

Dark Matter, Unification and Left-Right Symmetry

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SO(10) Unification

Many symmetry breaking paths to the Standard Model

Testable low energy predictions, $p \rightarrow e^+ + \pi^0$

Appealing from reductionist viewpoint

Electroweak WIMPs

Commonly studied dark matter candidate

Reproduces relic dark matter abundance

Can we build Electroweak WIMP SO(10) Models?

Embedding electroweak WIMPs contains challenges

Restricts symmetry breaking pathway from $SO(10)$ to the Standard Model



Standard model and dark matter embed into $SO(10)$



SM Higgs embedded into a **16** scalar in $SO(10)$

SM Fermions embedded into three **16** fermions in $SO(10)$

We consider an EW-charged DM Dirac fermion

$$\chi_L(\mathbf{1}, \mathbf{2}, -1/2)$$

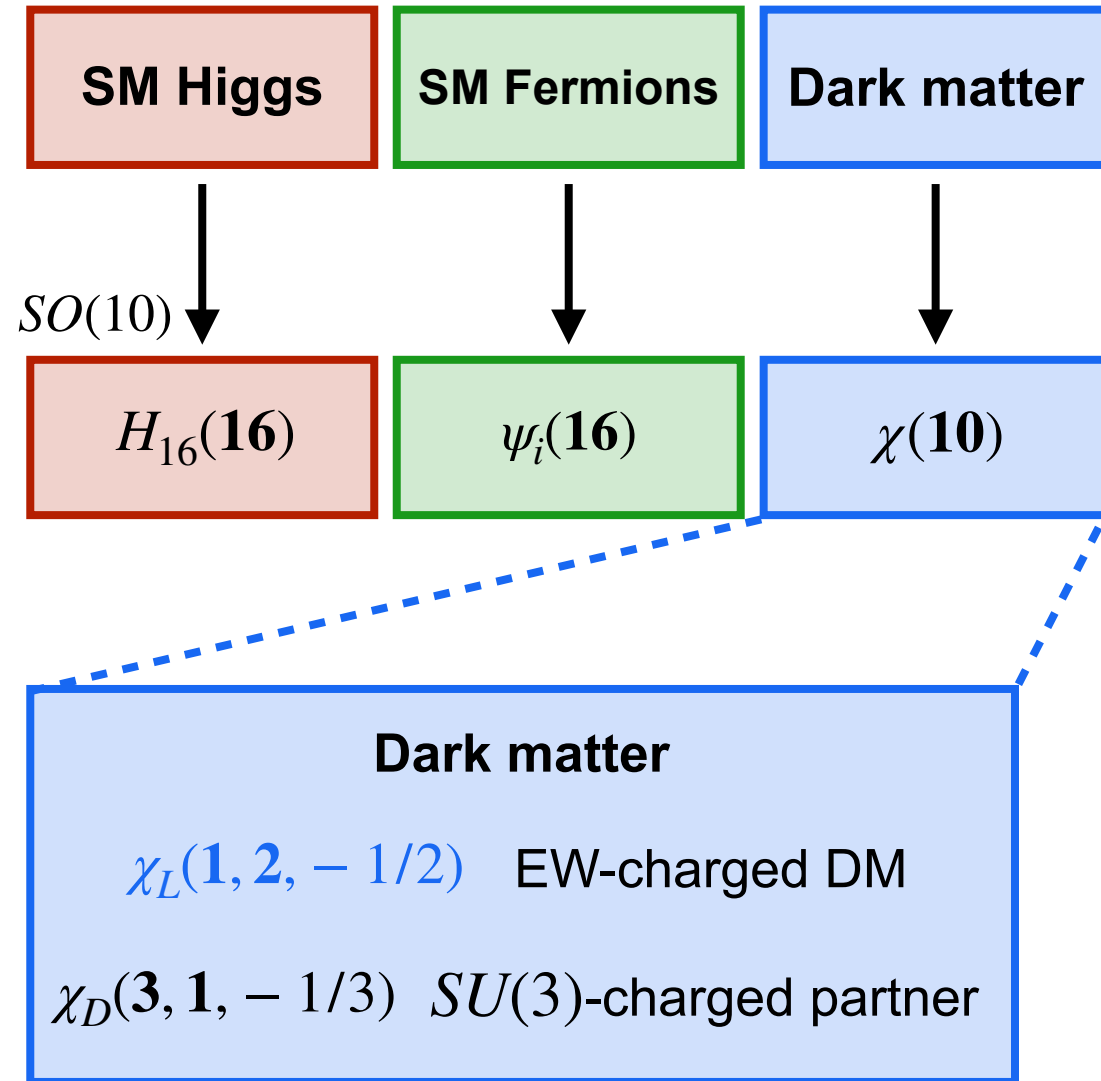
Embedding into **10** fermions of $SO(10)$ leads to an $SU(3)$ -charged partner fermion

$$\chi_D(\mathbf{3}, \mathbf{1}, -1/3)$$

We will need χ_D to be much heavier than χ_L

$$m_D > m_L$$

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$





Symmetry breaking via left-right intermediate gauge group

$m_D > m_L$ possible by breaking via the Left-Right symmetry group

$$SO(10)$$



$$G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$$



$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

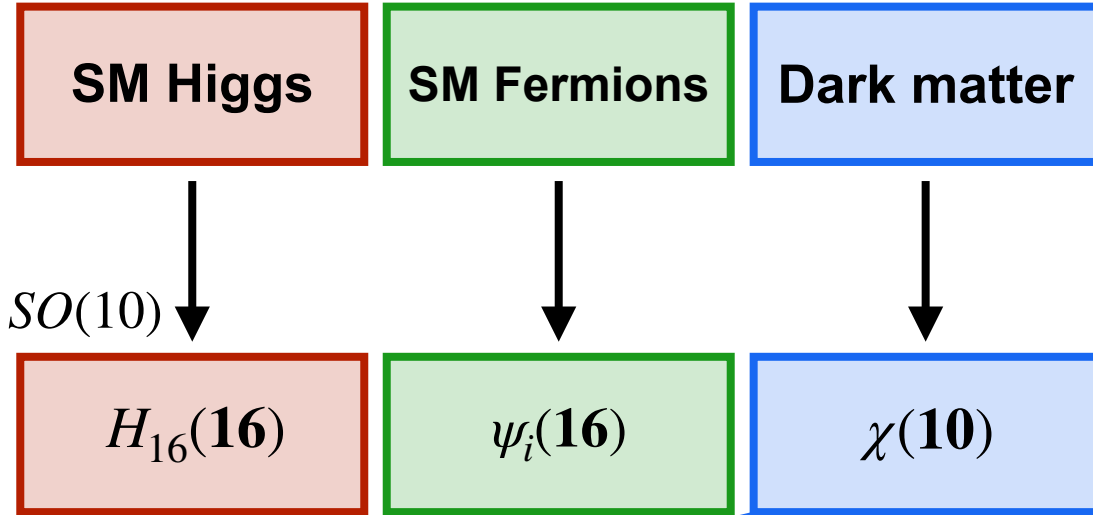
$H_{45}(45)$ of $SO(10)$ breaks to G_{LR}

$$SO(10) \xrightarrow{\langle H_{45} \rangle} G_{LR}$$

resulting in heavy XY gauge bosons with mass M_{XY} that mediate proton decay

Unification scale M_{XY}

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$



$SO(10)$

Dark matter

$\chi_L(1, 2, -1/2)$ EW-charged DM

$\chi_D(3, 1, -1/3)$ $SU(3)$ -charged partner



Missing VEV of $SO(10)$ Higgs generates mass splitting



How is $m_D > m_L$ achieved?

Non-zero VEV of H_{45}

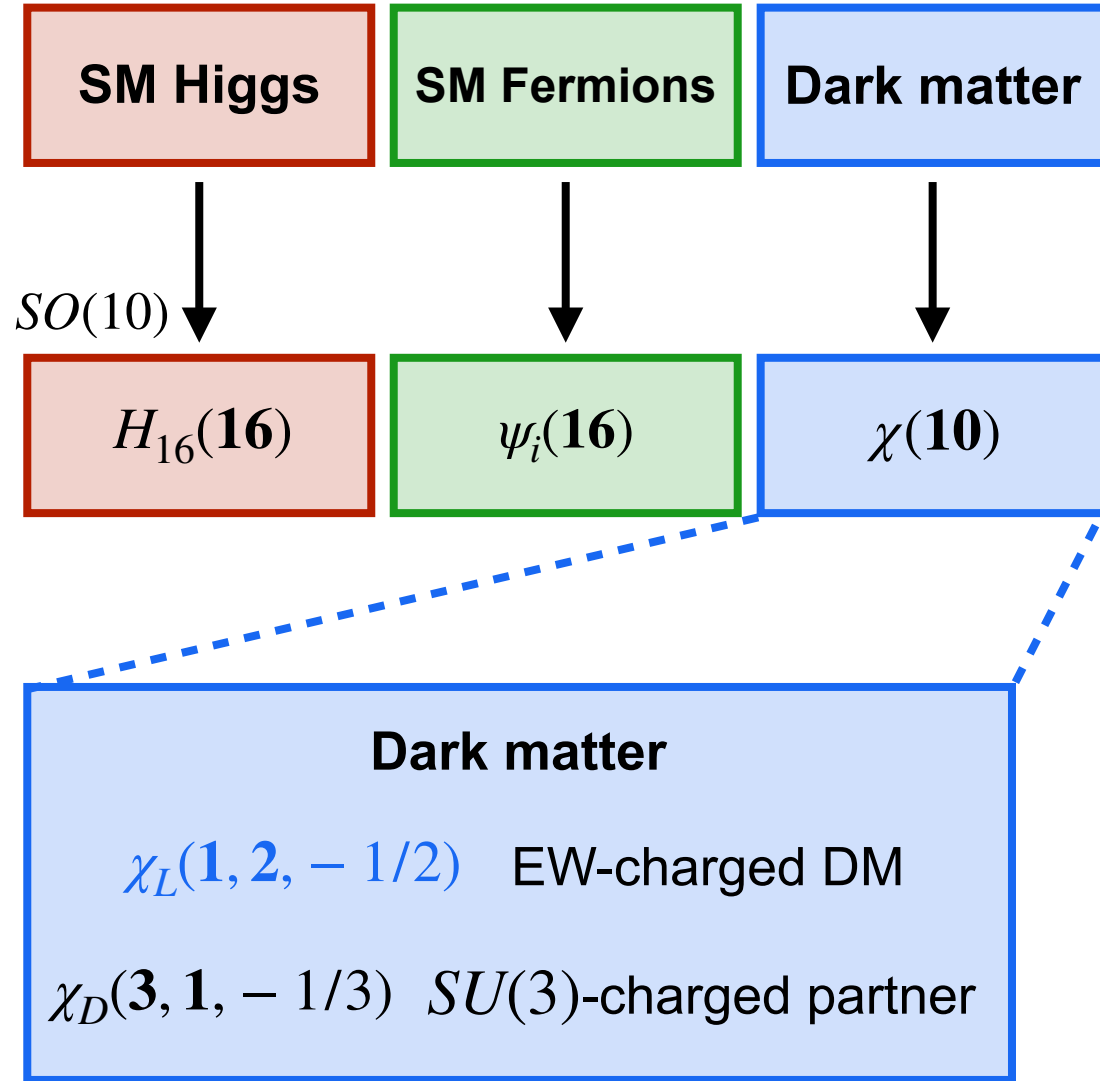
$$\langle H_{45} \rangle = -iv_{45} \begin{pmatrix} \sigma_2 & 0 & 0 & 0_{4 \times 6} \\ 0 & \sigma_2 & 0 & \\ 0 & 0 & \sigma_2 & \\ 0_{6 \times 4} & 0_{4 \times 4} & & \end{pmatrix}$$

Missing VEV of H_{45}

$$\left. \begin{array}{l} \chi_L \text{ coupled to missing VEV} \\ \chi_D \text{ coupled to non-zero VEV} \end{array} \right\} m_D > m_L$$

DM mass m_L , Partner mass m_D

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$





Left-right gauge groups breaks to standard model



H_R , contained in H_{16} , breaks G_{LR} to G_{SM}

$$G_{LR} \xrightarrow{\langle H_R \rangle} G_{SM}$$

G_{LR} with Parity symmetry also has the benefit of solving the strong CP problem

