

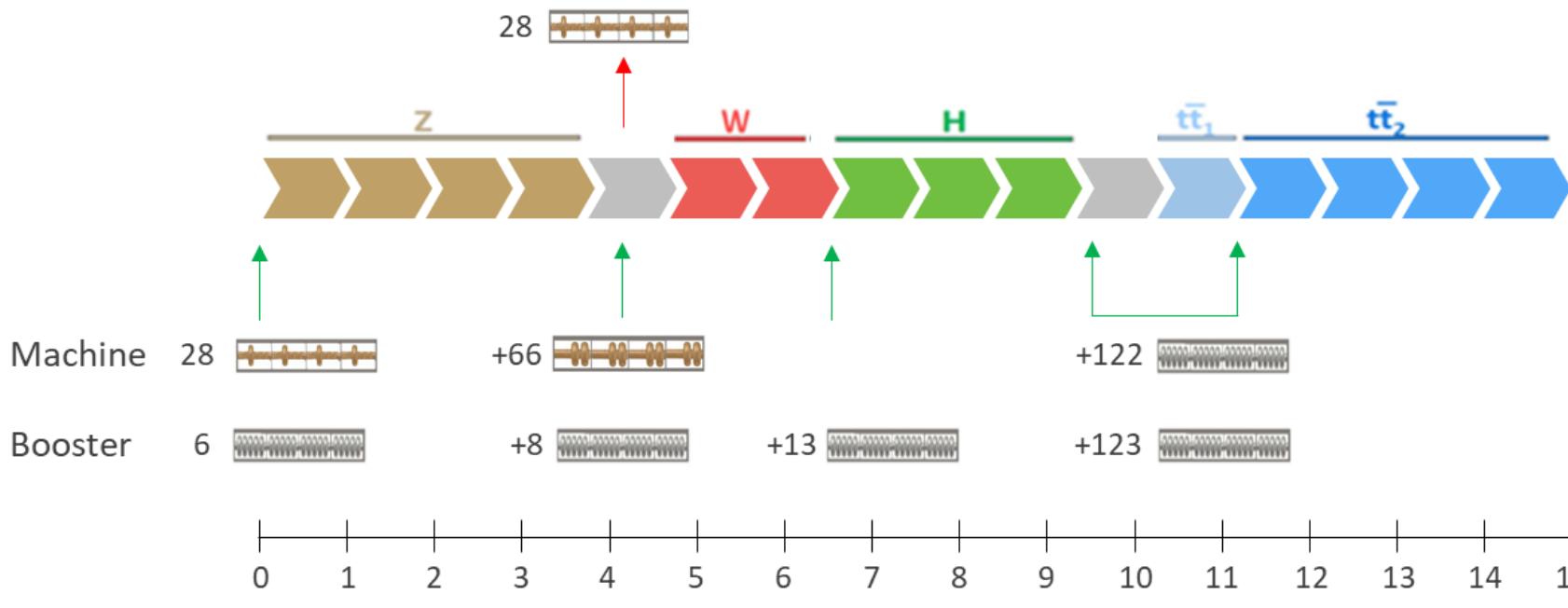
# Reverse-phasing mode for FCC-ee RF system

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Frank Gerigk, Wolfgang Höfle, Eric Montesinos, Igor Syratchev, Shahnam Gorgi Zadeh, Alice Vanel

# Baseline RF system configuration

	Energy (GeV)	Current (mA)	RF voltage (GV)
Z	45.6	1283	0.079
W	80	135	1.05
H	120	26.7	2.1
t̄t	182.5	5	11.67

