How Matter Matters:

The Story of Time Invariance Violation in Neutrino Oscillations

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Neutrino pictures: https://www.particlezoo.net/

Motivation from the CPT symmetry story

If we follow that CPT is a fundamental symmetry.

CPT

- ✓ Charge conjugation
- ✓ Parity
- ✓ Time reversal

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Charge parity transform (CP) alone is violated in the weak sector.

CPT + ~~CP~~

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- ✓ Parity
- ✓ Time reversal

Motivation from the CPT symmetry story

If we follow that CPT is a fundamental symmetry.

Charge parity transform (CP) alone is violated in the weak sector.

Time reversal transforms alone should also be violated in weak interactions, in order to preserve the overall symmetry.

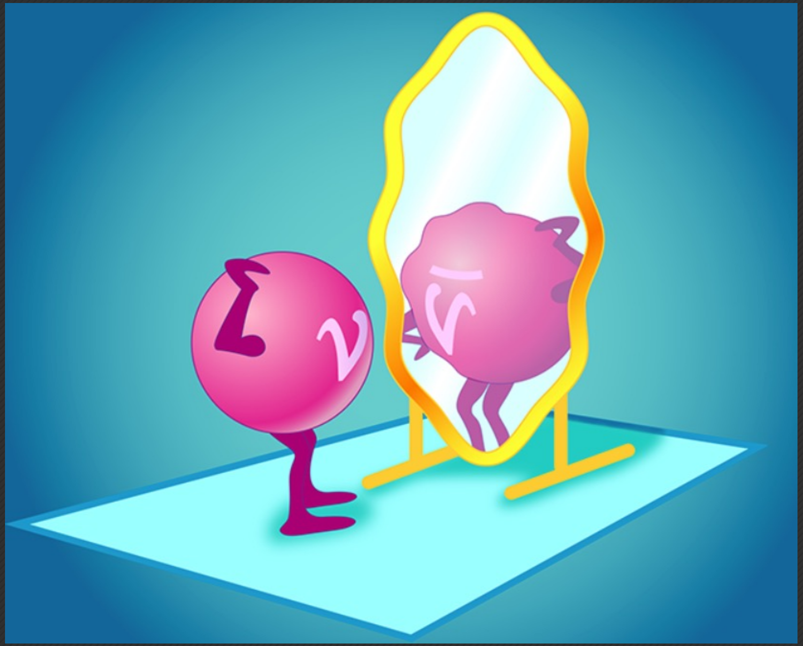
- ✓ Charge conjugation
- ✓ Parity
- ✓ Time reversal

Motivation from the CPT symmetry story

- ✓ Neutrino physics is a well motivated probe for CP violation, but we are limited to "improper tests" due to our inability to build experiments in an anti-Earth.

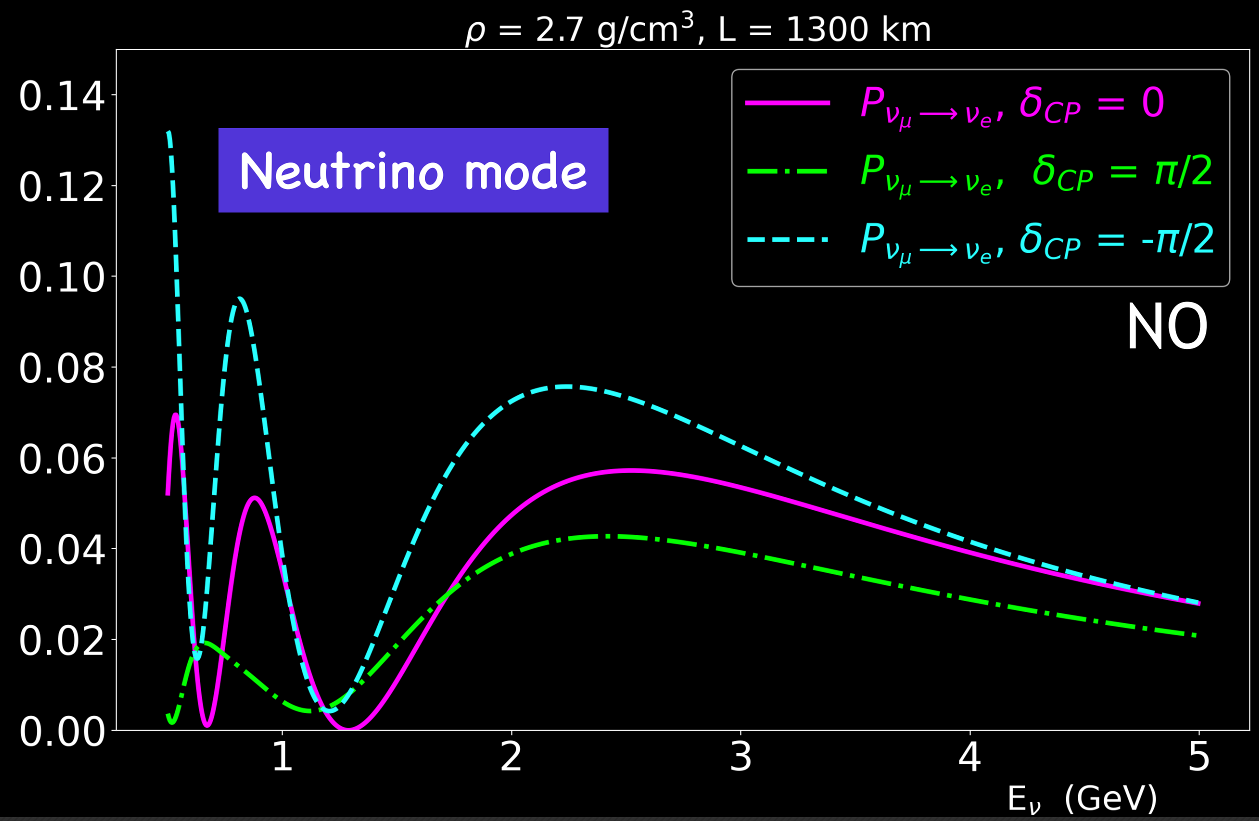
Motivation from the CPT symmetry story

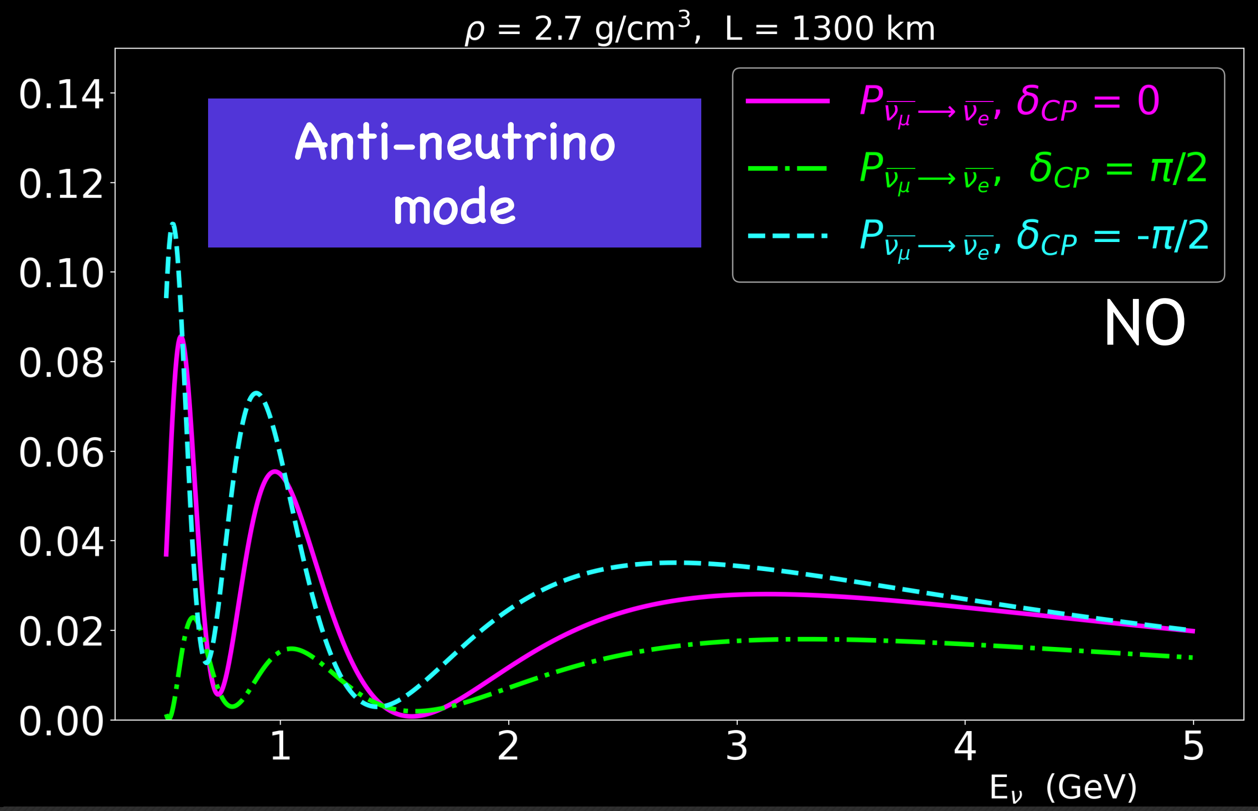
- ✓ Neutrino physics is a well motivated probe for CP violation, but we are limited to "improper tests" due to our inability to build experiments in an anti-Earth.
- ✓ Let us then consider to what extent time invariance violation occurs within the neutrino sector.
- ✓ Why? New physics may not impact both CP and time reversal in the same way.



Source:https://physics.aps.org/articles/v15/120
Credit:APS/Carin Cain

Motivation from the experiments story



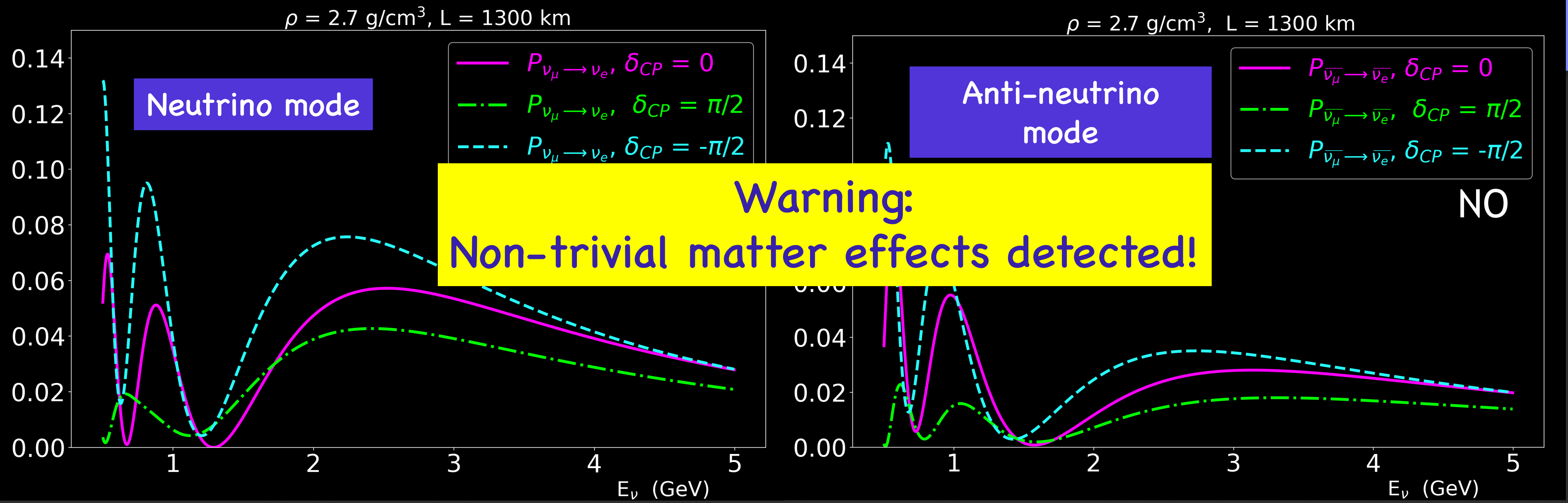


CP conjugate channels are the *most common probes,* as they are more accessible to experiments like long baselines.

DUNE DEEP UNDERGROUND NEUTRINO EXPERIMENT

Source:https://www.dunescience.org/

Motivation from the experiments story



CP conjugate channels are the *most common probes,* as they are more <u>accessible to experiments</u> like long baselines.

DUNE DEEP UNDERGROUND NEUTRINO EXPERIMENT

Source:https://www.dunescience.org/

Enter Time Invariance Violation Tests

- ✓ Time invariance violation tests may provide a clearer way* to aid in our understanding of how different matter profiles can affect neutrino oscillations

(i.e. distinguishing between intrinsic & induced time invariance violation)

*dependent upon matter potential profile

Source: (Time-turner) https://tenor.com/view/time-turner-harry-potter-moving-spinning-gif-16031036

Enter Time Invariance Violation Tests

Time invariance tests require comparing

$$P_{\nu_\mu \to \nu_e} \quad \text{vs} \quad P_{\nu_e \to \nu_\mu}$$

(or anti-neutrino versions)

We assume that a new beam capable of producing high energy ν_e's exists (i.e. muon storage rings as neutrino factories).

Source: https://www.symmetrymagazine.org/article/november-2012/how-to-make-a-neutrino-beam?language_content_entity=und

Enter Time Invariance Violation Tests

Time invariance tests are *not new*, but the full range of nuances with a variety of different matter profiles, is perhaps not as widely appreciated.

Our aim is to provide a fresh perspective and different insight in this pedagogical study.

For more details about previous work around this topic, please see backup slides.

Recalling 3-Flavor Neutrino Oscillations with Charge-Current Matter Effects

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

(PMNS matrix)

$$H = \frac{1}{2E_\nu} U M^2 U^\dagger + A$$

Sources: https://neutrino.syr.edu/research/neutrino-oscillations/ & https://neutrino-history.in2p3.fr/neutrino-oscillation/

$$M^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix}$$

$$A = \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

+/- for neutrinos/antineutrinos

Defining Time Invariance Measures

To clarify what "comparisons" we are making in looking for time invariance violation effects, we've specified two distinct measures:

1. proper time invariance: (aka: true time invariance violation)

    - Not a good observable, but certainly no harder to calculate than CP conjugate channels.

    - Requires comparing probabilities with final states exchanged and swapping the detector with source.

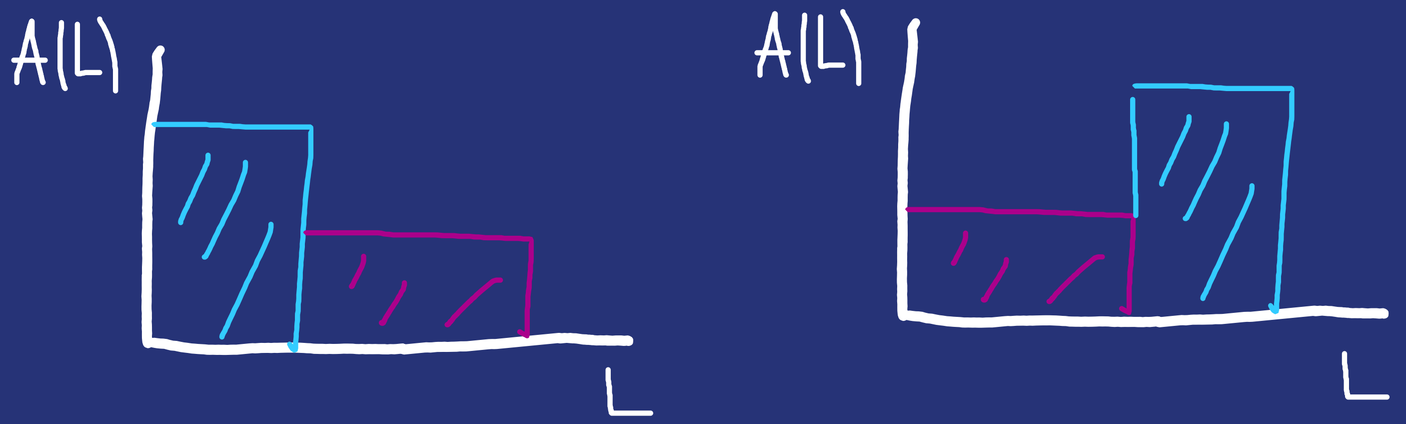
Defining Time Invariance Measures

To clarify what "comparisons" we are making in looking for time invariance violation effects, we've specified two distinct measures:

1. proper time invariance: (aka: true time invariance violation)

$P_{\nu_e \to \nu_\mu}$ vs $P_{\nu_\mu \to \nu_e}$

(decreasing potential) (increasing potential)



ex: 2-step piece-wise constant matter profiles
A_{00} = A(L) = $\sqrt{2} G_F N_e$

N_e changes depending on density

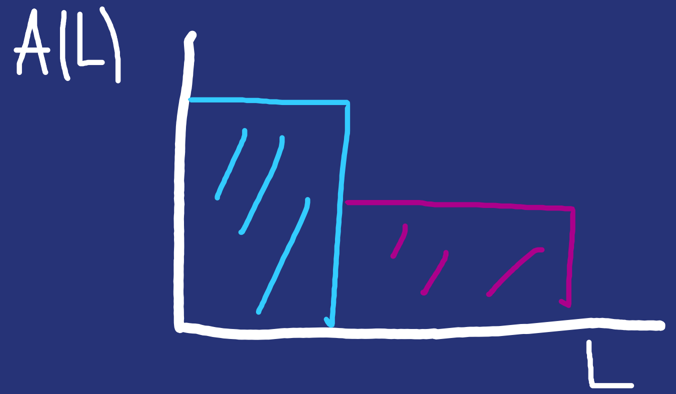
Defining Time Invariance Measures

2. improper time invariance: (next best thing!)

- Compares probabilities with only the final states exchanged.

$$P_{\nu_e \to \nu_\mu} \quad vs \quad P_{\nu_\mu \to \nu_e}$$

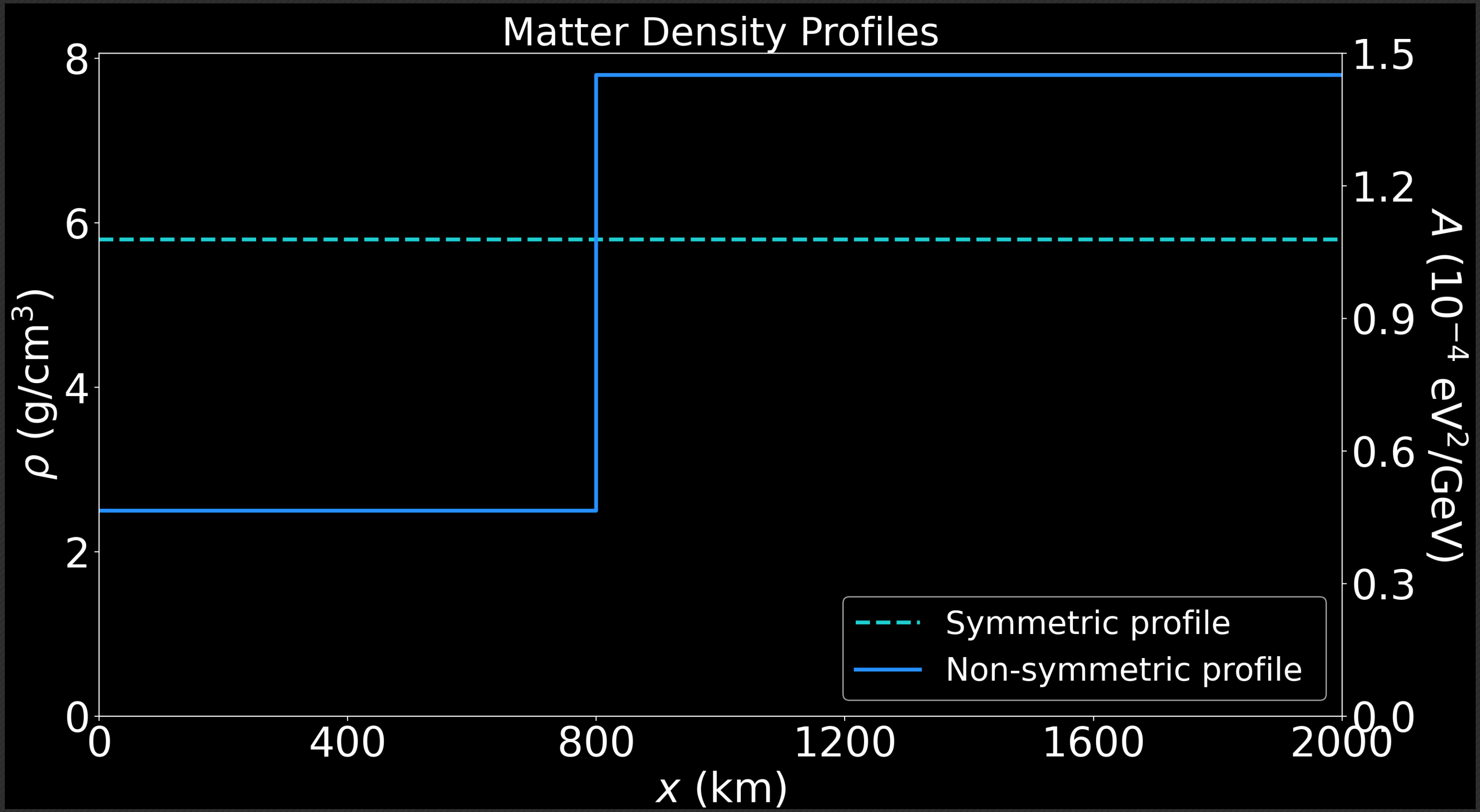
(decreasing potential) (decreasing potential)



This is what an experiment can measure!

Modeling Matter Effects for 3-Flavors

- ✓ For the purposes of our study, we separately two types of matter potential profiles.



