Gravitational waves induced by electromagnetic radiation

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Gravitational waves for a weak field

For the case of a weak field in Einstein's linearized theory of gravity, gravitational waves are considered as small perturbations of the Minkowski space-time metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1,$$

where $\eta_{00} = 1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$ and $\mu, \nu = 0, 1, 2, 3$.

Conservation laws and harmonic gauge

$$\partial^\mu T_{\mu\nu} = 0, \quad \partial^\mu \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h_\rho{}^\rho \right) = 0, \qquad \text{where} \quad \partial^\mu \equiv \frac{\partial}{\partial x^\mu}, \; h_\rho{}^\rho = h_{\rho\sigma}\eta^{\rho\sigma}.$$

Gravitational field equations

$$\Box h_{\mu\nu} = 2\kappa \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T_\rho{}^\rho \right), \quad \Box \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}, \; \kappa \equiv \frac{8\pi G}{c^4}.$$

Types of gravitational waves

When a gravitational wave propagates along the direction $x^1 = x$, the nonzero components of the $h_{\mu\nu}$ tensor determine three types of gravitational waves

- h_{22}, h_{23}, h_{33}: transverse-transverse (TT),
- h_{12}, h_{13}, h_{20}, h_{30}: longitudinal-transverse (LT),
- h_{11}, h_{10}, h_{00}: longitudinal–longitudinal (LL).

Possible types of polarization of TT-waves:

$$h_{23}, \quad h_{22} - h_{33}, \quad h_{22} + h_{33},$$

in empty space
$h_{23} \neq 0, \quad h_{22} - h_{33} \neq 0, \quad h_{22} + h_{33} = 0, \quad LT = 0, \quad LL = 0.$
Transverse-traceless (TT) gauge $h^{0\mu} = 0, \quad h^i{}_i = 0, \quad \partial^i h_i = 0.$

A. S. Eddington, The propagation of gravitational waves, Proc. Roy. Soc. Lond. A **102** (1922), 268-282. doi:10.1098/rspa.1922.0085.

TT-gravitational waves

Solutions of the equations of the gravitational field in the case of TT-gauge

$$h_{ij}^{TT}(t,\boldsymbol{x}) = \frac{4G}{rc^4}\Lambda_{ij,kl}(\boldsymbol{n})\int d^4x' T_{kl}\left(t - \frac{r}{c} + \frac{\boldsymbol{x'}\cdot\boldsymbol{n}}{c}, \boldsymbol{x'}\right),$$

where $r = |\boldsymbol{x} - \boldsymbol{x}'|$,

$$\begin{aligned}\Lambda_{ij,kl}(\boldsymbol{n}) = \quad & P_{ik}P_{jl} - \tfrac{1}{2}P_{ij}P_{kl} = \delta_{ik}\delta_{jl} - \tfrac{1}{2}\delta_{ij}\delta_{kl} - n_j n_l \delta_{ik} - n_i n_k \delta_{jl} \\ + \quad & \tfrac{1}{2}n_k n_l \delta_{ij} + \tfrac{1}{2}n_i n_j \delta_{kl} + \tfrac{1}{2}n_i n_j n_k n_l,\end{aligned}$$

$P_{jk}(\boldsymbol{n}) = \delta_{jk} - n_j n_k$,
n_k – unit vector in the direction of propagation of the gravitational wave.

TT-gravitational waves

In the framework of the multipole expansion h_{ij}^{TT}, the first nonzero moment is a quadrupole one

$$[h_{ij}^{TT}(t,\boldsymbol{x})]_{\text{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}(\boldsymbol{n})\ddot{Q}^{kl}(t-r/c),$$

$$Q^{ij} = M^{ij} - \frac{1}{3}\delta^{ij}M_{kk} \equiv \int d^3x\rho(x^i x^j - \frac{1}{3}r^2\delta^{ij}),$$

$$M^{ij} = \int d^3x\rho(t,\boldsymbol{x})x^i x^j.$$

Perturbations of a medium as the source of gravitational waves (shock waves model)

Irradiation of matter with the high-power laser radiation is considered as a possible mechanism for generating high-frequency gravitational waves.
The amplitude of flat gravitational waves induced by the **shock waves** was estimated as follows

$$h_0 = \frac{7}{2}\frac{GP_L\tau}{rc^4}\sqrt{\frac{\rho_c}{\rho_0}}, \tag{1}$$

where P_L is the laser power, τ is the laser pulse duration, ρ_0 is the density of the medium and

$$\rho_c = \frac{\epsilon_0 m_e m_i}{Ze^2}\frac{(2\pi c)^2}{\lambda_L^2}. \tag{2}$$

The amplitude of gravitational waves for realistic parameters of experiment is estimated at the distance $r = 10$ m as $h_0 \simeq 10^{-40}$.