K. Oide, 29.05.2024

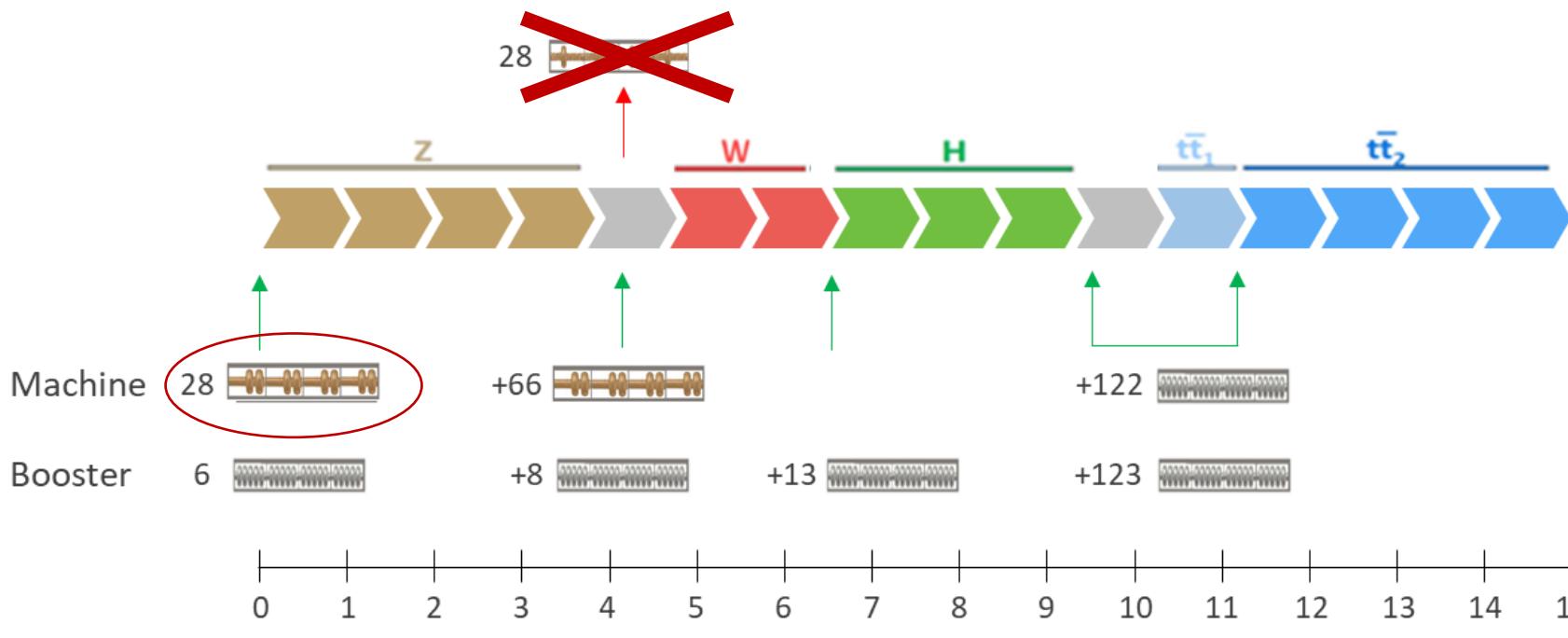


Courtesy of O. Brunner

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# Steady-state beam loading

RF power for SRF cavities with circulators is minimized for optimal parameters:

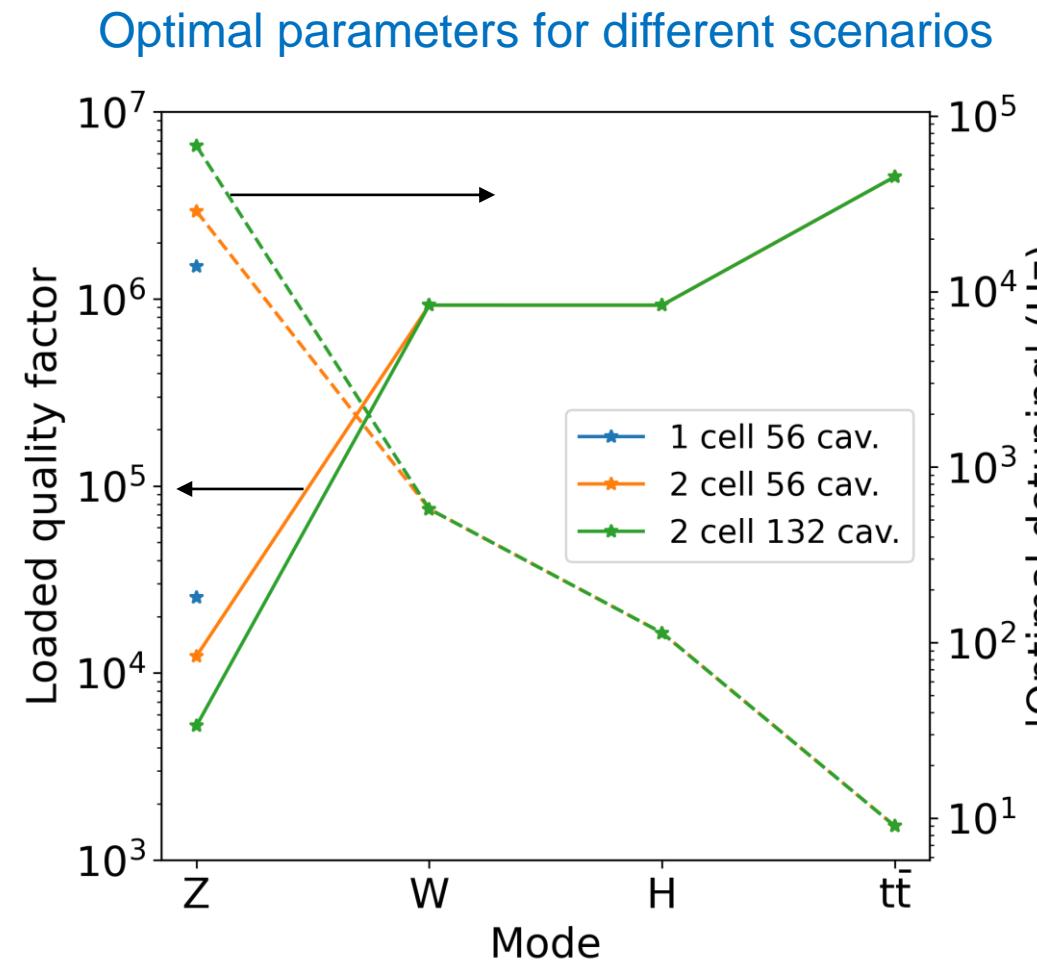
Optimal detuning  $\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}} \sin \phi_s}{2V_{\text{cav}}}$

Optimal quality factor  $Q_{\text{ext,opt}} = \frac{V_{\text{cav}}}{|F_b|(R/Q)I_{b,\text{dc}} \cos \phi_s}$

Keeping 2-cell cavities for Z, W, H, (and  $t\bar{t}$ ):

- Large range for  $Q_{\text{ext,opt}}$  adjustment (a factor of ~75-600) starting from  $\sim 5 \times 10^3$ : possible FPC solutions under study (*S. Gorgi Zadeh and E. Montesinos, CERN SRF, 2024; see also slides of F. Gerigk, FCC Week 2024*)
- Increased detuning enhances instability due to fundamental mode

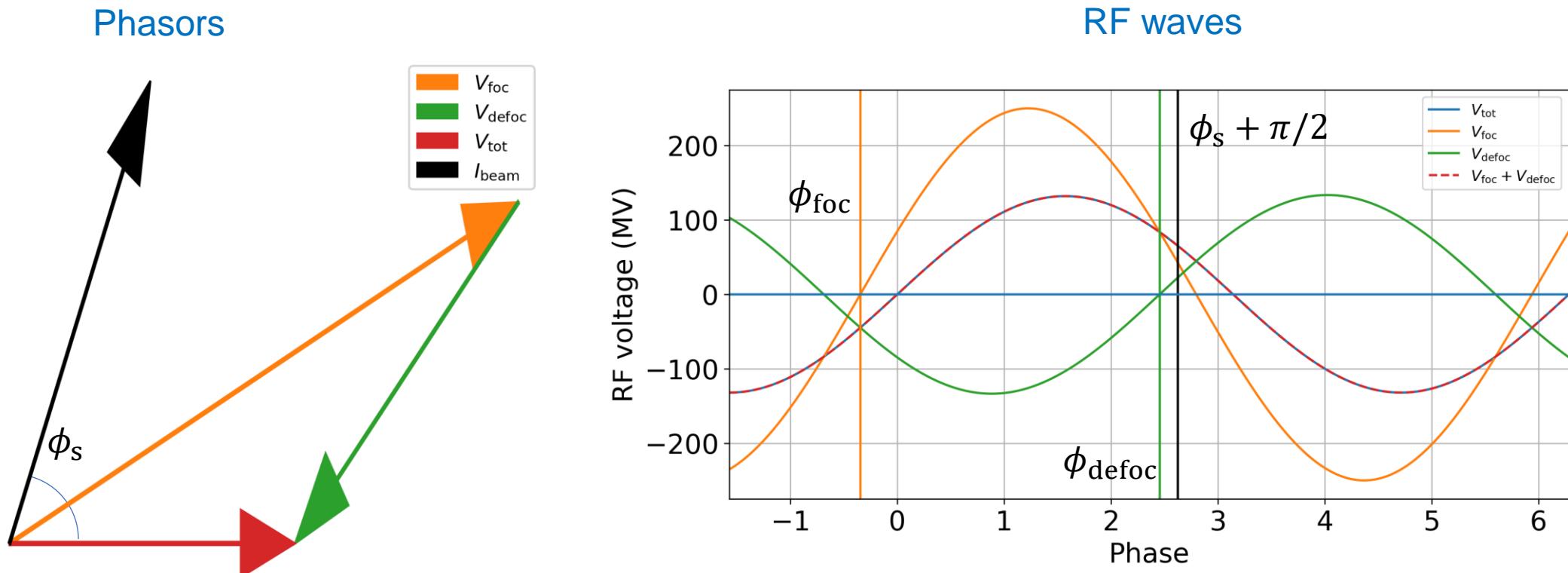
Can the voltage per cavity be increased for Z mode?