Symmetric: vacuum or single step constant matter potential profile

Non-symmetric: piece-wise matter potential profiles (increasing or decreasing)

Modeling Matter Effects for 3-Flavors

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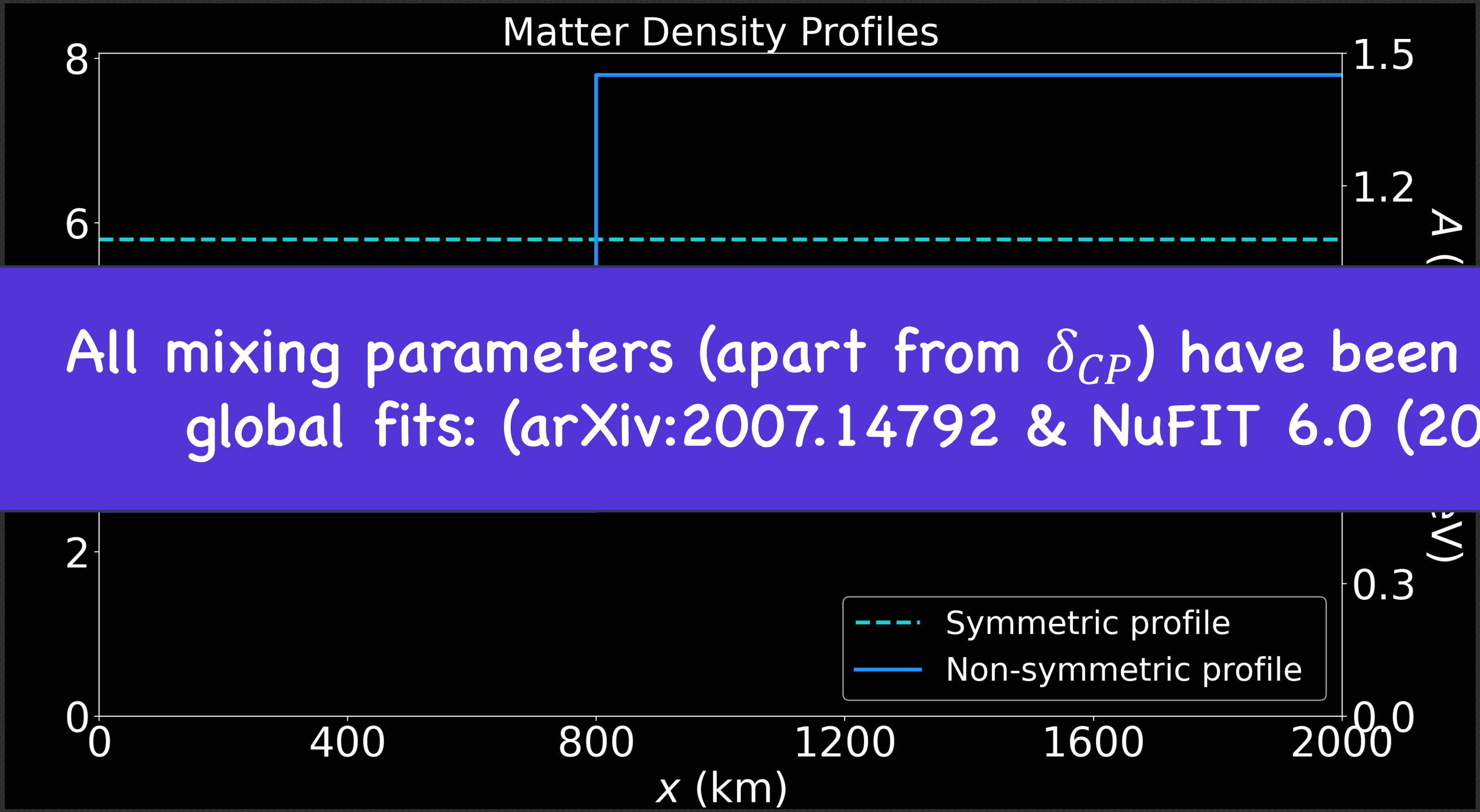


Symmetric: vacuum or single step constant matter potential profile

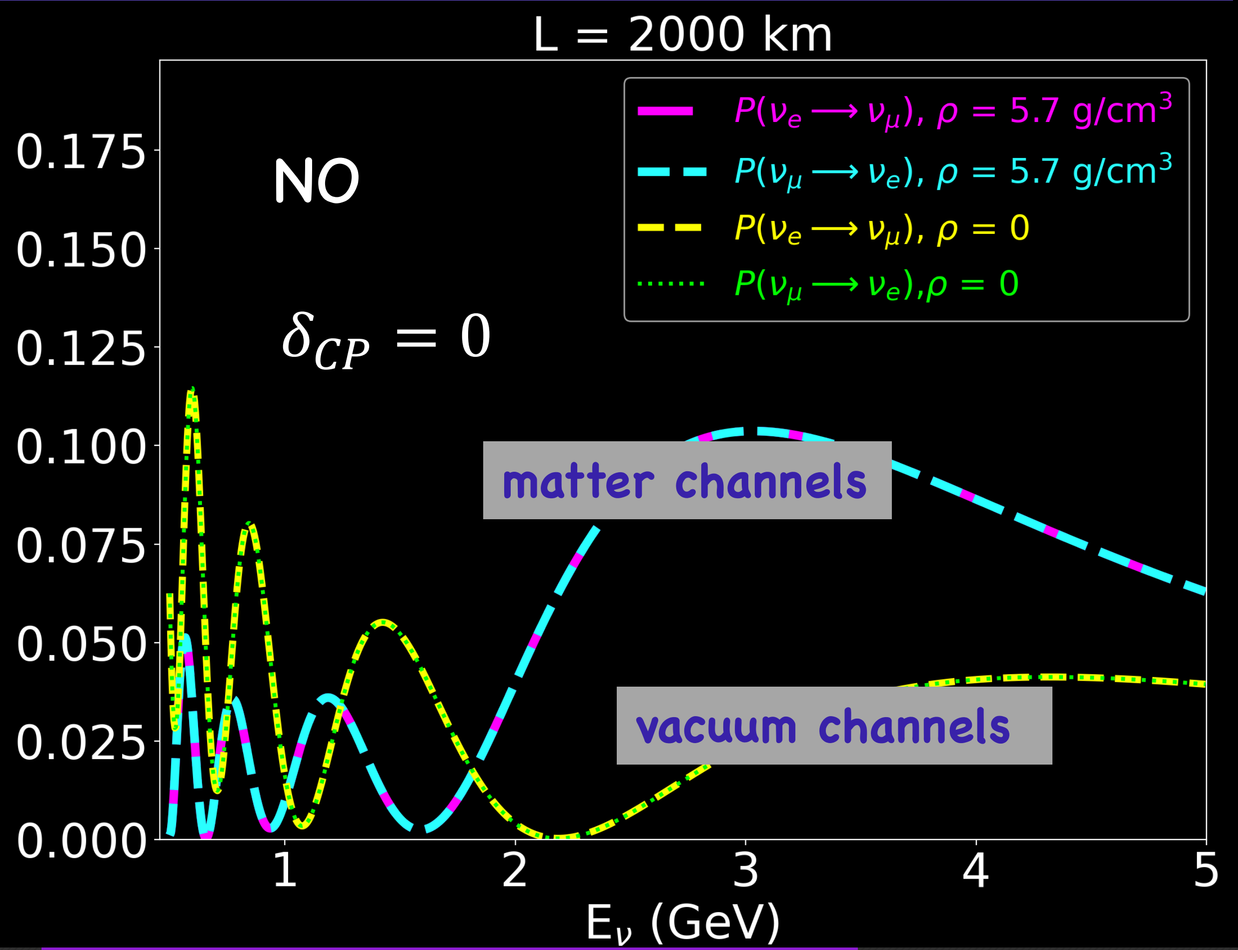
All mixing parameters (apart from δ_{CP}) have been drawn from NuFIT 2024 global fits: (arXiv:2007.14792 & NuFIT 6.0 (2024), www.nu-fit.org)

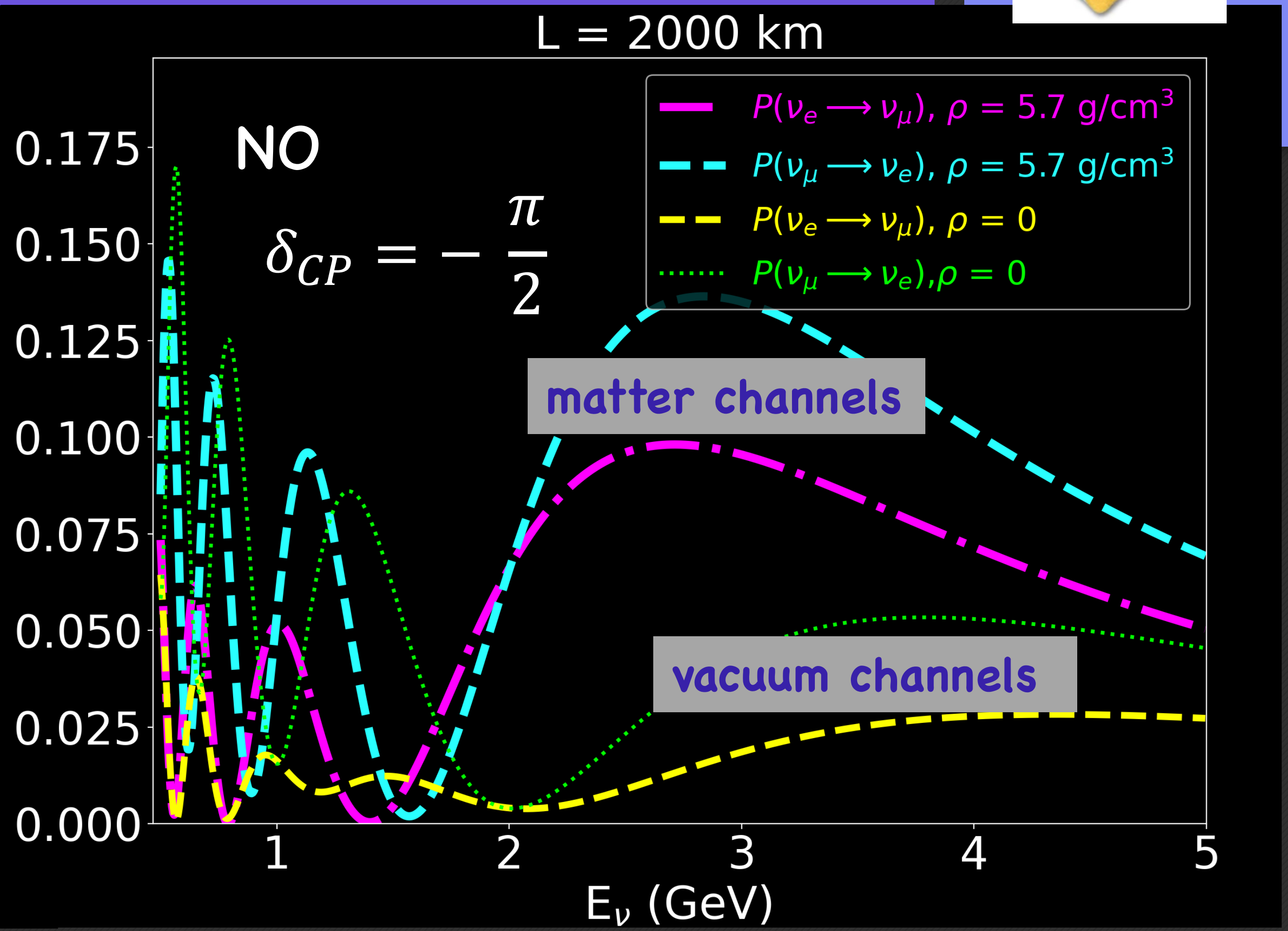
Disclaimer!

The first part of the following analysis is done with **hypothetical/fictious matter effects**, to get a general sense of how intrinsic versus induced time invariance violation behave with matter effects in cases that the have stronger oscillation differences.

Symmetric Matter
Potential Profiles

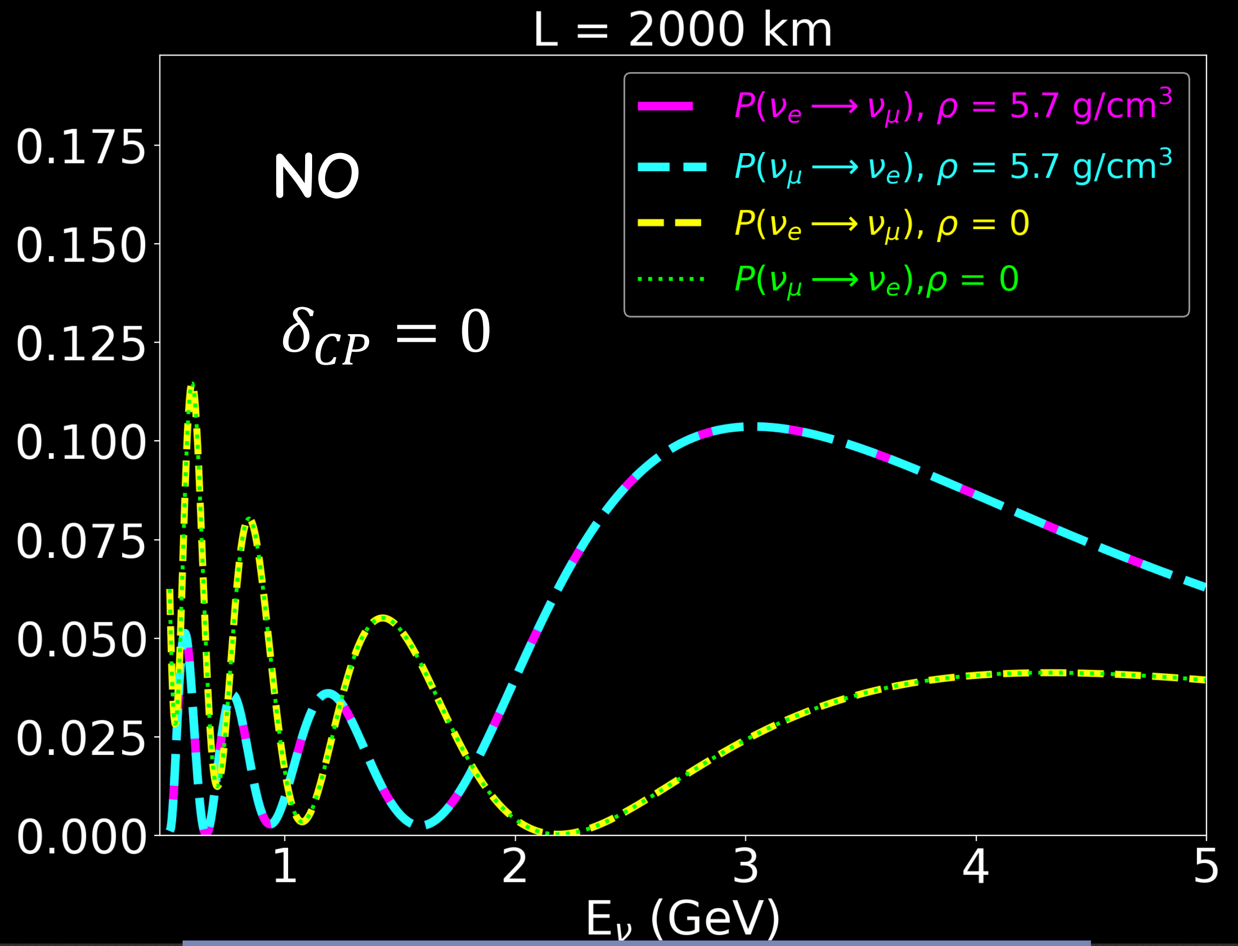
Symmetric matter effects for 3-Flavors

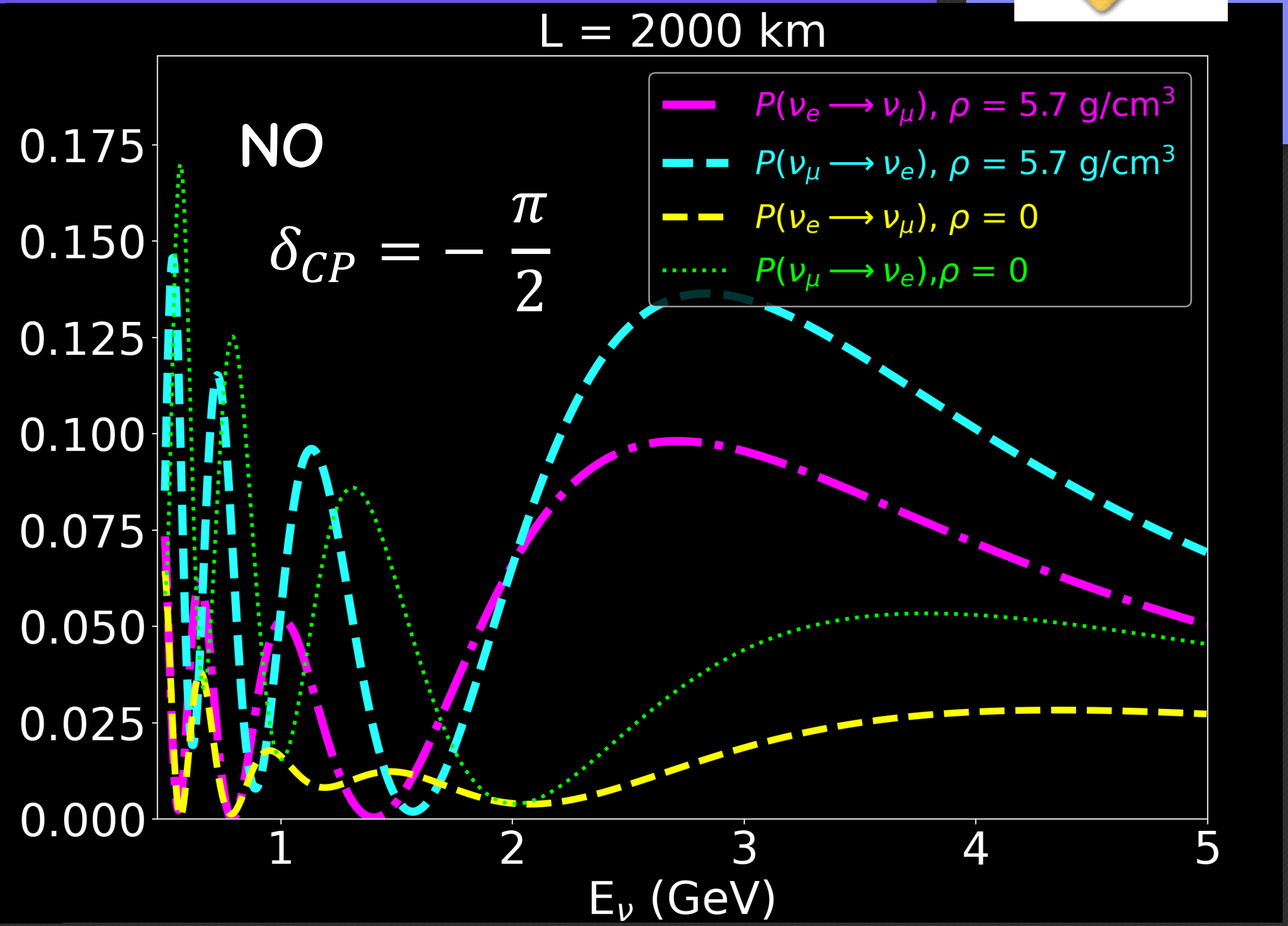




Constant matter potential:
1.1×10^{-3} eV^2/GeV (5.7 g/cm^3)