H. Kadlecová, O. Klimo, S. Weber and G. Korn, Gravitational wave generation by interaction of high power lasers with matter using shock waves, Eur. Phys. J. D **71** (2017) no.4, 89.

Perturbations of a medium as the source of gravitational waves (crystal interacting with an electromagnetic wave)

The other approach to generation of the high-frequency gravitational waves by the perturbations of the medium based on a model of an crystal interacting with an electromagnetic wave. The amplitude of flat gravitational waves induced by this method

$$h_0 = \frac{G}{rc^4}\gamma_\omega l^2 \omega_T^2 m_i N, \qquad \gamma_\omega = \frac{\epsilon_\infty - \epsilon_0}{2\pi}. \tag{3}$$

The amplitude of gravitational waves for realistic parameters of experiment is estimated at the distance $r = 10$ m as $h_0 \simeq 10^{-40}$, which is coincide with result in previous approach.

I. V. Fomin, V. O. Gladyshev, V. S. Gorelik, V. L. Kauts, A. V. Kaytenko and E. A. Sharandin, Laboratory sources of gravitational waves, J. Phys. Conf. Ser. **1705** (2020) no.1, 012004. doi:10.1088/1742-6596/1705/1/012004.

Direct Gertsenshtein effect

Conversion of an electromagnetic wave into a gravitational wave in the external magnetic field.

The gravitational waves induced by this method are

$$h_{22} = -h_{33} = \frac{2GH_0H_{00}}{c^3\omega} x \sin\left(\omega\left(t - \frac{x}{c}\right)\right). \tag{4}$$

The maximal amplitude of gravitational waves for realistic parameters of experiment is $h_0 \simeq 10^{-30} - 10^{-32}$.

N. I. Kolosnitsyn and V. N. Rudenko, Gravitational Hertz experiment with electromagnetic radiation in a strong magnetic field, Phys. Scripta **90** (2015) no.7, 074059. doi:10.1088/0031-8949/90/7/074059. [arXiv:1504.06548 [gr-qc]].

Gravitational waves from high-frequency laser pulses

Light pulse model: cylinder moving at speed c.
Optical frequency of GWs: $f_{gw} \sim 10^{-15}$
Maximal amplitude of GWs: $h_0 \sim 10^{-37}$

P. Lageyre, E. D'Humieres and X. Ribeyre, Gravitational influence of high power laser pulses, Phys. Rev. D **105** (2022) no.10, 104052. doi:10.1103/PhysRevD.105.104052. [arXiv:2108.00896 [gr-qc]].

Bessel pulses as sources of gravitational waves

$$h \sim 10^{-37} \left(\frac{P}{1PW}\right) \left(\frac{r}{1.36\mu m}\right)^{-1} \quad (5)$$

Optical frequency of GWs: $f_{gw} \sim 10^{-15}$
Maximal amplitude of GWs: $h_0 \sim 10^{-34}$

E. Atonga, K. Martineau, R. Aboushelbaya, A. Barrau, M. von der Leyen, S. Howard, A. James, J. Lee, C. Lin and H. Martin, *et al.* Gravitational waves from high-power twisted light, Phys. Rev. D **110** (2024) no.4, 044023. doi:10.1103/PhysRevD.110.044023. [arXiv:2309.04191 [gr-qc]].

Method for detection of the high-frequency GWs

Inverse Gertsenshtein effect: Conversion of an gravitational wave into an electromagnetic wave in the external magnetic field. Sensitivity $h \sim 10^{-31}$.

H. Zheng and L. F. Wei, Phys. Rev. D **106** (2022) no.10, 104003. doi:10.1103/PhysRevD.106.104003.

Detector	Sensitivity band	$h_{\min}$
ALPS	$(3-7.5)\times 10^{14}$ Hz	$h_{\min}(f_{gw}=3\times 10^{14}) \approx 3.64\times 10^{-25}$
OSQAR I	$(3-7.5)\times 10^{14}$ Hz	$h_{\min}(f_{gw}=3\times 10^{14}) \approx 4.92\times 10^{-25}$
OSQAR II	$(3-7.5)\times 10^{14}$ Hz	$h_{\min}(f_{gw}=3\times 10^{14}) \approx 1.04\times 10^{-25}$
CAST	$(5-15)\times 10^{18}$ Hz	$h_{\min}(f_{gw}=10^{18}) \approx 5.25\times 10^{-28}$
ALPS IIc	$\sim 4\times 10^{14}$ Hz	$h_{\min}(f_{gw}=4\times 10^{14}) \approx 2.84\times 10^{-30}$
JURA	$\sim 4\times 10^{14}$ Hz	$h_{\min}(f_{gw}=4\times 10^{14}) \approx 3.97\times 10^{-32}$
IAXO	$\sim (2.4-24)\times 10^{17}$ Hz	$h_{\min}(f_{gw}=10^{18}) \approx 9.78\times 10^{-30}$

Table 1: Sensitivities of the WISP detectors as GWs detectors.