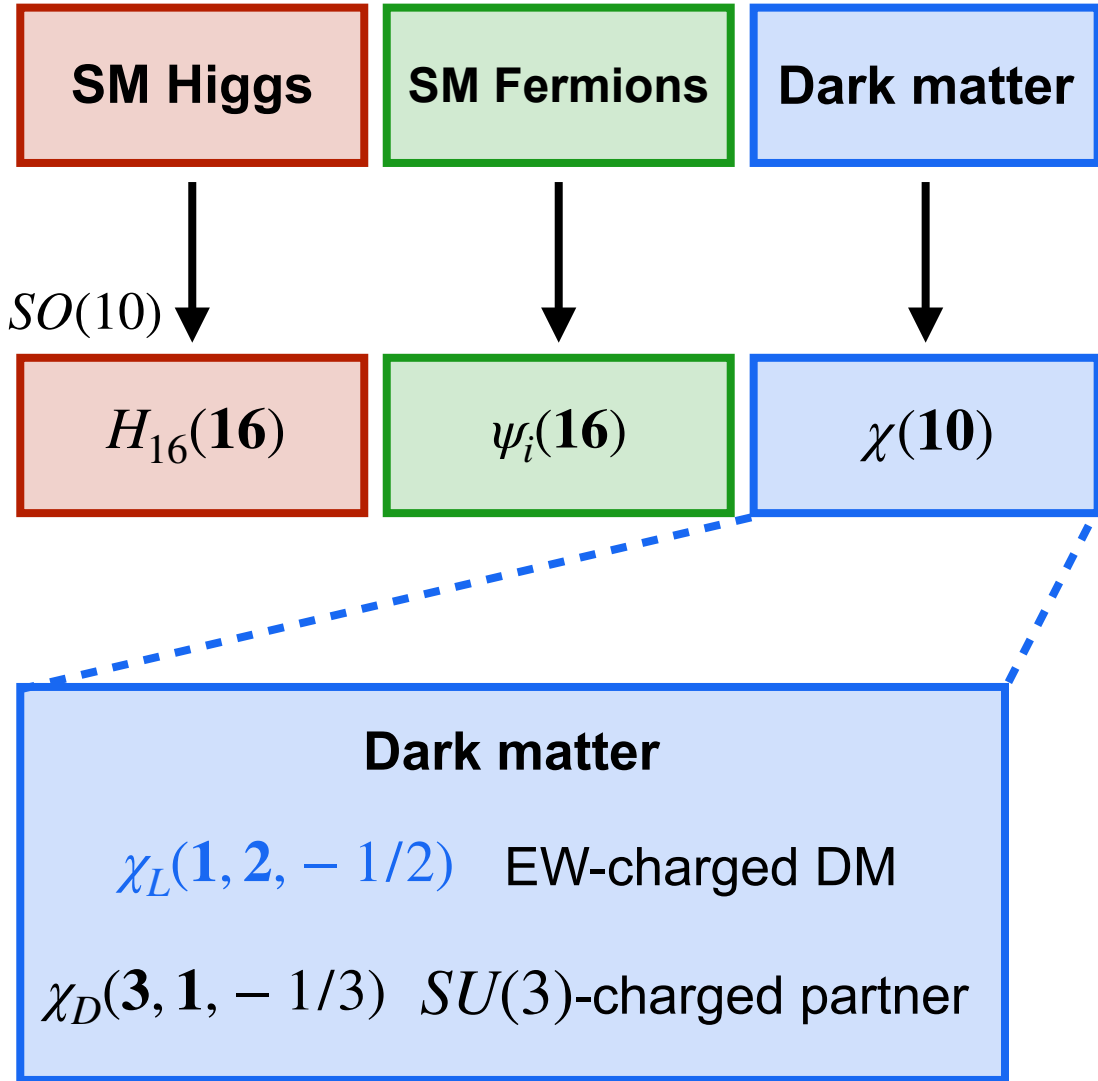
Left-right breaking scale v_R (VEV of H_R)

$m_D > m_L$ will modify v_R

$m_D > m_L$ will modify M_{XY}

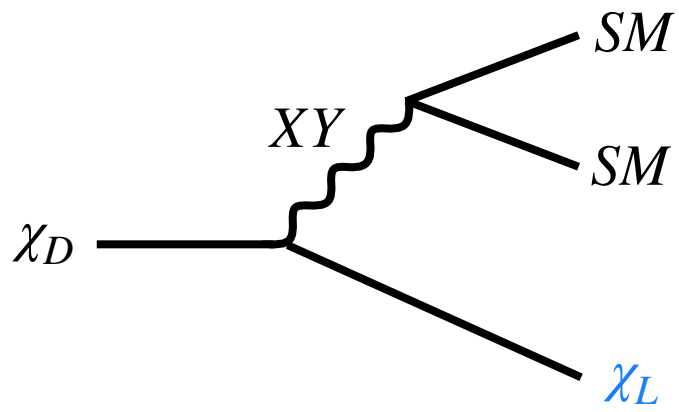
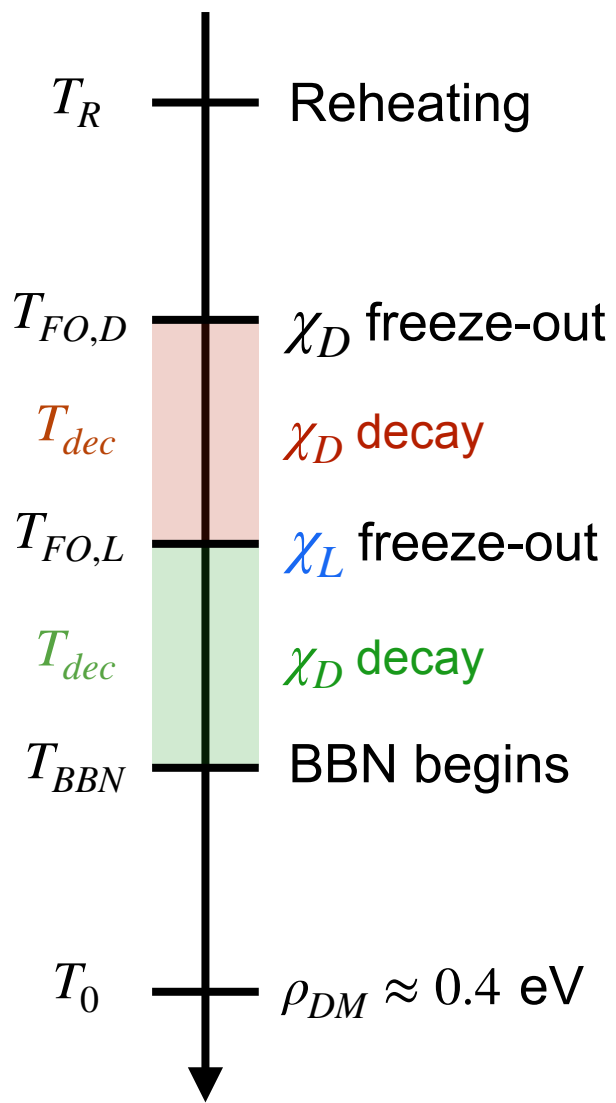
Why is $m_D > m_L$ required?

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$





Mass splitting necessary to avoid dark matter overproduction



$$\chi_L(1, 2, -1/2) \quad \chi_D(3, 1, -1/3)$$

$$\Gamma \sim \frac{1}{128\pi^3} \frac{m_D^5}{M_{XY}^4}$$

$T_{dec} > T_{FO,L}$ χ_L freeze-out gives ρ_{DM} for $m_L = 1$ TeV

$T_{dec} < T_{FO,L}$ χ_L freeze-out and $\chi_D \rightarrow \chi_L$ decay give ρ_{DM}

To avoid overproduction

$$m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{M_{XY}}{10^{16} \text{ GeV}} \right)^{\frac{4}{5}} \left(\frac{m_L}{100 \text{ GeV}} \right)^{\frac{6}{5}}$$

We will take $m_L = 1$ TeV



Gauge coupling unification modified by dark matter



2-loop RGE

$$\frac{d}{d \ln \mu} \left(\frac{2\pi}{\alpha_i} \right) = b_i + \sum_j b_{ij} \frac{\alpha_j}{2\pi}$$

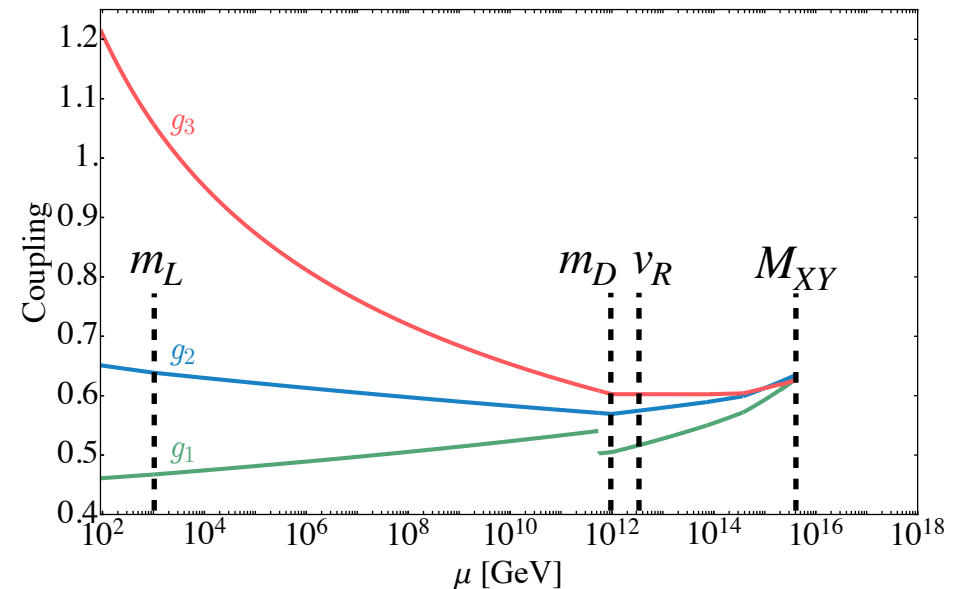
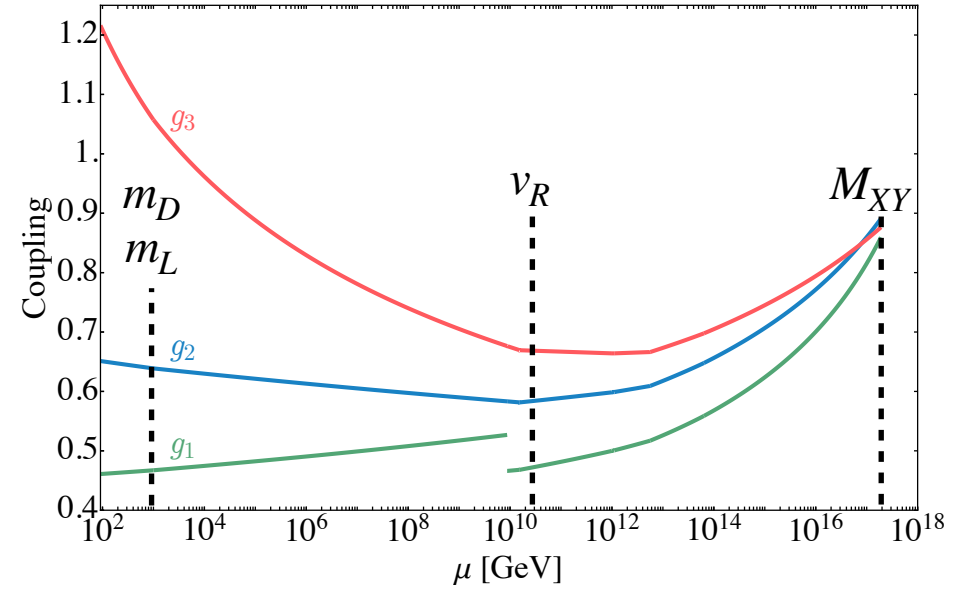
Quality of unification— **threshold correction**

$$\Delta(M_{XY}) \sim \max_{i,j=1,2,3} \left| \frac{2\pi}{\alpha_i(M_{XY})} - \frac{2\pi}{\alpha_j(M_{XY})} \right|$$

Smaller $\Delta(M_{XY})$ signals good unification

We consider $\Delta < 15^*$

Mass splitting between χ_L and χ_D modifies M_{XY} and v_R





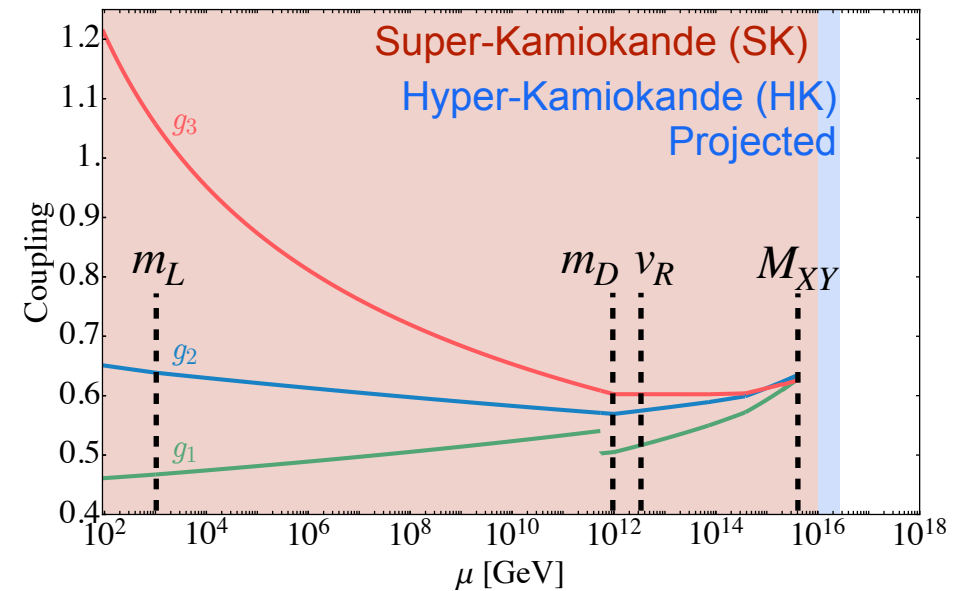
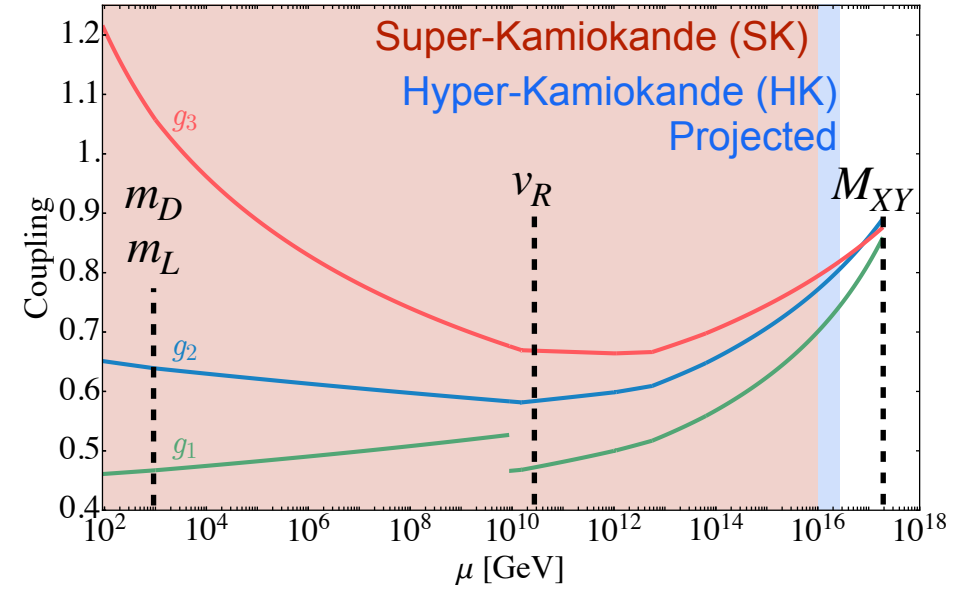
Unification scale bounded above by proton decay