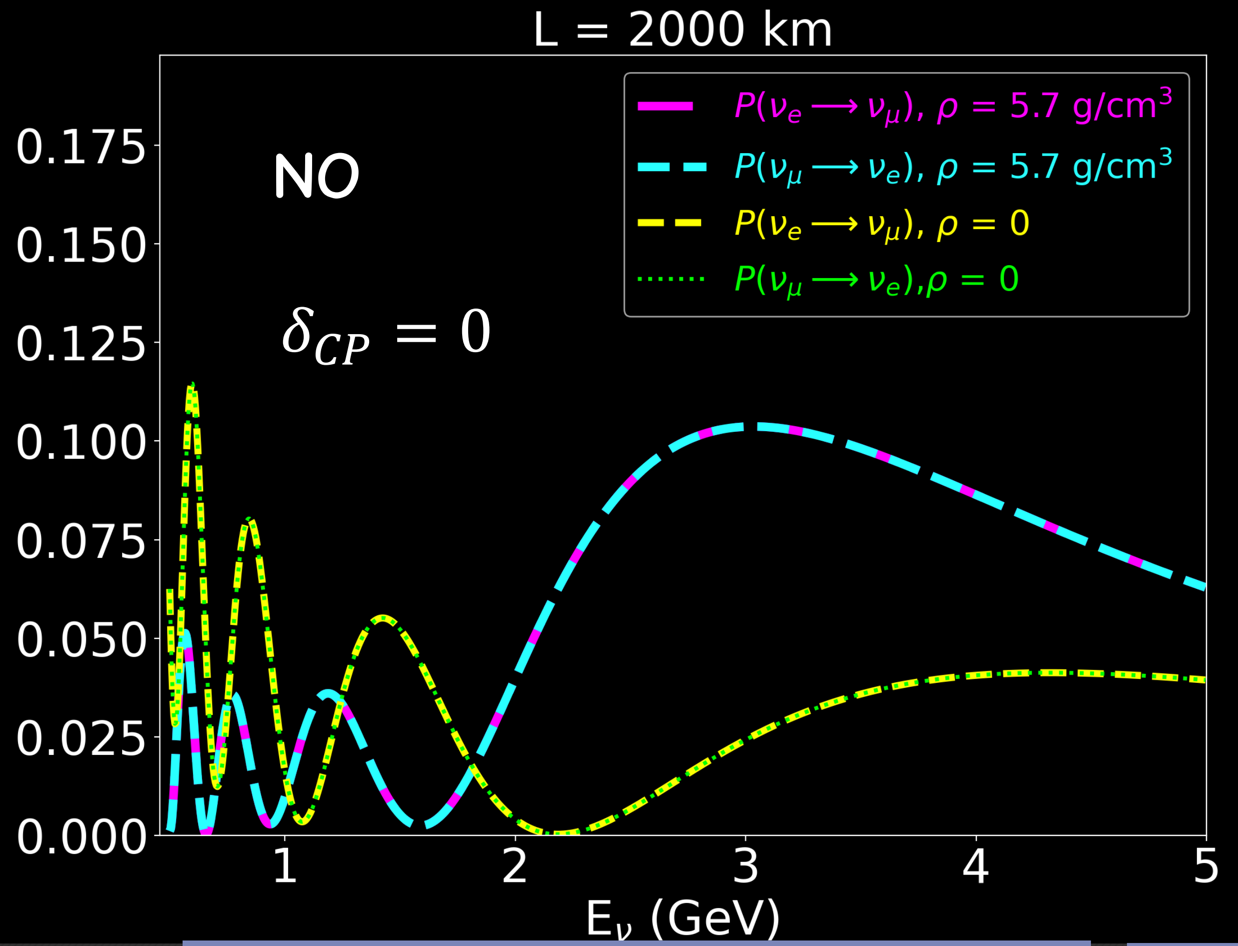
Symmetric matter effects for 3-Flavors

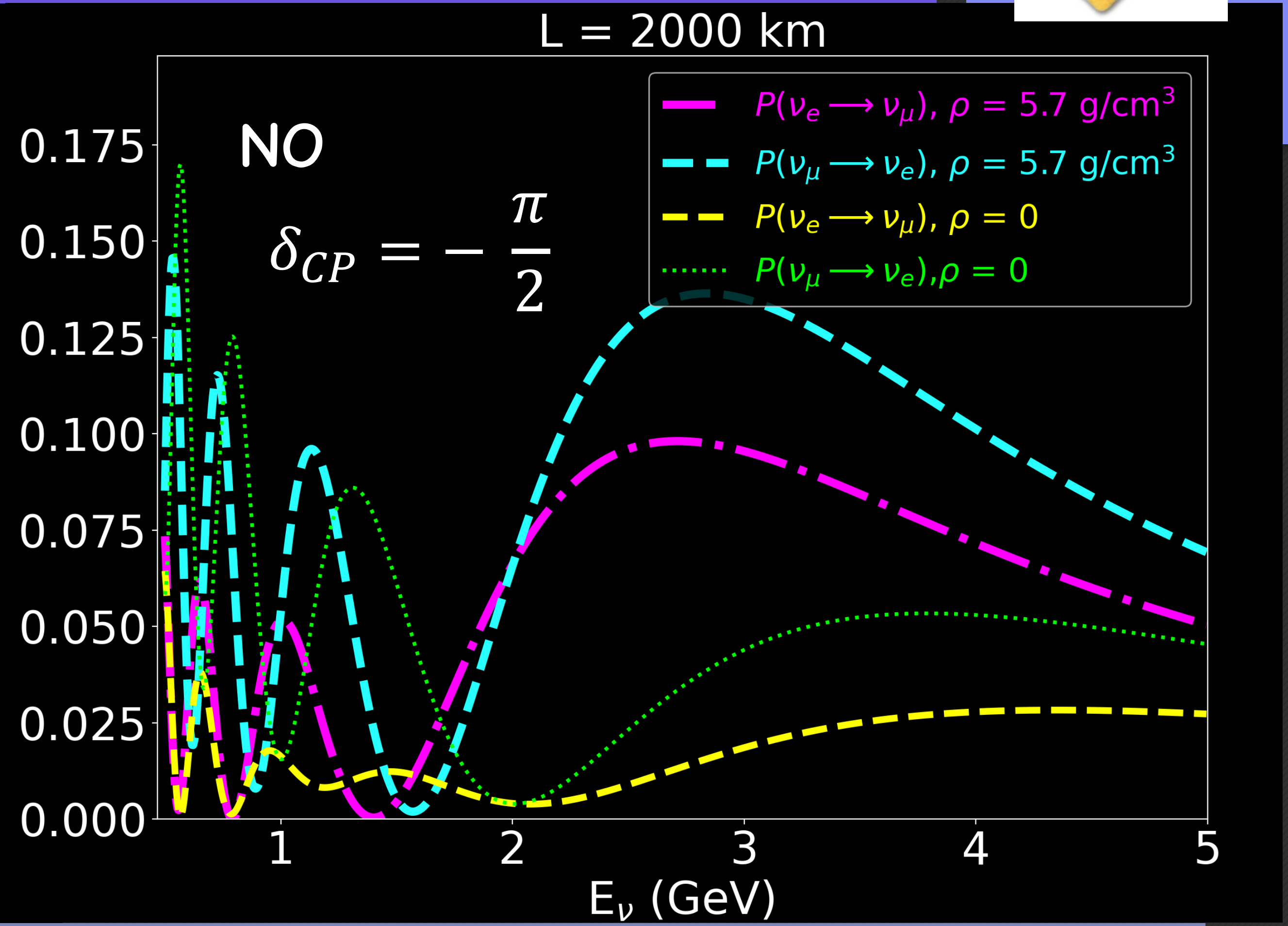




1. Symmetric matter potentials *cannot induce time invariance violation.*

Symmetric matter effects for 3-Flavors

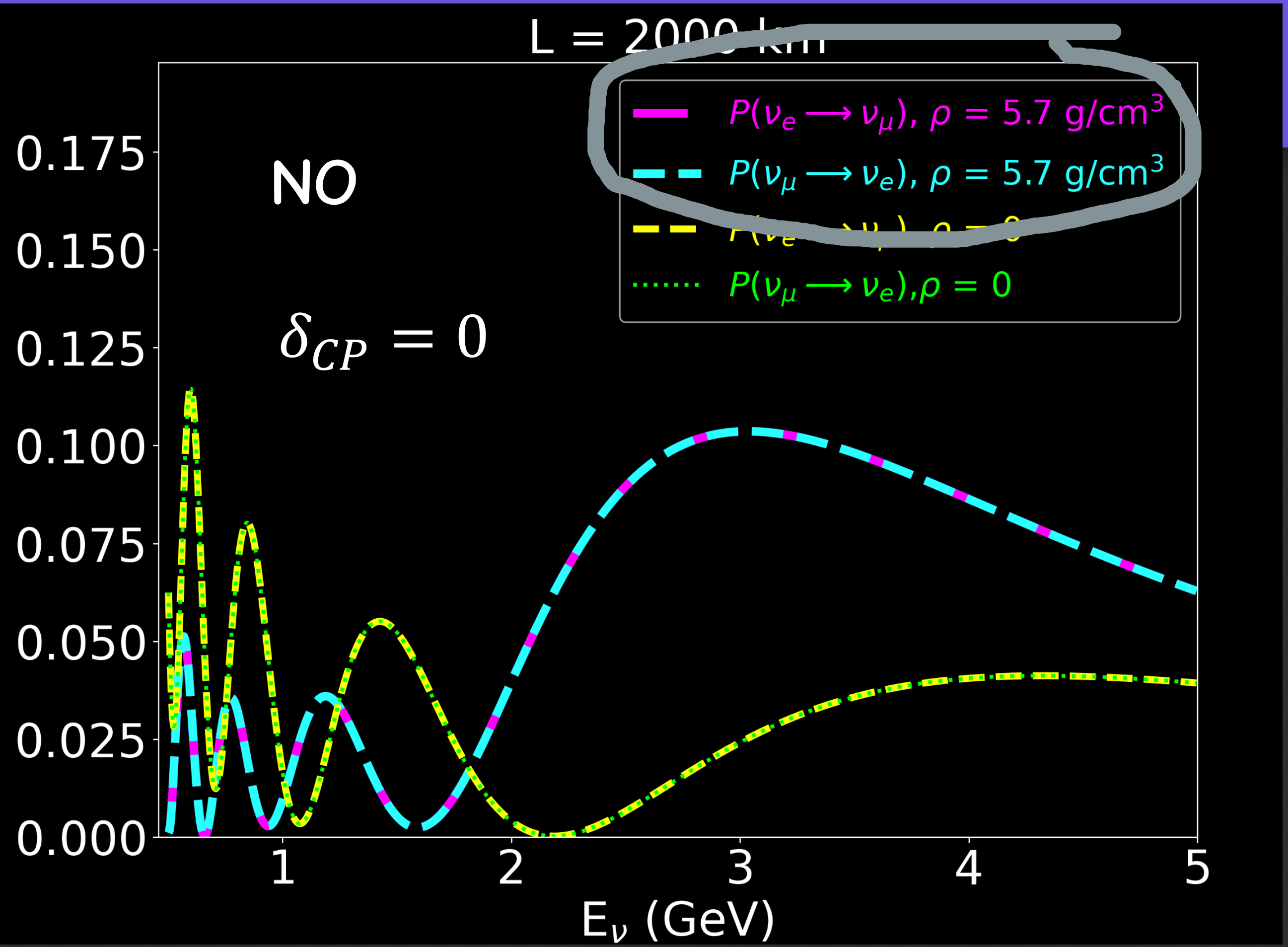




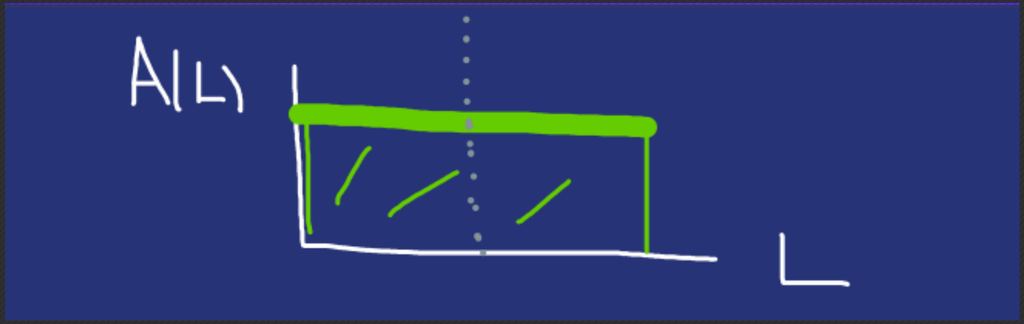
1. Symmetric matter potentials *cannot induce time invariance violation.*

2. If there is intrinsic time invariance violation from δ_{CP}, then the matter potential *modifies the size of the effects.*

Symmetric matter effects for 3-Flavors



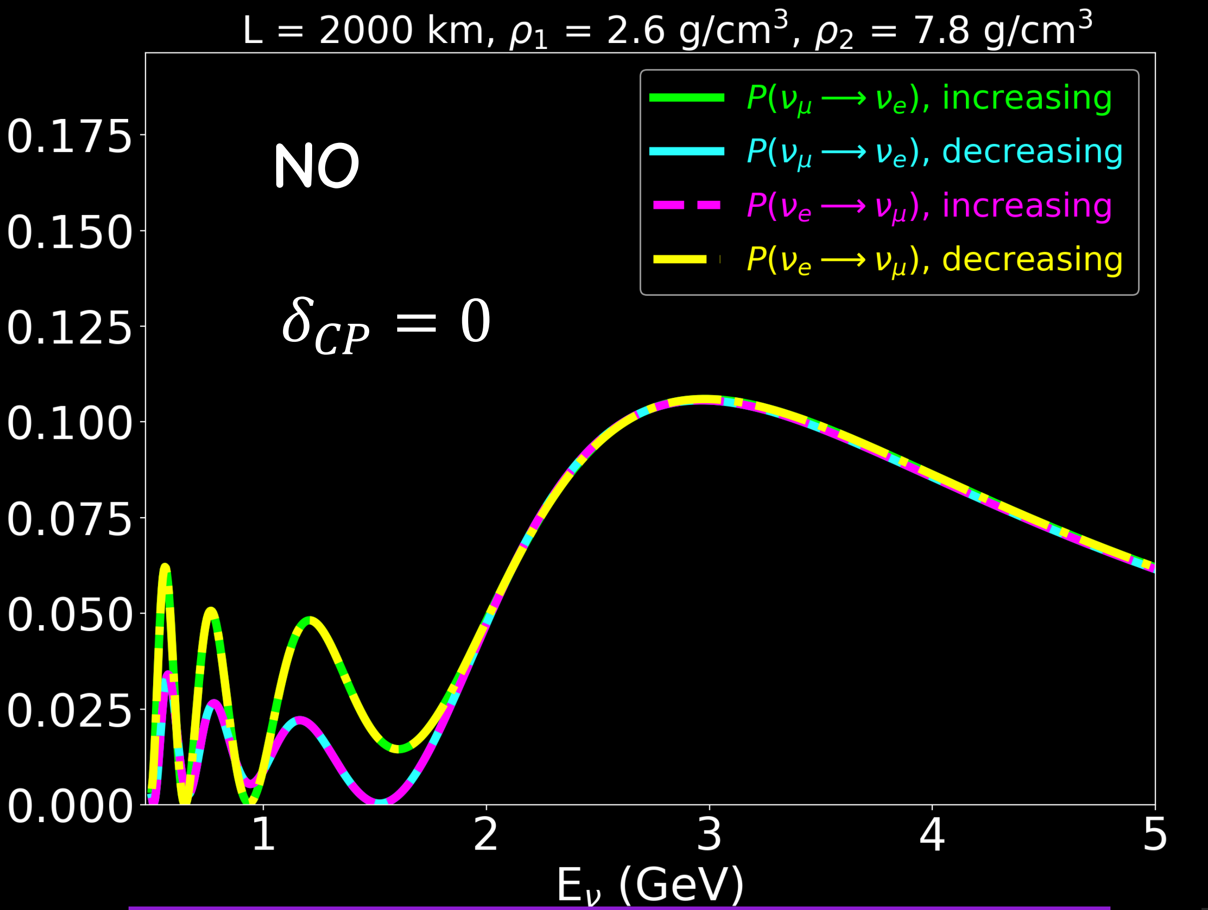
If we exchange final states, same matter potential profile.

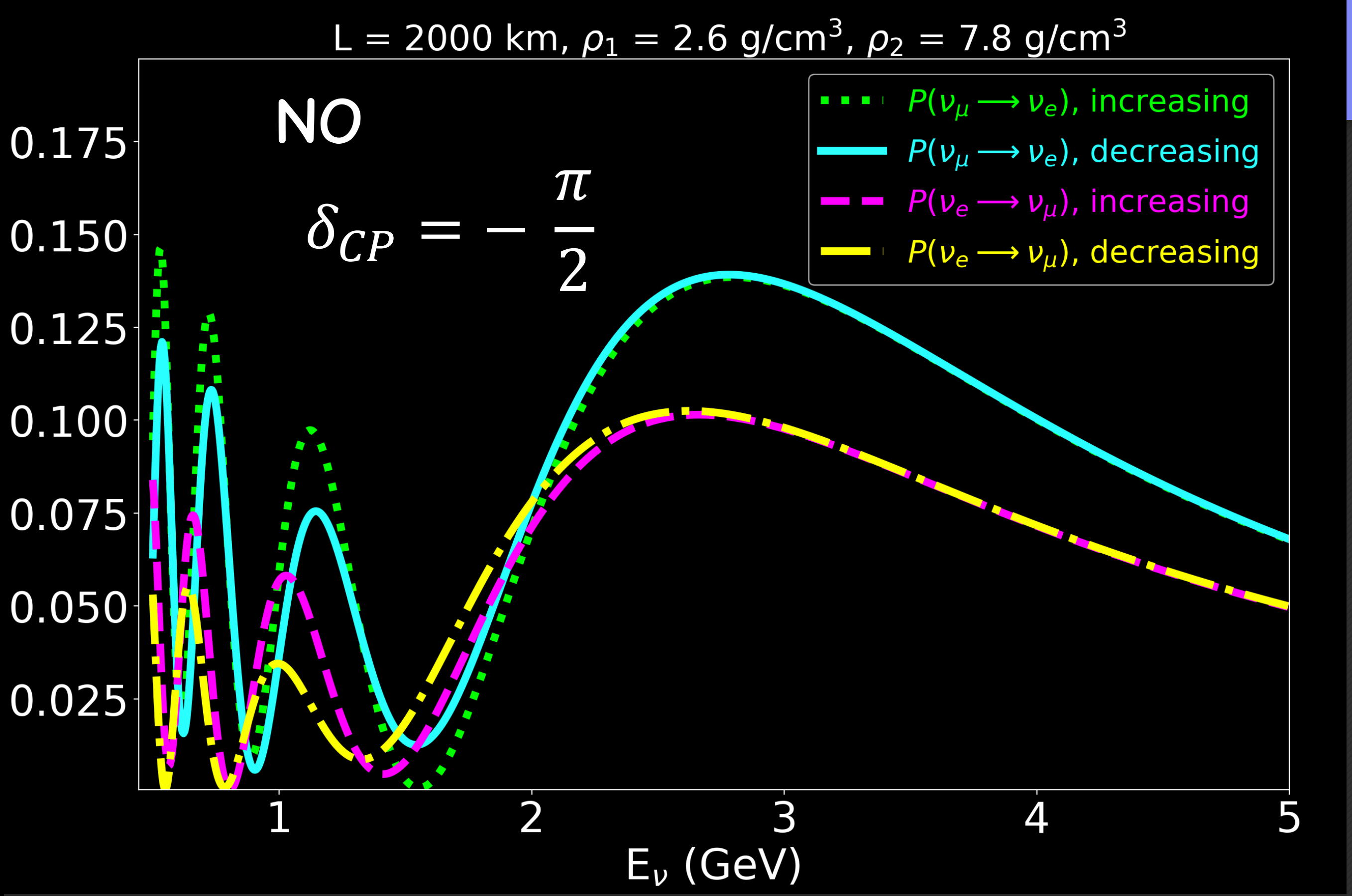


3. Improper and proper comparisons are the same if the *matter potential is symmetric*.

Non-Symmetric Matter
Potential Profiles

Non-symmetric matter effects for 3-Flavors



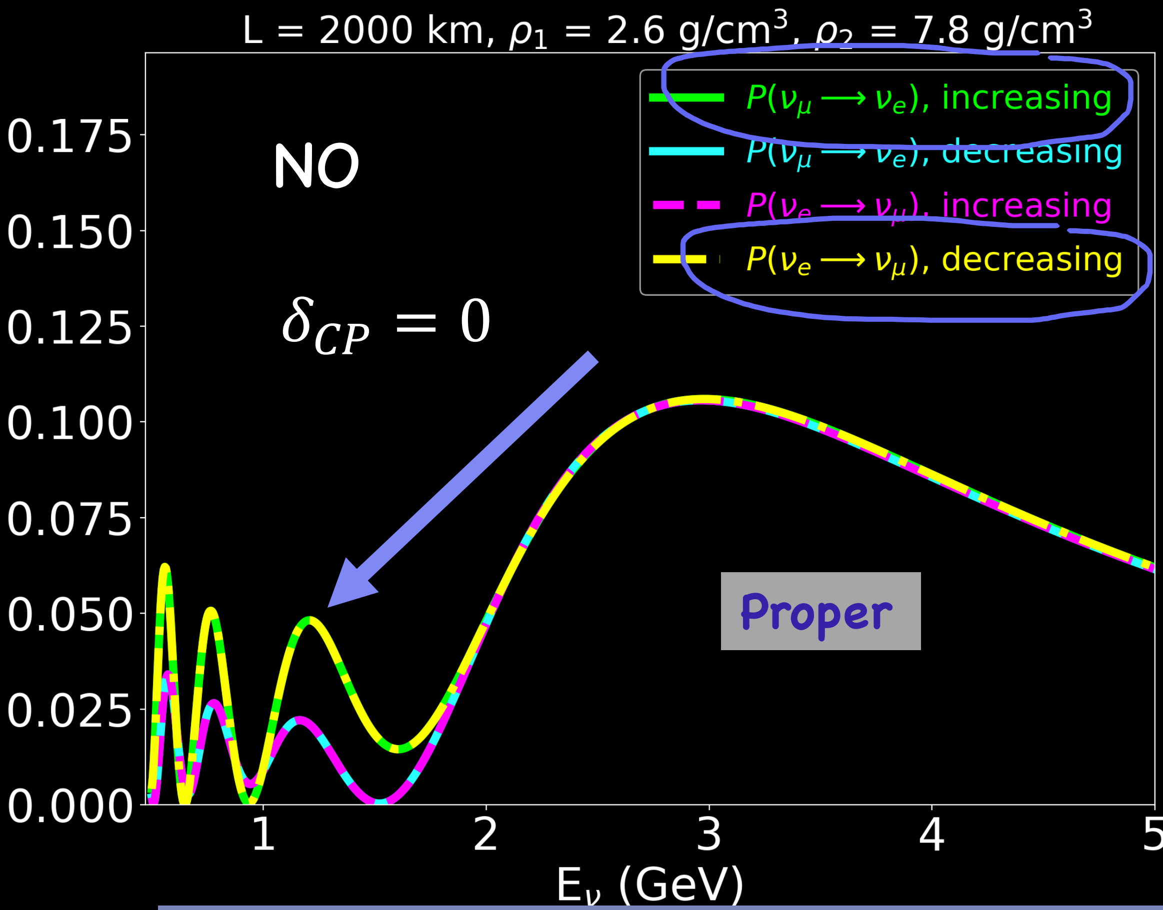


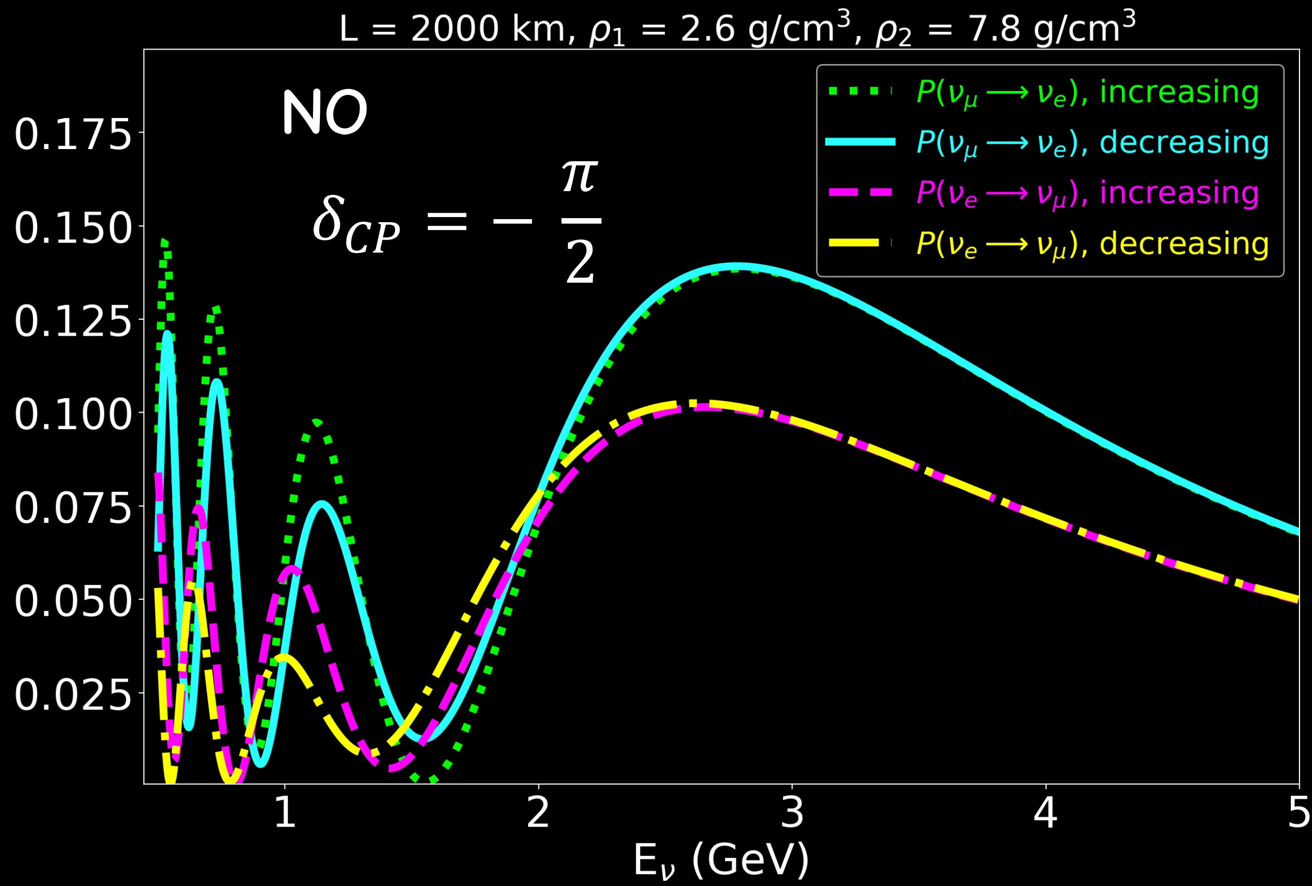
Piecewise matter potential (2 steps):