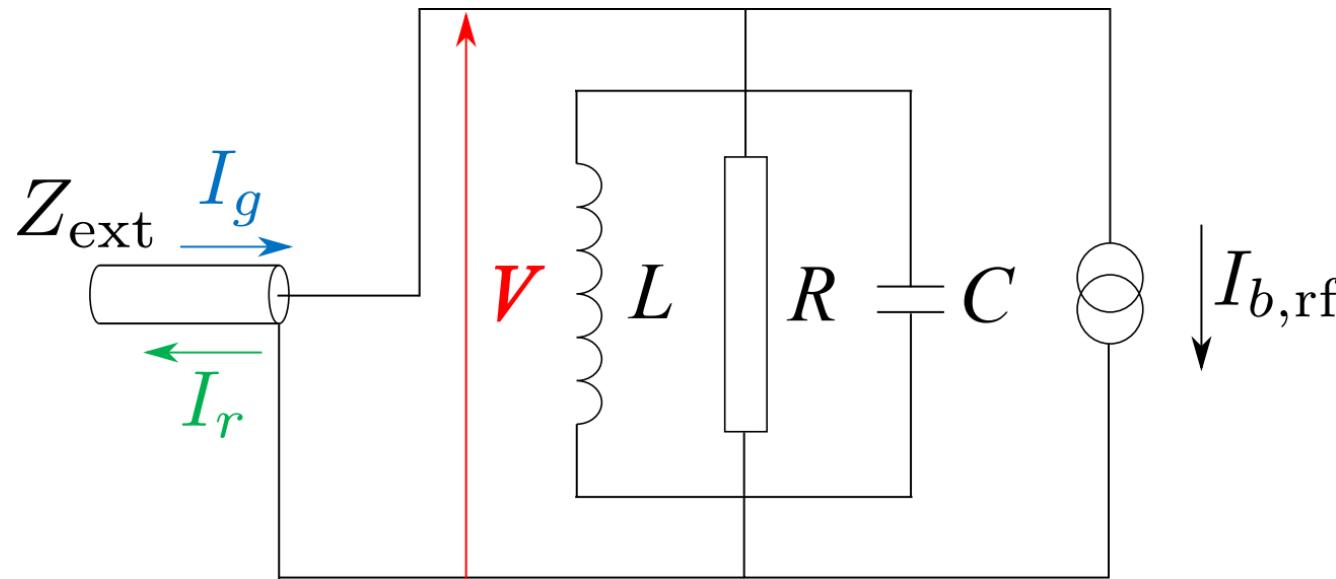
# Reverse phase operation

Reverse phase operation (RPO) mode allows increasing RF cavity voltage ([Y. Morita et al., SRF, 2009](#))

- Experimentally verified with high beam loading in KEKB ([Y. Morita et al., IPAC, 2010](#))
- Baseline solution for EIC ESR ([e.g., J. Guo et al., IPAC, 2022](#))



# Beam loading model: main equation



Generator current

$$I_g = \frac{V}{2(R/Q)} \left( \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{I_{b,\text{rf}}}{2}$$

Generator power

$$P_g = \frac{1}{2} Z_{\text{ext}} |I_g|^2 = \frac{1}{2} (R/Q) Q_{\text{ext}} |I_g|^2$$

