Top-quark charge asymmetry at hadron colliders within the SM

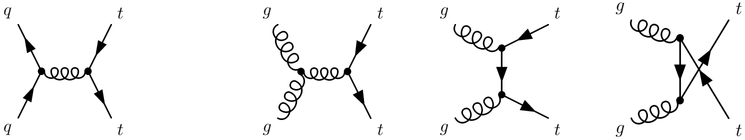
Ben Pecjak

Johannes Gutenberg Universität Mainz

Top 2011 Workshop, September 29, 2011

Introduction

Born level QCD graphs for $pp(\bar{p}) \to t\bar{t}X$



$$\frac{d\sigma_{q\bar{q}}^{\text{Born}}}{d\cos\theta_t} = f_{q\bar{q}}(\cos^2\theta_t); \qquad \frac{d\sigma_{gg}^{\text{Born}}}{d\cos\theta_t} = f_{gg}(\cos^2\theta_t)$$

Implies

- FB asymmetries are small in SM, so the Tevatron measurements are a legitimate hint at new physics
- quantifying "small in SM" is challenging but important

Outline

1) FB asymmetry at Tevatron

- total and binned FB asymmetries in inclusive $t\bar{t}X$ production
- FB asymmetry in $t\bar{t}X$ + jet
- leptonic asymmetries

2) Charge asymmetry at the LHC

- forward and central charge asymmetries
- CMS and ATLAS definitions
- other proposals

FB asymmetry definitions for Tevatron

1) Top-quark asymmetry in frame i

$$A^i_{\mathrm{FB}} = \frac{N_t(y^i_t > 0) - N_t(y^i_t < 0)}{N_t(y^i_t > 0) + N_t(y^i_t < 0)}$$

- frame-dependent quantity, usually measured in $p\bar{p}$ frame
- at Tevatron $N_t(y) = N_{\bar{t}}(-y)$ so FB asymmetry = charge asymmetry

2) Pair asymmetry (observe $t\bar{t}$ pair with $\Delta y = y_t - y_{\bar{t}}$)

$$A^{t\bar{t}}_{\mathrm{FB}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}$$

- Lorentz invariant, but same as top-quark asymmetry in $t\bar{t}$ frame

Asymmetric and symmetric cross sections

$$A^i_{\rm FB} = \frac{N_t(y^i_t > 0) - N_t(y^i_t < 0)}{N_t(y^i_t > 0) + N_t(y^i_t < 0)} \equiv \frac{\sigma^i_A}{\sigma^i_S}$$

In a QCD calculation

$$\frac{\sigma_A}{\sigma_S} = \frac{\left[\int\limits_{y_t>0} \frac{d\sigma}{dy_t} - \int\limits_{y_t<0} \frac{d\sigma}{dy_t}\right]}{\left[\int\limits_{y_t>0} \frac{d\sigma}{dy_t} + \int\limits_{y_t<0} \frac{d\sigma}{dy_t}\right]} = \frac{\alpha_s^3\sigma_A^{(0)} + \alpha_s^4\sigma_A^{(1)} + \dots}{\alpha_s^2\sigma_S^{(0)} + \alpha_s^3\alpha_s\sigma_S^{(1)} + \dots}$$

- FB asymmetry is ratio of asymmetric (σ_A) and symmetric (σ_S) cross sections
- must also decide whether or not to expand the ratio in α_s, which can make a (sometimes significant) numerical difference

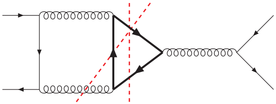
Components of A_{FB} calculations within SM

1) leading QCD contributions calculated in [Kuhn,Rodrigo 1998]

2) (mixed QCD)-electroweak corrections dealt with in [Kuhn, Rodrigo 1998], [Bernreuther, Si 2010], [Hollik, Pagani 2011]

