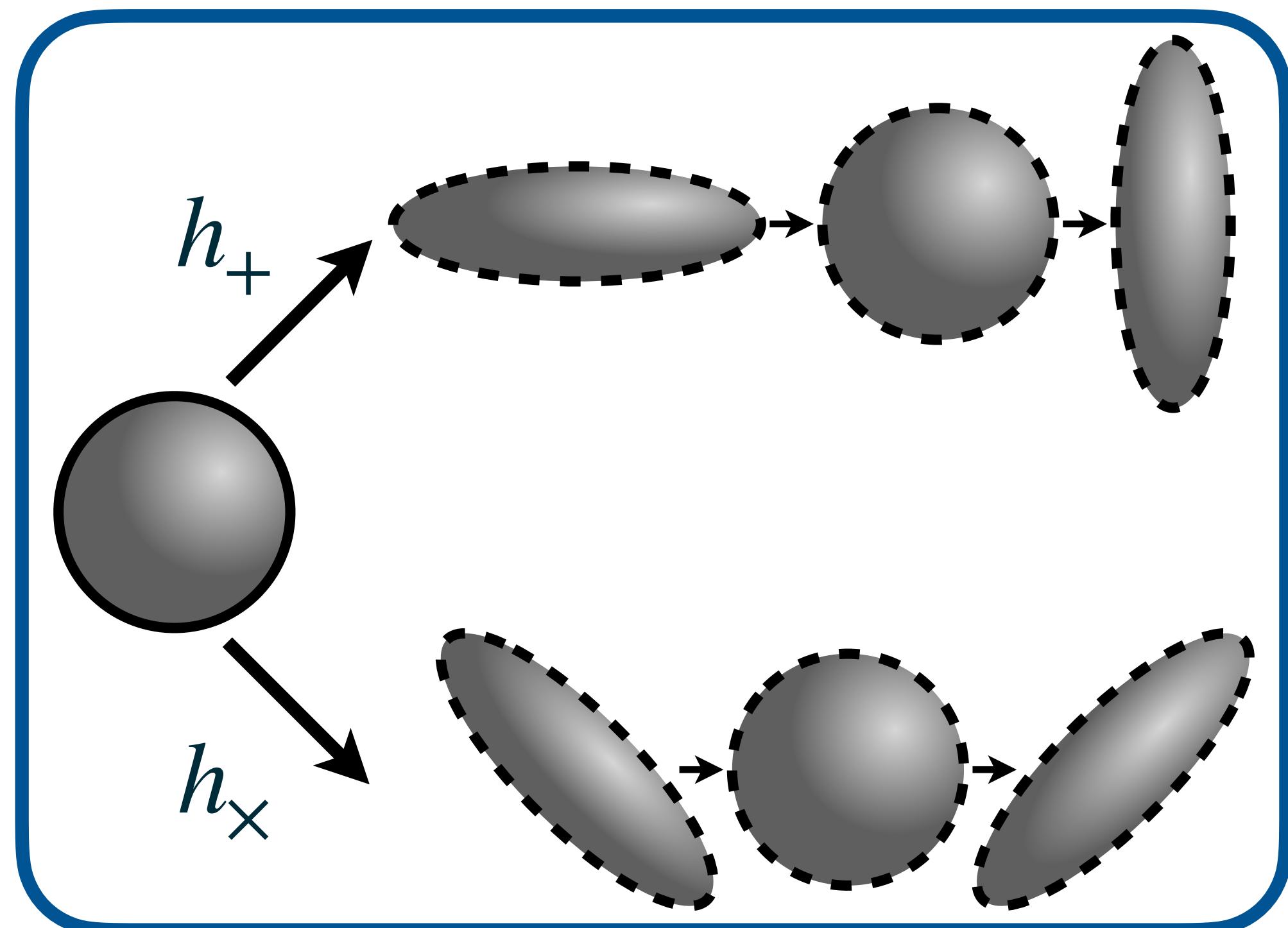
Is there a compact physical system where EM resonant enhancement can be realised?

$$\mathcal{T}_{\text{mech}}^2(\omega) = \frac{\omega_g^4 \omega_1^4}{\left((\omega_1^2 - \omega^2)^2 + \frac{\omega^2 \omega_1^2}{Q^2}\right) \left((\omega_m^2 - \omega_g^2)^2 + \frac{\omega_g^2 \omega_m^2}{Q_m^2}\right)}$$



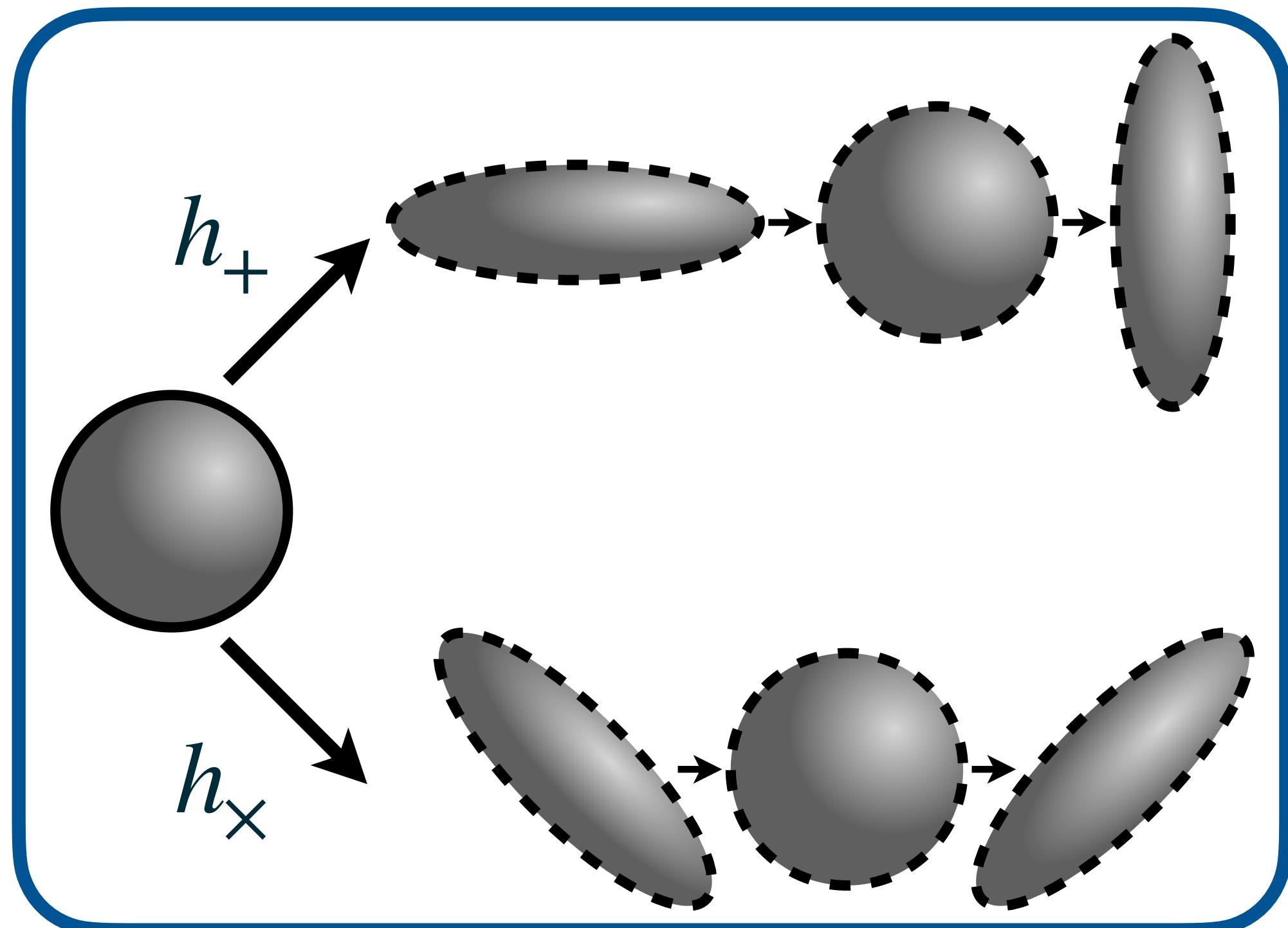
D'Agnolo, SARE (gr-qc/2412.17897)

Gravitational Wave and a Hollow Sphere



TT frame intuition

Gravitational Wave and a Hollow Sphere

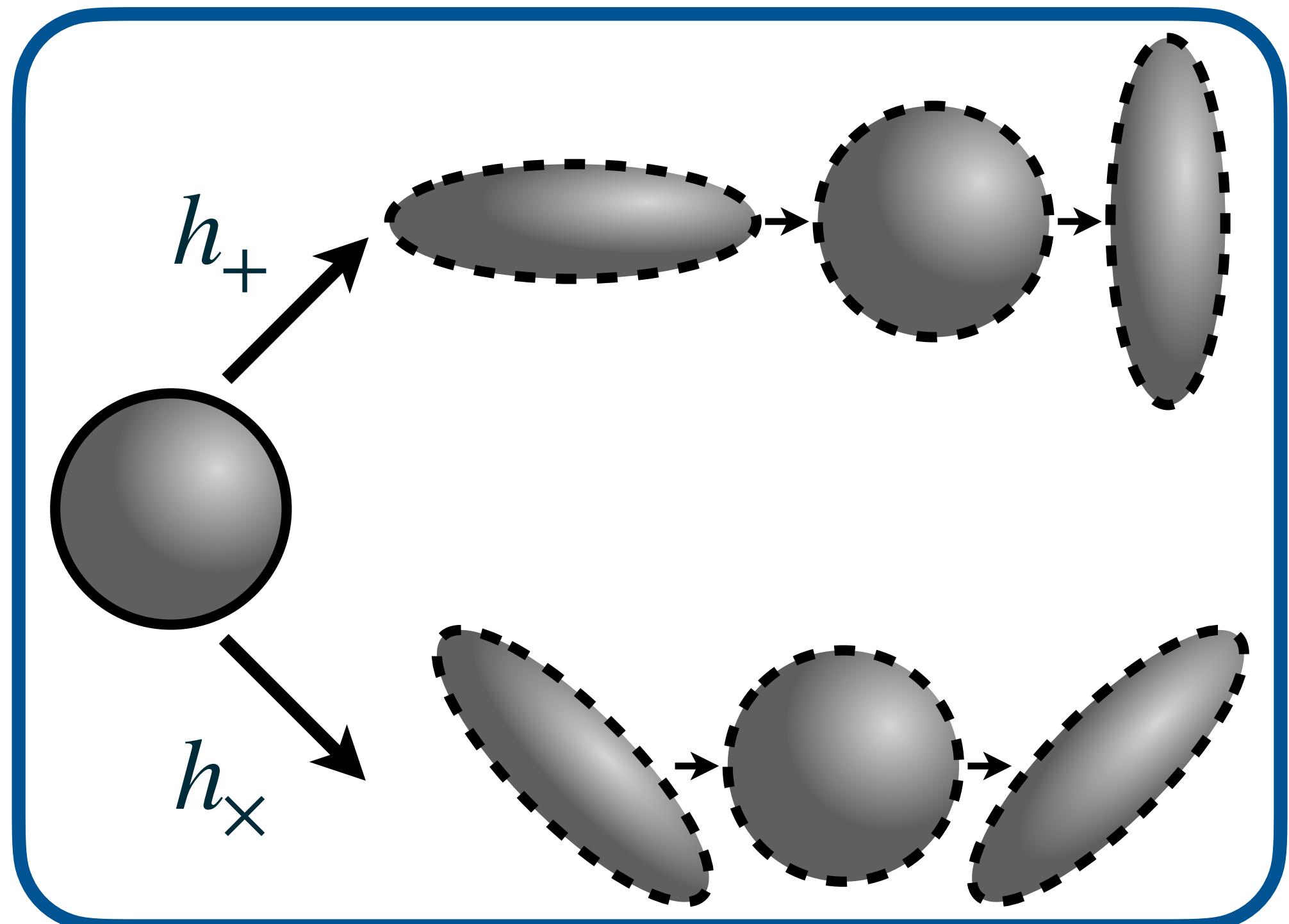


TT frame intuition

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\nabla\times\mathbf{L}\phi_{T_1} + i\mathbf{L}\phi_{T_2} .$$

Gravitational Wave and a Hollow Sphere

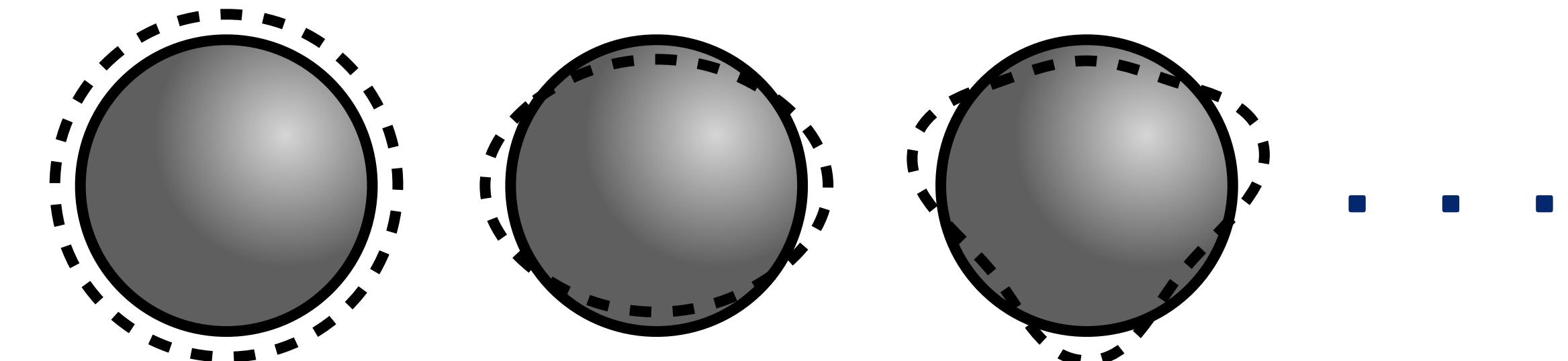


TT frame intuition

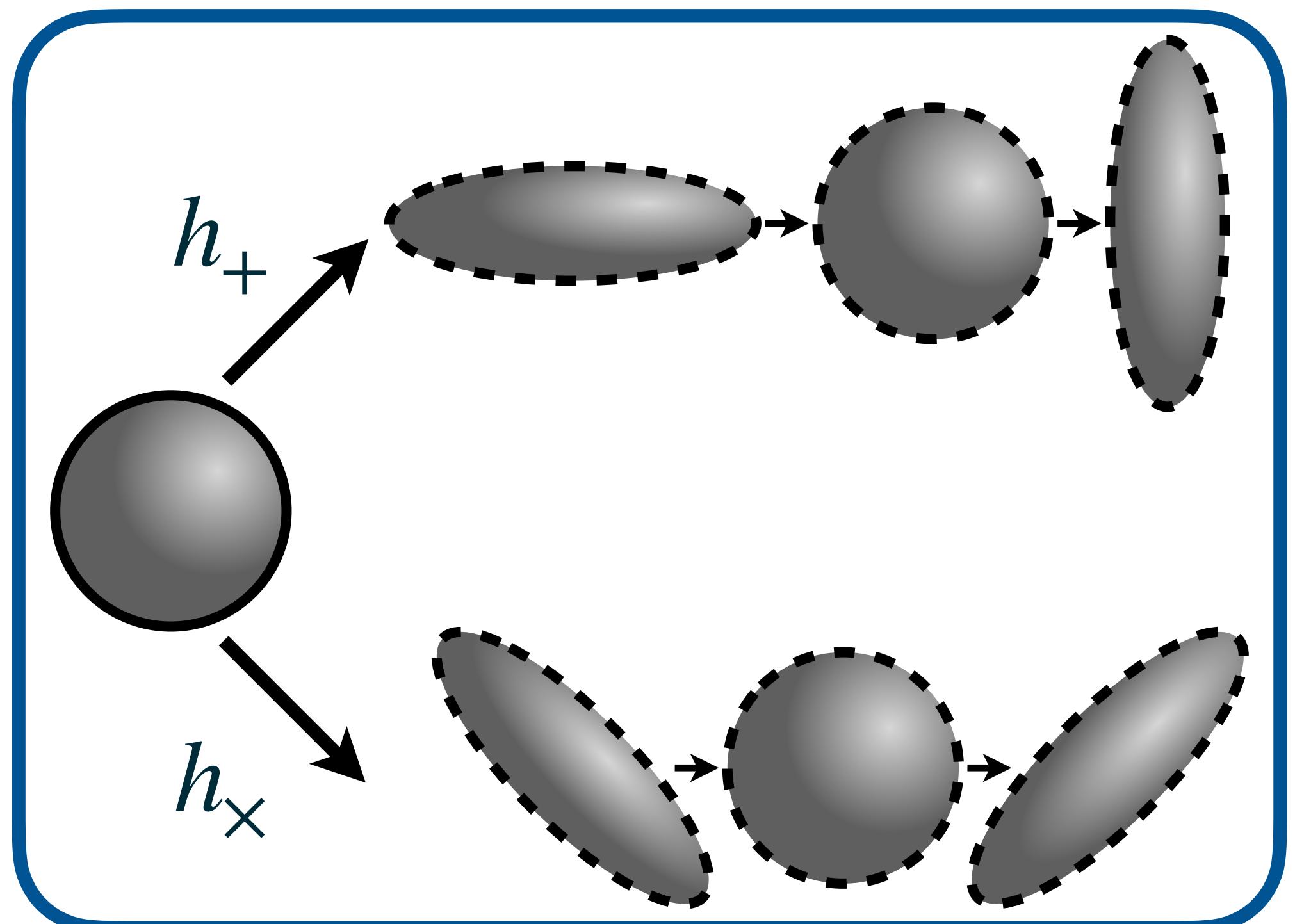
Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\nabla\times\mathbf{L}\phi_{T_1} + i\mathbf{L}\phi_{T_2} .$$

Spheroidal



Gravitational Wave and a Hollow Sphere



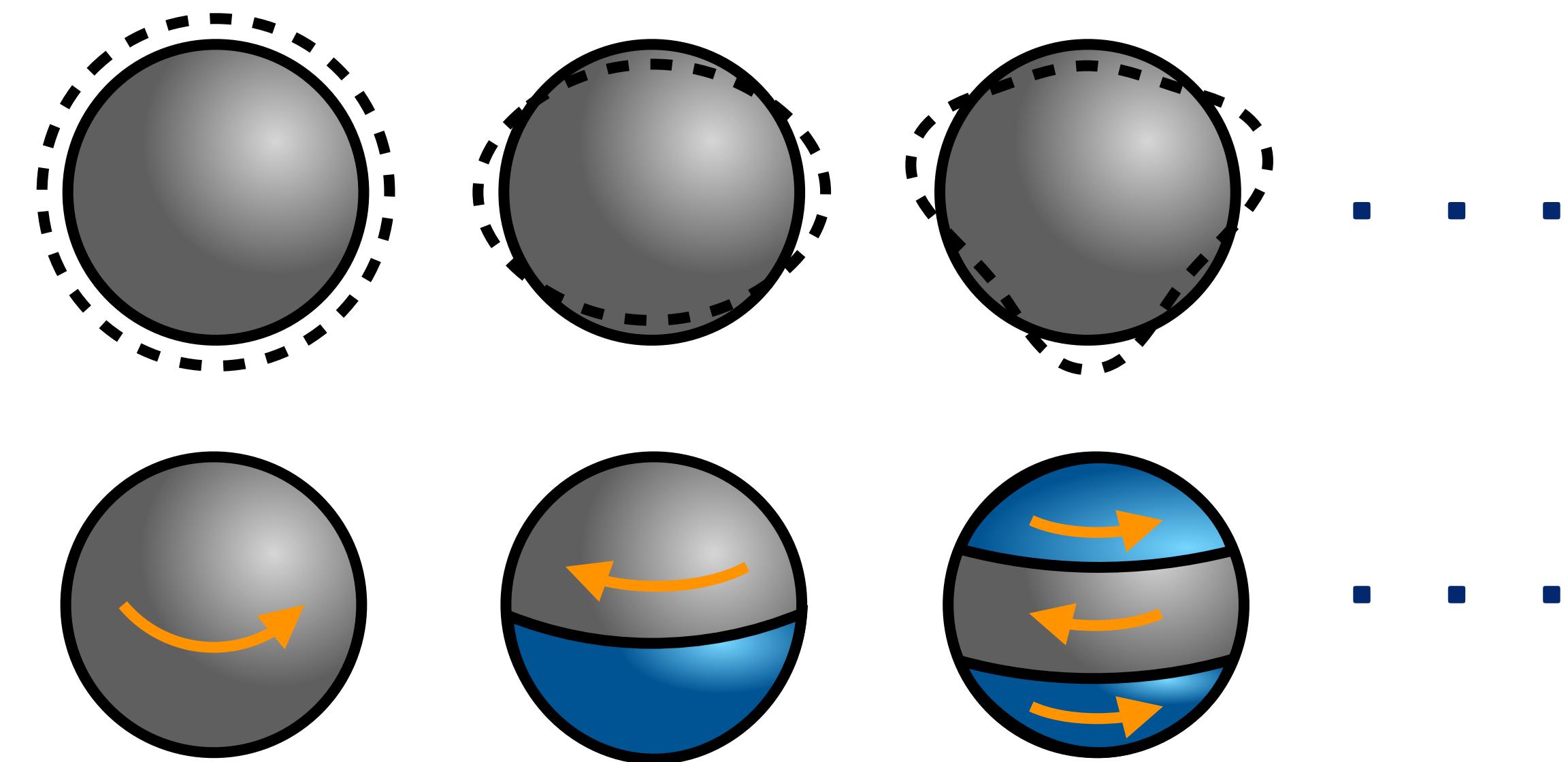
TT frame intuition

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\nabla\times\mathbf{L}\phi_{T_1} + i\mathbf{L}\phi_{T_2}.$$

Spheroidal

Toroidal



Gravitational Wave and a Hollow Sphere

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\nabla\times\mathbf{L}\phi_{T_1} + i\mathbf{L}\phi_{T_2} .$$

$$\mathbf{U}(\mathbf{x}, t) = u_p(t) \mathbf{U}_p(\mathbf{x})$$

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\nabla\times\mathbf{L}\phi_{T_1} + i\mathbf{L}\phi_{T_2} .$$

$$\mathbf{U}(\mathbf{x}, t) = u_p(t) \mathbf{U}_p(\mathbf{x})$$

Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p} \dot{u}_p + \omega_p^2 u_p \simeq -\frac{1}{2} \omega_g^2 V_{\text{cav}}^{1/3} \eta_{\text{mech}}^g h_0 e^{i\omega_g t}$$

$$\eta_{\text{mech}}^g = \frac{\hat{h}_{ij}^{TT}}{V_{\text{cav}}^{1/3} V_{\text{shell}}} \int_{V_{\text{shell}}} d^3\mathbf{x} U_p^{*i} x^j$$

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\nabla\times\mathbf{L}\phi_{T_1} + i\mathbf{L}\phi_{T_2} .$$

$$\mathbf{U}(\mathbf{x}, t) = u_p(t) \mathbf{U}_p(\mathbf{x})$$

Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p} \dot{u}_p + \omega_p^2 u_p \simeq -\frac{1}{2} \omega_g^2 V_{\text{cav}}^{1/3} \eta_{\text{mech}}^g h_0 e^{i\omega_g t}$$

$$\eta_{\text{mech}}^g = \frac{\hat{h}_{ij}^{TT}}{V_{\text{cav}}^{1/3} V_{\text{shell}}} \int_{V_{\text{shell}}} d^3\mathbf{x} U_p^{*i} x^j$$

$$\langle \mathbf{U}_p \rangle \sim h_0 V_{\text{cav}}^{1/3} \eta_{\text{mech}}^g \times \begin{cases} \frac{\omega_g^2}{\omega_g^2 - \omega_p^2} , & |\omega_g - \omega_p| \gg \omega_p/Q_p \\ Q_p , & |\omega_g - \omega_p| \ll \omega_p/Q_p \end{cases}$$

Tiny displacement $\ll \text{nm}$

MAGO 2.0: Mechanical and EM Signals

MAGO 2.0: Mechanical and EM Signals

On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro[†], E Picasso[‡] and L A Radicati^{‡§}

[†]Scuola Normale Superiore, Pisa, Italy

[‡]CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

MAGO 2.0: Mechanical and EM Signals

On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro[†], E Picasso[‡] and L A Radicati^{‡§}

[†]Scuola Normale Superiore, Pisa, Italy

[‡]CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

Microwave Apparatus for Gravitational Waves Observation

R. Ballantini, A. Chincarini, S. Cuneo, G. Gemme^{*}, R. Parodi, A. Podestà, and R. Vaccarone
INFN and Università degli Studi di Genova, Genova, Italy

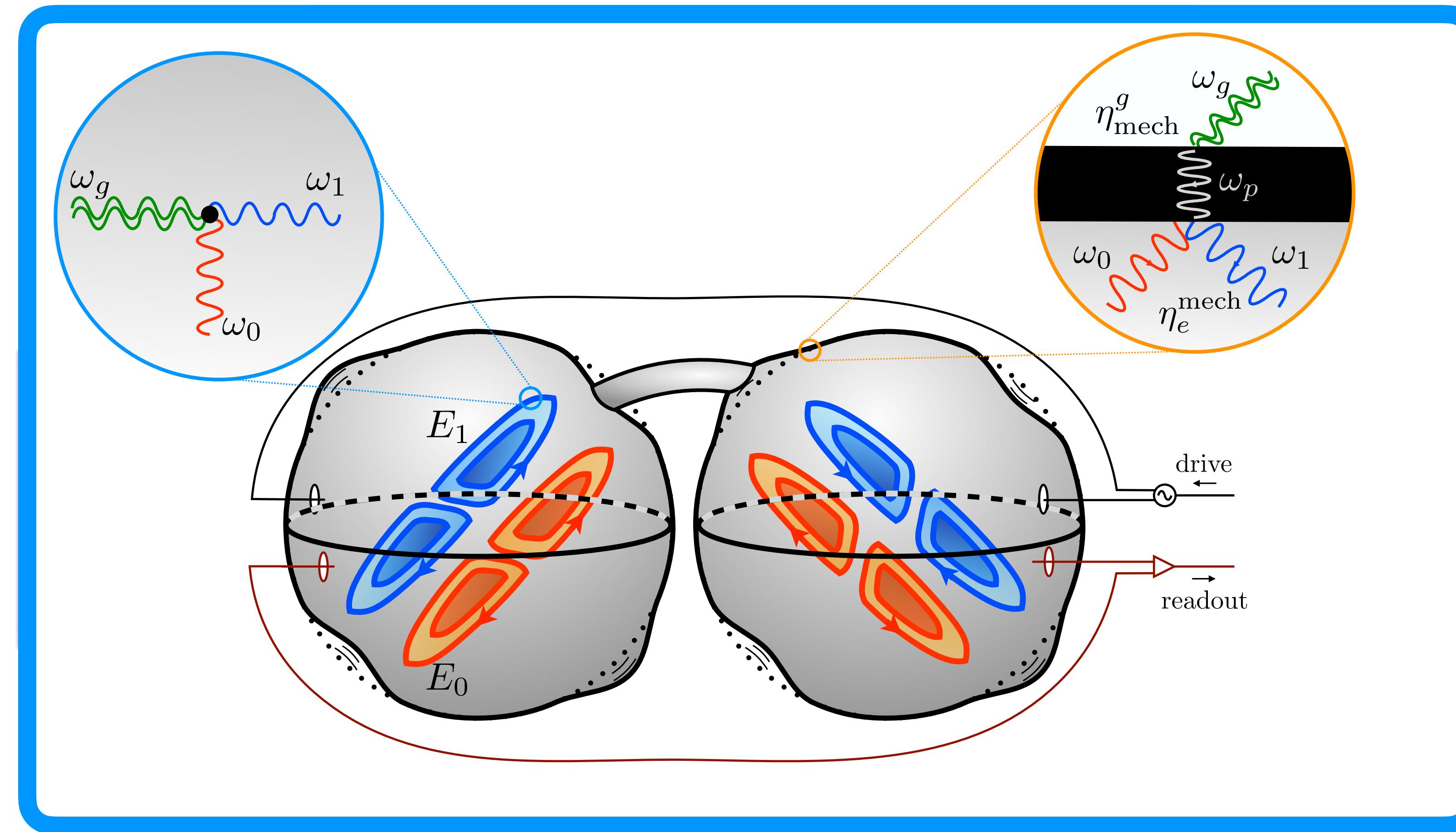
Ph. Bernard, S. Calatroni, E. Chiaveri, and R. Losito
CERN, Geneva, Switzerland

R.P. Croce, V. Galdi, V. Pierro, and I.M. Pinto
INFN, Napoli, and Università degli Studi del Sannio, Benevento, Italy

