Geometry and energy in EFT

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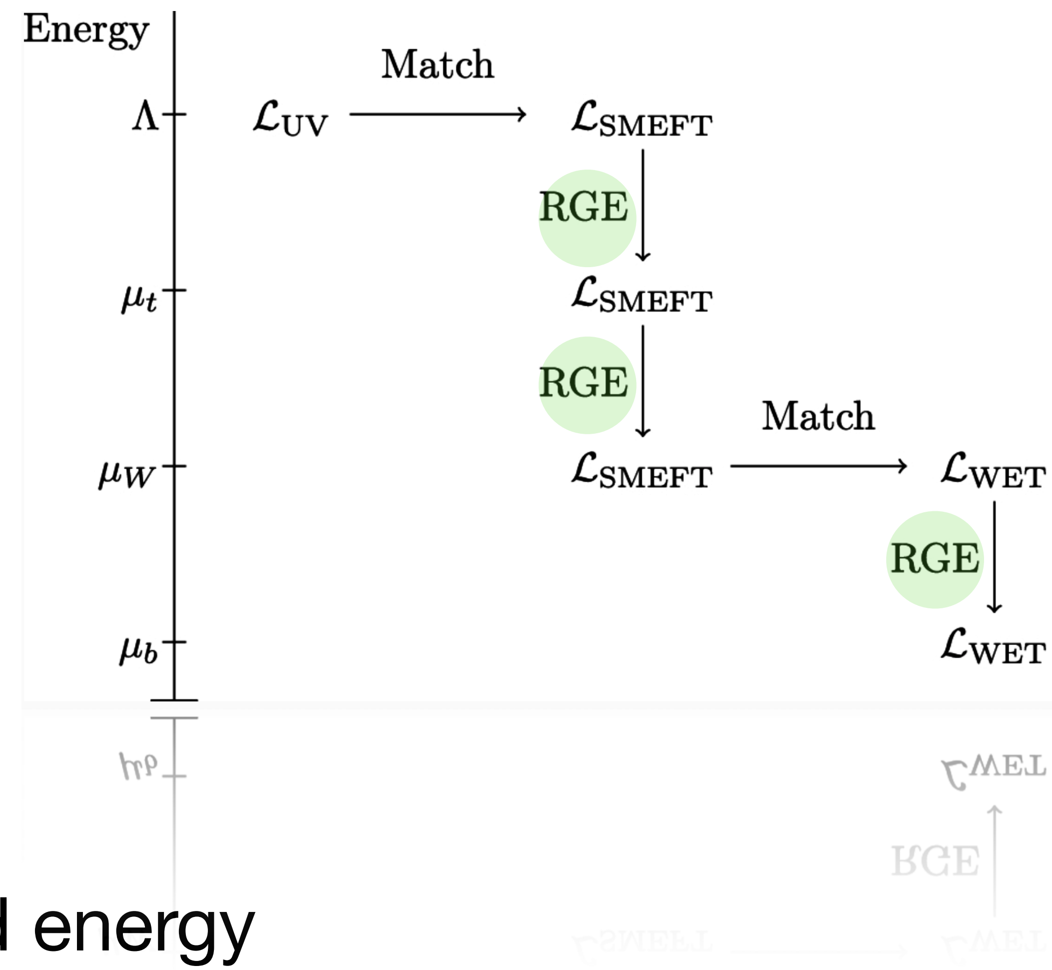
Mostly based on 2307.03187 and 2410.21563

Overview

EFT in a nutshell

A QFT describing low-energy limit of a 'more fundamental' theory (can also be an EFT…)

Allows calculation of experimental quantities with expansion to finite order in small parameter

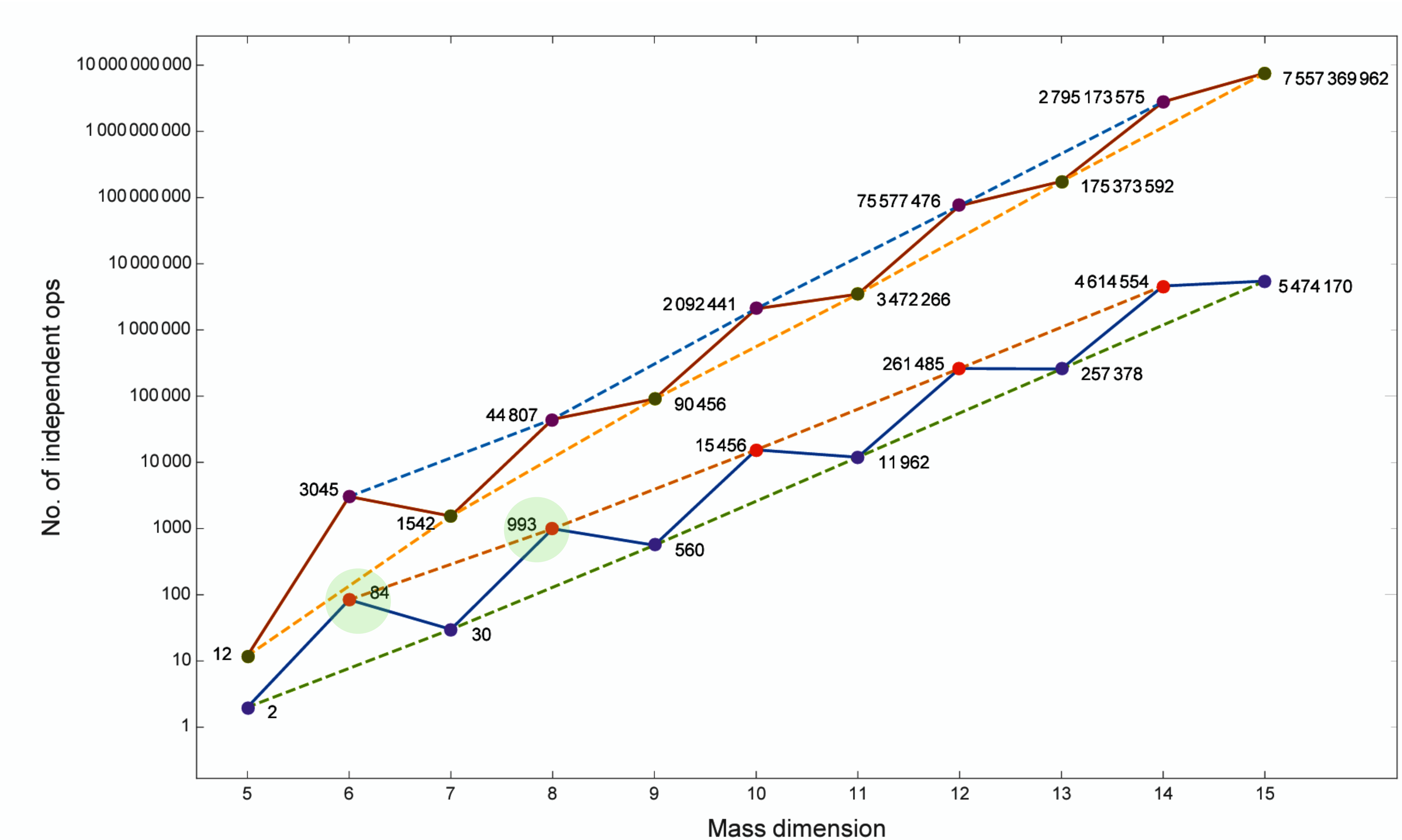


We will look at

1) Exploiting field-space geometry and energy
2) Scattering amplitudes
3) Mapping geometric quantities to SMEFT
4) Applications: RGEs and precision observables
5) Adding fermions to the geometric story

Motivation

Geometric story begins in practical pheno calculations for SMEFT…



[Henning et al. 1512.03433]

Number of operators **grows quickly** with increasing mass dimension

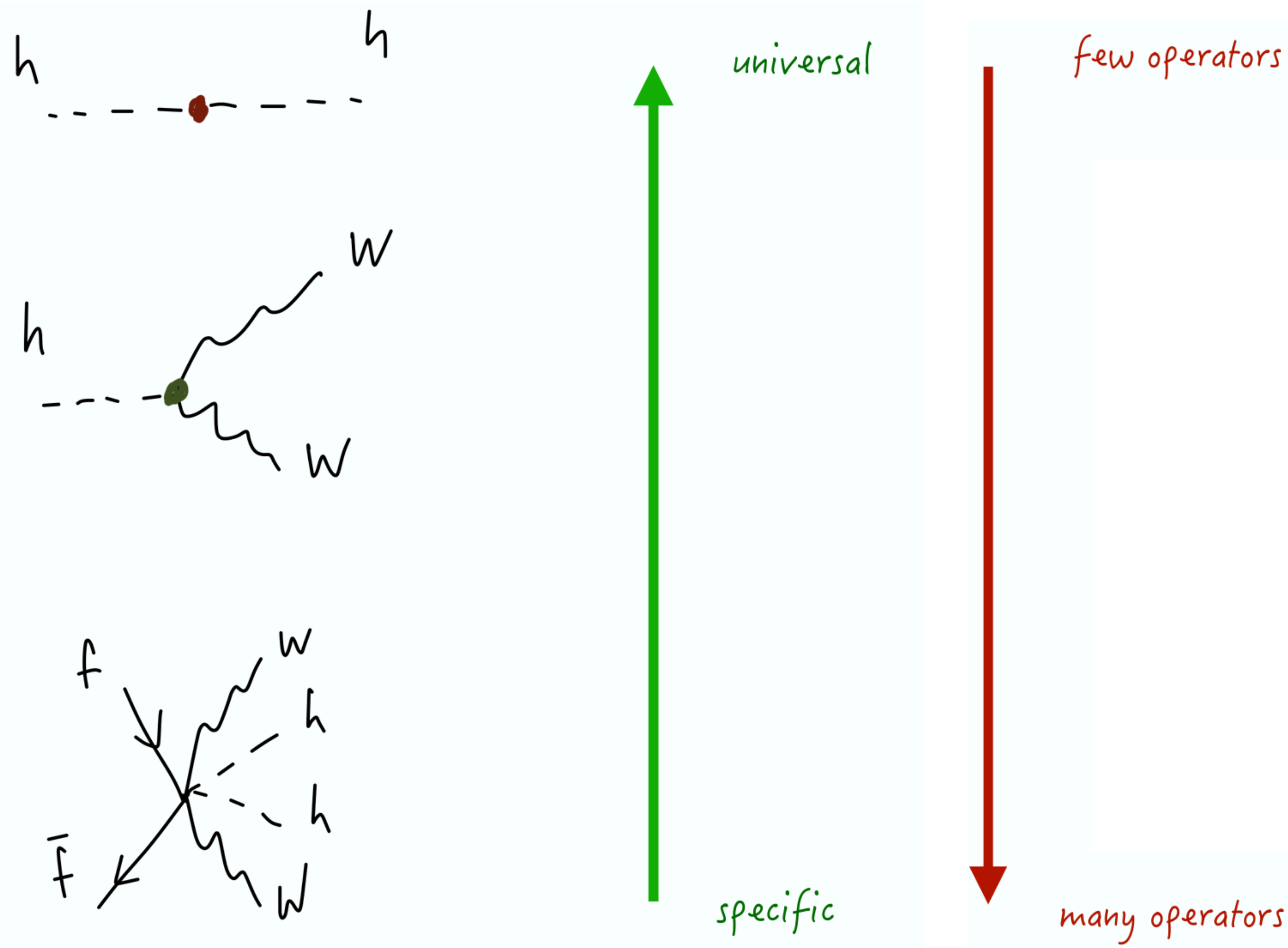
Motivation

And what do these operators do?

Change field strength normalization/inputs

Modify existing vertices

New multi-particle interactions



For 2- and 3-point interactions # of contributing SMEFT operators is **small** and **constant** with operator dimension ⇒ pheno can be done with small set of operators

Motivation

SMEFT observable up to $\mathscr{O}(1/\Lambda^4)$ corrections

$$\langle \mathcal{O}_i \rangle^{\rm SMEFT} = \int [{\rm dps}] \left(|A_{\rm SM}|^2 + 2\,{\rm Re}(A_{\rm SM})\, A_{\rm SMEFT}(\tilde{C}_i^{(6)}) + \left| A_{\rm SMEFT}(\tilde{C}_i^{(6)}) \right|^2 \right) + \int [{\rm dps}] \left(2\,{\rm Re}(A_{\rm SM})\, A_{\rm SMEFT}(\tilde{C}_i^{(8)}) \right).$$

Precision SMEFT analysis **going beyond** tree-level and $D = 6$ for many **resonant** processes e.g. $h \to \gamma\gamma, h \to \gamma Z, h \to GG, Z \to \psi\bar{\psi}, \ldots$ Why?

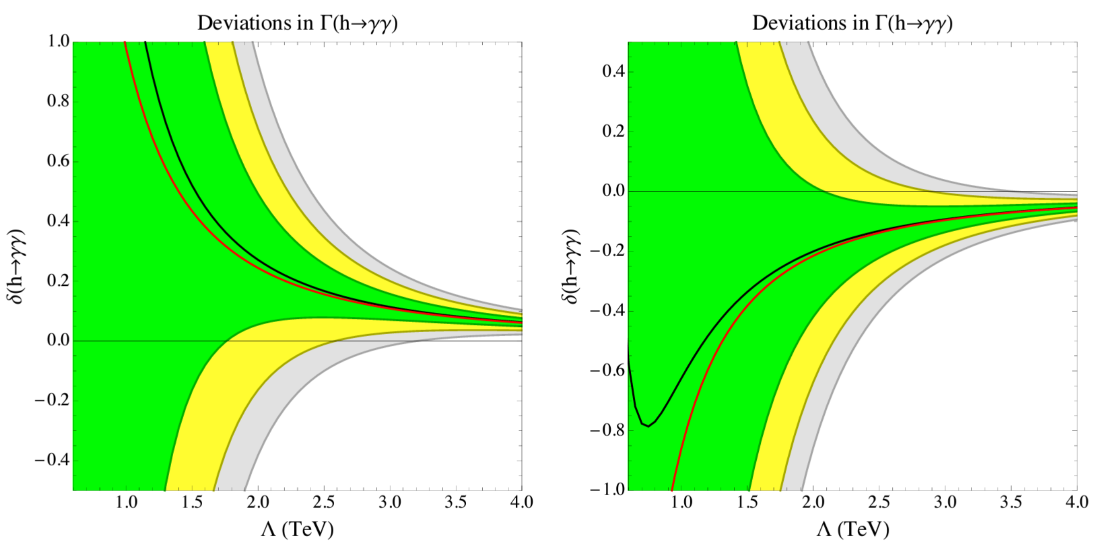


Figure 1. The deviations in $h \to \gamma\gamma$ from the $\mathcal{O}(v^2/\Lambda^2)$ (red line) and partial-square (black line) results, and the full $\mathcal{O}(v^4/\Lambda^4)$ results (green $\pm 1\,\sigma_\delta$, yellow $\pm 2\,\sigma_\delta$, and grey $\pm 3\,\sigma_\delta$ regions). In the left panel the coefficients determining the $\mathcal{O}(v^2/\Lambda^2)$ and partial-square results are $C_{HB}^{(6)} = -0.01$, $C_{HW}^{(6)} = 0.004$, $C_{HWB}^{(6)} = 0.007$, $C_{HD}^{(6)} = -0.74$, and $\delta G_F^{(6)} = -1.6$. In the right panel they are $C_{HB}^{(6)} = 0.007$, $C_{HW}^{(6)} = 0.007$, $C_{HWB}^{(6)} = -0.015$, $C_{HD}^{(6)} = 0.50$, and $\delta G_F^{(6)} = 1.26$.