Heavy XY gauge bosons from $SO(10) \rightarrow G_{LR}$ mediate proton decay $p \rightarrow e^+ + \pi^0$ with rate

$$\tau_{p \rightarrow e^+ + \pi^0} \propto M_{XY}^4$$

Non-observation of $p \rightarrow e^+ + \pi^0$ bounds M_{XY} from below





Parameters bounded in opposite directions by m_D/m_L and proton decay



As m_D/m_L increases, $\Delta = 0$ point moves towards larger ν_R , smaller M_{XY}

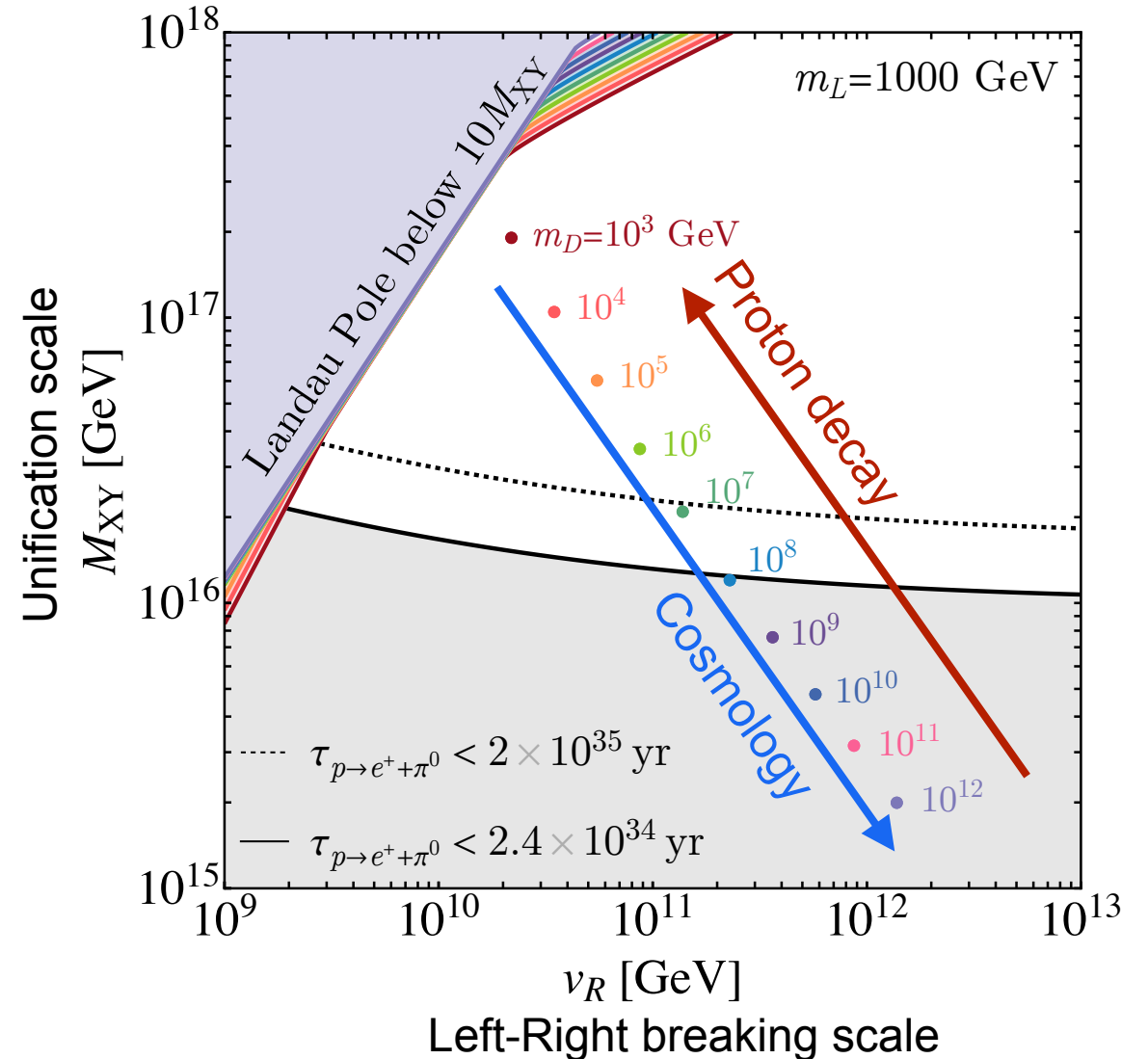
To avoid overabundance

$$m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{M_{XY}}{10^{16} \text{ GeV}} \right)^{\frac{4}{5}} \left(\frac{m_L}{100 \text{ GeV}} \right)^{\frac{6}{5}}$$

and have $\Delta = 0$ is excluded by SK

Proton decay bounds favour smaller m_D/m_L

Overabundance bounds favour larger m_D/m_L





Overabundance and proton decay bounds can be saturated



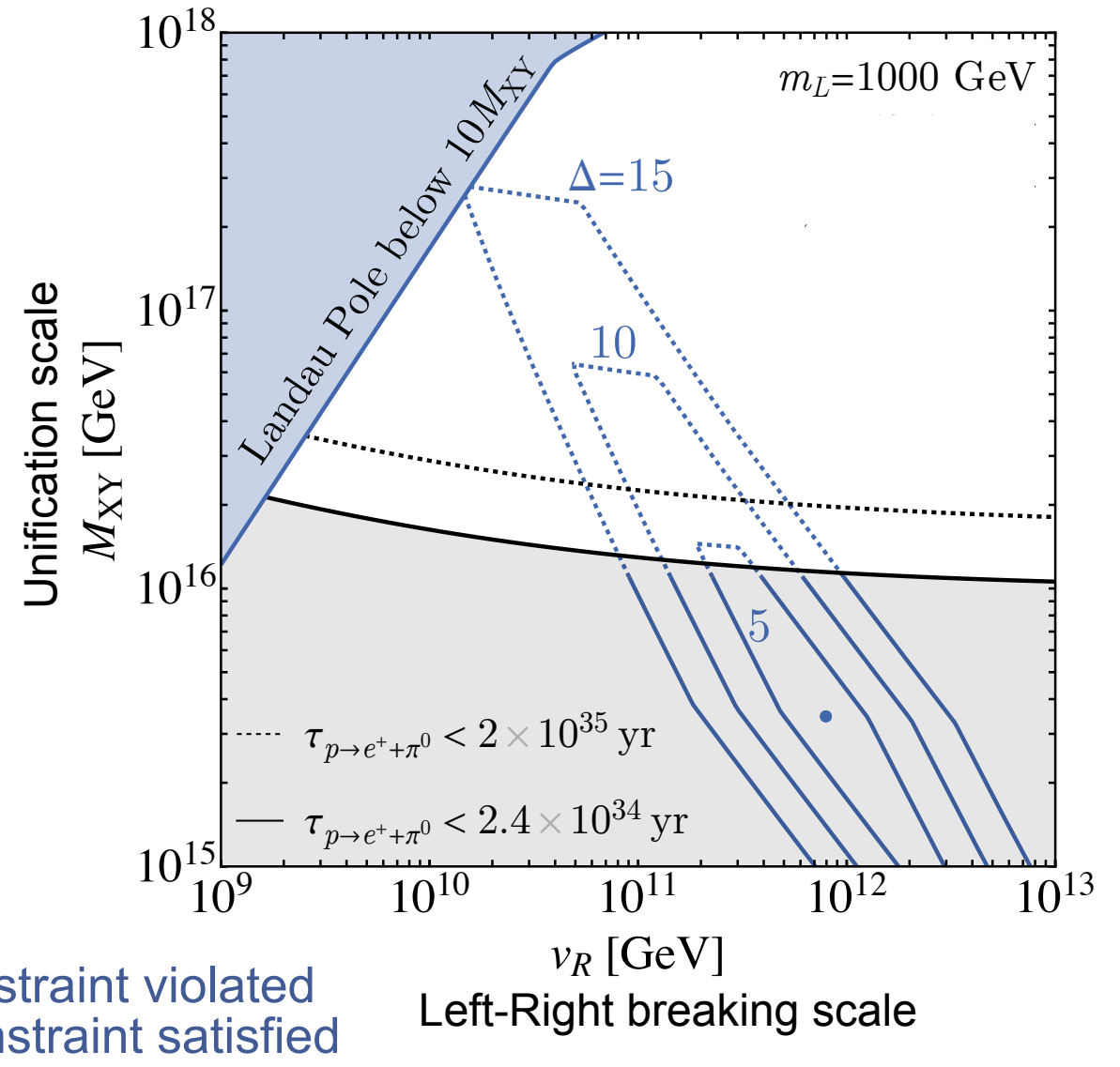
Proton decay bounds and overabundance bounds can be saturated

Minimum required thresholds for SK

$$\Delta_{SK} \gtrsim 4$$

Minimum required thresholds for HK

$$\Delta_{HK} \gtrsim 7$$





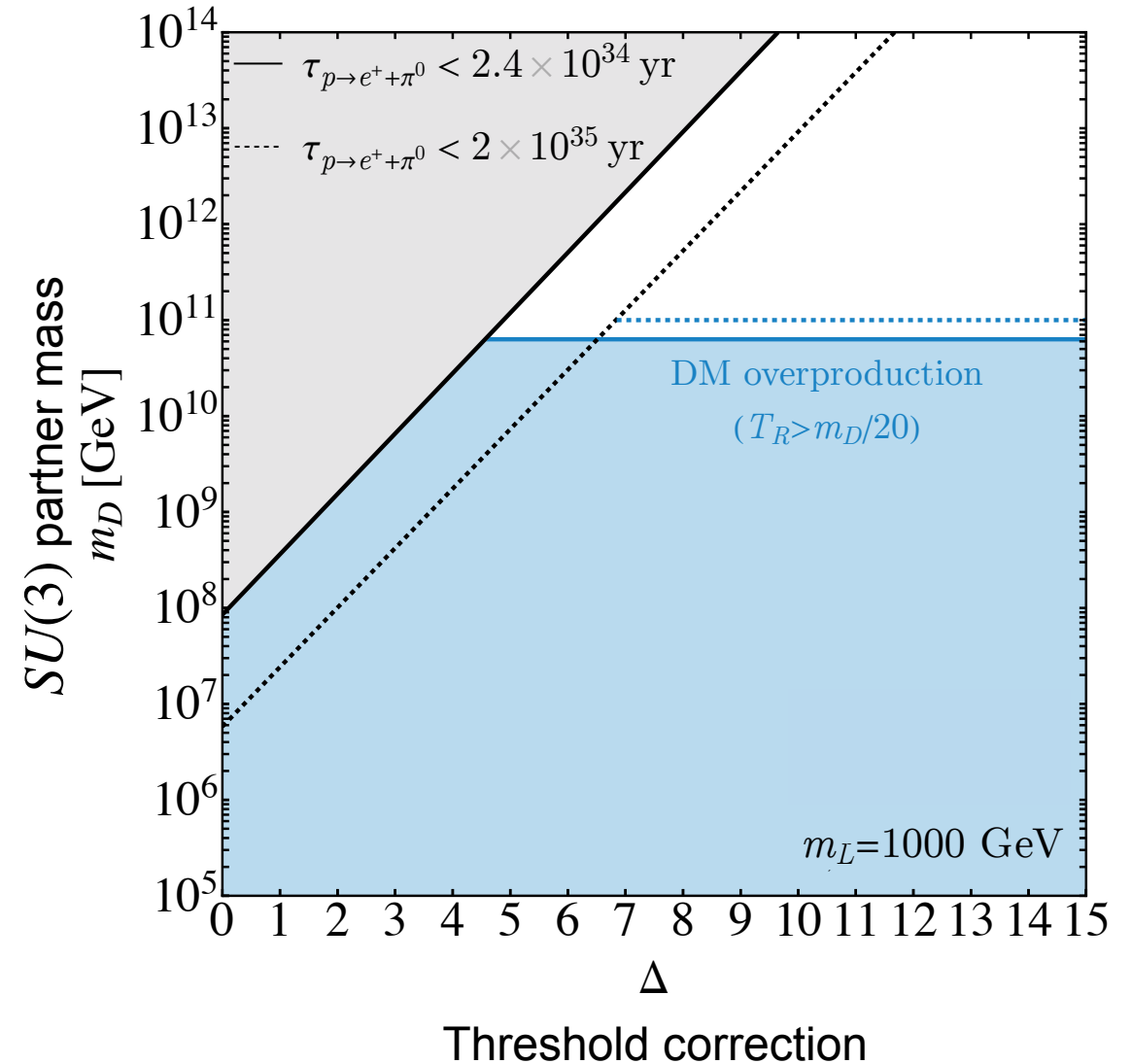
m_D constrained from below by cosmology



Proton decay bounds m_D from above for a given threshold correction Δ

$$m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{M_{XY}}{10^{16} \text{ GeV}} \right)^{\frac{4}{5}} \left(\frac{m_L}{100 \text{ GeV}} \right)^{\frac{6}{5}}$$