Fixed parameters are  $V$ ,  $(R/Q)$ ,  $Q_0$ ,  $\omega_{\text{rf}}$ ,  $I_{b,\text{rf}}$ , while  $V$ ,  $\Delta\omega$ , and  $Q_{\text{ext}}$  can be adjusted

# Derivations for arbitrary cavity phase (1/)

Generator current  $I_g = \frac{V}{2(R/Q)} \left( \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{I_{b,\text{rf}}}{2}$

Complex quantities:  $I_g$ ,  $V$ , and  $I_{b,\text{rf}}$   $\rightarrow$   $I_g = |I_g|e^{i\phi_L}$ ,  $V = |V_{\text{cav}}|e^{i\phi_c}$ ,  $I_{b,\text{rf}} = |F_b|I_{b,\text{dc}}e^{-i\phi_s}$

$F_b = 2 \frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\text{rf}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}}$

$$|I_g|e^{i\phi_L} = \frac{|V_{\text{cav}}|e^{i\phi_c}}{2(R/Q)} \left( \frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{|F_b|I_{b,\text{dc}}e^{-i\phi_s}}{2}$$

$\times e^{-i\phi_c}$

$$|I_g|e^{i\phi_L-i\phi_c} = \frac{|V_{\text{cav}}|}{2(R/Q)} \left( \frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{|F_b|I_{b,\text{dc}}e^{-i\phi_s-i\phi_c}}{2}$$

Then splitting in real and imaginary parts:

# Derivations for arbitrary cavity phase (2/2)

$$|I_g|e^{i\phi_L - i\phi_c} = \frac{|V_{\text{cav}}|}{2(R/Q)Q_{\text{ext}}} + \frac{|F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2} - i \left[ \frac{|V_{\text{cav}}| \Delta\omega}{(R/Q)\omega_{\text{rf}}} + \frac{|F_b|I_{b,\text{dc}} \sin(\phi_s + \phi_c)}{2} \right]$$

$$P_g = \frac{1}{2}(R/Q)Q_{\text{ext}}|I_g|^2$$

$$\begin{aligned} &= \frac{1}{2}(R/Q)Q_{\text{ext}} \left[ \frac{|V_{\text{cav}}|}{2(R/Q)Q_{\text{ext}}} + \frac{|F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2} \right]^2 \rightarrow \text{Minimized for } Q_{\text{ext,opt}} = \frac{|V_{\text{cav}}|}{|F_b|(R/Q)I_{b,\text{dc}} \cos(\phi_s + \phi_c)} \\ &\quad + \frac{1}{2}(R/Q)Q_{\text{ext}} \left[ \frac{|V_{\text{cav}}| \Delta\omega}{(R/Q)\omega_{\text{rf}}} + \frac{|F_b|I_{b,\text{dc}} \sin(\phi_s + \phi_c)}{2} \right]^2 \rightarrow = 0 \text{ for } \Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}} \sin(\phi_s + \phi_c)}{2|V_{\text{cav}}|} \end{aligned}$$

Setting  $\phi_c = 0$  recovers classical equations for optimal parameters  
 Adjusting  $\phi_c$ ,  $Q_{\text{ext,opt}}$  can be modified to meet certain constraints

The minimum power

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2} = \frac{P_{\text{SR}}}{N_{\text{tot}}}$$

# Reverse phasing mode equations

## Constraints:

- The same  $Q_{\text{ext,opt}}$  for all cavities to avoid a movable fundamental power coupler design
- The same  $P_{g,\text{opt}}$  to have the identical power sources and uniform power distribution (role of variations is under study)

→ Cavity voltage must be the same for all cavities:  $\cos(\phi_s + \phi_{\text{foc}}) = \cos(\phi_s + \phi_{\text{defoc}}) \rightarrow \phi_{\text{foc}} = -2\phi_s - \phi_{\text{defoc}}$

## Preservation of energy gain

$$N_{\text{foc}}|V_{\text{cav}}| \cos(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}| \cos(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \cos \phi_s$$