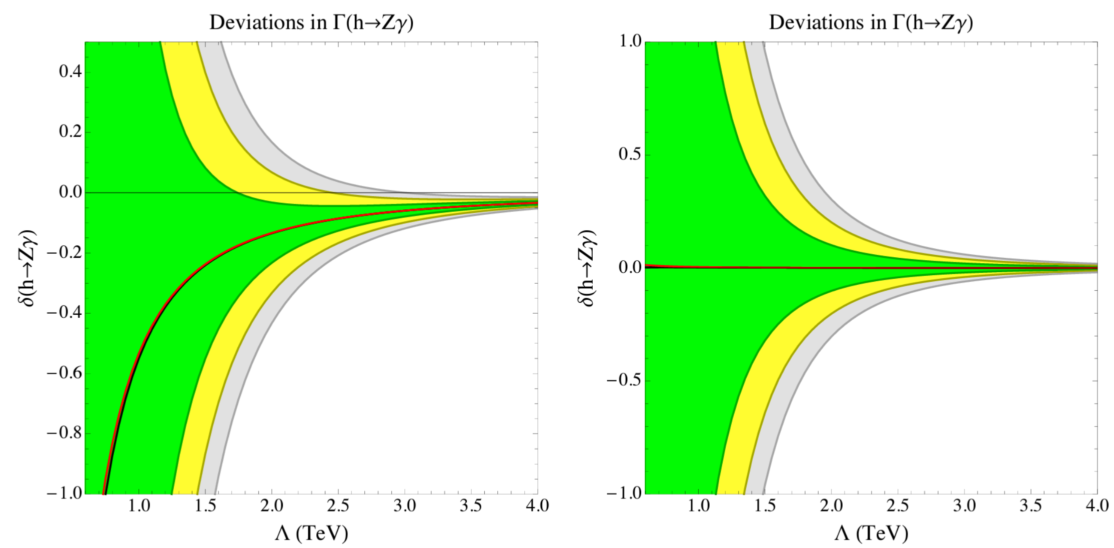


Figure 2. The deviations in $h \to \mathcal{Z}\gamma$ from the $\mathcal{O}(v^2/\Lambda^2)$ (red line) and partial-square (black line) results, and the full $\mathcal{O}(v^4/\Lambda^4)$ results (green $\pm 1\,\sigma_\delta$, yellow $\pm 2\,\sigma_\delta$, and grey $\pm 3\,\sigma_\delta$ regions). In the left panel the coefficients determining the $\mathcal{O}(v^2/\Lambda^2)$ and partial-square results are $C_{HB}^{(6)} = -0.01$, $C_{HW}^{(6)} = 0.02$, $C_{HWB}^{(6)} = -0.011$, $C_{HD}^{(6)} = 0.53$, and $\delta G_F^{(6)} = 0.13$. In the right panel they are $C_{HB}^{(6)} = 0.002$, $C_{HW}^{(6)} = 0.001$, $C_{HWB}^{(6)} = -0.001$, $C_{HD}^{(6)} = 0.28$, and $\delta G_F^{(6)} = -1.15$.

[Hays et al 2007.00565]

Many operators beyond $D = 6$ + loop-corrections for perturbative uncertainty of SMEFT

New calculation and organisational tools required $\Rightarrow$ **uncover geometric EFT structure**

Scalar field theory

NLSM: A scalar field theory can be written as

$$\mathscr{L} = \frac{1}{2} h_{IJ}(\boldsymbol{\phi})(\partial_\mu \phi)^I (\partial^\mu \phi)^J - V(\boldsymbol{\phi})$$

Riemannian metric in field-space is $h_{IJ}(\phi)$ wrt field multiplet ϕ^I

Expanding around flat-space ⇒ **higher-dim operators**

$$h_{IJ} = \delta_{IJ} + h_{IJ,K}\phi^K + h_{IJ,KL}\phi^K\phi^L + \ldots$$

Scalar EFT ↔ field theory on curved **scalar manifold**

Can include higher-derivative **metric-independent** operators E.g.

$$\lambda_{IJKL}(\phi)\partial_\mu\phi^I\partial^\mu\phi^J\partial_\nu\phi^K\partial^\nu\phi^L$$

[Cheung et al 2202.06972, Cohen et al 2202.06965, Craig et al 2307.15742]

Geometry

Key insight: S-matrix is **field re-definition invariant** ↔ Lagrangian can change but not physical observables

Field re-definition ↔ **coord change** on scalar field-space manifold

$$\phi^I \to \varphi^I(\boldsymbol{\phi})$$

Then the field-space metric **transforms as a tensor**

$$g_{IJ}(\boldsymbol{\phi}) \to g'_{IJ}(\boldsymbol{\varphi}) = \left(\frac{\partial \phi^K}{\partial \varphi^I}\right)\left(\frac{\partial \phi^L}{\partial \varphi^J}\right) g_{KL}(\boldsymbol{\phi})$$

and the derivative of the scalar **transforms as a vector**

$$\partial_\mu \phi^I \to \partial_\mu \varphi^I = \left(\frac{\partial \varphi^I}{\partial \phi^J}\right) \partial_\mu \phi^J$$

⇒ Lagrangian is also an **invariant scalar density**

Scalar amplitudes

Riemann curvature

$$R_{IJKL} = h_{IM}\left(\partial_K \Gamma^M_{LJ} + \Gamma^M_{KN}\Gamma^N_{LJ}\right) - (K \leftrightarrow L)$$

with **covariant derivative** ∇_I and **Christoffel symbol**

$$\Gamma^I_{JK} = \frac{1}{2} h^{IL}(h_{JL,K} + h_{LK,J} - h_{JK,L})$$

4-point Born amplitude $\phi_I \phi_J \to \phi_K \phi_L$ (massless fields)

$$\boxed{A^4_{IJKL} = R_{IJKL} s_{IK} + R_{IKJL} s_{IJ}}, \quad s_{ij} = (p_i + p_j)^2$$

Amplitudes depend on **geometric invariants**!

Bose symmetry $\leftrightarrow$ R_{IJKL} symmetries **Bianchi IDs**

$$R_{IJKL} + R_{IKLJ} + R_{ILJK} = 0 \qquad R_{IJMN;L} + R_{IJLM;N} + R_{IJNL;M} = 0$$

Gauge fields

Incorporating gauge fields in similar fashion [Helset, Manohar, Simons 2210.08000, 2212.03253]

$$\mathscr{L} = \frac{1}{2} h_{IJ}(\boldsymbol{\phi})(D_\mu \phi)^I (D^\mu \phi)^J - V(\boldsymbol{\phi}) - \frac{1}{4} g_{AB}(\boldsymbol{\phi}) F^A_{\mu\nu} F^{\mu\nu,B}$$

on scalar field manifolds with metrics $h_{IJ}(\boldsymbol{\phi})$and $g_{AB}(\boldsymbol{\phi})$

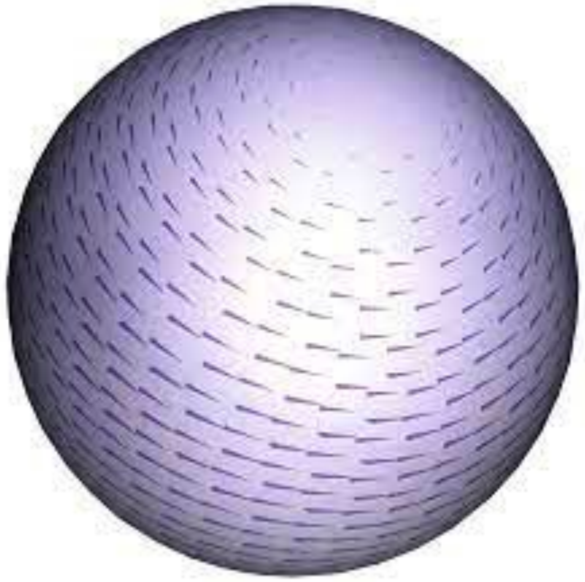
$$(D_\mu \phi)^I = \partial_\mu \phi^I + A^B_\mu t^I_B(\boldsymbol{\phi}) \qquad F^B_{\mu\nu} = \partial_\mu A^B_\nu - \partial_\nu A^B_\mu - f^B{}_{CD} A^C_\mu A^D_\nu$$

Killing vectors (isometric) of scalar manifold with null **Lie derivative**

$t^K_A h_{IJ,K} + t^K_{A,I} h_{KJ} + t^K_{A,J} h_{IK} = 0$ and **Lie bracket** $[t_A, t_B]^I = f^C_{AB} t^I_C$

Can also use **combined metric**

$$\tilde{g}_{ij} = \begin{pmatrix} h_{IJ} & 0 \\ 0 & -g_{AB}\eta_{\mu_A \mu_B} \end{pmatrix}$$



and combined geometric quantities

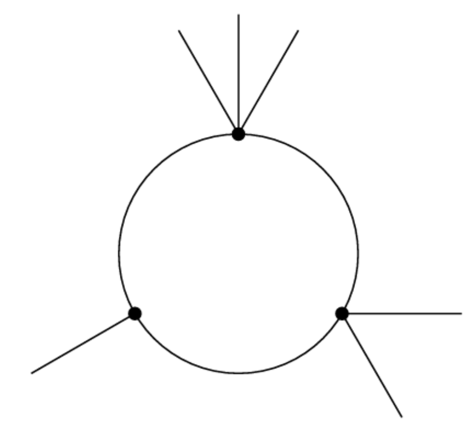
Application: RGEs

The 2nd variation has the form [t'Hooft '74, Alonso, Manohar et al '20]

$$\delta_{\eta\eta} S = \frac{1}{2}\int \mathrm{d}^4x \left\{ h_{IJ}(\mathscr{D}_\mu \eta)^I (\mathscr{D}_\mu \eta)^J + X_{IJ}\eta^I \eta^J \right\}$$

and 1-loop pole is given by

$$\boxed{\Delta S = \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \left\{ \frac{1}{12}\mathrm{Tr}\left[Y_{\mu\nu}Y^{\mu\nu}\right] + \frac{1}{2}\mathrm{Tr}\left[\mathcal{X}^2\right] \right\}}$$



applied to **scalar-gauge theory**

$$\left[\widetilde{\mathscr{D}}_\mu, \widetilde{\mathscr{D}}_\nu\right]^i{}_j = \left[\widetilde{Y}_{\mu\nu}\right]^i{}_j = \widetilde{R}^i{}_{jkl}(D_\mu Z)^k (D_\nu Z)^l + \widetilde{\nabla}_j \widetilde{t}^i_C F^C_{\mu\nu} \qquad Z^i_\mu = \begin{bmatrix} (D_\mu \phi)^I \\ F^{A\,\mu_A}_\mu \end{bmatrix}$$

$$\widetilde{\mathscr{D}}_\mu \begin{bmatrix} \eta^I \\ \zeta^A_\lambda \end{bmatrix} = \partial_\mu \begin{bmatrix} \eta^I \\ \zeta^A_\lambda \end{bmatrix} + \begin{bmatrix} t^I_{C,J}A^C_\mu + \Gamma^I_{LJ}(D_\mu\phi)^L & -\Gamma^I_{CB}F^C_{\mu\sigma} \\ \Gamma^A_{CJ}F^C_{\mu\lambda} & -f^A{}_{CB}A^C_\mu \eta_{\lambda\sigma} + \Gamma^A_{LB}(D_\mu\phi)^L \eta_{\lambda\sigma} \end{bmatrix} \begin{bmatrix} \eta^J \\ \zeta^B_\sigma \end{bmatrix}$$

with parts read from each 2nd variation

$$\mathcal{X}^I{}_J = h^{IK}X_{KJ} \qquad \mathcal{X} = \begin{bmatrix} [\mathcal{X}_{\eta\eta}]^I{}_J & [\mathcal{X}_{\eta\zeta}]^I{}_{(B\mu_B)} \\ [\mathcal{X}_{\eta\zeta}]^{(A\mu_A)}{}_J & [\mathcal{X}_{\zeta\zeta}]^{(A\mu_A)}{}_{(B\mu_B)} \end{bmatrix}$$

Fermions

General Lagrangian [BA, Helset, Manohar, Pagès, Shen 2307.03817]

$$\mathscr{L} = \frac{1}{2}h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J - V(\phi) - \frac{1}{4}g_{AB}(\phi)F^A_{\mu\nu}F^{B\mu\nu}$$
$$+\frac{1}{2}ik_{\bar{p}r}(\phi)\left(\bar{\psi}^{\bar{p}}\gamma^\mu \overleftrightarrow{D}_\mu\psi^r\right) + i\omega_{\bar{p}rI}(\phi)(D_\mu\phi)^I\bar{\psi}^{\bar{p}}\gamma^\mu\psi^r - \bar{\psi}^{\bar{p}}\mathscr{M}_{\bar{p}r}(\phi)\psi^r + \bar{\psi}^{\bar{p}}\sigma_{\mu\nu}\mathscr{T}^{\mu\nu}_{\bar{p}r}(\phi,F)\psi^r$$

All tensors are functions of **scalar fields** except $\mathscr{T}^{\mu\nu}_{\bar{p}r}(\phi, F)$