A. Ejlli, D. Ejlli, A. M. Cruise, G. Pisano and H. Grote, Eur. Phys. J. C **79** (2019) no.12, 1032. doi:10.1140/epjc/s10052-019-7542-5. [arXiv:1908.00232 [gr-qc]].

Manucharyan G. D., Fomin I. V. Analysis of the possibility of detecting high-frequency gravitational waves radiation on the basis of photon-graviton conversion. Space, Time and Fundamental Interactions, 2024, no. 1, pp. 78–83. https://stfi.ru/

Characteristics of TT-gravitational waves

Astrophysical sources (Neutron stars and black holes mergers)
Amplitude is $h_0 \sim 10^{-21}$, frequency is $f_{gw} \sim 10^2 - 10^3$ Hz.
Propagation speed: $|c_g - c| \le 5 \times 10^{-16}$ (GR $c_g = c$)
Detected based on interference registration methods [LIGO,VIRGO]

B.P. Abbott et al. Multi-messenger Observations of a Binary Neutron Star Merger. Astrophys. J. Lett. 848, L12 (2017)

Laboratory sources
Amplitude: $h_0^{(max)} \sim 10^{-32}$, frequency $f_{gw} \sim 10^{15}$ Hz.
Not registered.

GW Hertz experiment: controlled characteristics of the GW source and detector

Electromagnetic source of gravitational waves with controlled frequency.

Laboratory electromagnetic source of low-frequency gravitational waves.

Free and bound gravitational waves

Choice of coordinate system $\Rightarrow$ gauge artifacts in the form of LT and LL-gravitational waves in vacuum $\Rightarrow$ gauge-invariant equations of the gravitational field.

Representation of the gravitational wave tensor components

$$\begin{aligned} h_{00} &= 2\phi \,, \\ h_{0i} &= \beta_i + \partial_i \gamma \,, \\ h_{ij} &= h_{ij}^{\mathrm{TT}} + \frac{1}{3} H \delta_{ij} + \partial_i \varepsilon_j + \partial_j \varepsilon_i + \left(\partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \lambda \,, \end{aligned}$$

with conditions $\partial_i \beta_i = 0$, $\partial_i \varepsilon_i = 0$, $\partial_i h_{ij}^{\mathrm{TT}} = 0$, $\delta^{ij} h_{ij}^{\mathrm{TT}} = 0$.

Bardeen J. M. Gauge-invariant cosmological perturbations. Phys. Rev. D 22 1882 (1980)

E. E. Flanagan and S. A. Hughes, The Basics of gravitational wave theory, New J. Phys. **7** (2005), 204. doi:10.1088/1367-2630/7/1/204. [arXiv:gr-qc/0501041 [gr-qc]].

Free and bound gravitational waves

Gravitational field equations $G_{\mu\nu} = \kappa T_{\mu\nu}$ in gauge-invariant form:

$$\begin{aligned} \nabla^2\Theta &= -\kappa\rho\,, \\ \nabla^2\Phi &= \frac{\kappa}{2}\left(\rho + 3P - 3\dot{S}\right)\,, \\ \nabla^2\Xi_i &= -2\kappa S_i\,, \\ \Box h_{ij}^{\mathrm{TT}} &= -2\kappa\sigma_{ij}\,, \end{aligned} \qquad \begin{aligned} \nabla^2\Theta^{\mathrm{vac}} &= 0\,, \\ \nabla^2\Phi^{\mathrm{vac}} &= 0\,, \\ \nabla^2\Xi_i^{\mathrm{vac}} &= 0\,, \\ \Box h_{ij}^{\mathrm{TT,vac}} &= 0\,. \end{aligned}$$

E. E. Flanagan and S. A. Hughes, The Basics of gravitational wave theory, New J. Phys. **7** (2005), 204. doi:10.1088/1367-2630/7/1/204. [arXiv:gr-qc/0501041 [gr-qc]].

In vacuum only $\Box h_{ij}^{\mathrm{TT,vac}} = 0$ has wave solutions (free gravitational waves).