A_1 =5x10^{-4} eV^2/GeV (2.6 g/cm^3)

A_2 =1.5 x10^{-3} eV^2/GeV (7.8 g/cm^3)

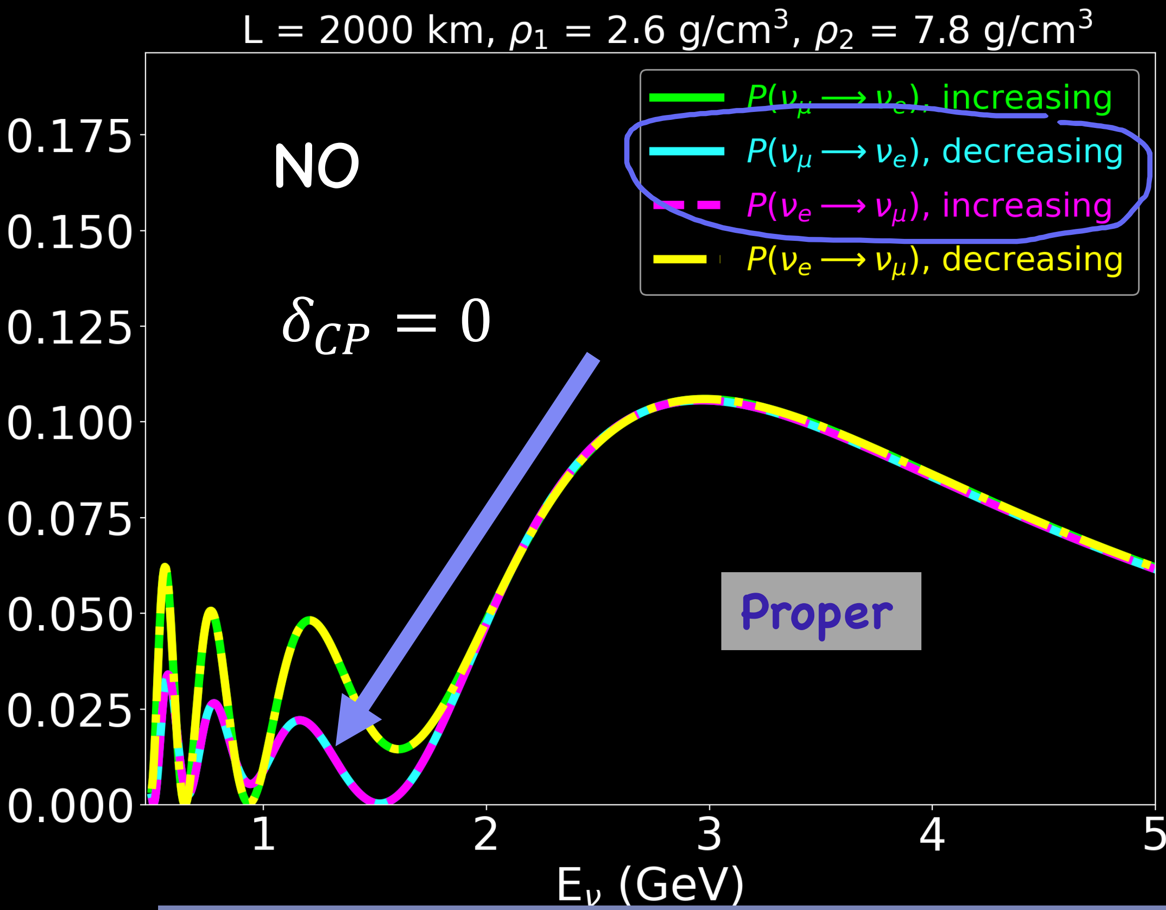
Non-symmetric matter effects for 3-Flavors

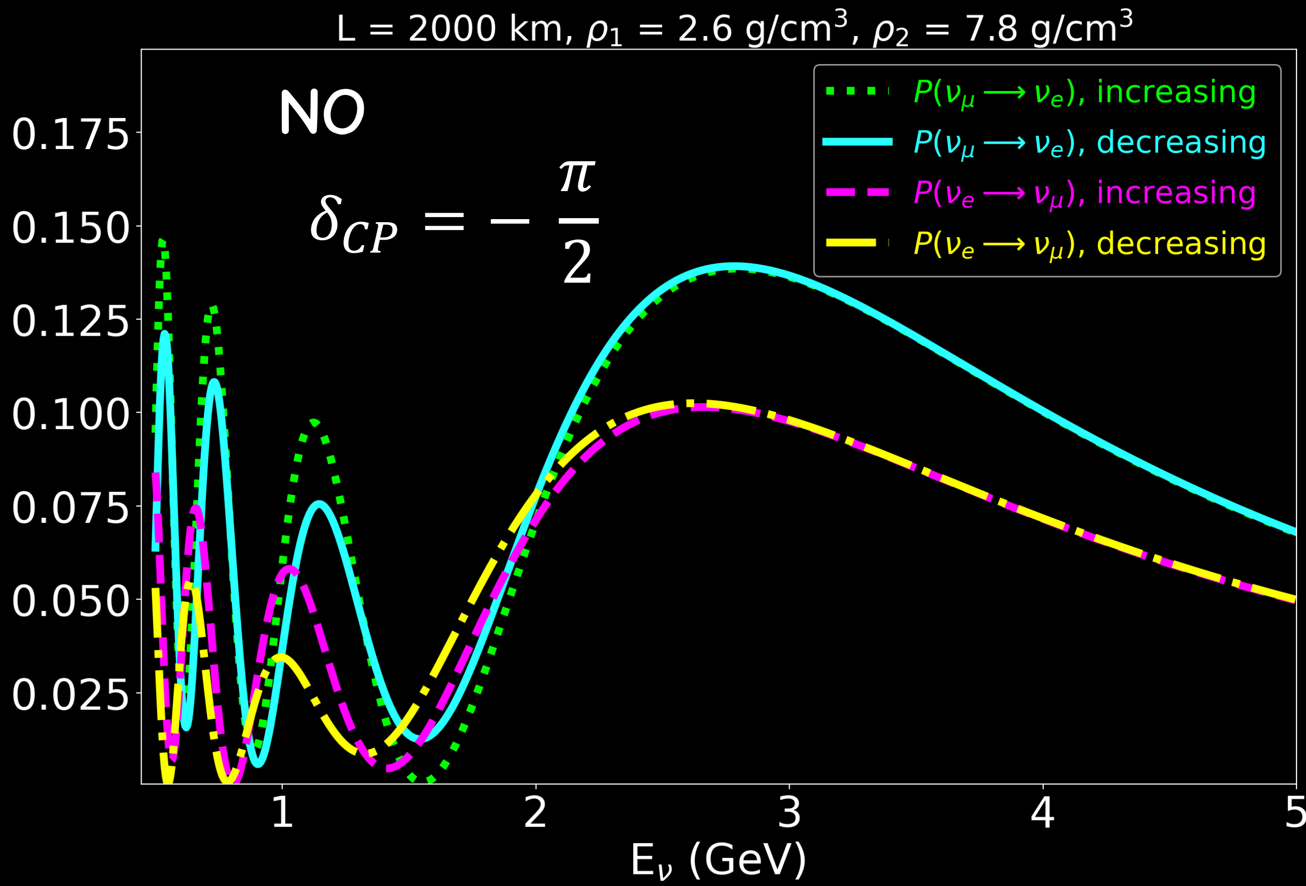




1. Non-symmetric matter effects are pairwise degenerate in the proper measure $(\delta_{CP} = 0)$

Non-symmetric matter effects for 3-Flavors





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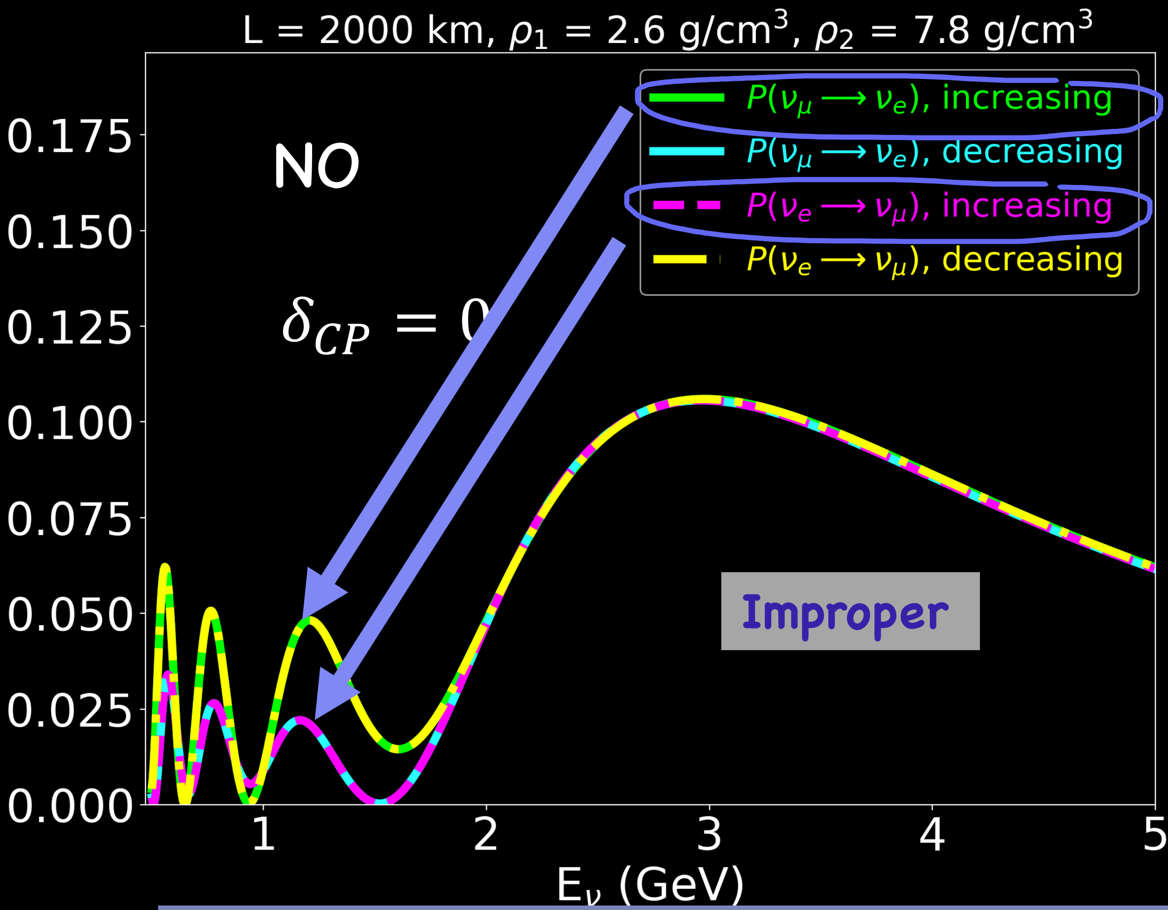
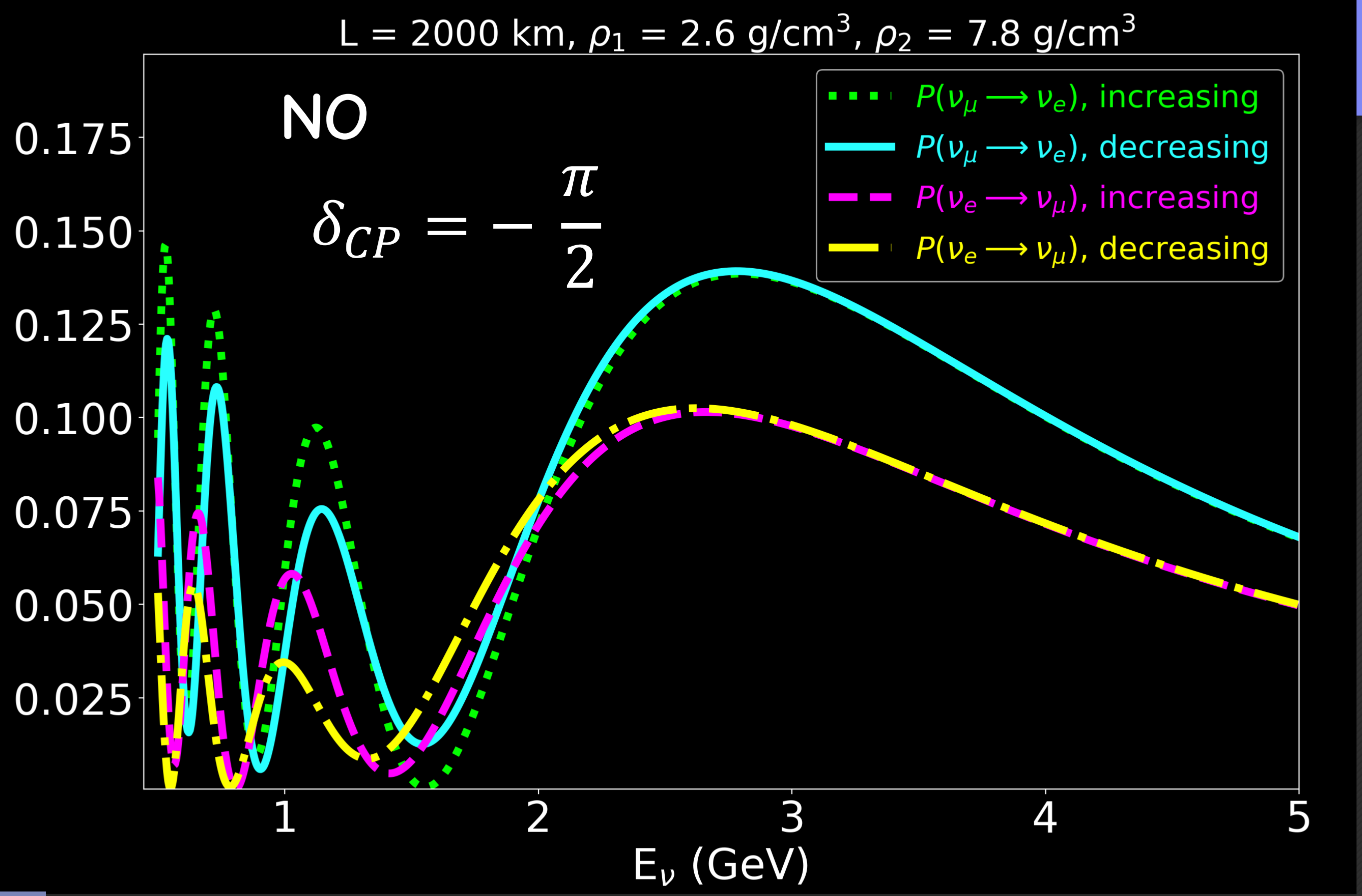
Non-symmetric matter effects for 3-Flavors