3) higher-order QCD contributions estimated with soft-gluon resummation

- [Almeida, Sterman, Vogelsang 2008] at NLL for $A_{\mathrm{FB}}^{t\bar{t}}$
- [Ahrens, Ferroglia, Neubert, BP, Yang 2010, 2011] at NNLL for $A_{\mathrm{FB}}^{t\bar{t}}$, $A_{\mathrm{FB}}^{p\bar{p}}$
- [Kidonakis 2011] at NNLL for $A_{\mathrm{FB}}^{p\bar{p}}$



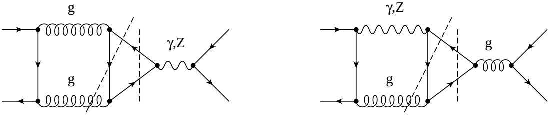
- diagrams involving interference of $C = 1$ and $C = -1$ amplitudes are odd under $t \leftrightarrow \bar{t}$ and generate asymmetry [Kuhn, Rodrigo '98]
- contributions arise from $q\bar{q}$ (dominant) and qg (suppressed) channels
- gg channel charge symmetric to all orders

$$A_{\rm FB}^{\rm QCD} = \frac{\sigma_A}{\sigma_S} = \frac{\alpha_s^3 \sigma_{A,q\bar{q}}^{(0)} + \alpha_s^3 \sigma_{A,qg}^{(0)} + \alpha_s^4 \sigma_{A,q\bar{q}}^{(1)} + \dots}{\alpha_s^2 \sigma_S^{(0)} + \alpha_s^3 \sigma_S^{(1)} + \dots} = \alpha_s A_{\rm FB}^{(0)} + \dots$$

- will call $A_{\rm FB}^{(0)}$ "NLO", since uses $d\sigma$ at NLO

Electroweak corrections

EW corrections also contribute and interfere with QCD diagrams



(a) (b)

$$A_{\mathrm{FB}} = \frac{\alpha^2 \tilde{N}_0 + \alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha_s^4 N_2 + \ldots}{\alpha^2 \tilde{D}_0 + \alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^2 \alpha \tilde{D}_1 + \ldots} = \alpha_s \frac{N_1}{D_0} + \alpha \frac{\tilde{N}_1}{D_0} + \frac{\alpha^2}{\alpha_s^2} \frac{\tilde{N}_0}{D_0} + \ldots$$

- (different) parts of the QCD/EW interference and pure EW effects known [Kuhn, Rodrigo 1998],[Bernreuther, Si 2010], [Hollik, Pagani 2011]
- [Hollik, Pagani 2011] include extra photonic corrections compared to [Bernreuther, Si 2010], but omit $b\bar{b} \to t\bar{t}$ corrections: these differences are important numerically

Higher-order QCD corrections and soft gluon resummation

$$A_{\rm FB} = \alpha_s A_{\rm FB}^{(0)} + \alpha_s^2 (A_{\rm FB}^{(1)} = ?) + \ldots$$

Soft gluon resummation seeks to obtain the dominant logarithmic pieces of the unknown higher-order corrections such as $A_{\rm FB}^{(1)}$

Recently, two types of differential cross sections were obtained at NNLL

- $d^2\sigma/dM_{t\bar{t}}d\Delta y \leftrightarrow A_{\rm FB}^{t\bar{t}}$ [Ahrens et. al. 2010]
- $d^2\sigma/dp_T dy_t \leftrightarrow A_{\rm FB}^{p\bar{p}}$ [Ahrens et. al. 2011, Kidonakis 2011]

Proper method of estimating uncertainties in resummed calculations is a subject of debate, but these resummed calculations give insight into higher order QCD corrections

NUMERICAL RESULTS

Results for total FB asymmetries

Theory: (NLO and NLO+NNLL use $\mu = m_t = 173.1$ GeV, MSTW2008 90% CL)

