

Single Higgs Precision at Muon Colliders

2203.09425, 2308.02633 with Patrick Meade

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Pathways to Innovation and Discovery in Particle Physics

Report of the 2023 Particle Physics Project Prioritization Panel



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So what's the big deal with muon colliders?

Muon colliders: the physics case

Muon colliders: energy and precision at the same time*!

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Large effort in community to develop physics case in the last ~ 5 years. A few highlights:

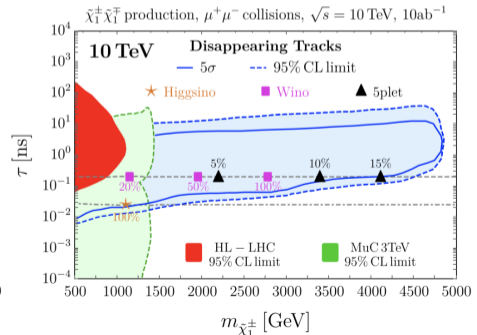
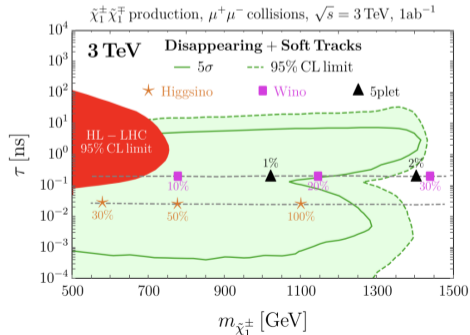
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- 5σ discovery for Higgsino (3 TeV) / Wino (10 TeV) thermal targets

(Capdevilla, Meloni, Zurita 2405.08858; Capdevilla, Meloni, Simoniello, Zurita 2102.11292; Han, Liu, Wang, Wang 2203.07351; . . .)



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- Unparalleled tests of Higgs compositeness

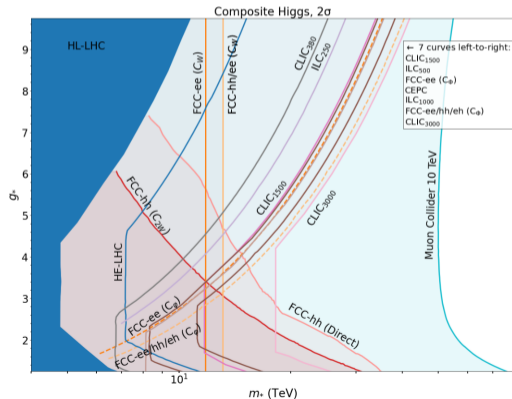
Chen, Glioti, Rattazzi, Ricci, Wulzer 2202.10509

EF report 2211.11084

Accettura et al. 2303.08533

Liu, Wang, Xie 2312.09117

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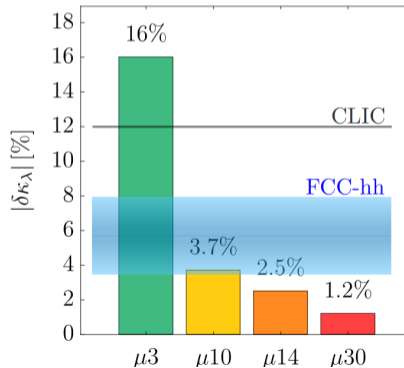
- $\sim 4\%$ level Higgs trilinear measurement
- $\mathcal{O}(1)$ measurement of Higgs quartic

Accettura et al. 2303.08533

Han, Liu, Low, Wang 2008.12204

Chiesa, Maltoni, Mantani, Mele, Piccinini, Zhao 2003.13628

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- Heavy EW BSM resonances up to $\sim E_{CM}/2$
- Lepton flavour universality
- Scalar singlets
- Dark sectors
- **Single Higgs precision** \leftarrow (This talk)

Muon smasher's guide 2103.14043
Muon collider forum report 2209.01318
Towards a muon collider 2303.08533
IMCC report 2407.12450
...

and many more!

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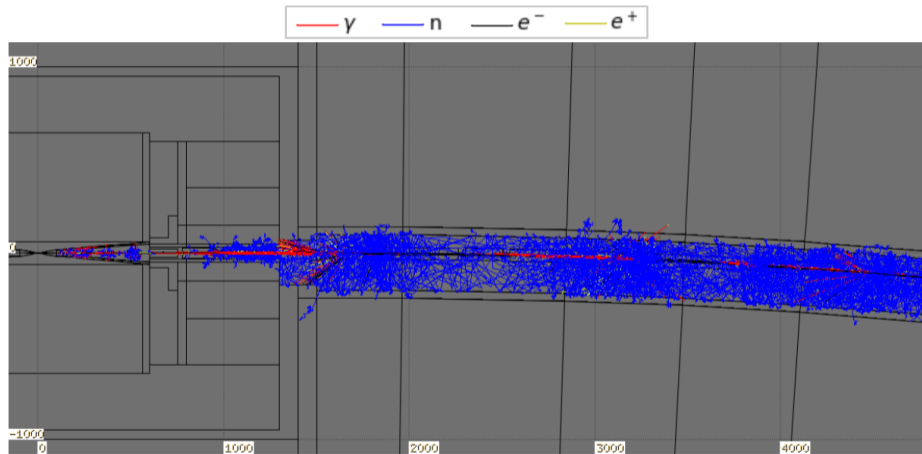
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However, (3) affects physics performance in the detector!