## Preservation of synchrotron tune

$$N_{\text{foc}}|V_{\text{cav}}| \sin(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}| \sin(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \sin \phi_s$$

$$|V_{\text{cav}}| = \frac{V_{\text{tot}}}{N_{\text{tot}}} \sqrt{\frac{U_0^2}{V_{\text{tot}}^2} + \left(1 - \frac{U_0^2}{V_{\text{tot}}^2}\right) \frac{N_{\text{tot}}^2}{(N_{\text{foc}} - N_{\text{defoc}})^2}}$$

See, also [A. Blednykh et al, EIC-ADD-TN-33, 2022](#)

The aim is to keep  $V_{\text{cav}}$ ,  $P_{g,\text{opt}}$ , and  $Q_{\text{ext,opt}}$  for Z, W, and H modes

$$Q_{\text{ext,opt}} = \frac{|V_{\text{cav}}|}{|F_b|(R/Q)I_{b,\text{dc}} \cos(\phi_s + \phi_c)}$$

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}| |F_b| I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2}$$

# Preliminary results

Total number of cavities 132, change wrt baseline 0.00 %

Number of focusing cavities 71.0, number of defocusing 61.0

RF power per cavity 378.79 kW, change wrt baseline 0.00 %

Q\_L opt 9.21e+05 for V\_cav 7.95 MV change to baseline -0.087 %

V\_Z actual 0.088 GV, change wrt baseline 12.000 %

V\_W actual 1.049 GV, change wrt baseline 4.908 %

V\_H actual 2.098 GV, change wrt baseline 0.391 %

Q\_s Z: computed 0.0311, change wrt baseline 7.54 %

Q\_s W: computed 0.0833, change wrt baseline 2.86 %

Q\_s H: computed 0.0343, change wrt baseline 0.93 %

RF acceptance Z: computed 1.20, change wrt baseline 14.20 %

RF acceptance W: computed 3.50, change wrt baseline 4.57 %

RF acceptance H: computed 2.10, change wrt baseline 2.56 %

Total number of cavities 152, change wrt baseline 15.15 %

Number of focusing cavities 81.0, number of defocusing 71.0

RF power per cavity 328.95 kW, change wrt baseline -13.16 %

Q\_L opt 7.93e+05 for V\_cav 6.88 MV, change wrt baseline -13.571 %

V\_Z actual 0.079 GV, change wrt baseline 0.000 %

V\_W actual 1.045 GV, change wrt baseline 4.500 %

V\_H actual 2.090 GV, change wrt baseline 0.000 %

Q\_s Z: computed 0.0289, change wrt baseline 0.00 %

Q\_s W: computed 0.0832, change wrt baseline 2.62 %

Q\_s H: computed 0.0340, change wrt baseline 0.00 %

RF acceptance Z: computed 1.05, change wrt baseline 0.00 %

RF acceptance W: computed 3.49, change wrt baseline 4.20 %

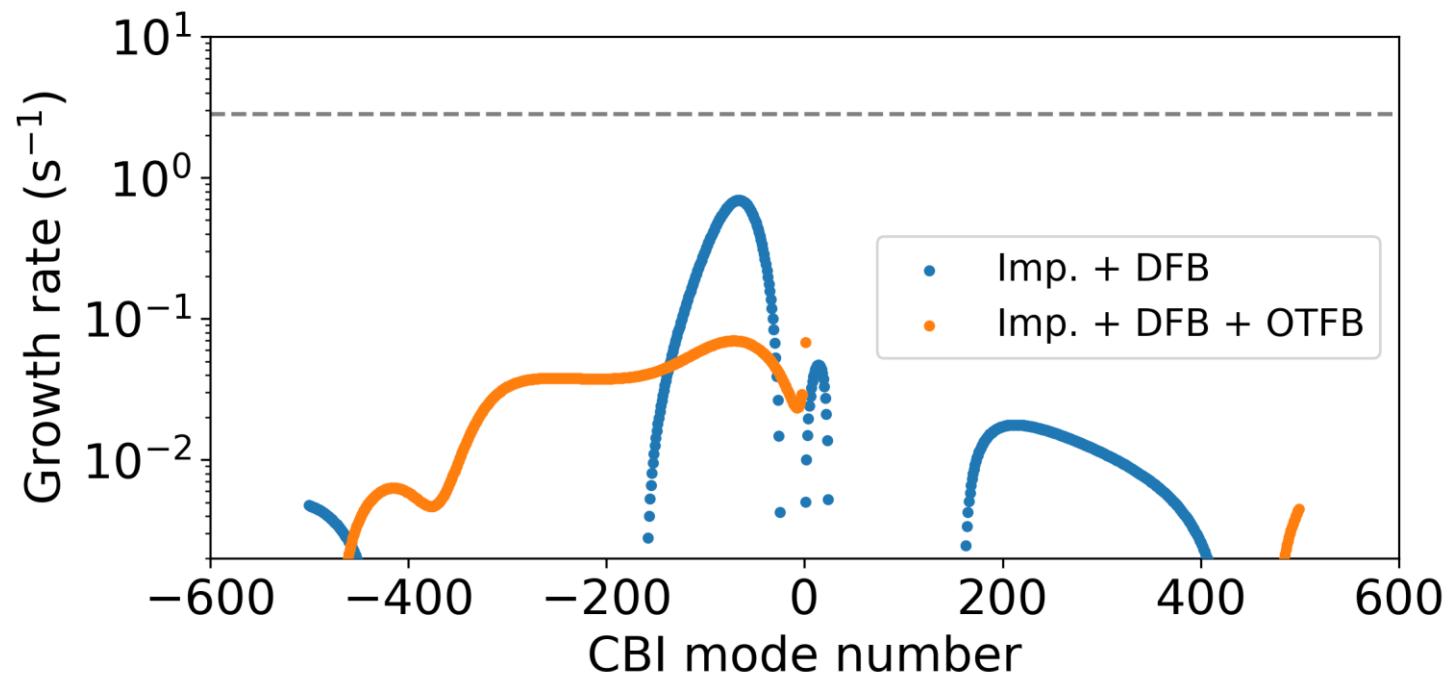
RF acceptance H: computed 2.05, change wrt baseline 0.00 %

