Ameen Ismail

PIKIMO Fall 2024

(energy correlators, confinement, and holography)

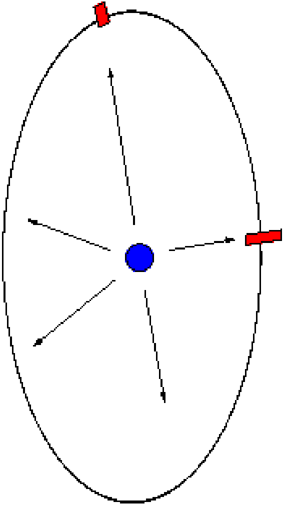
arXiv:2403.12123 w/ C. Csáki
and WIP w/ C. Csáki, S. Ferrante, L. Kiriliuk
PIKIMO Fall 2024

Energy correlators

Correlation functions of energy flow operators $\langle \mathcal{E}(\vec{n}) \rangle$, $\langle \mathcal{E}(\vec{n}) \mathcal{E}(\vec{n}') \rangle$

$\mathcal{E}(\vec{n})$: energy deposited in a calorimeter

CFT context by Hofman + Maldacena '08

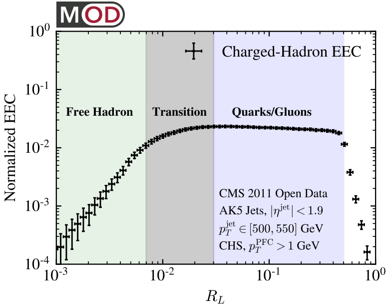


Energy correlators and confinement

Two-point function visualizes confinement transition

The big idea:

Can we reproduce this in a holographic model?



Komiske, Moult, Thaler, Zhu 2201.07800

Calculating energy correlators

Idealized calorimeter: energy flow operators

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt T_{0i}(t, x^i = rn^i)n^i$$

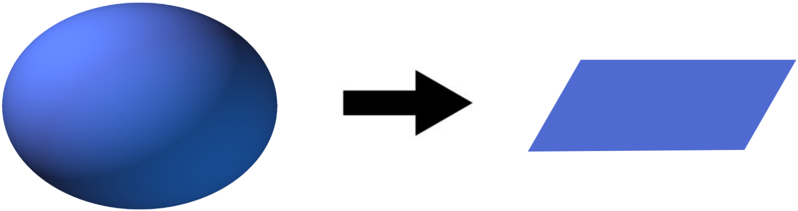
Lightcone coordinates $x^\pm = x^0 \pm x^3$, $x^\perp = x^{1,2}$:

$$\mathcal{E}(\vec{n}) = \lim_{x^+\to\infty} \frac{(x^+)^2}{4} \int_{-\infty}^\infty dx^- T_{--}(x^+, x^-, x^\perp)$$

i.e. integral over future null infinity

Conformal transformation

Map celestial sphere (at $x^+ \to \infty$) to null plane $x^+ = 0$:

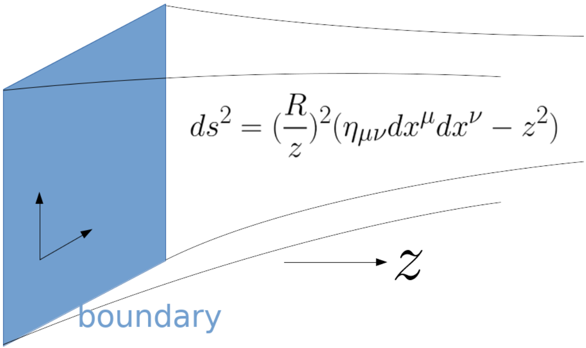


Energy flow on the null plane

$$\mathcal{E}(\vec{n}) = \left(1 + |x^\perp|^2\right)^3 \int_{-\infty}^{\infty} dx^- \, T_{--}(x^+ = 0, x^-, x^\perp)$$

(mapping to celestial sphere: $x^1 + ix^2 = e^{i\phi} \tan\theta/2$)