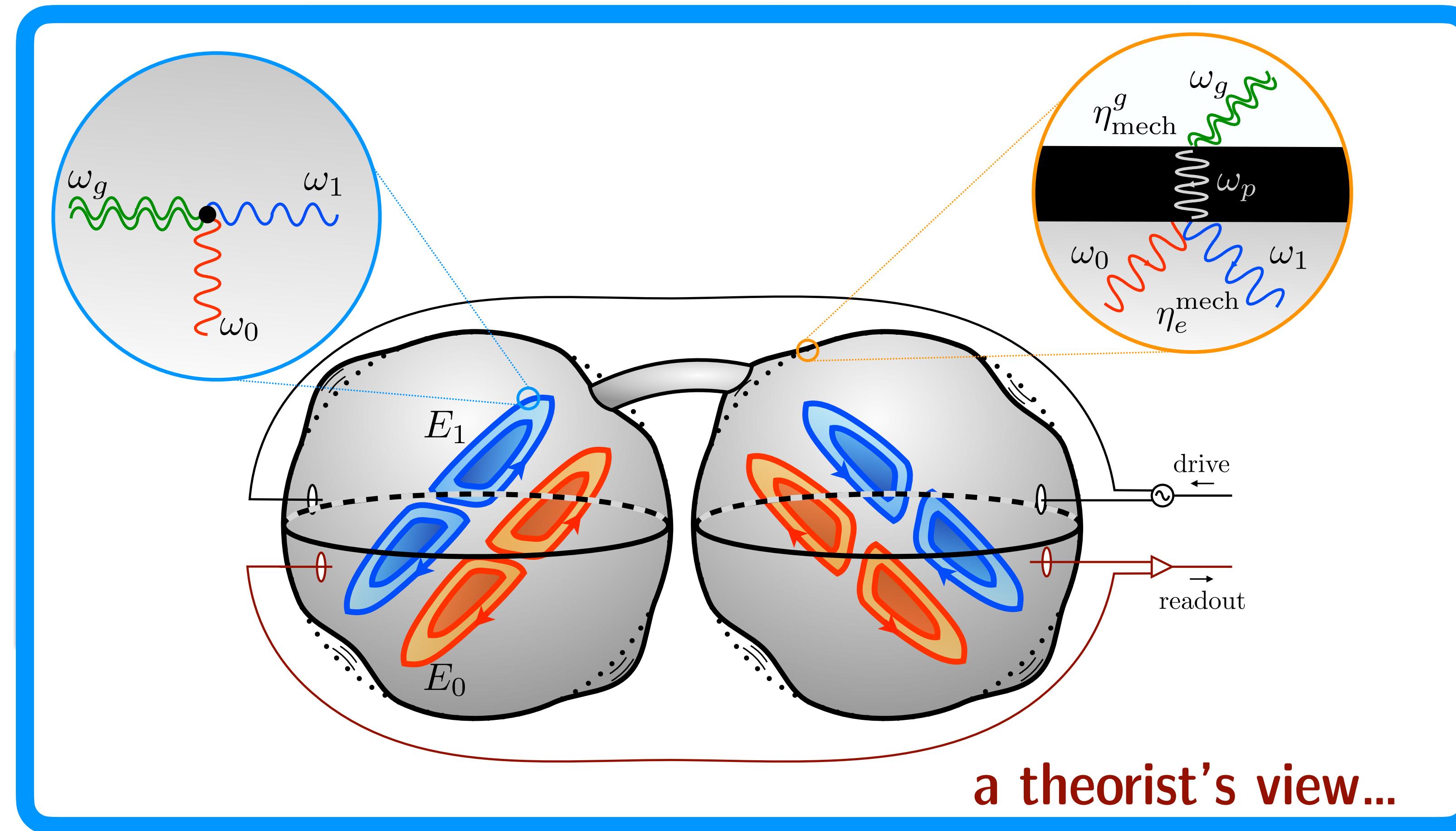
E. Picasso
*INFN and Scuola Normale Superiore, Pisa, Italy and
CERN, Geneva, Switzerland*



