Software for Muon Cooling

International
MUON Collider
Collaboration

C. T. Rogers
Rutherford Appleton Laboratory

Science & Technology Facilities Council
ISIS

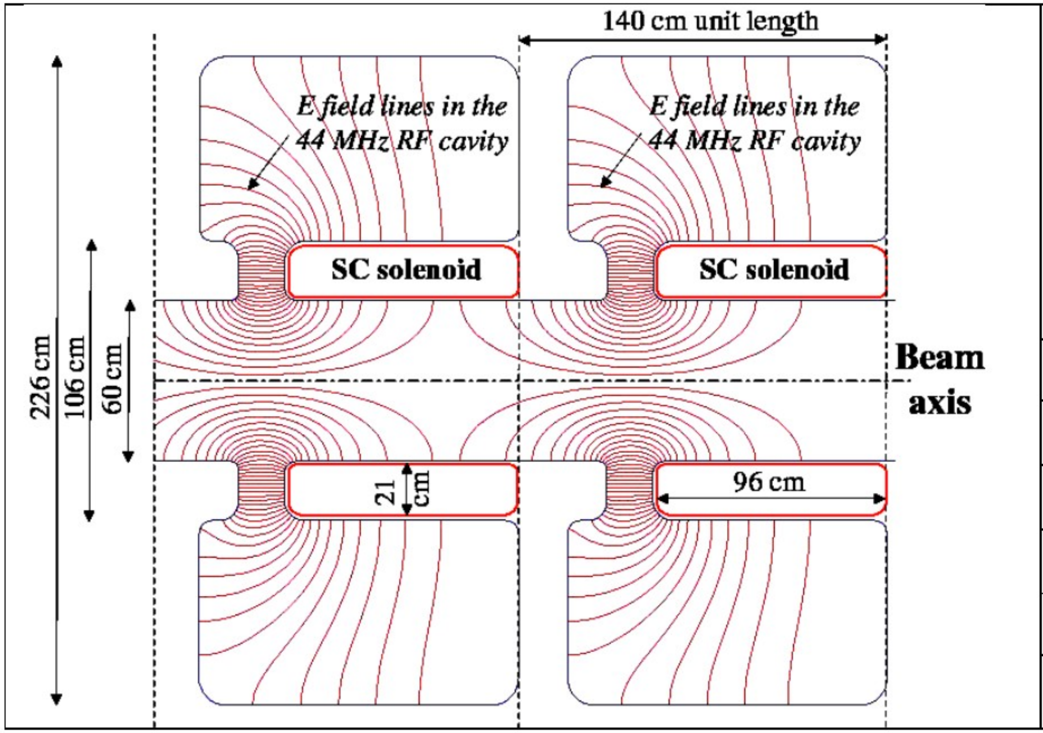
Origins

International
MUON Collider
Collaboration

- Start in the late 90s/early 2000s
- Interest in Neutrino Factory and Muon Collider
- Clear that muon capture and cooling were big challenges
- Requirement for challenging compact lattices
- Handling for multiple particle species
  - e.g. pions and muons
- Simulation codes had to handle
  - Overlapping fields
  - Decays
  - Materials
- Very brief/potted history of lattice development
  - Apologies for missed things
- Overview of modelling considerations
- Some important experimental results

CERN design - capture

The Study of a European Neutrino Factory Complex, Gruber et al. 2002

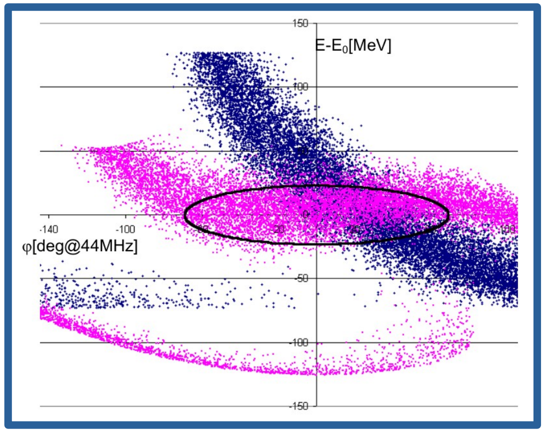


RF characteristics	
E_{0T}	2 MV/m
Z_{TT}	3 MΩ/m
R/Q	103 Ω
Filling time	296 μs
P_{PEAK}	1.86 MW/cavity
P_{MEAN} (50 Hz)	89 kW/m

Fig. 5.9 Sketch of a 44 MHz cavity with solenoid

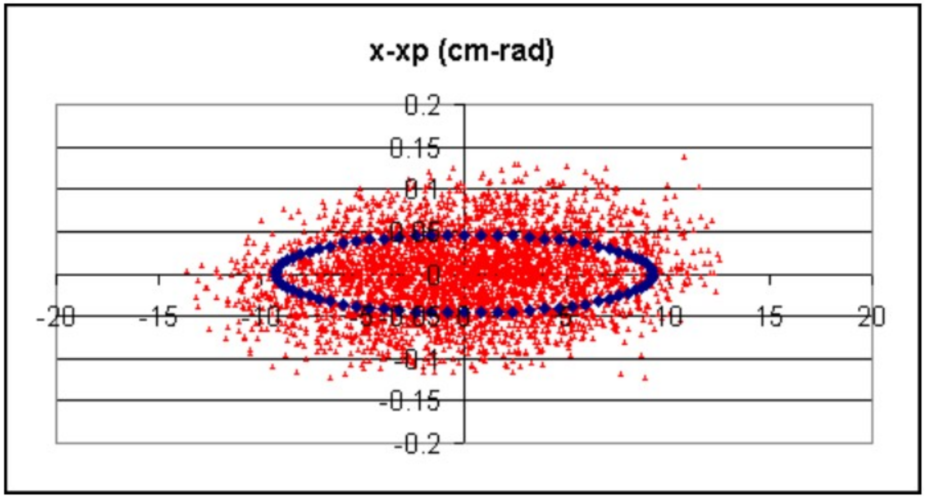
Tab. 5.1 Components of the 40–80 MHz scheme and their characteristics

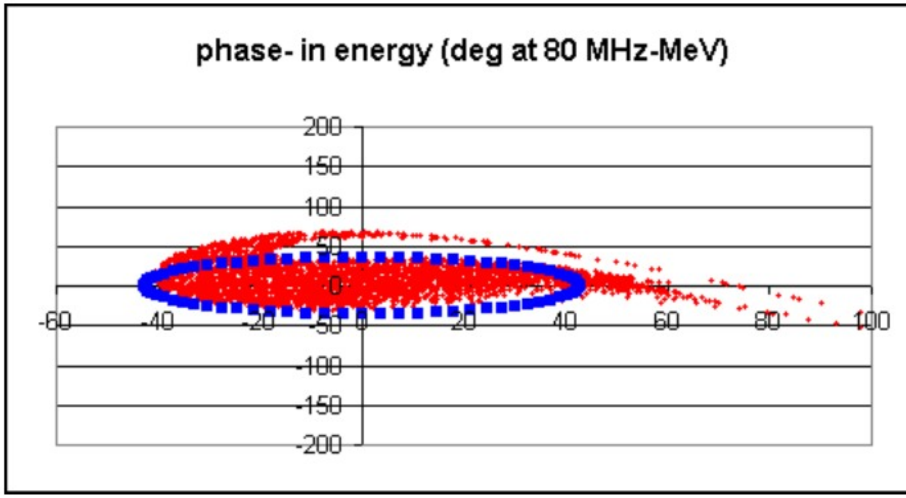
	Decay	Rotation	Cooling I	Accel	Cooling II	Accel
Length [m]	30	30	46	32	112	~450
Diameter [cm]	60	60	60	60	30	20
B-field [T]	1.8	1.8	2.0	2.0	5.0	5.0
Frequency [MHz]	–	44	44	44	88	88–220
Gradient [MV/m]	–	2	2	2	4	4–10
Kinetic energy [MeV]	–	200	200	200–280	280–300	300–3000



CERN design - cooling

The Study of a European Neutrino Factory Complex, Gruber et al. 2002

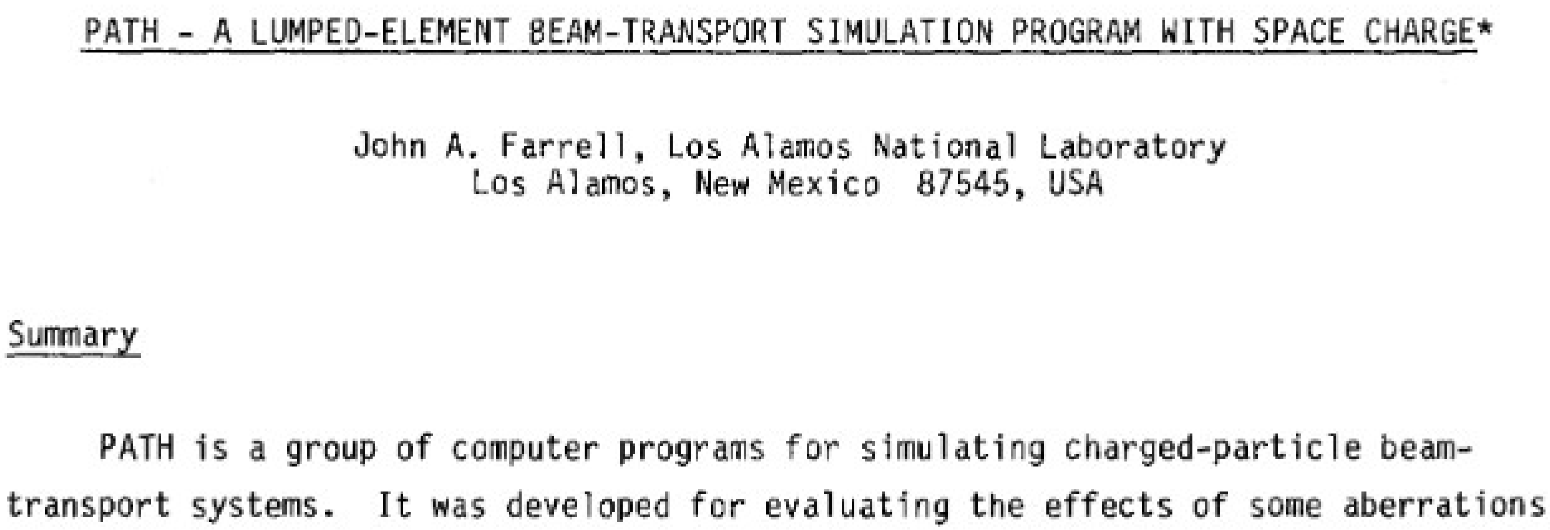




RF 24 cm gap 1 m long	RF 24 cm gap 1 m long	RF 24 cm gap 1 m long	RF 24 cm gap 1 m long	H_2 24 cm