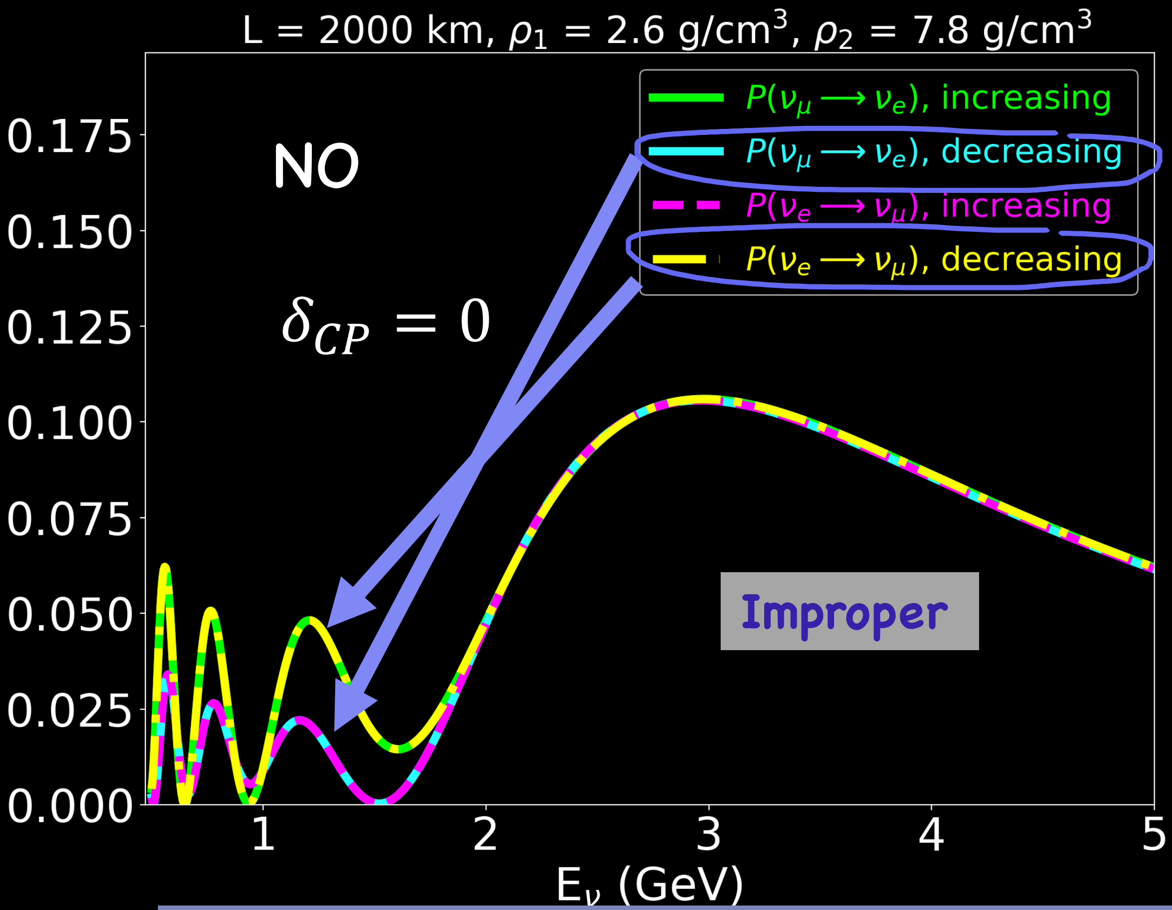


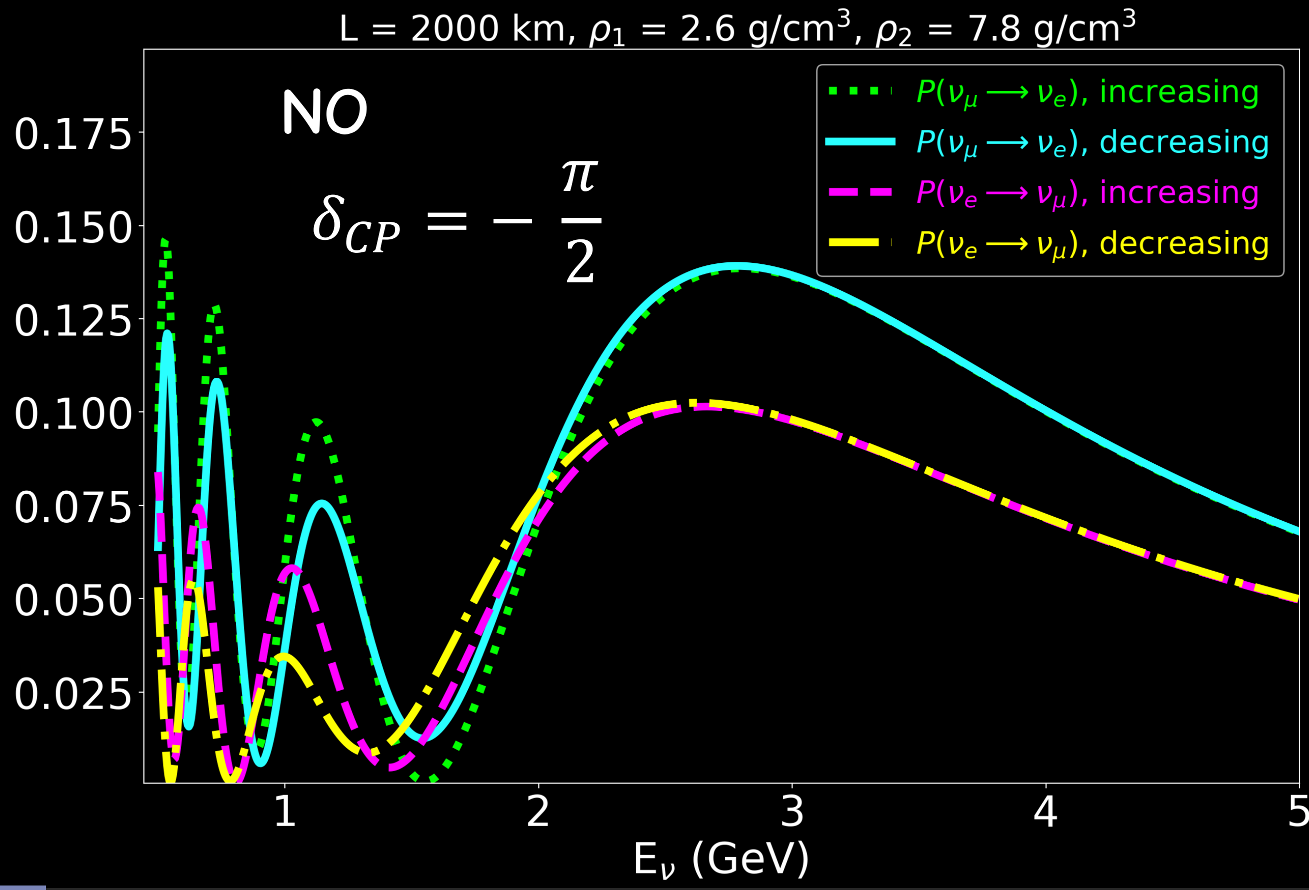


1. Non-symmetric matter effects are pairwise degenerate in the proper measure ($\delta_{CP} = 0$)

2. Proper time *remains invariant*, but improper channels are *different*.

Non-symmetric matter effects for 3-Flavors





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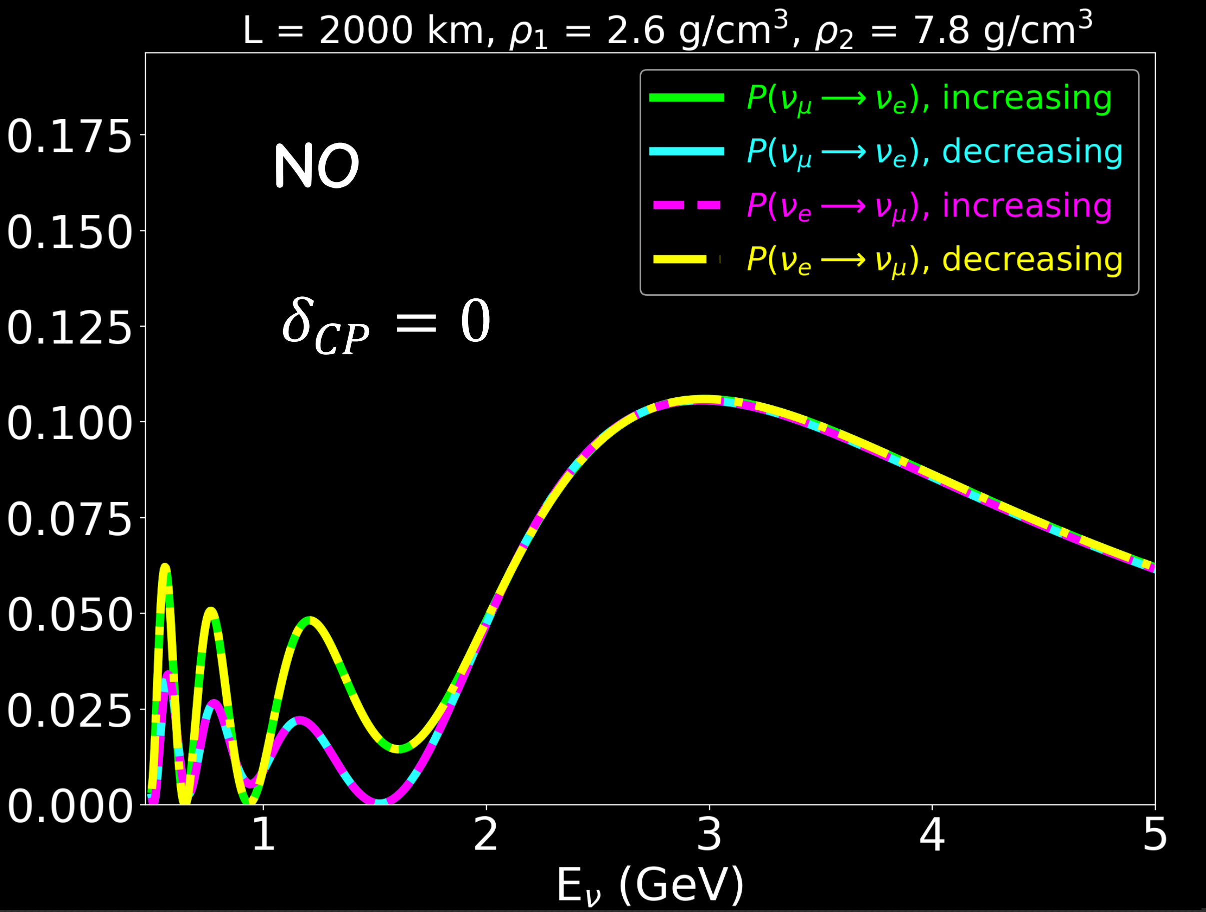
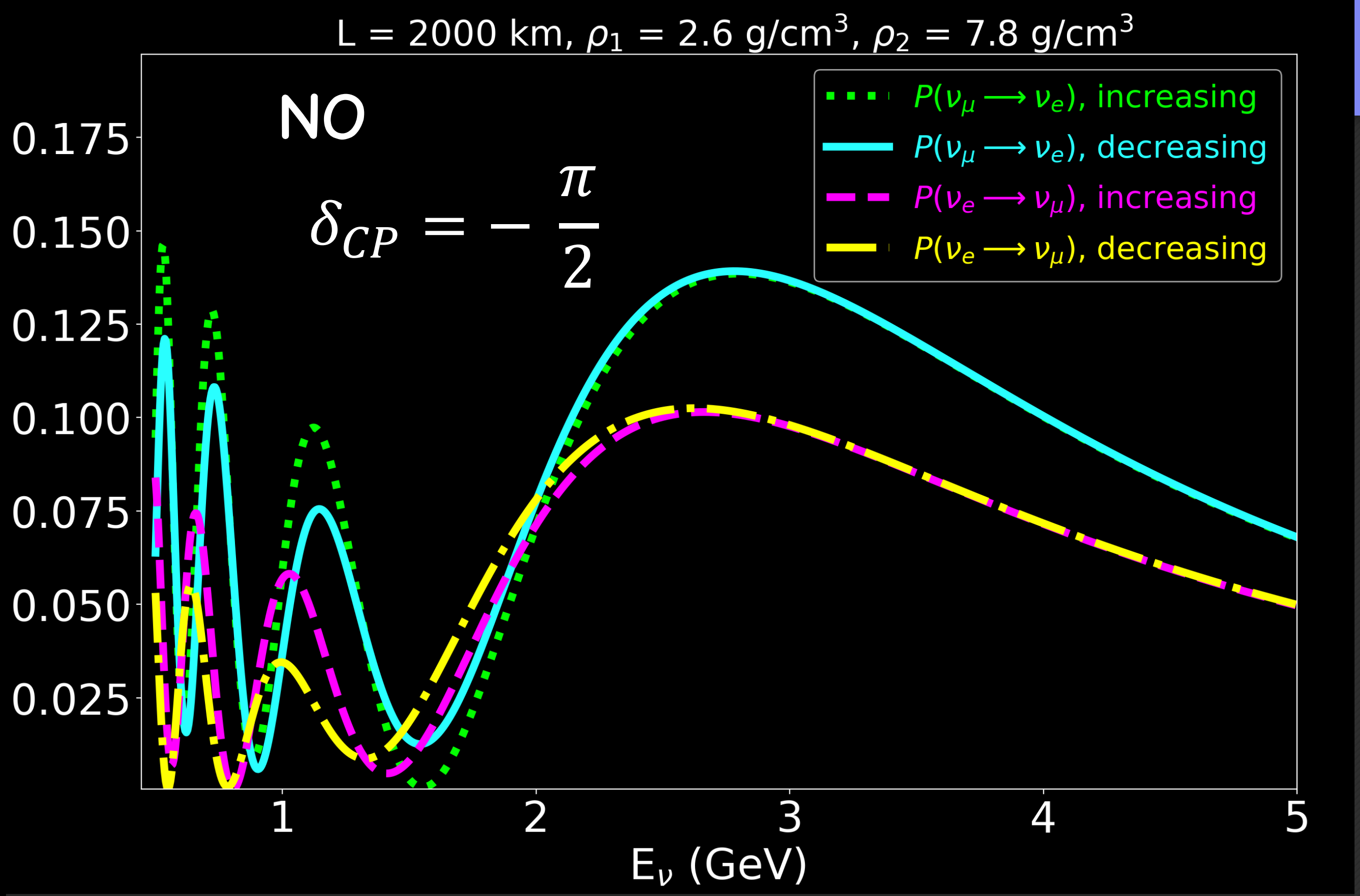
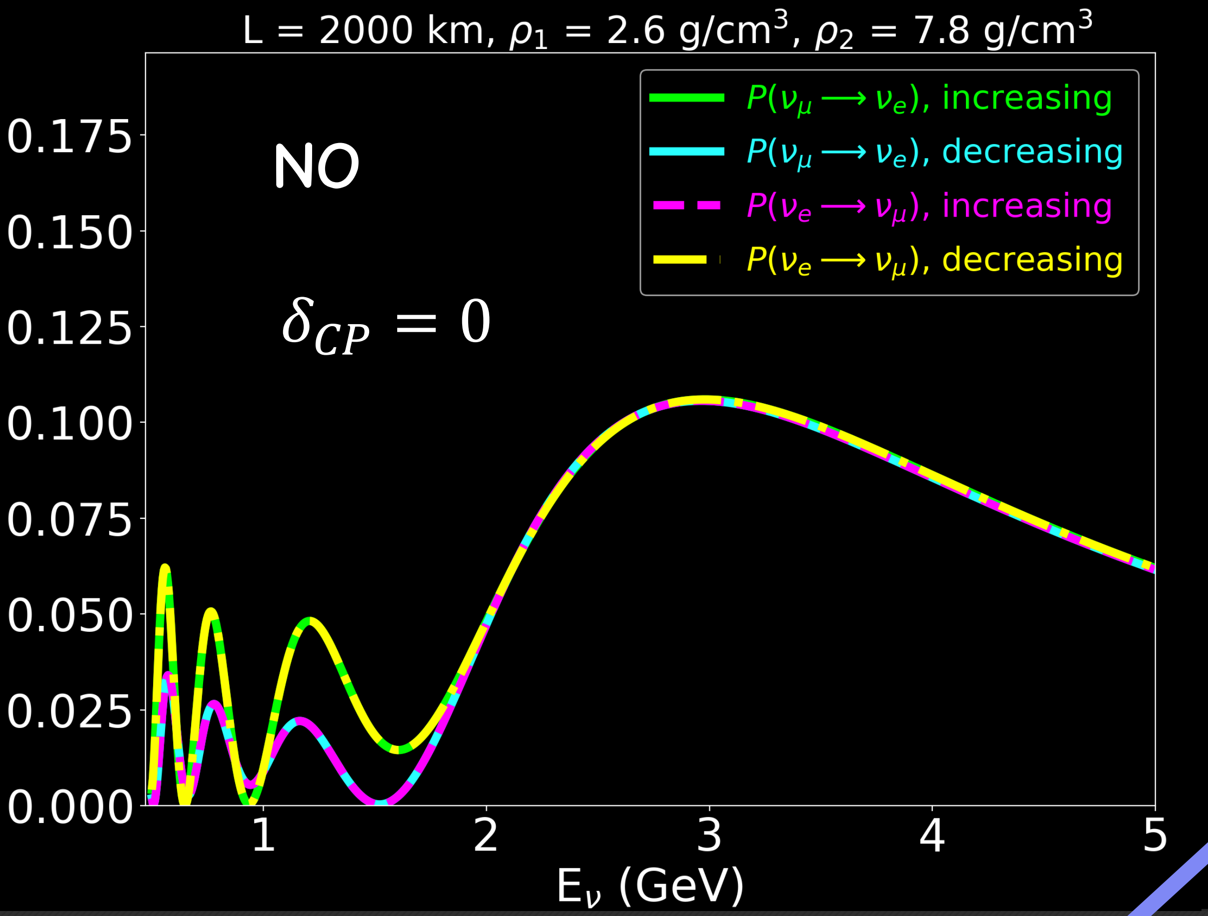
Non-symmetric matter effects for 3-Flavors

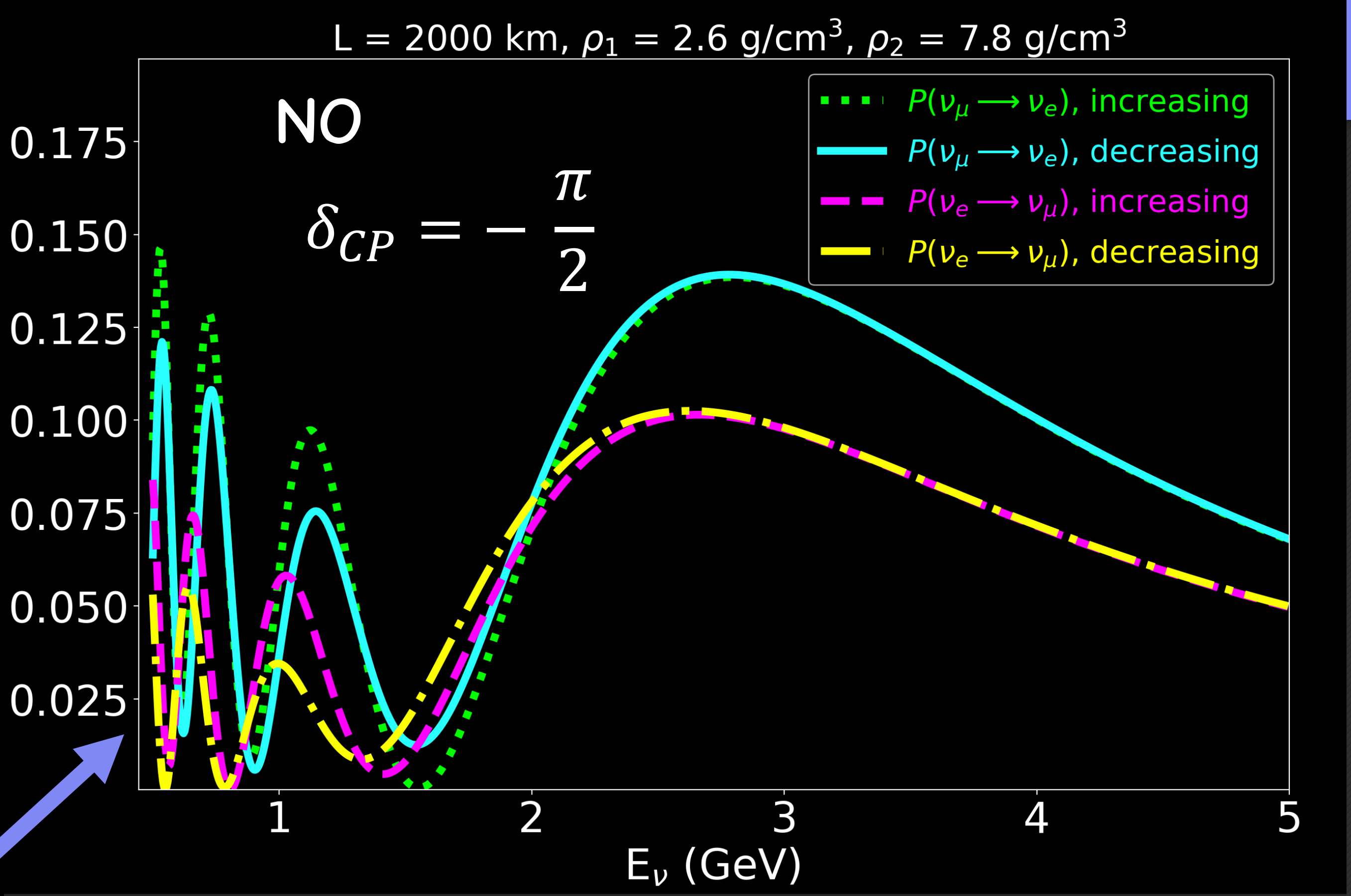




3. No intrinsic time invariance violation but *matter induced time invariance violation occurs.*

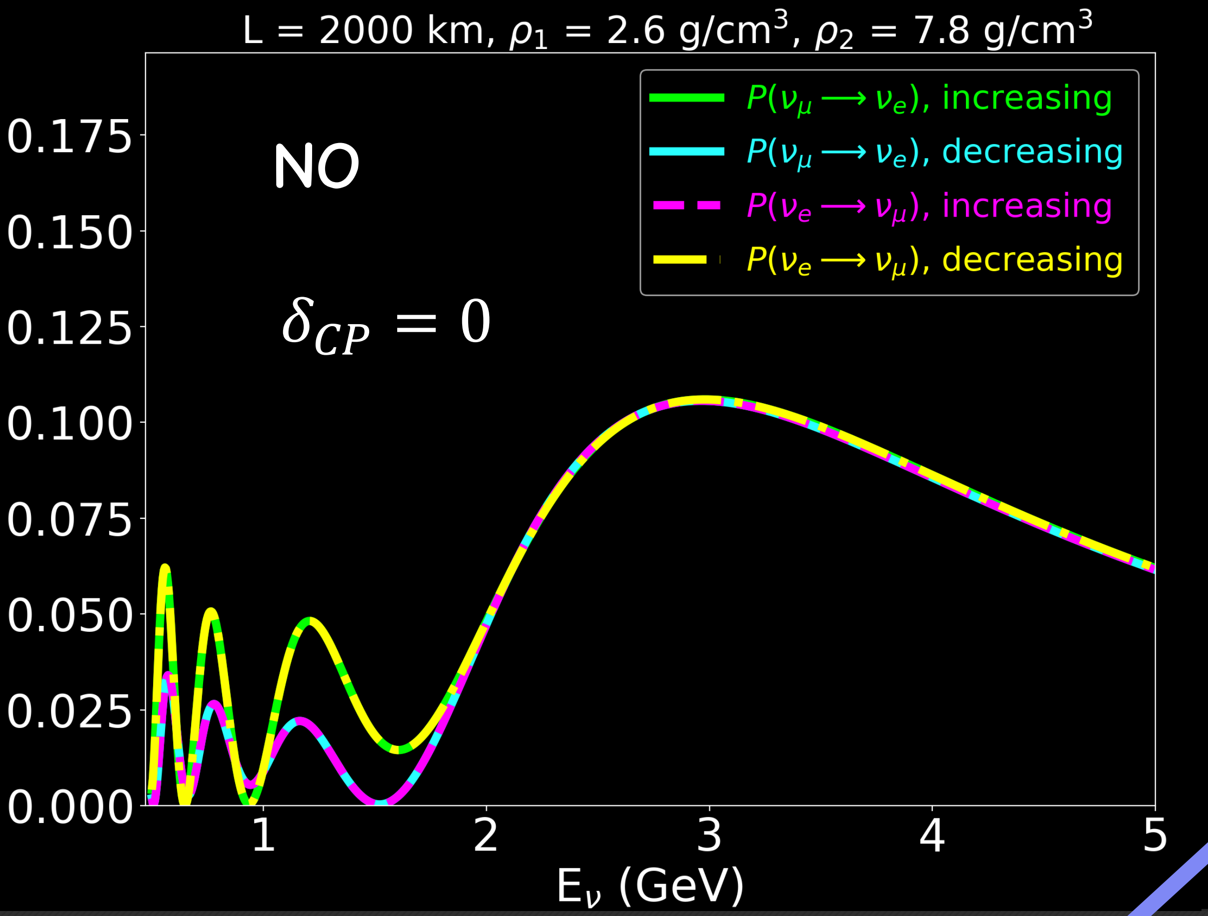
Non-symmetric matter effects for 3-Flavors