→ RPO is under evaluation potentially allowing for the same optimal quality factor for Z, W, and H modes

Thank you for your attention!

# Backup slides

# Coupled-bunch instability due to fundamental mode



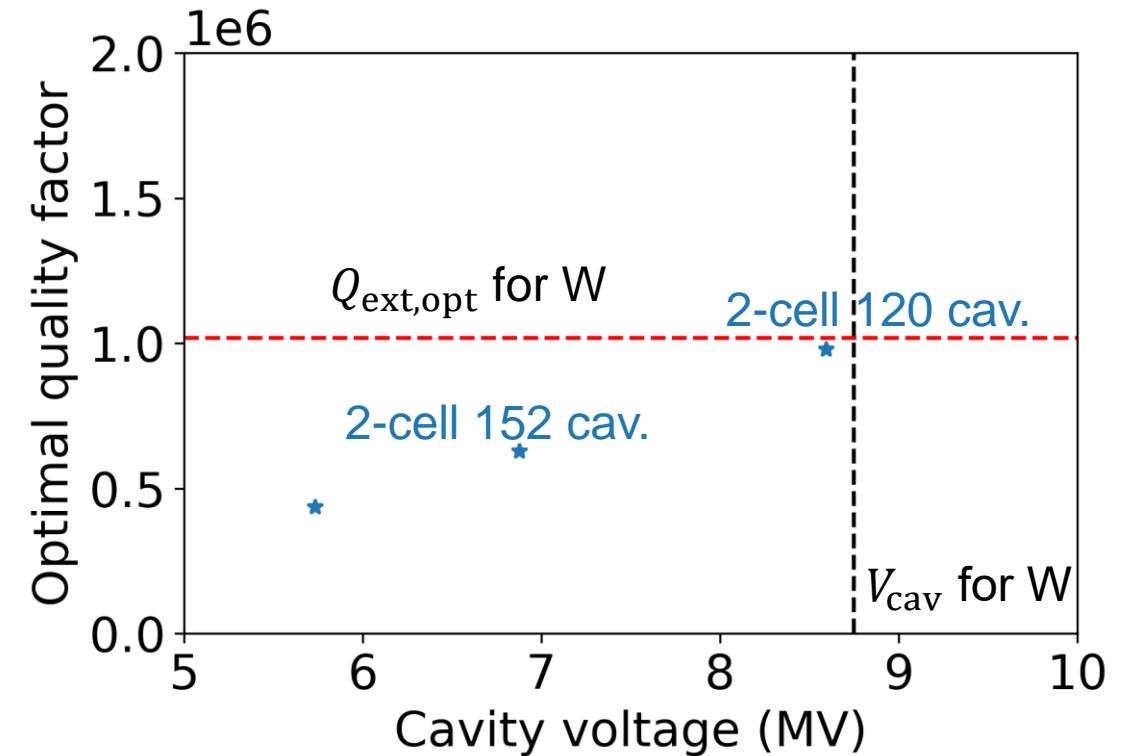
A factor of 4 margin with DRF only, ~40 with additional OTFB