Beam induced backgrounds (2203.07964)

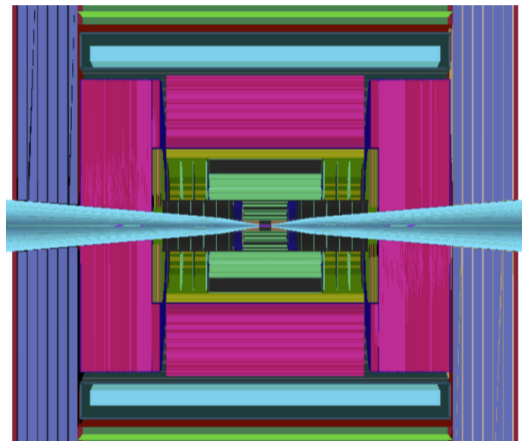


BIB mitigation (IMCC 2407.12450)

Simulations at 1.5 TeV: tungsten nozzles with $\theta = 10^\circ$ reduces BIB to tolerable level

Limiting factor in i.e. jet energy resolution

BIB more forward at higher energies

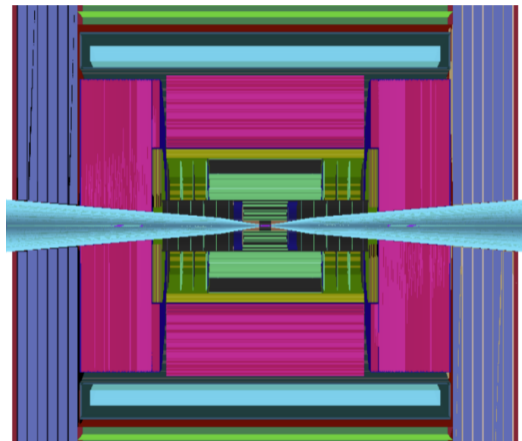


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The effects of BIB on precision studies must be included eventually, still under development

Our input to the physics case:

(single) Higgs precision at muon colliders

The Higgs Precision Landscape (de Blas et al, 1905.03764)

κ -0 fit	HL- LHC	LHeC	HE-LHC		ILC			CLIC			CEPC	FCC-ee		FCC-ee/ eh/hh
			S2	S2'	250	500	1000	380	1500	3000		240	365	
κ_W	1.7	0.75	1.4	0.98	1.8	0.29	0.24	0.86	0.16	0.11	1.3	1.3	0.43	0.14
κ_Z	1.5	1.2	1.3	0.9	0.29	0.23	0.22	0.5	0.26	0.23	0.14	0.20	0.17	0.12
κ_g	2.3	3.6	1.9	1.2	2.3	0.97	0.66	2.5	1.3	0.9	1.5	1.7	1.0	0.49
κ_γ	1.9	7.6	1.6	1.2	6.7	3.4	1.9	98*	5.0	2.2	3.7	4.7	3.9	0.29
$\kappa_{Z\gamma}$	10.	—	5.7	3.8	99*	86*	85*	120*	15	6.9	8.2	81*	75*	0.69
κ_c	—	4.1	—	—	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3	0.95
κ_t	3.3	—	2.8	1.7	—	6.9	1.6	—	—	2.7	—	—	—	1.0
κ_b	3.6	2.1	3.2	2.3	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67	0.43
κ_μ	4.6	—	2.5	1.7	15	9.4	6.2	320*	13	5.8	8.9	10	8.9	0.41
κ_τ	1.9	3.3	1.5	1.1	1.9	0.70	0.57	3.0	1.3	0.88	1.3	1.4	0.73	0.44

κ -0:

$$BR_{BSM} = 0$$

$$\kappa_i \equiv g_i/g_i^{SM}$$

$$\delta\kappa_V \sim 0.1\%$$

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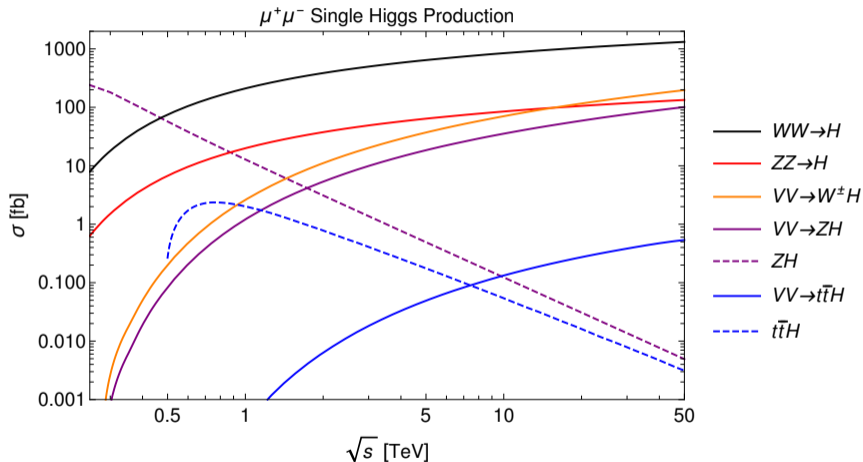
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Single Higgs Production at Muon Colliders

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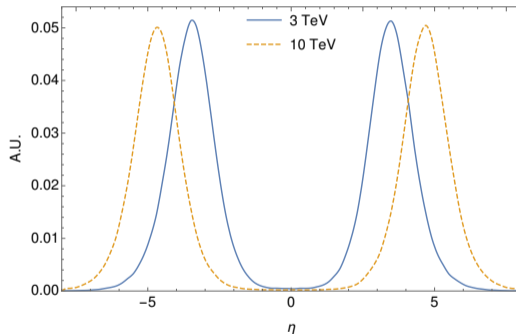
High energies dominated by $WW \rightarrow H$ and $ZZ \rightarrow H$.

Forward Muons

To distinguish between WW -fusion and ZZ -fusion, must be able to tag the forward muons beyond the $|\eta| \approx 2.5$ nozzles.

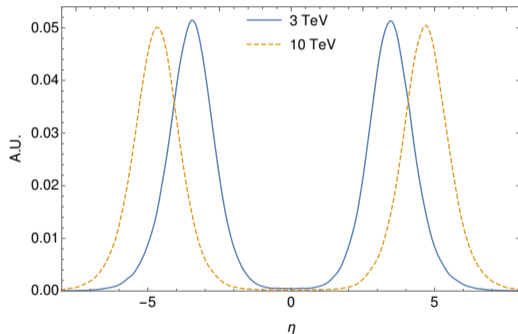
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For ZZ -fusion, we include results considering (optimistic) tagging up to $|\eta| \leq 6$.