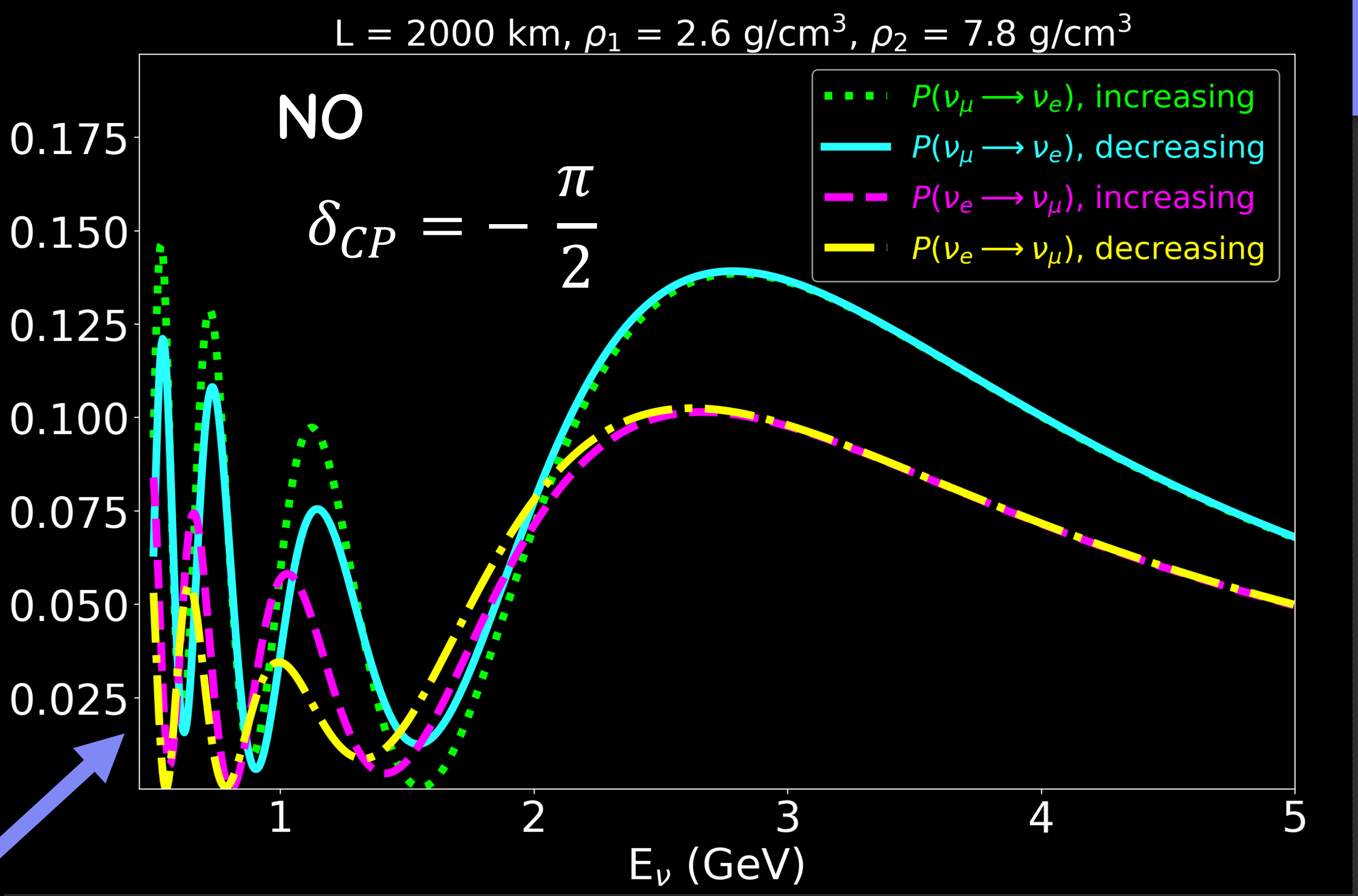




4. No longer pairwise degenerate if (δ_{CP} =- $\frac{\pi}{2}$).

Non-symmetric matter effects for 3-Flavors





4. No longer pairwise degenerate if ($\delta_{CP} =- \frac{\pi}{2}$).

5. Matter potential changes magnitude of oscillations.

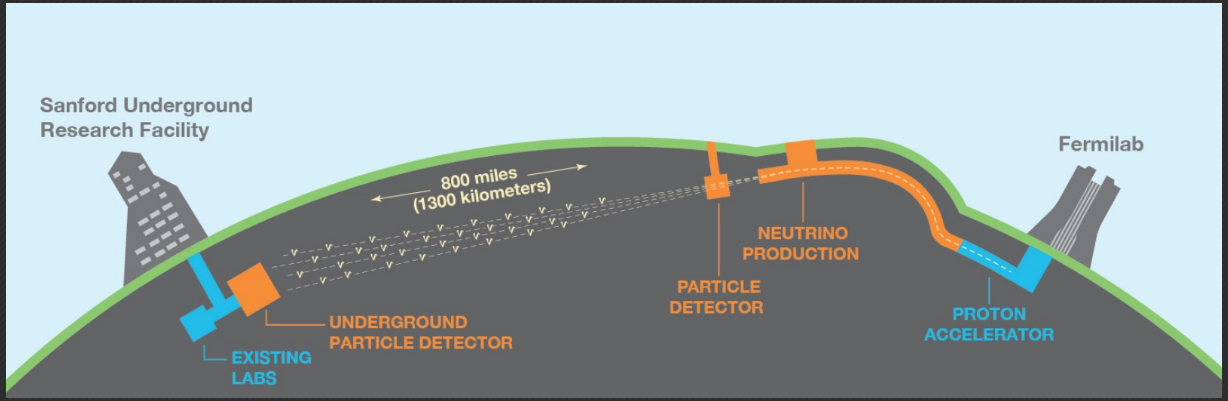
Realistic Matter
Potential Profiles

Realistic Considerations

We can play the same games for a more realistic model!

Our main model ingredients:

- ✓ Assume the high energy neutrino factory beamline as before
- ✓ Pick a well motivated baseline to test (ex: Fermilab DUNE baseline at 1300 km)
- ✓ Find the variations in Earth's crust densities



Source:https://www.dunescience.org/

Realistic Considerations

Remember!

We are testing for possible time invariance violation

(i.e. when $P_{\nu_\mu \to \nu_e} - P_{\nu_e \to \nu_\mu} \neq 0$).

Let's call this "derived" observable:

$$\Delta P = P_{\nu_\mu \to \nu_e} - P_{\nu_e \to \nu_\mu}$$

Realistic Considerations

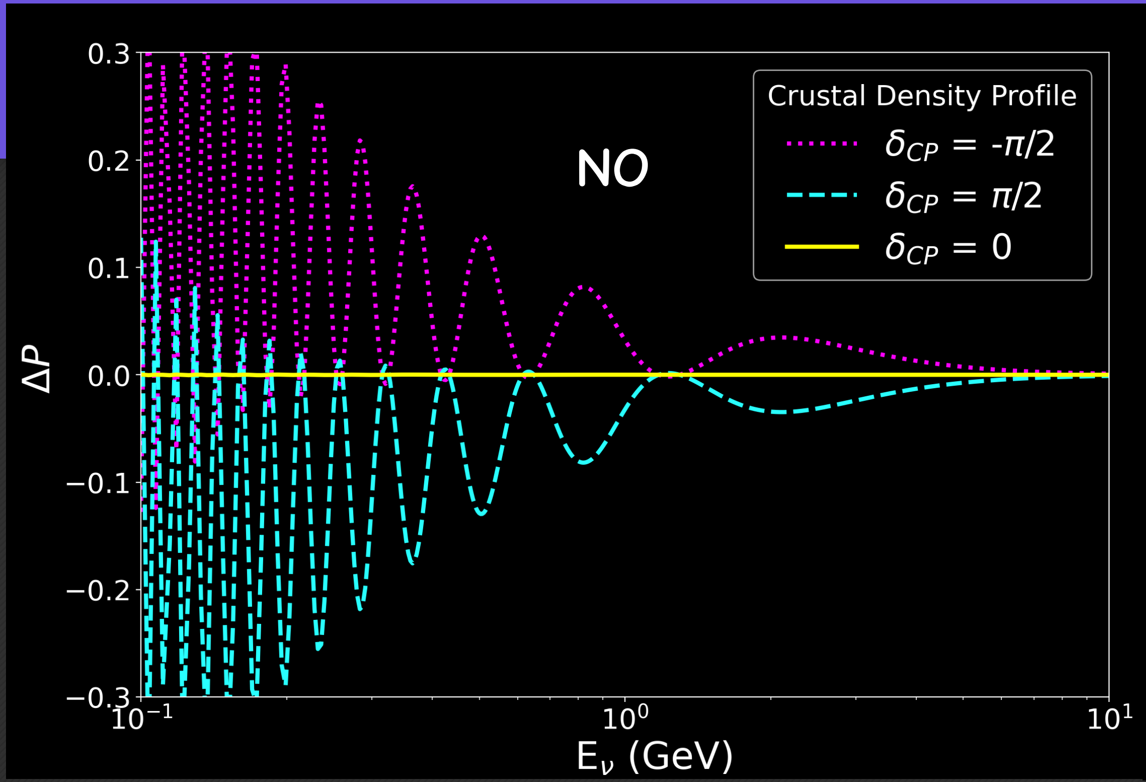
But let's also be clever in how we ask our question!

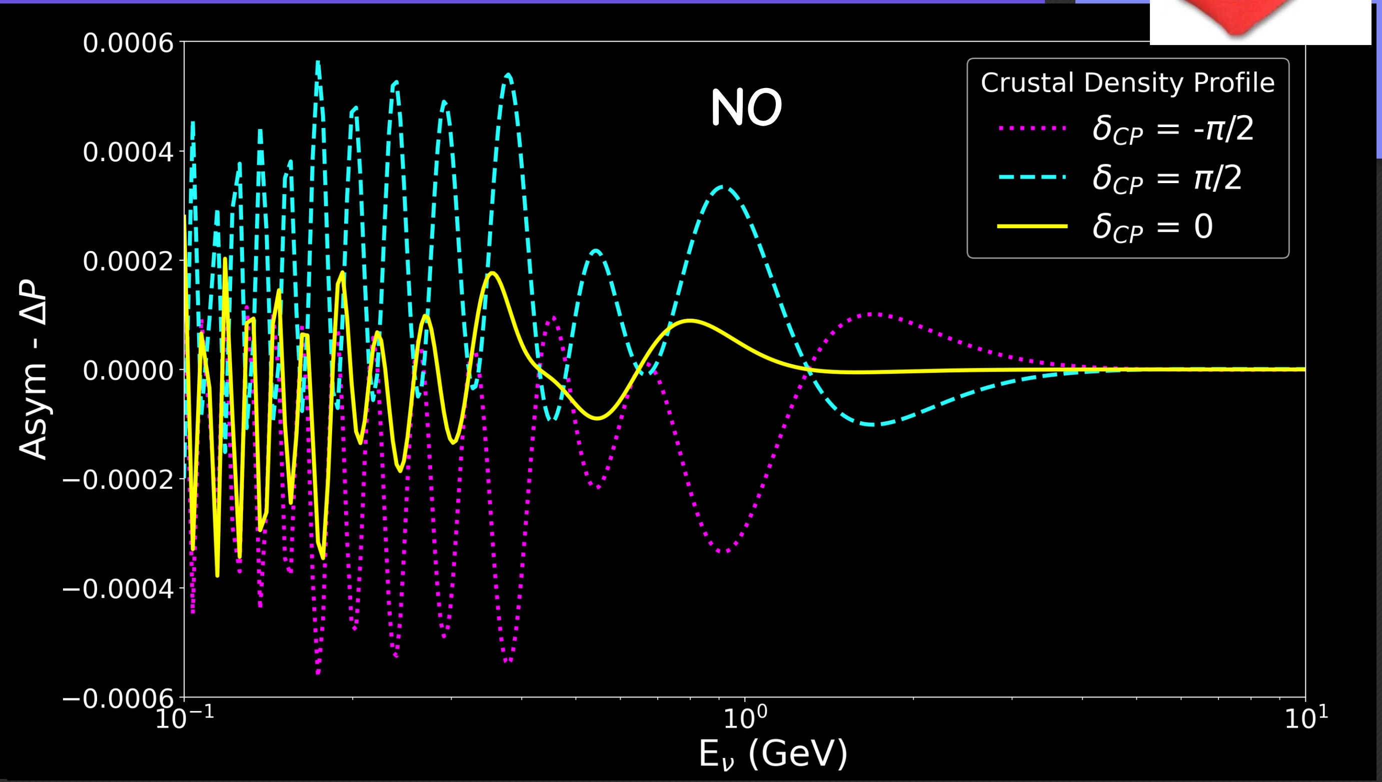
Can we meaningfully extract how much a possible time invariance violation is dependent upon the asymmetry in the potential? YES!

Let's call this: Asymmetrically-Induced - ΔP

or Asym - ΔP

Realistic Matter Effects for 3-Flavors

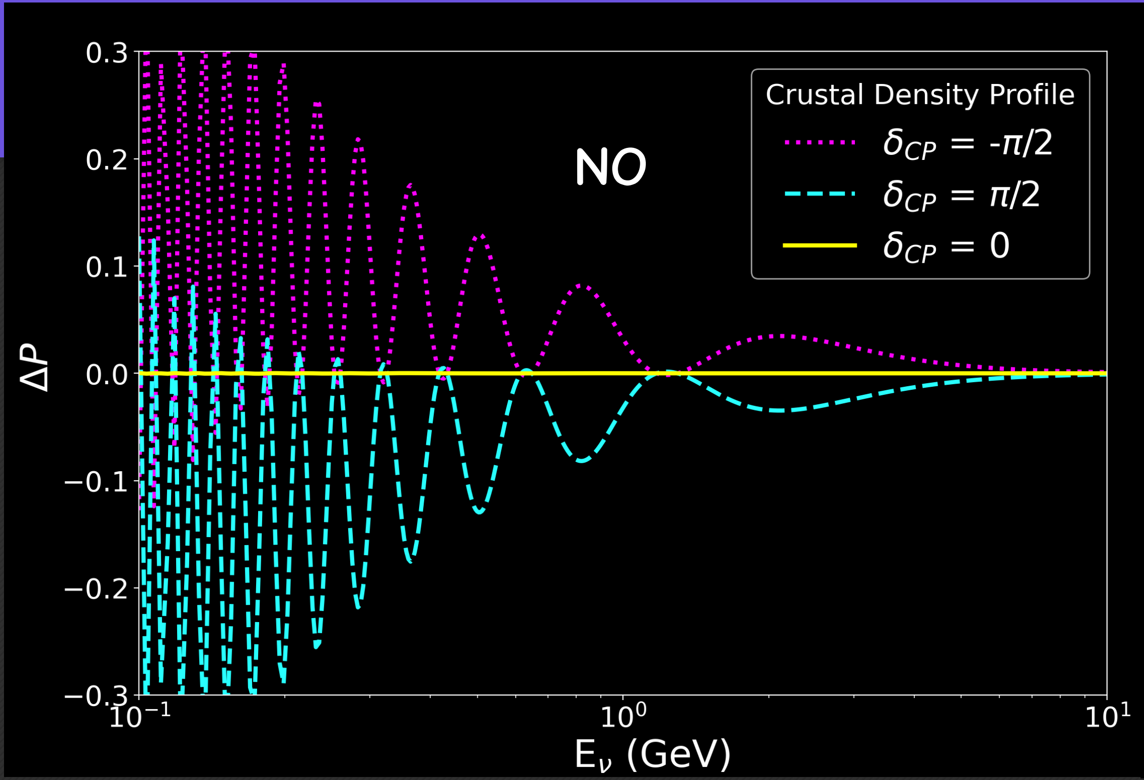


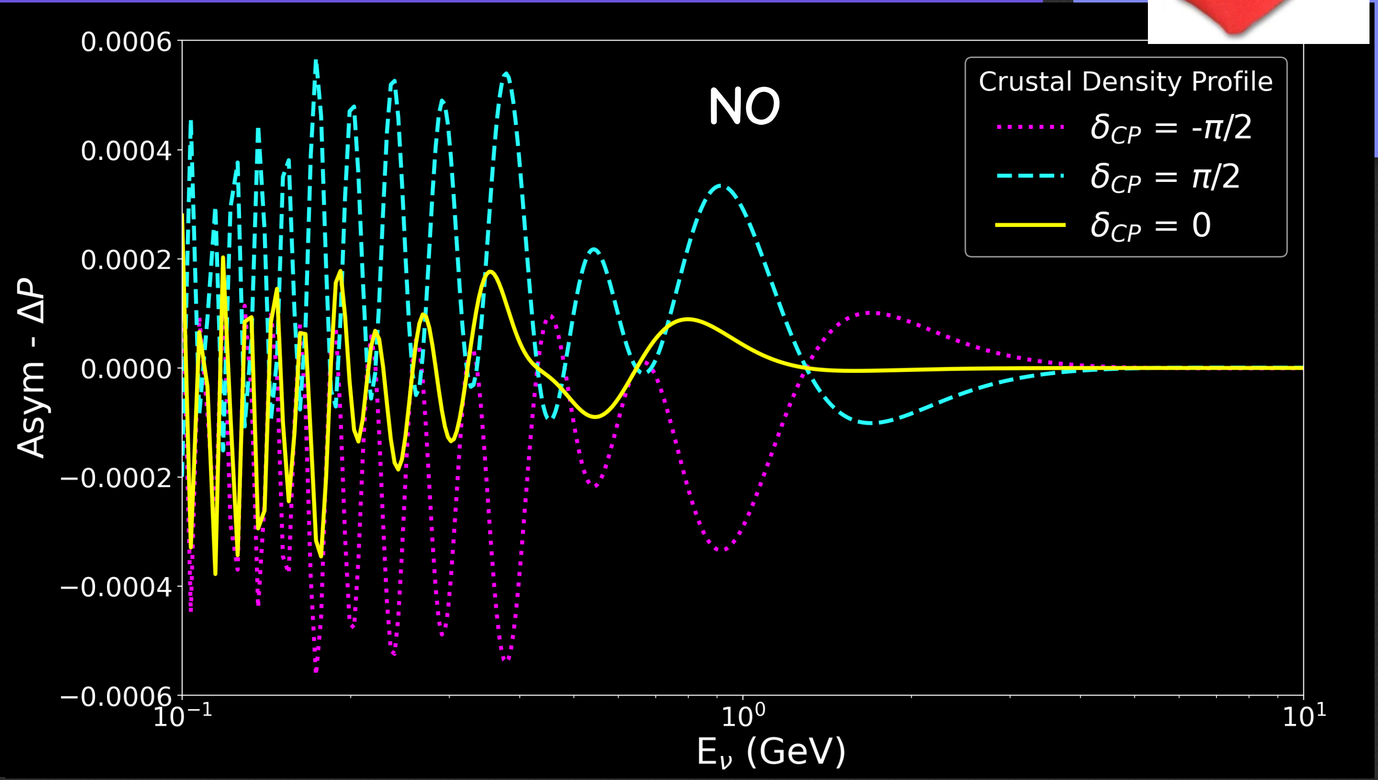


Details for Crustal Density Profile:
1) https://ui.adsabs.harvard.edu/abs/2013EGUGA..15.2658L/abstract
2) K J Kelly and S J Parke (2018) arXiv:1802.06784