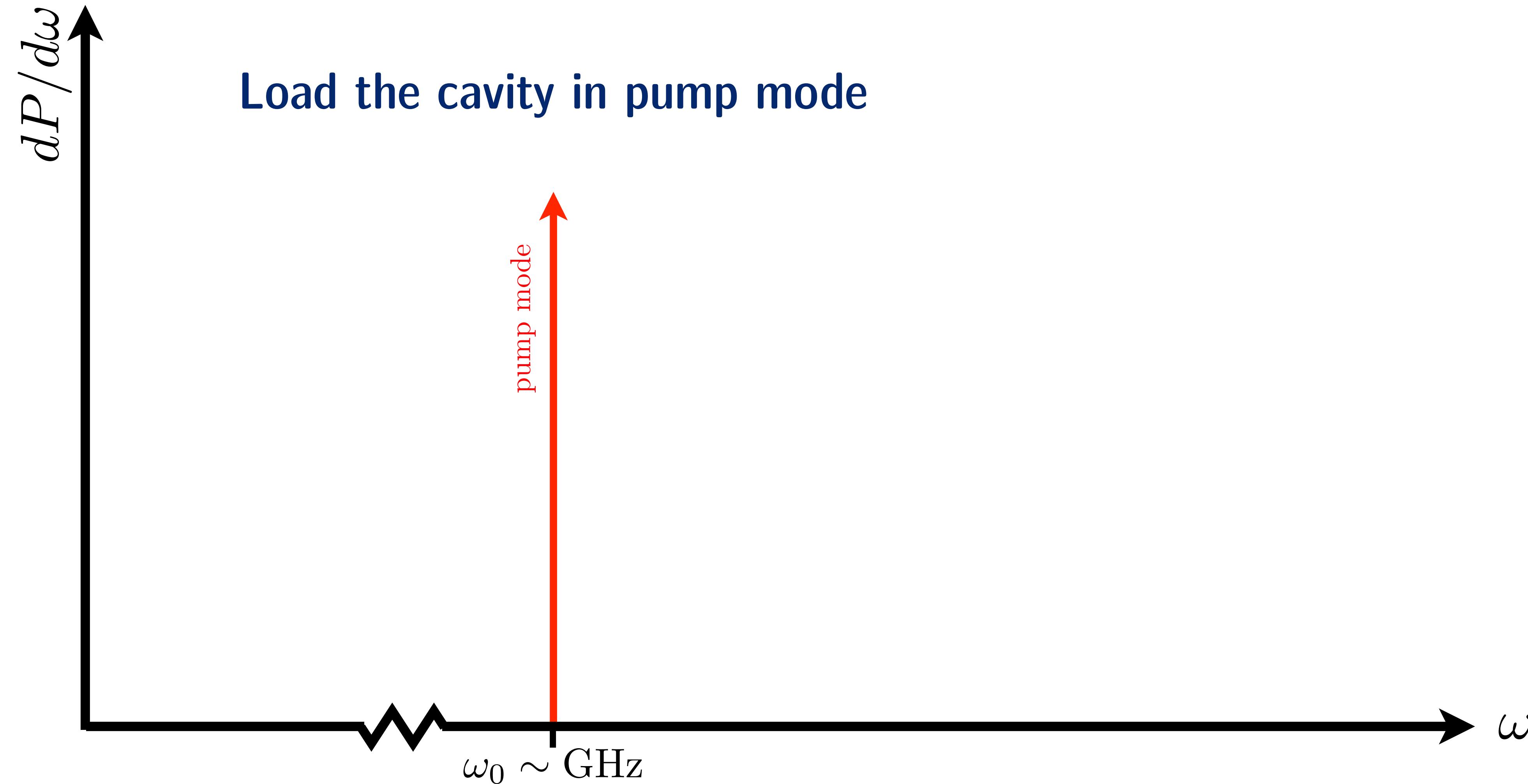
MAGO 2.0: Mechanical and EM Signals



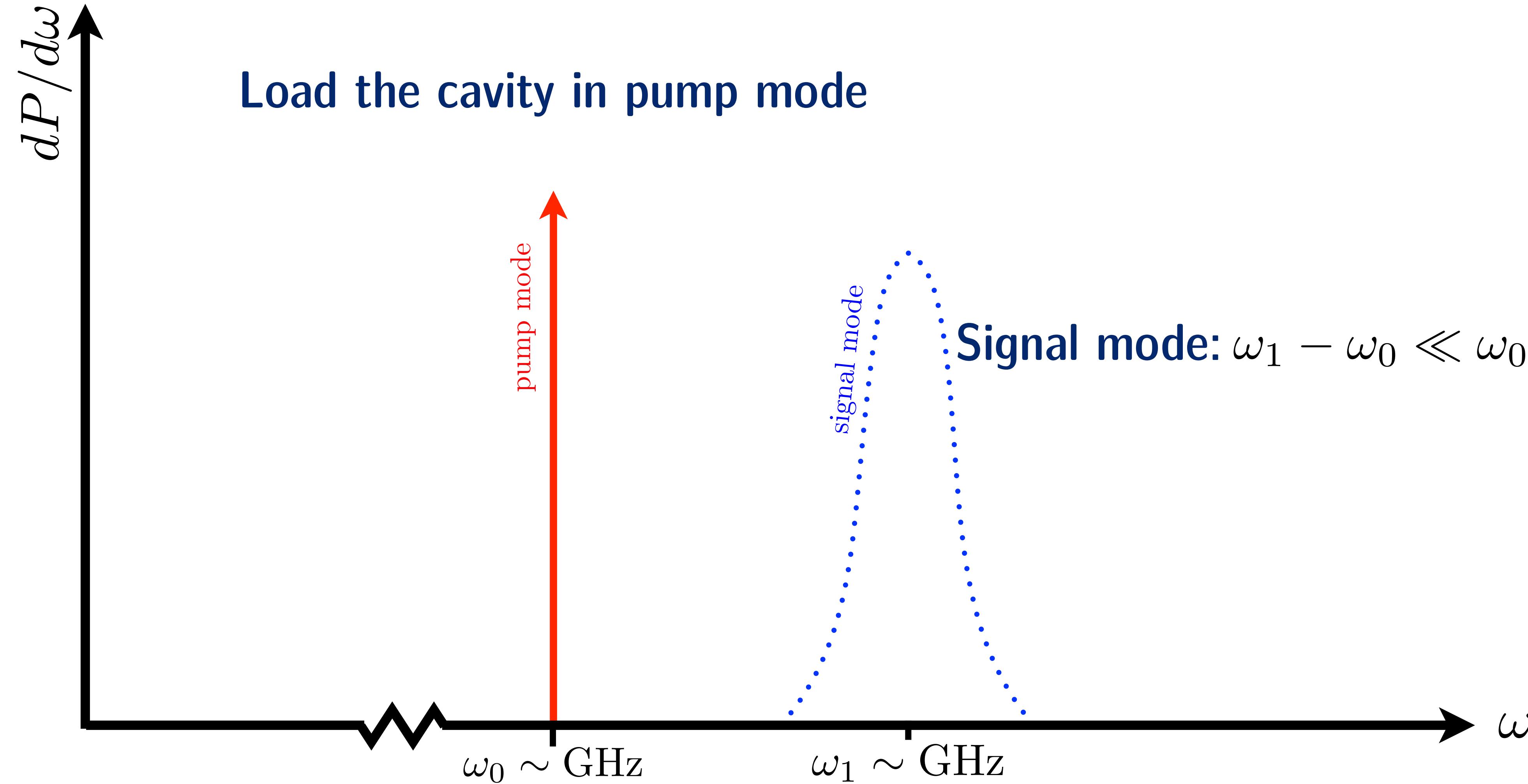
MAGO 2.0: Mechanical and EM Signals



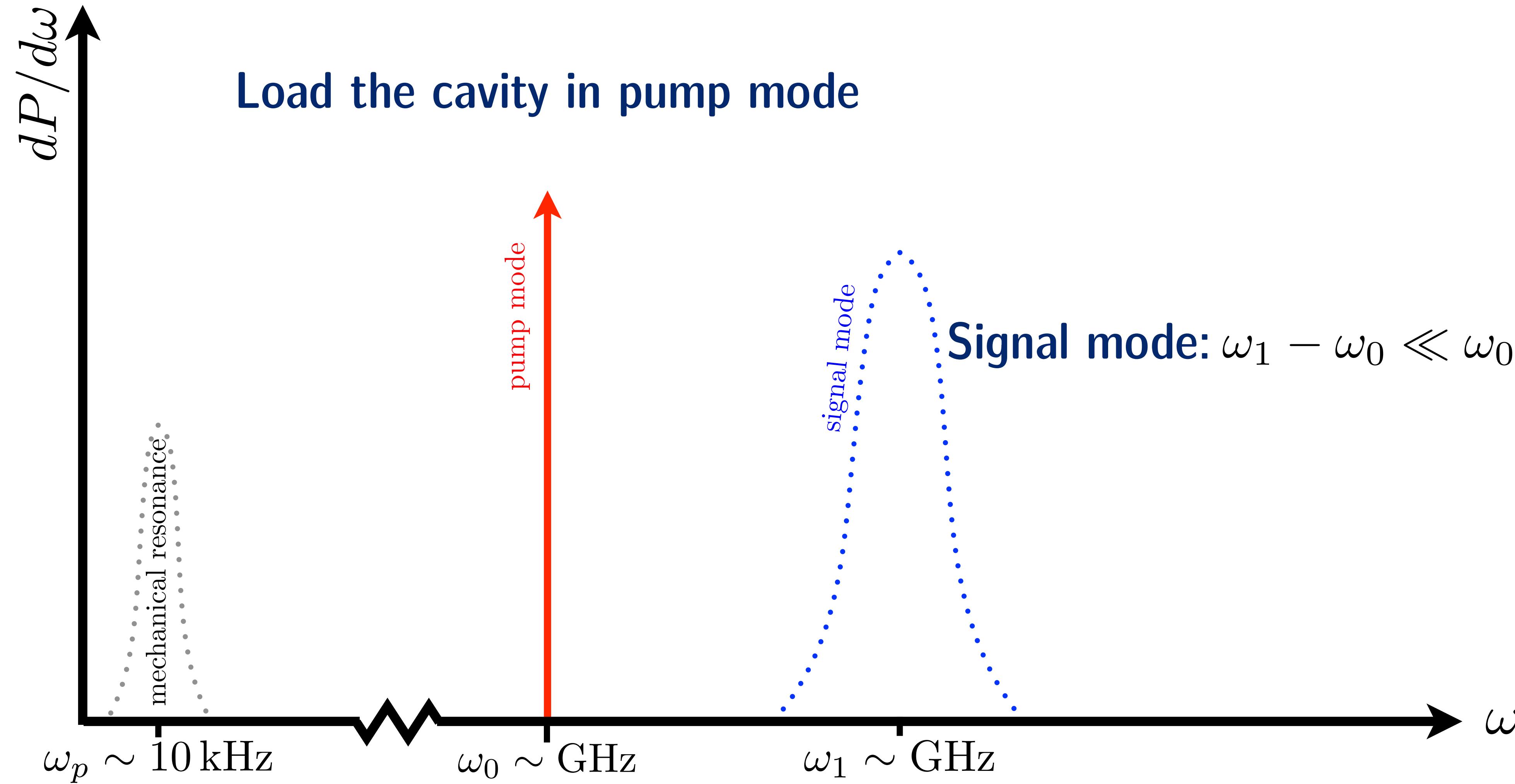
MAGO 2.0



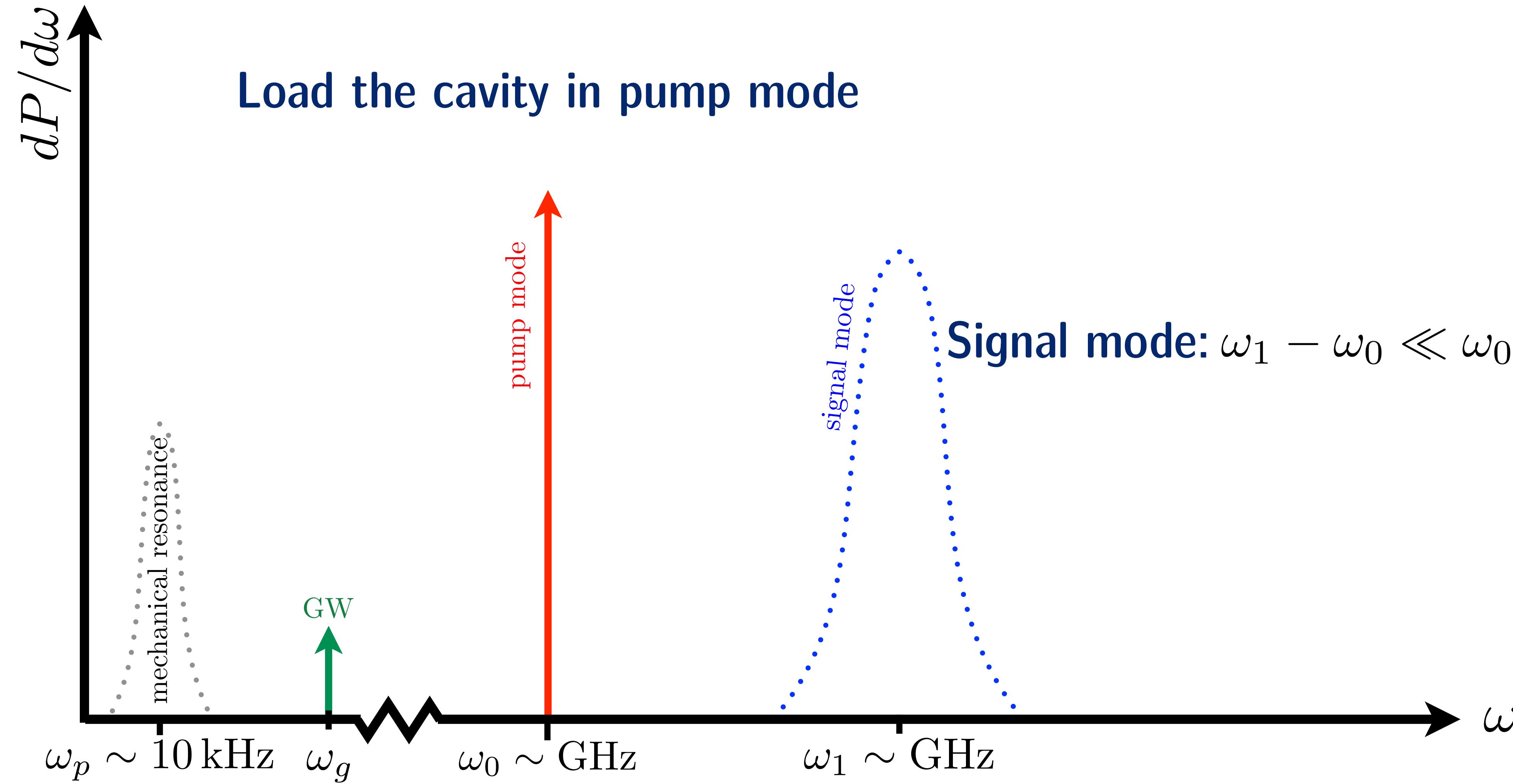
MAGO 2.0



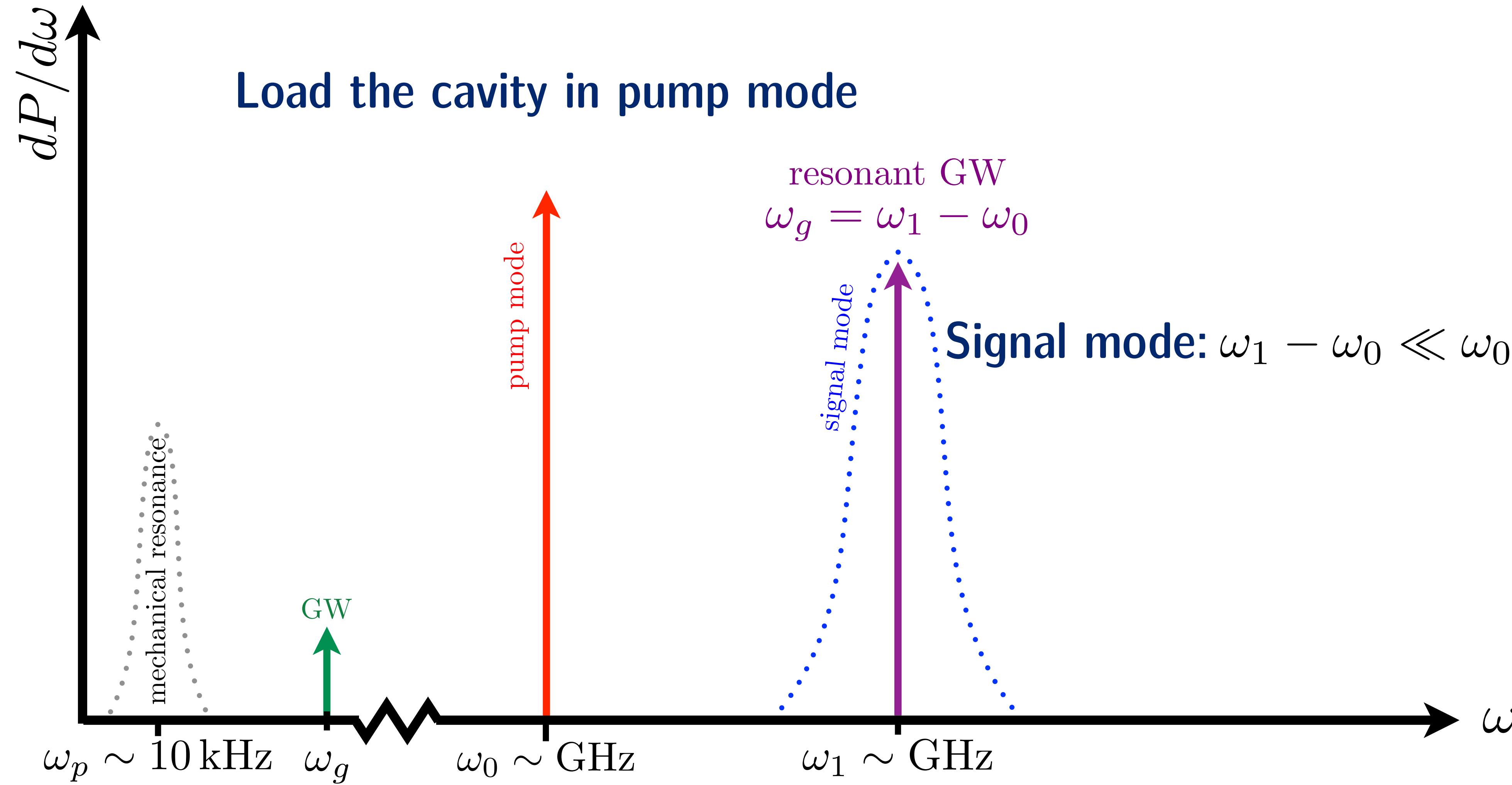
MAGO 2.0



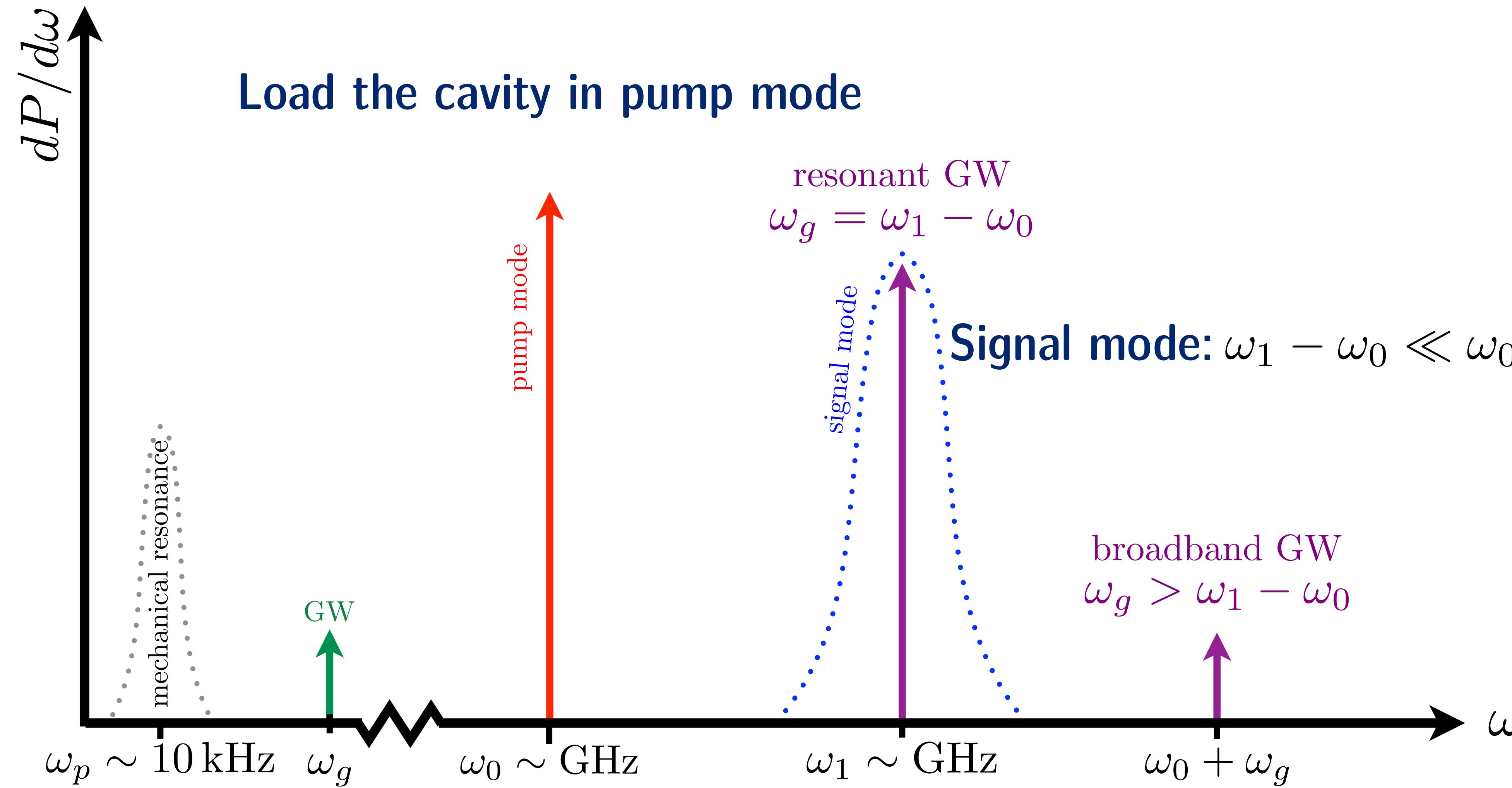
MAGO 2.0



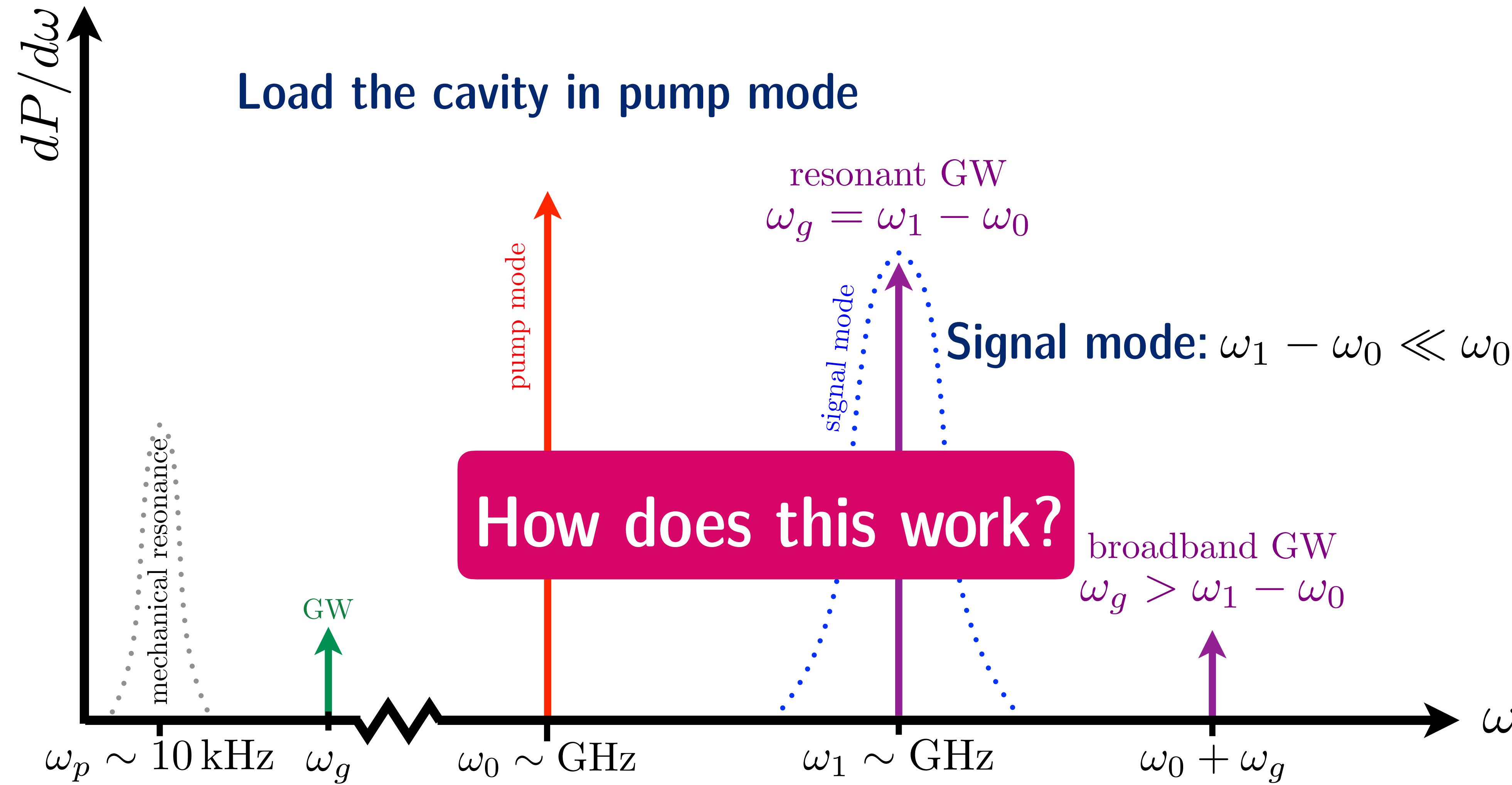
MAGO 2.0



MAGO 2.0

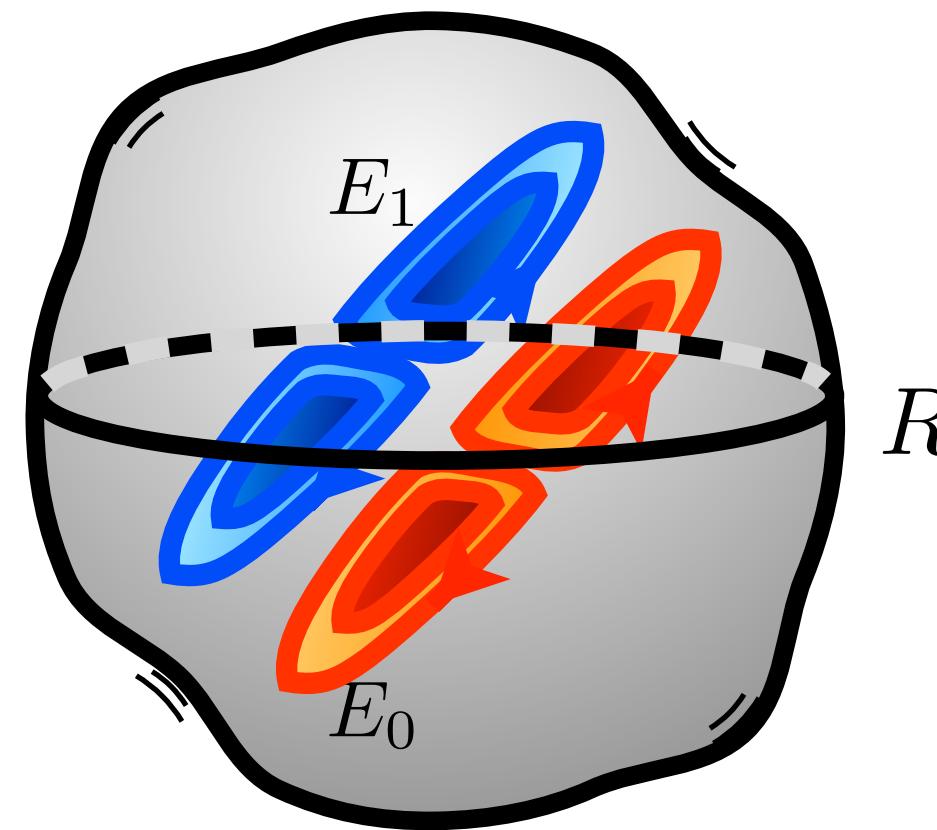


MAGO 2.0

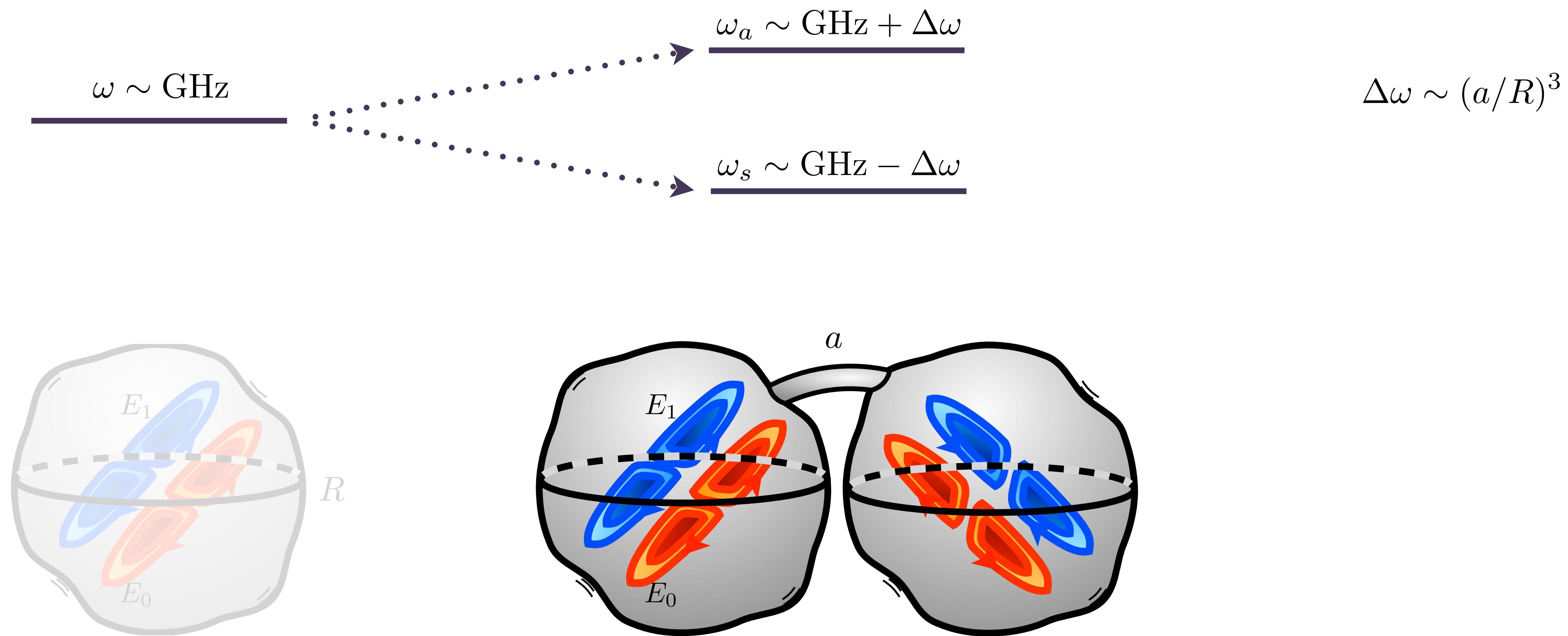


MAGO 2.0

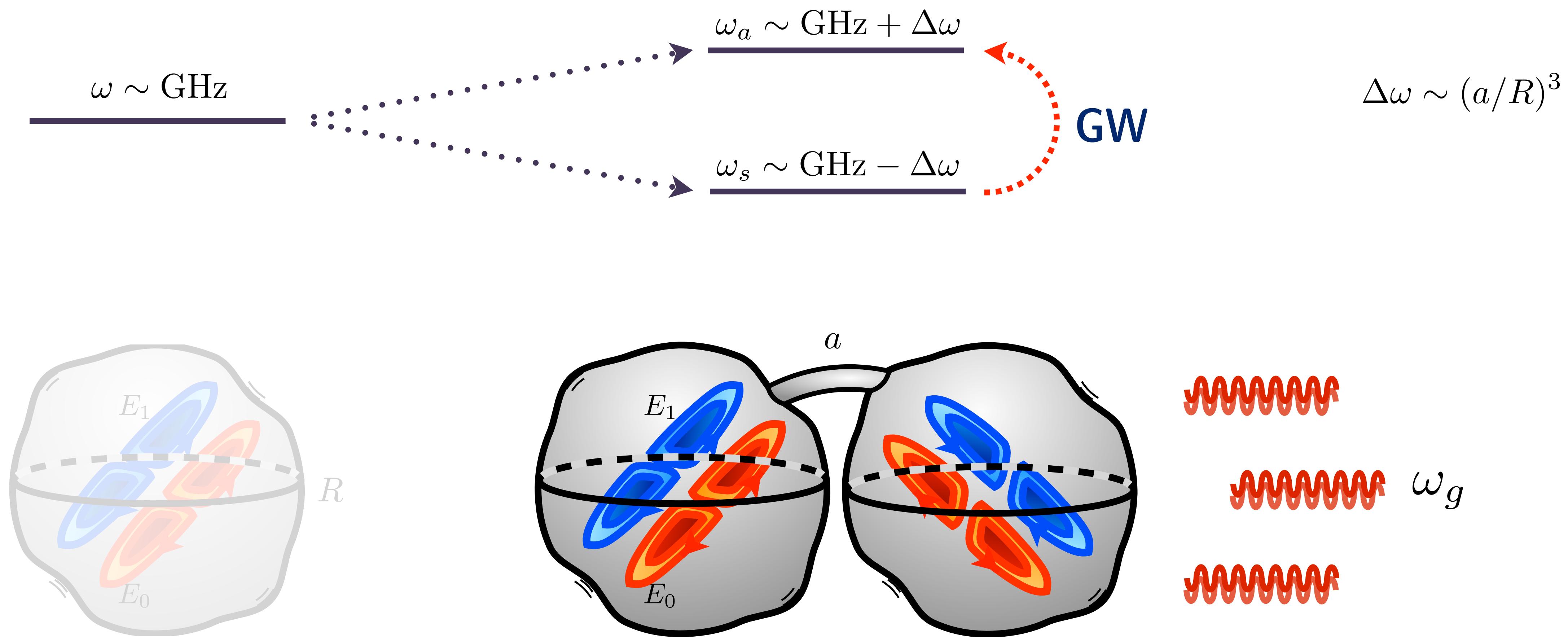
$\omega \sim \text{GHz}$



MAGO 2.0



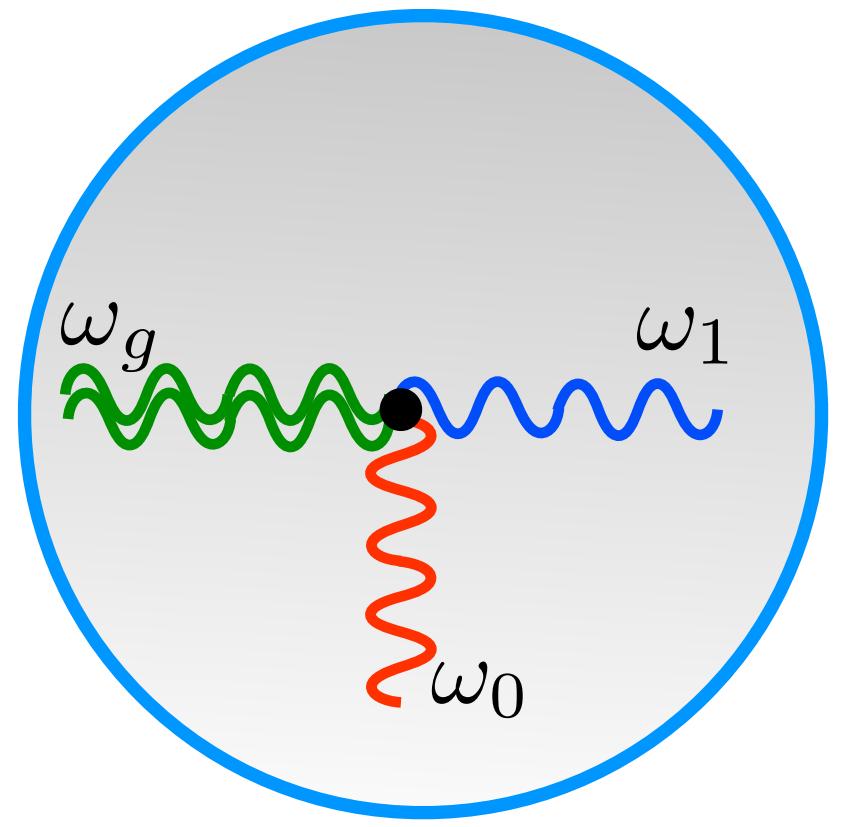
MAGO 2.0



EM and Mechanical signals

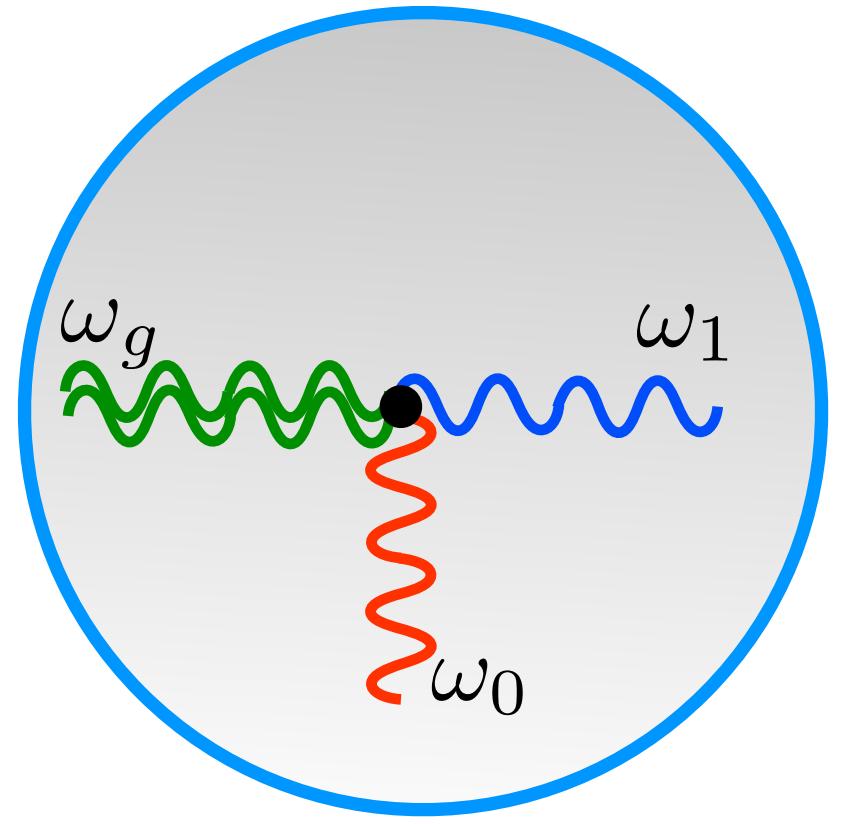
EM and Mechanical signals

Parametrics of the EM signal: $E_{\text{sig}}^{(\text{EM})} \sim Q_{\text{em}} (\omega_g L_{\text{cav}})^2 h^{\text{TT}} E_0$



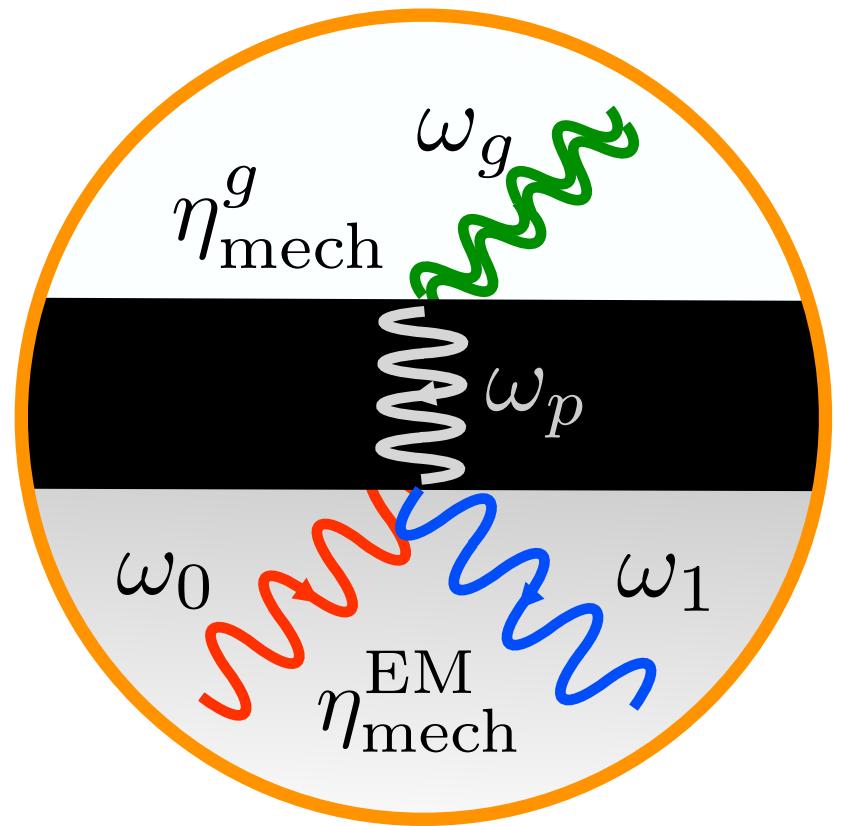
EM and Mechanical signals

Parametrics of the EM signal: $E_{\text{sig}}^{(\text{EM})} \sim Q_{\text{em}} (\omega_g L_{\text{cav}})^2 h^{\text{TT}} E_0$



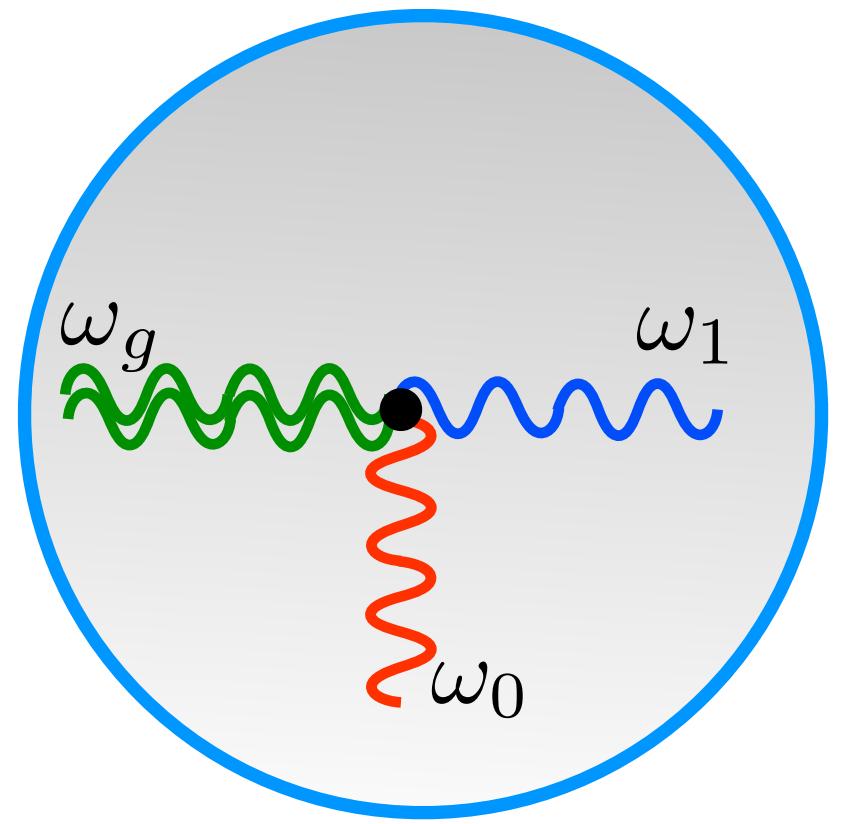
Mechanical signal:

$$E_{\text{sig}}^{(\text{mech})} \sim Q_{\text{em}} h^{\text{TT}} E_0 \min \left(1, \frac{\omega_g L_{\text{cav}}}{c_s} \right)^2$$



EM and Mechanical signals

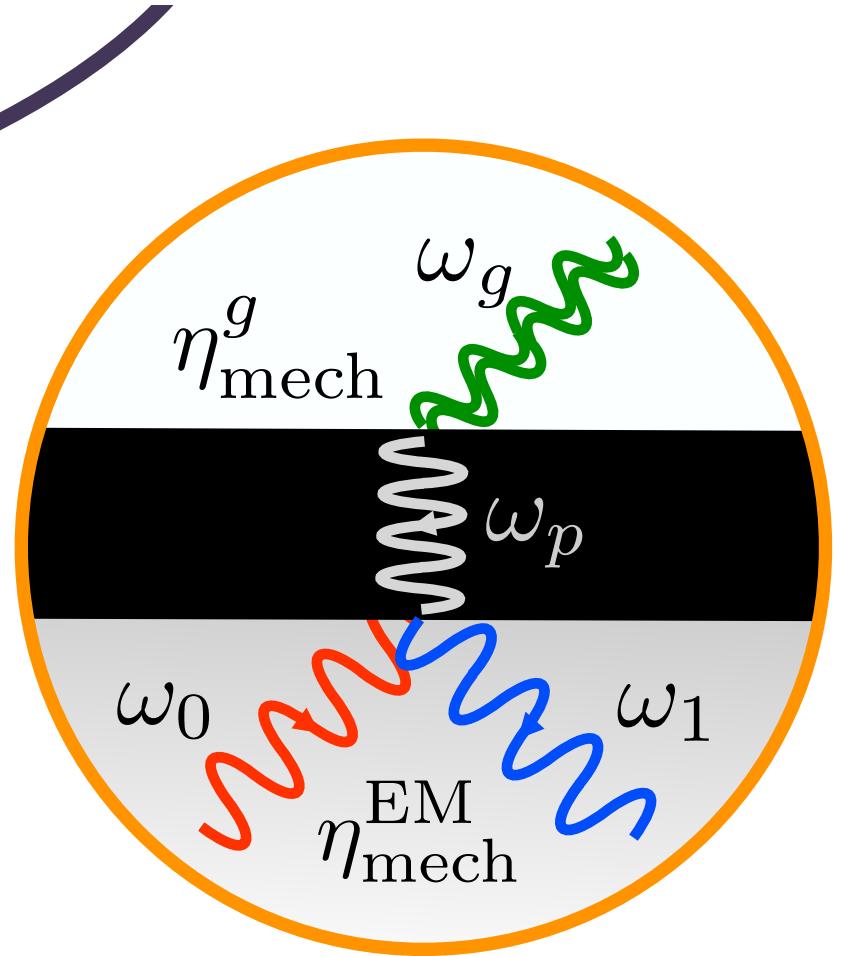
Parametrics of the EM signal: $E_{\text{sig}}^{(\text{EM})} \sim Q_{\text{em}} (\omega_g L_{\text{cav}})^2 h^{\text{TT}} E_0$



Enhanced by up to $1/c_s^2 \gg 1$ (!)

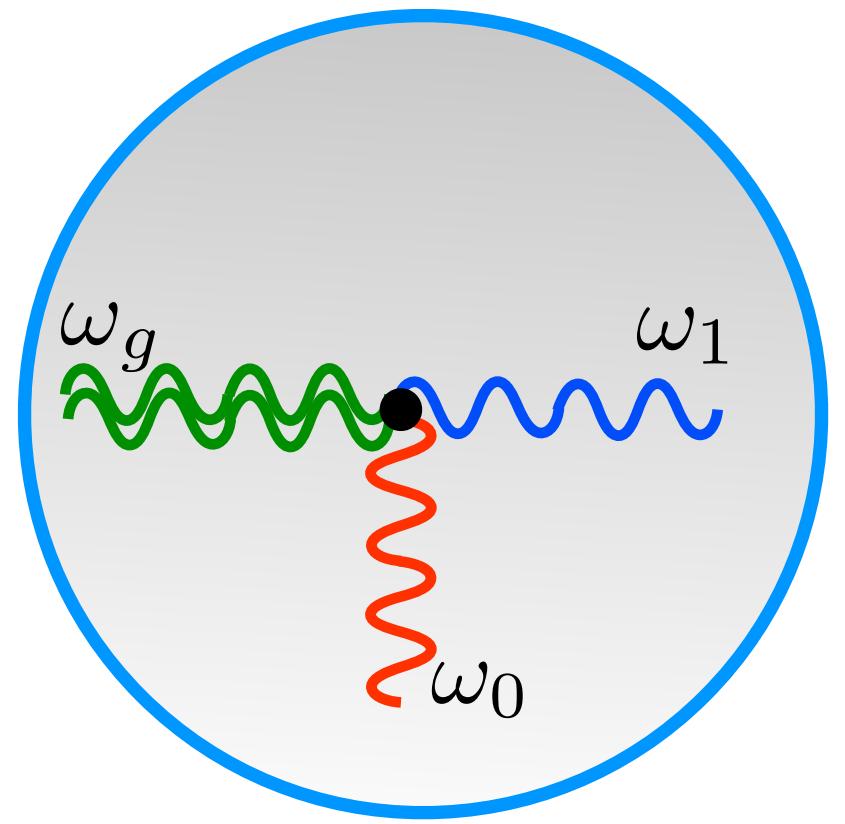
Mechanical signal:

$$E_{\text{sig}}^{(\text{mech})} \sim Q_{\text{em}} h^{\text{TT}} E_0 \min \left(1, \frac{\omega_g L_{\text{cav}}}{c_s} \right)^2$$



EM and Mechanical signals

Parametrics of the EM signal: $E_{\text{sig}}^{(\text{EM})} \sim Q_{\text{em}} (\omega_g L_{\text{cav}})^2 h^{\text{TT}} E_0$

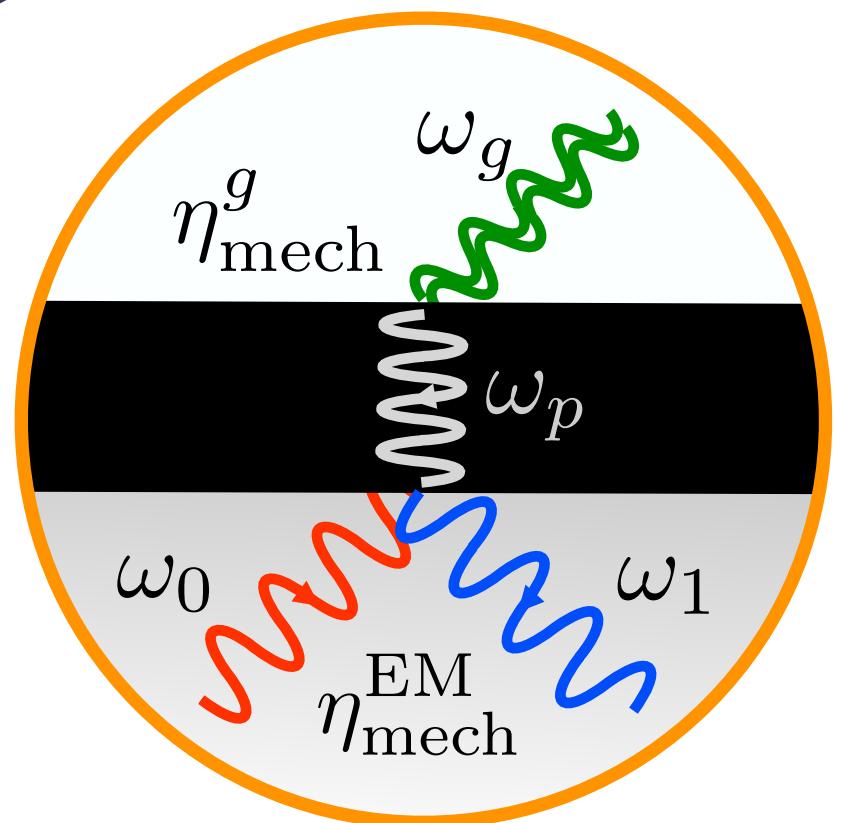


Enhanced by up to $1/c_s^2 \gg 1$ (!)