Cosmology bounds m_D from below for a given threshold correction Δ





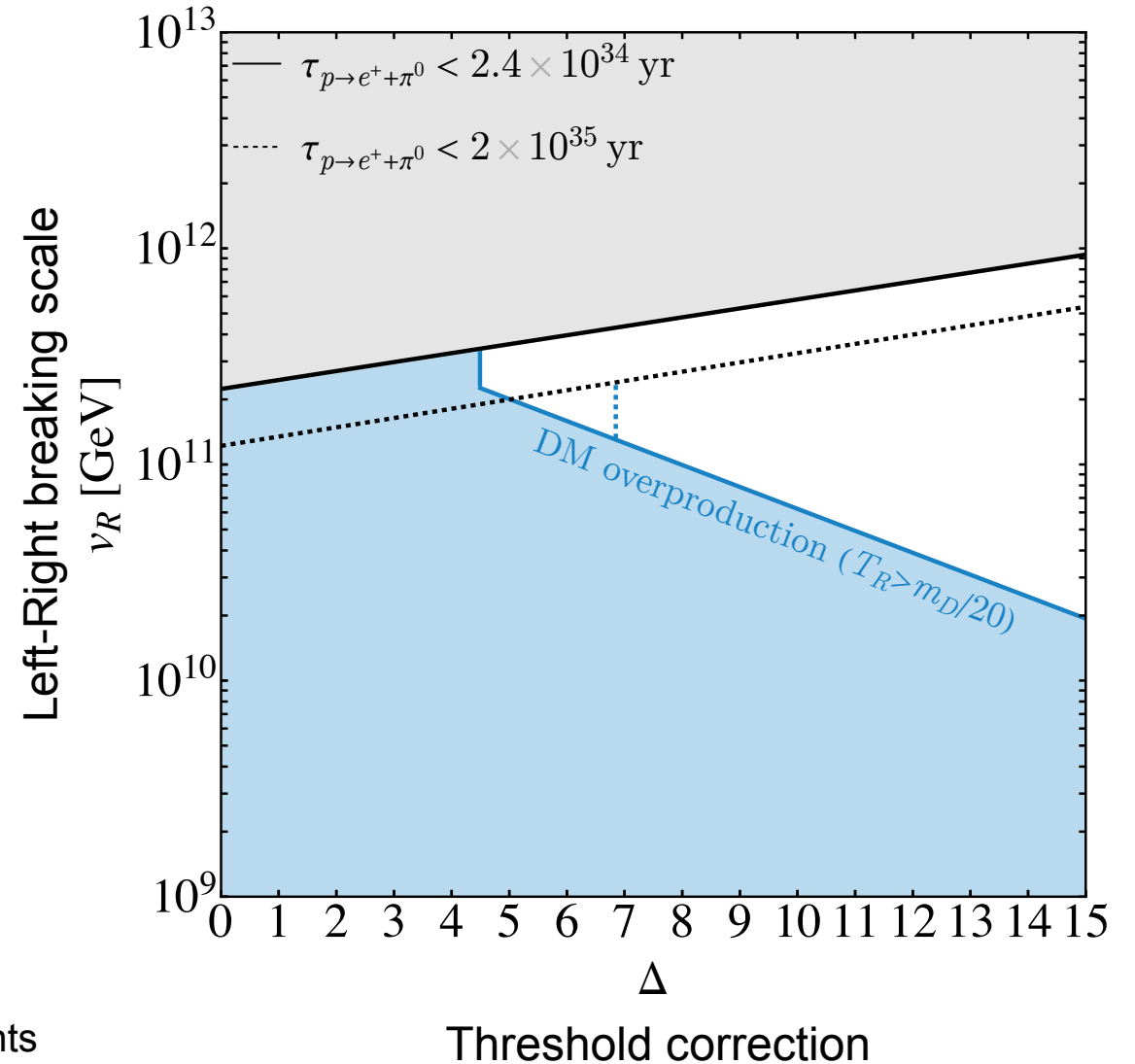
ν_R constrained from below by cosmology



Proton decay bounds ν_R from above for a given threshold correction Δ

$$m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{M_{XY}}{10^{16} \text{ GeV}} \right)^{\frac{4}{5}} \left(\frac{m_L}{100 \text{ GeV}} \right)^{\frac{6}{5}}$$

Cosmology bounds ν_R from below for a given threshold correction Δ



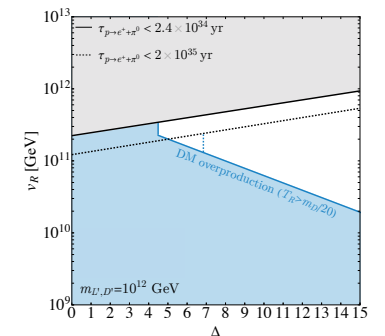
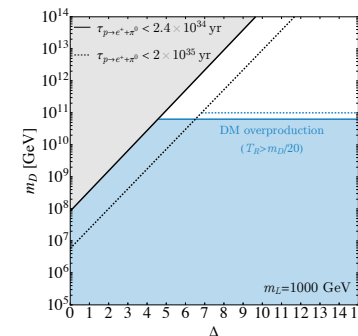
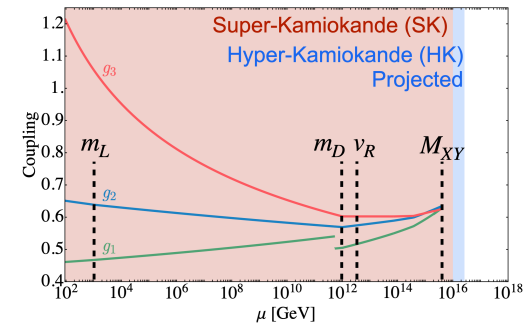
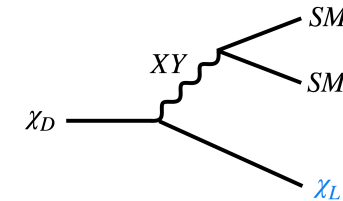
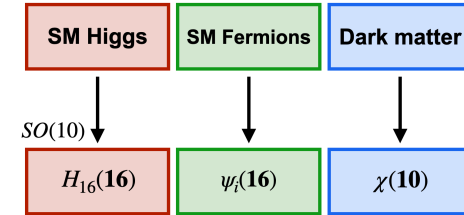
Electroweak-charged WIMPs can be successfully embedded into $SO(10)$

Overabundance bounds require mass splitting between DM and $SU(3)$ -charged particles

Successful unification and proton decay bounds constrain $SO(10)$ breaking scale

Combined, these bounds provide new constraints on model parameters

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$





Questions?





Backup Slides





VEV hierarchy of breaking scales



$$V(H_R, H_L) = \lambda \left(|H_R|^2 + |H_L|^2 - f^2 \right)^2 + \Delta\lambda |H_R|^2 |H_L|^2$$

$\Delta\lambda > 0$ vacua: $(v_L, v_R) = (f, 0)$ and $(0, f)$ \longrightarrow Not phenomenologically viable

$\Delta\lambda < 0$ vacua: $(v_L, v_R) = (f, f)/\sqrt{2}$ \longrightarrow Not phenomenologically viable

$\Delta\lambda \simeq 0$ vacua: $(v_L, v_R) = (\cos\theta, \sin\theta)f$ \longrightarrow Tree level degeneracy lifted by quantum corrections
Obtain $v_L \simeq 173 \text{ GeV} \ll v_R \simeq f$ by tuning $\Delta\lambda$ with accuracy v_L^2/v_R^2

No additional fine tuning that SM due to cancellation of the v_R scale in the total fine tuning

$$\frac{v_L^2}{v_R^2} \times \frac{v_R^2}{\Lambda^2} = \frac{v_L^2}{\Lambda^2} \text{ SM fine tuning}$$

v_R from cutoff Λ tuning



Other $SO(10)$ breaking scenarios



$$SO(10) \xrightarrow{\langle H_{45} \rangle} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{\langle H_{54} \rangle} G_{LR} \quad \langle H_{45} \rangle = v_{45}, \langle H_{54} \rangle = v_{54}$$

XY gauge bosons

Mediate proton decay

$$M_{XY}^2 = g_{10}^2 (v_{45}^2 + v_{54}^2)$$

PS gauge bosons

Do not mediate proton decay

$$M_{PS}^2 = 4g_{10}^2 v_{45}^2$$

$$r_{XY} \equiv \frac{M_{PS}}{M_{XY}}$$

When $SO(10) \times CP$ is broken only by H_{45} then $r_{XY} = 2$

The $m_L = 1$ TeV results shown in the main slides correspond to $r_{XY} = 2$

$r_{XY} < 2$ is possible by more complicated breaking with multiple Higgs, e.g. H_{54} (**54**)



Tree level instability of SO(10) breaking H_{45}



$$\langle H_{45} \rangle = -iv_{45} \begin{pmatrix} \sigma_2 & 0 & 0 & \\ 0 & \sigma_2 & 0 & 0_{4 \times 6} \\ 0 & 0 & \sigma_2 & \\ 0_{6 \times 4} & & & 0_{4 \times 4} \end{pmatrix}$$

Missing VEV of H_{45}

$\langle H_{45} \rangle$ unstable at tree level but stabilized by loop corrections [1]

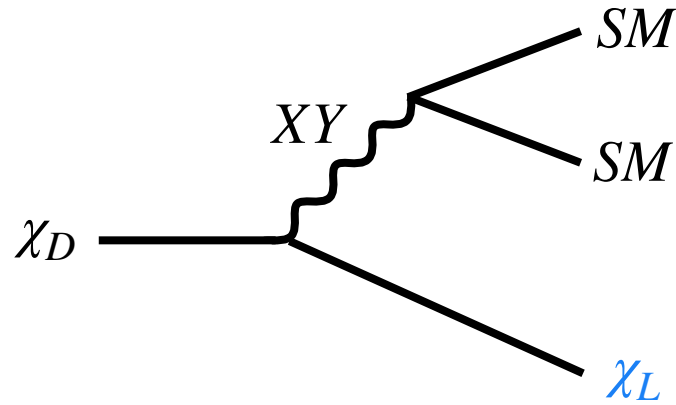
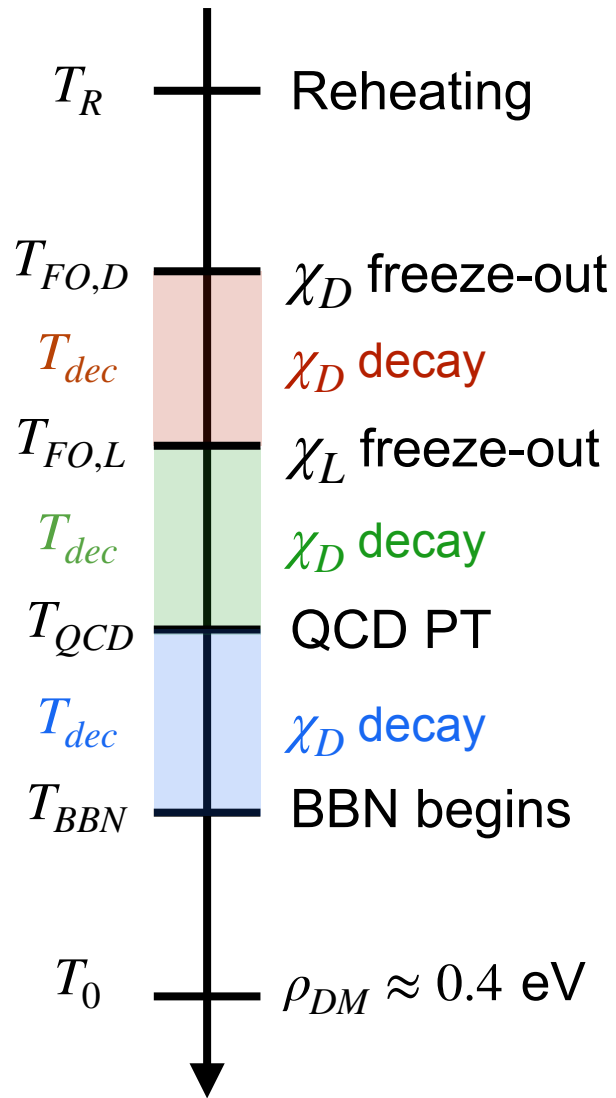
Alternatively we can stabilize the VEV by introducing an additional H_{54} Higgs to SO(10)

$$\langle H_{54} \rangle = \frac{1}{5} v_{54} \begin{pmatrix} 2 \times 1_{6 \times 6} & 0 \\ 0 & -3 \times 1_{4 \times 4} \end{pmatrix}$$

H_{54} couples to H_{45} to stabilize the vacuum



χ_D forms bound states if it decays after QCD phase transition



$$\chi_L(1, 2, -1/2) \quad \chi_D(3, 1, -1/3)$$

$$\Gamma \sim \frac{1}{128\pi^3} \frac{m_D^5}{M_{XY}^4}$$

$T_{QCD} < T_{FO,L} < T_{dec}$ χ_L freeze-out gives ρ_{DM} for $m_L = 1$ TeV

$T_{QCD} < T_{dec} < T_{FO,L}$ χ_L freeze-out and $\chi_D \rightarrow \chi_L$ decay give ρ_{DM}