Under fermion **field re-definition** $\psi^p \to R^p{}_s(\phi)\psi^s$

$$k_{\bar{p}r} \to \left[(R^\dagger)^{-1}kR^{-1}\right]_{\bar{p}r},$$

$$\omega_{\bar{p}rI} \to \left[(R^\dagger)^{-1}\omega_I R^{-1}\right]_{\bar{p}r} + \frac{1}{2}\left[(R^\dagger)^{-1}k(\partial_I R^{-1})\right]_{\bar{p}r} - \frac{1}{2}\left[(\partial_I(R^\dagger)^{-1})kR^{-1}\right]_{\bar{p}r}$$

$\Rightarrow k_{\bar{p}r}$ transforms as a **Hermitian** metric and $\omega_{\bar{p}rI}$ transforms as an **anti-Hermitian** connection

Scalar-fermion metric

Promoting **Riemannian** scalar manifold to a graded supermanifold

[DeWitt '12, Rogers '07]

We can group the fields into a **multiplet** $\Phi^a = \begin{pmatrix} \phi^I \\ \psi^p \\ \bar{\psi}^{\bar{p}} \end{pmatrix}$ and **metric**

$$\bar{g}_{ab}(\phi, \psi) = \begin{pmatrix} h_{IJ} & -\left(\frac{1}{2}k_{\bar{s}r,I} - \omega_{\bar{s}rI}\right)\bar{\psi}^{\bar{s}} & \left(\frac{1}{2}k_{\bar{r}s,I} + \omega_{\bar{r}sI}\right)\psi^s \\ \left(\frac{1}{2}k_{\bar{s}p,J} - \omega_{\bar{s}pJ}\right)\bar{\psi}^{\bar{s}} & 0 & k_{\bar{r}p} \\ -\left(\frac{1}{2}k_{\bar{p}s,J} + \omega_{\bar{p}sJ}\right)\psi^s & -k_{\bar{p}r} & 0 \end{pmatrix}$$

Derived by requiring metric **transforms as tensor** under field redef

[BA, Helset, Manohar, Pagès, Shen 2307.03817, 2411.XXXX]

Scattering amplitudes

The 4-point $\psi^p \phi^I \to \psi^{\bar{r}} \phi^J$ **massless** scattering amplitude

$$\boxed{\mathcal{A}_{pI\bar{r}J} = (\bar{u}_{\bar{r}} \not{\phi}_I u_p) \bar{R}_{\bar{r}pJI}}$$

The 5-point $\psi^p \phi^I \to \psi^{\bar{r}} \phi^J \phi^K$

$$\mathcal{A}_{pI\bar{r}JK} = (\bar{u}_{\bar{r}} \not{\phi}_J u_p) \bar{\nabla}_K \bar{R}_{\bar{r}pIJ} + (\bar{u}_{\bar{r}} \not{\phi}_K u_p) \bar{\nabla}_J \bar{R}_{\bar{r}pIK}$$

$$\bar{\nabla}_K \bar{R}_{\bar{r}pIJ} = \bar{R}_{\bar{r}pIJ,K} - \bar{\Gamma}^{\bar{s}}_{\bar{r}K} \bar{R}_{\bar{s}pIJ} - \bar{\Gamma}^{s}_{pK} \bar{R}_{\bar{r}sIJ} - \bar{\Gamma}^{L}_{IK} \bar{R}_{\bar{r}pLJ} - \bar{\Gamma}^{L}_{JK} \bar{R}_{\bar{r}pIL}$$

Turning on the scalar potential and fermion mass matrix

$$\begin{aligned} \mathcal{A}_{pI\bar{r}J} =& (\bar{u}_{\bar{r}} \not{\phi}_I u_p) \left(\bar{R}_{\bar{r}pJI} + k^{s\bar{t}} \left(\frac{\mathcal{M}_{\bar{r}s;I} \mathcal{M}_{\bar{t}p;J}}{s_{\bar{r}I}} - \frac{\mathcal{M}_{\bar{r}s;J} \mathcal{M}_{\bar{t}p;I}}{s_{pI}} \right) \right) \\ & - (\bar{u}_{\bar{r}} u_p) \left(\mathcal{M}_{\bar{r}p;IJ} - h^{LK} \frac{\mathcal{M}_{\bar{r}p;L} V_{;IJK}}{s_{IJ}} \right) , \end{aligned}$$

Renormalisation

One-loop RGE from **2nd variation** of action $\psi^a \to \psi^a + \chi^a$

$$\delta_{\bar\chi\chi} S = \int \mathrm{d}^4x \left\{ \frac{1}{2} i k_{\bar p r} \left(\bar\chi^{\bar p} \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \chi^r \right) - \bar\chi^{\bar p} \mathcal{M}_{\bar p r} \chi^r + \bar\chi^{\bar p} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar p r} \chi^r \right\}$$

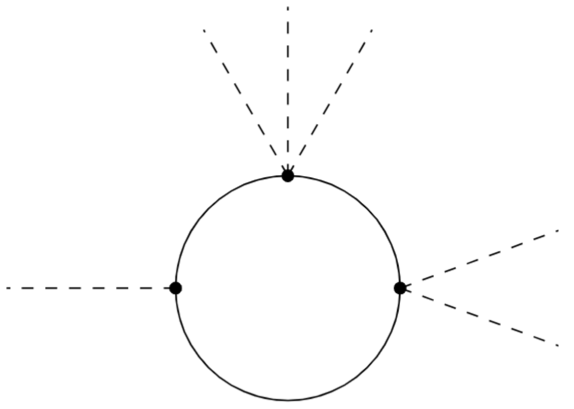
with **covariant derivative** $\mathscr{D}_\mu = \partial_\mu \mathbf{1} + \omega_\mu$ and fermion fluctuations $\chi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$

The metric, mass and dipole terms

$$k = \begin{pmatrix} \kappa_L & 0 \\ 0 & \kappa_R \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix} \quad \mathcal{T}^{\mu\nu} = \begin{pmatrix} 0 & T^{\mu\nu} \\ T^{\mu\nu\dagger} & 0 \end{pmatrix} \quad \omega_{\bar p r I} = \begin{pmatrix} \omega_{L,\bar p r I} & 0 \\ 0 & \omega_{R,\bar p r I} \end{pmatrix}$$

gives **covariant** result for $\chi\bar\chi$-variation

$$\begin{aligned} \Delta S = & \frac{1}{32\pi^2\epsilon} \int \mathrm{d}^4x \left\{ \frac{1}{3} \mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] \right. \\ & - \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] \\ & \left. - 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2 \right\} \end{aligned}$$

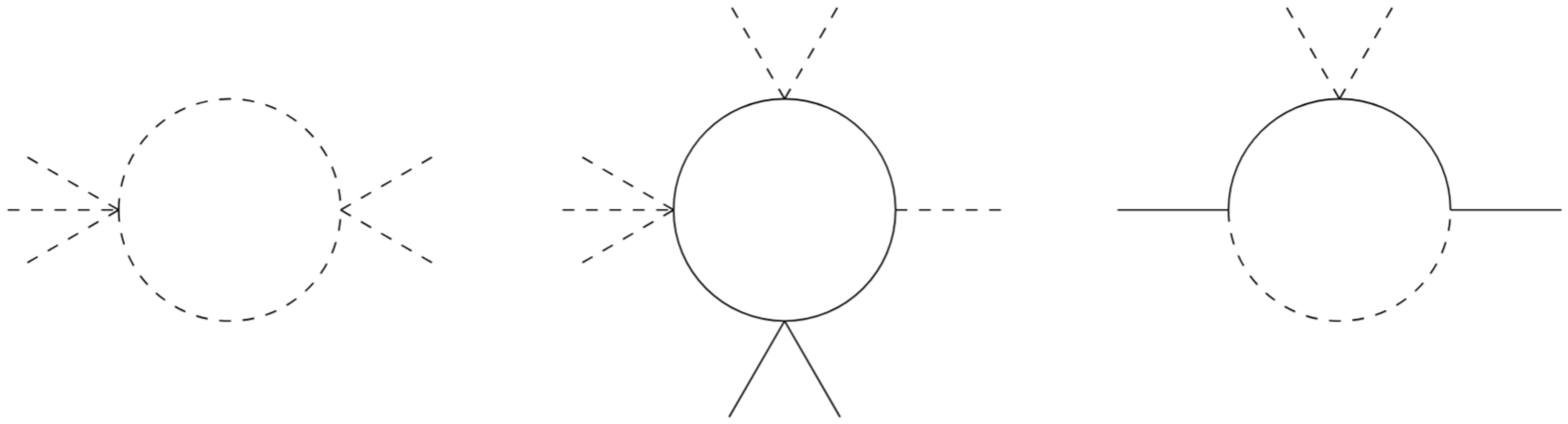


Renormalisation

with identified **covariant parts**

$$
\begin{aligned}
[\mathcal{Y}_{\mu\nu}]^p{}_r &= [\mathcal{D}_\mu, \mathcal{D}_\nu]^p{}_r = \bar{R}^p{}_{rIJ}(D_\mu\phi)^I(D_\nu\phi)^J + \left(\bar{\nabla}_r t^p_A\right) F^A_{\mu\nu} \,, \\
(\mathcal{D}_\mu \mathcal{M})^p{}_r &= k^{p\bar{t}}(\mathcal{D}_\mu \mathcal{M}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu \mathcal{M}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I \mathcal{M}_{\bar{s}r} - \bar{\Gamma}^{s}_{Ir}(D_\mu\phi)^I \mathcal{M}_{\bar{t}s}\right] \,, \\
(\mathcal{M}\mathcal{M})^p{}_r &= k^{p\bar{t}}\mathcal{M}_{\bar{t}q} k^{q\bar{s}} \mathcal{M}_{\bar{s}r} \,, \\
(\mathcal{D}_\mu \mathcal{T}^{\alpha\beta})^p{}_r &= k^{p\bar{t}}(\mathcal{D}_\mu \mathcal{T}^{\alpha\beta}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu \mathcal{T}^{\alpha\beta}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I \mathcal{T}^{\alpha\beta}_{\bar{s}r} - \bar{\Gamma}^{s}_{Ir}(D_\mu\phi)^I \mathcal{T}^{\alpha\beta}_{\bar{t}s}\right] \,, \\
(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^p{}_r &= k^{p\bar{t}}\mathcal{T}^{\mu\nu}_{\bar{t}q} k^{q\bar{s}} \mathcal{T}^{\alpha\beta}_{\bar{s}r} \,.
\end{aligned}
$$

Next: Pure boson and mixed variations $\eta\chi$, $\eta\zeta$, $\eta\eta$, $\zeta\zeta$ requires more understanding of supergeometry [BA, Helset, Pagès, Shen, 2411.XXXX]



SMEFT: bosons

We can apply formalism to the SMEFT by **identification**

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^4 - i\phi^3 \end{pmatrix} \qquad A^B_\mu = \begin{pmatrix} G^{\mathscr{A}}_\mu \\ W^a_\mu \\ B_\mu \end{pmatrix}$$

with **scalar metric**

$$\begin{aligned} h_{IJ} =& \delta_{IJ} \left[1 + \frac{1}{4} \left({}^8C^{(1)}_{H^6 D^2} - {}^8C^{(2)}_{H^6 D^2} \right) (\phi^K \phi^K)^2 \right] + \left(-2 \;\; {}^6C_{H^4 \Box} \right) \phi^I \phi^J \\ & + \frac{1}{2} \left[{}^6C_{H^4 D^2} + {}^8C^{(2)}_{H^6 D^2} (\phi^K \phi^K) \right] \mathcal{H}_{IJ}(\phi) \,, \end{aligned}$$

$$\mathcal{H}_{IJ}(\phi) = \phi_I \phi_J + \begin{bmatrix} \phi_2^2 & -\phi_1\phi_2 & -\phi_2\phi_4 & \phi_2\phi_3 \\ -\phi_1\phi_2 & \phi_1^2 & \phi_1\phi_4 & -\phi_1\phi_3 \\ -\phi_2\phi_4 & \phi_1\phi_4 & \phi_4^2 & -\phi_3\phi_4 \\ \phi_2\phi_3 & -\phi_1\phi_3 & -\phi_3\phi_4 & \phi_3^2 \end{bmatrix}$$

and **gauge metric**

$$g_{AB} = \begin{bmatrix} [g_{GG}]_{\mathscr{A}\mathscr{B}} & 0 & 0 \\ 0 & [g_{WW}]_{ab} & [g_{WB}]_a \\ 0 & [g_{BW}]_b & g_{BB} \end{bmatrix}$$

SMEFT RGEs

E.g. of **bosonic RGE** at dimension six

$$
{}^{6}\dot{C}_{H^4\Box} = m_H^2 \Big\{ -24\left({}^{6}C_{H^4\Box}\right)^2 + \frac{3}{4}\left({}^{6}C_{H^4D^2}\right)^2 + 8\ {}^{6}C_{H^4\Box}{}^{6}C_{H^4D^2} - 64\left({}^{6}C_{G^2H^2}\right)^2
- 24\left({}^{6}C_{W^2H^2}\right)^2 - 8\left({}^{6}C_{B^2H^2}\right)^2 + 4\left({}^{6}C_{WBH^2}\right)^2 - 3\ {}^{8}C^{(1)}_{H^6D^2} + 2\ {}^{8}C^{(2)}_{H^6D^2} \Big\}
$$

And dimension eight

$$
\begin{aligned}
{}^{8}\dot{C}^{(1)}_{H^6D^2} =& -96\ {}^{6}C_{H^6}{}^{6}C_{H^4\Box} - 12\ {}^{6}C_{H^6}{}^{6}C_{H^4D^2} + \left(352\lambda + 20g_1^2 + \frac{20}{3}g_2^2\right)\left({}^{6}C_{H^4\Box}\right)^2 \\
&+ \left(-23\lambda + \frac{1}{8}g_1^2 + \frac{161}{24}g_2^2\right)\left({}^{6}C_{H^4D^2}\right)^2 + \left(-64\lambda - 2g_1^2 + 12g_2^2\right){}^{6}C_{H^4\Box}{}^{6}C_{H^4D^2} \\
&- 22g_2^2\ {}^{6}C_{H^4\Box}{}^{6}C_{W^2H^2} + 6g_1^2\ {}^{6}C_{H^4\Box}{}^{6}C_{B^2H^2} - \frac{32}{3}g_1g_2\ {}^{6}C_{H^4\Box}{}^{6}C_{WBH^2} \\
&+ 8g_2^2\ {}^{6}C_{H^4D^2}{}^{6}C_{W^2H^2} + 6g_1^2\ {}^{6}C_{H^4D^2}{}^{6}C_{B^2H^2} + \frac{43}{3}g_1g_2\ {}^{6}C_{H^4D^2}{}^{6}C_{WBH^2} \\
&+ 512\lambda\left({}^{6}C_{G^2H^2}\right)^2 + \left(192\lambda + 4g_2^2\right)\left({}^{6}C_{W^2H^2}\right)^2 + \left(64\lambda + 12g_1^2\right)\left({}^{6}C_{B^2H^2}\right)^2 \\
&+ \left(-3g_1^2 - 3g_2^2\right)\left({}^{6}C_{WBH^2}\right)^2 + \frac{80}{3}g_1g_2\ {}^{6}C_{W^2H^2}{}^{6}C_{WBH^2} + \frac{8}{3}g_1g_2\ {}^{6}C_{B^2H^2}{}^{6}C_{WBH^2} \\
&+ \left(68\lambda + \frac{1}{2}g_1^2 - \frac{31}{6}g_2^2\right){}^{8}C^{(1)}_{H^6D^2} + \left(-8\lambda + 7g_1^2 + \frac{17}{3}g_2^2\right){}^{8}C^{(2)}_{H^6D^2}\,,
\end{aligned}
$$