# Preliminary results

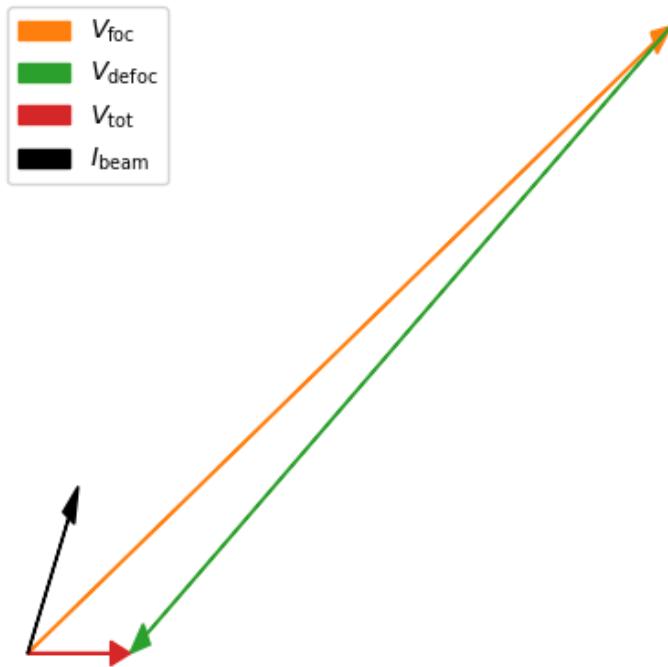
$$|V_{\text{cav}}| = \frac{V_{\text{tot}}}{N_{\text{tot}}} \sqrt{\frac{U_0^2}{V_{\text{tot}}^2} + \left(1 - \frac{U_0^2}{V_{\text{tot}}^2}\right) \frac{N_{\text{tot}}^2}{(N_{\text{foc}} - N_{\text{defoc}})^2}}$$

Optimal quality factor with RPO

$$Q_{\text{ext,opt}} = \frac{V_{\text{cav}}^2 N_{\text{tot}}}{V_{\text{tot}} (R/Q) |F_b| I_{b,\text{dc}} \cos \phi_s}$$



→ RPO is under evaluation potentially allowing for the same optimal quality factor for Z, W, and H modes



# Reverse phasing mode equations

Constraints:  $|V_{\text{cav}}|$  and  $P_{g,\text{opt}}$  are the same for focusing and defocusing cavities  
 $\rightarrow \cos(\phi_s + \phi_{\text{foc}}) = \cos(\phi_s + \phi_{\text{defoc}}) \rightarrow \phi_{\text{foc}} = -2\phi_s - \phi_{\text{defoc}}$

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}| |F_b| I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2}$$

Preservation of energy gain

$$N_{\text{foc}} |V_{\text{cav}}| \cos(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}} |V_{\text{cav}}| \cos(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \cos \phi_s$$

Preservation of synchrotron tune

$$N_{\text{foc}} |V_{\text{cav}}| \sin(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}} |V_{\text{cav}}| \sin(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \sin \phi_s$$

RPO

Optimal quality factor

$$Q_{\text{ext,opt}} = \frac{V_{\text{cav}}^2 N_{\text{tot}}}{V_{\text{tot}} (R/Q) |F_b| I_{b,\text{dc}} \cos \phi_s}$$

Classical

$$Q_{\text{ext,opt}} = \frac{V_{\text{cav}}}{|F_b| (R/Q) I_{b,\text{dc}} \cos \phi_s}$$

Optimal detuning

$$\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}} (R/Q) |F_b| I_{b,\text{dc}}}{2 V_{\text{cav}}} \sqrt{1 - \frac{\cos^2 \phi_s V_{\text{tot}}^2}{V_{\text{cav}}^2 N_{\text{tot}}^2}}$$

$$\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}} (R/Q) |F_b| I_{b,\text{dc}} \sin \phi_s}{2 V_{\text{cav}}}$$