Fig. 5.4 Cooling cell at 44 MHz, total length 4.24 m

CERN design - code



PATH - A LUMPED-ELEMENT BEAM-TRANSPORT SIMULATION PROGRAM WITH SPACE CHARGE*

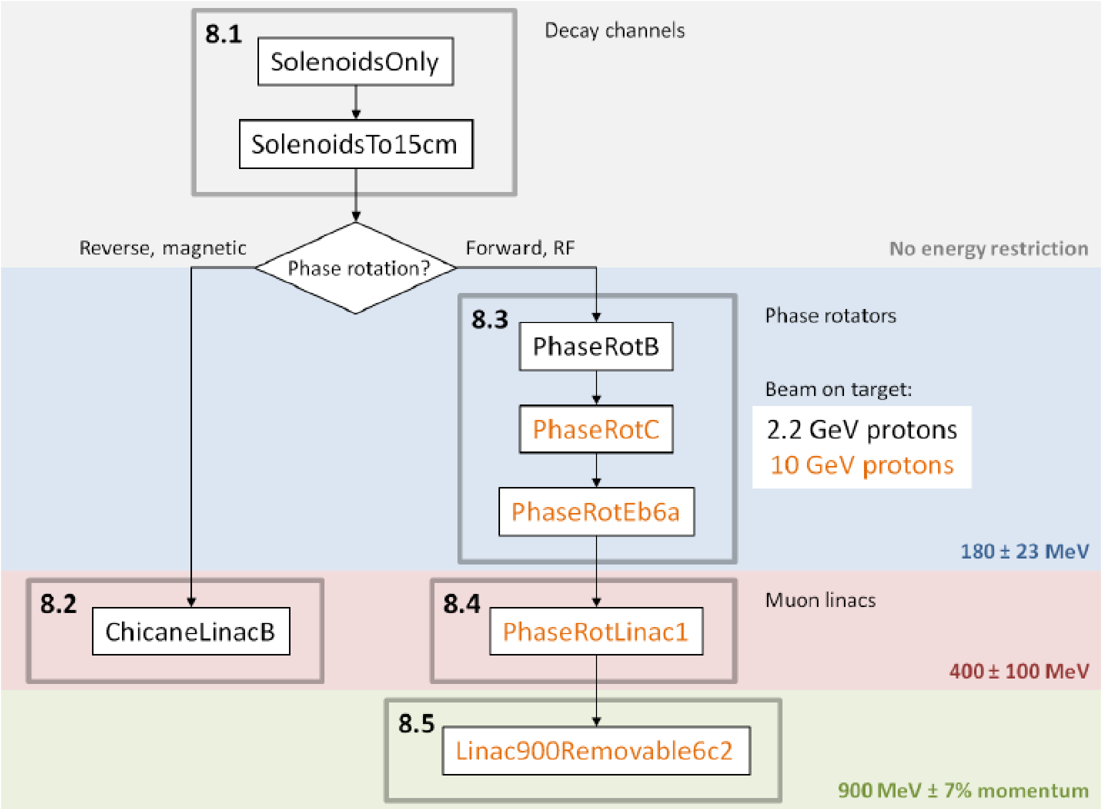
John A. Farrell, Los Alamos National Laboratory
Los Alamos, New Mexico 87545, USA

Summary

PATH is a group of computer programs for simulating charged-particle beam-transport systems. It was developed for evaluating the effects of some aberrations

- Simulated using **PATH**
  - Matrix code (to 3rd order)
  - With pion decay

UK design



Decay channels

8.1

SolenoidsOnly

SolenoidsTo15cm

Phase rotation?

Reverse, magnetic

Forward, RF

No energy restriction

Phase rotators

8.3

PhaseRotB

PhaseRotC

PhaseRotEb6a

Beam on target:

2.2 GeV protons

10 GeV protons

180 ± 23 MeV

Muon linacs

8.2

ChicaneLinacB

8.4

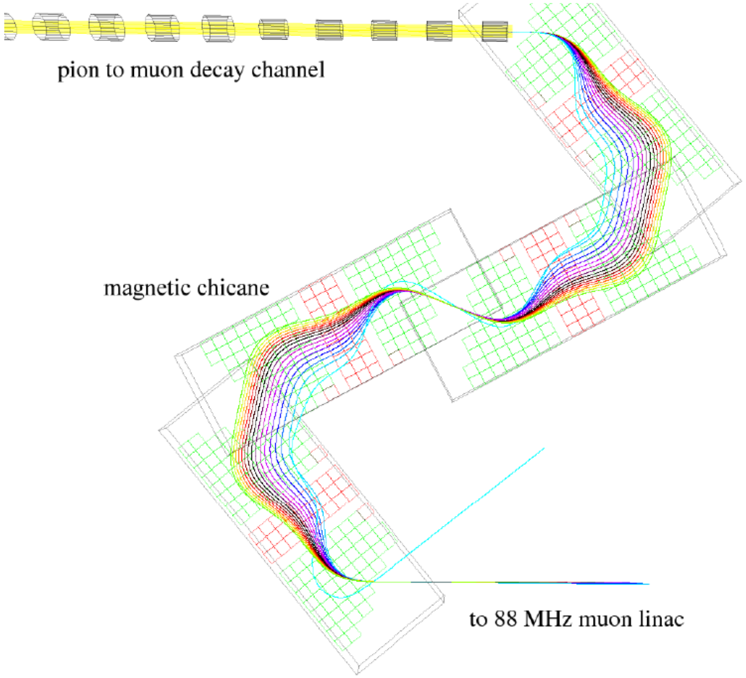
PhaseRotLinac1

400 ± 100 MeV

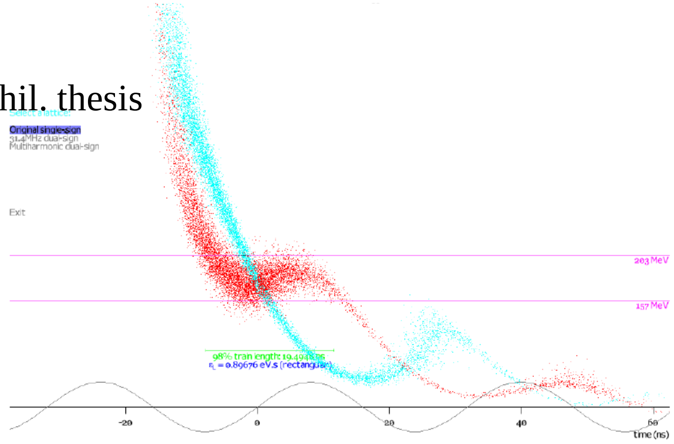
8.5

Linac900Removable6c2

900 MeV ± 7% momentum

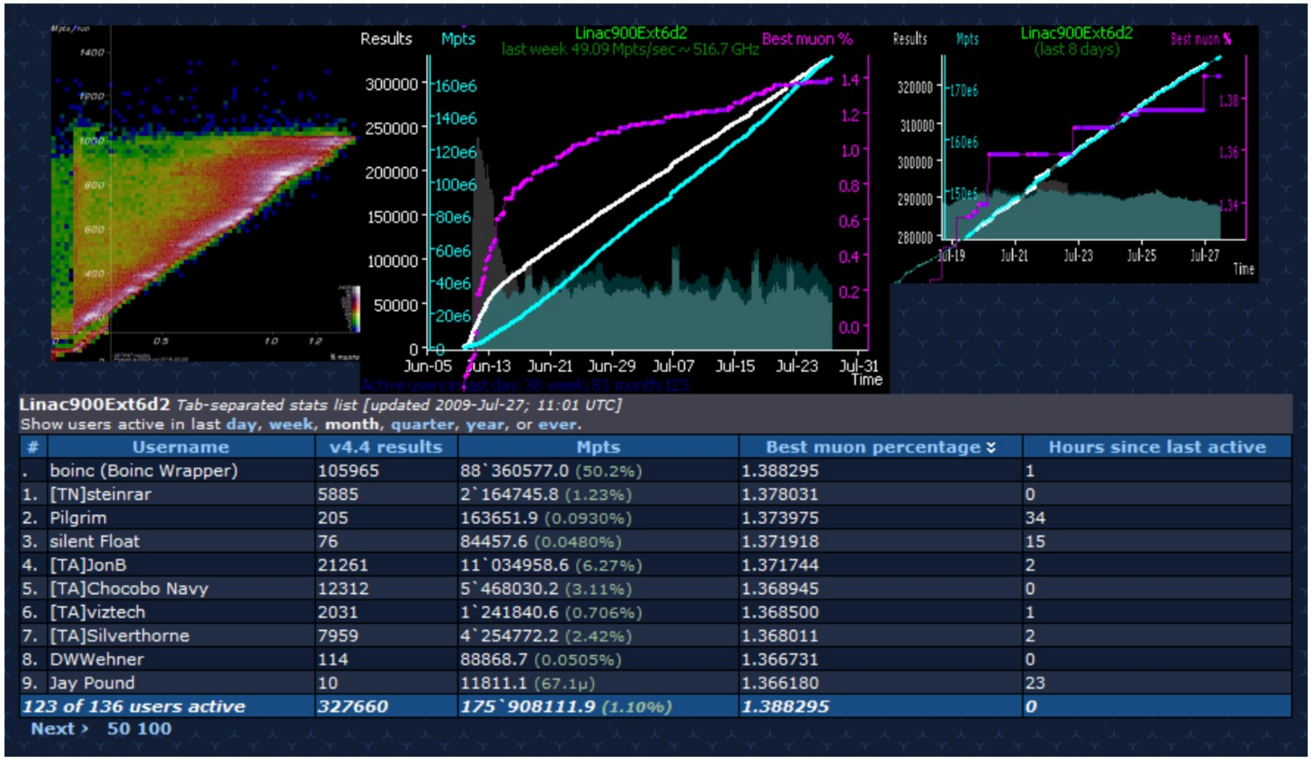


Muon Capture Schemes for the Neutrino Factory, Stephen Brooks, D.Phil. thesis



- Cooling-less design

Muon1



Linac900Ext6d2 *Tab-separated stats list [updated 2009-Jul-27; 11:01 UTC]*
Show users active in last **day**, **week**, **month**, **quarter**, **year**, or **ever**.