SMEFT: fermonic

Again applying formalism to the SMEFT

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^4 - i\phi^3 \end{pmatrix} \qquad A^B_\mu = \begin{pmatrix} G^{\mathscr{A}}_\mu \\ W^a_\mu \\ B_\mu \end{pmatrix} \qquad \psi^p = \begin{pmatrix} \ell^p_L \\ q^p_L \\ e^p_R \\ u^p_R \\ d^p_R \end{pmatrix}$$

with SM Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu} + (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 + \delta_{\bar{p}r} i \bar{\psi}^{\bar{p}} \gamma^\mu D_\mu \psi^r - \bar{\psi}^{\bar{p}} \mathcal{M}_{\mathrm{SM},\bar{p}r} \psi^r$$

and identifying e.g. for RH electrons in SMEFT

$$M_{\bar{p}r} \supset [Y_e]^\dagger_{\bar{p}r} H - {}^6C_{\underset{\bar{p}r}{leH^3}} H (H^\dagger H) - {}^8C_{\underset{\bar{p}r}{leH^5}} H (H^\dagger H)^2$$

$$T^{\mu\nu}_{\bar{p}r} \supset {}^6C_{\underset{\bar{p}r}{leBH}} H \frac{1}{2} \left(B^{\mu\nu} - i\tilde{B}^{\mu\nu} \right) + {}^8C_{\underset{\bar{p}r}{leBH^3}} H (H^\dagger H) \frac{1}{2} \left(B^{\mu\nu} - i\tilde{B}^{\mu\nu} \right)$$

$$\omega_{R,\bar{p}rI} \supset + i (\phi \gamma_4)_I \, {}^6Q^{(1)}_{\underset{\bar{p}r}{e^2 H^2 D}}$$

Bosonic fermion loop corrections

$$
\begin{aligned}
{}^{8}\dot{C}_{H^8} =& \lambda\left(-\frac{4}{3}g_1^2\,{}^{6}C_{H^4D^2}-\frac{8}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1\\
&+\left(-8g_2^2\,{}^{6}C_{H^6}+\lambda\left(\frac{64}{3}g_2^2\,{}^{6}C_{H^4\Box}-4g_2^2\,{}^{6}C_{H^4D^2}-\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\right)\kappa_2\\
&+\left(6\,{}^{6}C_{H^6}-16\lambda\,{}^{6}C_{H^4\Box}+2\lambda\,{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})\\
&-\frac{4}{3}\lambda g_1^2\kappa_1^{(8)}-\frac{4}{3}\lambda g_2^2\kappa_2^{(8)}-\frac{4}{3}\lambda g_2^2\kappa_3-\frac{4}{3}\lambda g_2^2\kappa_4-\frac{8}{3}\lambda g_1^2\kappa_5+\frac{4}{3}\lambda g_2^2\kappa_5+\frac{1}{3}\lambda(g_1^2-g_2^2)\kappa_6\\
&+4\lambda\kappa_7^{(8)}-8\lambda\kappa_8+4\lambda\kappa_9^{(8)}+4\lambda\kappa_{10}^{(8)}+4\lambda\kappa_{12}-4\lambda\kappa_{13}-4\lambda\kappa_{14}-4\lambda\kappa_{15}-4\lambda\kappa_{16}\\
&-4\lambda\kappa_{17}-4\kappa_{21}^{(8)}+2\kappa_{22}-\frac{20}{3}\lambda g_1g_2\tau_2-\frac{8}{3}\lambda g_2^2\tau_3'+4\lambda g_2\tau_{18}+8\lambda g_1\tau_{20}+2\lambda g_2\tau_{26}\,.
\end{aligned}
$$

$$
\begin{aligned}
{}^{8}\dot{C}^{(1)}_{H^6D^2} =& \left(2g_1^2\,{}^{6}C_{H^4D^2}+\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1\\
&+\left(-\frac{32}{3}g_2^2\,{}^{6}C_{H^4\Box}+\frac{2}{3}g_2^2\,{}^{6}C_{H^4D^2}+8g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_2\\
&+\left(8\,{}^{6}C_{H^4\Box}+{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})\\
&+2g_1^2\kappa_1^{(8)}+\frac{10}{3}g_2^2\kappa_2^{(8)}+2g_2^2\kappa_3+\frac{8}{3}g_2^2\kappa_4+4g_1^2\kappa_5-\frac{10}{3}g_2^2\kappa_5-\frac{1}{2}g_1^2\kappa_6+g_2^2\kappa_6\\
&+2\kappa_8-6\kappa_9^{(8)}-10\kappa_{10}^{(8)}\underbrace{-2}_{+4}\kappa_{11}^{(8)}-6\kappa_{12}+6\kappa_{13}+6\kappa_{14}+10\kappa_{15}+6\kappa_{16}+10\kappa_{17}\\
&\underbrace{+2}_{-2}\kappa_{18}-\kappa_{19}+4\kappa_{20}+\frac{32}{3}g_1g_2\tau_2+\frac{20}{3}g_2^2\tau_3'-8g_2\tau_{18}-12g_1\tau_{20}-6g_2\tau_{26}
\end{aligned}
$$

More beyond geometry?

Recall: Higher-dim operators suppressed by $1/\Lambda$ so amp-squared SMEFT series

$$|\mathscr{A}|^2 = |A_{\rm SM}|^2 \left\{ 1 + \frac{2\text{Re}(A^*_{\rm SM}A_6)}{\Lambda^2 |A_{\rm SM}|^2} + \frac{1}{\Lambda^4}\left(\frac{|A_6|^2}{|A_{\rm SM}|^2} + \frac{2\text{Re}(A^*_{\rm SM}A_8)}{|A_{\rm SM}|^2} \right) + \cdots \right\}$$

Key Insight: Higher-dim operator effects can grow with $E \Rightarrow$ overcome suppression by powers of $1/\Lambda$ when $E \sim \Lambda$

Geometry $\leftrightarrow$ metric re-summation of higher-dimensional operators in $(\phi^2 \sim (HH^\dagger) \sim v^2)/\Lambda^2$ but **not** $E/\Lambda \Rightarrow$ **need more** for $E \gg v$

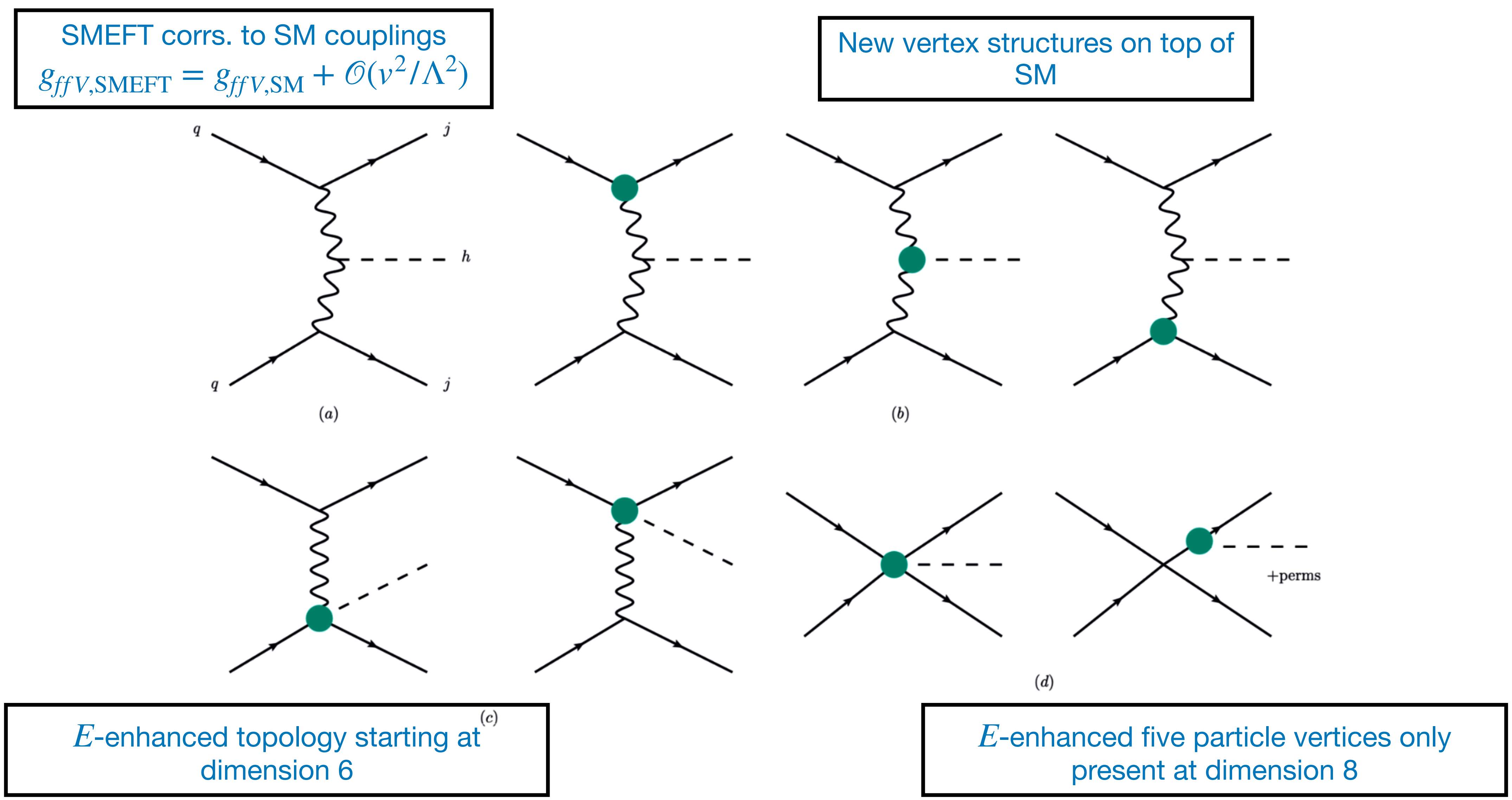
ID higher-dim **multi-particle operators** that grow with energy and have the most significant impact on high-energy processes

VBF Higgs production

Need process with **high E kinematics** $\leftrightarrow$ amplify effects of high-dim operators

Previous work found leading operators up to $\mathcal{O}(1/\Lambda^2)$ in VBF and VH
[Araz et al '20, Corbett and Martin '23]

Our aim: Argue which operators are E-enhanced and push to **unconstrained** $\mathcal{O}(1/\Lambda^4)$ [BA and Martin 2410.25163]



Energy-enhanced geoSMEFT operators

In regime $E \gg v$ the terms in $\mathscr{A}_6$ and $\mathscr{A}_8$ that incorporate the highest powers of E carry the largest impact

$2 \to 3$ amplitudes have mass dimension -1 with naive **scaling**

[BA, Martin, In preparation]

$$\mathscr{A}_{\rm SM} \sim g^3_{\rm SM}\frac{v}{E^2}, \quad \mathscr{A}_{Hq}, \mathscr{A}_{Hu,d} \sim g^2_{\rm SM}\frac{c_6\, v}{\Lambda^2}, \quad \mathscr{A}_{q^2H^2XD}, \mathscr{A}_{q^2H^2D^3} \sim g^2_{\rm SM}\frac{c_8\, v\, E^2}{\Lambda^4}, \quad \mathscr{A}_{q^4H^2} \sim \frac{c_8\, v\, E^2}{\Lambda^4}$$

The ratio of $D = 8$ interference piece to the $D = 6$

$$\frac{\mathscr{A}^*_{\rm SM}\mathscr{A}_8}{\mathscr{A}^*_{\rm SM}\mathscr{A}_6} \sim \left(\frac{c_8}{c_6}\right)\left(\frac{E^2}{\Lambda^2}\right)$$

For fixed $\Lambda \sim \mathrm{TeV}$ the Wilson coefficients for E-enhanced $D = 6$ operators such as $c^{(3)}_{Hq} \ll 1$ to be consistent with LEP

[Ellis et al. ’20]

Energy-enhanced contributions to VBF

Geometry-driven basis simplifies energy counting

Lacks extra D's and allows expansion only in v/Λ

Energy counting at a vertex is dictated by the lowest-dim geoSMEFT operator

Only impacts three-particle vertices or less $\Rightarrow$ look **beyond the geoSMEFT** operator set for E-enhanced

Operator set **process-dependent** requiring interference with SM - same chirality, color, Lorentz

Dimension 6

	Operator	relevant ψ
$Q^{(1)}_{H\psi}$	$i(\bar{\psi}_p\gamma^\nu\psi_r)H^\dagger \overleftrightarrow{D}_\mu H$	$\psi=\{q,u,d\}$
$Q^{(3)}_{H\psi}$	$i(\bar{\psi}\gamma^\nu\sigma^I\psi)\,H^\dagger \overleftrightarrow{D}_\mu \sigma_I H$	$\psi=\{q\}$

Remaining HVV and ffV vertices suppressed [Araz et al '20]