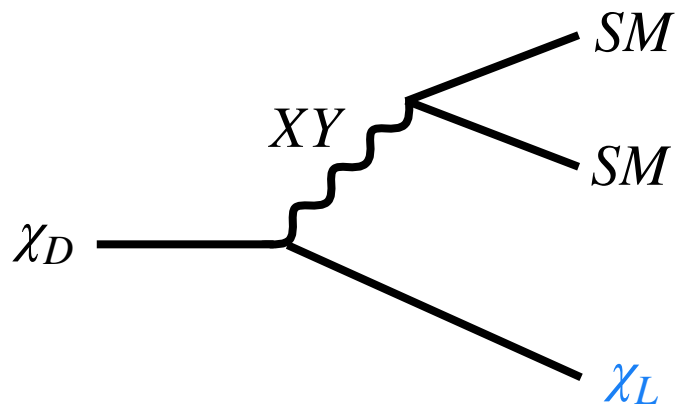
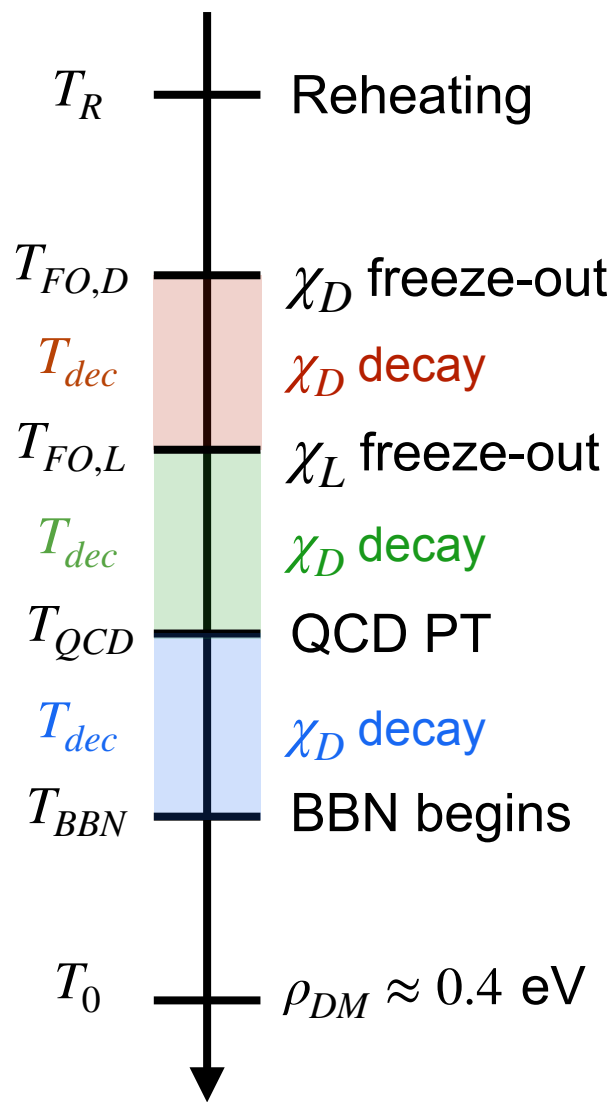
$T_{dec} < T_{QCD} < T_{FO,L}$ χ_L freeze-out and $\chi_D \rightarrow$ bound states $\rightarrow \chi_L$ decay give ρ_{DM}

To avoid overproduction

$$m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{M_{XY}}{10^{16} \text{ GeV}} \right)^{\frac{4}{5}} \left(\frac{m_L}{100 \text{ GeV}} \right)^{\frac{6}{5}}$$



Derivation of the overabundance bound



$$\chi_L(1, 2, -1/2) \quad \chi_D(3, 1, -1/3)$$

$$\Gamma \sim \frac{1}{128\pi^3} \frac{m_D^5}{M_{XY}^4}$$

χ_D decay occurs around temperature

$$T_{dec} \simeq 2 \text{ MeV} \left(\frac{m_D}{10^9 \text{ GeV}} \right)^{5/2} \left(\frac{10^{16} \text{ GeV}}{m_{XY}} \right)^2$$

Even for $T_{dec} < T_{FO,L}$, number density n decreases by annihilation to $n \approx H/(\sigma v)$, giving DM density

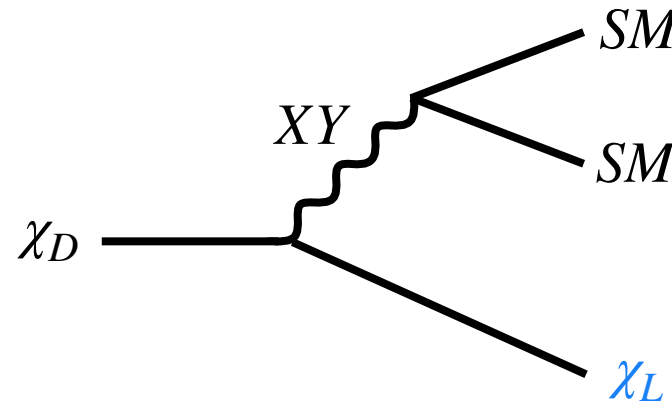
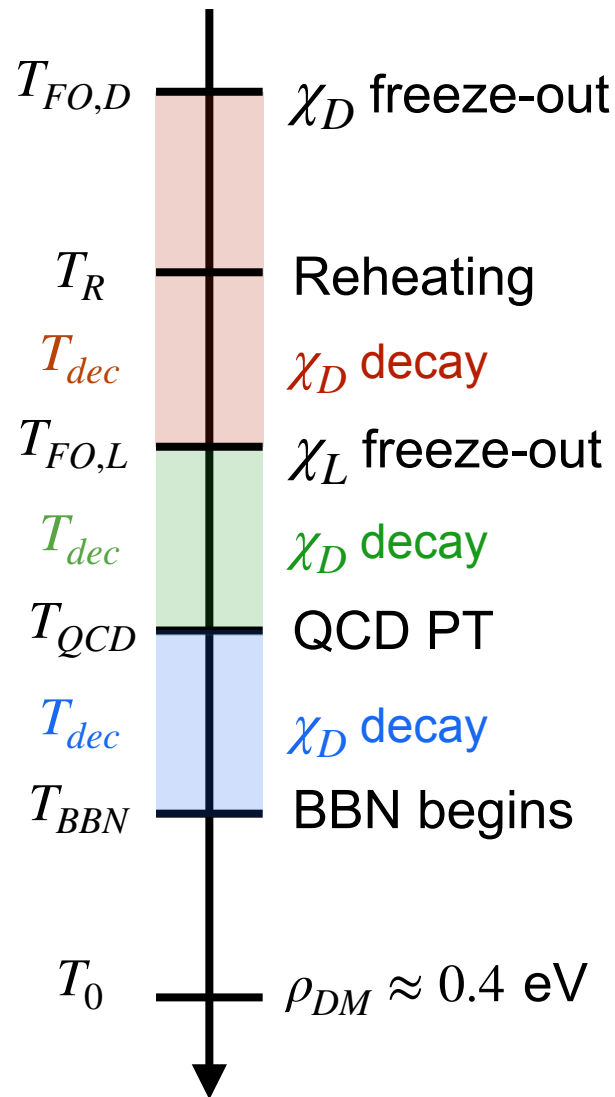
$$\frac{\rho_{DM}}{s} \simeq 0.4 \text{ eV} \left(\frac{m_L}{100 \text{ GeV}} \right)^3 \frac{0.05 \text{ GeV}}{T_{dec}}$$

Inserting T_{dec} and requiring that we do not overproduce DM

$$\rho_{DM} \leq 0.4 \text{ eV leads to } m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{m_{XY}}{10^{16} \text{ GeV}} \right)^{4/5} \left(\frac{m_L}{100 \text{ GeV}} \right)^{6/5}$$



$T_R < T_{FO,D}$ reheating scenario can be achieved



$$\chi_L(1, 2, -1/2) \quad \chi_D(3, 1, -1/3)$$

$$\Gamma \sim \frac{1}{128\pi^3} \frac{m_D^5}{M_{XY}^4}$$

Abundance of χ_D can be suppressed compared to $T_R > T_{FO,D}$

Cosmological bound on m_D avoided by taking $T_R \ll T_{FO,D}$
[CITE]

Conservative estimate

Maximal temperature of universe $T_{max} \leq T_R \sim T_{FO,L}$

Even if $m_D = \text{few } m_L$, abundance of χ_D exponentially suppressed compared to χ_L



$SO(10)$ needs Yukawa and Singlet sectors



	Higgs	SM Fermions	Yukawa	Singlets	Dark Matter
$\langle H_{45} \rangle = v_{45}$	$H_{45}(45) \quad H_{16}(16)$	$\psi_i(16)$ $i = 1, 2, 3$	$X_{10,i}(10)$ $X_{45,i}(45)$	$S(1)$	$\chi_{10,n}(10)$ Weyl $n = 1, 2$
$\langle H_R \rangle = v_R$	$H_R(1, 1, 2, 1/2)$ $H_L(1, 2, 1, -1/2)$	$q(3, 2, 1, 1/6) \quad \bar{q}(\bar{3}, 1, 2, -1/6)$ $l(1, 2, 1, -1/2) \quad \bar{l}(1, 1, 2, 1/2)$			$D_n(3, 1, 1, -1/3)$ $\bar{D}_n(\bar{3}, 1, 1, 1/3)$ $\Delta_n(1, 2, 2, 0)$
G_{SM}	$h_L(1, 2, -1/2)$	$Q(3, 2, 1/6) \quad L(1, 2, -1/2)$ $\bar{d}(\bar{3}, 1, 1/3) \quad \bar{u}(\bar{3}, 1, -2/3)$ $\bar{e}(1, 1, 1) \quad \bar{N}(1, 1, 0)$			$\chi_D(3, 1, -1/3) \quad \chi_{D'}(\bar{3}, 1, 1/3)$ $\chi_L(1, 2, -1/2) \quad \chi_{L'}(1, 2, 1/2)$ Dirac



Dark matter embedding and coupling to H_{45}



Successful embedding of electroweak WIMPs into $SO(10)$ requires two Weyl fermions χ_1 and χ_2 with

$$\left[\chi_{10_1}^a H_{45}^{ab} \chi_{10_2}^b + \text{h.c.} \right] + m_2 \chi_{10_2} \chi_{10_2} \quad a, b = 1, 2, \dots, 10$$

↓

$m_L \leq m_D$

Avoid overabundance

↓

$m_L \ll m_{L'}, m_{D'}$

Successful unification

$$\langle H_{45} \rangle = -iv_{45} \begin{pmatrix} \sigma_2 & 0 & 0 & \\ 0 & \sigma_2 & 0 & 0_{4 \times 6} \\ 0 & 0 & \sigma_2 & \\ 0_{6 \times 4} & & & \boxed{0_{4 \times 4}} \end{pmatrix}$$

Missing VEV* of H_{45}



Standard model Yukawa sector obtained from $SO(10)$



SM Yukawas generated by $SO(10) \times CP$ interaction terms with $X_{10}(\mathbf{10})$ and $X_{45}(\mathbf{45})$ fermions

$$\mathcal{L} = -x_{10}^{ij} H_{16} \psi_i X_{10,j} - i x_{10}^{\prime ij} H_{16} \psi_i X_{10,j} H_{45} - (M_{10}^{ij} + i \lambda_{10}^{ij} H_{45}) X_{10,i} X_{10,j} \\ - x_{45}^{ij} H_{16}^\dagger \psi_i X_{45,j} - i x_{45}^{\prime ij} H_{16}^\dagger \psi_i X_{45,j} H_{45} - (M_{45}^{ij} + i \lambda_{45}^{ij} H_{45}) X_{45,i} X_{45,j} + \text{h.c.}$$

$SO(10) \times CP \rightarrow G_{LR}$ yields quark Yukawa terms

$$\mathcal{L} = -x_d^{ij} H_L q_i \bar{D}_j - x_d^{*ij} H_R \bar{q}_i D_j - M_d^{ij} D_i \bar{D}_j + \text{h.c.}$$

Yukawas are complex with Parity ensures no strong CP phase

Under Parity $q(t, \mathbf{x}) \leftrightarrow i\sigma_2 \bar{q}^*(t, -\mathbf{x}), D(t, \mathbf{x}) \leftrightarrow i\sigma_2 \bar{D}^*(t, -\mathbf{x})$

Mass matrix $(d_i \ D_i) \begin{pmatrix} 0 & x_d^{ij} v_L \\ x_d^{*ji} v_R & M_d^{ij} \end{pmatrix} \begin{pmatrix} \bar{d}_j \\ \bar{D}_j \end{pmatrix}$ Real determinant \longrightarrow Strong CP phase **0** or π

Strong CP Problem Solved

Similar terms obtained for lepton Yukawas

$$\mathcal{L} = -x_e^{ij} H_R \ell_i \Delta_j - x_e^{*ij} H_L \bar{\ell}_i \Delta_j - \frac{1}{2} M_e^{ij} \Delta_i \Delta_j + \text{h.c.}$$



Singlet sector mixes dark matter via Majorana mass term

Majorana mass allows dark matter to mix

Necessary to avoid non-velocity-suppressed nucleon scattering via Z -exchange— excluded by direct-detection constraints.