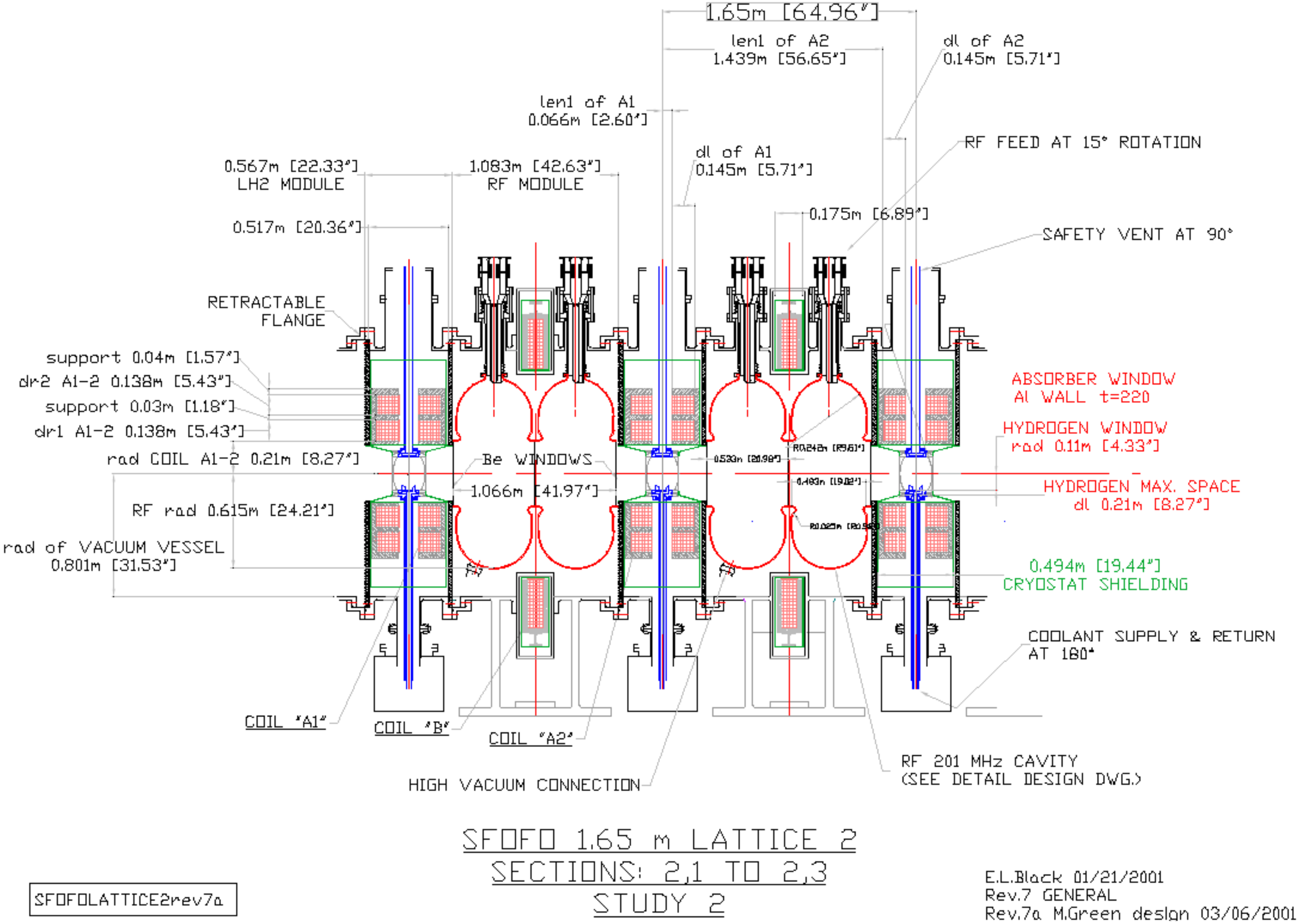
#	Username	v4.4 results	Mpts	Best muon percentage ≚	Hours since last active
.	boinc (Boinc Wrapper)	105965	88`360577.0 (50.2%)	1.388295	1
1.	[TN]steinrar	5885	2`164745.8 (1.23%)	1.378031	0
2.	Pilgrim	205	163651.9 (0.0930%)	1.373975	34
3.	silent Float	76	84457.6 (0.0480%)	1.371918	15
4.	[TA]JonB	21261	11`034958.6 (6.27%)	1.371744	2
5.	[TA]Chocobo Navy	12312	5`468030.2 (3.11%)	1.368945	0
6.	[TA]viztech	2031	1`241840.6 (0.706%)	1.368500	1
7.	[TA]Silverthorne	7959	4`254772.2 (2.42%)	1.368011	2
8.	DWWehner	114	88868.7 (0.0505%)	1.366731	0
9.	Jay Pound	10	11811.1 (67.1μ)	1.366180	23
123 of 136 users active		*327660*	*175`908111.9 (1.10%)*	*1.388295*	*0*

Next › **50** **100**

- Simulation using **muon1**
  - Homebrew code
- Massively distributed optimisations
  - Tracking through fairly realistic field maps

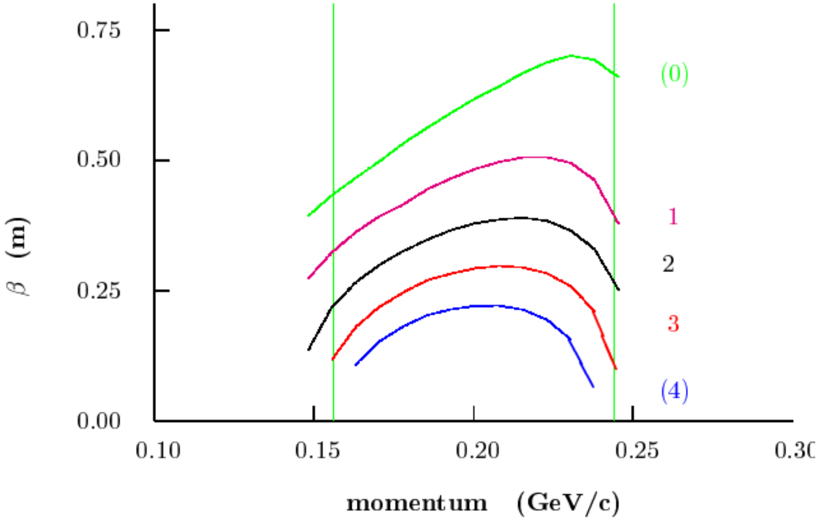
FS2 design

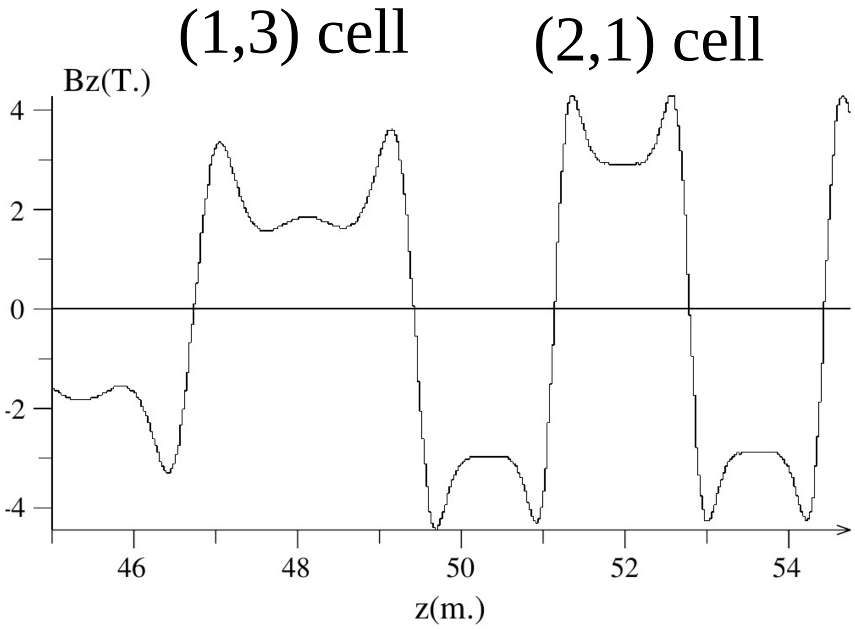
- Advanced cooling lattices developed “Feasibility Study 2”
  - “SFoFo” lattice



Feasibility Study-II of a Muon Based Neutrino Source, Ed. S. Geer et al

US design - lattice



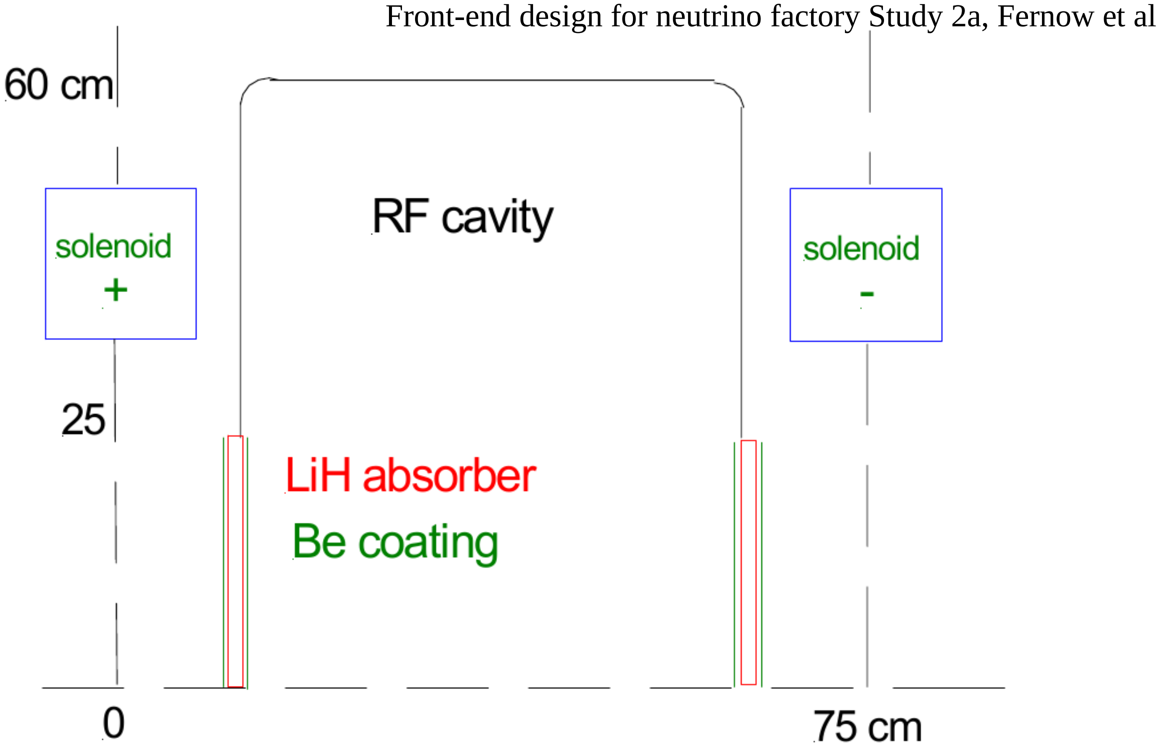


- Introduce complicated lattice concepts e.g. intricate solenoid arrangements
  - Nb: basis of MICE design