Dimension 8

	Operator	relevant ψ
$Q^{(1)}_{\psi^2H^2D^3}$	$i(\bar{\psi}_p\gamma^\mu\psi_r)\left[(D_\nu H)^\dagger(D^2_{(\mu,\nu)}H)-(D^2_{(\mu,\nu)}H)^\dagger(D_\nu H)\right]$	$\psi=\{q,u,d\}$
$Q^{(2)}_{\psi^2H^2D^3}$	$i(\bar{\psi}_p\gamma^\mu \overleftrightarrow{D}_\nu\psi_r)\left[(D_\mu H)^\dagger(D_\nu H)+(D_\nu H)^\dagger(D_\mu H)\right]$	$\psi=\{q,u,d\}$
$Q^{(3)}_{\psi^2H^2D^3}$	$i(\bar{\psi}_p\gamma^\mu\sigma^I\psi_r)\left[(D_\nu H)^\dagger\tau^I(D^2_{(\mu,\nu)}H)-(D^2_{(\mu,\nu)}H)^\dagger\sigma^I(D_\nu H)\right]$	$\psi=\{q\}$
$Q^{(4)}_{\psi^2H^2D^3}$	$i(\bar{\psi}_p\gamma^\mu\sigma^I \overleftrightarrow{D}_\nu\psi_r)\left[(D_\mu H)^\dagger\tau^I(D_\nu H)+(D_\nu H)^\dagger\tau^I(D_\mu H)\right]$	$\psi=\{q\}$

	Operator
$Q^{(1)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_p\gamma_\mu q_r)(H^\dagger H)$
$Q^{(2)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_p\gamma_\mu\sigma^I q_r)(H^\dagger\sigma^I H)$
$Q^{(3)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu\sigma^I q_r)(\bar{q}_p\gamma_\mu\sigma^I q_r)(H^\dagger H)$
$Q^{(1)}_{u^4H^2}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_p\gamma_\mu u_r)(H^\dagger H)$
$Q^{(1)}_{d^4H^2}$	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger H)$
$Q^{(1)}_{u^2d^2H^2}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger H)$
$Q^{(1)}_{q^2u^2H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{u}_p\gamma_\mu u_r)(H^\dagger H)$
$Q^{(2)}_{q^2u^2H^2}$	$(\bar{q}_p\gamma^\mu\sigma^I q_r)(\bar{u}_p\gamma_\mu u_r)(H^\dagger\sigma^I H)$
$Q^{(1)}_{q^2d^2H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger H)$
$Q^{(2)}_{q^2d^2H^2}$	$(\bar{q}_p\gamma^\mu\sigma^I q_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger\sigma^I H)$

	Operator	relevant ψ
$Q^{(1)}_{\psi^2BH^2D}$	$(\bar{\psi}_p\gamma^\nu\psi_r)D^\mu(H^\dagger H)B_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q^{(2)}_{\psi^2BH^2D}$	$i(\bar{\psi}_p\gamma^\nu\psi_r)(H^\dagger \overleftrightarrow{D}^\mu H)B_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q^{(3)}_{\psi^2BH^2D}$	$(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)D^\mu(H^\dagger\sigma^I H)B_{\mu\nu}$	$\psi=\{q\}$
$Q^{(4)}_{\psi^2BH^2D}$	$i(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)(H^\dagger \overleftrightarrow{D}^{I\mu} H)B_{\mu\nu}$	$\psi=\{q\}$
$Q^{(1)}_{\psi^2WH^2D}$	$(\bar{\psi}_p\gamma^\nu\psi_r)D^\mu(H^\dagger\sigma^I H)W^I_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q^{(2)}_{\psi^2WH^2D}$	$i(\bar{\psi}_p\gamma^\nu\psi_r)(H^\dagger \overleftrightarrow{D}^{I\mu} H)W^I_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q^{(3)}_{\psi^2WH^2D}$	$(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)D^\mu(H^\dagger H)W^I_{\mu\nu}$	$\psi=\{q\}$
$Q^{(4)}_{\psi^2WH^2D}$	$i(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)(H^\dagger \overleftrightarrow{D}^\mu H)W^I_{\mu\nu}$	$\psi=\{q\}$
$Q^{(5)}_{\psi^2WH^2D}$	$\epsilon_{IJK}(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)D^\mu(H^\dagger\sigma^J H)W^K_{\mu\nu}$	$\psi=\{q\}$
$Q^{(6)}_{\psi^2WH^2D}$	$i\epsilon_{IJK}(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)(H^\dagger \overleftrightarrow{D}^{J\mu} H)W^K_{\mu\nu}$	$\psi=\{q\}$

From 993 to 41 E-enhanced operators for VBF up to $D=8$

[BA and Martin 2410.25163]

Numerical analysis and resonant operators

Implemented LHC VBF selection cuts on $m_{j_1 j_2}$ and $\Delta\eta_{j_1 j_2}$ and restricted $p_{T,H} \in [200{,}400]$ GeV

[Araz et al '20]

Numerical analysis needed to confirm **EFT validity** up to $(D=8)^2$ terms; **minimum** $\Lambda \approx 1.2$ TeV

ID'd $D=8$ operators with **largest contributions** consistent with analysis: $c^{(3)}_{q^2H^2D^3}$ and $c^{(3)}_{q^2H^4}$

Operator $c^{(4)}_{q^2H^2D^3}$ is significant but causes **EFT breakdown** at $\Lambda = 1.2$ TeV due to $\hat{s}^3$ scaling $\Rightarrow$ **exclude** since requires $\Lambda > 3$ TeV

Type	(480 GeV, 2.5)	SM Deviation (%)	(600 GeV, 3.0)	SM Deviation (%)
SM	0.1375(2)	-	0.1239(2)	-
$D=6$	$0.1357(7)^{+0.0089}_{-0.0090}$	$[-7.9, +5.2]$	$0.1219(6)^{+0.0077}_{-0.0063}$	$[-6.8, +4.5]$
$D=6+(6\times 6)$	$0.1355(7)^{+0.0087}_{-0.0077}$	$[-7.1, +4.9]$	$0.1221(6)^{+0.0080}_{-0.0065}$	$[-6.8, +4.9]$

Type	(480 GeV, 2.5)	SM Deviation (%)	(600 GeV, 3.0)	SM Deviation (%)
SM	0.1375(2)	-	0.1239(2)	-
Coefficients at $D=8$				
$c^{(1)}_{q^4H^2}$	**0.1396(2)**	**+1.5**	**0.1261(2)**	**+1.8**
$c^{(2)}_{q^4H^2}$	0.1367(3)	0.6	0.1234(2)	-0.4
$c^{(3)}_{q^4H^2}$	**0.1512(3)**	**10.0**	**0.1359(2)**	**+9.7**
$c^{(1)}_{d^4H^2}$	0.1376(2)	+0.1	0.1240(2)	+0.1
$c^{(1)}_{u^4H^2}$	0.1380(3)	+0.4	0.1250(2)	+0.9
$c^{(1)}_{u^2d^2H^2}$	0.1374(3)	-0.1	0.1238(2)	-0.1
$c^{(1)}_{q^2d^2H^2}$	0.1377(3)	+0.1	0.1222(3)	-1.4
$c^{(2)}_{q^2d^2H^2}$	0.1370(3)	-0.4	0.1237(3)	-0.2
$c^{(1)}_{q^2u^2H^2}$	0.1372(2)	-0.2	0.1239(3)	0.0
$c^{(2)}_{q^2u^2H^2}$	0.1385(2)	+0.7	0.1252(3)	+1.0
$c^{(1)}_{q^2BH^2D}$	0.1374(3)	-0.1	0.1243(3)	+0.3
$c^{(3)}_{q^2BH^2D}$	0.1374(3)	0.0	0.1243(2)	+0.2
$c^{(1)}_{q^2WH^2D}$	0.1375(2)	+0.2	0.1241(2)	+0.2
$c^{(3)}_{q^2WH^2D}$	**0.1408(3)**	**+2.4**	**0.1270(2)**	**+2.5**
$c^{(5)}_{q^2WH^2D}$	0.1372(3)	-0.2	0.1240(3)	+0.1
$c^{(1)}_{u^2WH^2D}$	0.1381(2)	+0.4	0.1241(3)	+0.2
$c^{(1)}_{u^2BH^2D}$	0.1375(3)	0.0	0.1242(2)	+0.2
$c^{(1)}_{d^2WH^2D}$	0.1373(3)	-0.1	0.1239(2)	0.0
$c^{(1)}_{d^2BH^2D}$	0.1375(3)	0.0	0.1241(2)	+0.2
$c^{(1)}_{q^2H^2D^3}$	0.1376(3)	+0.1	0.1240(2)	+0.1
$c^{(2)}_{q^2H^2D^3}$	0.1372(3)	-0.2	0.1240(2)	+0.1
$c^{(3)}_{q^2H^2D^3}$	**0.1439(3)**	**+4.7**	**0.1299(2)**	**+4.8**
$c^{(4)}_{q^2H^2D^3}$ (*)	**0.1419(3)**	**+3.2**	**0.1280(3)**	**+3.3**
$c^{(1)}_{u^2H^2D^3}$	0.1380(3)	+0.4	0.1244(3)	+0.4
$c^{(1)}_{d^2H^2D^3}$	0.1371(2)	-0.3	0.1239(2)	0.0

$(D=8)^2 > (D=8) \times \text{SM}$

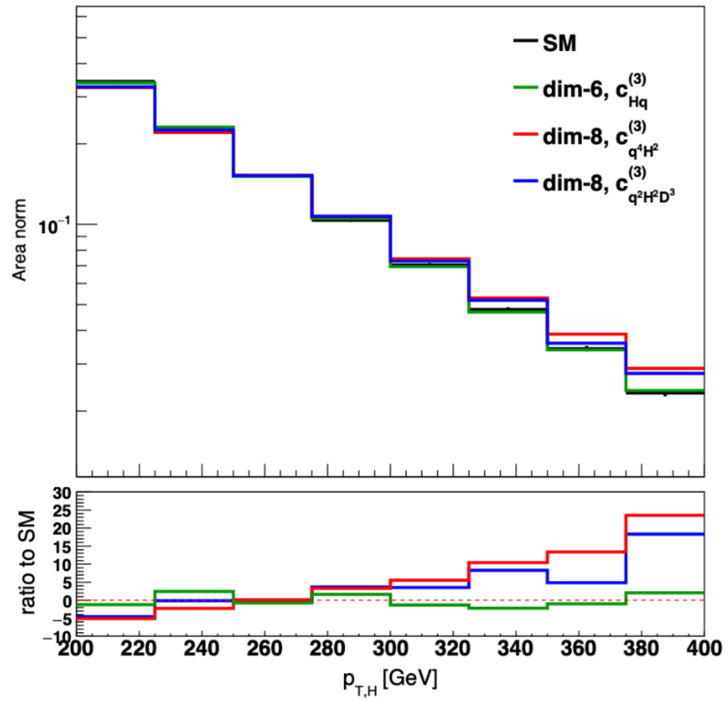
Observable distributions

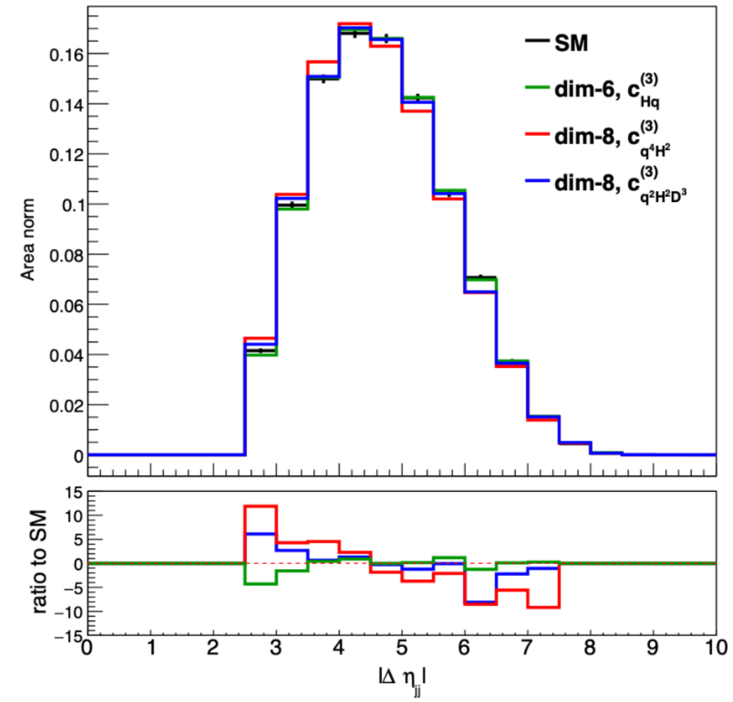
$D = 8$ operators influence high p_T^H regions more than $D = 6$ operators

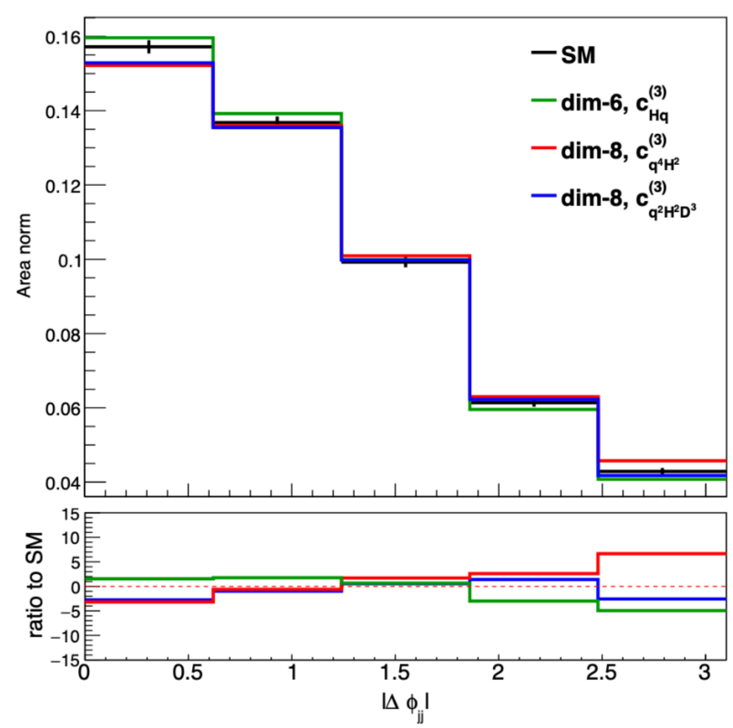
Small c_6 **LEP constrained** values largely suppress $D = 6$ impacts

Angular distributions **subtle differences** among SMEFT operators

Operators $c_{Hq}^{(3)}$ and $c_{q^2H^2}^{(3)}$ minimally affect angular distributions while $c_{q^2H^2D^3}^{(3)}$ causes noticeable shifts