For specific source: $\rho \sim \frac{\partial^2}{\partial t^2}\Theta$, $\left(\rho + 3P - 3\dot{S}\right) \sim \frac{\partial^2}{\partial t^2}\Phi$, $S_i \sim \frac{\partial^2}{\partial t^2}\Xi_i$ $\Rightarrow$ wave equation (coupled gravitational waves).

Method for analyzing the gravitational field of electromagnetic waves

Gravitational field equations for the case of an electromagnetic wave propagating along the direction $x^1 = x$:

$$
\begin{aligned}
&-h_{11,00} + 2h_{01,01} - h_{00,11} + h_{22,00} + h_{33,00} = 2\kappa T_{00},\\
&h_{11,00} - 2h_{01,01} + h_{00,11} + h_{22,11} + h_{33,11} = 2\kappa T_{11},\\
&h_{22,01} + h_{33,01} = 2\kappa T_{01},\\
&h_{22,11} - h_{22,00} = 2\kappa T_{22},\\
&h_{33,11} - h_{33,00} = 2\kappa T_{33}.
\end{aligned}
$$

On the right side of the equations, the material source of the gravitational field: the energy-momentum tensor of the electromagnetic field $T^{EM}_{\mu\nu}$.

Construction of exact solutions (internal solutions) of gravitational field equations for coupled gravitational waves $h^{(c)}_{\mu\nu}$.

Method for analyzing the gravitational field of electromagnetic waves

Calculation of the energy flux density of gravitational waves along the direction $x^1 = x$ based on the obtained solutions

$$ct^{01} = -\frac{c}{4\kappa}\left(\partial_0 h_{\alpha\beta}\partial_1 h^{\alpha\beta}\right).$$

The continuity condition for the energy flux density ct^{01} determines the boundary conditions.
Gravitational field equations in empty space (external solutions):

$$\begin{aligned}
&-h_{11,00} + 2h_{01,01} - h_{00,11} + h_{22,00} + h_{33,00} = 0,\\
&h_{11,00} - 2h_{01,01} + h_{00,11} + h_{22,11} + h_{33,11} = 0,\\
&h_{22,01} + h_{33,01} = 0, \quad h_{22,11} - h_{22,00} = 0, \quad h_{33,11} - h_{33,00} = 0.
\end{aligned}$$

Reconstruction of the characteristics of external solutions of the equations of the gravitational field $h^{TT}_{\mu\nu}$ (plane gravitational waves) based on the continuity of the energy flux density at the boundaries of the resonator.

Electromagnetic waves as a source of gravitational waves

The energy-momentum tensor of an electromagnetic field in vacuum

$$T_{00} = \frac{1}{8\pi}\left(E^2 + H^2\right),$$

$$T_{0k} = T_{k0} = -\frac{1}{4\pi}\varepsilon^{kij}E_iH_j,$$

$$T_{ij} = -\frac{1}{4\pi}\left[E_iE_j + H_iH_j - \frac{1}{2}\delta_{ij}\left(E^2 + H^2\right)\right],$$

where $i, j, k = 1, 2, 3$, ε^{kij} is the Levi-Civita symbol and δ_{ij} is the Kronecker symbol.

Coupled gravitational-electromagnetic waves in vacuum

Consider a plane electromagnetic wave

$$E_y = E_0 \cos\left(\omega\left(t - \frac{x}{c}\right)\right),$$
$$H_z = H_0 \cos\left(\omega\left(t - \frac{x}{c}\right)\right),$$

where $E_0 = H_0$.
Solutions of gravitational field equations (coupled gravitational waves)

$$h_{00} = h_{11} = -\frac{GE_0^2}{2c^3\omega} x \sin\left(2\omega\left(t - \frac{x}{c}\right)\right),$$
$$h_{01} = h_{10} = \frac{GE_0^2}{2c^3\omega} x \sin\left(2\omega\left(t - \frac{x}{c}\right)\right),$$
$$h_{22} = h_{33} = -\frac{GE_0^2}{4c^2\omega^2} \cos\left(2\omega\left(t - \frac{x}{c}\right)\right), \quad h_{22} + h_{33} \neq 0.$$

Reconstruction of characteristics of free TT-gravitational waves

Gravitational wave energy flux density along the axis $x^1 = x$:

$$ct^{01} = -\frac{c}{4\kappa}\left(\partial_0 h_{\alpha\beta}\partial_1 h^{\alpha\beta}\right).$$