Event Generation and Detector Assumptions

Focus on benchmarks of 3 TeV @ 1 ab⁻¹ and 10 TeV @ 10 ab⁻¹

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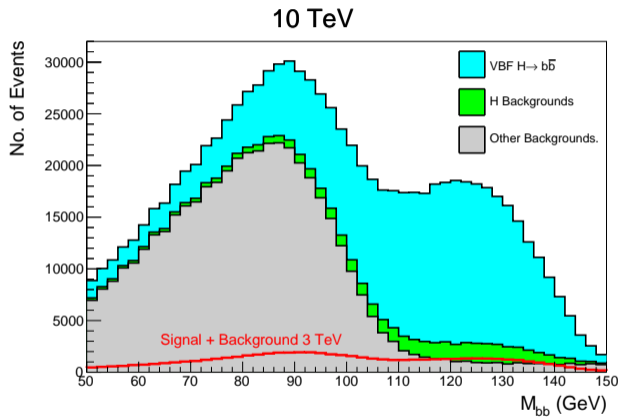
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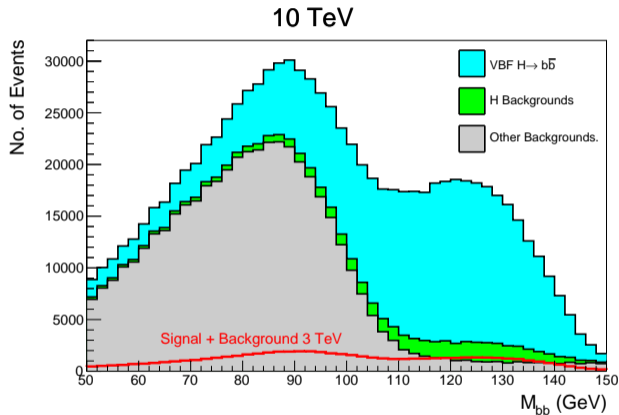
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Estimate precision as $\frac{\delta\sigma}{\sigma} = \frac{\sqrt{S+B}}{S}$ (stat only!)

Hadronic Processes: $b\bar{b}$



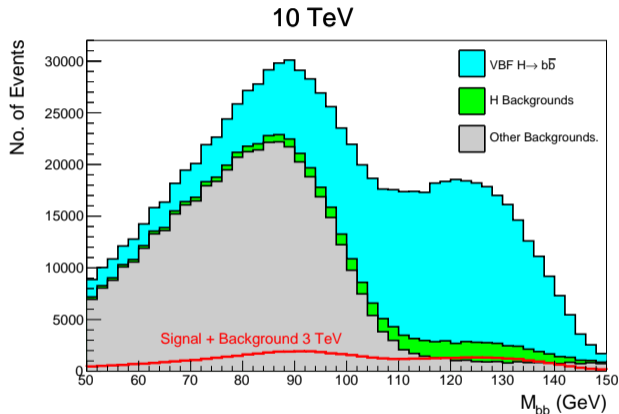
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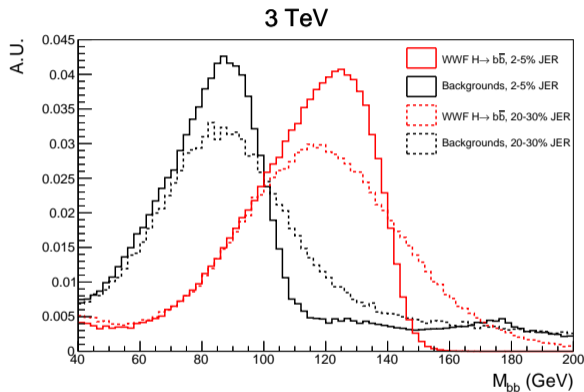
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$c\bar{c}$ and gg channels are very similar, with mistagged $H \rightarrow b\bar{b}$ contributing as a large background

Estimating the Effects of the BIB

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Worse JER based on fullsim – additional spreading roughly doubles the background contribution from the Z peak: 0.76% \rightarrow 0.86% precision.

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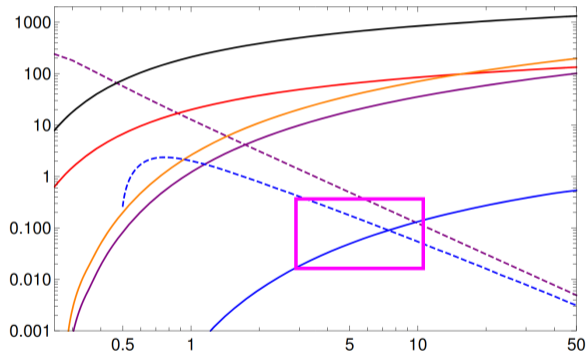
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Process	Number of Events					
	3 TeV			10 TeV		
	$4j$	$2j2\ell$	4ℓ	$4j$	$2j2\ell$	4ℓ
$\mu^+\mu^- \rightarrow \nu_\mu\bar{\nu}_\mu H; H \rightarrow ZZ^* \rightarrow X$	124	103	5	2910	1590	66
$\mu^+\mu^- \rightarrow \mu^+\mu^- H; H \rightarrow ZZ^* \rightarrow X$	3	9	0	315	151	8
Others	6700	50	0	208000	1370	2

The top Yukawa

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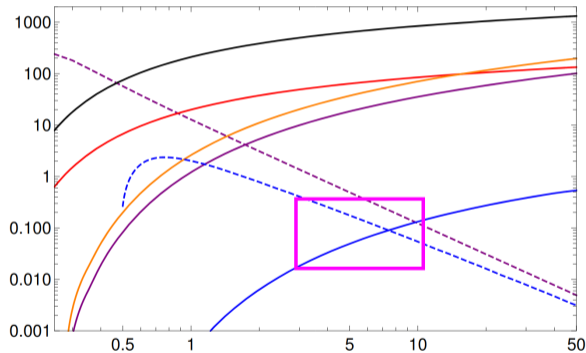
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Find $\delta\sigma = 53\%$ at 10 TeV ($\delta y_t \sim 11\%$)