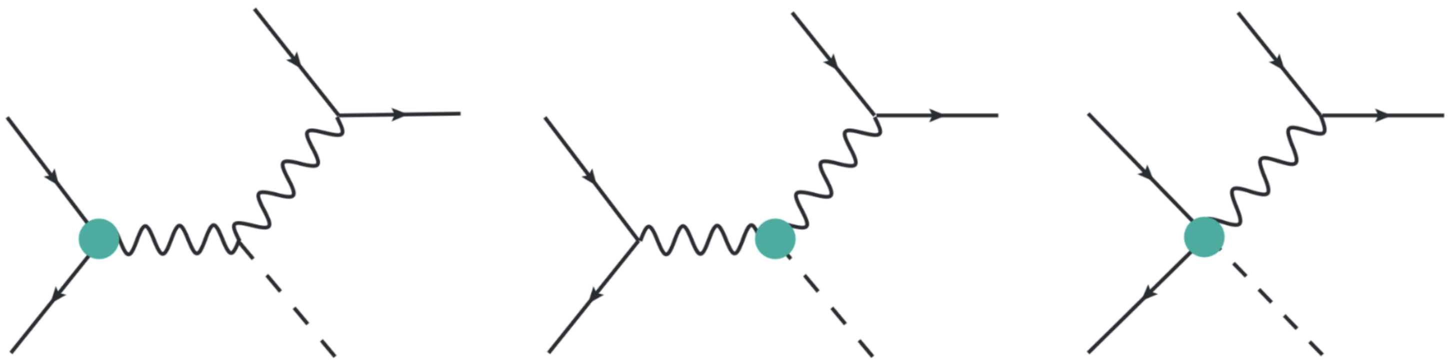




Takeaway: Observables at high p_T^H, optimized kinematic cuts and observable correlations **needed to distinguish** $D = 8$ operators

Crossed-process: Associated production $pp \to V(\bar{q}q)H$

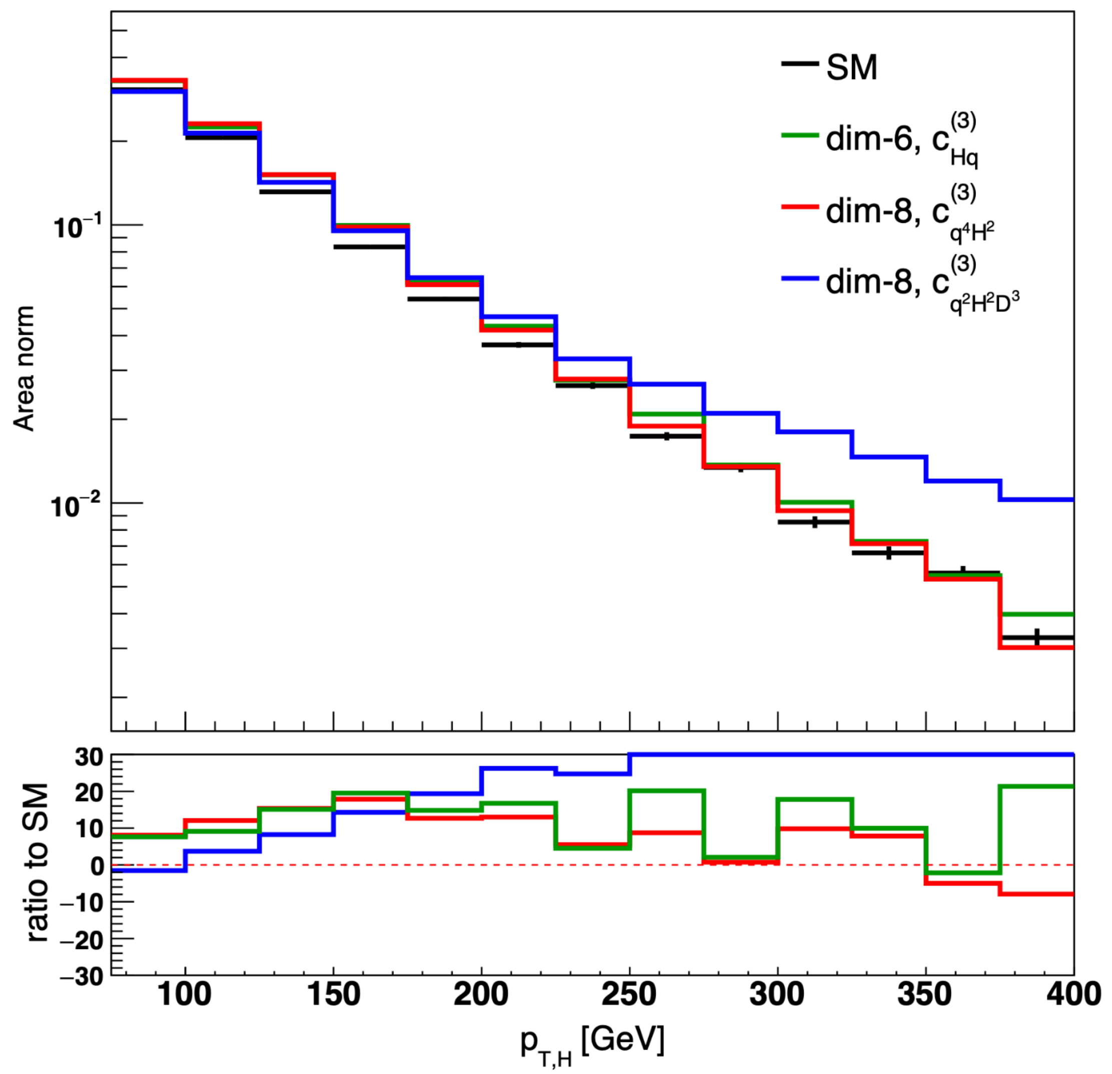
Crossing initial fermion transforms VBF topology to $pp \to V(\bar{q}q)H$



Simulated $pp \to Z(\bar{q}q)H$ with $75\,\mathrm{GeV} \le \mathrm{p}_{\mathrm{T,Z}} \le 400\,\mathrm{GeV}$ and $70 \le m_{jj} \le 110\,\mathrm{GeV} \leftrightarrow$ STSX binning strategy [Corbett et al ’23]

Operator $c^{(3)}_{H^2Q^2D^3}$ significantly impacts $p_{T,H}$ **affecting both** VBF and VH production

Operator $c^{(3)}_{H^2Q^4}$ **negligible** effect on VH production since analysis cuts break crossing symmetry
$\Rightarrow$ **deviations only in VBF**



Recap

Provided geometric framework for both bosons and fermions

Applied geometric formulation to calculate one-loop bosonic RGEs up to $D = 8$

Dimension-eight operators significantly impact VBF Higgs production when dimension-six operators are constrained

Developed E-enhanced arguments $\rightarrow$ small subset of operators have large impact at high-E offsetting their higher-dimensional suppression

What next

1) Completing fermion story in super-geometry and obtain remaining 1-loop RGEs - fermonic, boson and mixed [BA, A. Helset, J.Pagès, C.Shen, 2411.XXXX]

2) Understanding higher-derivative geometry

3) Fully incorporating gauge bosons gauge-invariantly [Cohen et al. '22, Craig et al. '23,…]

4) Provide a more general prescription to identify energy-enhanced operators [BA, Martin, In preparation]

5) Combined VBF di-Higgs and single Higgs analyses to enhance sensitivity to dimension-eight operators

Back-up

Geometric quantities

As before we have **Christoffel symbols**

$$\bar{\Gamma}^I_{JK} = \Gamma^I_{JK}$$

$$\bar{\Gamma}^p_{Is} = \bar{\Gamma}^p_{sI} = k^{p\bar{r}}\left(\frac{1}{2}k_{\bar{r}s,I} + \omega_{\bar{r}sI}\right)$$

$$\bar{\Gamma}^{\bar{p}}_{I\bar{s}} = \bar{\Gamma}^{\bar{p}}_{\bar{s}I} = \left(\frac{1}{2}k_{\bar{s}r,I} - \omega_{\bar{s}rI}\right)k^{r\bar{p}}$$

Satisfying **metric compatibility**

$$\nabla_I k_{\bar{b}a} = \partial_I k_{\bar{b}a} - k_{\bar{c}a}\Gamma^{\bar{c}}_{I\bar{b}} - k_{\bar{b}d}\Gamma^d_{Ia} = 0$$

and **Riemann curvature**

$$\bar{R}_{\bar{p}rIJ} = \omega_{\bar{p}rJ,I} - \left(\frac{1}{2}k_{\bar{p}s,I} - \omega_{\bar{p}sI}\right)k^{s\bar{t}}\left(\frac{1}{2}k_{\bar{t}r,J} + \omega_{\bar{t}rJ}\right) - (I \leftrightarrow J)$$

Summary

Dimension-eight operators significantly impact VBF Higgs production when dimension-six operators are constrained

Developed E-enhanced arguments $\rightarrow$ small subset of operators have large impact at high-E offsetting their higher-dimensional suppression

Identified operators of type $q^2H^2D^3$ and q^4H^2 which cause significant deviations in high-E distributions

Outlook

1) Provide a more general prescription to identify energy-enhanced operators [BA, Martin, In preparation]

2) Study more high-E processes e.g. di-Higgs where dimension-six operators are constrained to uncover dimension-eight effects

3) Combined VBF di-Higgs and single Higgs analyses to enhance sensitivity to dimension-eight operators

Scattering amplitudes

Similarly for 5-point amplitude

$$
\begin{aligned}
\mathcal{A}_{pI\bar{r}JK} = {} & (\bar{u}_{\bar{r}}\not{p}_J u_p)\bar{\nabla}_K \bar{R}_{\bar{r}pIJ} + (\bar{u}_{\bar{r}}\not{p}_K u_p)\bar{\nabla}_J \bar{R}_{\bar{r}pIK} \\
& + k^{s\bar{t}}\left\{ \frac{\mathcal{M}_{\bar{t}p;J}}{s_{pJ}} \bar{R}_{\bar{r}sIK}(\bar{u}_{\bar{r}}\not{p}_K\not{p}_J u_p) + \frac{\mathcal{M}_{\bar{r}s;J}}{s_{\bar{r}J}} \bar{R}_{\bar{t}pKI}(\bar{u}_{\bar{r}}\not{p}_J\not{p}_K u_p) + (IJK) \right\} \\
& + \left\{ \frac{k^{s\bar{t}}k^{n\bar{o}}}{s_{pJ}s_{\bar{r}I}} \mathcal{M}_{\bar{r}n;I}\mathcal{M}_{\bar{o}s;K}\mathcal{M}_{\bar{t}p;J}(\bar{u}_{\bar{r}}\not{p}_I\not{p}_J u_p) + (I \leftrightarrow J \leftrightarrow K) \right\} \\
& + \left\{ (\bar{u}_{\bar{r}}\not{p}_K u_p)\frac{1}{2}\left[-\frac{V_{;IJM}}{s_{IJ}} h^{ML} \left[\bar{R}_{\bar{r}pLK} + k^{s\bar{t}} \left(\frac{\mathcal{M}_{\bar{r}s;K}\mathcal{M}_{\bar{t}p;L}}{s_{\bar{r}K}} - \frac{\mathcal{M}_{\bar{r}s;L}\mathcal{M}_{\bar{t}p;K}}{s_{pK}} \right) \right] \right. \right. \\
& \qquad \left. \left. + k^{s\bar{t}} \left(\frac{\mathcal{M}_{\bar{r}s;K}\mathcal{M}_{\bar{t}a;IJ}}{s_{\bar{r}K}} - \frac{\mathcal{M}_{\bar{r}s;IJ}\mathcal{M}_{\bar{t}p;K}}{s_{pK}} \right) \right] + (I \leftrightarrow J \leftrightarrow K) \right\} \\
& + (\bar{u}_{\bar{r}}u_p)\left\{ -\mathcal{M}_{\bar{r}p;KJI} + \frac{\mathcal{M}_{\bar{r}p;L}}{s_{p\bar{r}}} h^{LM} \bar{R}_{KJIM}(s_{IK} - s_{JI}) + \frac{\mathcal{M}_{\bar{r}p;M}}{s_{p\bar{r}}} h^{ML} V_{;KJIL} \right. \\
& \qquad \left. + \left[\frac{1}{2}\frac{V_{;IJM}}{s_{IJ}} h^{ML} \left(\mathcal{M}_{\bar{r}p;KL} - \frac{\mathcal{M}_{\bar{r}p;N}}{s_{p\bar{r}}} h^{NO} V_{;OKL} \right) + (I \leftrightarrow J \leftrightarrow K) \right] \right\}
\end{aligned}
$$

Again amplitudes group into **geometric invariants**!

Scalar-gauge scattering

Some **Born amplitudes for** massless fields $\phi_I \phi_J \to \phi_K \phi_L$

$$\mathcal{A}_{IJKL} = R_{IJKL} s_{IK} + R_{IKJL} s_{IJ}$$

$$+ \frac{(t_{I;J} \cdot t_{K;L})(s_{IL} - s_{IK})}{s_{IJ}} + \frac{(t_{I;K} \cdot t_{J;L})(s_{IL} - s_{IJ})}{s_{IK}} + \frac{(t_{I;L} \cdot t_{K;J})(s_{IJ} - s_{IK})}{s_{IL}}$$

and $\phi_I \phi_J \to A_A A_B$

$$\mathcal{A}_{IJAB} = \left(\nabla_I \nabla_J g_{AB} - \frac{1}{2} (\nabla_I g_{AC}) g^{CD} (\nabla_J g_{BD}) - \frac{1}{2} (\nabla_J g_{AC}) g^{CD} (\nabla_I g_{BD}) \right) B_1$$
$$- \left(\frac{(\nabla_I g_{AC}) g^{CD} (\nabla_J g_{BD})}{s_{IA}} + \frac{(\nabla_J g_{AC}) g^{CD} (\nabla_I g_{BD})}{s_{JA}} \right) B_2 + \ldots$$

Again amplitudes depend on **geometric invariants**!