Achieved by introducing heavy $SO(10)$ singlet S

$$\mathcal{L} = -\frac{1}{2M}S\chi_{10}H_{16}H_{16} - \frac{1}{2M'}S\chi_{10}H_{16}^\dagger H_{16}^\dagger - \frac{1}{2}m_S S^2 + \text{h.c.}$$

Integrate out S

$$\mathcal{L} = \frac{v_R^2}{2M^2 m_S} \chi_L \chi_L H_L H_L$$

Now DM multiplet contains two Majorana fermions χ_1, χ_2 with mass splitting

$$\Delta m_0 = m_{\chi_2} - m_{\chi_1} \simeq \frac{v_R^2 v_L^2}{M^2 m_S} = 100 \text{ keV} \frac{10 \text{ TeV}}{m_S} \left(\frac{v_R/M}{0.006} \right)^2$$



$m_L < 1$ TeV possible for $T_{dec} < T_{FO,L}$



$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} \quad SU(2) \text{ doublet}$$

χ_L^+ decays to χ_L^0 leave disappearing tracks in collider detectors

This probes the mass difference Δm^\pm between χ_L^+ and χ_L^0

$\Delta m^\pm \simeq 340$ MeV if dominated by EW quantum corrections

This corresponds to $m_L > 200$ GeV LHC bound

High Luminosity LHC will improve this to $m_L > 500$ GeV

If $\Delta m^0 \simeq$ EW correction then $\Delta m^\pm > 340$ MeV and the $m_L > 100$ GeV LEP bound applies



Matching conditions relate couplings of different gauge groups



Match to SM values at m_t

$$g_1(m_t) = 0.4626, \quad g_2(m_t) = 0.64779, \quad g_3(m_t) = 1.1666$$

Match G_{SM} to G_{LR} at $M_{W_R} = g_R(v_R)v_R/\sqrt{2}$

$$\frac{2\pi}{\alpha_1^{SM}(M_{W_R})} = \frac{2}{5} \frac{2\pi}{\alpha_1^{LR}(M_{W_R})} + \frac{3}{5} \frac{2\pi}{\alpha_2^{LR}(M_{W_R})} - \frac{1}{10},$$

$$\frac{2\pi}{\alpha_2^{SM}(M_{W_R})} = \frac{2\pi}{\alpha_2^{LR}(M_{W_R})}, \quad \frac{2\pi}{\alpha_3^{SM}(M_{W_R})} = \frac{2\pi}{\alpha_3^{LR}(M_{W_R})}.$$

Threshold corrections

Match G_{LR} to $SO(10) \times CP$ at M_{XY}

$$\frac{2\pi}{\alpha_1(M_{XY})} = \frac{2\pi}{\alpha_{10}(M_{XY})} + \Delta_{1,G} + \Delta_{1,H} + \Delta_1,$$

$$\frac{2\pi}{\alpha_2(M_{XY})} = \frac{2\pi}{\alpha_{10}(M_{XY})} + \Delta_{2,G} + \Delta_{2,H} + \Delta_2,$$

$$\frac{2\pi}{\alpha_3(M_{XY})} = \frac{2\pi}{\alpha_{10}(M_{XY})} + \Delta_{3,G} + \Delta_{3,H} + \Delta_3,$$



Threshold corrections contain gauge and Higgs contributions



$$\Delta(M_{XY}) \equiv \max_{i,j} |\Delta_i - \Delta_j| = \max_{i,j} \left| \left(\frac{2\pi}{\alpha_i} - \Delta_{i,G} - \Delta_{i,H} \right) - \left(\frac{2\pi}{\alpha_j} - \Delta_{j,G} - \Delta_{j,H} \right) \right|$$

Gauge boson contribution $\Delta_{1,G} = 14 \ln r_{XY} - \frac{4}{3}$, $\Delta_{2,G} = -1$, $\Delta_{3,G} = \frac{7}{2} \ln r_{XY} - \frac{5}{6}$

Higgs contribution $\Delta_{1,H} = 0$, $\Delta_{2,H} = -\frac{1}{3} \ln \frac{M_{(1,3,1,0)}}{M_{XY}} = -\frac{1}{3} \ln \frac{M_{(1,1,3,0)}}{M_{XY}}$, $\Delta_{3,H} = -\frac{1}{2} \ln \frac{M_{(8,1,1,0)}}{M_{XY}}$

where [1] $\frac{M_{1,3,1,0}^2}{M_{XY}^2} = \frac{M_{1,1,3,0}^2}{M_{XY}^2} = \frac{19g^2}{4\pi^2}$, $\frac{M_{8,1,1,0}^2}{M_{XY}^2} = \frac{22g^2}{4\pi^2}$



Integrating out heavy field gives $B-$ and $L-$ violating terms



Effective Lagrangian

$$\mathcal{L} = \frac{g_{10}^2}{M_{XY}^2} \left[2A_L(QL)(\bar{u}\bar{d})^\dagger + A_R(QQ)(\bar{u}\bar{e})^\dagger \right] + \text{h.c.} \quad A_{R,L} = A_{R,L}^{SM} \times A_{R,L}^{LR}$$

1-loop renormalization [2]

$$A_R^{SM} = \prod_n \left(\frac{\alpha_3(\mu_{n+1})}{\alpha_3(\mu_n)} \right)^{-\frac{2}{b_3^{\eta_3}}} \left(\frac{\alpha_2(\mu_{n+1})}{\alpha_2(\mu_n)} \right)^{-\frac{9}{4b_2^{\eta_2}}} \left(\frac{\alpha_1(\mu_{n+1})}{\alpha_1(\mu_n)} \right)^{-\frac{11}{12b_1^{\eta_1}}},$$

$$A_L^{SM} = \prod_n \left(\frac{\alpha_3(\mu_{n+1})}{\alpha_3(\mu_n)} \right)^{-\frac{2}{b_3^{\eta_3}}} \left(\frac{\alpha_2(\mu_{n+1})}{\alpha_2(\mu_n)} \right)^{-\frac{9}{4b_2^{\eta_2}}} \left(\frac{\alpha_1(\mu_{n+1})}{\alpha_1(\mu_n)} \right)^{-\frac{23}{12b_1^{\eta_1}}},$$

$$A_R^{LR} = \prod_n \left(\frac{\alpha_3(\mu_{n+1})}{\alpha_3(\mu_n)} \right)^{-\frac{2}{b_3^{\eta_3}}} \left(\frac{\alpha_2(\mu_{n+1})}{\alpha_2(\mu_n)} \right)^{-\frac{9}{2b_2^{\eta_2}}} \left(\frac{\alpha_1(\mu_{n+1})}{\alpha_1(\mu_n)} \right)^{-\frac{1}{4b_1^{\eta_1}}},$$

$$A_L^{LR} = A_R^{LR},$$

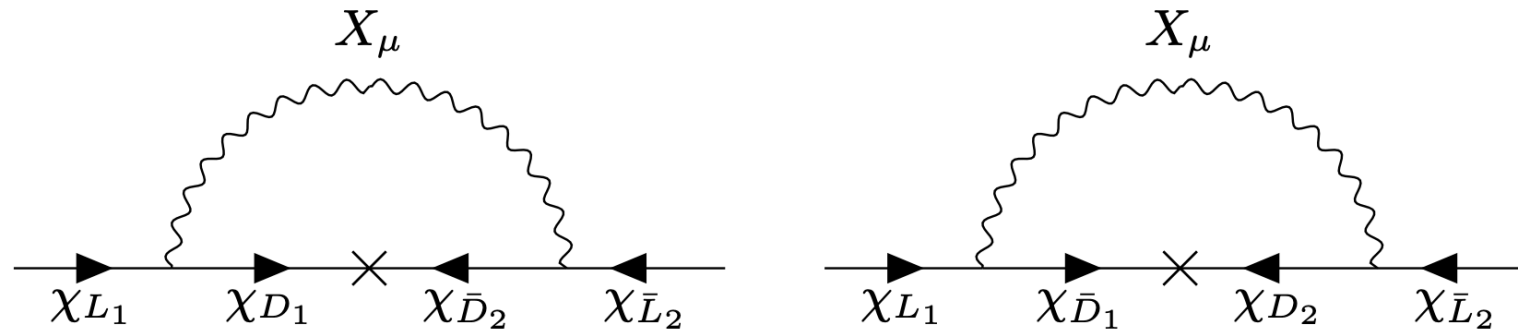
$$p \rightarrow e^+ + \pi^0 \text{ decay rate } \tau_{p \rightarrow e^+ + \pi^0} = \left[\frac{1}{32\pi} m_p \left(1 - \frac{m_{\pi^0}^2}{m_p^2} \right)^2 \frac{g_{10}^4}{M_{XY}^4} (4A_L^2 + A_R^2) |W_0|^2 \right]^{-1}$$



One-loop corrections to the mass of χ_L vanish



1-loop corrections to mass of χ_L



Sum of diagrams is zero due to opposite masses of $\chi_{D1}\chi_{\bar{D}2}$ and $\chi_{\bar{D}1}\chi_{D2}$



One Weyl Fermion Case



One Weyl fermion can couple to H_{45} via

$$\chi_{10}^a H_{45}^{ab} H_{45}^{bc} \chi_{10}^c$$

If H_{45} real, mass splitting unstable due to quadratically divergent corrections generating $\chi_{10}^a \chi_{10}^a$

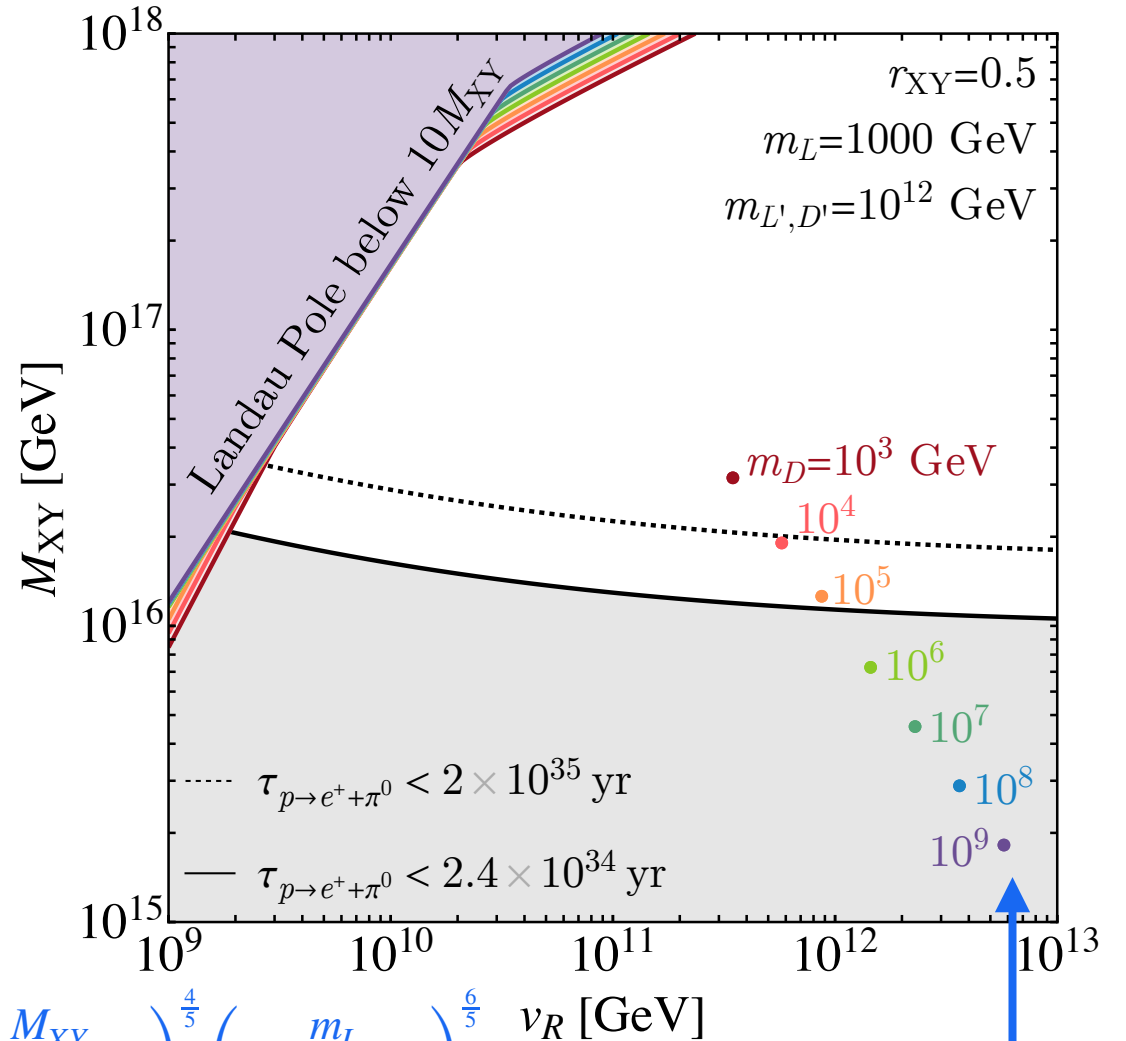
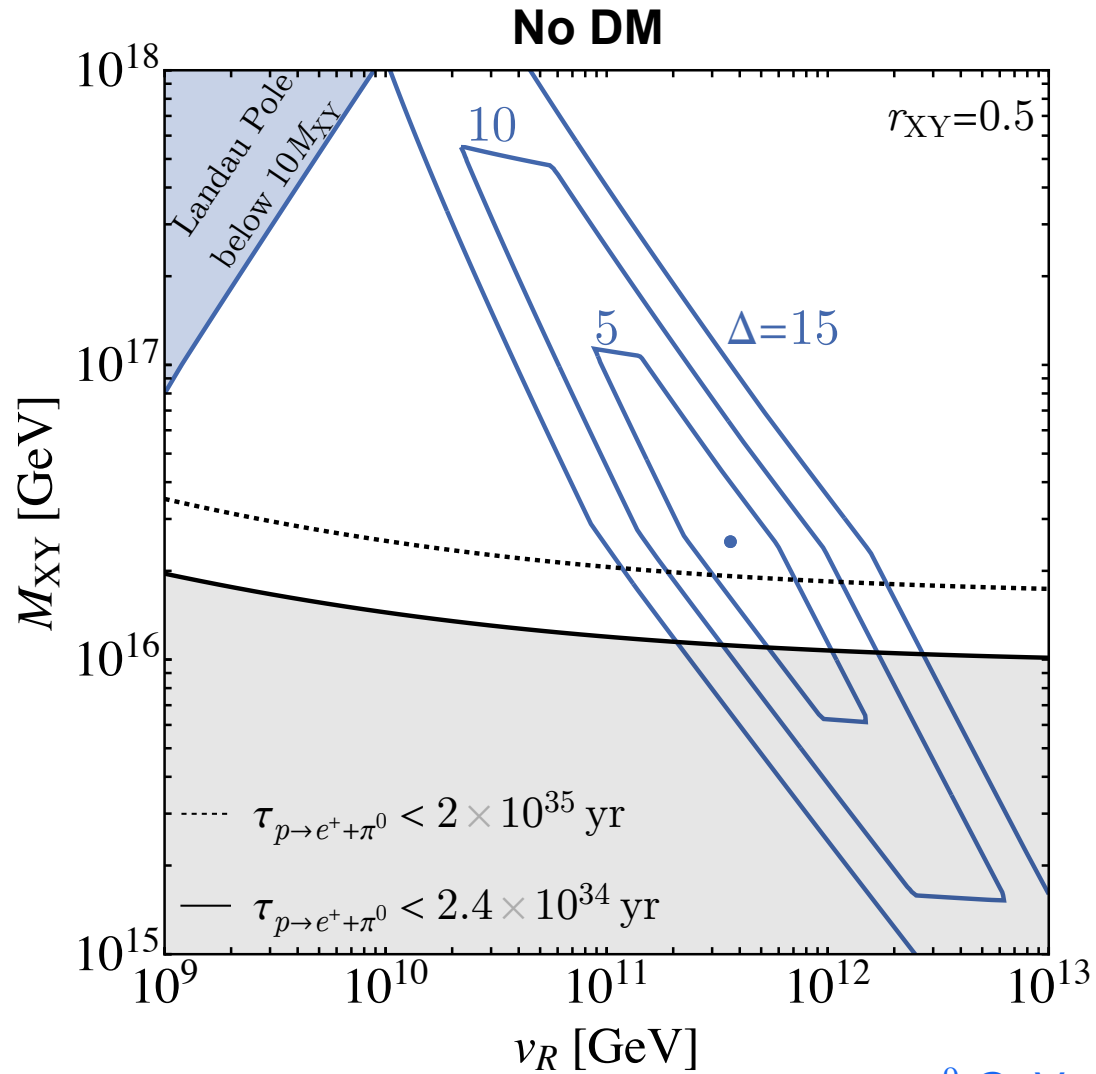
If H_{45} complex, no quadratically divergent corrections, but the following is generated by quantum corrections