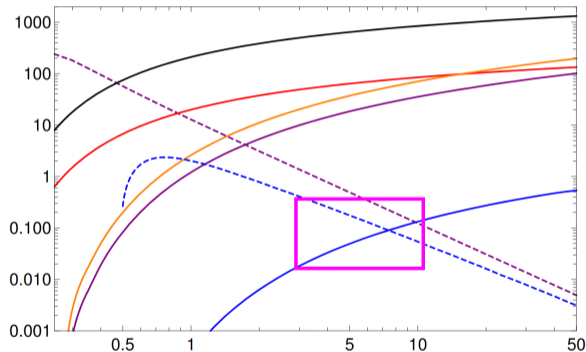
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Off-shell y_t measurement from VBF tt could give $\delta y_t \sim 1.5\%$ at 10 TeV

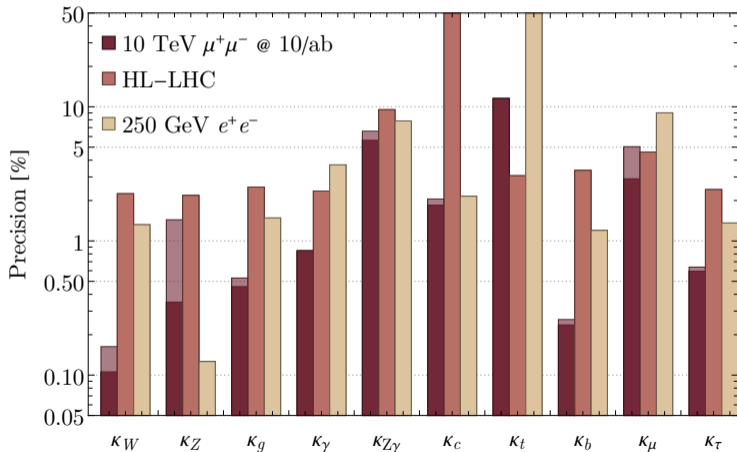
Liu, Lyu, Mahbub, Wang 2308.06323; Chen, Liu 2212.11067



Repeat for all other channels...

Where do we stand?

$BR_{BSM}=0$ Fit Comparisons



Shaded:
forward tagging

$|\kappa_V| < 1$ fit
very similar

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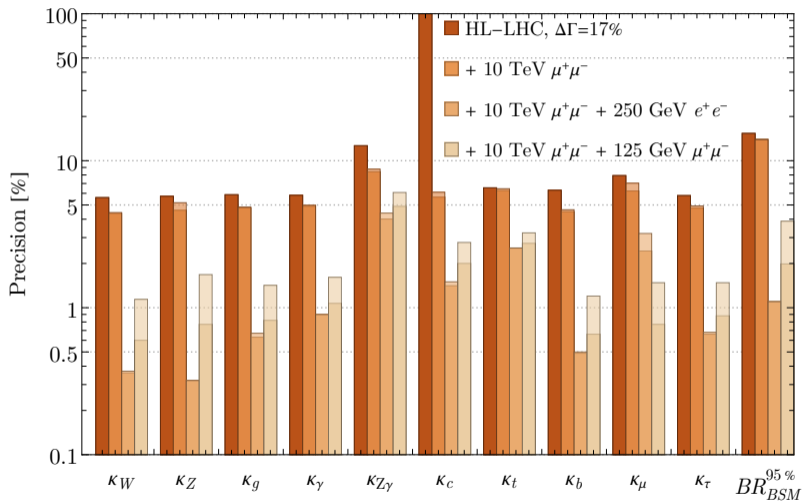
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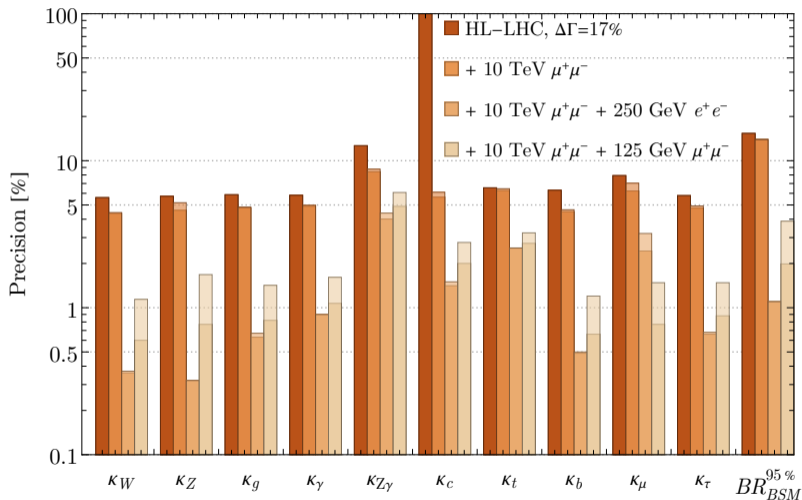
For a width precision of $\delta\Gamma$, can't obtain a coupling precision better than $\delta\kappa \sim (1/4)\delta\Gamma$.

Using other colliders to fix $\delta\Gamma_H$

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Would like to constrain the width at the MC itself!

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Let's look in more detail

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At e^+e^- colliders, measure the inclusive $e^+e^- \rightarrow ZH$ cross section via the recoil mass method:

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→ Can measure σ_{Incl}^{ZH} by *only* measuring the Z decay products!