SMEFT

Goal: bottom-up EFT to systematically classify "all" BSM physics (knowledge of UV **not required**)

Assumptions: new nearly physics decoupled ⇒ $\Lambda \sim$ few TeV $\gg v$ and at the accessible scale only SM fields + symmetries

Violate B/L number

params known to all $\mathcal{O}$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \ldots : \qquad \mathcal{L}_n = \sum_i C_i \mathcal{O}_i^{(d=n)}$$

"Leading" SM deviations

Majorana ν masses

SMEFT

Extensive studies done for $\mathscr{L}_6$ and much available:

1) Complete RGEs and various 1-loop results

2) Tools for matching and numerical analysis

3) Many tree-level calculations of EW, Higgs, & flavour observables

Similarly but to much lesser extent for $\mathscr{L}_8$ (RGEs and tree-level)

Bosonic

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Fermionic

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

SMEFT

The gauge metric entries

$$
\begin{aligned}
g_{GG} &= \left[1 - 2\ \,^6C_{G^2H^2}(\phi^I\phi^I) - \,^8C_{G^2H^4}(\phi^I\phi^I)^2\right] \mathbb{1}_{8\times 8}\,, \\
[g_{WW}]_{ab} &= \left[1 - 2\ \,^6C_{W^2H^2}(\phi^I\phi^I) - \,^8C^{(1)}_{W^2H^4}(\phi^I\phi^I)^2\right] \mathbb{1}_{3\times 3} - 4\ \,^8C^{(2)}_{W^2H^4} x_a(\phi) x_b(\phi)\,, \\
[g_{WB}]_a &= [g_{BW}]_a = \left(2\ \,^6C_{WBH^2} + \,^8C_{WBH^4}\right) x_a\,, \\
g_{BB} &= \left[1 - 2\ \,^6C_{B^2H^2}(\phi^I\phi^I) - \,^8C_{B^2H^4}(\phi^I\phi^I)^2\right]\,.
\end{aligned}
$$

and **Riemann curvature**

$$
\begin{aligned}
R_{IJKL} = &- 2\ \,^6C_{H^4\Box}(\delta_{IK}\delta_{JL} - \delta_{IL}\delta_{JK}) - \frac{1}{2}\,^6C_{H^4D^2} \sum_{a=1}^{4} \left([\Upsilon_a]_{IK}[\Upsilon_a]_{JL} - [\Upsilon_a]_{IL}[\Upsilon_a]_{JK}\right) \\
&- \left(4\left(^6C_{H^4\Box}\right)^2 + \,^8C^{(1)}_{H^6D^2} - \,^8C^{(2)}_{H^6D^2}\right)(\phi^R\phi^R)(\delta_{IK}\delta_{JL} - \delta_{IL}\delta_{JK}) \\
&+ \left(^8C^{(1)}_{H^6D^2} - \,^8C^{(2)}_{H^6D^2}\right)(\delta_{JK}\phi_I\phi_L + \delta_{IL}\phi_J\phi_K - \delta_{JL}\phi_I\phi_K - \delta_{IK}\phi_J\phi_L) \\
&+ \text{dimension-eight } \Upsilon \text{ terms}\,,
\end{aligned}
$$

SMEFT

Some **bosonic** operators at **dimension six**

$$
\begin{aligned}
{}^{6}\mathcal{L}^{(6)}_{\mathrm{SMEFT}} = \; & C_{H^6}(H^\dagger H)^3 + C_{H^4 D^2}(H^\dagger H)\Box(H^\dagger H) + C_{H^4 D^2}(D_\mu H^\dagger H)(H^\dagger D^\mu H) \\
& + C_{H^2 B^2}(H^\dagger H) B_{\mu\nu} B^{\mu\nu} + \ldots
\end{aligned}
$$

Some operators at **dimension eight**

$$
\begin{aligned}
{}^{8}\mathcal{L}_{\mathrm{SMEFT}} = \; & C_{H^8}(H^\dagger H)^4 + C^{(1)}_{H^6 D^2}(H^\dagger H)^2(D_\mu H^\dagger D^\mu H) \\
& + C^{(2)}_{H^6 D^2}(H^\dagger H)(H^\dagger \tau^a H)(D_\mu H^\dagger \tau^a D^\mu H) + C_{H^4 B^2}(H^\dagger H)^2 B_{\mu\nu} B^{\mu\nu} + \ldots
\end{aligned}
$$

Dimension 6 and 8 **matching coefficients** in Lagrangian

$$
{}^{6}C_{H^6},\; {}^{6}C_{H^4\Box},\; {}^{6}C_{H^4 D^2},\; {}^{6}C_{G^2 H^2},\; {}^{6}C_{W^2 H^2},\; {}^{6}C_{B^2 H^2},\; {}^{6}C_{WBH^2},
$$

$$
{}^{8}C_{H^8},\; {}^{8}C^{(1)}_{H^6 D^2},\; {}^{8}C^{(2)}_{H^6 D^2},\; {}^{8}C^{(1)}_{G^2 H^4},\; {}^{8}C^{(1)}_{W^2 H^4},\; {}^{8}C^{(3)}_{W^2 H^4},\; {}^{8}C^{(1)}_{B^2 H^4},\; {}^{8}C^{(1)}_{WBH^4}
$$

The **RGEs** dependent on coefficients above were determined

Renormalisation

One-loop RGE from 2nd variation of action [t'Hooft '74, Alonso, Manohar et al '20]

$$A^{B\mu_B} = \mathsf{A}^{B\mu_B} + \zeta^{B\mu_B} - \frac{1}{2}\widetilde{\Gamma}^{(B\mu_B)}_{jk}\eta^j\eta^k + \dots$$

$$\phi^I = \Phi^I + \eta^I - \frac{1}{2}\widetilde{\Gamma}^I_{jk}\eta^j\eta^k + \dots$$

$$\eta^i = \begin{pmatrix} \eta^I \\ \zeta^{A\mu_A} \end{pmatrix}$$

in **geodesic coordinates**

$$\frac{\mathrm{d}^2\phi^i}{\mathrm{d}\lambda^2} + \Gamma^i_{jk}(\phi)\frac{\mathrm{d}\phi^j}{\mathrm{d}\lambda}\frac{\mathrm{d}\phi^k}{\mathrm{d}\lambda} = 0$$

$$\phi^i = \phi^i_0 + \lambda\eta^i - \frac{1}{2}\lambda^2\,\Gamma^i_{jk}(\phi_0)\,\eta^j\eta^k + \dots$$

$$\phi^i \to \phi^i + \eta^i - \frac{1}{2}\Gamma^i_{jk}\eta^j\eta^k + \mathcal{O}(\eta^3)$$

gives **covariant** result e.g. $\eta\eta$-variation

$$\delta_{\eta\eta}S = \frac{1}{2}\int d^4x\,\Big\{h_{IJ}\left(\widetilde{\mathscr{D}}_\mu\eta\right)^I\left(\widetilde{\mathscr{D}}_\mu\eta\right)^J + \Big[-\widetilde{R}_{IKJL}(D_\mu\phi)^K(D^\mu\phi)^L - (\nabla_I\nabla_J V)$$
$$- \frac{1}{4}\left(\nabla_I\nabla_J g_{AB} - \Gamma^C_{IA}g_{CB,J} - \Gamma^C_{IB}g_{AC,J}\right)F^{A\mu\nu}F^B_{\mu\nu} - h_{IK}h_{JL}g^{AB}t^K_A t^L_B\Big]\eta^I\eta^J\Big\}$$

Scalar amplitudes

5-point amplitude $\phi_I \phi_J \to \phi_K \phi_L \phi_M$

$$A^5_{IJKLM} = \nabla_M R_{IJKL}(s_{LM} + s_{JL}) + \nabla_K R_{ILJM} s_{LM} + \nabla_L R_{IKJM} s_{KM} + \nabla_L R_{IJKM} s_{JM} + \nabla_M R_{IKJL} s_{KL}$$

Including **4-derivative** interactions

$$A^4_{IJKL} \supset \frac{1}{2}\lambda_{IJKL} s_{IJ} s_{KL} + \frac{1}{2}\lambda_{IKJL} s_{IK} s_{JL} + \frac{1}{2}\lambda_{JKIL} s_{JK} s_{IL}$$

$$A^5_{IJKLM} \supset \frac{1}{2}\nabla_M \lambda_{IJKL} s_{IJ} s_{KL} + \frac{1}{2}\nabla_M \lambda_{IKJL} s_{IK} s_{JL} + \frac{1}{2}\nabla_M \lambda_{JKIL} s_{JK} s_{IL} + \text{cylic}$$

New **soft theorem** for theory of scalars with no potential [Alonso et al '20]

$$\lim_{q_i \to 0} A^{i_1 \ldots i_n i}_{n+1} = \nabla^i A^{i_1 \ldots i_n}_n$$

Plus double- and triple-soft theorems - generalises the double-soft theorem for pions [Arkani-Hamed et al '08]

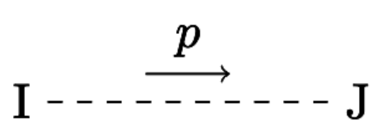
Scalar field EFT

Scalar field theory up to two-derivatives [Alonso, Manohar et al 1605.0360]

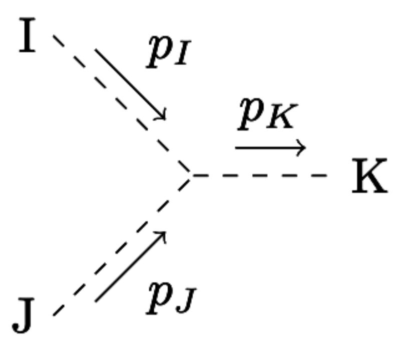
$$\mathscr{L} = \frac{1}{2} h_{IJ}(\boldsymbol{\phi})(\partial_\mu \phi)^I (\partial^\mu \phi)^J - I(\boldsymbol{\phi})$$

Expanding metric $h_{IJ}(\phi) \Rightarrow$ higher-dim operators $\leftrightarrow$ vertices

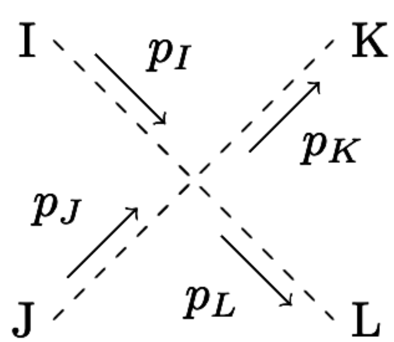
$$h_{IJ} = h_{IJ} + h_{IJ,K}\phi^K + h_{IJ,KL}\phi^K\phi^L + \dots$$



$$= i\,\frac{h^{IJ}}{p^2}$$



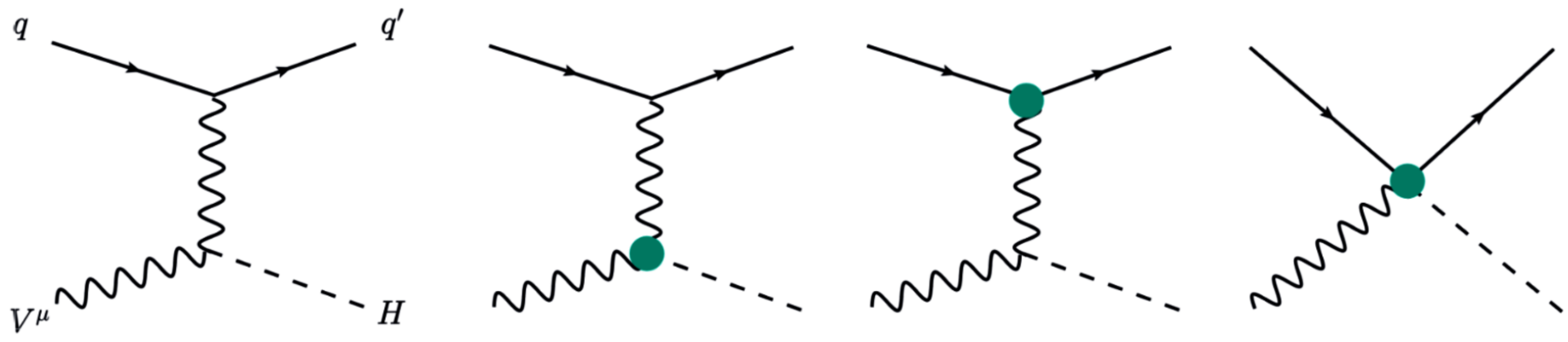
$$= i\,(h_{JK,I}(p_J \cdot p_K) + h_{IK,J}(p_I \cdot p_K) - h_{IJ,K}(p_I \cdot p_J)) + i\,\mathcal{I}_{,IJK}$$



$$= i\,(h_{IK,LJ}\, p_I \cdot p_K + h_{IL,KJ}\, p_I \cdot p_L - h_{IJ,LK}\, p_I \cdot p_J$$
$$+ h_{JK,LI}\, p_J \cdot p_K + h_{JL,IK}\, p_J \cdot p_L - h_{KL,IJ}\, p_K \cdot p_L) + i\,\mathcal{I}_{,IJKL}$$

Analysis of energy-enhanced contributions to VBF

Consider $qV \to q'H$ as proxy for VBF to ID most enhanced SMEFT operators



High-E limit $\hat{t} \gg m_V$ with V_L effects grow the strongest with E once $qV \to q'H$ embedded in VBF

$$\mathcal{A}(qZ_{L,\mu} \to qH) \underset{\hat{t} \gg m^2_{Z,H}}{=} -i\langle \bar{q} | \gamma_\mu p^\mu_H | q] \frac{1}{\hat{t}} \left(g_{Zq_Lq_L} g^{(1)}_{HZZ} + g^{(1)}_{ZHq_Lq_L} \frac{\hat{t}}{\Lambda^2} + (g^{(2)}_{ZHq_Lq_L} - g^{(3)}_{ZHq_Lq_L}) \frac{\hat{t}^2}{2\Lambda^4} \right)$$

4-particle contact terms scale with higher powers of $\hat{t}$

$$\mathcal{A}(qW_{L,\mu} \to q'H) \underset{\hat{t} \gg m^2_{W,H}}{=} -i\langle \bar{q} | \gamma_\mu p^\mu_H | q] \frac{1}{\hat{t}} \left(g_{Wq_Lq'_L} g^{(1)}_{HWW} + g^{(1)}_{WHq_Lq'_L} \frac{\hat{t}}{\Lambda^2} + (g^{(2)}_{WHq_Lq'_L} - g^{(4)}_{WHq_Lq'_L}) \frac{\hat{t}^2}{2\Lambda^4} - g^{(3)}_{WHq_Lq'_L} \frac{\hat{t}\,(2\hat{s} + \hat{t})}{2\Lambda^4} \right)$$

New terms involving quark momenta $\propto \hat{s}\hat{t}$ and **dominate** when $\hat{s}$ is large but $\hat{t}$ remains small; other SMEFT contributions are **suppressed** by $\hat{t}$

Total cross-sections

Effective W approximation: treating incoming W as proton constituent in the $2 \to 3$ process ⇒ convolving the W-boson PDF with the $qV \to q'H$ in the limit $\hat{t} \to 0$

[Dawson '84]