US design

- Simulation done in **ICOOL**
    - Based on heavily modified version of Geant3
    - Analysis tool ecalc9f
- Tracking done by numerical integration through space
    - E.g. RK4
    - Custom models for physics processes
    - Dedicated model for passage of particles through liquid hydrogen
        - Based on tracking individual scatters and tabulating results
    - Several models for solenoids and multipoles
    - Supports elaborate geometries (e.g. wedges, etc)
- See talk by Scott

FS2a


Front-end design for neutrino factory Study 2a, Fernow et al

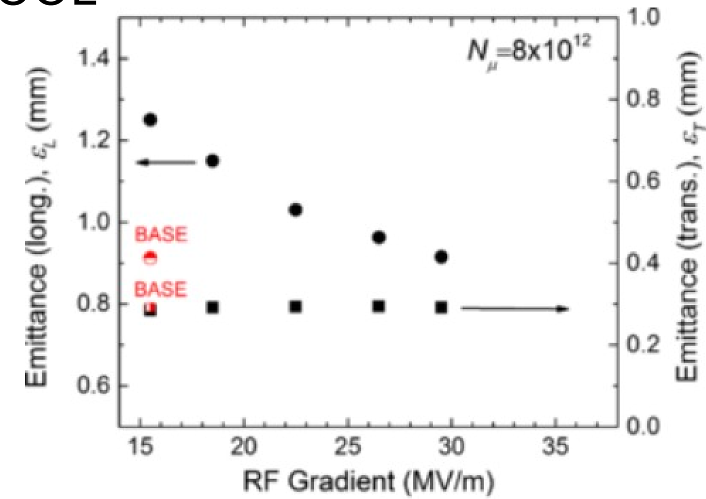
- Ps:
  - FS2 design was considered too elaborate and simplified to so-called Feasibility Study 2a design
  - Ring lattices were investigated but considered too difficult
  - FS2a lattice and ring concept merged into “rectilinear” concept

Enter Geant4

- Geant4 adopted as a useful tool sometime early 2000s
  - Daniel V Elvira’s “BeamTools” package
    - Simulation of common beam elements e.g. RF, solenoids, dipoles …
  - Used by MICE project to build a simulation of the experiment
  - Hard coded (C++) lattice description “**G4MICE**” package
    - Includes provision for reconstruction etc
  - Adopted into **G4Beamline**
    - Convenient user interface – soft-coded lattice files
    - Rather versatile lattice geometry building tools
    - Convenient visualisation & GUIs
    - Talk by Dan

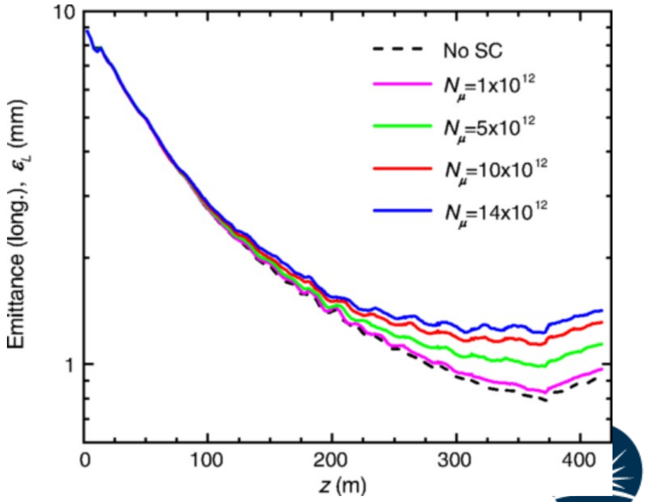
Collective effects

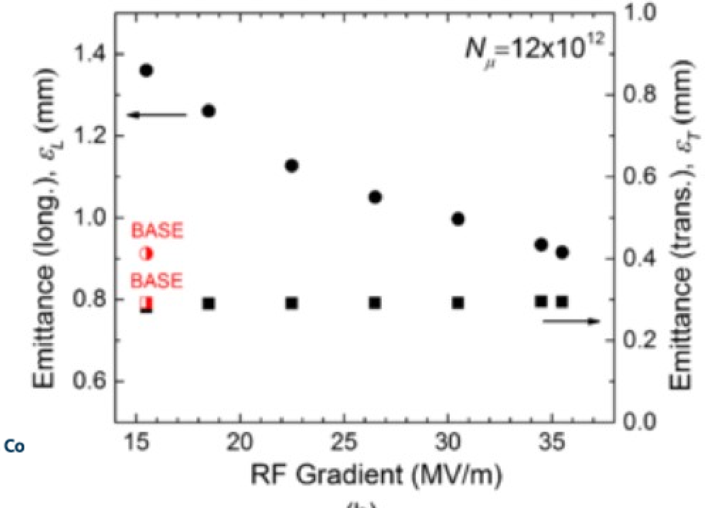
- Space charge simulated using **WARP** models
  - Binary? interface between WARP and ICOOL
  - Good agreement for low beam current



(a)

Influence of space-charge fields on the cooling process of muon beams, Stratakis et al





(b)

IMCC

- IMCC is now looking at two codes for simulations
- **BDSIM** widely used for beamline simulations
  - Interesting mix of matrix simulations and particle tracking
  - Talk by Paul & Rohan
- **RFTrack** linac simulation tool
  - Support for some collective effects
  - Talk by Bernd

Models – Challenges

- Several “novel” challenges of simulation tools

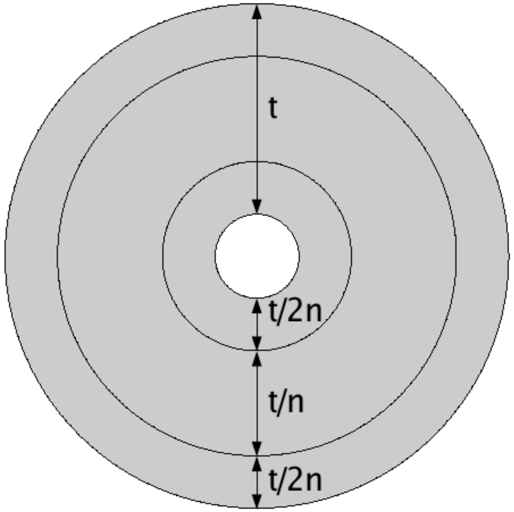
- Overlapping field elements & long fringe fields
- Slightly exotic RF cavity arrangements
- Dipoles
- Passage of particles through matter
  - Energy loss
  - Scattering
- Decays
- Multiple and exotic particle species
- (Lately) collective effects
  - Space charge
  - Beam loading

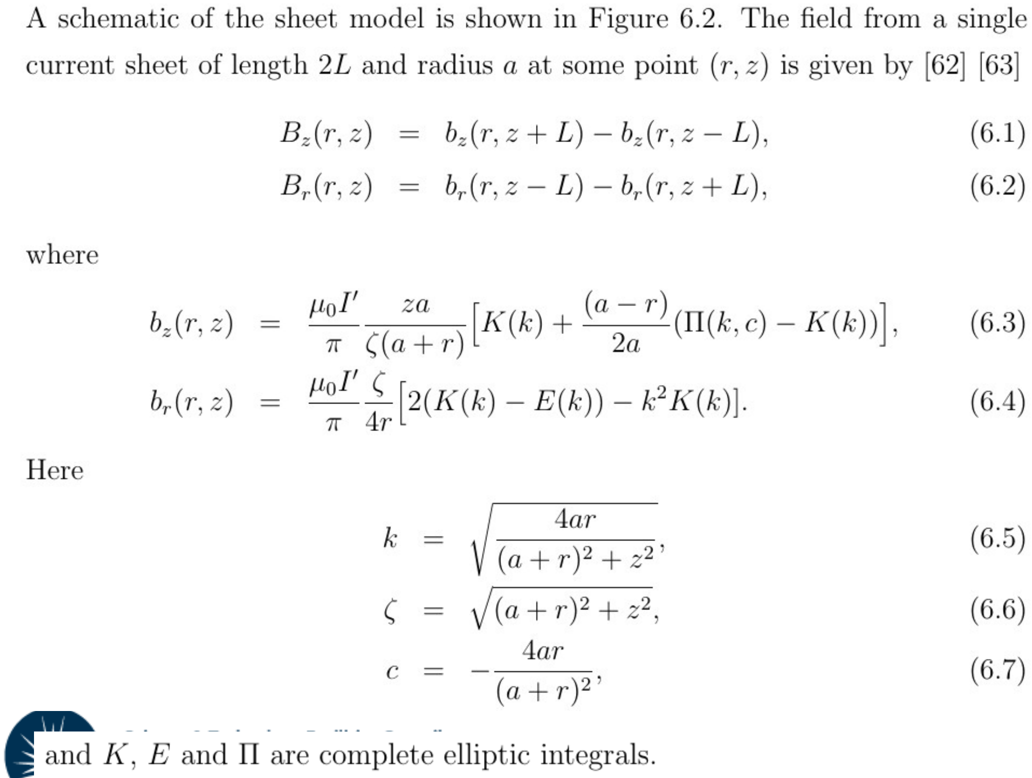
Models – Solenoid

- Solenoids are a key lattice element
- Typically a few different options for solenoid models
  - Analytical solution for “Block” conductor
  - Current sheet model
  - Maxwellian expansion
  - Field maps

Current sheet model

- Semi-analytical solution exists for an infinitely thin sheet of current
- In the limit that *n* sheets → infinity, becomes a block conductor





A schematic of the sheet model is shown in Figure 6.2. The field from a single current sheet of length $2L$ and radius a at some point (r, z) is given by [62] [63]

$$B_z(r,z) = b_z(r,z+L) - b_z(r,z-L), \tag{6.1}$$

$$B_r(r,z) = b_r(r,z-L) - b_r(r,z+L), \tag{6.2}$$

where

$$b_z(r,z) = \frac{\mu_0 I'}{\pi}\frac{za}{\zeta(a+r)}\Big[K(k) + \frac{(a-r)}{2a}(\Pi(k,c) - K(k))\Big], \tag{6.3}$$

$$b_r(r,z) = \frac{\mu_0 I'}{\pi}\frac{\zeta}{4r}\big[2(K(k) - E(k)) - k^2 K(k)\big]. \tag{6.4}$$

Here

$$k = \sqrt{\frac{4ar}{(a+r)^2 + z^2}}, \tag{6.5}$$

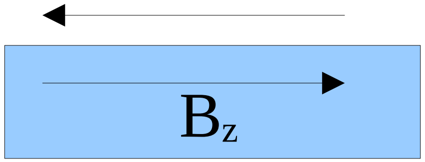
$$\zeta = \sqrt{(a+r)^2 + z^2}, \tag{6.6}$$