Mechanical signal:

$$E_{\text{sig}}^{(\text{mech})} \sim Q_{\text{em}} h^{\text{TT}} E_0 \min \left(1, \frac{\omega_g L_{\text{cav}}}{c_s} \right)^2$$

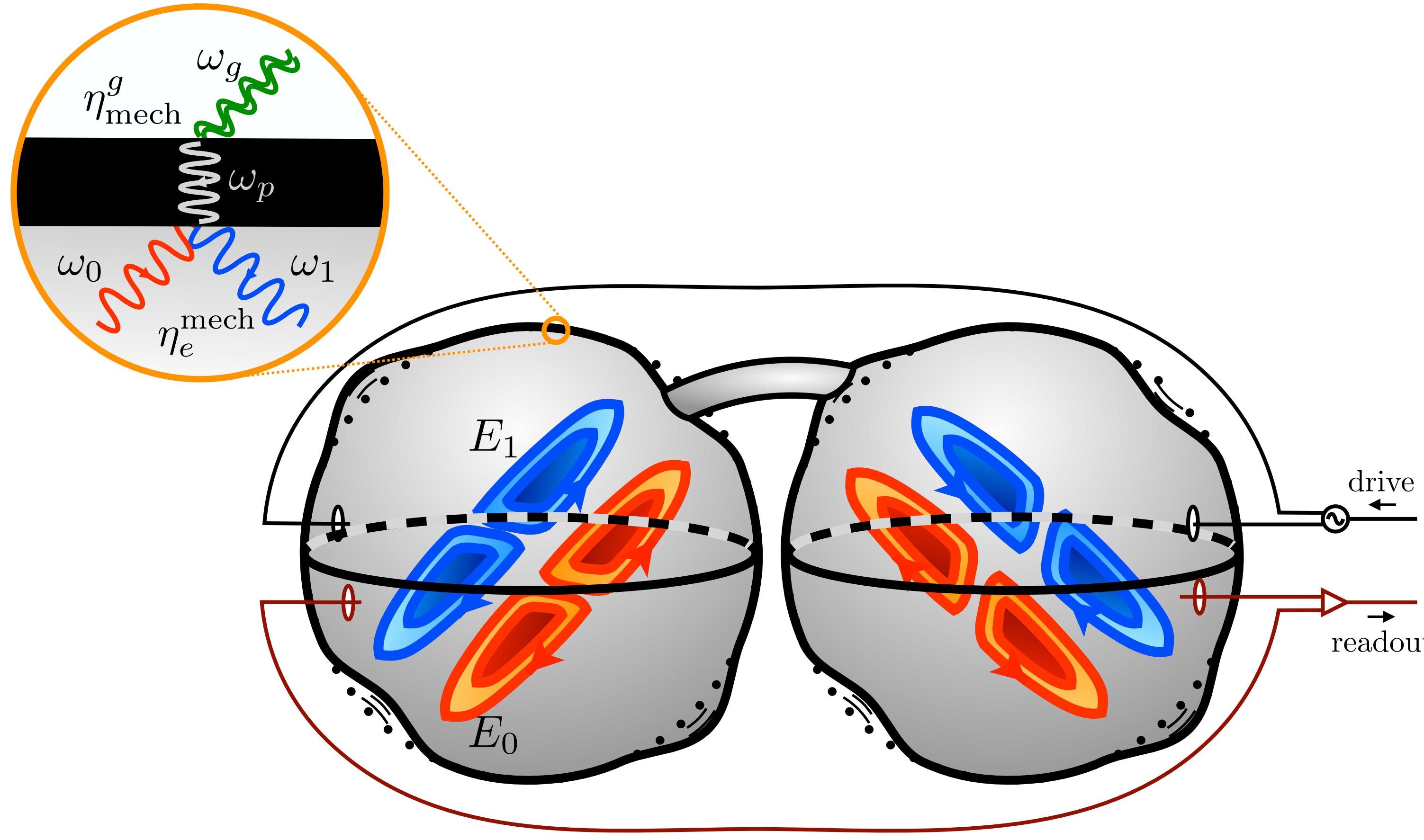
Mechanical modes less “rigid” than EM modes



Gravitational Wave Signal: Mechanical

Mechanical modes of a hollow sphere:

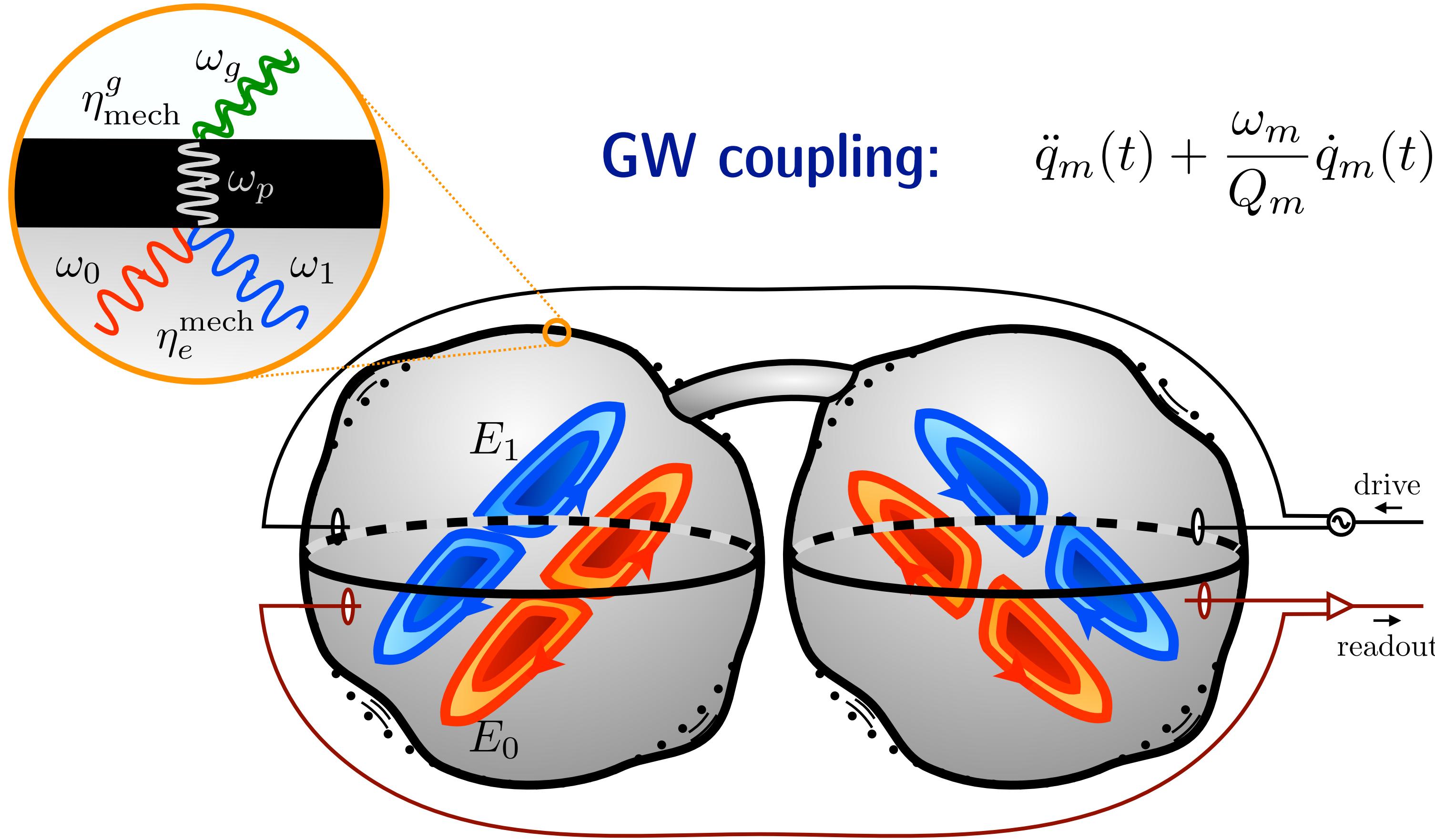
$$\mathbf{q}_{mnl} = q_{0,mn} [f_{mn}(r)Y_{ml}(\theta, \varphi)\hat{r} - ig_{mn}(r)\hat{r} \times \mathbf{L}Y_{ml}(\theta, \varphi)]$$



Gravitational Wave Signal: Mechanical

Mechanical modes of a hollow sphere:

$$\mathbf{q}_{mnl} = q_{0,mn} [f_{mn}(r)Y_{ml}(\theta, \varphi)\hat{r} - ig_{mn}(r)\hat{r} \times \mathbf{L}Y_{ml}(\theta, \varphi)]$$



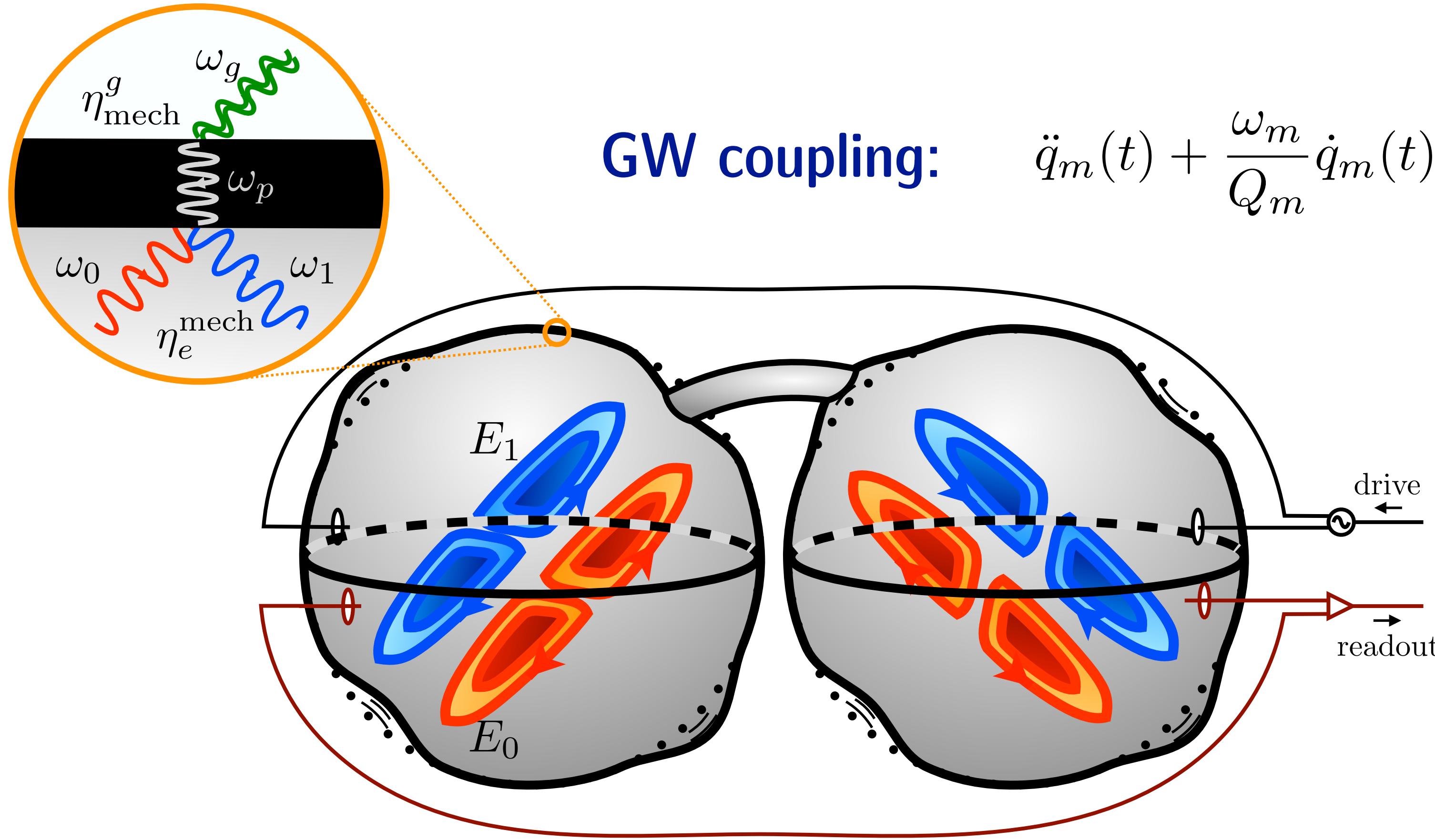
GW coupling:

$$\ddot{q}_m(t) + \frac{\omega_m}{Q_m} \dot{q}_m(t) + \omega_m^2 q_m(t) = \frac{1}{2} \ddot{h}_{ij}^{\text{TT}} \int_V d^3 \mathbf{x} x_j (q_{mnl})_i$$

Gravitational Wave Signal: Mechanical

Mechanical modes of a hollow sphere:

$$\mathbf{q}_{mnl} = q_{0,mn} [f_{mn}(r)Y_{ml}(\theta, \varphi)\hat{r} - ig_{mn}(r)\hat{r} \times \mathbf{L}Y_{ml}(\theta, \varphi)]$$



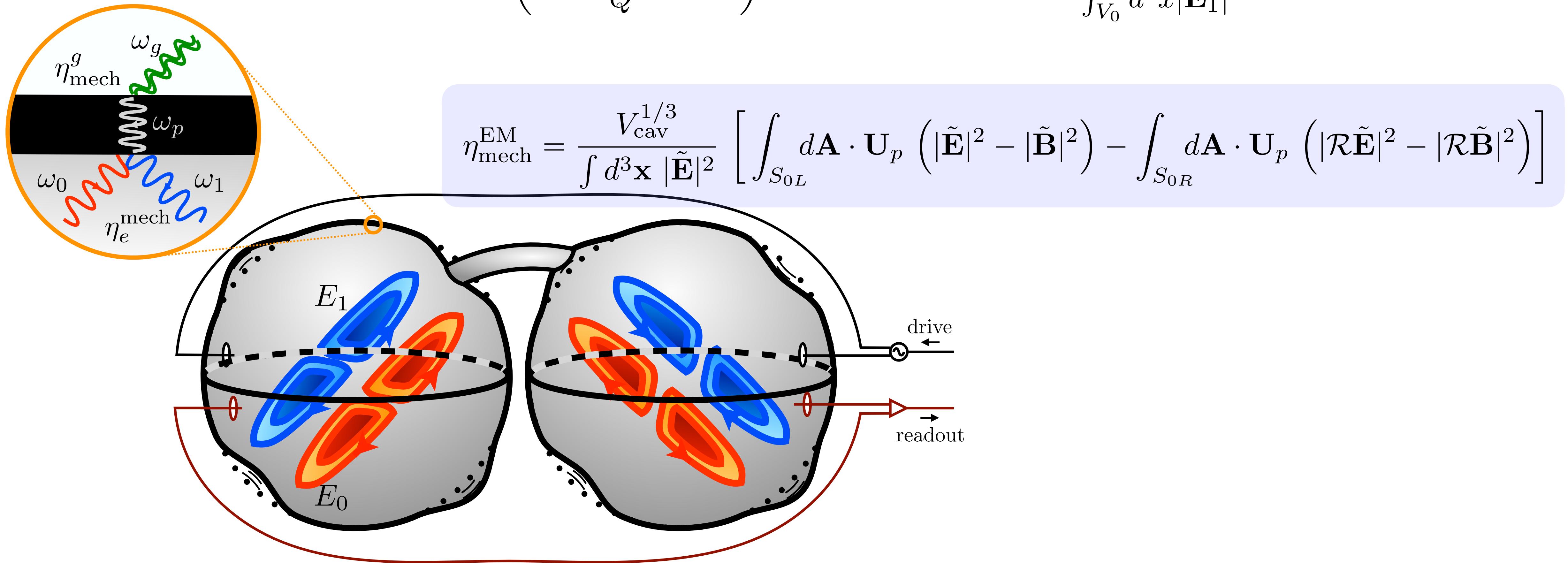
GW coupling:

$$\ddot{q}_m(t) + \frac{\omega_m}{Q_m} \dot{q}_m(t) + \omega_m^2 q_m(t) = \frac{1}{2} \ddot{h}_{ij}^{\text{TT}} \int_V d^3x x_j (q_{mnl})_i$$

$$\underbrace{\eta_u^{\text{mech}}}_{\text{GW coupling}}$$

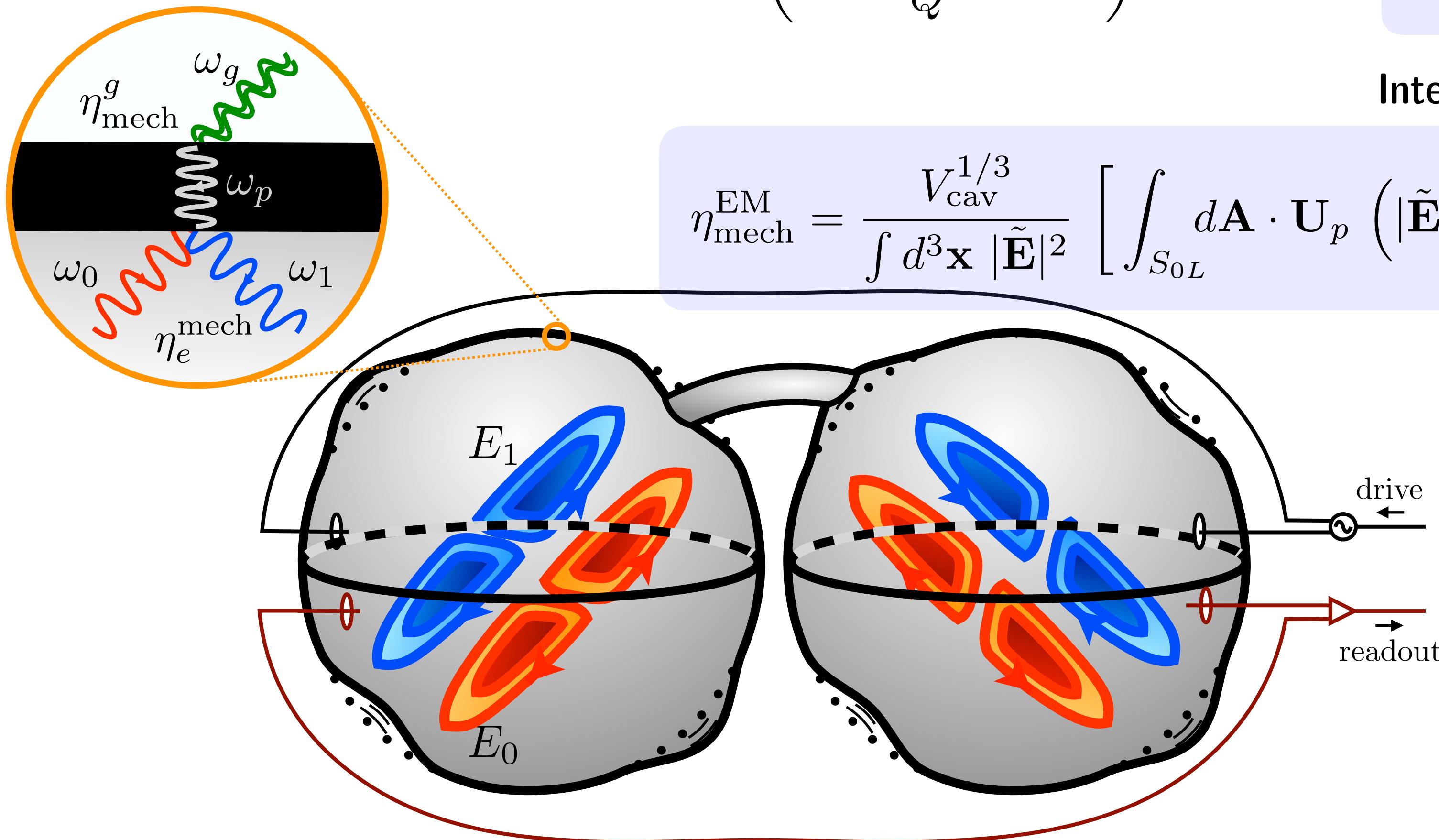
Gravitational Wave Signal: Mechanical

Mechanical coupling to EM: $\left(\partial_t^2 + \frac{\omega_1}{Q} \partial_t + \omega_1^2 \right) e_1 = -\omega_1^2 e_0 \frac{\int_{\Delta V} d^3x (\mathbf{E}_0 \cdot \mathbf{E}_1^* - (\omega_0/\omega_1)\mathbf{B}_0 \cdot \mathbf{B}_1^*)}{\int_{V_0} d^3x |\mathbf{E}_1|^2} + \mathcal{O}(h^2)$



Gravitational Wave Signal: Mechanical

Mechanical coupling to EM: $\left(\partial_t^2 + \frac{\omega_1}{Q} \partial_t + \omega_1^2 \right) e_1 = -\omega_1^2 e_0 \frac{\int_{\Delta V} d^3x (\mathbf{E}_0 \cdot \mathbf{E}_1^* - (\omega_0/\omega_1)\mathbf{B}_0 \cdot \mathbf{B}_1^*)}{\int_{V_0} d^3x |\mathbf{E}_1|^2} + \mathcal{O}(h^2)$

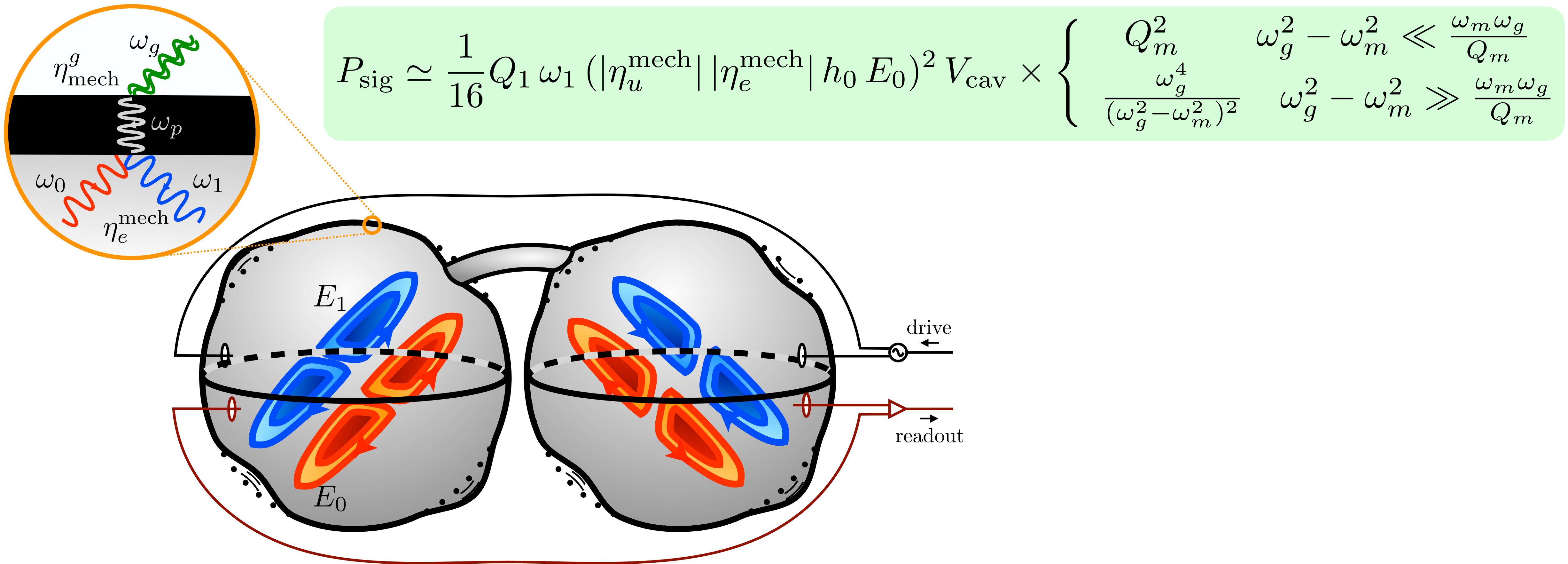


Integral over deformation to cavity volume

$$\eta_{\text{mech}}^{\text{EM}} = \frac{V_{\text{cav}}^{1/3}}{\int d^3x |\tilde{\mathbf{E}}|^2} \left[\int_{S_{0L}} d\mathbf{A} \cdot \mathbf{U}_p \left(|\tilde{\mathbf{E}}|^2 - |\tilde{\mathbf{B}}|^2 \right) - \int_{S_{0R}} d\mathbf{A} \cdot \mathbf{U}_p \left(|\mathcal{R}\tilde{\mathbf{E}}|^2 - |\mathcal{R}\tilde{\mathbf{B}}|^2 \right) \right]$$

Gravitational Wave Signal: Mechanical

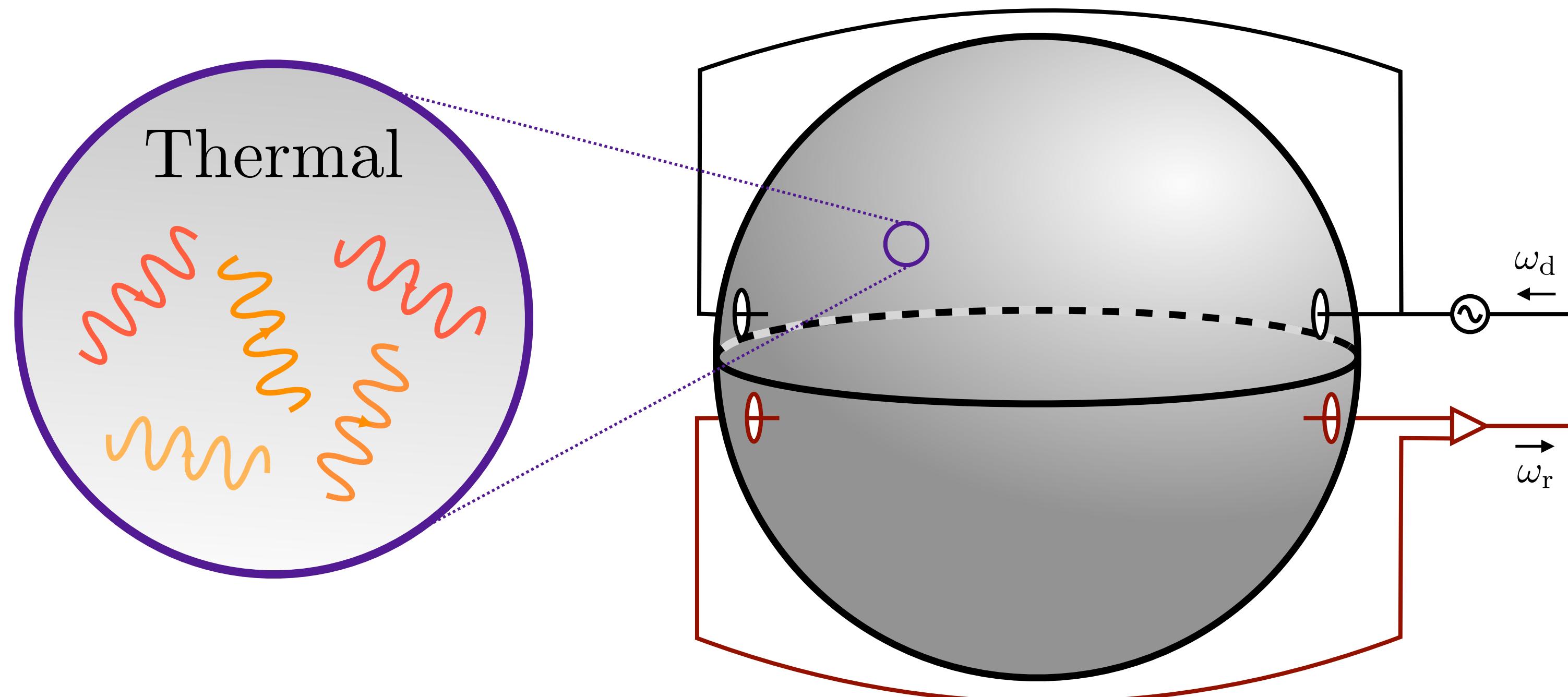
Power in a heterodyne setup from monochromatic source



Thermal Noise

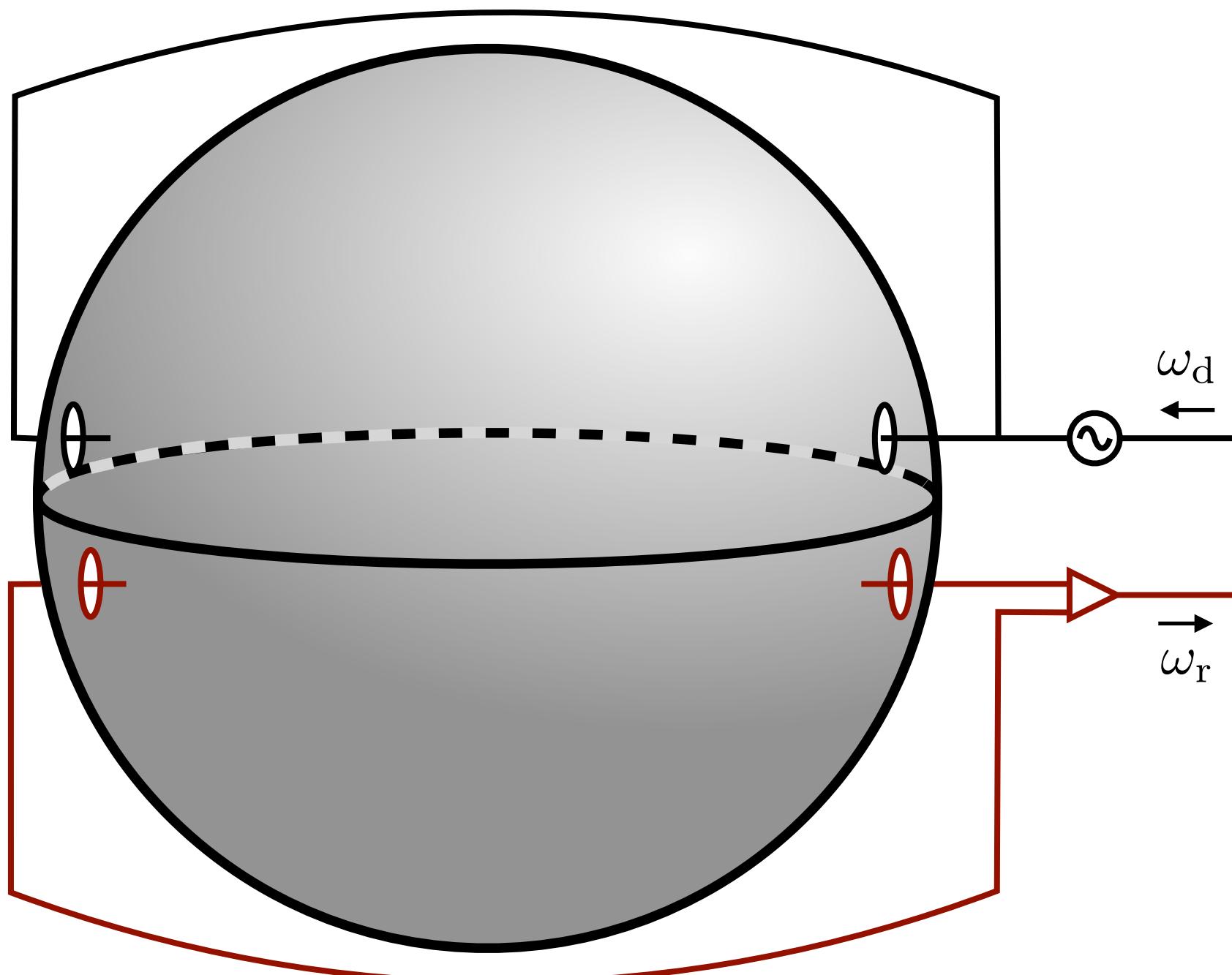
Power spectral density:

$$S_{\text{th}}(\omega) = \frac{Q_1}{Q_{\text{int}}} \frac{4\pi T (\omega \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1/Q_1)^2}$$



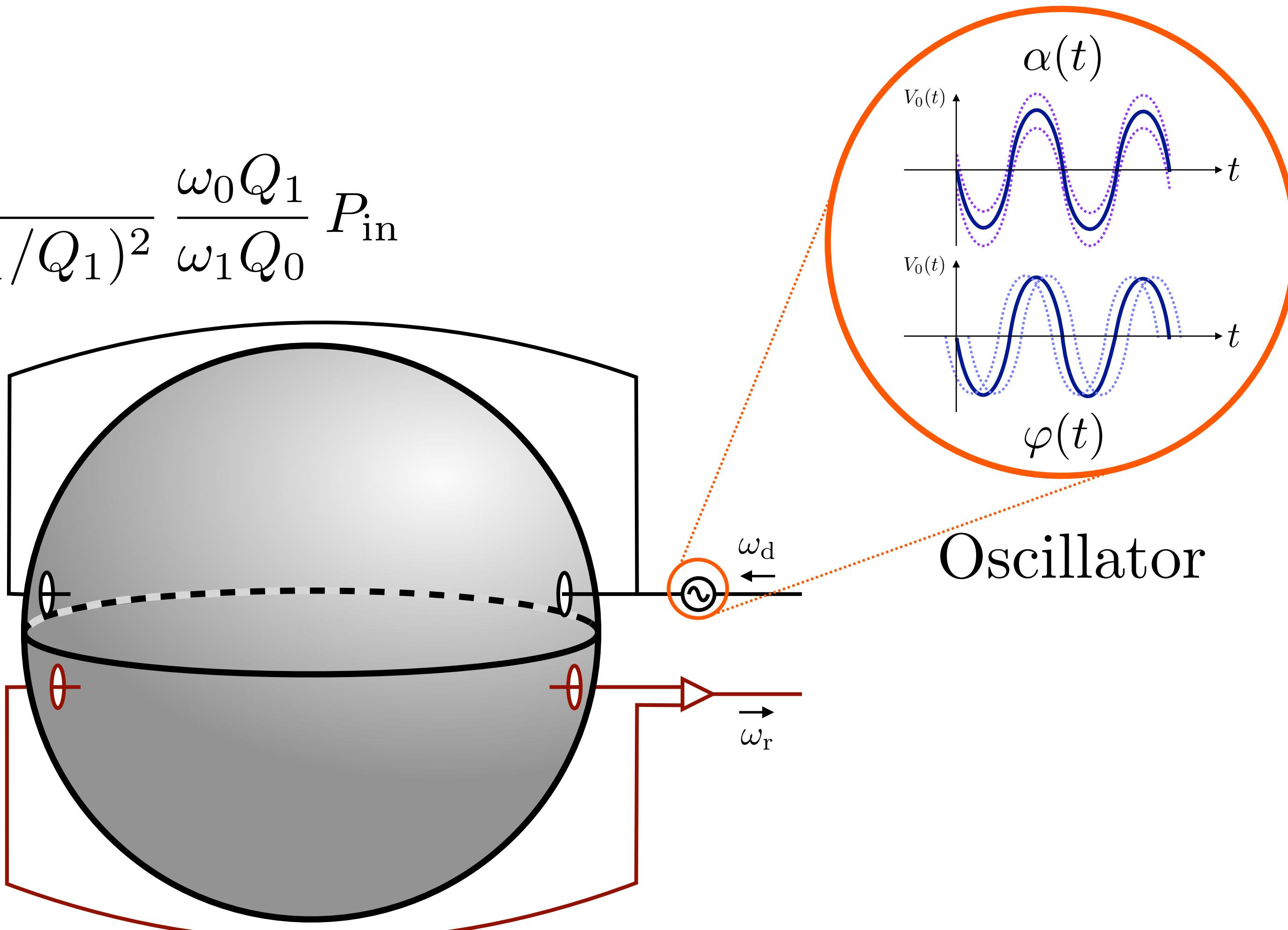
Phase Noise

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_\varphi(\omega - \omega_0) \times \frac{(\omega \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1/Q_1)^2} \frac{\omega_0 Q_1}{\omega_1 Q_0} P_{\text{in}}$$