For a coupled gravitational wave

$$ct^{01} = \frac{GE_0^4}{64\pi c\omega^2}\sin\left(2\omega\left(t-\frac{x}{c}\right)\right).$$

With complete absorption of an electromagnetic wave by the wall located at a distance $x = L$, based on the continuity of the energy flux of the gravitational wave $ct^{01}_{(L\leq x)} = ct^{01}_{(x\geq L)}$ we obtain the characteristics of free TT–gravity waves

$$h_{22} = -h_{33} = \pm\frac{GE_0^2}{4c^2\omega^2}\cos\left(2\omega\left(t-\frac{x}{c}\right)\right).$$

Coupled gravitational waves in dielectric media

Plane electromagnetic wave in a dielectric medium

$$E_y = E_0 \cos\left(\omega\left(t - \frac{nx}{c}\right)\right),$$
$$H_z = H_0 \cos\left(\omega\left(t - \frac{nx}{c}\right)\right),$$

where $n = \sqrt{\varepsilon}$ and $H_0 = \sqrt{\varepsilon} E_0 = n E_0$.

Components of the energy-momentum tensor for the case $\varepsilon = 1 + \kappa\rho$

$$T_{00} = \frac{1}{8\pi}\left(\varepsilon E_y^2 + H_z^2\right) = \frac{n^2 E_0^2}{8\pi}\left(1 + \cos\left(2\omega\left(t - \frac{nx}{c}\right)\right)\right),$$
$$T_{01} = T_{10} = -\frac{1}{4\pi}\varepsilon E_y H_z = -\frac{n E_0^2}{8\pi}\left(1 + \cos\left(2\omega\left(t - \frac{nx}{c}\right)\right)\right),$$
$$T_{\alpha\alpha} = \frac{1}{4\pi}\left[-\varepsilon E_\alpha E_\alpha - H_\alpha H_\alpha + \frac{1}{2}\left(\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_{T_0}\right)\left(E_y^2 + \frac{H_z^2}{\varepsilon}\right)\right].$$

Coupled gravitational waves in dielectric media

Solutions of the gravitational field equations

$$h_{22} = h_{33} = -\frac{GE_0^2}{4c^2\omega^2}\cos\left(2\omega\left(t - \frac{nx}{c}\right)\right),$$

$$h_{00} = -\frac{GE_0^2}{2c^3\omega}x\sin\left(2\omega\left(t - \frac{nx}{c}\right)\right) + \frac{GE_0^2}{4c^2\omega^2}\cos\left(2\omega\left(t - \frac{nx}{c}\right)\right),$$

$$h_{01} = h_{10} = \frac{nGE_0^2}{2c^3\omega}x\sin\left(2\omega\left(t - \frac{nx}{c}\right)\right) - \frac{nGE_0^2}{4c^2\omega^2}\cos\left(2\omega\left(t - \frac{nx}{c}\right)\right),$$

$$h_{11} = -\frac{n^2GE_0^2}{2c^3\omega}x\sin\left(2\omega\left(t - \frac{nx}{c}\right)\right) + \frac{GE_0^2}{4c^2\omega^2}\cos\left(2\omega\left(t - \frac{nx}{c}\right)\right).$$

Velocity of propagation of coupled gravitational waves $c_g = \frac{c}{n} \neq c$.

Coupled gravitational waves in a magnetic field and dielectric media

In a constant magnetic field H_{00} (along the $x^3 = z$ axis) and a dielectric medium

$$E_y = E_0 \cos\left(\omega\left(t - \frac{nx}{c}\right)\right),$$
$$H_z = H_0 \cos\left(\omega\left(t - \frac{nx}{c}\right)\right) + H_{00}.$$

Solutions of the gravitational field equations

$$h_{00} = -\frac{2GE_0H_{00}}{nc^3\omega} x \sin\left(\omega\left(t - \frac{nx}{c}\right)\right) + \frac{2GE_0H_{00}}{nc^2\omega^2} \cos\left(\omega\left(t - \frac{nx}{c}\right)\right),$$
$$h_{01} = h_{10} = \frac{2GE_0H_{00}}{c^3\omega} x \sin\left(\omega\left(t - \frac{nx}{c}\right)\right) - \frac{2GE_0H_{00}}{c^2\omega^2} \cos\left(\omega\left(t - \frac{nx}{c}\right)\right),$$
$$h_{11} = -\frac{2nGE_0H_{00}}{c^3\omega} x \sin\left(\omega\left(t - \frac{nx}{c}\right)\right) + \frac{2GE_0H_{00}}{nc^2\omega^2} \cos\left(\omega\left(t - \frac{nx}{c}\right)\right).$$