$$\chi_{10}^a \chi_{10}^a H_{45}^{bc} H_{45}^{bc}$$

In this case $m_L/m_D \sim 10^{-3}$ which is too small to satisfy overproduction bounds



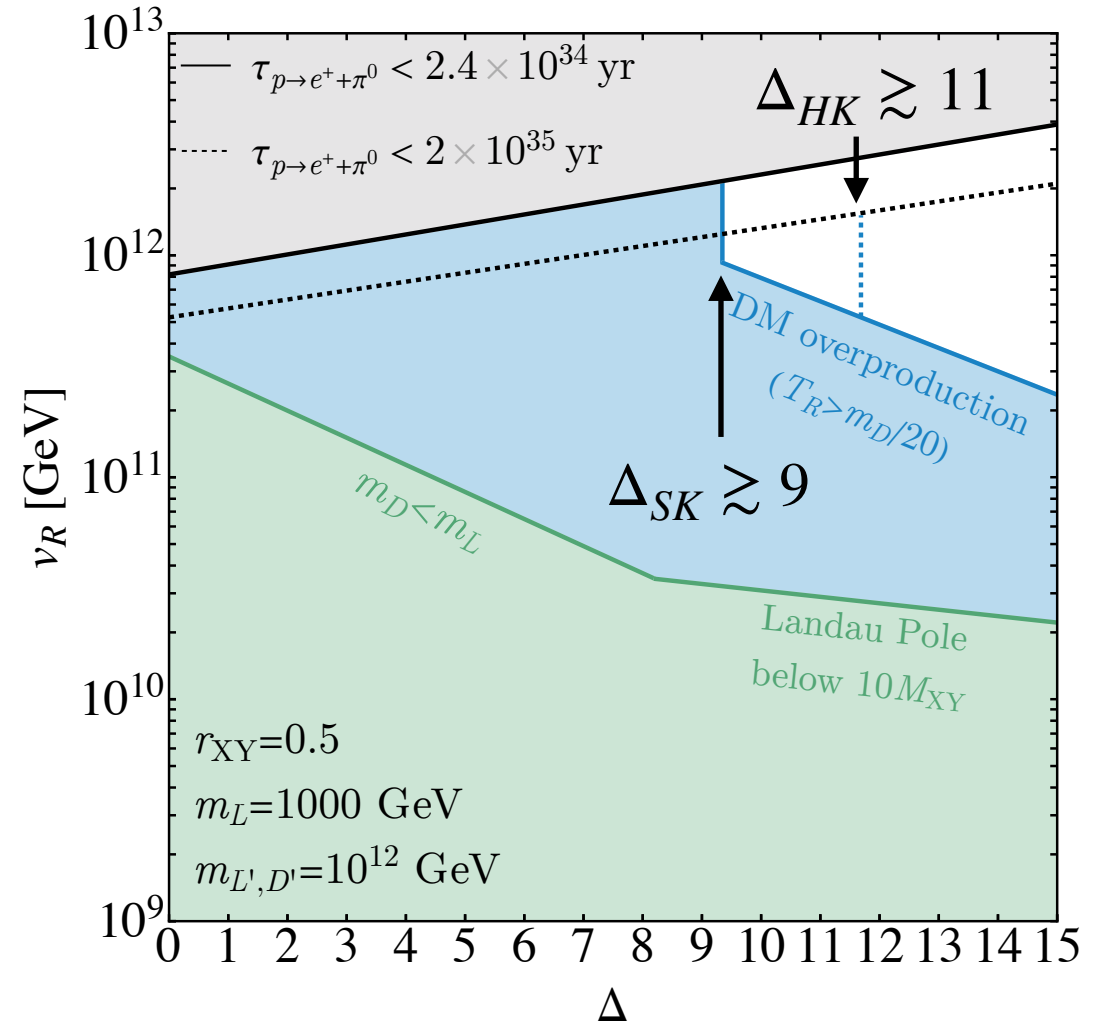
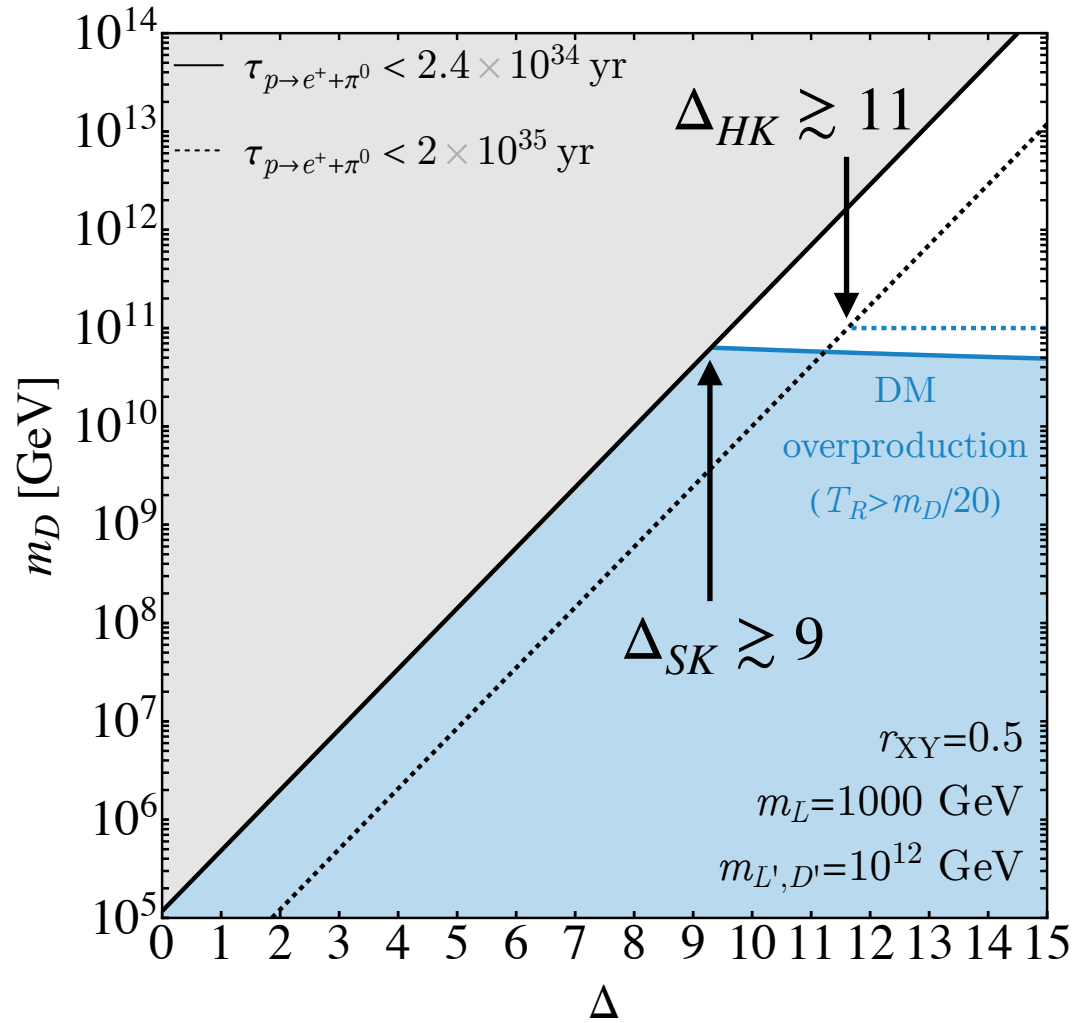
$\Delta = 0$ for $r_{XY} = 1/2$ shifted to higher v_R , smaller M_{XY} relative to $r_{XY} = 2$



$$m_D > 3 \times 10^9 \text{ GeV} \times \left(\frac{M_{XY}}{10^{16} \text{ GeV}} \right)^{\frac{4}{5}} \left(\frac{m_L}{100 \text{ GeV}} \right)^{\frac{6}{5}} v_R [\text{GeV}] \quad \Delta > 0 \text{ to satisfy } m_D, m_L \text{ splitting}$$