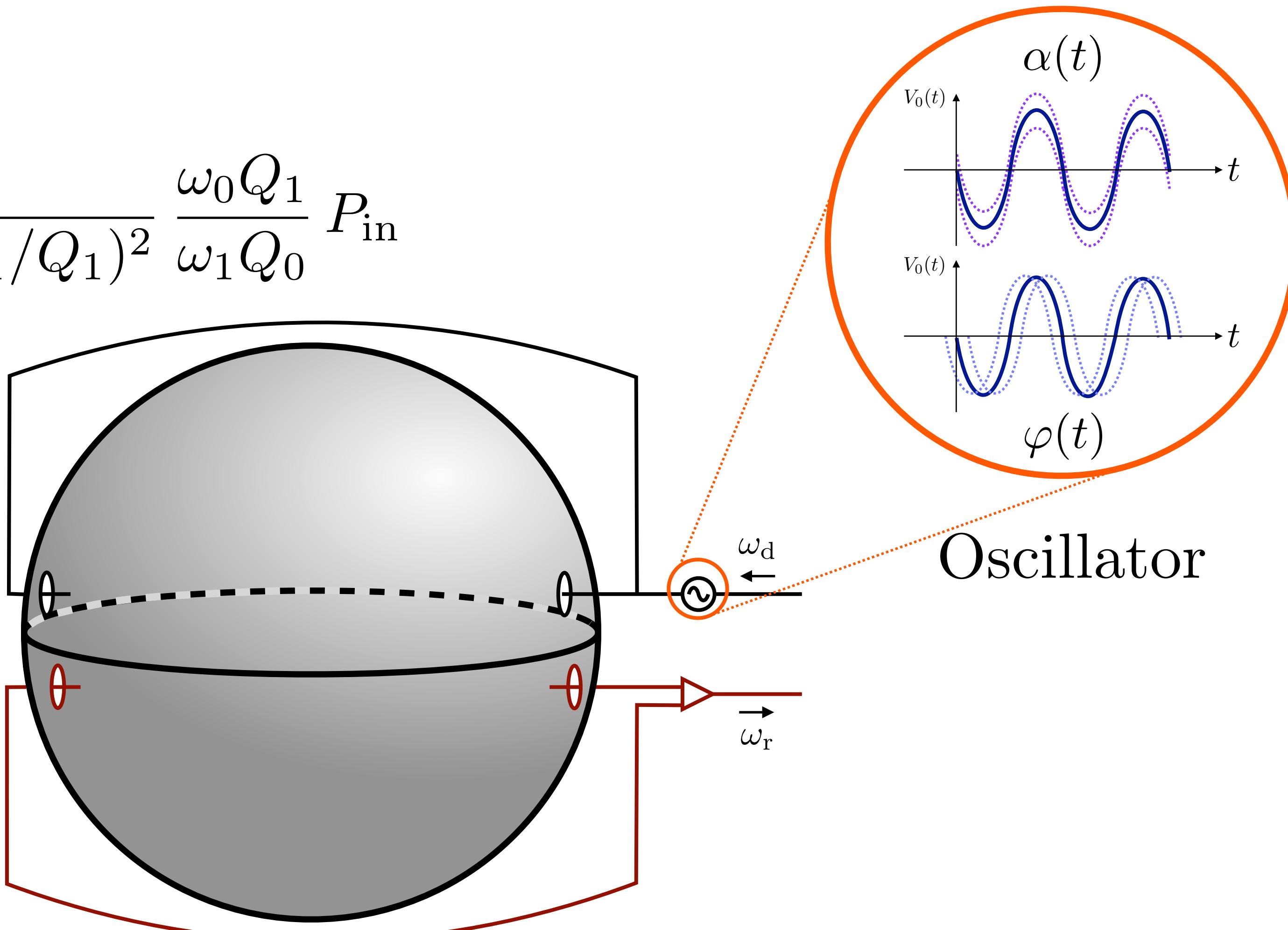
Phase Noise

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_\varphi(\omega - \omega_0) \times \frac{(\omega \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1/Q_1)^2} \frac{\omega_0 Q_1}{\omega_1 Q_0} P_{\text{in}}$$



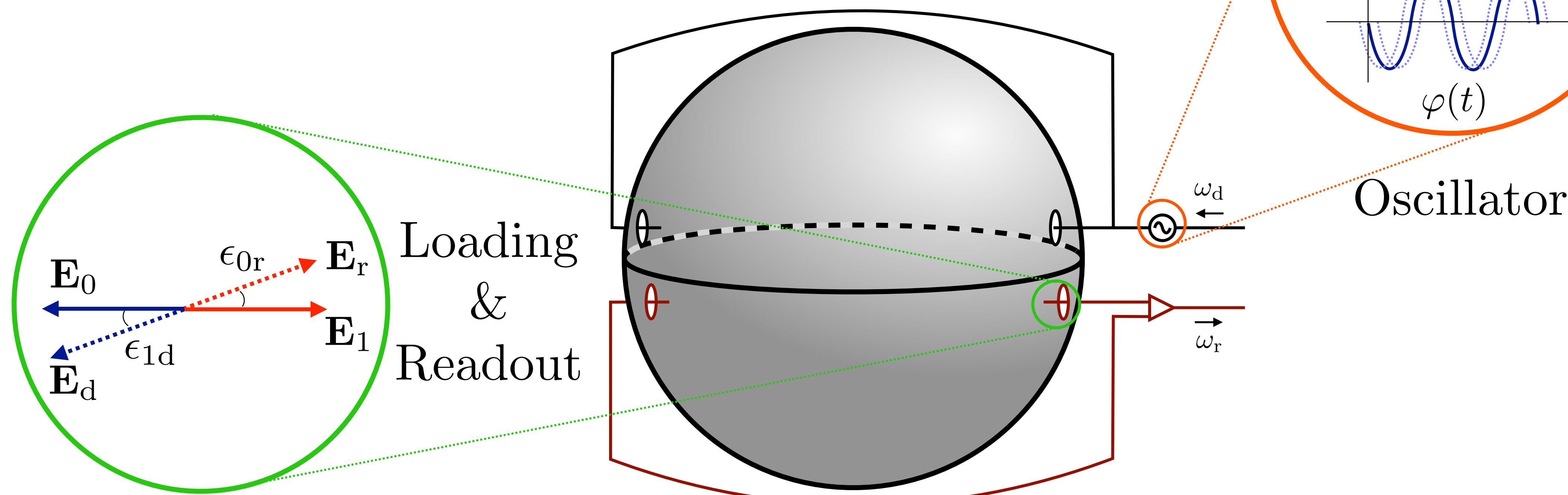
Phase Noise

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_\varphi(\omega - \omega_0) \times \frac{(\omega \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1/Q_1)^2} \frac{\omega_0 Q_1}{\omega_1 Q_0} P_{\text{in}}$$



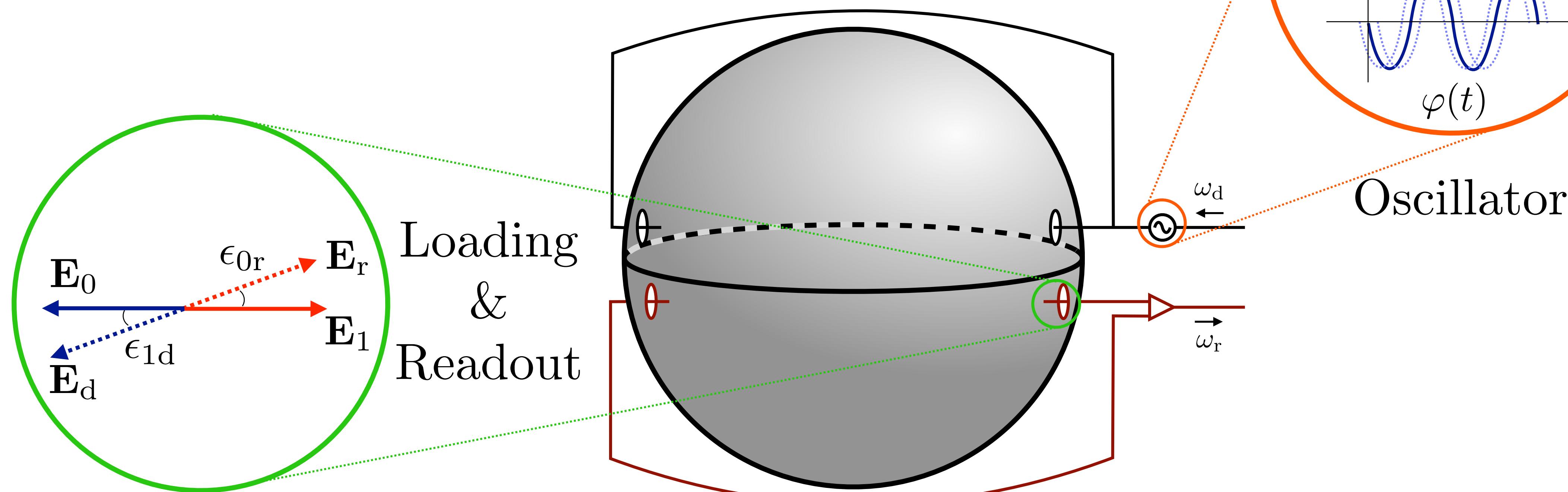
Phase Noise

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_\varphi(\omega - \omega_0) \times \frac{(\omega \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1/Q_1)^2} \frac{\omega_0 Q_1}{\omega_1 Q_0} P_{\text{in}}$$



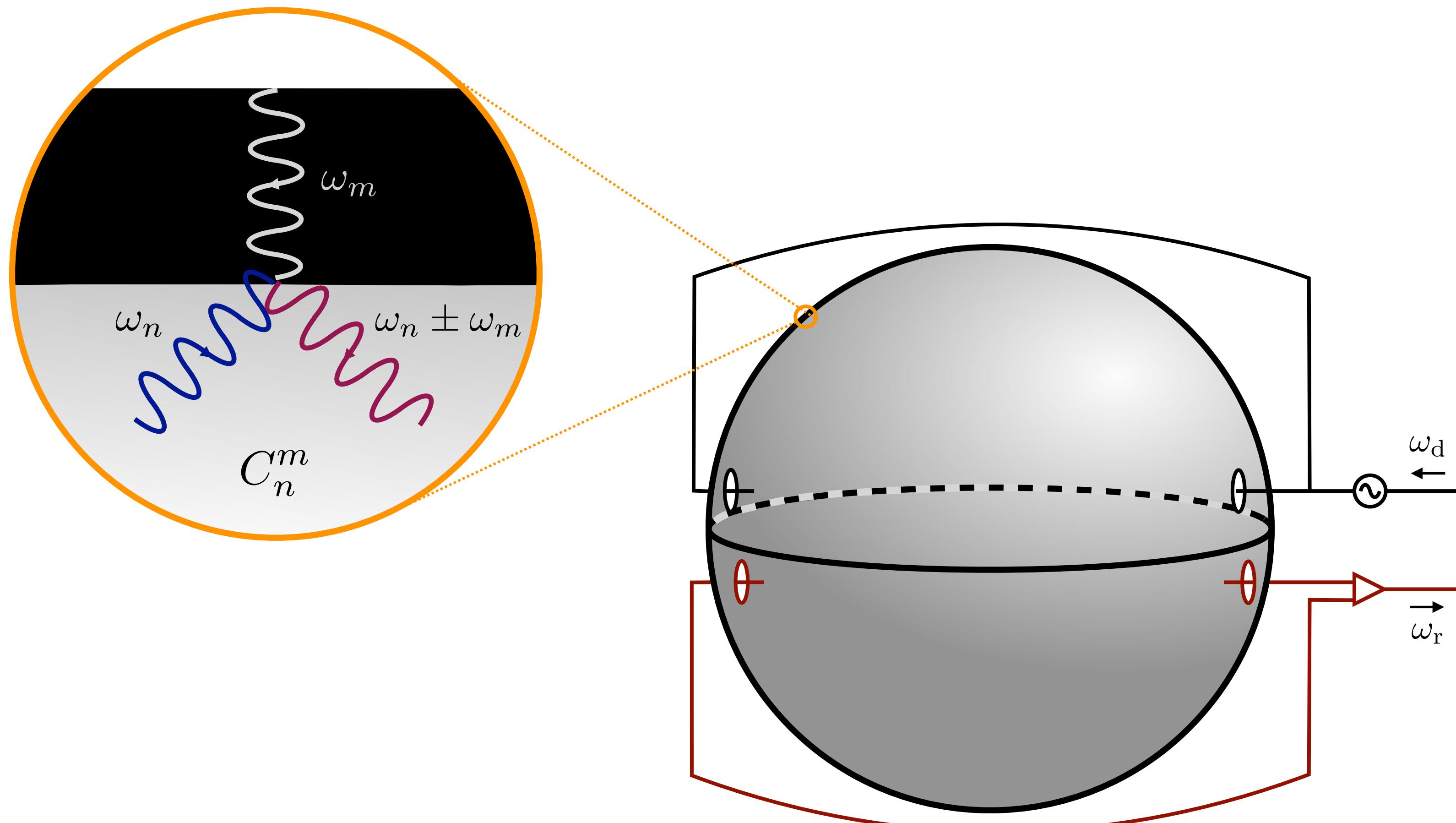
Phase Noise

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 S_\varphi(\omega - \omega_0) \times \frac{(\omega \omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1/Q_1)^2} \frac{\omega_0 Q_1}{\omega_1 Q_0} P_{\text{in}}$$



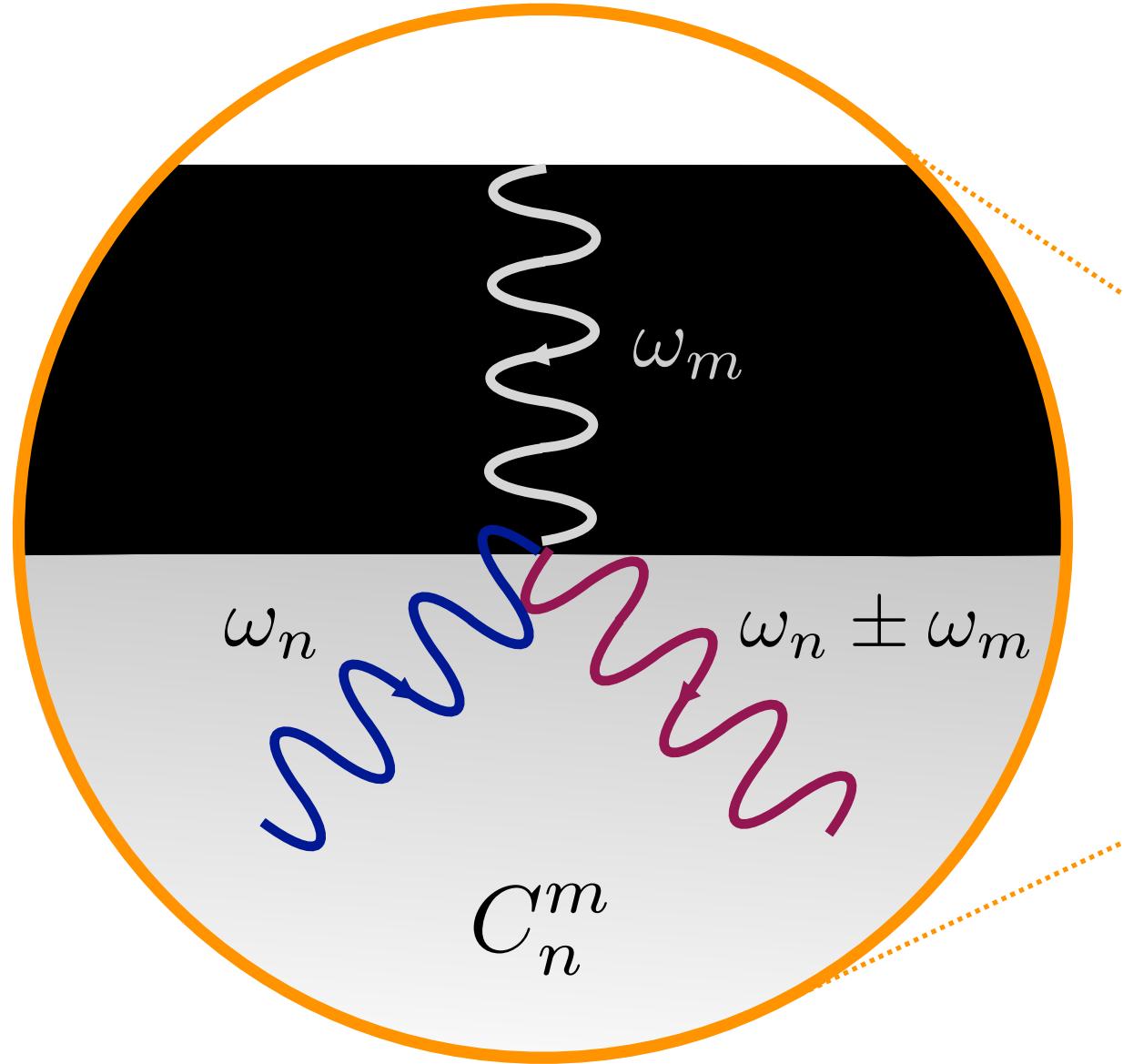
Mechanical Noise

Vibrations



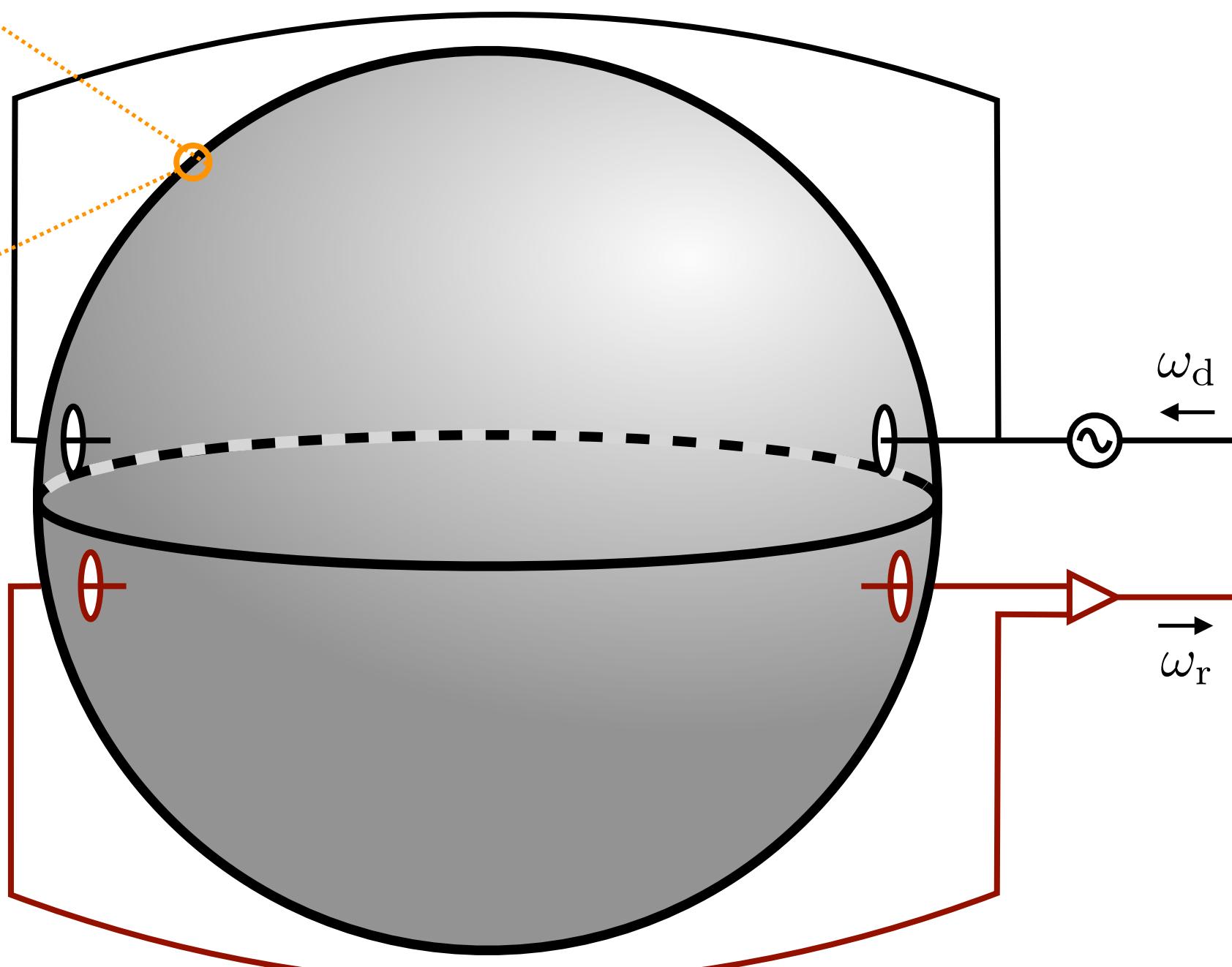
Mechanical Noise

Vibrations



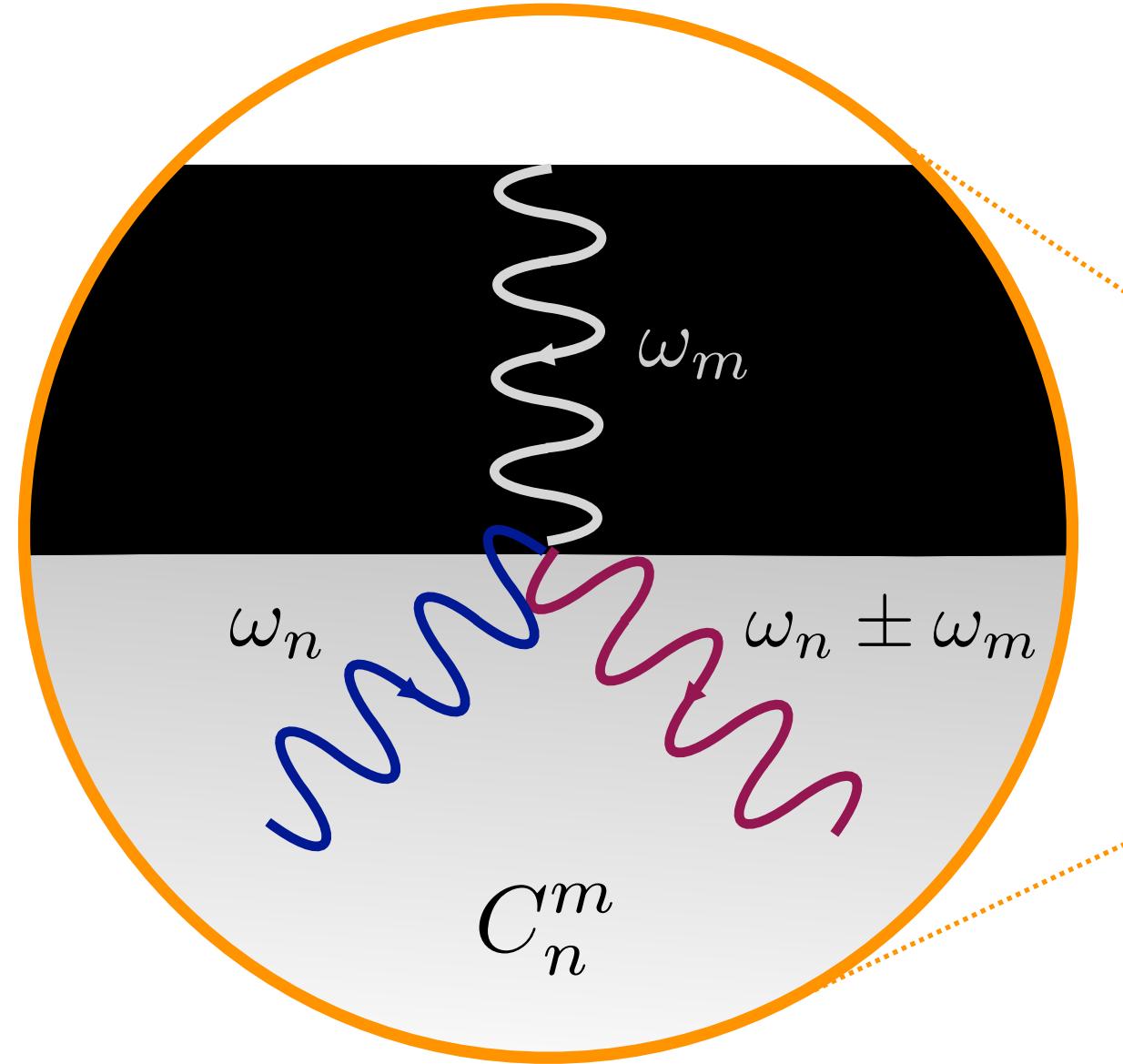
$$S_{\text{mech}}(\omega) \simeq \frac{\epsilon_{1d}^2}{4} \frac{\omega_0}{Q_0} P_{\text{in}} \times$$

$$\sum_{n=0,1} \frac{(S_{q_m}(\omega - \omega_0)/V^{2/3}) (\omega_n/Q_n) \omega_n^4 \omega^2}{[(\omega^2 - \omega_n^2)^2 + (\omega \omega_n/Q_n)^2] [(\omega_0^2 - \omega_n^2)^2 + (\omega_0 \omega_n/Q_n)^2]}$$



Mechanical Noise

Vibrations

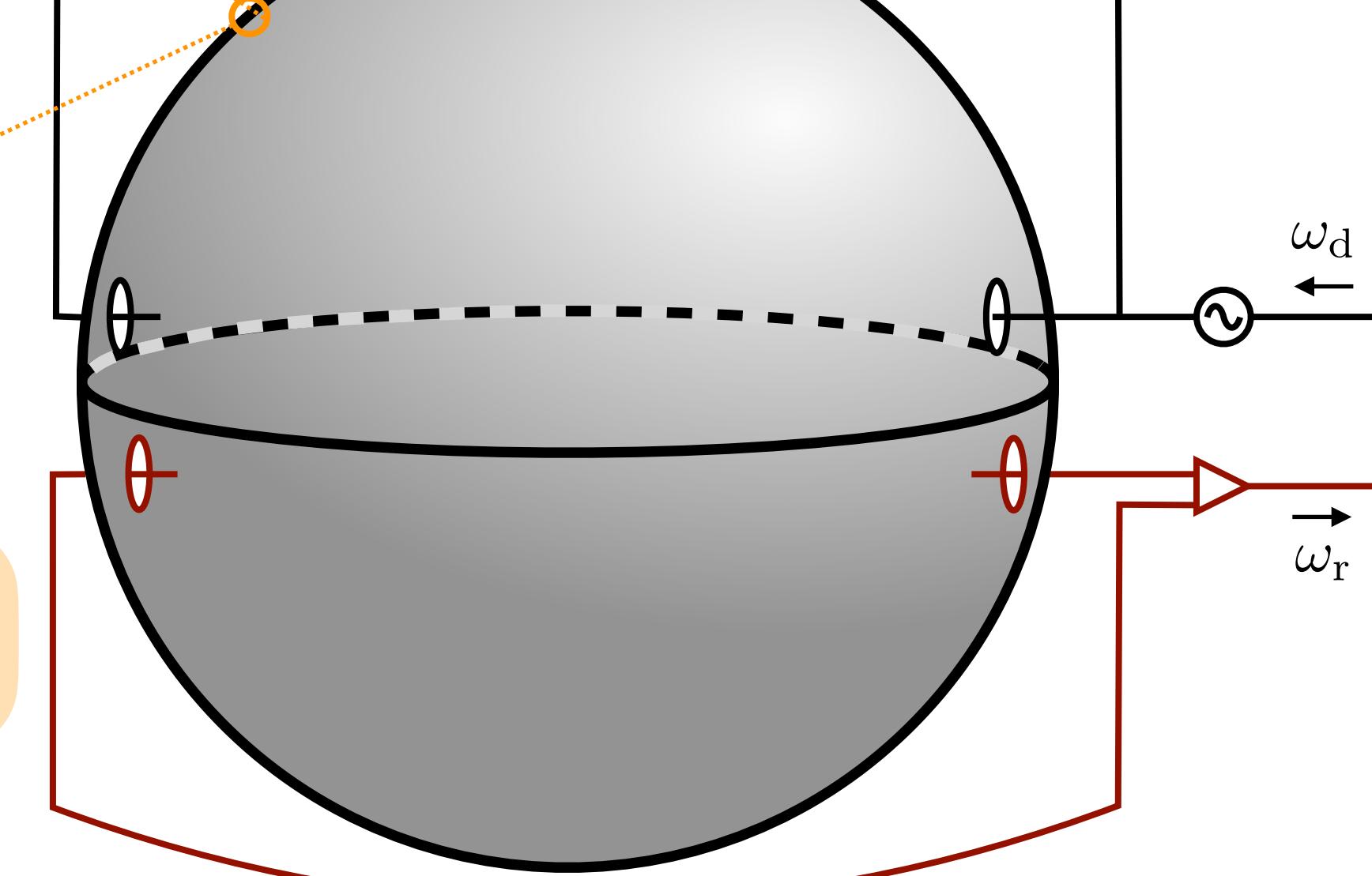


$$S_{\text{mech}}(\omega) \simeq \frac{\epsilon_{1d}^2}{4} \frac{\omega_0}{Q_0} P_{\text{in}} \times$$