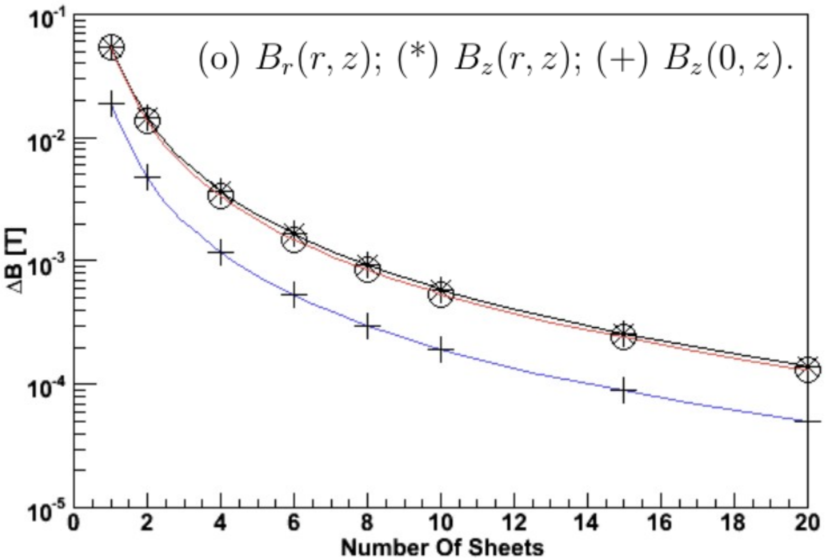
$$c = -\frac{4ar}{(a+r)^2}, \tag{6.7}$$

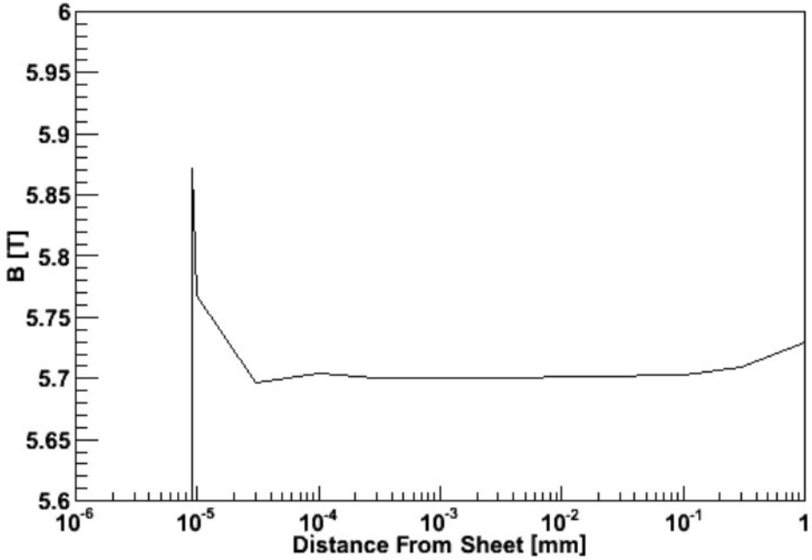
and K, E and Π are complete elliptic integrals.

Current sheet model (2)

- Convergence is reasonably rapid
- But note that there is a field discontinuity very close to the sheet

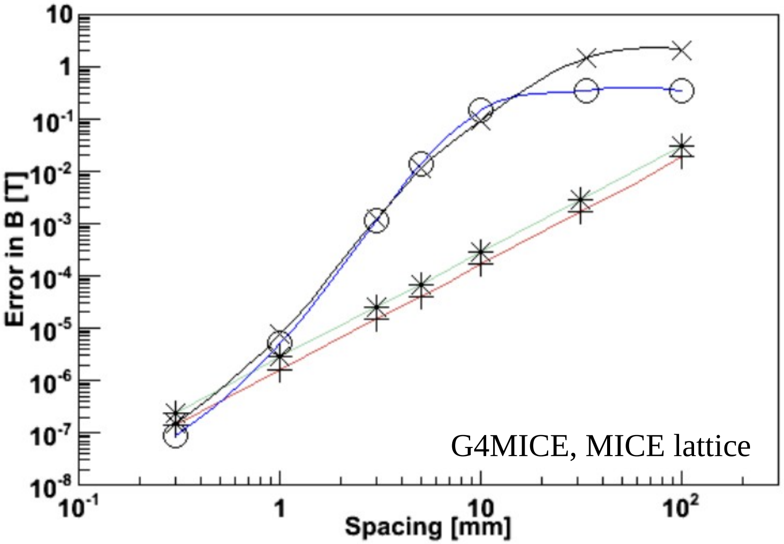






Field map

- Elliptical integrals can be slow to calculate
  - Field calculation CPU time goes as N(sheets)
  - For many thousands of muons this can be slow
- An optimisation is to:
  - Write field map
    - 2D because rotational symmetry
  - Do a look up at run time
    - Quick
  - Introduces a source of error (field map granularity)



Maxwellian expansion

- If we know the Bz on axis, can approximate the field off-axis
  - Assumes Maxwell's laws to get a recursion relation

From $\nabla \times \vec{B}$, we have

$$\partial_z B_r = \partial_r B_z$$
$$\partial_r \partial_z B_r = \partial_r^2 B_z$$

and from $\nabla . \vec{B}$, we have

$$\partial_r B_r + \frac{B_r}{r} + \partial_z B_z = 0$$
$$\partial_z \partial_r B_r + \frac{\partial_z B_r}{r} + \partial_z^2 B_z = 0$$

substituting curl into div

$$\partial_r^2 B_z + \frac{\partial_r B_z}{r} + \partial_z^2 B_z = 0$$

Maxwellian expansion

- Standard accelerator thing
  - Nb: problem in SY Lee “Accelerator Physics”

Try writing fields as a series expansion

$$B_r = \sum_{i=0} a_i(z) r^i$$

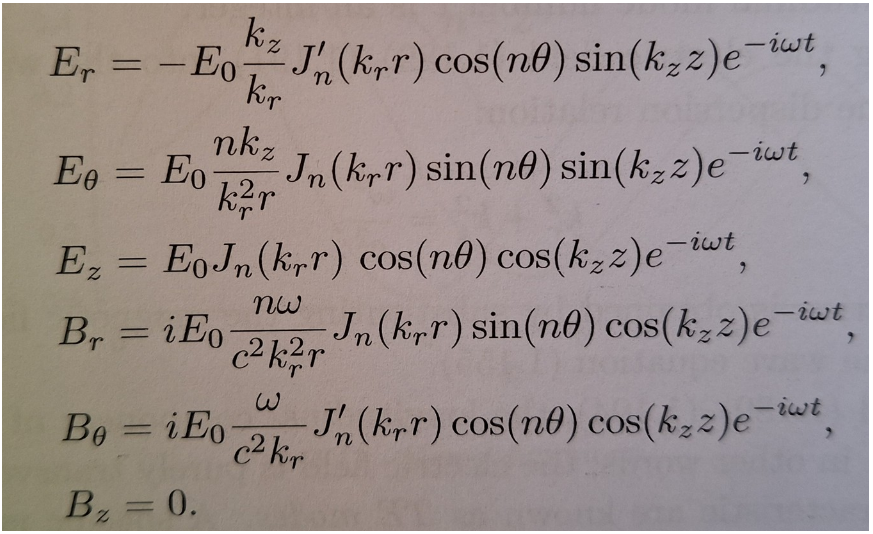
$$B_z = \sum_{i=0} b_i(z) r^i$$

$$a_i = -\frac{1}{i+1} \partial_z b_{i-1}(z).$$

$$b_{i+2}(z) = -\frac{1}{(i+2)^2} \partial_z^2 b_i(z)$$

RF

- RF cavities are typically assumed to be standing EM waves
- Cooling systems → typically cylindrical cavity
- If the cavity is a perfect cylinder, field can be calculated analytically:



$$E_r = -E_0 \frac{k_z}{k_r} J_n'(k_r r)\cos(n\theta)\sin(k_z z)e^{-i\omega t},$$

$$E_\theta = E_0 \frac{n k_z}{k_r^2 r} J_n(k_r r)\sin(n\theta)\sin(k_z z)e^{-i\omega t},$$

$$E_z = E_0 J_n(k_r r)\,\cos(n\theta)\cos(k_z z)e^{-i\omega t},$$

$$B_r = iE_0 \frac{n\omega}{c^2 k_r^2 r} J_n(k_r r)\sin(n\theta)\cos(k_z z)e^{-i\omega t},$$

$$B_\theta = iE_0 \frac{\omega}{c^2 k_r} J_n'(k_r r)\cos(n\theta)\cos(k_z z)e^{-i\omega t},$$

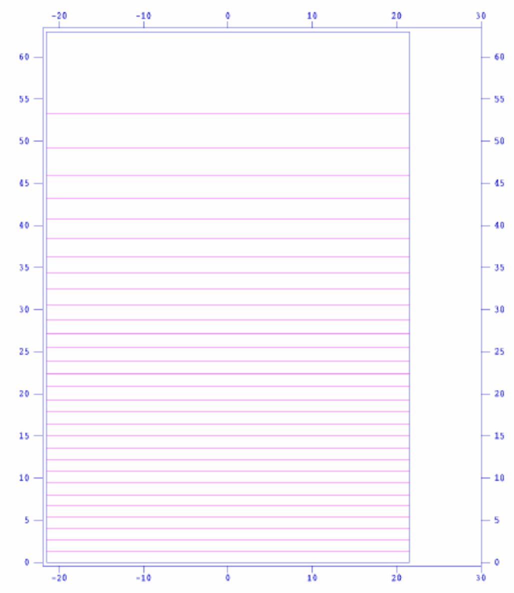
$$B_z = 0.$$

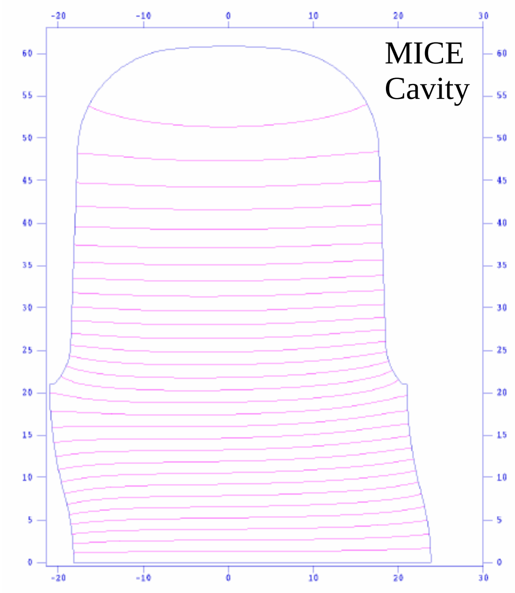
A. Wolski, Beam Dynamics in High Energy Particle Accelerators, Imperial College Press

- For some cooling systems, field can be assumed to follow TM010 mode (Tmnml)
  - Assumes cavity is a perfect cylinder