Coupled gravitational waves in a magnetic field and dielectric media

$$h_{22} = -\frac{4nGE_0H_{00}}{(n^2-1)\,c^2\omega^2}\left(\cos\left(\omega\left(t-\frac{nx}{c}\right)\right)-\cos\left(\omega\left(t-\frac{x}{c}\right)\right)\right)-$$
$$\frac{2GE_0H_{00}}{nc^2\omega^2}\cos\left(\omega\left(t-\frac{nx}{c}\right)\right),$$
$$h_{33} = \frac{4nGE_0H_{00}}{(n^2-1)\,c^2\omega^2}\left(\cos\left(\omega\left(t-\frac{nx}{c}\right)\right)-\cos\left(\omega\left(t-\frac{x}{c}\right)\right)\right)-$$
$$\frac{2GE_0H_{00}}{nc^2\omega^2}\cos\left(\omega\left(t-\frac{nx}{c}\right)\right).$$

Coupled gravitational waves in a magnetic field and dielectric media

Based on the continuity of the energy flux density ct^{01}, we restore the characteristics of TT–gravity waves in empty space (with a magnetic field, $n \to 1$)

$$h_{22} = -h_{33} = -\frac{4nGE_0H_{00}}{(n+1)\,c^2\omega^2}\frac{\cos\left(\omega\left(t-\frac{nx}{c}\right)\right)-\cos\left(\omega\left(t-\frac{x}{c}\right)\right)}{n-1}\Bigg|_{n\to1} =$$

$$-\frac{2GE_0H_{00}}{c^3\omega}x\sin\left(\omega\left(t-\frac{x}{c}\right)\right)$$

A.N. Morozov, V.I. Pustovoit, I.V. Fomin Bound gravitational waves in a dielectric medium and a constant magnetic field. The European Physical Journal Plus. 135, 950 (2020).

Standing electromagnetic wave in Fabry-Perot interferometer

Standing electromagnetic wave

$$E_y = E_0 \left\{ \cos\left[\omega\left(t - \frac{x}{c}\right)\right] + \cos\left[\omega\left(t + \frac{x}{c}\right)\right] \right\},$$
$$H_z = H_0 \left\{ \cos\left[\omega\left(t - \frac{x}{c}\right)\right] - \cos\left[\omega\left(t + \frac{x}{c}\right)\right] \right\},$$

between mirrors located at points $x = -L/2$ and $x = L/2$.

$$u_1(t,x) \equiv \cos\left[2\omega\left(t - \frac{x}{c}\right)\right], \quad u_2(t,x) \equiv \cos\left[2\omega\left(t + \frac{x}{c}\right)\right],$$
$$\tilde{u}_1(t,x) \equiv \cos\left(\frac{2\omega x}{c}\right), \qquad \tilde{u}_2(t,x) \equiv \cos\left(2\omega t\right),$$
$$v_1(t,x) \equiv \sin\left[2\omega\left(t - \frac{x}{c}\right)\right], \quad v_2(t,x) \equiv \sin\left[2\omega\left(t + \frac{x}{c}\right)\right],$$
$$\tilde{v}_1(t,x) \equiv \sin\left(\frac{2\omega x}{c}\right), \qquad \tilde{v}_2(t,x) \equiv \sin\left(2\omega t\right).$$

Standing electromagnetic wave in Fabry-Perot interferometer

Internal solutions (coupled gravitational waves)

$$h_{00} = h_{11} = -\frac{GE_0^2}{2c^3\omega}\left(\frac{L}{2}+x\right)v_1(t,x) - \frac{GE_0^2}{2c^3\omega}\left(\frac{L}{2}-x\right)v_2(t,x),$$
$$h_{01} = h_{10} = \frac{GE_0^2}{2c^3\omega}\left(\frac{L}{2}+x\right)v_1(t,x) - \frac{GE_0^2}{2c^3\omega}\left(\frac{L}{2}-x\right)v_2(t,x),$$
$$h_{22} = \frac{GE_0^2}{c^2\omega^2}\left[\tilde{u}_1(t,x) - \tilde{u}_2(t,x)\right] - \frac{GE_0^2}{4c^2\omega^2}\left[u_1(t,x) + u_2(t,x)\right],$$
$$h_{33} = -\frac{GE_0^2}{c^2\omega^2}\left[\tilde{u}_1(t,x) - \tilde{u}_2(t,x)\right] - \frac{GE_0^2}{4c^2\omega^2}\left[u_1(t,x) + u_2(t,x)\right].$$