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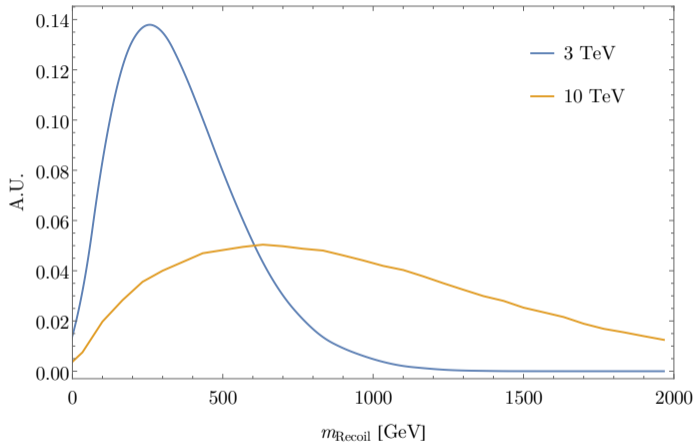
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Nevertheless, could this be done at a muon collider via the forward muons in ZZF?

Can we do this for $\mu^+\mu^- \rightarrow \mu^+\mu^-H$?



Maybe (Li, Liu, Lyu 2401.08756), but highly sensitive to forward detector properties

LHC techniques

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(Same method as the off-shell y_t measurement)

Off-shell $VV \rightarrow VV$ scattering

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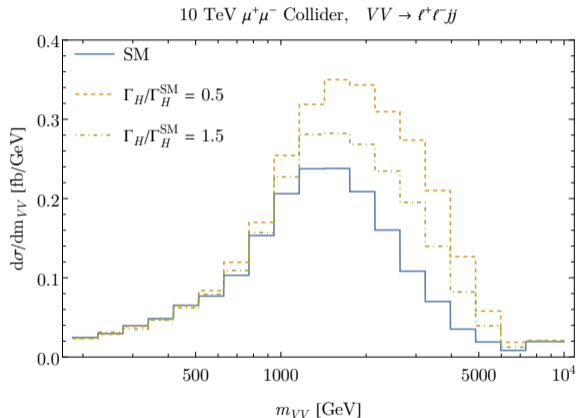
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Fit each bin to a function $a + b\kappa_i\kappa_j + c\kappa_i^2\kappa_j^2$ by varying κ_V .



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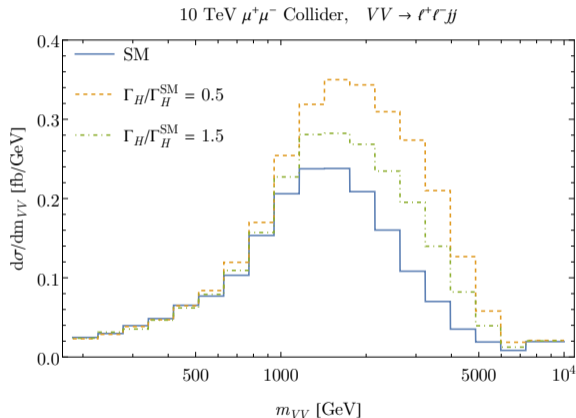
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Fitting κ_W , κ_Z , and $\delta\Gamma$ yields:

$\delta\Gamma = 4.0\%$ at 10 TeV

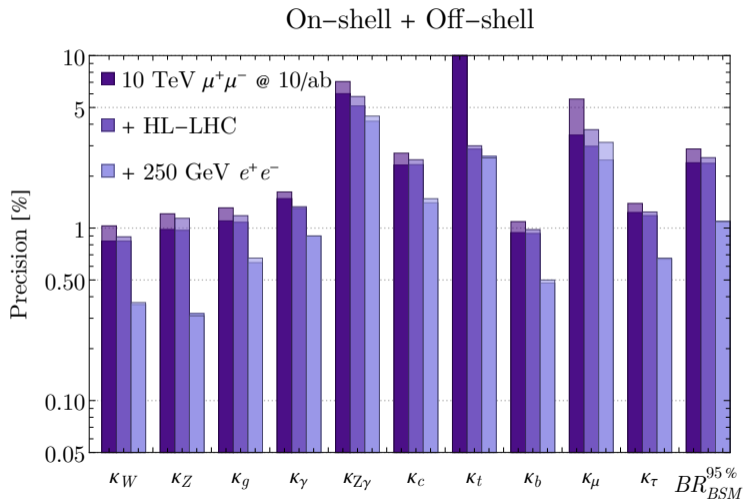
$\delta\Gamma = 58\%$ at 3 TeV
(not competitive with LHC)



Off-shell fit

Shaded:
forward tagging

Find
 $\delta\kappa_V \approx \delta\Gamma/4$



What UV models need the off-shell measurement?

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Are there concrete examples of models that can do this?

Higher multiplet scalars

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These models all have singly and doubly charged scalars. How do direct searches enter the story?

The minimal example: Georgi-Machacek Model

Add to the SM two scalar triplets in a custodial bi-triplet

$$X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

This is custodially symmetric if $\langle \chi^0 \rangle = \langle \xi^0 \rangle$.

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Most general scalar potential with the added field content (Φ is SM Higgs doublet):

$$\begin{aligned} V(\Phi, X) = & \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 \text{Tr}[(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\ & + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) + \lambda_4 \text{Tr}[(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau_a \Phi \tau_b) \text{Tr}(X^\dagger t_a X t_b) \\ & - M_1 \text{Tr}(\Phi^\dagger \tau_a \Phi \tau_b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t_a X t_b) (UXU^\dagger)_{ab} \end{aligned}$$

Georgi-Machacek model

After SSB, obtain a custodial fiveplet, a triplet, and two singlets

$$(H_5^0, H_5^\pm, H_5^{\pm\pm}), (H_3^0, H_3^\pm), h, H$$

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$$\kappa_f = \frac{\cos \alpha}{\cos \theta}, \quad \kappa_V = \cos \alpha \cos \theta - \sqrt{\frac{8}{3}} \sin \alpha \sin \theta$$

with α the $h - H$ mixing angle, and $\cos \theta = \frac{v_\phi}{v}$ the SM Higgs doublet contribution to EWSB.