Dominant $D = 6$ terms are suppressed at large $\hat{s}$ with W_T ⇒ Focus on W_L

$$\int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}} A^{(6)})_{W_L} \sim \frac{v^2\,\hat{s}}{\Lambda^2\, m_W^2} \qquad \int^{\theta_{\max}} d\theta^* \, |A^{(6)}|^2_{W_L} \sim \frac{v^2\,\hat{s}}{\Lambda^4},$$

Dominant $D = 8$ **interference terms** from operators leads to **different scaling** for $\sim$ $c^{(3)}_{q^2H^2D^3}, c^{(3)}_{q^2H^2WD}$ vs. $c^{(4)}_{q^2H^2D^3}$ ↔ operators with different Lorentz structures

$$\int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}} A_3^{(8)})_{W_L} \sim \frac{v^2\,\hat{s}^2}{\Lambda^4\, m_W^2} \qquad \int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}} A_{24}^{(8)})_{W_L} \sim \frac{v^2\,\hat{s}}{\Lambda^4}$$

Squared terms exhibit larger differences

$$\int^{\theta_{\max}} d\theta^* \, |A_3^{(8)}|^2_{W_L} \sim \frac{v^2\,\hat{s}^3}{\Lambda^8} \qquad \int^{\theta_{\max}} d\theta^* \, |A_{24}^{(8)}|^2_{W_L} \sim \frac{v^2\,\hat{s}\, m_W^4}{\Lambda^8}$$

Effective W approximation

Additionally: The operator $c^{(3)}_{q^2H^2D^3}$ interferes with the SM for W_T

$$\int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}}A^{(8)}_{24})_{W_T} \sim \frac{v^2\,\hat{s}}{\Lambda^4} \qquad \int^{\theta_{\max}} d\theta^* \, |A^{(8)}_{24}|^2_{W_T} \sim \frac{v^2\,\hat{s}\,m_W^4}{\Lambda^8}$$

This weaker interference is **offset** by larger transverse W PDFs

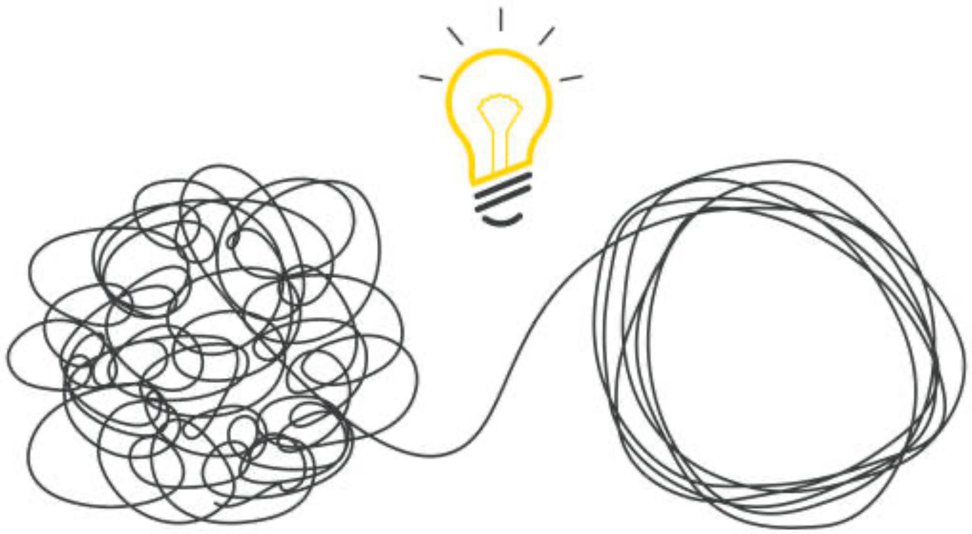
[Dawson '84]

Determining whether T or L effects dominate requires **numerical analysis** beyond $2 \to 2$ approximations

New pure contact $D = 8$ vertices from q^4H^2 operators contribute in VBF with largest effect from $(LL)(LL)$ helicity structures

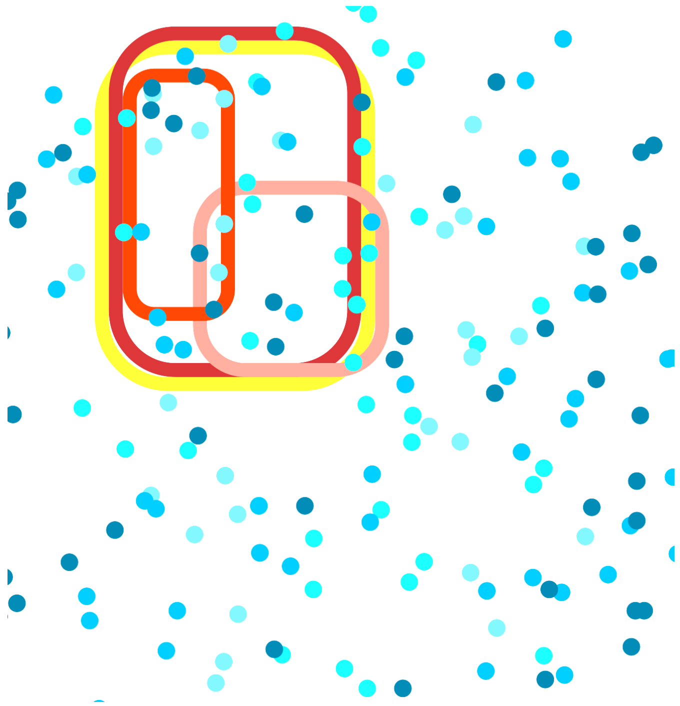
$$\mathscr{A}(u_L d_L \to u_L d_L H) \sim v c^{(3)}_{q^4H^2}\langle 34\rangle[12]$$

SMEFT



Large # of operators ⇒ many operators can contribute to same observable

Ideal: global SMEFT fit to very precise measurement, all C_i free parameters



Reality: only partial fits are feasible since too many operators to constrain

Aim: come up with set of **observables** sensitive to a close manageable set of operators

Dominant effect: the tree-level interference e.g. $|\mathscr{A}_{\rm SM}\mathscr{A}^*_{d=6}| \sim C_i/\Lambda^2$ ⇒ if suppressed can neglect C_i

E.g. Four-fermion operator in Drell-Yan via Z-resonance

N.B. many studies along this vein, interesting to think up new observables

Renormalisation

One-loop RGE from 2nd variation of action in geodesic coordinates

$$A^{B\mu_B} = \mathsf{A}^{B\mu_B} + \zeta^{B\mu_B} - \frac{1}{2}\widetilde{\Gamma}^{(B\mu_B)}_{jk}\eta^j\eta^k + \dots$$

$$\phi^I = \Phi^I + \eta^I - \frac{1}{2}\widetilde{\Gamma}^I_{jk}\eta^j\eta^k + \dots$$

$$\eta^i = \begin{pmatrix} \eta^I \\ \zeta^{A\mu_A} \end{pmatrix}$$

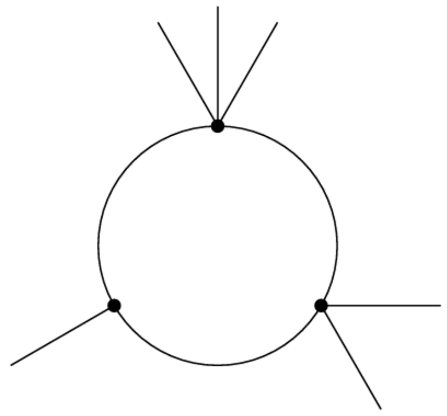
gives **covariant** result e.g. $\eta\eta$-variation

$$\delta_{\eta\eta}S = \frac{1}{2}\int d^4x \left\{ h_{IJ}\left(\widetilde{\mathscr{D}}_\mu\eta\right)^I\left(\widetilde{\mathscr{D}}_\mu\eta\right)^J + \left[-\widetilde{R}_{IKJL}(D_\mu\phi)^K(D^\mu\phi)^L - (\nabla_I\nabla_J V) - \frac{1}{4}\left(\nabla_I\nabla_J g_{AB} - \Gamma^C_{IA}g_{CB,J} - \Gamma^C_{IB}g_{AC,J}\right)F^{A\mu\nu}F^B_{\mu\nu} - h_{IK}h_{JL}g^{AB}t^K_A t^L_B \right]\eta^I\eta^J \right\}$$

with covariant derivative

$$Z^i_\mu = \begin{bmatrix} (D_\mu\phi)^I \\ F^{A\;\mu_A}_\mu \end{bmatrix}$$

$$(\widetilde{\mathscr{D}}_\mu\eta)^I = \partial_\mu\eta^I + t^I_{B,K}A^B_\mu\eta^K + \widetilde{\Gamma}^I_{jk}Z^j_\mu\eta^k$$



similarly for gauge $\zeta\zeta$ and mixed $\zeta\eta$ variation

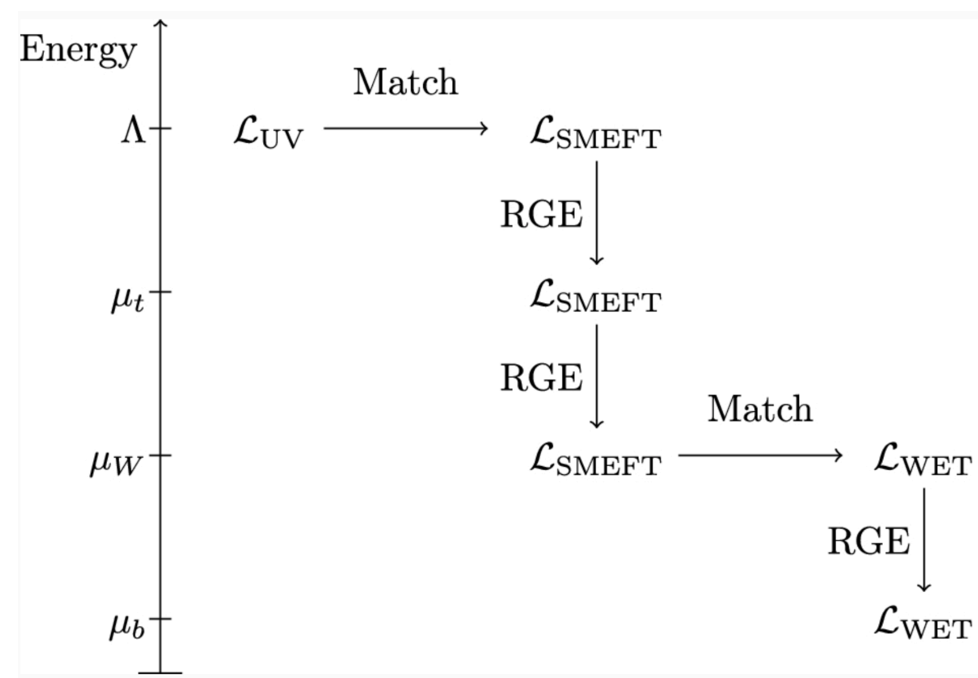
HEFT LEFT and ALP-SMEFT

HEFT: SMEFT ⊂ HEFT with HEFT a fusion of ChPT in scalar sector and SMEFT in gauge & fermion sector, HEFT has 3 goldstones embedded in matrix plus one gauge singlet Higgs ⇒ HEFT = SMEFT + no assumptions about Higgs scalar being in doublet

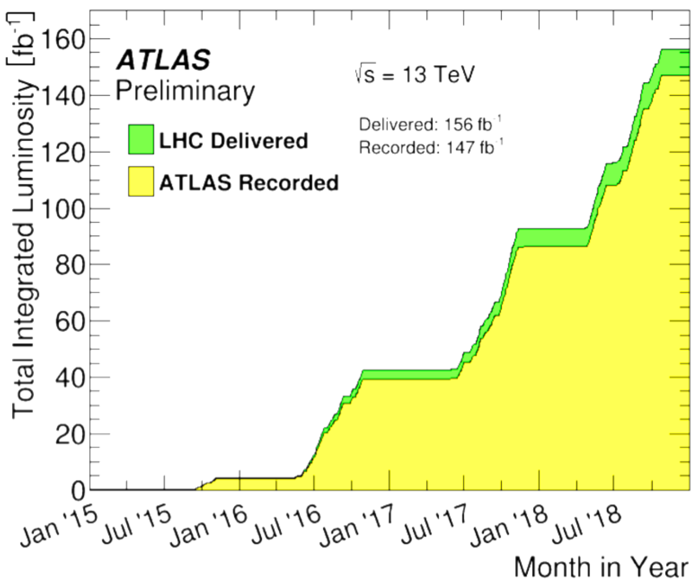
ALP-SMEFT: EFTs to describe interactions of axion our axion-like particles which are not present in SMEFT or HEFT

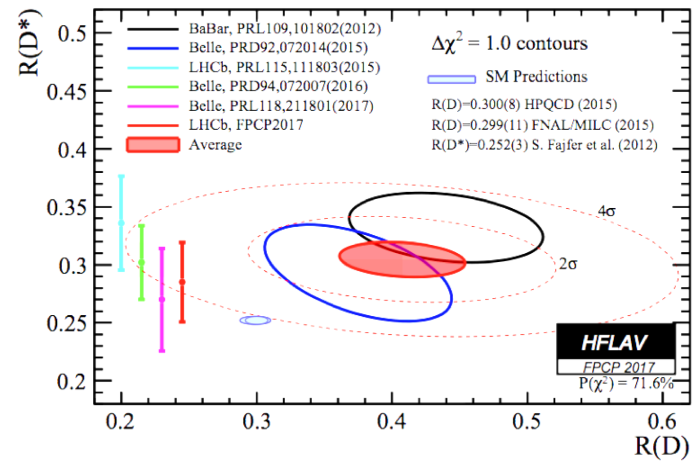
Below EW scale: can write low energy effective theory (LEFT) with quark and lepton fields, and only QCD and QED gauge fields

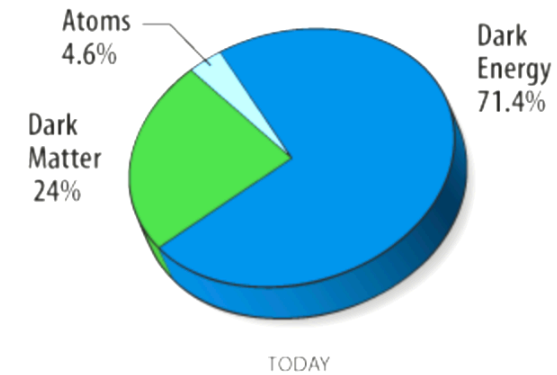
Combining EFTs: If scales widely separated can match and run repeatedly between EFTs systematically



Data rich era spanning multiple scales







Credit: NASA/WMAP Science Team