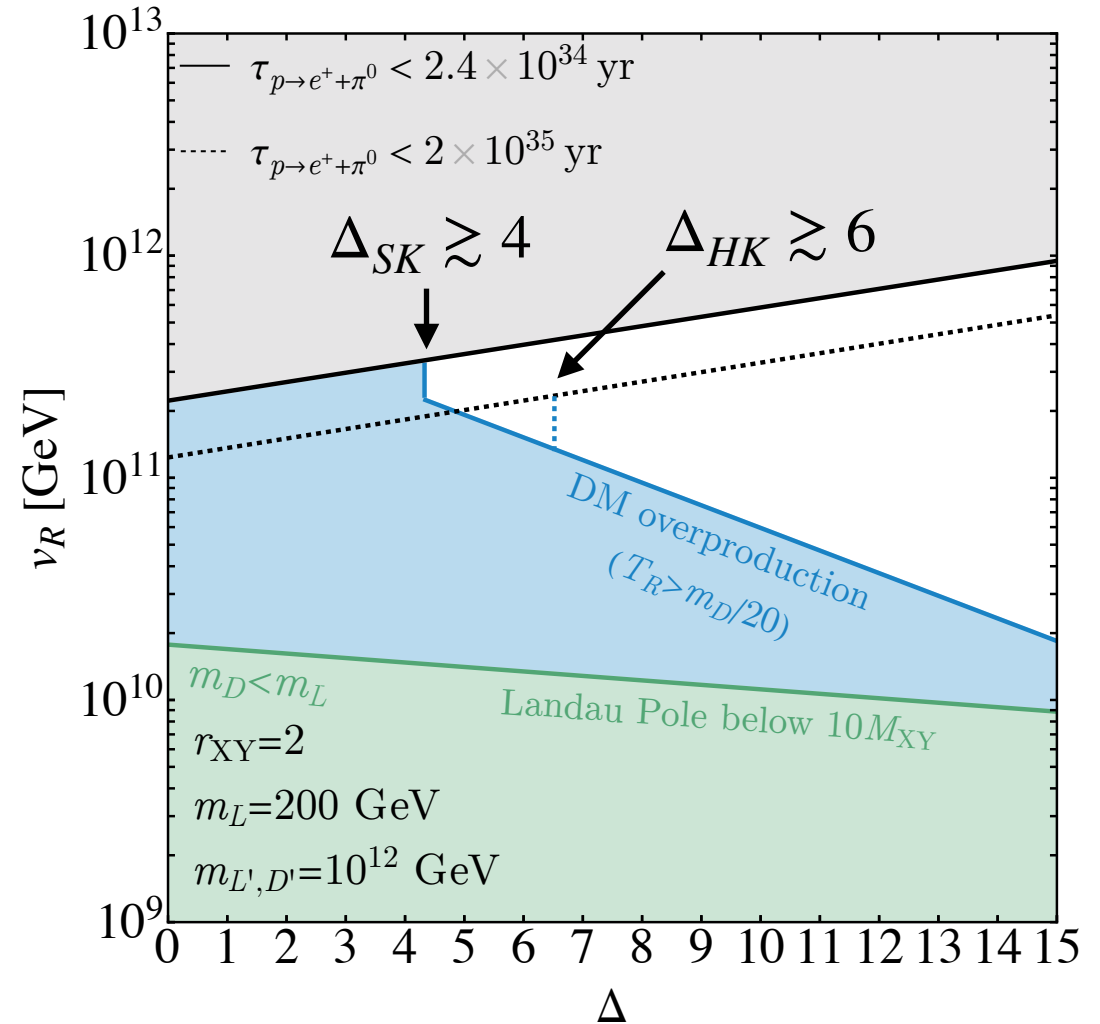
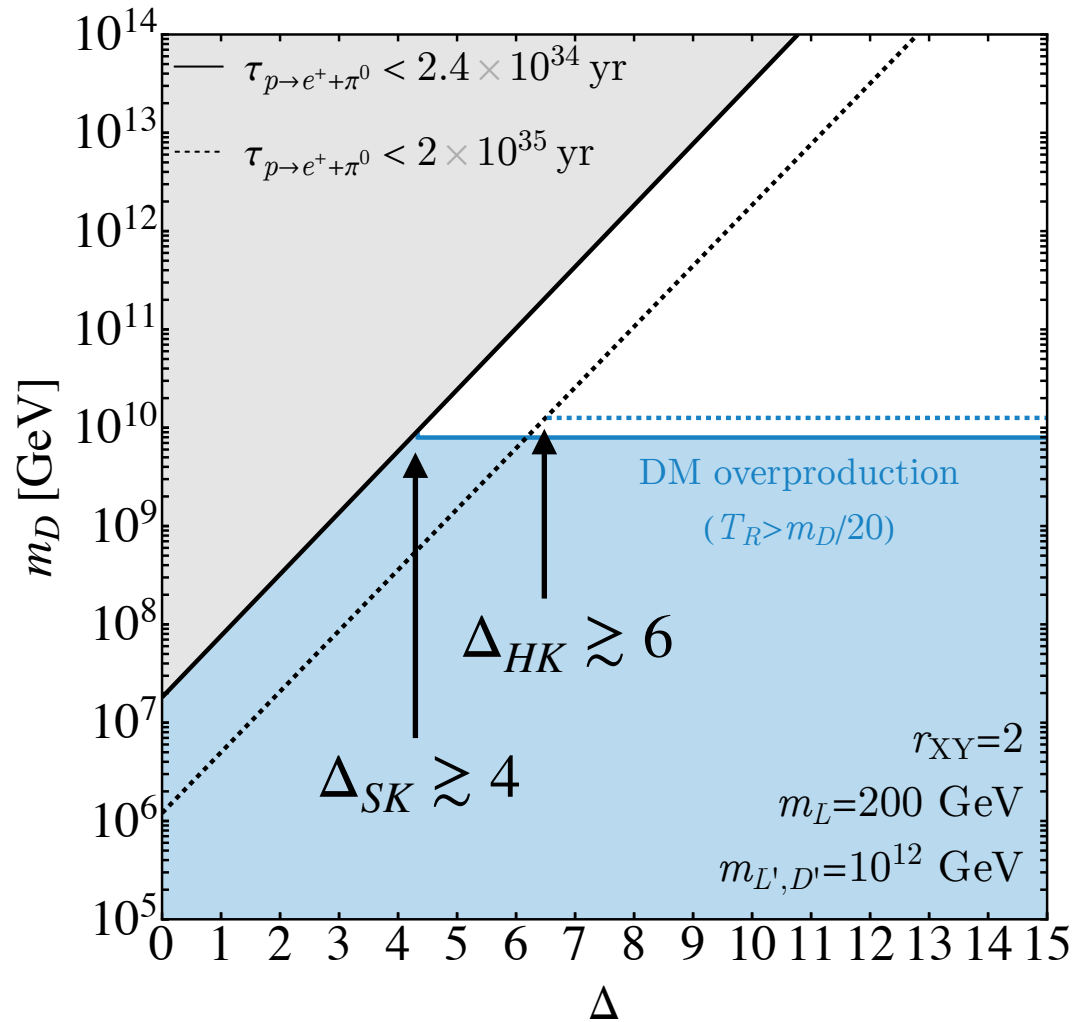


$$m_L = 1 \text{ TeV}, r_{XY} = 1/2$$



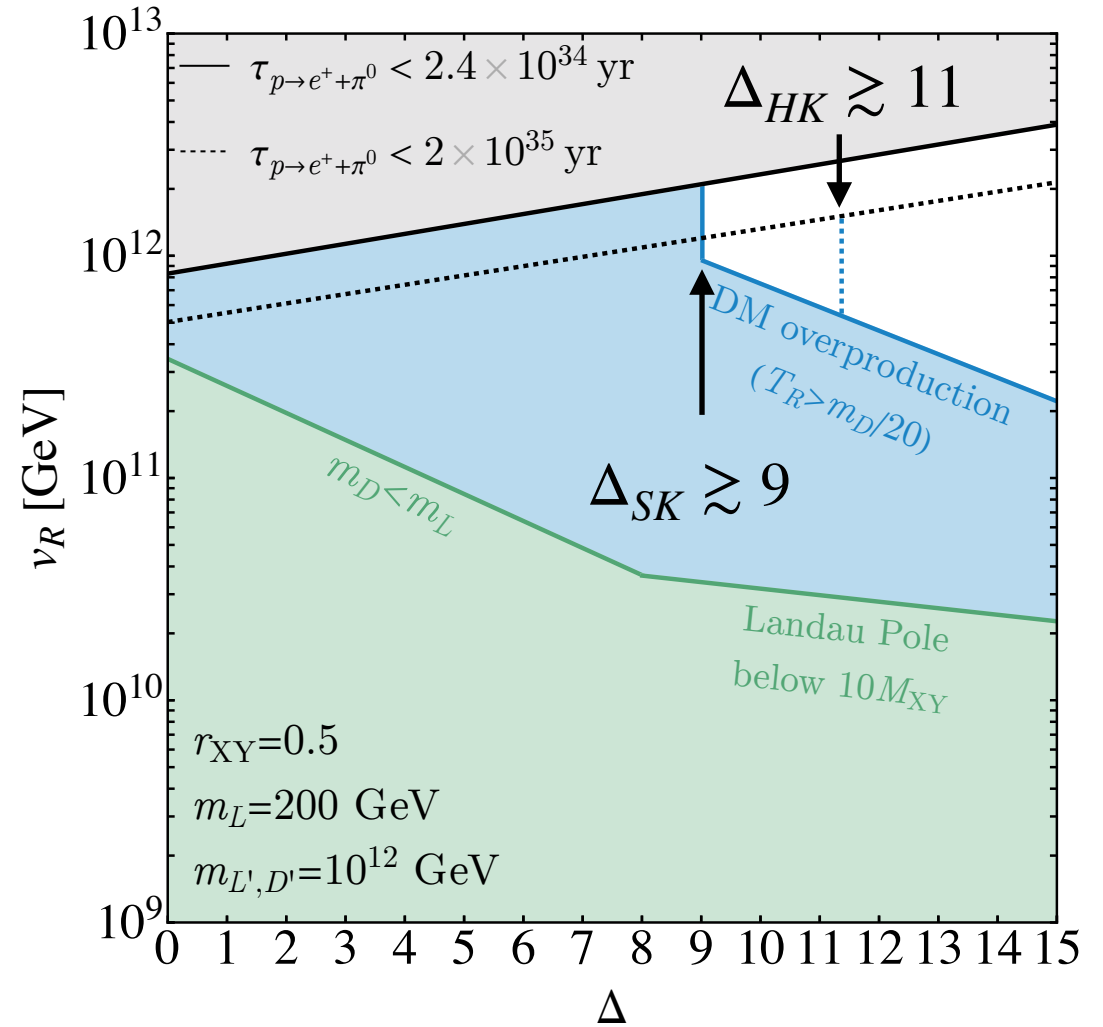
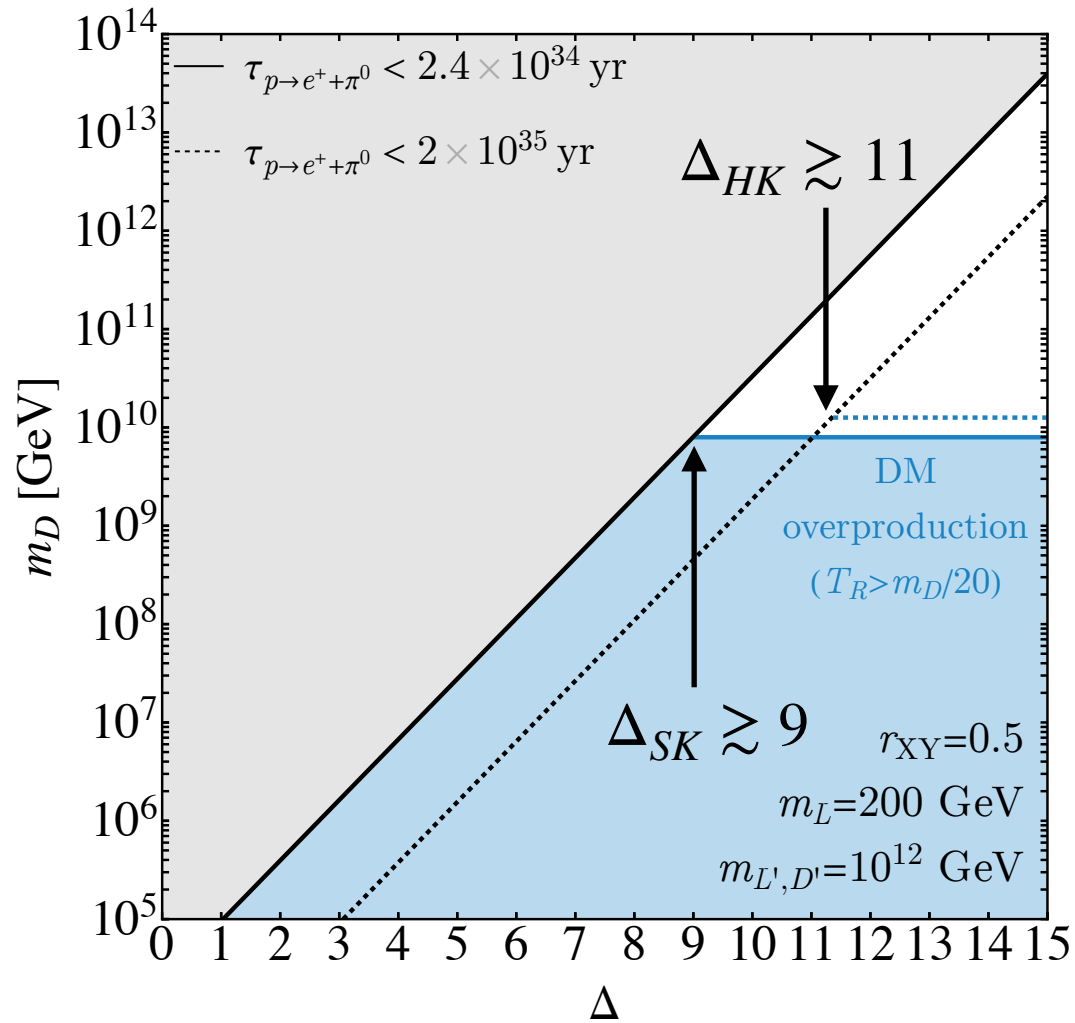


$$m_L = 200 \text{ GeV}, r_{XY} = 2$$



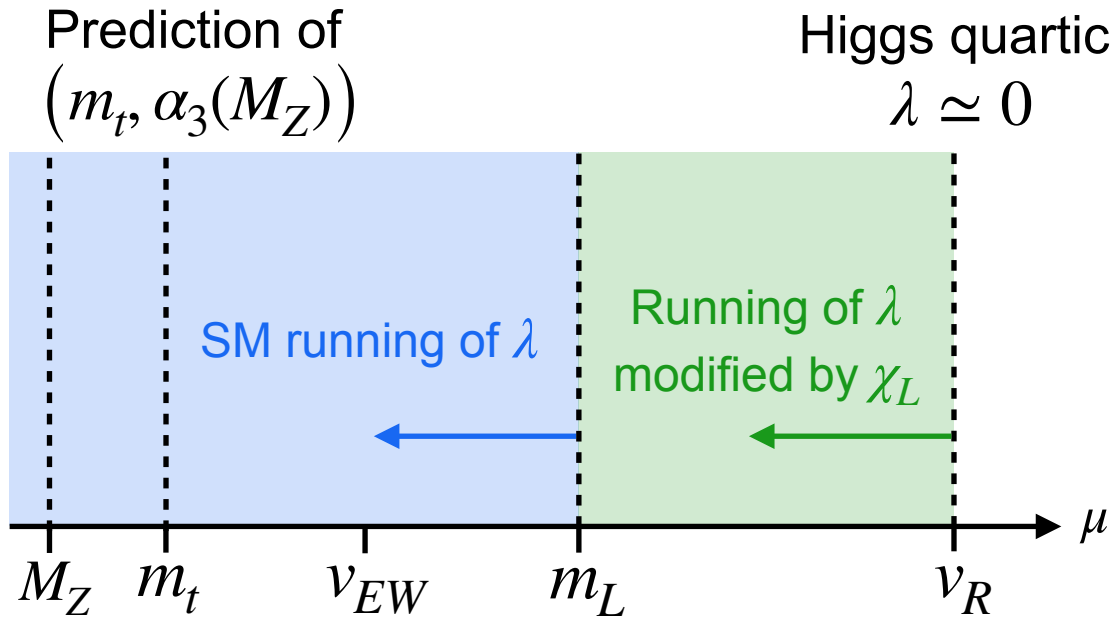


$$m_L = 200 \text{ GeV}, r_{XY} = 1/2$$





Standard Model parameters are precisely predicted



$(m_t, \alpha_3(M_Z))$ constrained by unification, cosmological and proton decay bounds