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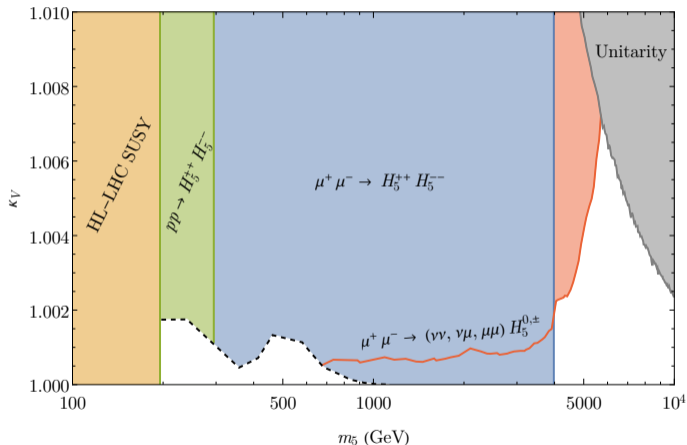
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In the decoupling limit $\mu_3 \gg \mu_2$, can match onto the SMEFT and find:

$$\kappa_f^{dec} \approx 1 - \frac{1}{8} \frac{M_1^2 v^2}{\mu_3^4}, \quad \kappa_V^{dec} \approx 1 + \frac{3}{8} \frac{M_1^2 v^2}{\mu_3^4}$$

Enter direct searches



With exception of rare (easily excludable) points: direct searches push to decoupling limit

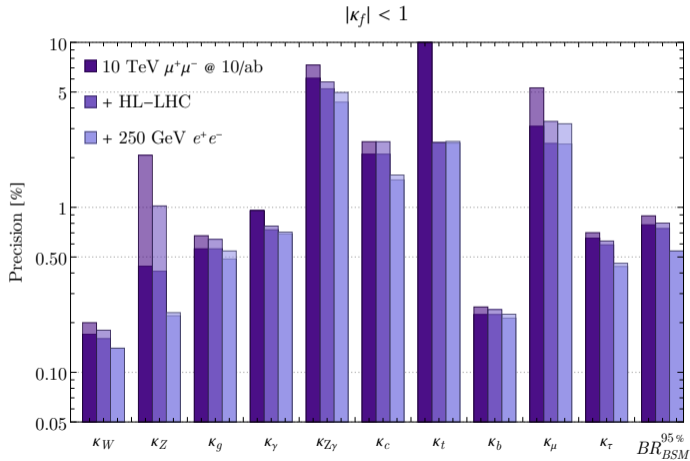
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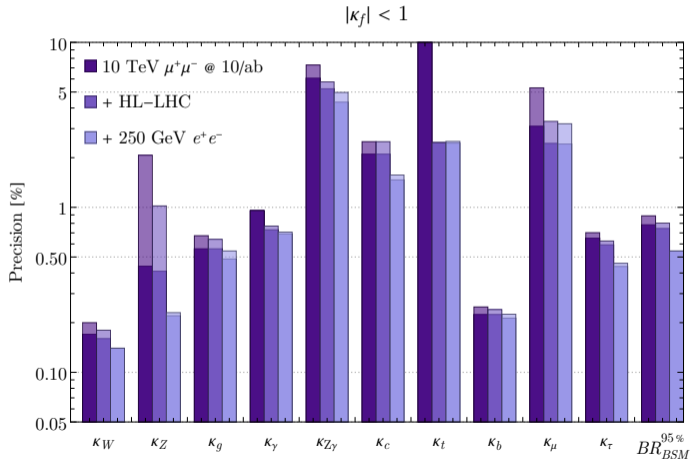
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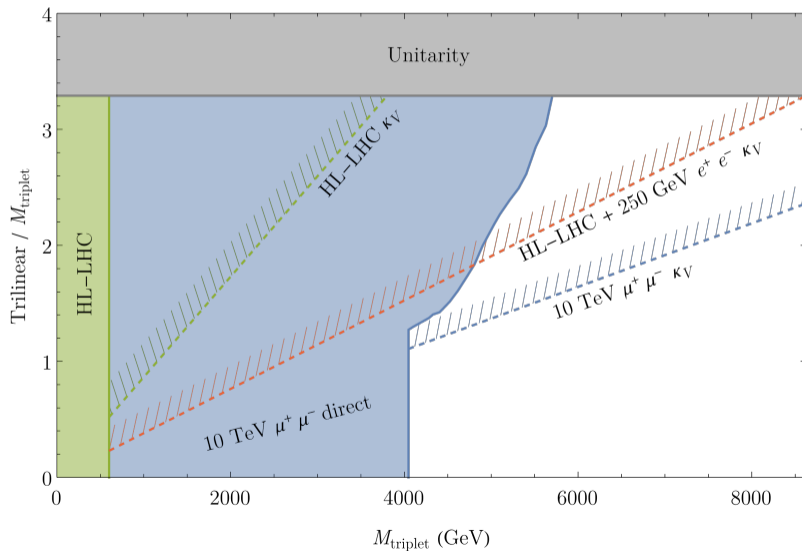
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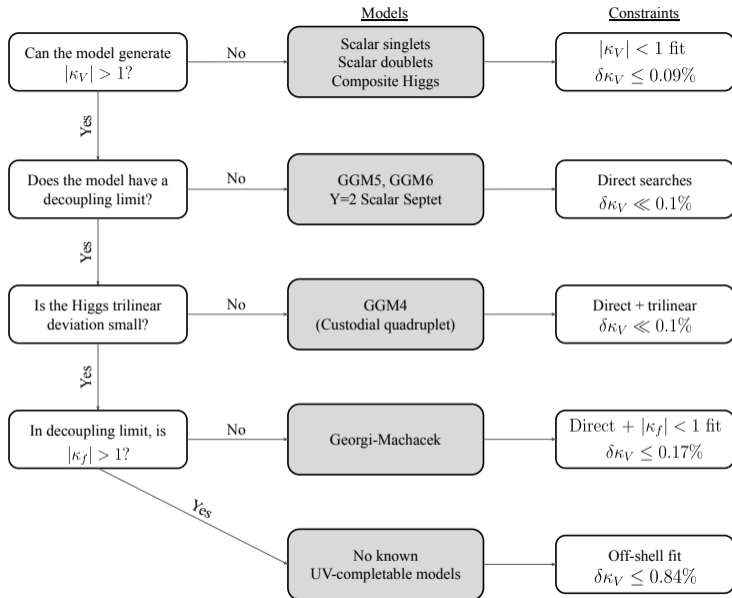
All other GM models (and 7-plet) would exclusively be *more* constrained

(Of these, only custodial quartet has a decoupling limit)



Putting the pieces together





What about probing BR_{BSM} ?