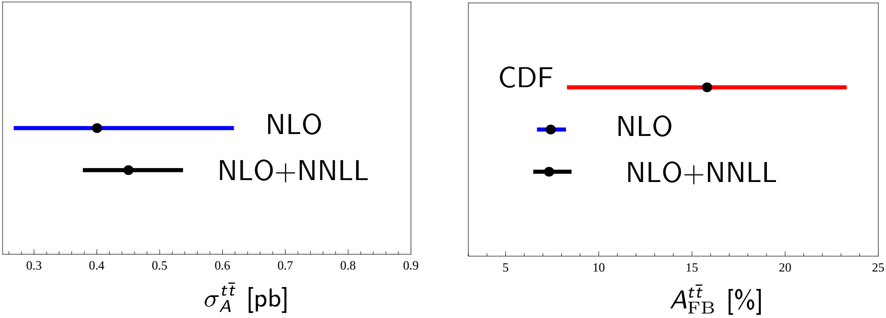
	$A_{\rm FB}^{t\bar{t}}$ [%]	$A_{\rm FB}^{p\bar{p}}$ [%]
NLO	$7.32^{+0.69+0.18}_{-0.59-0.19}$	$4.81^{+0.45+0.13}_{-0.39-0.13}$
NLO+NNLL [Ahrens et. al.'11]	$7.24^{+1.04+0.20}_{-0.67-0.27}$	$4.88^{+0.20+0.17}_{-0.23-0.18}$
NNLO$_{\rm approx}$ [Kidonakis '11]		$5.2^{+0.0}_{-0.6}$
EW'/NLO' ($\mu = m_t$) [Bernreuther, Si '10]	0.05	0.04
EW/NLO ($\mu = m_t$) [Hollik, Pagani '10]	0.22	0.22

CDF with 5.3 fb^{-1}: $A_{\rm FB}^{p\bar{p}} = (15.0 \pm 5.5)\%$, $A_{\rm FB}^{t\bar{t}} = (15.8 \pm 7.5)\%$

- theory and experiment agree at roughly $1(2)\sigma$ for $A_{\rm FB}^{t\bar{t}}(A_{\rm FB}^{p\bar{p}})$
- soft-gluon resummation moderate effect for $\mu_f = m_t$
- EW corrections deserve attention

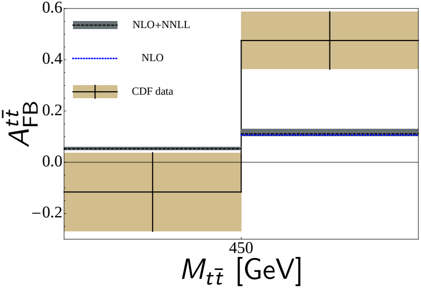
Scale uncertainties in $\sigma_A^{t\bar{t}}$ and $A_{\rm FB}^{t\bar{t}}$

$m_t = 173.1$ GeV, $m_t/2 < \mu_f = \mu_r < 2m_t$, MSTW2008 90% CL



- resummation roughly halves scale dependence in σ_A and σ compared to NLO, but scale dependence of $A_{\rm FB}$ somewhat larger
- NLO calculation can easily underestimate uncertainties of higher orders, although at present the experimental uncertainty is more important

Binned $A_{\rm FB}^{t\bar{t}}$ as function of $M_{t\bar{t}}$

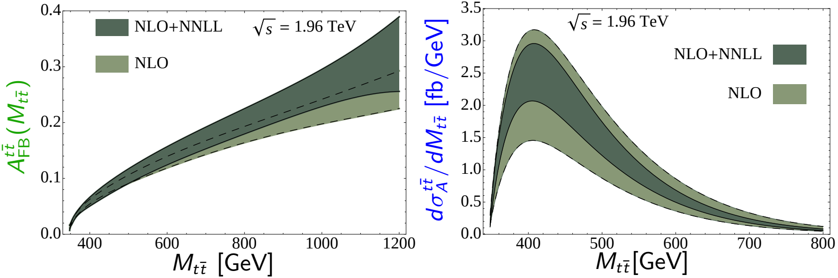


(NLO and NLO+NNLL use $\mu = m_t = 173.1$ GeV, MSTW2008 90% CL)

$A_{\rm FB}^{t\bar{t}}$ [%]	$M_{t\bar{t}} < 450$ GeV	$M_{t\bar{t}} > 450$ GeV
NLO	$5.3^{+0.3+0.1}_{-0.4-0.1}$	$10.6^{+1.1+0.3}_{-0.8-0.1}$
NLO+NNLL [Ahrens et al]	$5.2^{+0.7+0.1}_{-0.5-0.0}$	$11.1^{+1.9+0.3}_{-1.0-0.0}$
EW/NLO ($\mu = m_t$) [Hollik et al]	–	0.23