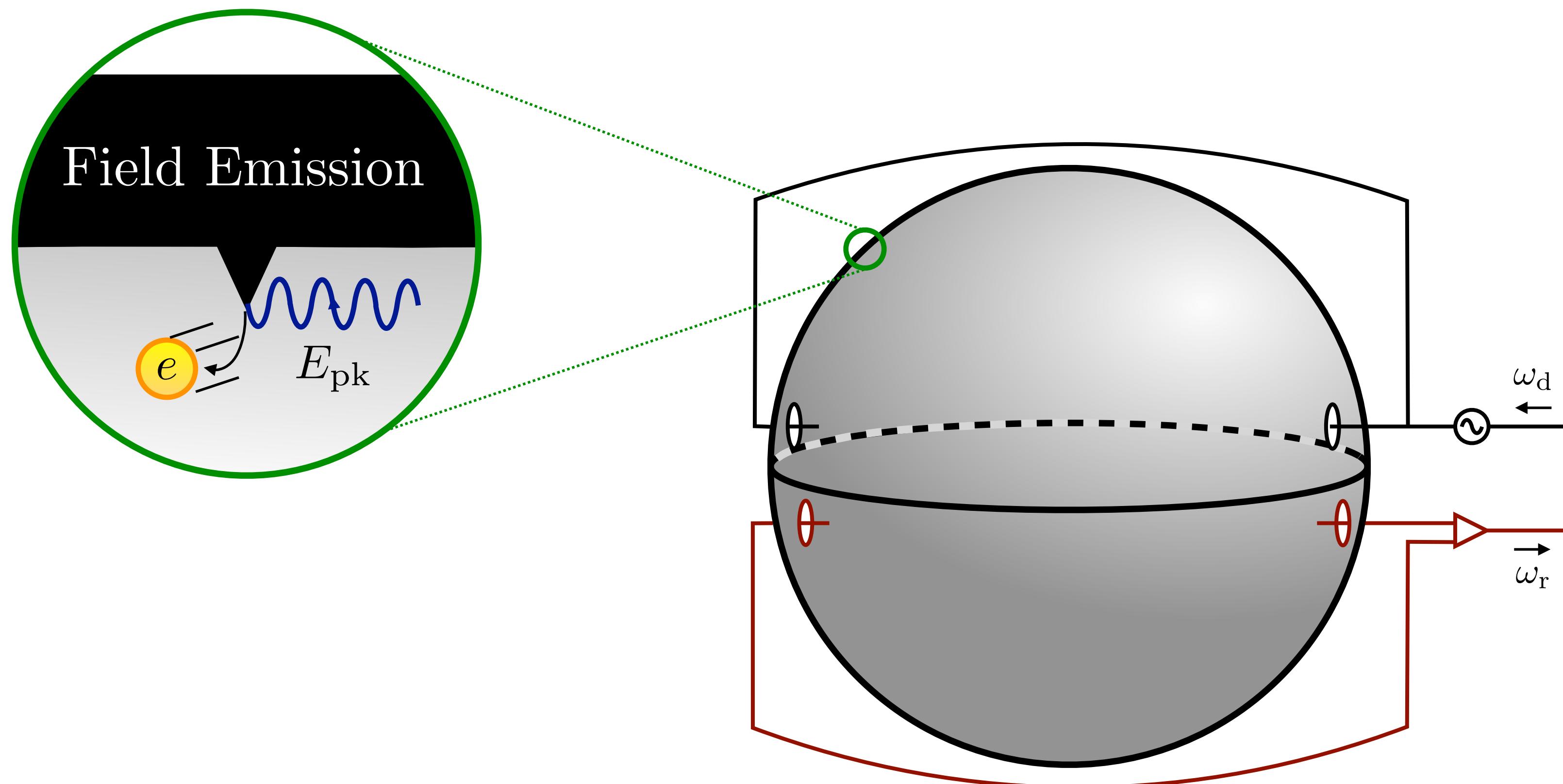
$$\sum_{n=0,1} \frac{(S_{q_m}(\omega - \omega_0)/V^{2/3}) (\omega_n/Q_n) \omega_n^4 \omega^2}{[(\omega^2 - \omega_n^2)^2 + (\omega \omega_n/Q_n)^2] [(\omega_0^2 - \omega_n^2)^2 + (\omega_0 \omega_n/Q_n)^2]}$$

Displacement PSD:

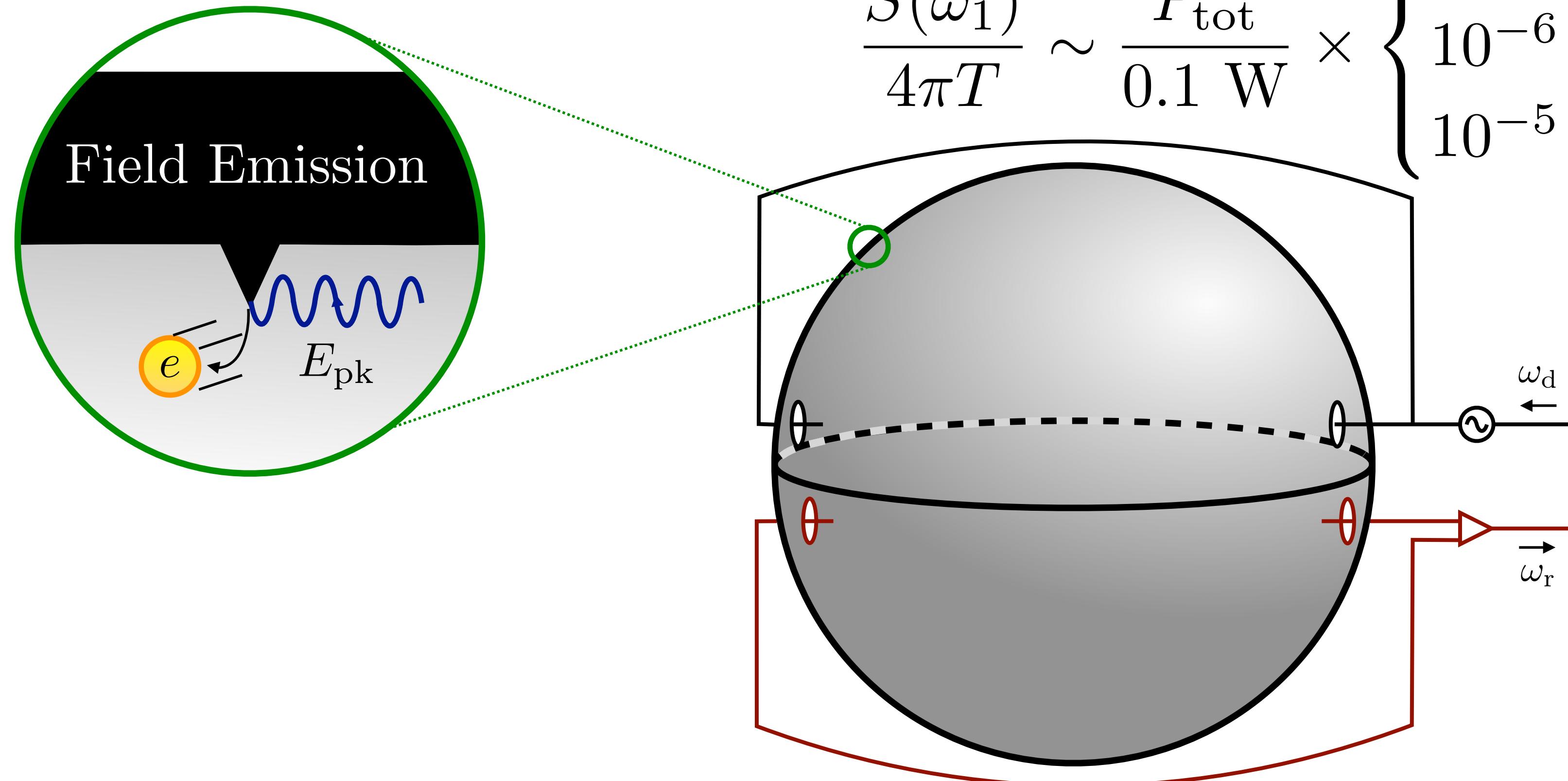
$$S_{q_m}(\omega) \simeq \frac{1}{M^2} \frac{S_{f_m}(\omega)}{(\omega^2 - \omega_m^2)^2 + (\omega_m \omega/Q_m)^2}$$



Non-standard Noise Sources: Field Emission



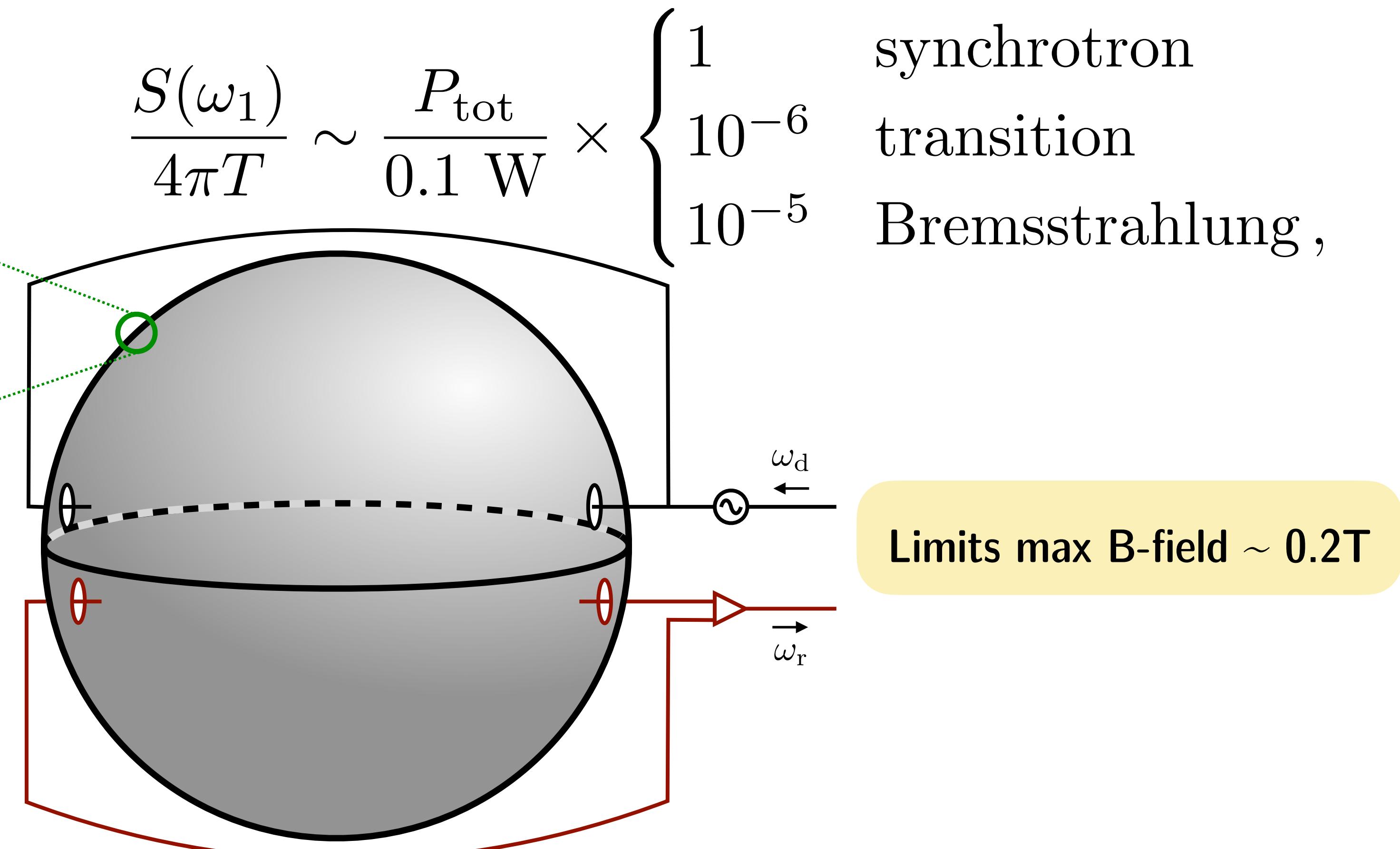
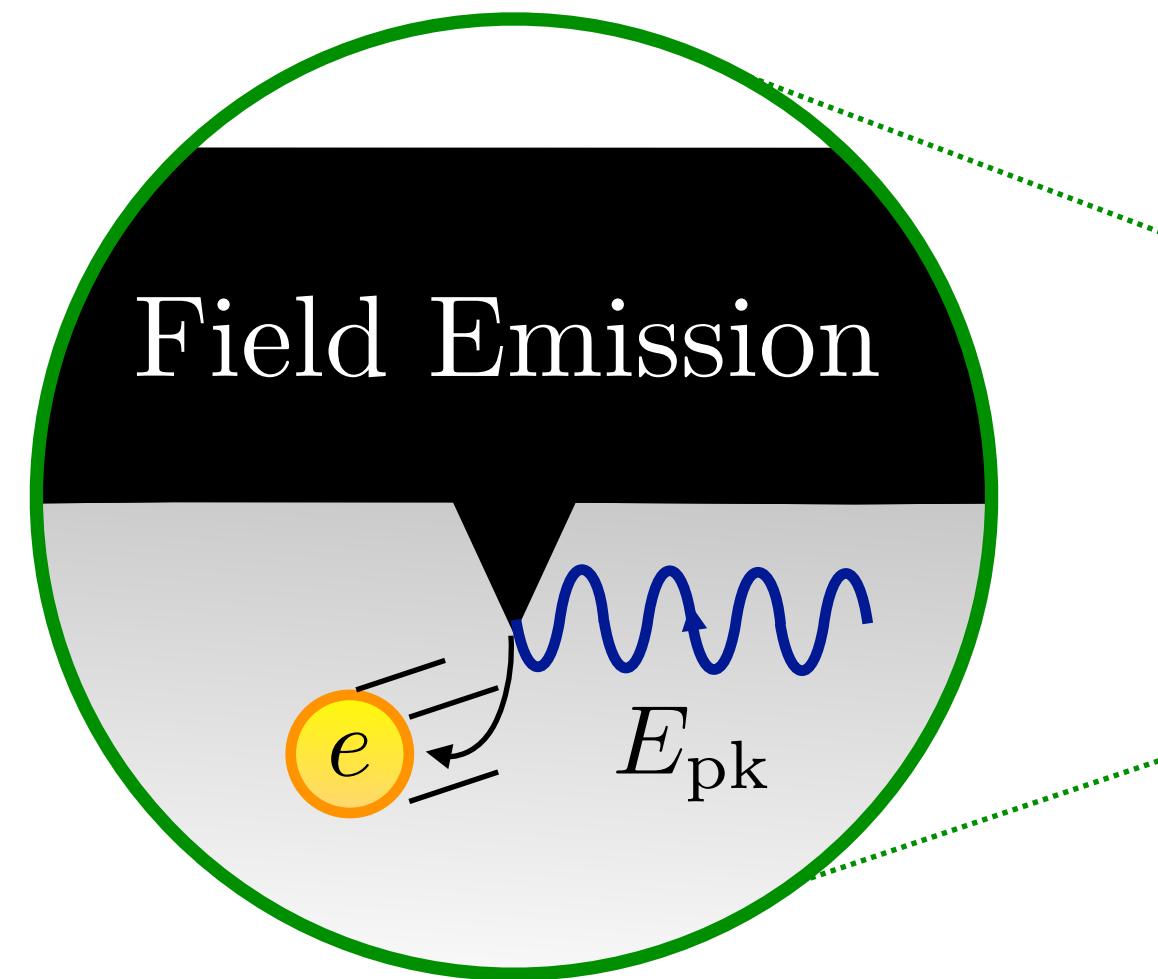
Non-standard Noise Sources: Field Emission



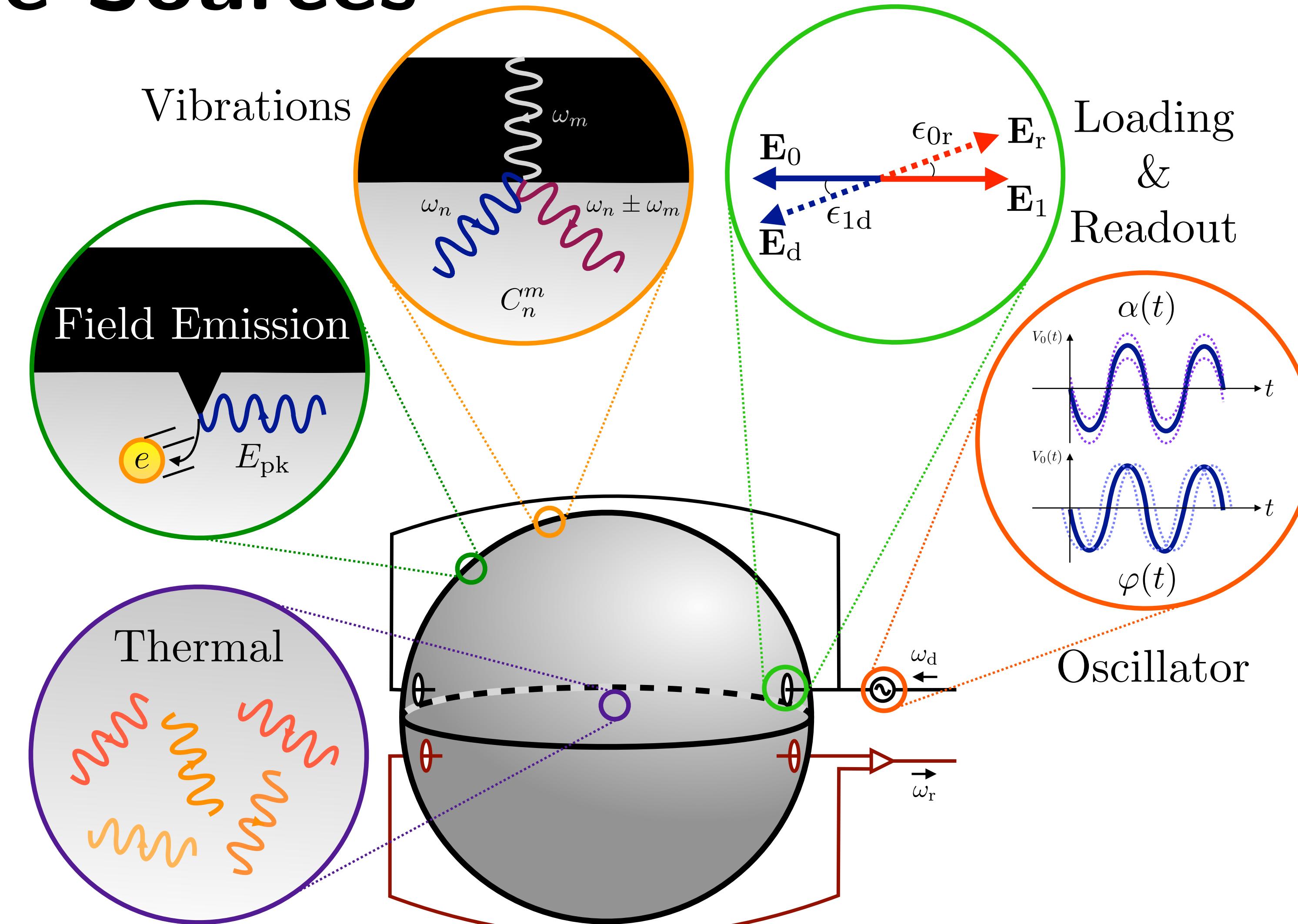
$$\frac{S(\omega_1)}{4\pi T} \sim \frac{P_{\text{tot}}}{0.1 \text{ W}} \times \begin{cases} 1 \\ 10^{-6} \\ 10^{-5} \end{cases}$$

synchrotron
transition
Bremsstrahlung ,

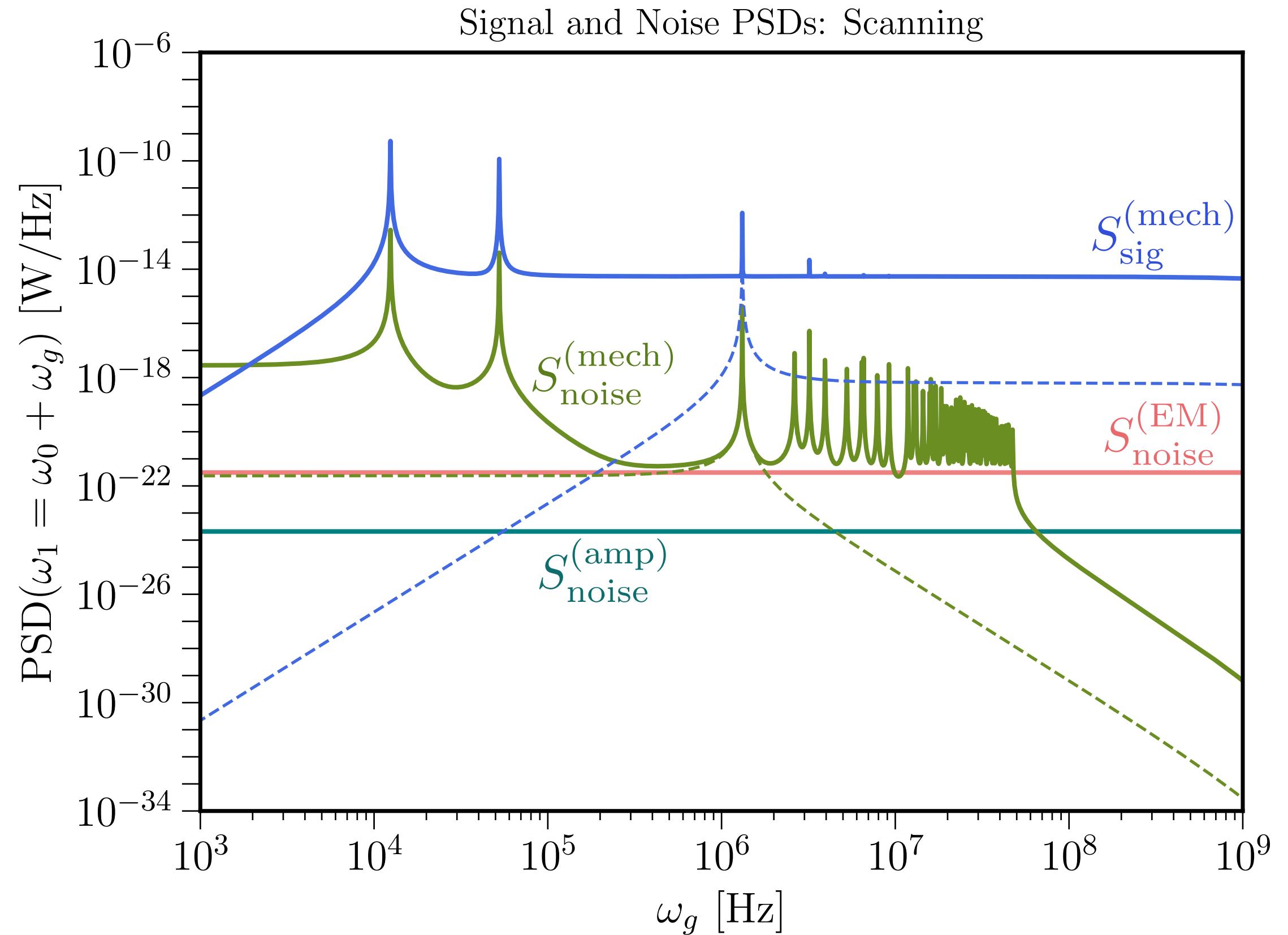
Non-standard Noise Sources: Field Emission