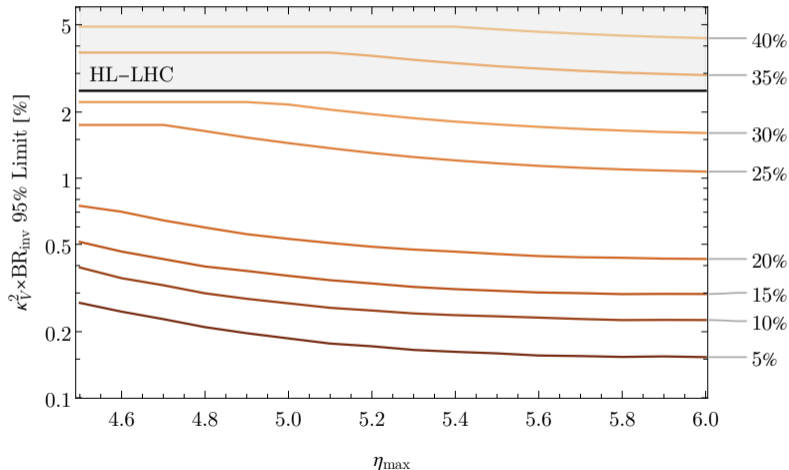
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Measure forward μ 's in
 $\mu^+\mu^- \rightarrow \mu^+\mu^- H, H \rightarrow inv$

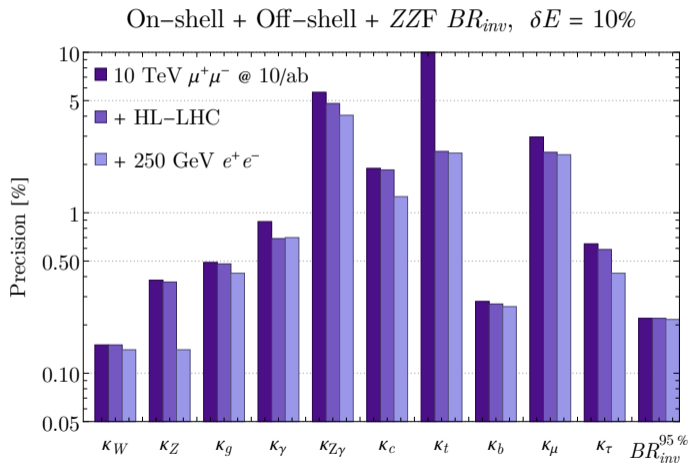
Highly sensitive to forward
detector energy resolution



(see also M. Ruhdorfer, E. Salvioni, A. Wulzer 2303.14202)

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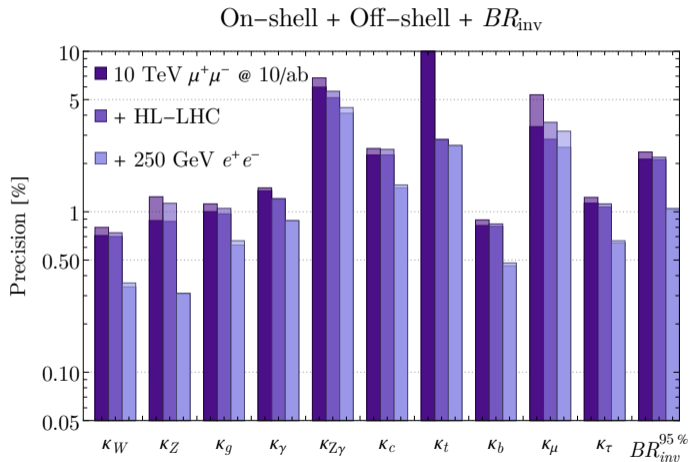
Once again, restored to $\sim 0.1\%$ level,
if δE is good enough



Without forward muons?

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Results from VBF HZ , HW^\pm , $H\gamma$
without ZZF forward tags not nearly
as good



Conclusion

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Great complementary between $\mu^+\mu^-$ collider and Higgs factories

Thank you!

BACKUPS

Flavour Tagging

b -tagging is done using the tight working point (50%) inspired by CLIC (1812.07337)

- c -quark mistagging rate $\leq 3\%$
- light quark mistagging rate $\leq 0.5\%$

For c -tagging, take 20% as our working point inspired by ILC studies (1506.08371).

- b -quark mistagging rate of flat 1.3%
- light quark mistagging rate of flat 0.66%

For $H \rightarrow \tau\tau$, we take a τ -tagging efficiency of 80% with a jet mistag rate of 2%.