- neither soft-gluon nor EW effects produce large corrections at high $M_{t\bar{t}}$

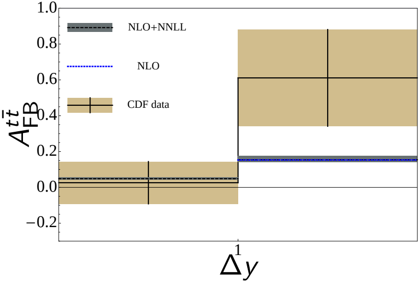
Invariant-mass dependent asymmetry



$$A_{FB}^{t\bar{t}}(M_{t\bar{t}}) = \frac{\left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_{\Delta y>0} - \left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_{\Delta y<0}}{\left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_{\Delta y>0} + \left(\frac{d\sigma}{dM_{t\bar{t}}}\right)_{\Delta y<0}} \equiv \frac{\frac{d\sigma_A^{t\bar{t}}}{dM_{t\bar{t}}}}{\frac{d\sigma}{dM_{t\bar{t}}}}$$

- a two-bin analysis below and above $M_{t\bar{t}} = 450$ GeV roughly divides the number of asymmetric events in two

BINNED $A_{\rm FB}^{t\bar{t}}$ AS FUNCTION OF Δy

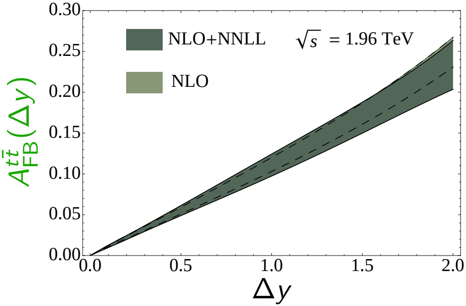


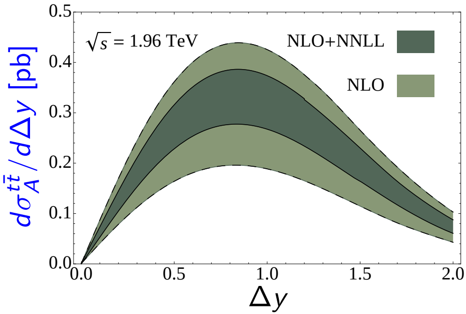
(NLO and NLO+NNLL use $\mu = m_t = 173.1$ GeV, MSTW2008 90% CL)

$A_{\rm FB}^{t\bar{t}}$ [%]	$\Delta y < 1$	$\Delta y > 1$
NLO	$4.86^{+0.42}_{-0.35}$	$15.29^{+1.26}_{-1.11}$
NLO+NNLL [Ahrens et al]	$4.77^{+0.39}_{-0.35}$	$14.59^{+2.16}_{-1.30}$
EW/NLO ($\mu = m_t$) [Hollik et al]	–	0.22

- neither soft-gluon nor EW effects give large corrections at high Δy

Rapidity-dependent FB asymmetry





$$A_{FB}^{t\bar{t}}(\Delta y) = \frac{\left(\frac{d\sigma}{d\Delta y}\right) - \left(\frac{d\sigma}{d\bar{\Delta}y}\right)\Big|_{\bar{\Delta}y=-\Delta y}}{\left(\frac{d\sigma}{d\Delta y}\right) + \left(\frac{d\sigma}{d\bar{\Delta}y}\right)\Big|_{\bar{\Delta}y=-\Delta y}} \equiv \frac{\frac{d\Delta\sigma_{\rm FB}^{p\bar{p}}}{d\Delta y}}{\left(\frac{d\sigma}{d\Delta y}\right) + \left(\frac{d\sigma}{d\bar{\Delta}y}\right)\Big|_{\bar{\Delta}y=-\Delta y}}$$