External solutions (free TT - gravitational waves)

$$h_{22} = -h_{33} = h_0\left[u_1(t,x) + u_2(t,x)\right].$$

Standing electromagnetic wave in Fabry-Perot interferometer

Conditions for the continuity of the energy flux density of gravitational waves $\left[ct^{01}\right]_{in} = \left[ct^{10}\right]_{out}$ at the borders $x = \pm L/2$:

$$v_2(t,x)u_1(t,x) - v_1(t,x)u_2(t,x) = 0,$$
$$\tilde{v}_1(t,x)\tilde{v}_2(t,x) = 0,$$

are satisfied when

$$\sin\left(\frac{\omega L}{c}\right) = 0.$$

Longitudinal optical modes in a Fabry-Perot resonator

$$\omega = q\left(\frac{\pi c}{L}\right), \qquad q = 1, 2, 3, ...$$

Standing electromagnetic wave in Fabry-Perot interferometer

External solutions (free TT-gravity waves in empty space) are explicitly defined as follows

$$h_{22} = -h_{33} = h_0 \cos\left[\omega_g\left(t \mp \frac{x}{c}\right)\right],$$

where

$$h_0 = \pm\frac{GE_0^2}{4\omega^2 c^2} = \pm\frac{G}{c^4}\left(\frac{E_0 L}{2\pi q}\right)^2,$$
$$\omega_g = 2\omega = \frac{2\pi q c}{L}.$$

A.N. Morozov, V.I. Pustovoit, I.V. Fomin. Generation of Gravitational Waves by a Standing Electromagnetic Wave. Gravitation and Cosmology. 27, 24-29 (2021).

Standing electromagnetic wave in Fabry-Perot interferometer

Approximate solutions of the gravitational field equations

$$h_{\mu\nu} \approx \frac{G}{c^4} \int \frac{(T_{\mu\nu})_{t'}}{r} dV,$$

where $t' = t - r/c$ is the delayed time, V is the volume occupied by the electromagnetic field.

At the boundary of the wave zone $\lambda_g \sim r$, approximate solutions

$$h_0^{approx} \approx \frac{G}{c^4} \sigma_e \lambda_g^2 \left(\frac{\lambda_g}{r} \right) \approx \frac{G}{c^4} \sigma_e \lambda_g^2, \qquad h_0^{approx} = \left(\frac{\pi}{2} \right) h_0^{exact} \approx h_0^{exact},$$

where $\sigma_e = E_0^2 / 8\pi c^2$ is the reduced energy density of the electromagnetic field, the length of the gravitational wave $\lambda_g = \pi c/\omega$.

Standing electromagnetic wave in Fabry-Perot interferometer

In the optical frequency range $\omega \sim 10^{14} - 10^{15}$ Hz for an electromagnetic wave with intensity $E_0 \sim 10^6$ V/m in the Fabry-Perrot resonator, we obtain the following amplitude estimate corresponding free gravitational waves

$$h_0 \sim 10^{-44} - 10^{-46} \ll 10^{-20}.$$

The ratio of the amplitudes of longitudinal coupled gravitational waves to the amplitude of transverse free gravitational waves

$$r_{gw} \equiv \frac{h_0^{(c)}}{h_0^{TT}} \simeq \frac{\omega L}{c} = \pi \frac{L}{\lambda_g} = L \times (10^6 - 10^7)\,[1/\mathrm{m}] \gg 1.$$

$$h_0^{(c)} = L \times (10^{-38} - 10^{-39})\,[1/\mathrm{m}]\,.$$

Generation of gravitational waves at the difference frequency

Consider a system of two standing electromagnetic waves in a Fabry-Perot interferometer with frequencies ω_1 and ω_2 $(\omega_1 > \omega_2)$.

$$E_y = E_1 \left[\cos\left(\omega_1\left(t - \frac{x}{c}\right)\right) + \cos\left(\omega_1\left(t + \frac{x}{c}\right)\right)\right] +$$
$$E_2 \left[\cos\left(\omega_2\left(t - \frac{x}{c}\right)\right) + \cos\left(\omega_2\left(t + \frac{x}{c}\right)\right)\right],$$
$$H_z = H_1 \left[\cos\left(\omega_1\left(t - \frac{x}{c}\right)\right) - \cos\left(\omega_1\left(t + \frac{x}{c}\right)\right)\right] +$$
$$H_2 \left[\cos\left(\omega_2\left(t - \frac{x}{c}\right)\right) - \cos\left(\omega_2\left(t + \frac{x}{c}\right)\right)\right],$$

where $E_1 = H_1$ and $E_2 = H_2$.