Event Selection ($b\bar{b}$, $c\bar{c}$, $gg(+s\bar{s})$)

Apply an additional correction to b -jet p_T to account for energy losses during reconstruction (1811.02572)

- Smoothly scales 4-momentum by up to ~ 1.16 at low p_T
- Rough approximation to ATLAS $ptcorr$ correction (1708.03299)
- Reproduces a Higgs peak centered near 125 GeV

Apply a similar correction to c -jets

Events that pass the P_T and η cuts are then selected based on an invariant mass cut:

- $100 < M_{b\bar{b}} < 150$ for $b\bar{b}$
- $105 < M_{c\bar{c}} < 145$ for $c\bar{c}$
- $95 < M_{jj} < 135$ for $gg(+s\bar{s})$

$$c\bar{c}, gg(+s\bar{s}), \tau^+\tau^-$$

The dominant backgrounds for $c\bar{c}$ and $gg(+s\bar{s})$ are mostly the same as for $b\bar{b}$ and primarily removed via an M_{jj} cut

$H \rightarrow b\bar{b}$ becomes a large irreducible background

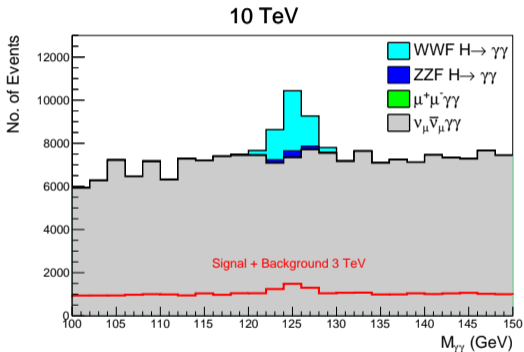
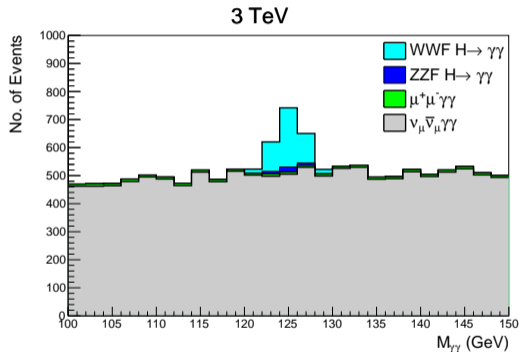
Following the same procedure as in $b\bar{b}$, we obtain results for $c\bar{c}$ and $gg(+s\bar{s})$:

Energy	Precision (%)	
	$c\bar{c}$	$gg(+s\bar{s})$
3 TeV	13	3.3
10 TeV	4.0	0.89

$\tau^+\tau^-$ follows a similar strategy with similar backgrounds, adding $\theta_{\tau\tau} > 15(20)$ cuts, to get 4.0(1.1)% precision.

$\gamma\gamma$ and $Z\gamma$

For $\gamma\gamma$, require no isolated leptons and a cut of $122 < M_{\gamma\gamma} < 128$.



The $Z(jj)\gamma$ process has similar backgrounds as the hadronic modes, but with more complicated cuts.

Full list of cuts: off-shell analysis

For $4j$, same cuts at 3 and 10 TeV:

- $p_{Tj} > 60$ GeV, $|\eta_j| < 2.5$, $30 < m_V^{min} < 100$ GeV, $40 < m_V^{max} < 115$ GeV

For $\ell^+ \ell^- jj$:

- $p_{T_{\ell,j}} > 20$ GeV, $|\eta_{j,\ell}| < 2.5$, $70 < m_{\ell\ell} < 115$ GeV, $40 < m_{jj} < 115$ GeV
- $\theta_{\ell\ell}, \theta_{jj} < 25^\circ$ (10 TeV)

For $\ell^\pm \nu_{\ell} jj$:

3 TeV:

- $p_{T_{\ell,j}} > 20$ GeV, $|\eta_{j,\ell}| < 2.5$, $p_{T_\ell} < 200$ GeV, $p_{T_{jj}} < 500$ GeV, $40 < m_{jj} < 115$ GeV

10 TeV:

- $p_{T_{\ell,j}} > 20$ GeV, $|\eta_{j,\ell}| < 2.5$, $p_{T_\ell} < 750$ GeV, $p_{T_{jj}} < 1200$ GeV, $40 < m_{jj} < 115$ GeV

Perturbative unitarity

There is a delicate cancellation between the Higgs diagrams and the W/Z continuum diagrams that prevents the longitudinal pieces from growing like $\mathcal{M} \sim E^2$

In extended scalar sectors, this requirement becomes a sum rule for each process

$$(\kappa_{VV}^h)^2 + \sum_i \alpha_i (\kappa_{VV}^i)^2 = 1$$

For example, for the Georgi-Machacek model, $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ yields

$$(\kappa_W^h)^2 + (\kappa_W^H)^2 + (\kappa_W^{H_5^0})^2 - (\kappa_W^{H_5^{++}})^2 = 1$$

Therefore if m_H and m_5 are below our off-shell analysis window, everything appears the same as in the SM, even if $\kappa_V \neq 1$.

A loophole in the off-shell measurement

Even if both the on-shell and off-shell regions appear SM-like, there is still a loophole.

We assumed the off-shell region scaled like the SM, but this is not true if additional scalars contribute to electroweak symmetry breaking.

When these additional scalars contribute to $VV \rightarrow VV$, combination with SM will restore perturbative unitarity of off-shell region, making it appear to be SM, even if $\kappa_V \neq 1$.

This restoration only occurs above resonance: must be lighter than our off-shell analysis window – direct searches probe them

Full list of cuts: BR_{inv}

For γH , and $W^\pm H \rightarrow \ell^\pm \nu_\ell H$, only one observed particle, so only one set of cuts:

- $p_{T_{\gamma,\ell}} > 40 \text{ GeV}$, $|\eta_{\gamma,\ell}| < 2.5$

For $ZH \rightarrow \ell^+ \ell^- H$:

- $p_{T_\ell} > 20 \text{ GeV}$, $|\eta_\ell| < 2.5$, $80 < m_{\ell\ell} < 100 \text{ GeV}$, $R_{\ell\ell} > 0.2$

For $VH \rightarrow jjH$:

- $p_{T_j} > 40 \text{ GeV}$, $|\eta_j| < 2.5$, $60 < m_{jj} < 100 \text{ GeV}$

For $\mu^+ \mu^- H$ (forward tagging, only 10 TeV):

- $p_{T_\mu} > 20 \text{ GeV}$, $p_{T_{\mu\mu}} > 100 \text{ GeV}$, $R_{\mu\mu} > 9$, $m_{\mu\mu} > 8000 \text{ GeV}$