- a two-bin analysis below and above $\Delta y = 1$ roughly divides the number of asymmetric events in two

ASYMMETRY IN $t\bar{t}X$ + JET

Differential cross section $t\bar{t}X$ + jet known at NLO
[Dittmaier, Uwer, Weinzierl '07], [Melnikov, Schulze '10]

For $p_T^{\rm jet} > 20$ GeV with theory errors estimated through $m_t/2 < \mu < 2m_t$:

$$A_{\rm FB}^{p\bar{p},{\rm LO}} = \text{-}7.7 \pm 0.1 \qquad A_{\rm FB}^{p\bar{p},{\rm NLO}} = \text{-}1.8^{+0.6}_{-0.3}$$

- large NLO corrections wash out the asymmetry and not estimated through scale variation of LO result
- Melnikov and Schulze argue that this is because of hard corrections first appearing at NLO, and that NNLO corrections should be small
- $A_{\rm FB}$ in $t\bar{t}X$ receives both hard and soft at leading order so this argument implies higher order corrections are of normal size

Leptonic observables

In real life, the top quarks decay (take dilepton as an example)

$$p\bar{p} \to t\bar{t}X \to \ell^+\ell^- + j_b + j_{\bar{b}} + \cdots + E_T^{\text{miss}}$$

Can also define lepton asymmetries

$$A^i_{\text{FB}} = \frac{N_{\ell^+}(y^i_{\ell^+} > 0) - N_{\ell^+}(y^i_{\ell^+} < 0)}{N_{\ell^+}(y^i_{\ell^+} > 0) + N_{\ell^+}(y^i_{\ell^+} < 0)}$$

The asymmetries (with acceptances) can be obtained with NLO+EW calculations [Bernreuther, Si '10] (in on-shell approx.)

$$A^{p\bar{p}}_{\text{FB}} = 3.4(5)\%, \quad A^{\ell^+\ell^-}_{\text{FB}} = 4.4(4)\%$$

- full NLO calculation with off-shell top-quarks shows corrections to on-shell approx. are small [Bevilacqua et. al. 2010]
- measurement of leptonic asymmetries with acceptances is a good consistency check on $t\bar{t}$-level results

Outline

1) FB asymmetry at Tevatron

- total and binned FB asymmetries in inclusive $t\bar{t}X$ production
- FB asymmetry in $t\bar{t}X$ + jet
- leptonic asymmetries

2) Charge asymmetry at the LHC

- forward and central charge asymmetries
- CMS and ATLAS definitions
- other proposals

Charge asymmetry at LHC

At LHC pp initial state is symmetric so $A_{\rm FB} = 0$

But $N_t(y) \neq N_{\bar{t}}(-y)$, so charge asymmetry is not A_{FB}, and even though

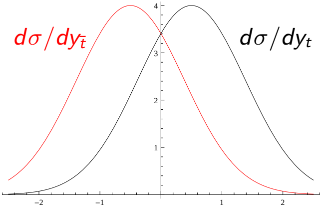
$$A^{pp}_{\rm charge} = \frac{N_t(y_t > 0) - N_{\bar{t}}(y_{\bar{t}} > 0)}{N_t(y_t > 0) + N_{\bar{t}}(y_{\bar{t}} > 0)} = 0$$

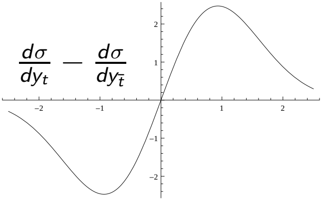
rapidity-distributions of t and $\bar{t}$ not the same locally, due to same partonic graphs which generate $A_{\rm FB}$ at Tevatron

- simplest way to study LHC charge asymmetries is with rapidity cuts (intuitive picture in next two slides)
- optimal way to study LHC charge asymmetry depends on theory model, integrated luminosity and/or $\sqrt{s}$, and systematic errors in different regions of phase space

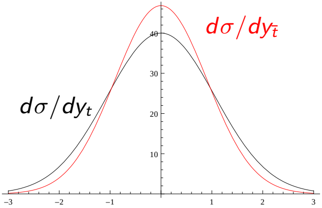
Rapidity distributions at Tevatron and LHC (schematic)