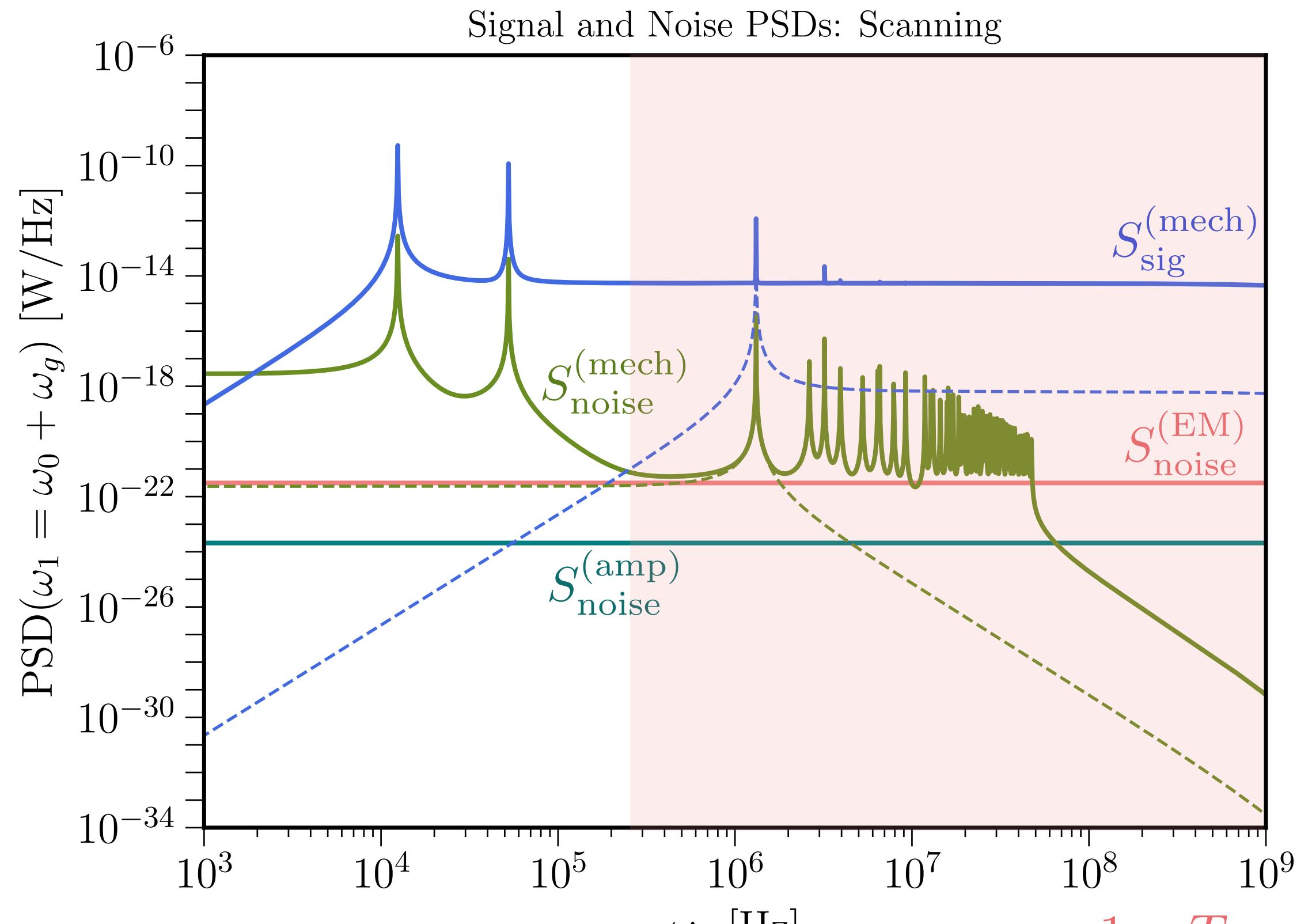
All Noise Sources



Noise in MAGO 2.0



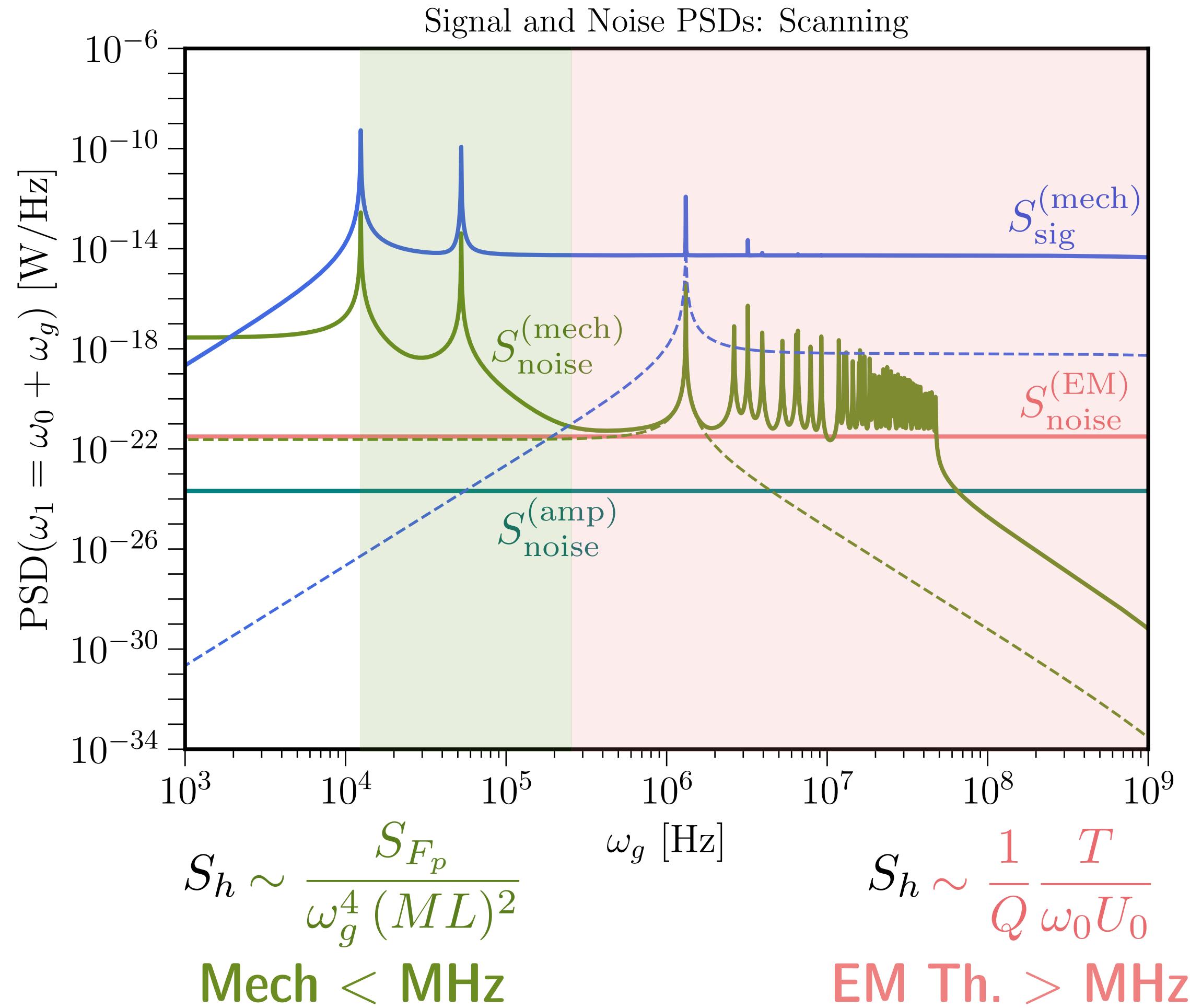
Noise in MAGO 2.0



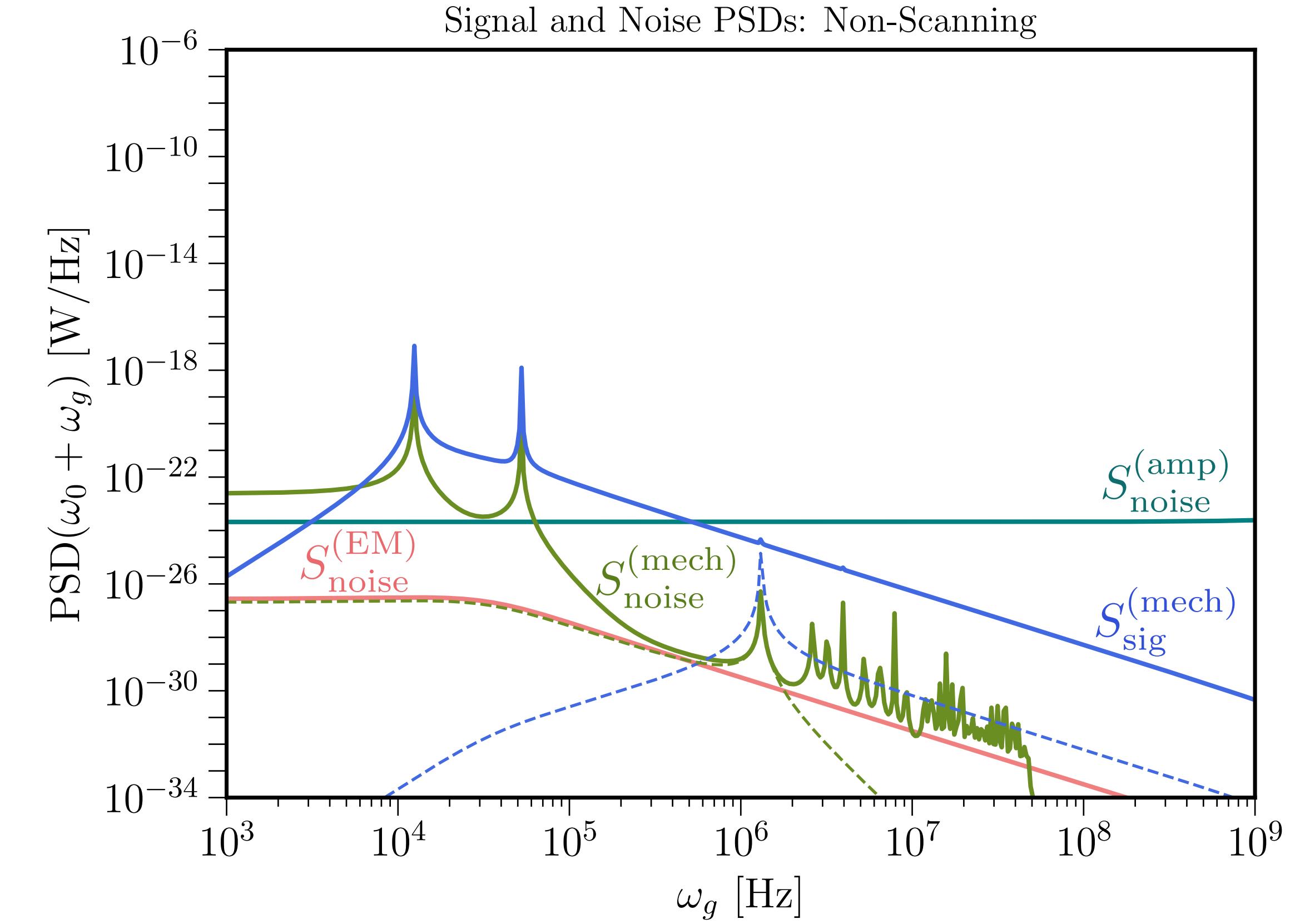
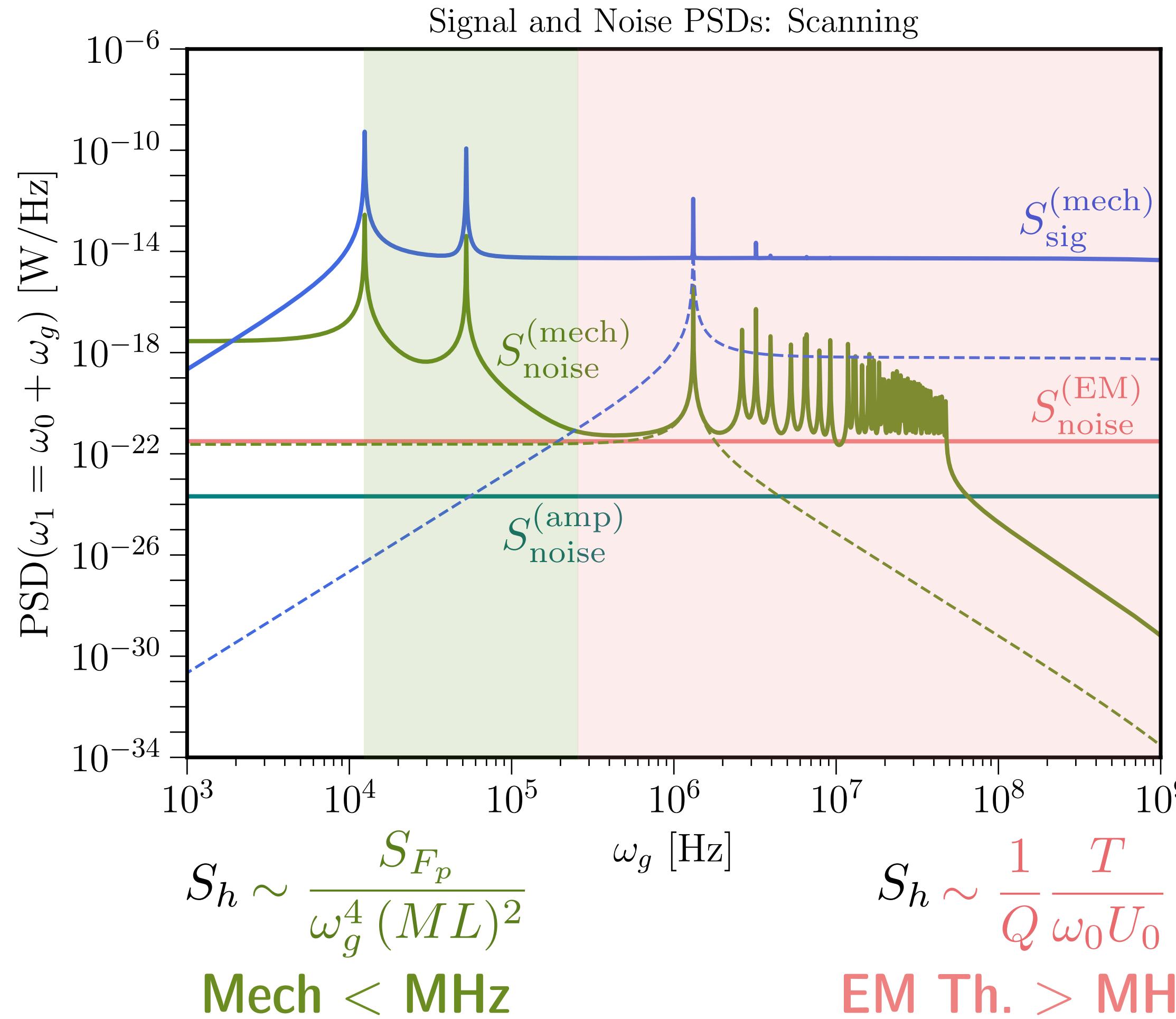
$$S_h \sim \frac{1}{Q} \frac{T}{\omega_0 U_0}$$

EM Th. > MHz

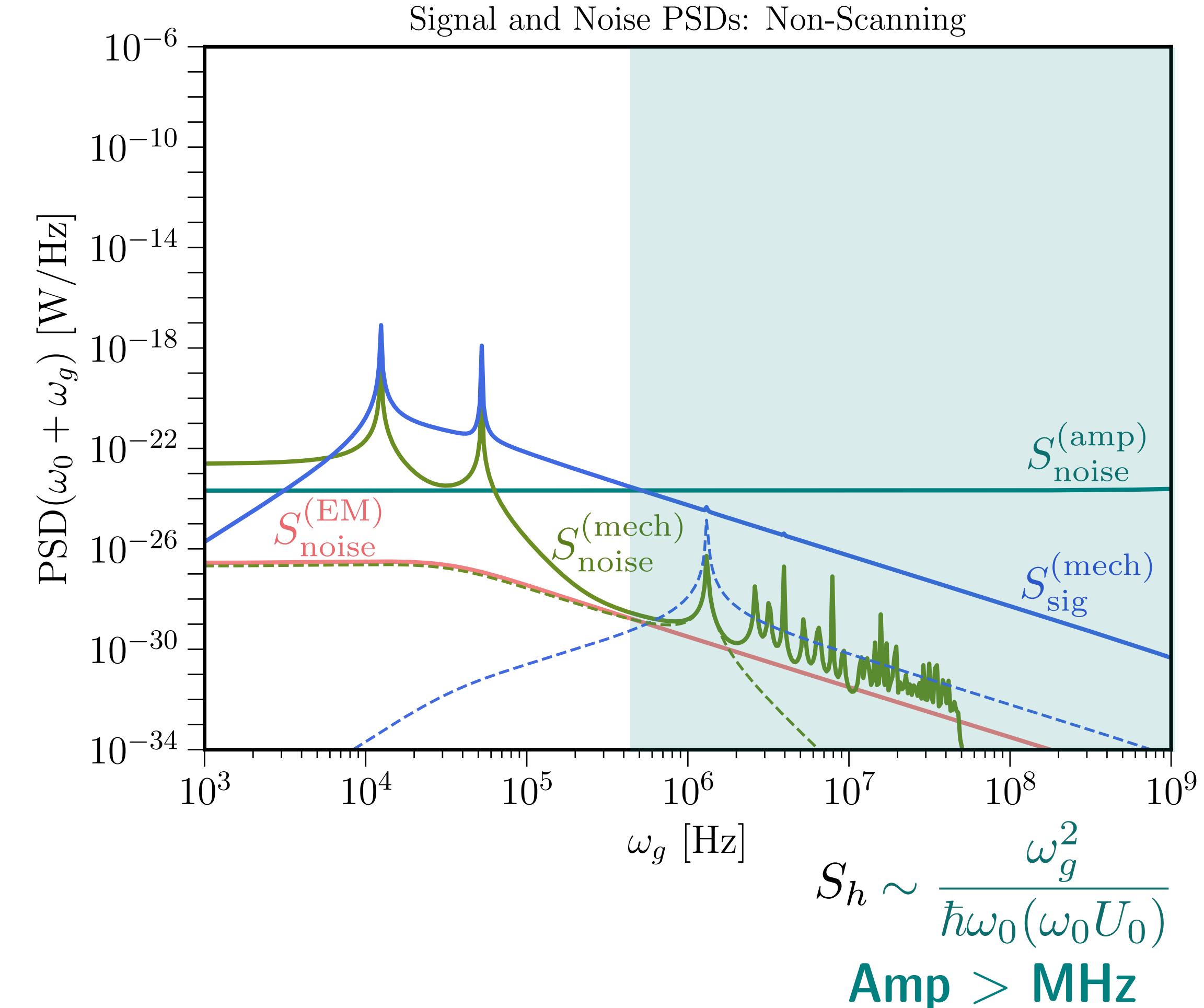
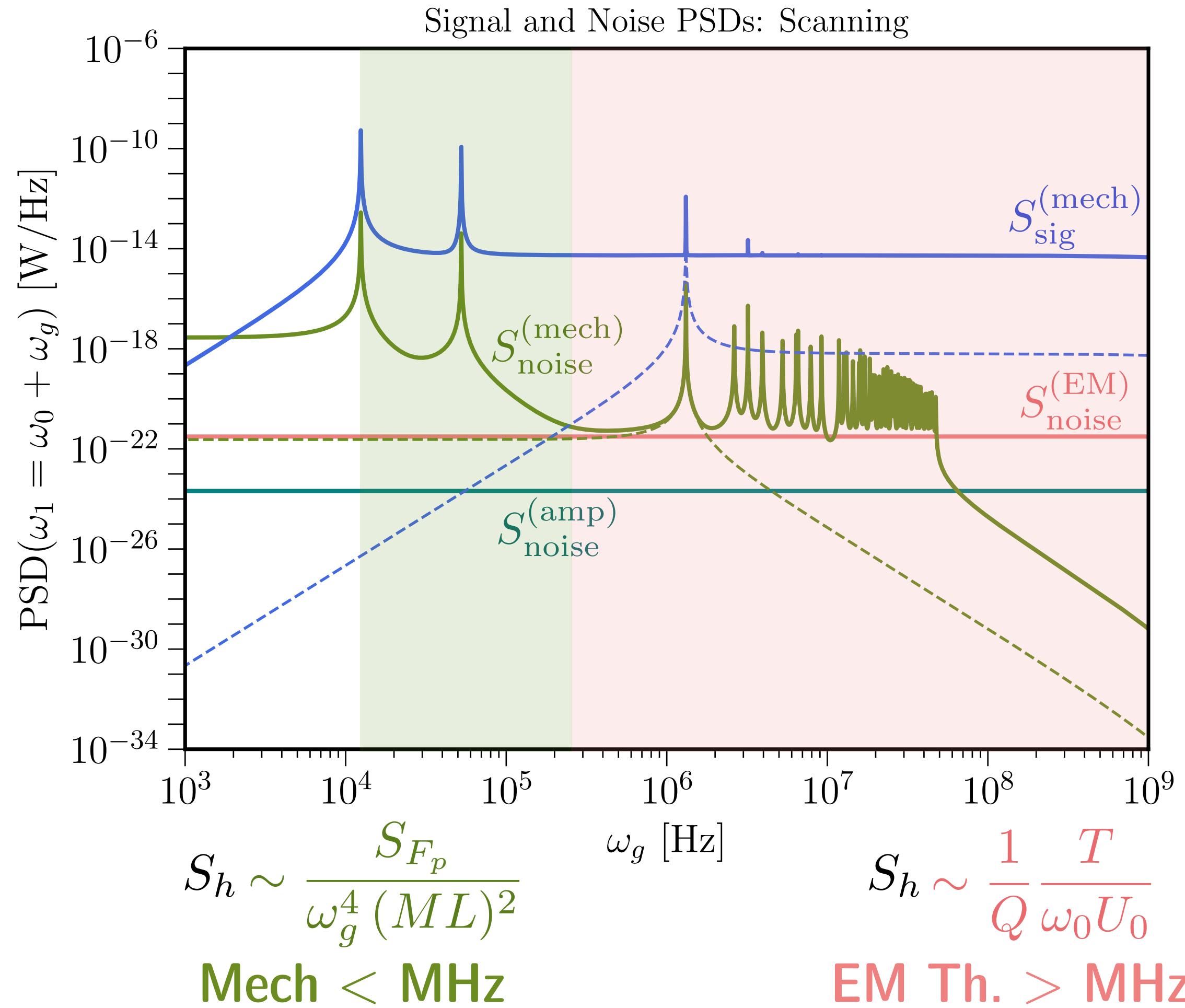
Noise in MAGO 2.0



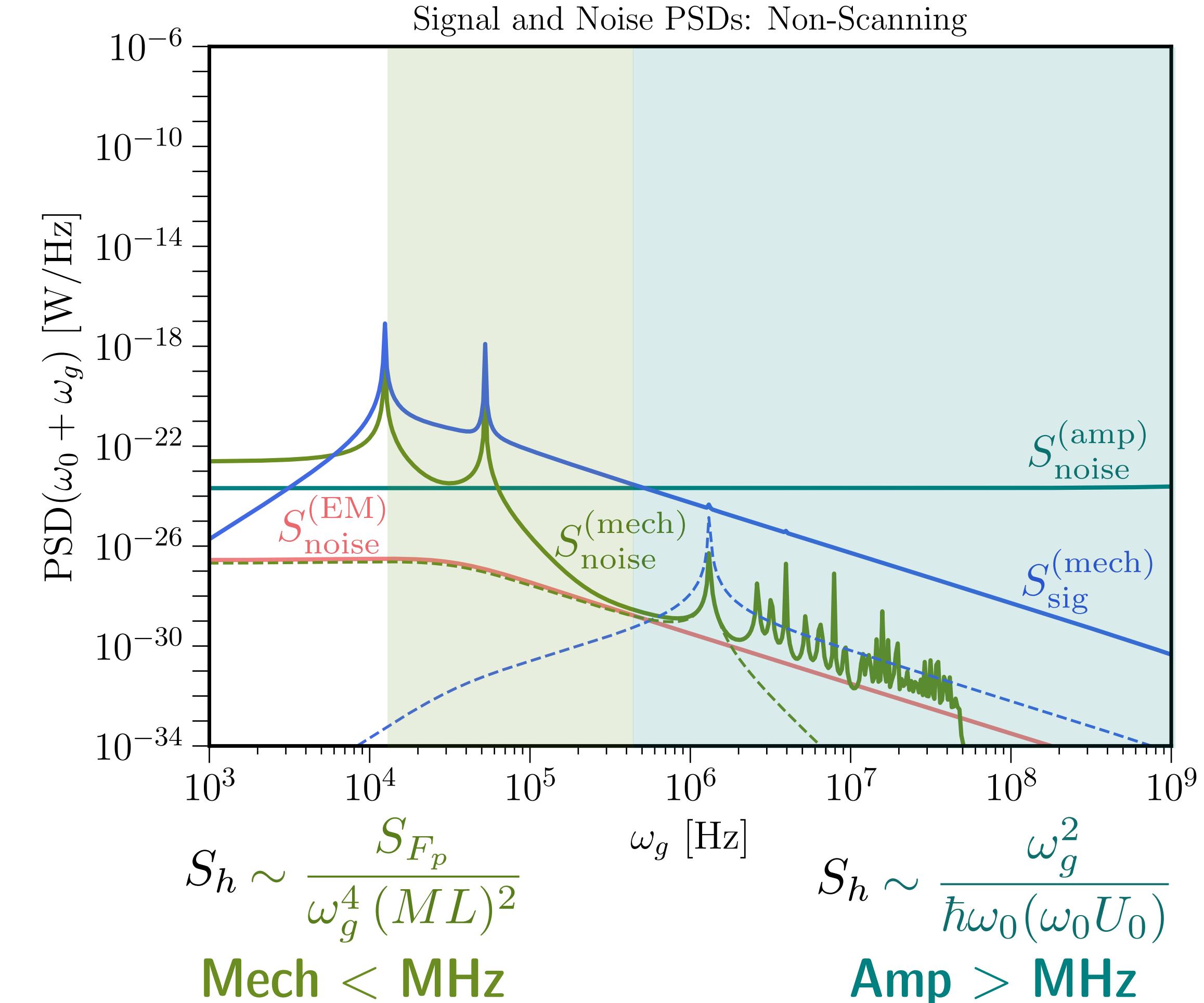
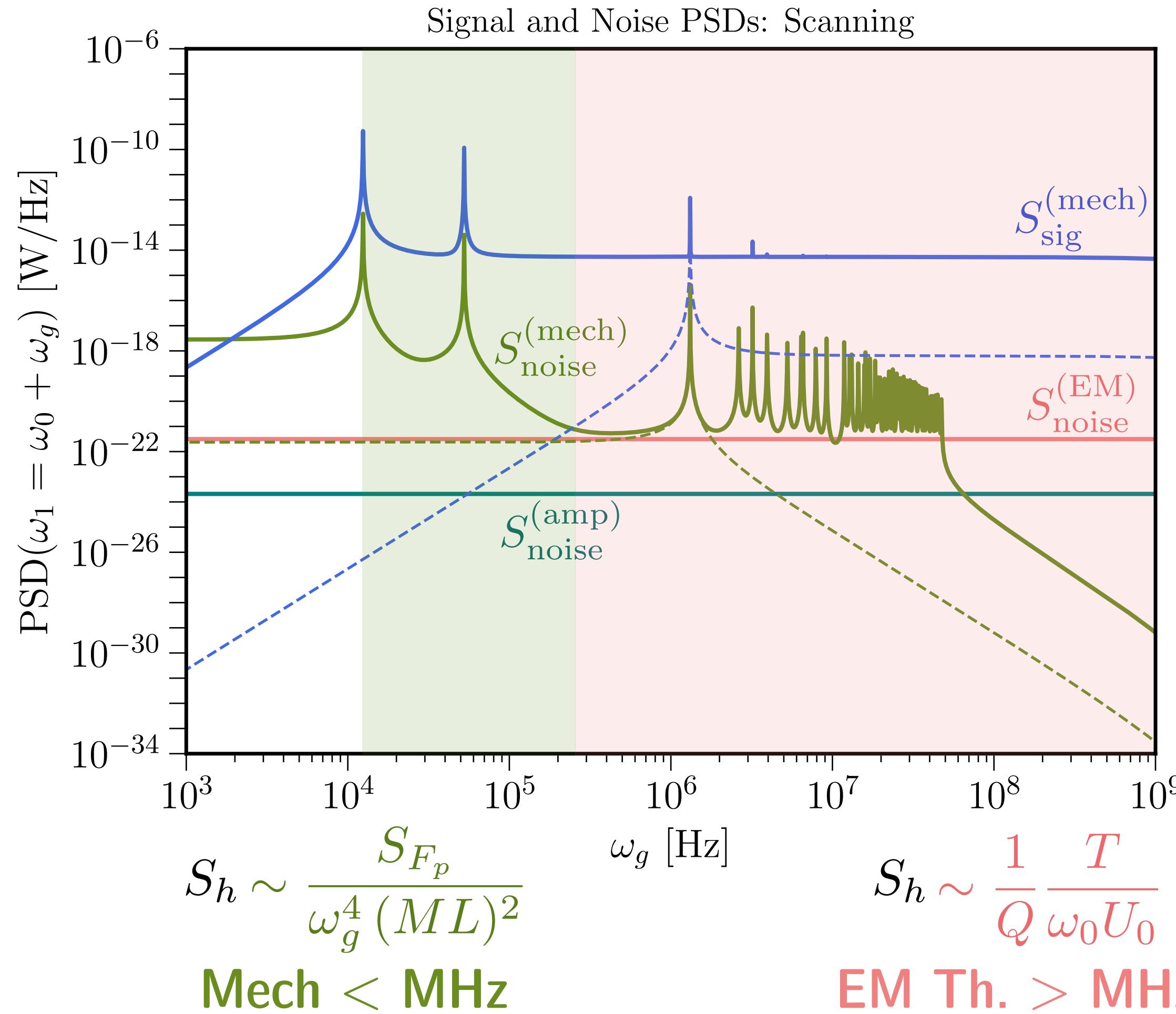
Noise in MAGO 2.0



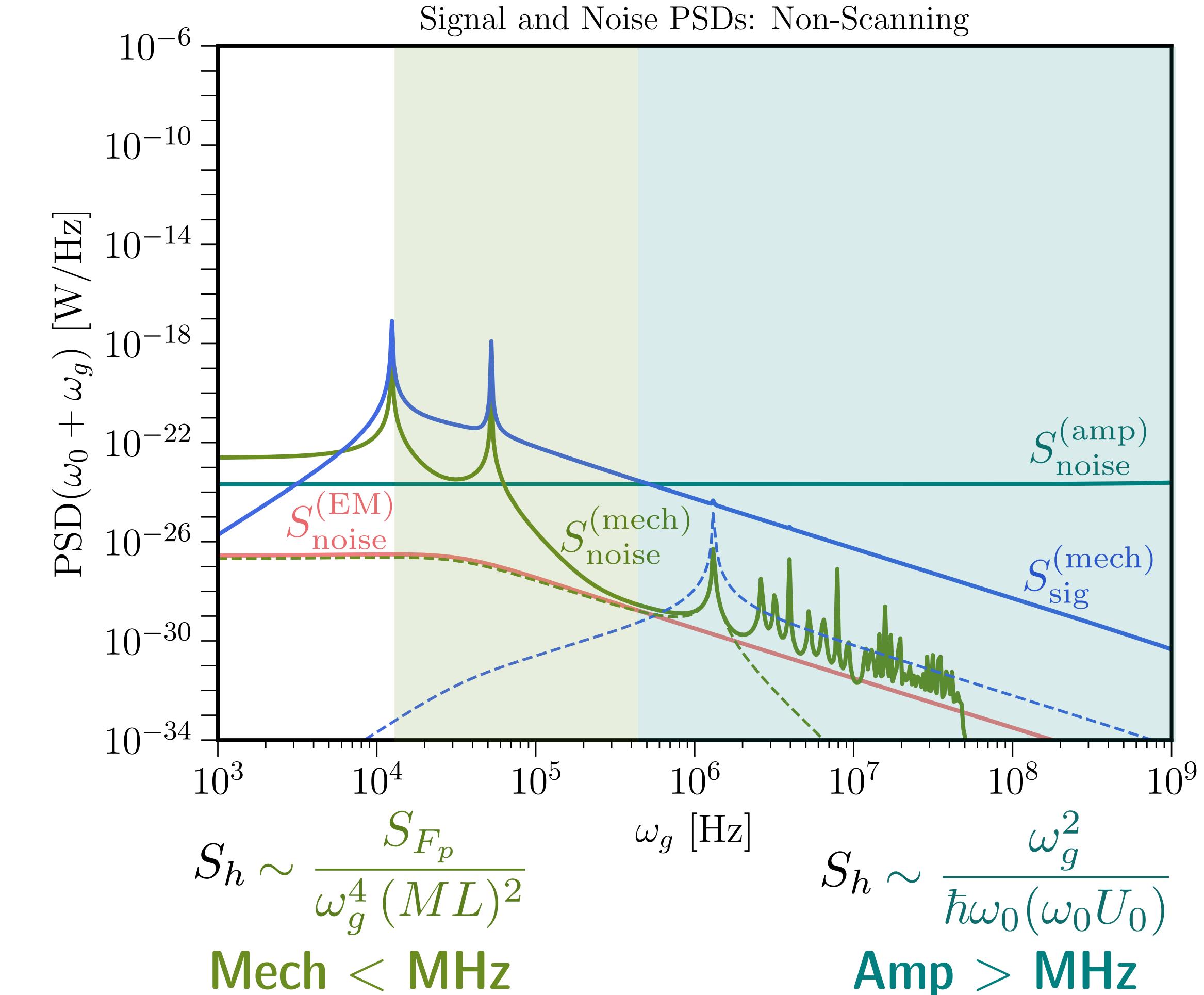
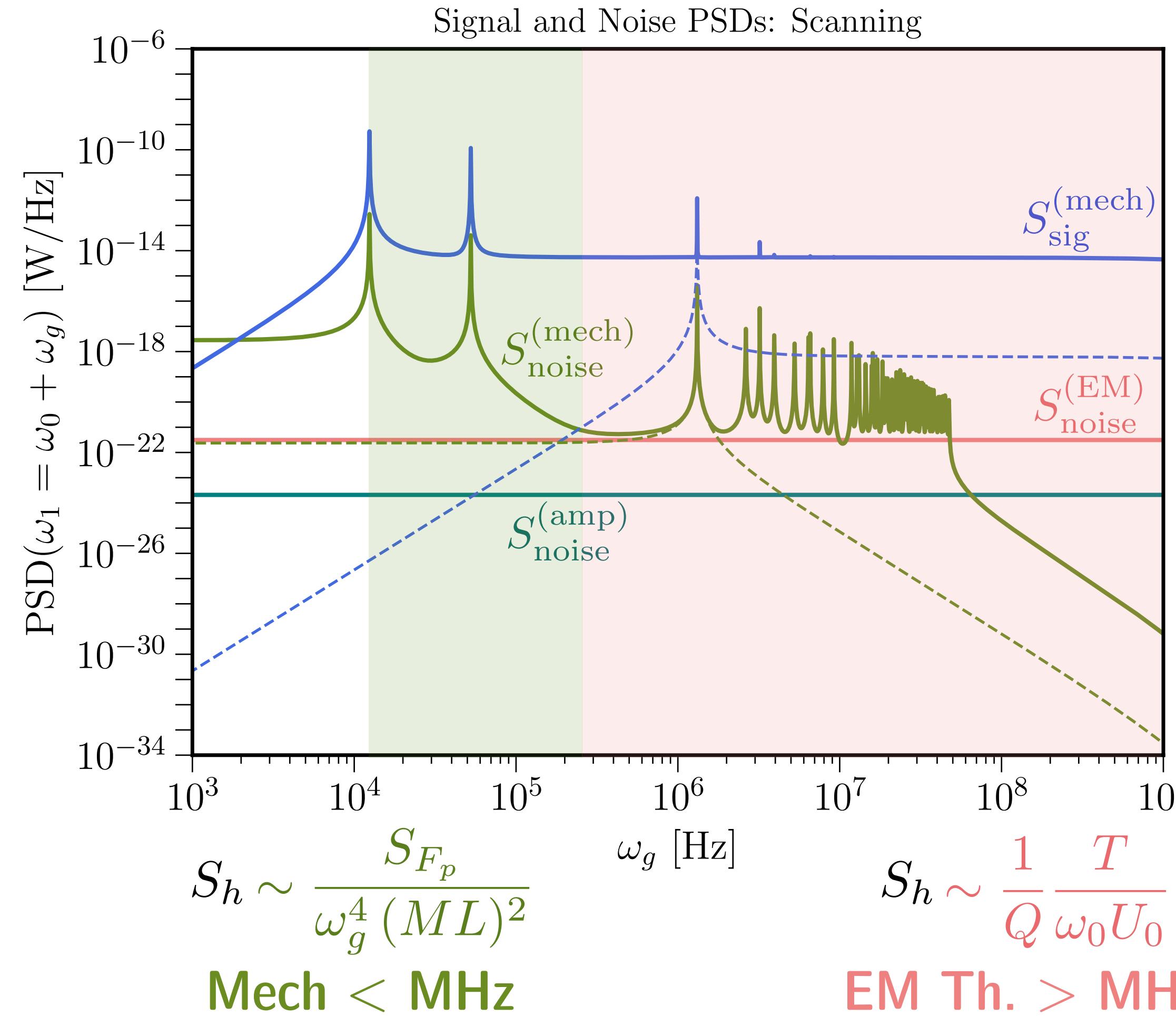
Noise in MAGO 2.0