More realistic RF

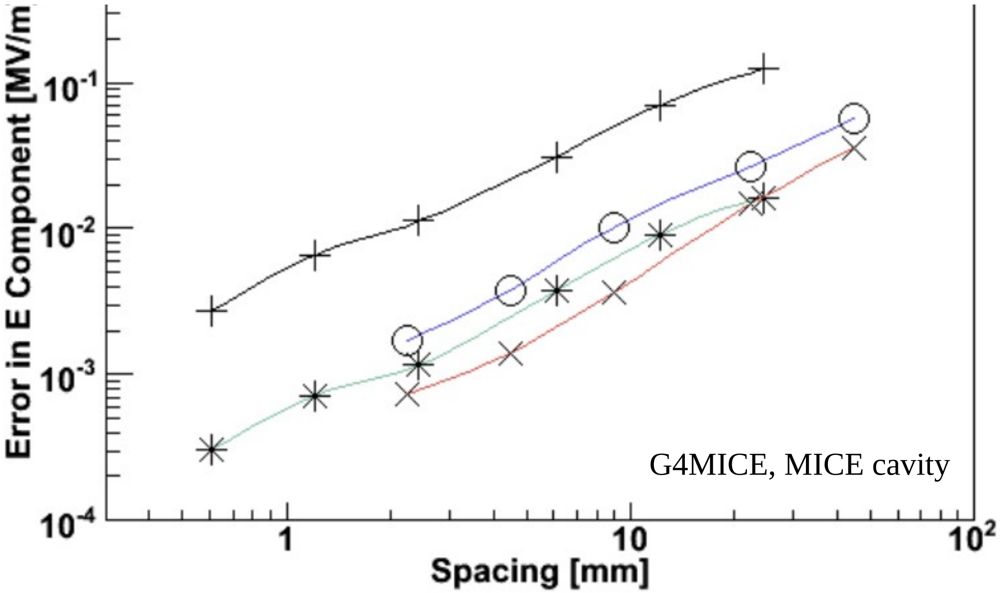
- For greater realism, one can use 2D field map
  - RF designers don’t like parallel walls
  - RF designers don’t like sharp corners
- Cylindrical symmetry is a very good approximation
  - Anisotropy near e.g. power couplers is negligible near the beam
- Ps: it is also possible to use a field expansion like solenoids but I don’t think any of the cooling codes do this





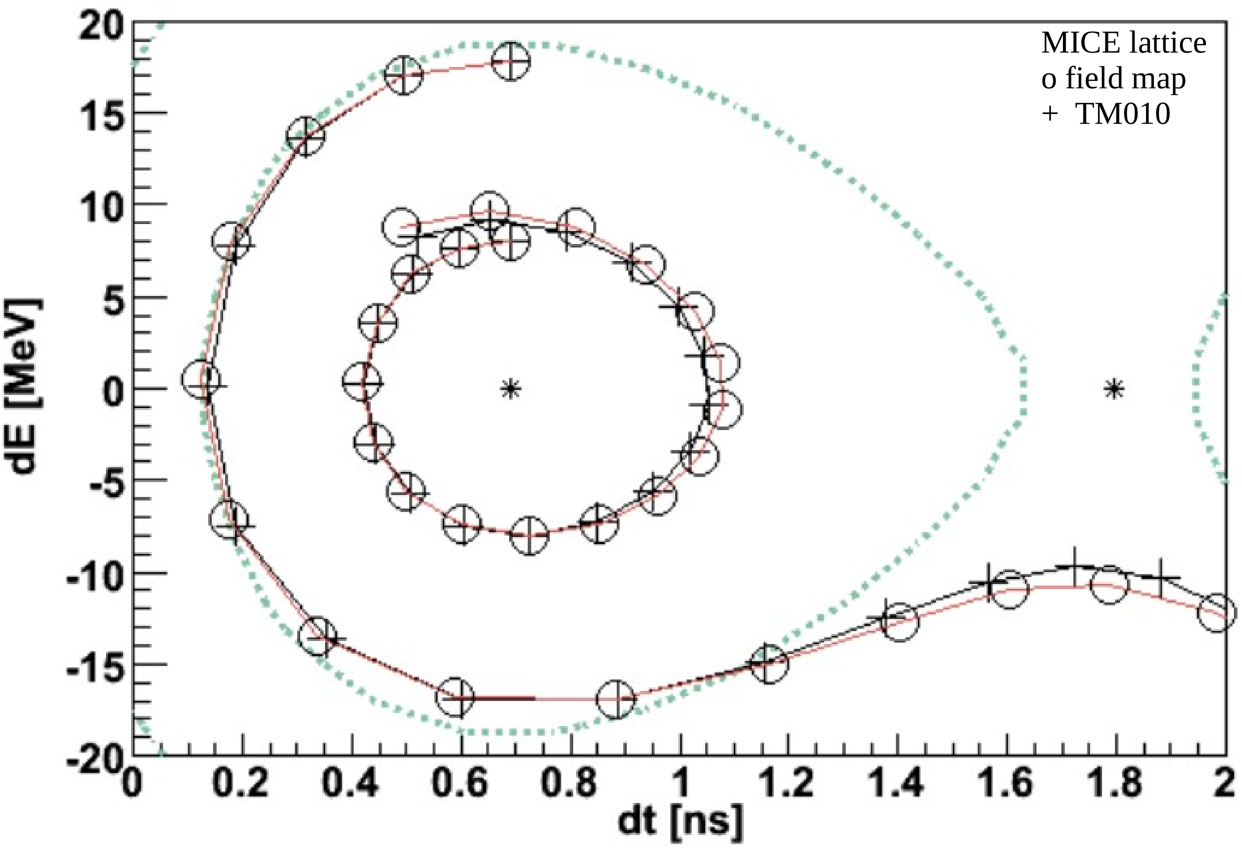
Field map granularity

- Granularity in the field map can again introduce errors



TM010 vs field map

- Practically the difference can be small



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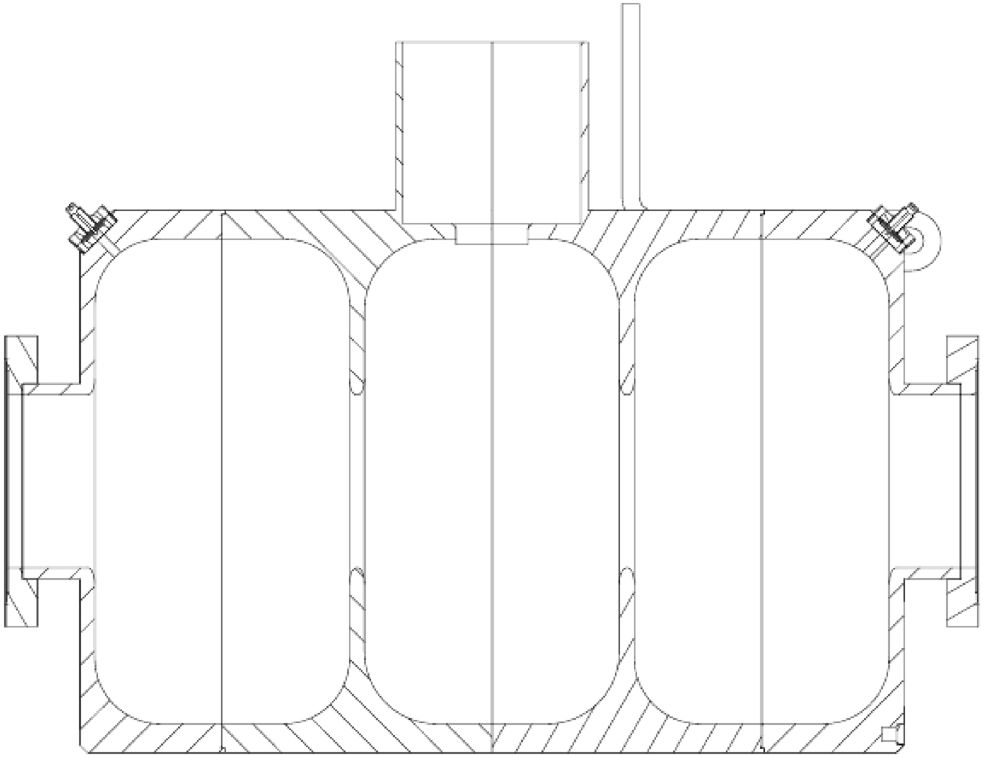
Demonstrator RF structure

- Note that an RF structure like this may have bigger effect
  - Windowless



Dipoles (quads, etc)

- No analytical model exists for dipoles and multipoles
- Either:
  - Use a field map generated in an external code (OPERA, COMSOL, etc)
- Or:
  - Use a field expansion based on Maxwell
  - Typically field on axis is assumed to follow a “Enge” function
  - $B_y = C_{n,0}(d)$

$$C_{n,0}(z) = \frac{G_0}{1 + exp[P(d(z))]},$$

$$P(d) = C_0 + C_1\left(\frac{d}{\lambda}\right) + C_2\left(\frac{d}{\lambda}\right)^2 + \dots + C_{k-1}\left(\frac{d}{\lambda}\right)^{k-1}$$

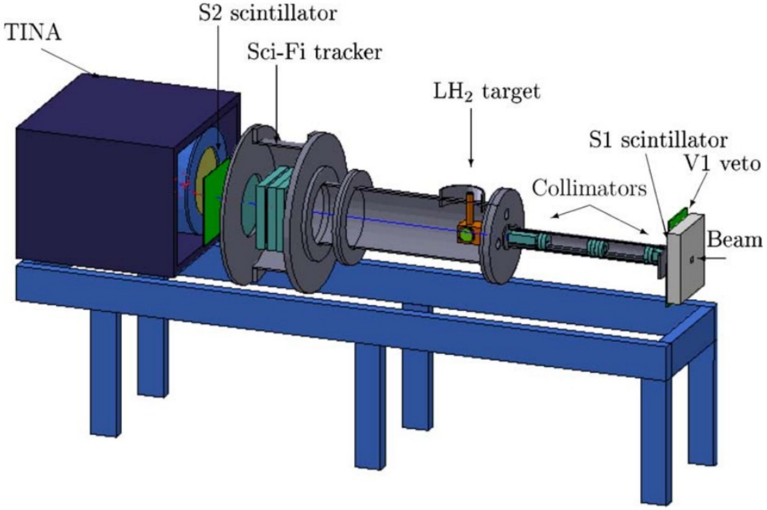
  - To first order this is tanh function
  - Find coefficients C_i by fitting to existing dipoles
  - For suitable C_i, lambda, derivatives are continuous and exponentially tend to 0 at high d
  - For complex d, get trigonometric functions appearing
    - Ruihu lattice