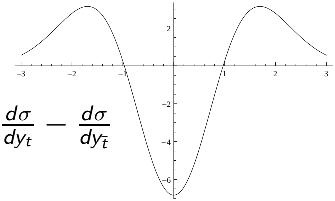
Tevatron





LHC



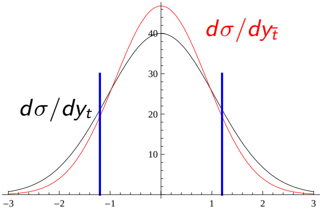


- total charge asymmetry vanishes at LHC, but asymmetry with cuts doesn't because t-quarks are more forward than $\bar{t}$ quarks [Kuhn, Rodrigo '98]

Central and forward charge asymmetries at LHC

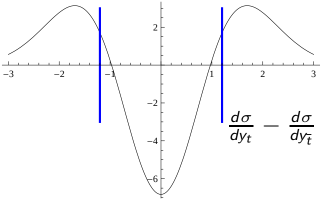
Central charge asymmetry

$$A_C(y_{\rm cut}) = \frac{N_t(|y| < y_{\rm cut}) - N_{\bar t}(|y| < y_{\rm cut})}{N_t(|y| < y_{\rm cut}) + N_{\bar t}(|y| < y_{\rm cut})}$$



Forward charge asymmetry

$$A_F(y_{\rm cut}) = \frac{N_t(|y| > y_{\rm cut}) - N_{\bar t}(|y| > y_{\rm cut})}{N_t(|y| > y_{\rm cut}) + N_{\bar t}(|y| > y_{\rm cut})}$$

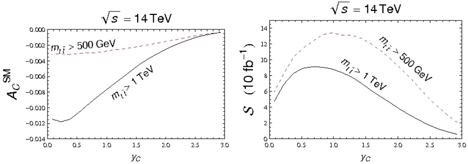


Reducing gg background

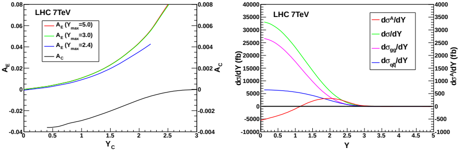
Can enhance charge asymmetries by making cuts which probe large x, where $q\bar{q}$ luminosity is higher compared to gg (high $M_{t\bar{t}}$, p_T, etc)

Two examples

- central charge asymmetry ($y < y_C$) with cut $M_{t\bar{t}} > M_{t\bar{t}}^{\min}$ [Ferrario and Rodgrigo 2008]



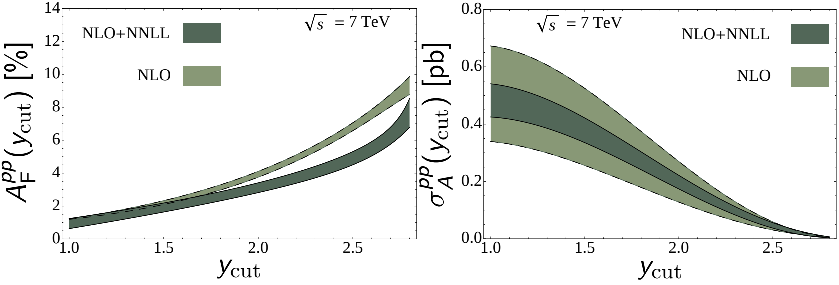
- forward asymmetry ($y_C < y < y^{\max}$) with large y_C [Xiao et. al. 2011]



QCD corrections to the forward asymmetry

$$A_F(y_{\rm cut}) = \frac{N_t(|y| > y_{\rm cut}) - N_{\bar{t}}(|y| > y_{\rm cut})}{N_t(|y| > y_{\rm cut}) + N_{\bar{t}}(|y| > y_{\rm cut})}$$

- QCD corrections can become important in corners of phase space



- NLO very sensitive to whether σ^A/σ^S expanded in α_s, NLO+NNLL more stable [Ahrens et. al. 2011]