Gladyshev V. O., Kauts V. L., Kayutenko A. V., Morozov A. N., Nikolaev P. P., Fomin I. V., Sharandin E. A. The analysis of coupled gravitational and electromagnetic waves. Space, Time and Fundamental Interactions, 2023, no. 3-4, pp. 99–107. https://stfi.ru/

Generation of gravitational waves at the difference frequency

Internal solutions (coupled gravitational waves, frequency $\tilde{\omega} = \omega_1 - \omega_2$)

$$h_{00} = h_{11} = -\frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2} + x\right)\sin\left(\tilde{\omega}\left(t - \frac{x}{c}\right)\right) -$$

$$\frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2} - x\right)\sin\left(\tilde{\omega}\left(t + \frac{x}{c}\right)\right),$$

$$h_{01} = h_{10} = \frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2} + x\right)\sin\left(\tilde{\omega}\left(t - \frac{x}{c}\right)\right) -$$

$$\frac{4GE_1E_2}{c^3\tilde{\omega}}\left(\frac{L}{2} - x\right)\sin\left(\tilde{\omega}\left(t + \frac{x}{c}\right)\right),$$

$$h_{22} = -\frac{2GE_1E_2}{c^2\tilde{\omega}^2}\left[\cos\left(\tilde{\omega}\left(t - \frac{x}{c}\right)\right) + \cos\left(\tilde{\omega}\left(t + \frac{x}{c}\right)\right)\right],$$

$$h_{33} = -\frac{2GE_1E_2}{c^2\tilde{\omega}^2}\left[\cos\left(\tilde{\omega}\left(t - \frac{x}{c}\right)\right) + \cos\left(\tilde{\omega}\left(t + \frac{x}{c}\right)\right)\right].$$

Generation of gravitational waves at the difference frequency

External solutions (free TT - gravitational waves)

$$h_{22} = -h_{33} = \pm \frac{2GE_1E_2}{c^2\tilde{\omega}^2} \cos\left(\tilde{\omega}\left(t - \frac{x}{c}\right)\right).$$

In this case, the ratio of the amplitudes of coupled longitudinal and free transverse gravity waves is equal to

$$r_{gw} \equiv \frac{h_0^{(c)}}{h_0^{TT}} \simeq \frac{\tilde{\omega}L}{c} \ll 1,$$

for a small difference frequency $\tilde{\omega}$.

The main part of the gravitational field during the further propagation of the gravitational wave after absorption of the electromagnetic one is outside the source in the form of low-frequency transverse gravitational waves with a frequency of $\omega_{gw} = \tilde{\omega}$.

Relationship between the parameters of the GW source and detector

Within the framework of the problem of generating and detecting gravitational waves, the controlled parameters of the source are the electric field strengths $E_1 = E_2 = E_0$ in the resonator and their difference frequency $\tilde{\omega}$, and the controlled parameter of the detector h_0 is the amplitude of registered gravitational waves (detector sensitivity).

$$E_0^2 = \frac{8\pi}{c} I_L \times Q = \frac{8\pi}{c} \left(\frac{P_S}{S} \right) \times Q.$$

$$h_0 = \left(\frac{4G}{\pi c^3} \right) \times \left(\frac{I_L}{f_{gw}^2} \right) \times Q.$$

Relationship between the parameters of the GW source and detector

For the quality factor of the electromagnetic resonator $Q \simeq 10^6$ and the laser radiation intensity $I_L \sim 10^7$ (W/m^2) we obtain

$$h_0 \simeq 3 \times 10^{-23} \times \left(\frac{\mathrm{Hz}}{f_{gw}}\right)^2.$$

For gravitational waves in the frequency range $f_{gw} \simeq 10^2 - 10^3$ Hz, the amplitude $h_0 \simeq 10^{-27} - 10^{-29}$.

The sensitivity of operating detectors (LIGO, KAGRA) in the frequency range $f_{gw} \simeq 10^2 - 10^3$ Hz is estimated as $h_0 \simeq 10^{-23} - 10^{-22}$.

Sensitivity of perspective detectors: Einstein Telescope for $f_{gw} \simeq 10^2$ Hz is estimated as $h_0 \simeq 10^{-25}$ and NEMO for $f_{gw} \simeq 10^3$ Hz evaluates to $h_0 \simeq 10^{-26}$.

Conclusion

- Model of an electromagnetic source of gravitational waves with adjustable frequency.
- Consideration of the other configuration instead of plane standing electromagnetic waves inside resonator.
- Analysis of the optimal method and optimal frequency range for detecting gravitational waves induced by electromagnetic waves.

Thank you for your attention!