1300 km (DUNE-like baseline)
Asymmetric matter density profile

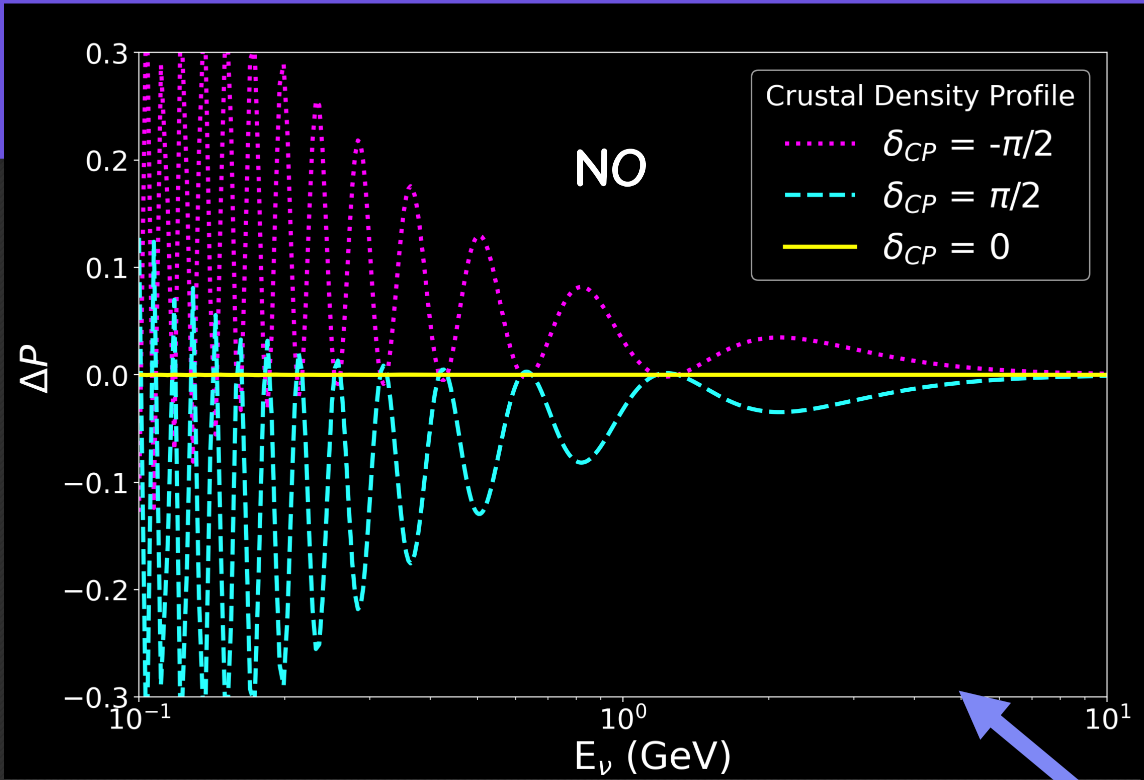
Realistic Matter Effects for 3-Flavors

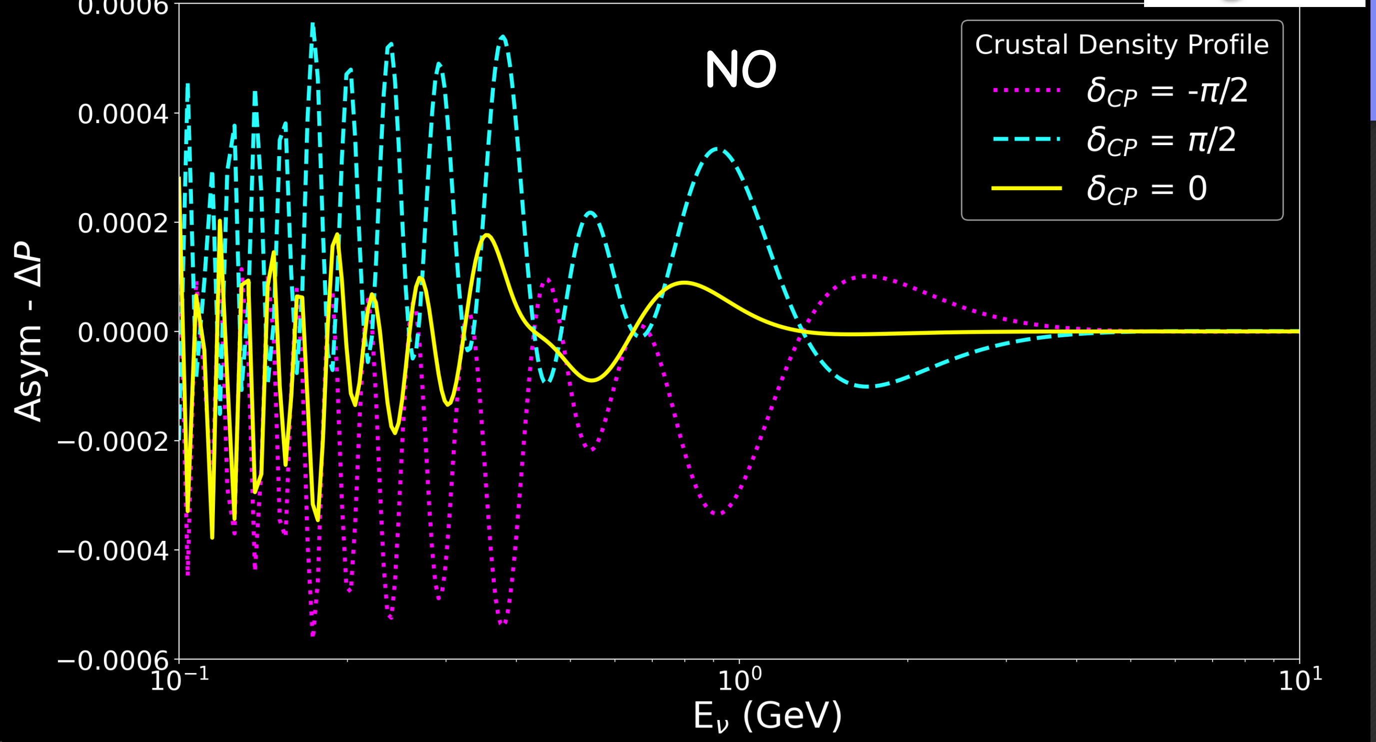




1. No T violation if ($\delta_{CP} = 0$) but matter induced T violation if ($\delta_{CP} \not\equiv 0$).

Realistic Matter Effects for 3-Flavors

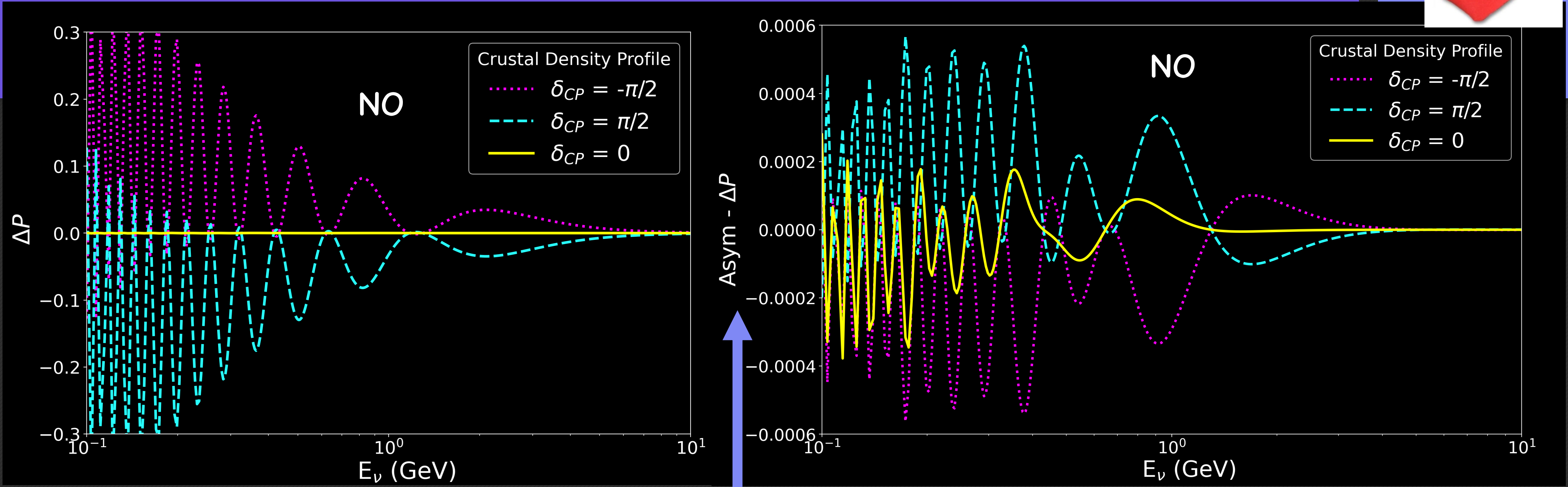




1. No T violation if $(\delta_{CP} = 0)$ but matter induced T violation if $(\delta_{CP} \neq 0)$.

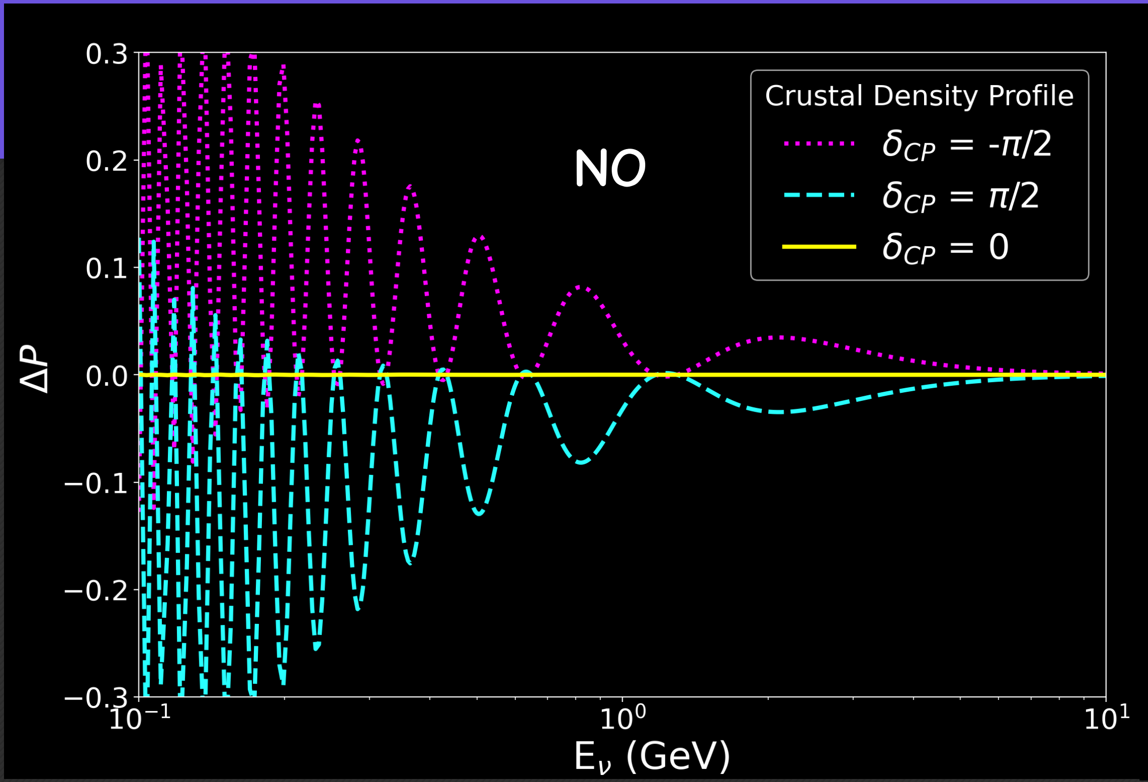
2. Meaning: for *DUNE-like parameters*, even in an extreme asymmetric matter, the density profile is <u>symmetric enough not to induce any meaningful T-invariance violation</u> if there is no intrinsic CP violation.

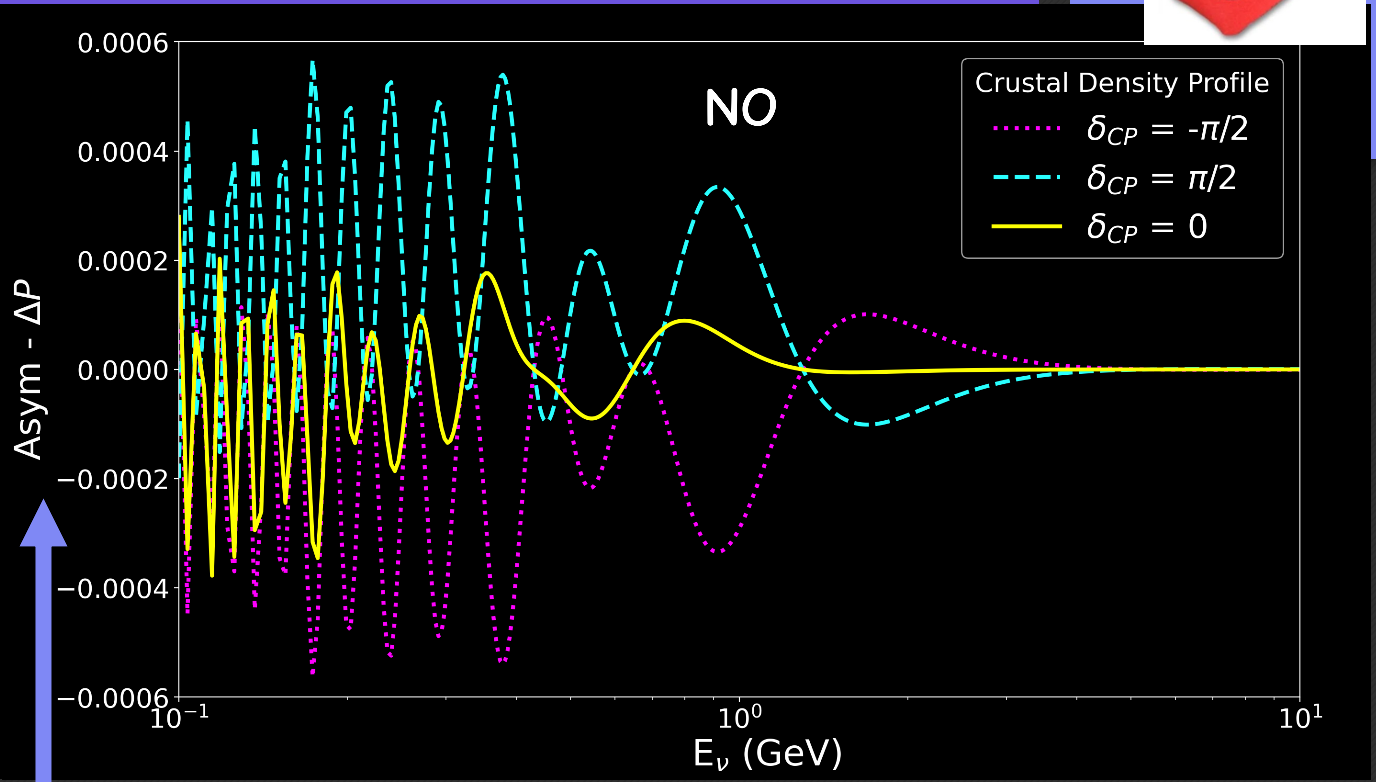
Realistic Matter Effects for 3-Flavors



3. T-invariance violation that is induced by the matter profile's asymmetry.

Realistic Matter Effects for 3-Flavors





3. T-invariance violation that is induced by the matter profile's asymmetry.

4. The amount of T-violation due to the matter profile's asymmetry, regardless of intrinsic CP violation, is insignificant and immeasurable at DUNE-like experiments.

Main Takeaways

- This study looked at how much time invariance violation (if at all) can be observed in neutrino oscillations, and their relation to simple neutrino matter potential models.
- Symmetric potentials cannot induce time violation, but if time invariance is intrinsically violated, the matter potential simply changes the degree of the observed effects.
- Symmetric potentials provide probes into proper time invariance.
- Non-symmetric matter potentials can induce improper time violation, while proper time violation is more protected.
- Realistic matter profiles induce immeasurable T violation effects for DUNE-like parameters.

ν_μ

ν_τ

Thank You!

ν_e

Backups

Past T invariance studies include:

This list is far from complete, but just to get a sense of the progress made over the years…

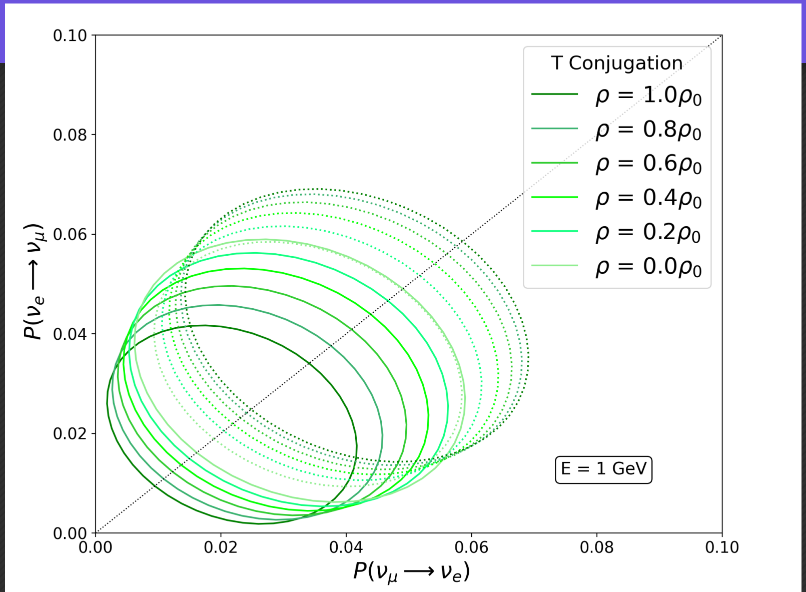
Formalism developed in the late 1980s:
T.K. Kuo & J. Pantaleone (Phys. Lett. B 198 (1987))
S. Toshev (Phys. Lett. B 226 (1989))

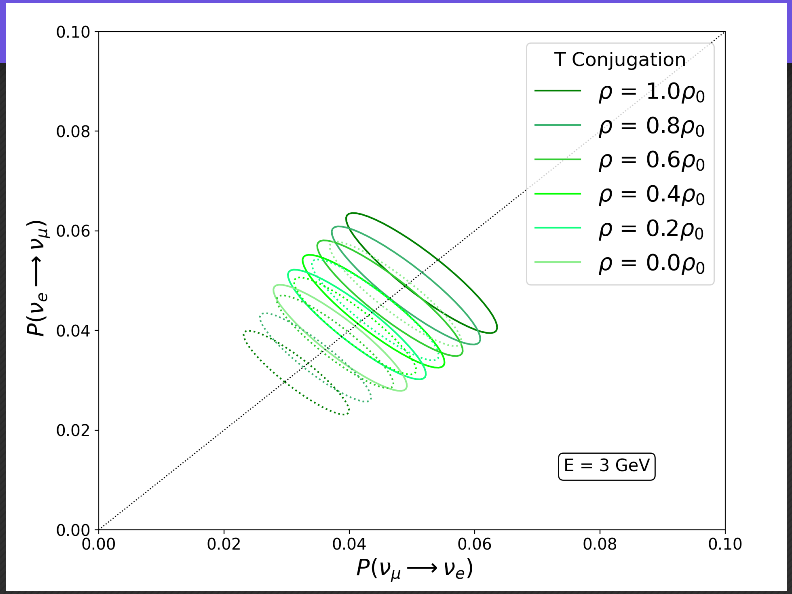
Details relating to characteristics extracted from neutrino probabilities developed (including oscillation parameters, Jarlskog invariant etc):
M. Blom & H. Minakata (arXiv: 0404142)
P. F. Harrison & W. G. Scott (arXiv: 9912435)
S. T. Petcov & Y-L. Zhou (arXiv: 1806.09112)
Z-Z. Xing (arXiv: 1304.7606)
S. J. Parke & T. J. Weiler (arXiv: 0011247)
E.Kh. Akhmedov, P. Huber, M. Lindner, T. Ohlsson (arXiv: 0105029)
J. Bernabeu & A. Segarra (arXiv: 1901.02761)

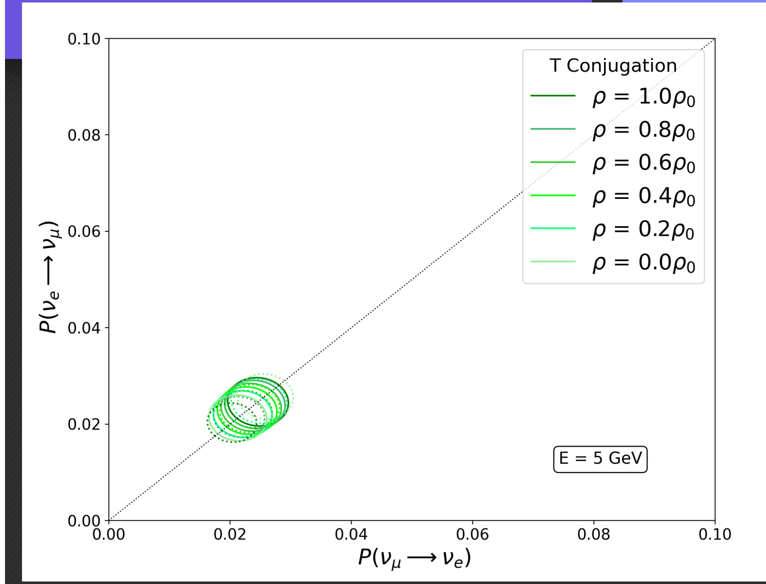
Discussions on possible Long-Baseline tests and future neutrino factories:
J. Arafune & J. Sato (arXiv: 9607437))
T. Miura, E. Takasugi, Y. Kuno, M. Yoshimura (arXiv: 0102111)
T. Schwetz & A. Segarra (arXiv : 2106.16099 & arXiv: 2112.08801)
S. S. Chatterjee, S. Patra, T. Schwetz, K. Sharma (arXiv: 2408.06419)

Exact formulaic derivations:
T K. Kimura , A. Takamura , H. Yokomakura (Physics Letters B 537 (2002))
O. Yasuda (https://doi.org/10.3390/e26060472)

Bi-Probability Plots







Solid lines: NO
Dashed lines: IO
ρ_0 = 2.84 g/cm^3

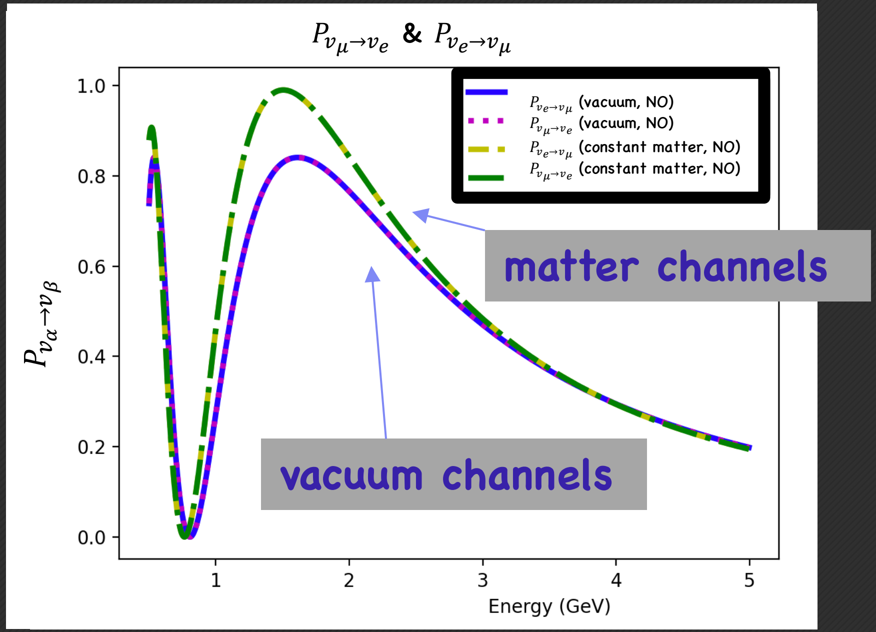
Showing the impact of neutrino mass ordering and energy on T-conjugation via bi-probability plots