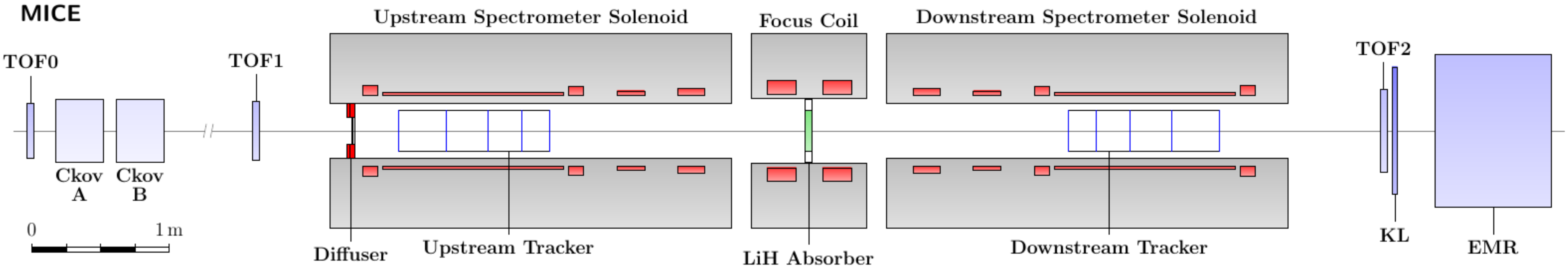
Materials

- In materials, two main processes
  - Multiple Coulomb scattering
  - Ionisation energy loss
- Multiple Coulomb scattering (MCS)
  - Muons strike atomic nucleus and scatter
  - Mostly causes random kicks transverse to particle motion
  - Does not change energy but tends to increase emittance
- Ionisation energy loss (dE/dx)
  - Muons strike atomic electrons and ionise them
  - Mostly causes kicks along particle motion
  - Does not increase emittance but absorbers muon energy
  - Leads to a reduction in *normalised* emittance

Multiple Coulomb Scattering

- Two data sets for MCS in the region of interest
  - MuScat – mid 2000s measurement at TRIUMF
  - MICE – mid 2010s measurement at RAL





MuScat

- Simulation showed early version of Geant4 (6.1) had big discrepancy especially in liquid Hydrogen

D. Attwood et al, Nuclear Instruments and Methods in Physics Research B 251 (2006) 41–55

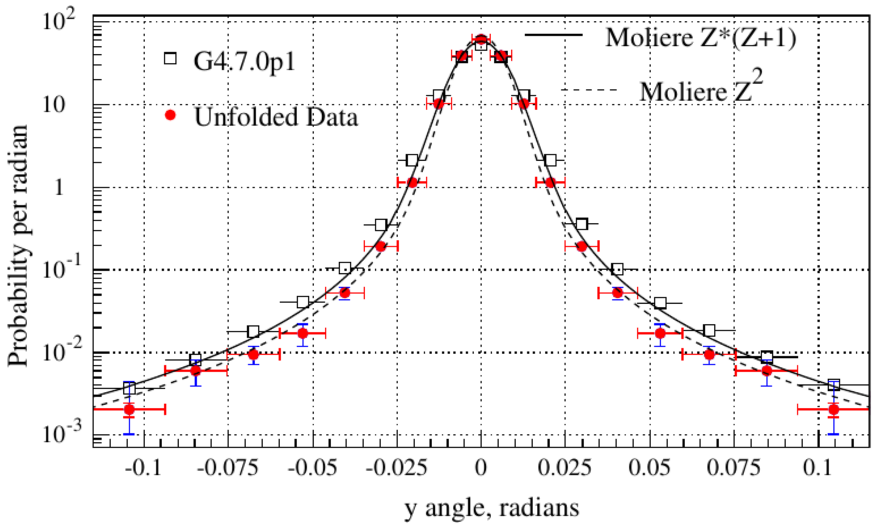


Fig. 21. The projected scattering angle distribution in data and simulation for thick lithium, both targets combined.

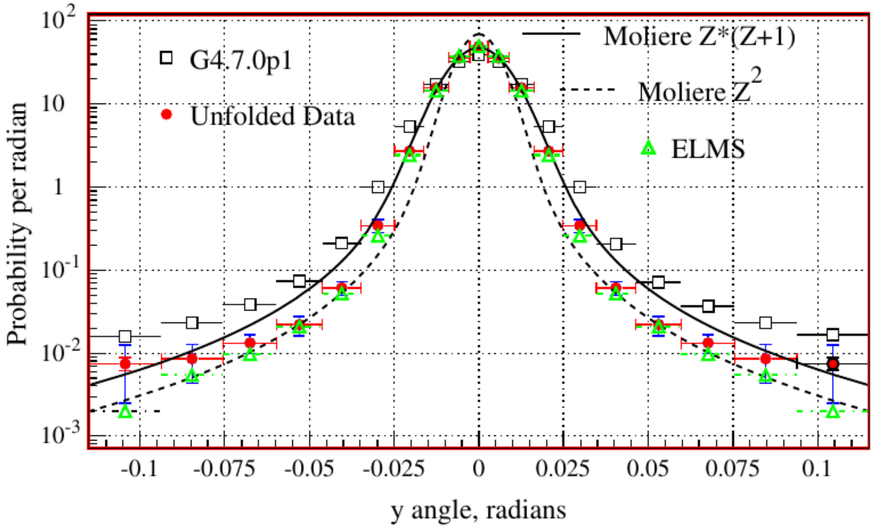
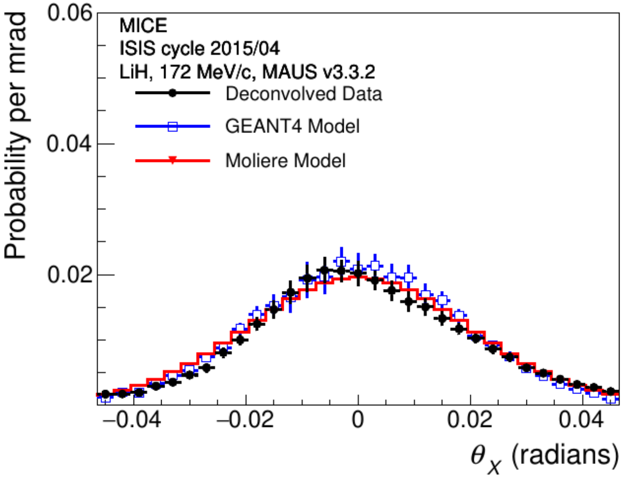


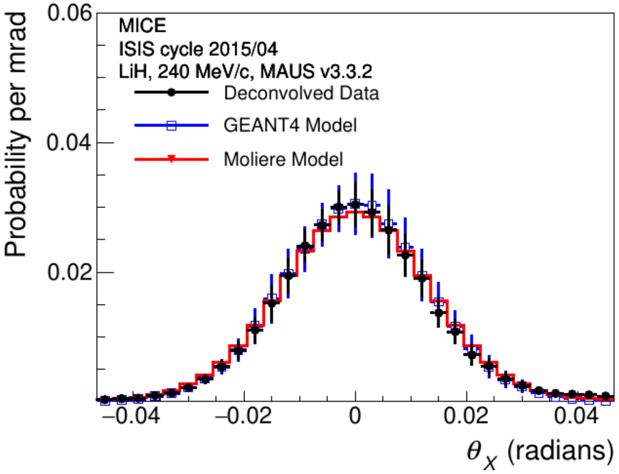
Fig. 22. The projected scattering angle distribution in data and simulation for 109 mm of liquid H_2.

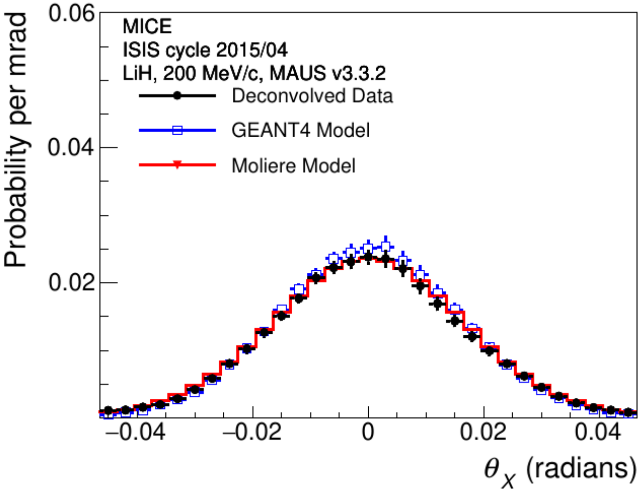
MICE

- Later versions of Geant4 (9.6) show improvement

M. Bogomilov et al, Phys.Rev.D 106 (2022) 9, 092003







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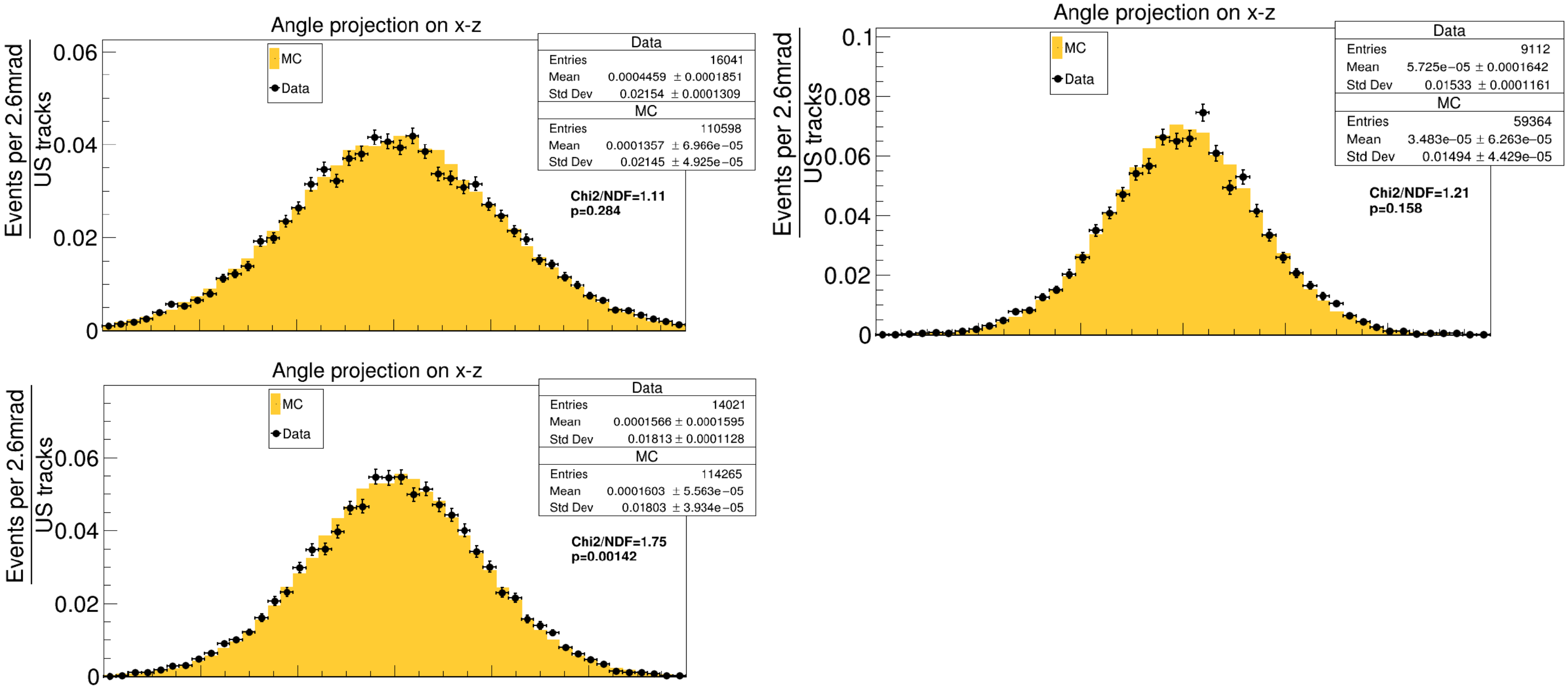
MICE (lH2)