The ATLAS/CMS charge asymmetry

ATLAS definition (similar in CMS but with pseudorapidity η)

$$A_C = \frac{N(|y_t| > |y_{\bar{t}}|) - N(|y_t| < |y_{\bar{t}}|)}{N(|y_t| > |y_{\bar{t}}|) + N(|y_t| < |y_{\bar{t}}|)}$$
$$= \frac{1}{\sigma} \times \int_0^{y_{\max}} d|y_t| \int_0^{|y_t|} d|y_{\bar{t}}| \left\{ \frac{d^2\sigma}{d|y_t|d|y_{\bar{t}}|} - \frac{d^2\sigma}{d|y_t|d|y_{\bar{t}}|}\bigg|_{|y_t|\leftrightarrow|y_{\bar{t}}|} \right\}$$

- probes antisymmetric part of cross section under $|y_t| \leftrightarrow |y_{\bar{t}}|$
- asymmetry is diluted by the gluon channel

$$\Rightarrow A_C^{\rm SM} \approx 1\%$$

- can enhance A_C with cuts to reduce gg background, for instance on minimum $\beta_t = |p^z_{t\bar{t}}|/E_{t\bar{t}}$ [Aguilar-Saavedra, Juste, Rubbo 2011]

Other proposals

- One-sided FB asymmetry [Wang, Xiao, Zhu 2010]

$$A_O^{FB} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}\bigg|_{P^z_{t\bar{t}} > P^z_{\rm cut}, M_{t\bar{t}} > M^{\rm cut}_{t\bar{t}}}$$

- FB asymmetry with redefined angle [Krohn, Liu, Shelton, Wang 2011]

$$\mathcal{A}^{t\bar{t}} = \frac{N(0 < \hat{\theta}_t < \pi/2) - N(\pi/2 < \hat{\theta}_t < \pi)}{N(0 < \hat{\theta}_t < \pi/2) + N(\pi/2 < \hat{\theta}_t < \pi)}$$

($\hat{\theta}_t$ production angle of t-quark in $t\bar{t}$ frame w.r.t direction of boost of $t\bar{t}$ system)

- many others. . .
- leptonic asymmetries

Which is best?

No single answer. Requires a systematic study at generator level rather than a simple parton analysis, and also depends on NP model

<u>Example:</u> study of A_C vs. A_F in lepton + jets channel, gg background reduced through high-p_T cut on jet

[Hewett, Shelton, Spannowsky, Tait, Takeuchi]

$\sqrt{s} = 7$ TeV	$(N_+ + N_-)$,	$(N_+ - N_-)$,	ΔN	$\mathcal{A}_{(F,C)10\text{fb}^{-1}}$	$S_{10\text{fb}^{-1}}$
<u>SM</u>					
$1.5 < \|y\| < 2.5$	163.8	1.43	12.8	0.0087 ± 0.025	0.35
$0 < \|y\| < 1.5$	1227.1	-1.43	35.0	-0.0012 ± 0.0090	0.13
<u>Z' model</u>					
$1.5 < \|y\| < 2.5$	140.3	10.55	11.8	0.075 ± 0.027	2.81
$0 < \|y\| < 1.5$	1054.5	-10.55	32.5	-0.010 ± 0.0097	1.03

Table: Expected number of events after an integrated luminosity of 1 fb^{-1}. The total cross section is normalized to 150 pb (NLO). We give the error for the resulting asymmetry and its significance, S, for 10 fb^{-1}.

Summary

Current predictions for $A_{\rm FB}$ at Tevatron not in good agreement with measurements. The SM calculations involve

- leading QCD contributions
- leading EW-QCD interference and pure EW
- higher order QCD estimated through soft gluon resummation

There are indications that SM asymmetry is stable under higher-order QCD corrections, although there is no substitute for calculating the next term in fixed order

Can probe the same physics at the LHC with charge asymmetries, but is harder

- no global direction in a *pp* collider
- *gg* contributions form a large background

Many proposals for LHC asymmetries exist in literature, optimal definition depends on integrated luminosity, systematic errors, and new physics model