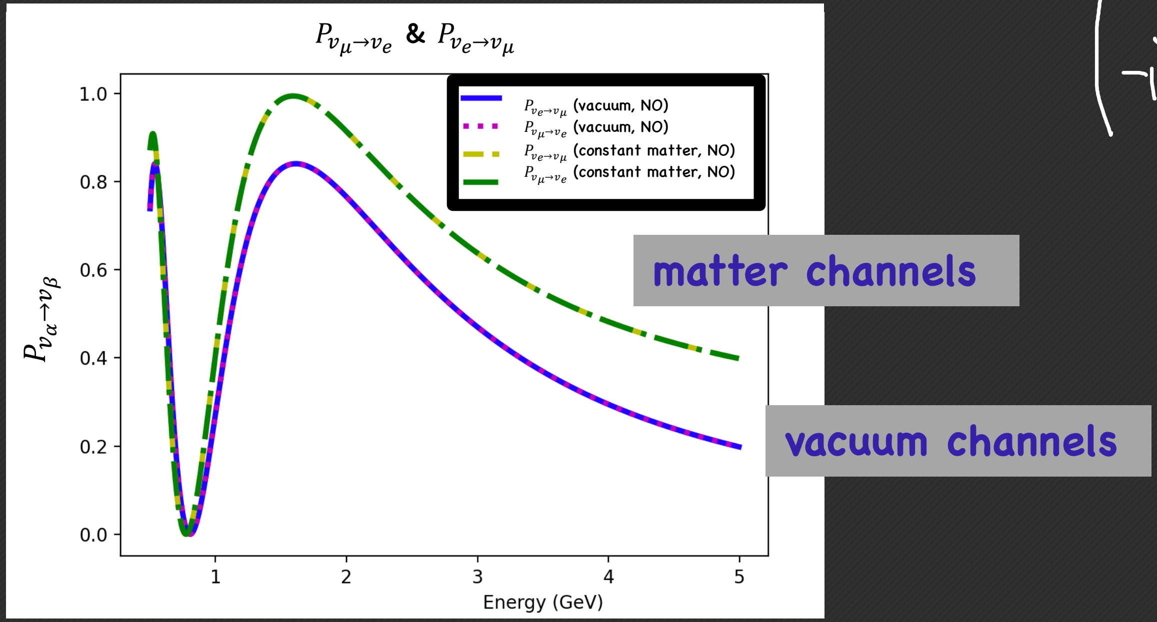
Symmetric matter effects for 2-Flavors
(Normal Ordering, Baseline: 2000 km)

constant matter potential:
A ≈ $5*10^{-4}$ eV^2/GeV (2.6 g/cm^3)

NSI + constant matter potential:
A ≈ $5*10^{-4}$ eV^2/GeV (2.6 g/cm^3)
B ≈ i * $2.5*10^{-4}$ eV^2/GeV (1.3 g/cm^3)

$$\begin{pmatrix} A & iB \\ -iB & 0 \end{pmatrix}$$

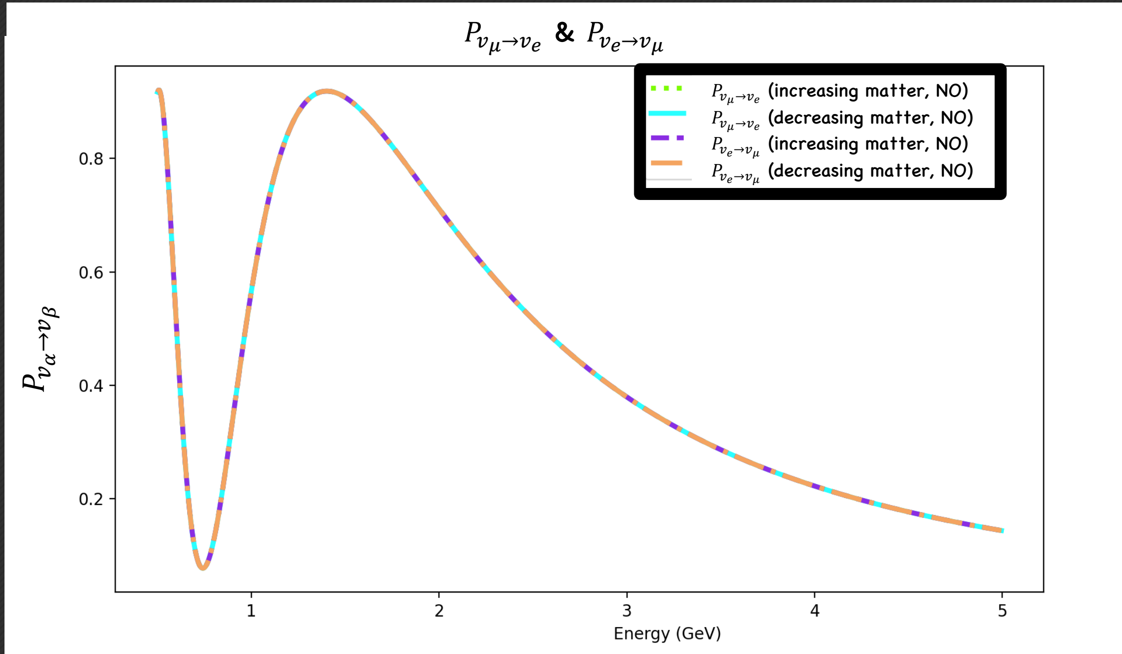




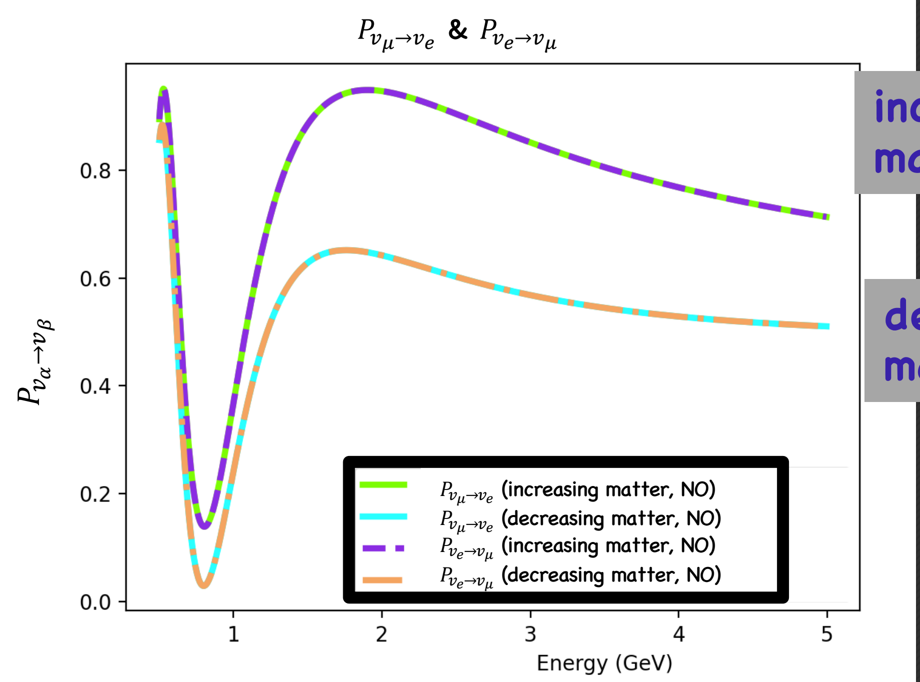
α, β -> $[\mu, e]$

Non-symmetric matter effects for 2-Flavors
(Normal Ordering, Baseline: 2000 km)

2 step constant matter potential:
A1 ≈ 5*10^{-4} eV^2/GeV (2.6 g/cm^3)
A2 ≈ 1.5*10^{-3} eV^2/GeV (7.8 g/cm^3)



NSI + 2 step constant matter potential:
A1 ≈ 5*10^{-4} eV^2/GeV (2.6 g/cm^3)
B1 ≈ i * 2.5*10^{-4} eV^2/GeV (1.3 g/cm^3)
A2 ≈ 1.5*10^{-3} eV^2/GeV (7.8 g/cm^3)
B2 ≈ i * 7.5*10^{-4} eV^2/GeV (3.9 g/cm^3)

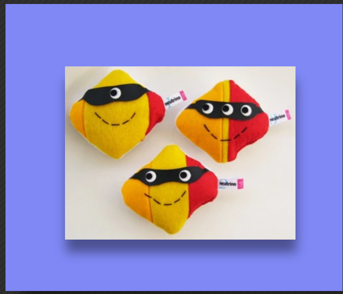


increasing matter

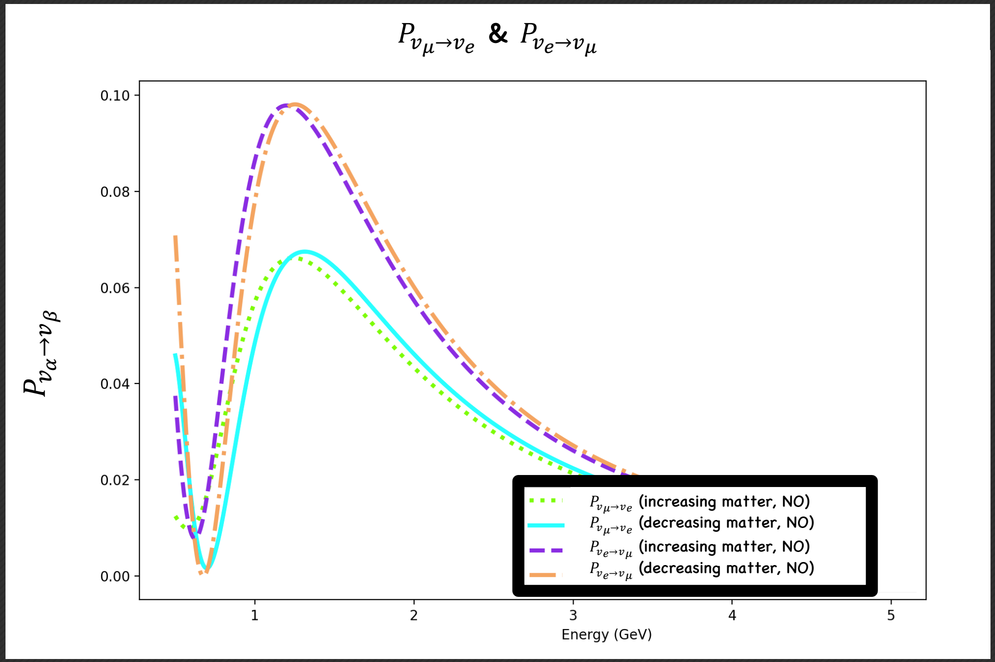
decreasing matter

α, β -> $[\mu, e]$

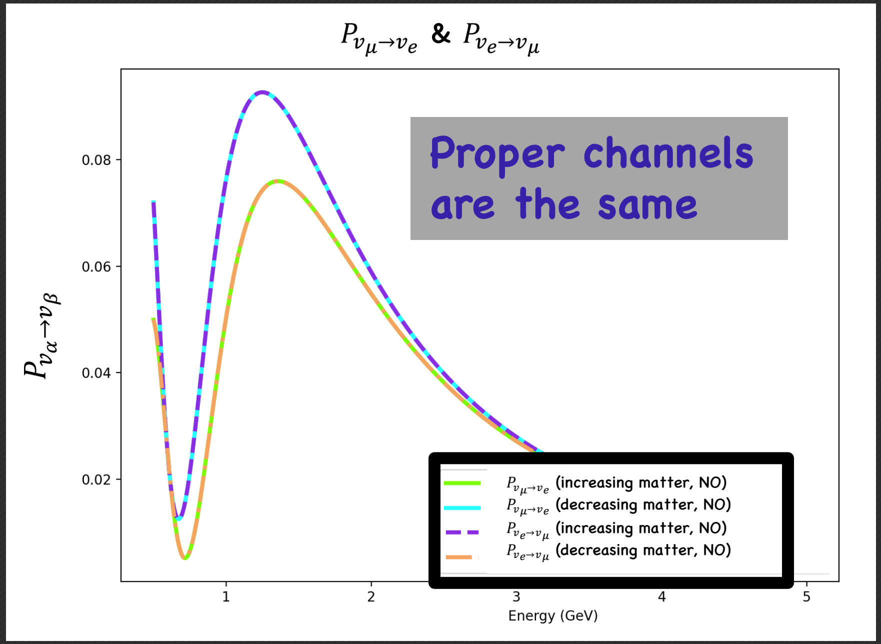
Non-symmetric matter effects for 3-Flavors, 3-Steps
(Normal Ordering, Baseline: 2000 km)



3 step constant matter potential:
A1 ≈ 5.0*10^{-4} eV^2/GeV (2.6 g/cm^3)
A2 ≈ 1.5*10^{-3} eV^2/GeV (7.8 g/cm^3)
A3 ≈ 3.0*10^{-3} eV^2/GeV (15.6 g/cm^3)



δ_{CP} = 90



δ_{CP} = 0

α, β -> $[\mu, e]$

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects

We include matter effects explicitly from the following prescription for the probabilities (useful in the context of this study):

$$P_{v_\alpha \to v_\beta} = |< \mathbf{v}_\beta(\mathbf{0})|\mathbf{v}_\alpha(\mathbf{L}) >|^2$$

where

$$|\mathbf{v}_\alpha(\mathbf{L}) > = \mathrm{U}\ |\mathbf{v}_\alpha(\mathbf{0}) >$$

with $\mathrm{U} = e^{-iLH}$

$(H = H_{vacuum} + H_{matter})$

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects

As an example for a Baseline L, let's break L up into 2 steps: L_1 and L_2, where each evolution U(L) will model different matter potentials A_1 and A_2 respectively.

Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects

As an example for a Baseline L, let's break L up into 2 steps: L_1 and $L_{2,}$ where each evolution U(L) will model different matter potentials A_1 and A_2 respectively.

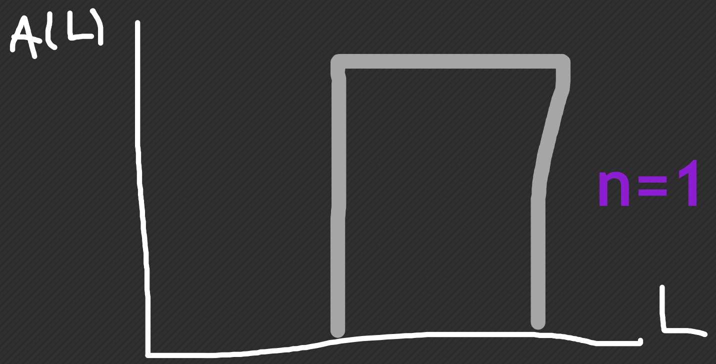
$$U_1 = e^{-iL_1H_1} \quad \& \quad U_2 = e^{-iL_2H_2}$$

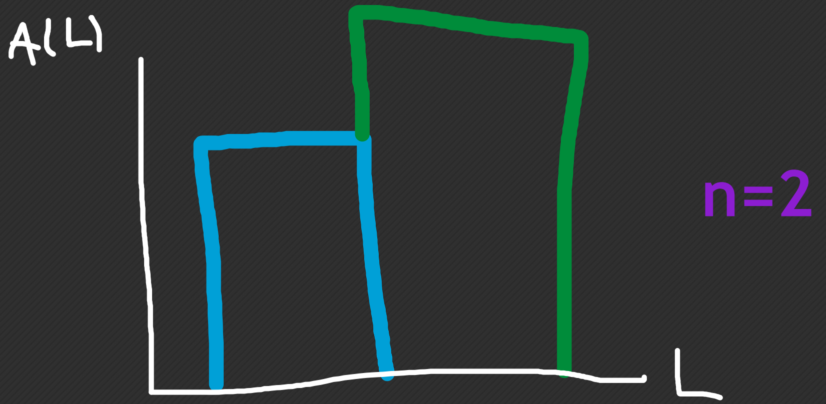
where H_i contain different matter effects in the form:

$A_i = \sqrt{2}G_FN_e$ -> constant

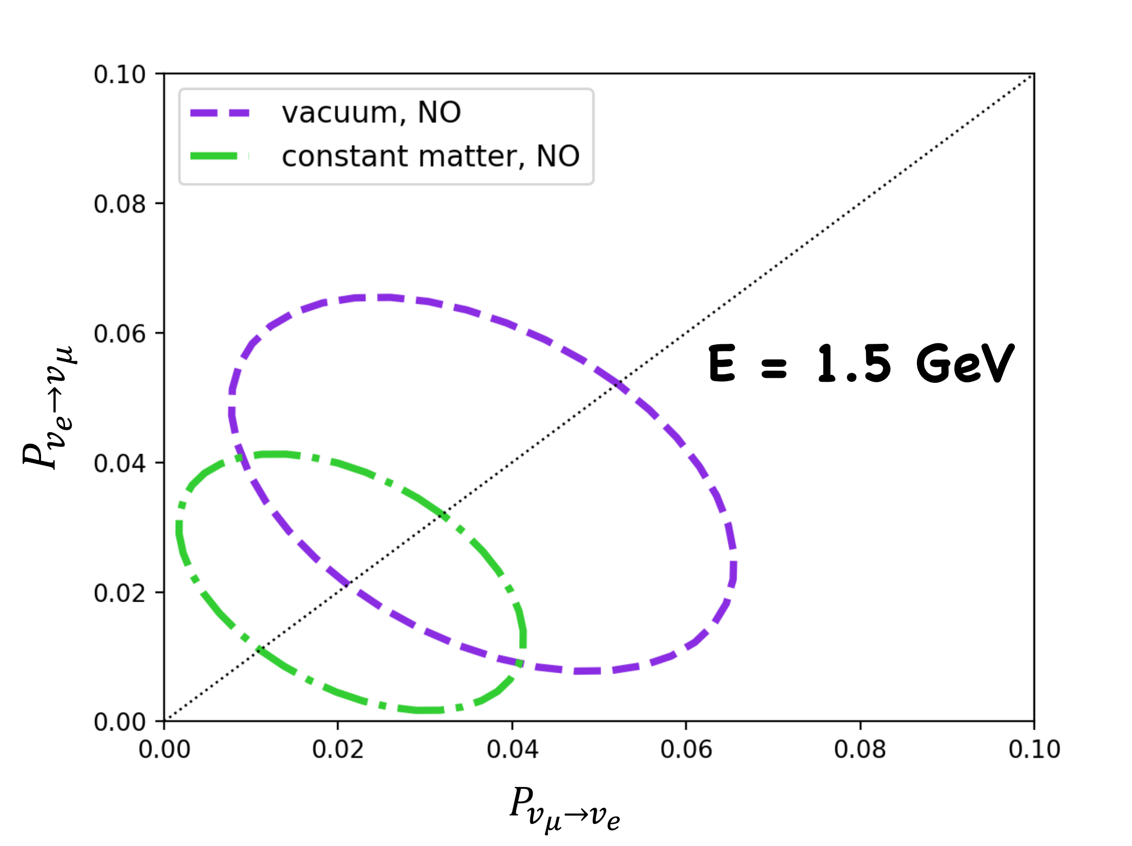
Recalling 3-Flavor Neutrino Oscillations with charge-current matter effects

In principle, one could consider an n multistep matter potential $A = \sum_{i=1}^{n} A_i$ where each step is itself a constant matter potential (piecewise constant).



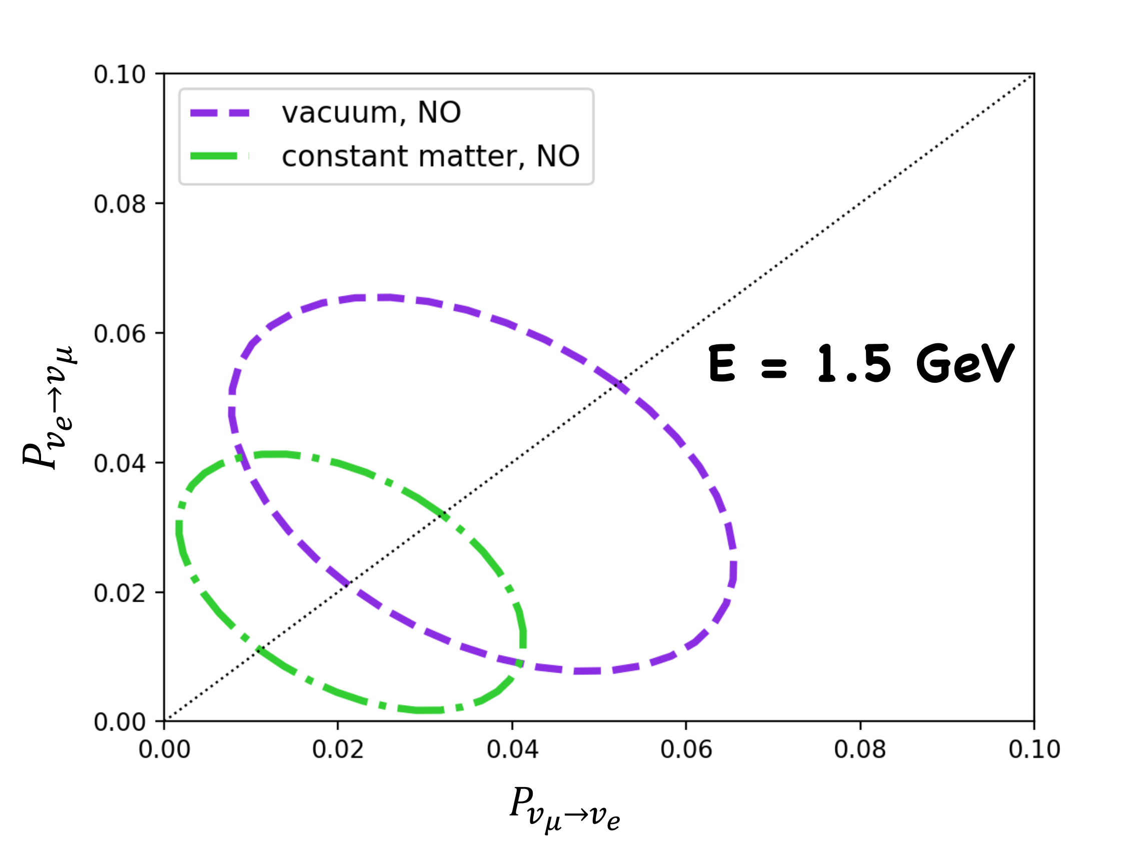


Symmetric matter effects for 3-Flavors



- ✓ Symmetric matter potentials *cannot induce time violation.*
- ✓ If there is intrinsic $P_{\nu_\mu \to \nu_e} \neq P_{\nu_e \to \nu_\mu}$ from δ_{CP}, then the matter potential simply *changes the degree of the observed effects.*

Symmetric matter effects for 3-Flavors



- ✓ Note: **Improper and proper comparisons are the same if the matter potential is symmetric.**
- ✓ *Why?*
  - ➔ A single constant matter potential is by construction symmetric.
  - ➔ We cannot tell the two measures apart if "exchanging source and detector" gives the *same results.*

Future Studies

- ✓ Interesting probes in cases where matter induced time violation occurs and/or realistic models are non-symmetric:
  - Center of the Earth (annihilating dark matter to neutrinos scenario)
  - Geo neutrinos (properties/applications)
- ✓ Next steps include *NSI time invariance* probes applicable to DUNE, (a follow-up to previous work).