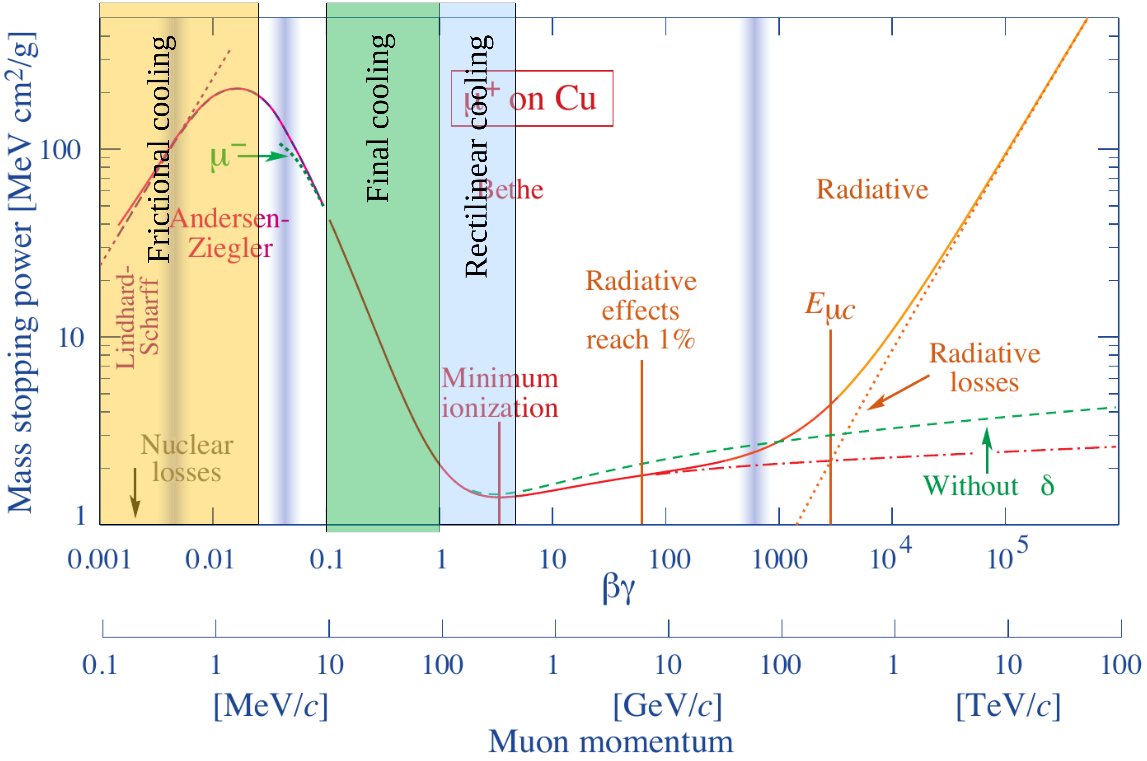
- Also lH2



Gavriil Chatzitheodoridis, Analysis of multiple Coulomb scattering of muons in liquid hydrogen, PhD Thesis (Strathclyde/Glasgow)

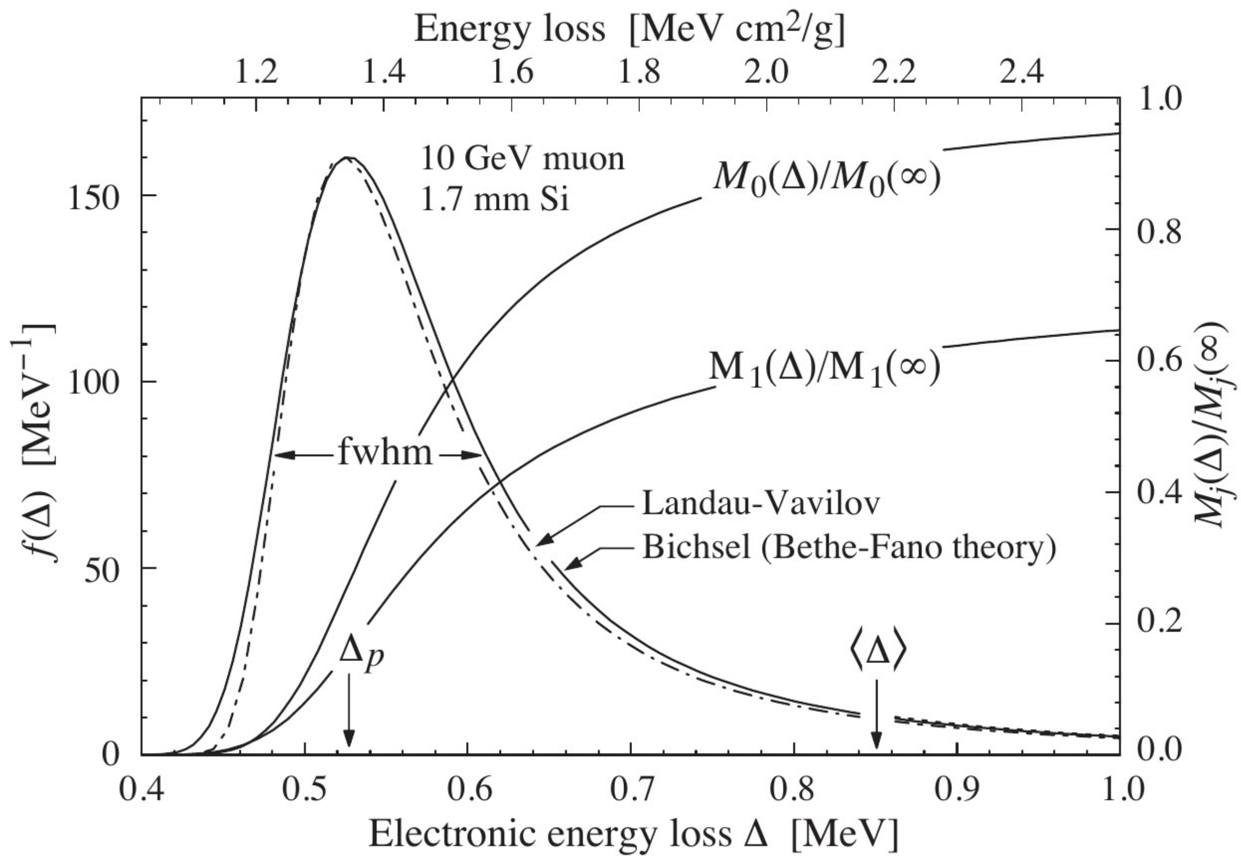
Energy loss

- Typically muon cooling is done around minimum ionising

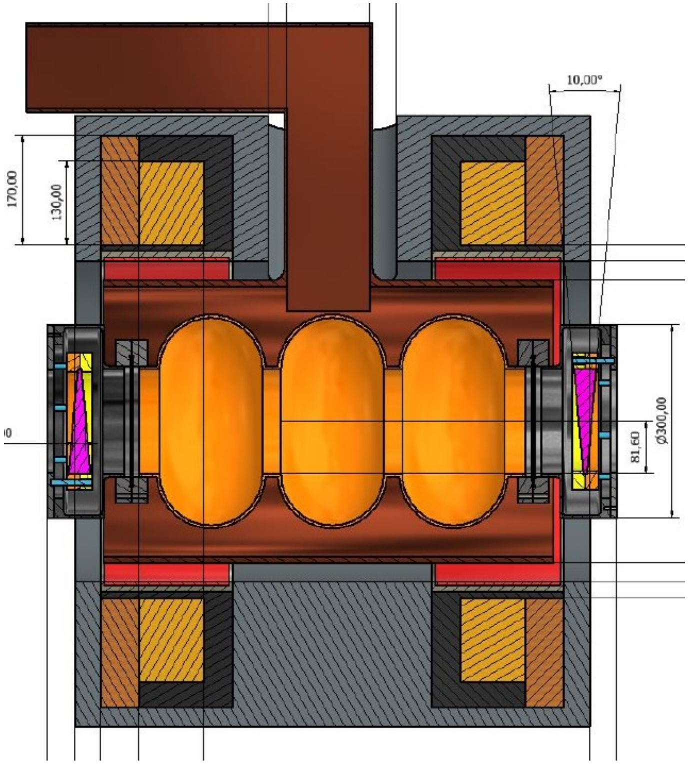


Energy straggling

- Random processes in energy straggling
- Note that everyone uses mean energy loss
  - But mean energy loss is ill-defined



Cooling lattices



- Challenge now to model this!
    - Overlapping solenoids with RF, dipoles
    - Complicated intersecting material geometry
    - Difficult beam dynamics

Extra Info

C. T. Rogers
Rutherford Appleton Laboratory

Collective Effects

- Two collective effects that are of immediate concern
- Space charge
  - Particle beam tries to self-annihilate due to internal charge of the beam
- Beam loading
  - Particle beam induces E-field in RF cavities
  - This causes the tail of the beam to experience a different voltage than the head of the beam
- Other collective effects may exist

Space charge

- Aim to solve Poisson equation for an evolving charge distribution (rho)

$$\nabla^2 \phi = -\rho/\epsilon_0,$$

- Solution looks like

$$\phi(x,y,z) = \int\int\int dx'dy'dz'\rho(x',y',z')G(x-x',y-y',z-z'),$$

- Discretising

$$\phi_{i,j,k} = h_x h_y h_z \sum_{i'=1}^{M_x}\sum_{j'=1}^{M_y}\sum_{k'=1}^{M_t} \rho_{i',j',k'} G_{i-i',j-j',k-k'},$$

$$G(u,v,w) = \frac{1}{\sqrt{u^2+v^2+w^2}}.$$

Adelmann et al, OPAL a Versatile Tool for Charged Particle Accelerator Simulations, arxiv 1905 06654v1