Noise in MAGO 2.0



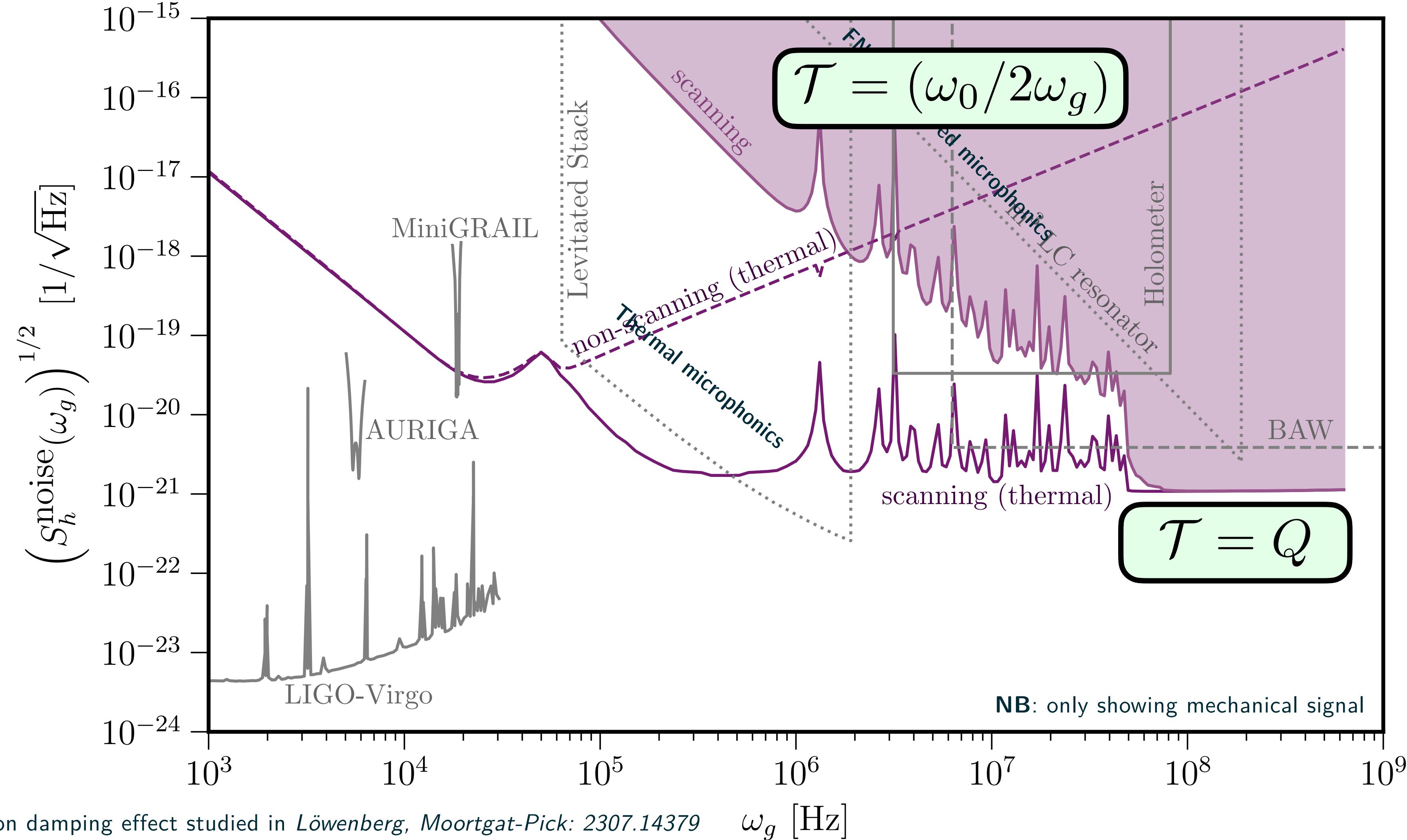
Noise in MAGO 2.0



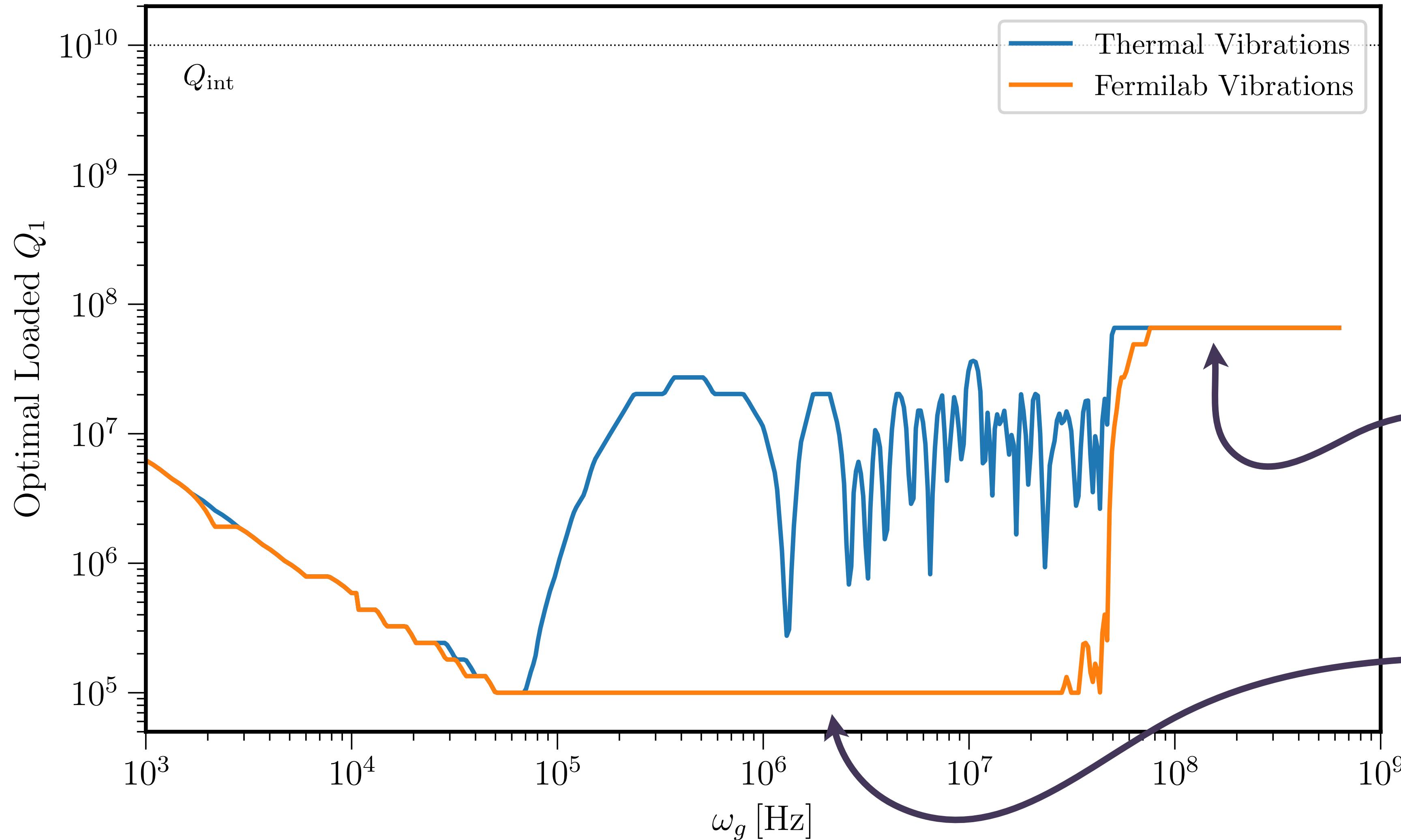
NB: missing radiation damping effect studied in Löwenberg, Moortgat-Pick: 2307.14379

Noise-equivalent strain

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel, Wentzel (PRD 2023)



Optimal Scanning



Integration time:

$$t_{\text{int}} \sim t_e \min \left(\frac{\omega_1}{Q_1 \omega_g}, 1 \right)$$

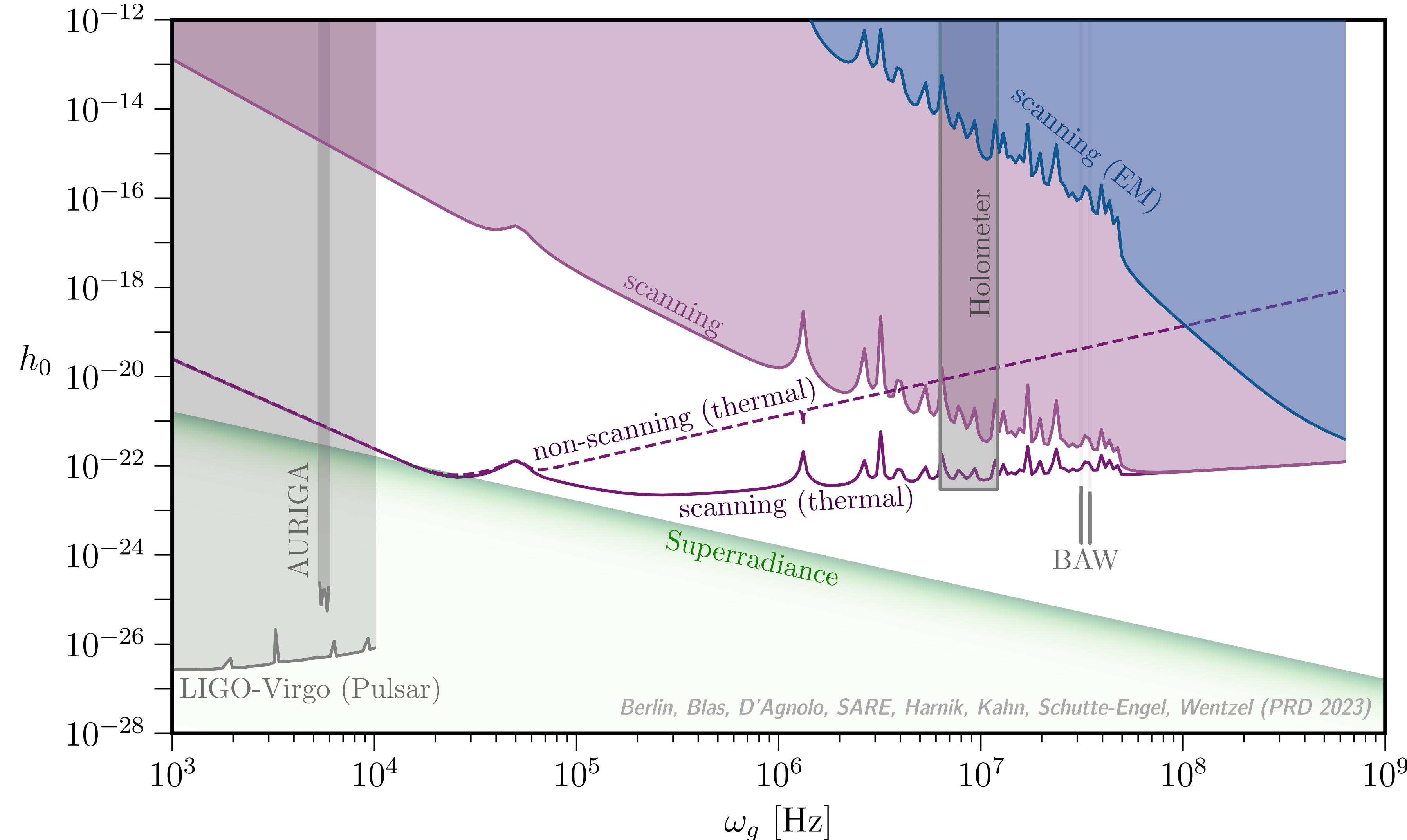
Thermal:

$$Q_1 \sim Q_{\text{int}}(\omega_1/T)$$

Vibrations:

$$Q_1 \sim Q_1^{\min}$$

MAGO 2.0 sensitivity to coherent GWs

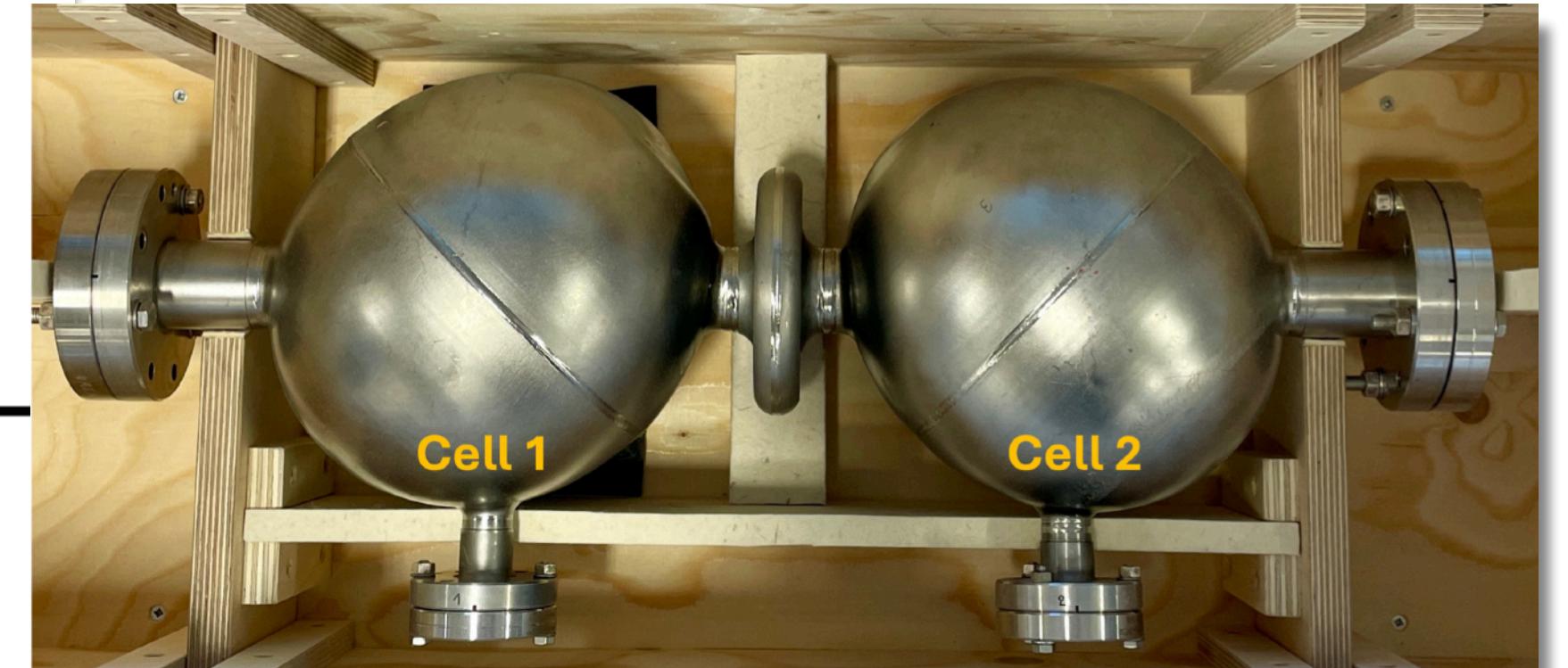


From Proposal to Practice

First characterisation of the MAGO cavity, a superconducting RF detector for kHz-MHz gravitational waves

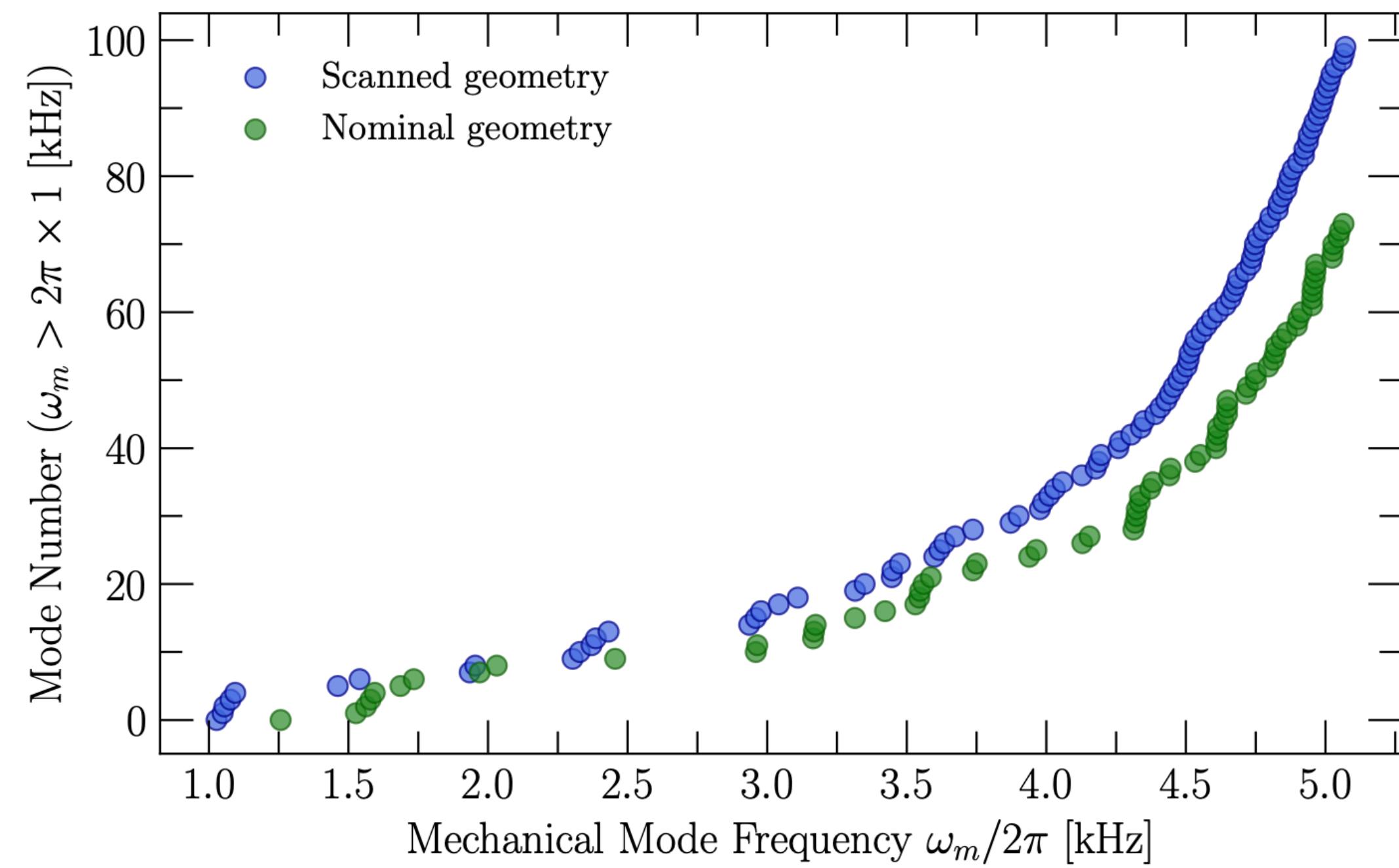
Lars Fischer^a Bianca Giaccone^b Ivan Gonin^b Anna Grassellino^b Wolfgang Hillert^a
Timergali Khabiboulline^b Tom Krokotsch^a Gudrid Moortgat-Pick^{a,c} Andrea Muhs^c
Yuriy Orlov^b Krisztian Peters^c Sam Posen^b Oleg Pronitchev^b Marc Wenskat^{a,c}

<https://arxiv.org/abs/2411.18346>



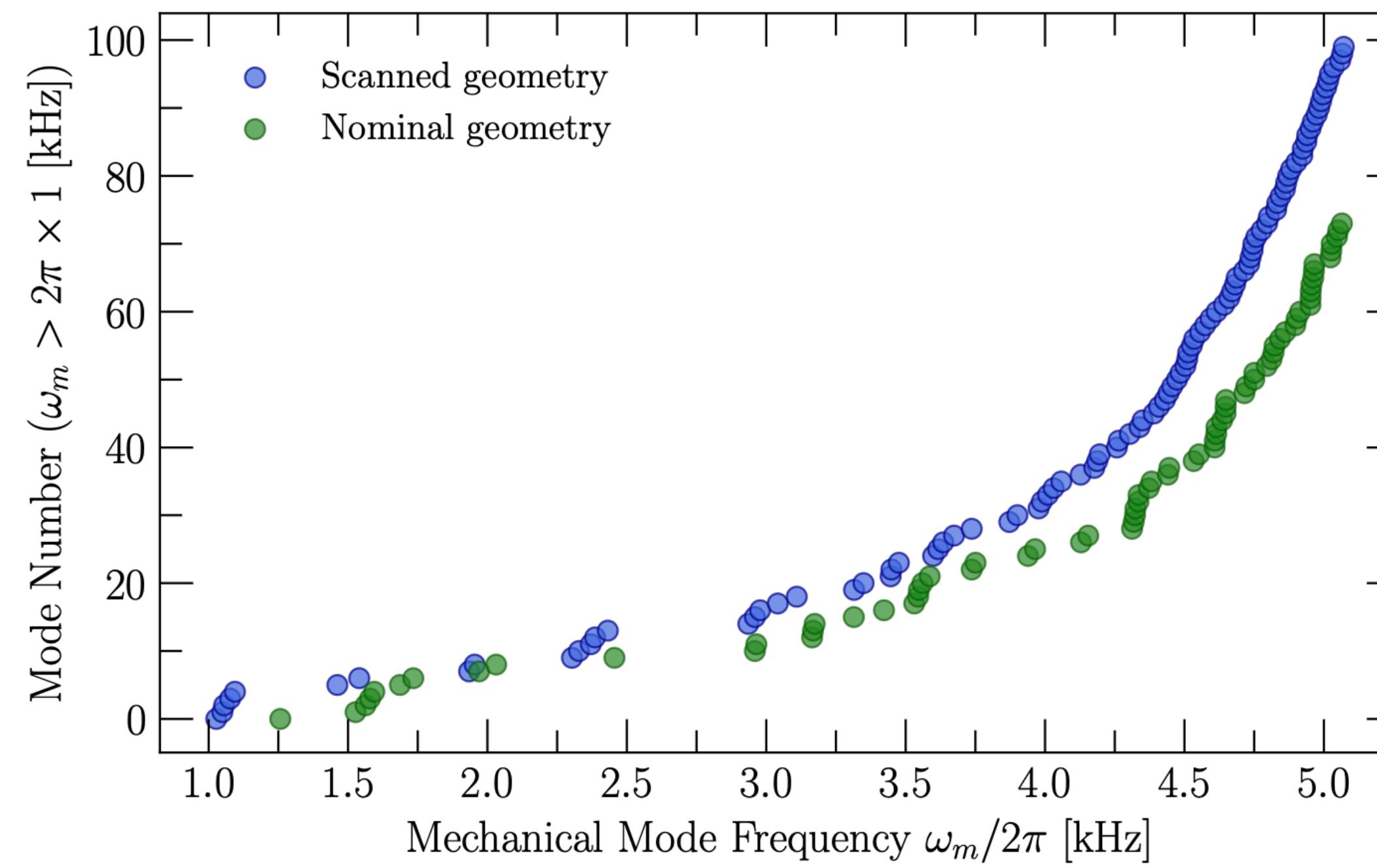
Ongoing collaboration between INFN/DESY/FNAL

Characterising the Prototype

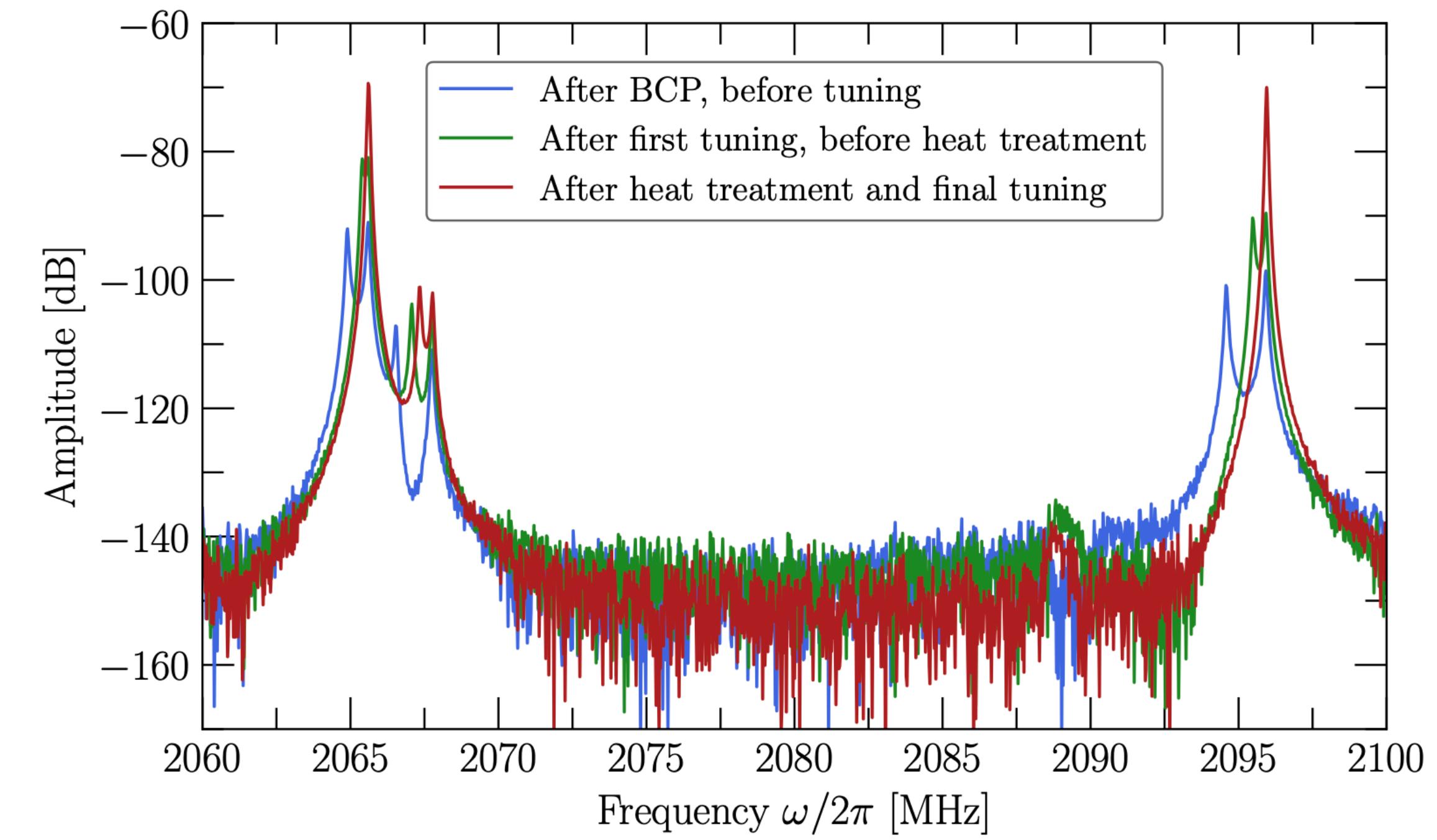


Mechanical modes

Characterising the Prototype



Mechanical modes



RF spectrum

Conclusion

MAGO operates in:

- Low-frequency regime: $1/L \gg \omega_g$ — use PDF and account for current & boundary changes

Renewed collaboration between INFN/DESY/
FNAL

First data expected in 2025