Holographic calculation setup



We want a generating functional for correlators

Inserting an energy flow operator

Perturb

Belin et al. 2011.13862

$$\delta S_{\mathrm{CFT}} = \epsilon \int dx^- T_{--}(x^+ = 0, x^-, x^\perp = y^\perp)$$
$$= \epsilon \int d^4x T_{--} \delta(x^+) \delta^2(x^\perp - y^\perp)$$

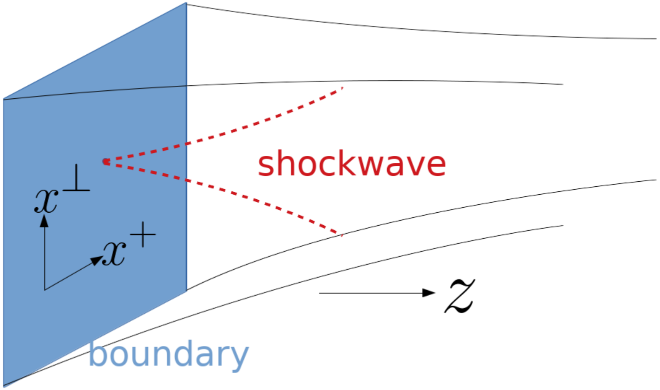
Inserts $e^{\epsilon \mathcal{E}}$ in path integral

Dual picture: shockwave,

$$\delta ds^2 = \frac{\epsilon}{z^2} \delta(x^+) f(x^\perp - y^\perp, z) \left(dx^+\right)^2$$

(boundary condition: $f(x^\perp, z = 0) = \delta^2(x^\perp)$)

Shockwave picture



Shockwaves in AdS

Field equations are linear: $\left(3/z\partial_z - \partial_z^2 - \partial_\perp^2\right) f = 0$

Shockwave

$$f(x^\perp, z) = \frac{z^4}{\left(z^2 + |x^\perp|^2\right)^3}$$

Shockwaves are linear $\Rightarrow$ superpose them: $ds^2 =$
$ds^2_{\mathrm{AdS}} + \frac{\delta(x^+)}{z^2}\left[\epsilon_1 f(x^\perp - y_1^\perp, z) + \epsilon_2 f(x^\perp - y_2^\perp, z)\right]\left(dx^+\right)^2$

Exact solution of field equations

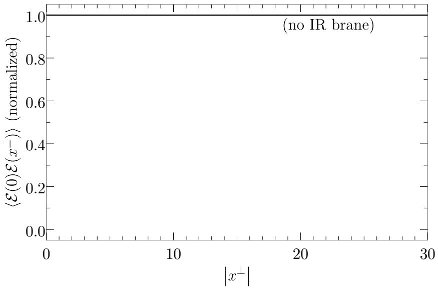
Recipe for two-point correlator

Assume scalar source with momentum $q^\mu == (q, \vec{0})$:

$$\langle \mathcal{E}(0)\mathcal{E}(y^\perp) \sim \left(1 + (y^\perp)^2\right)^3 f(y^\perp, z = 1) = \text{constant}$$

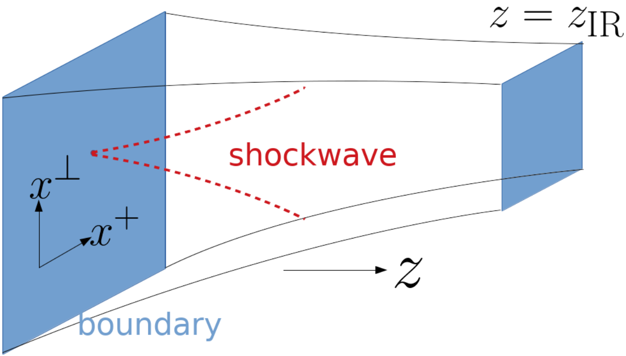
Upshot

Two-point correlator probes shockwave @ $z = 1$



Cutting off AdS

Simplest "hard-wall" confinement: IR brane at $z = z_{\text{IR}}$
(essentially RSI model)



Shockwaves with a brane

Shockwave EOM unchanged, $(3/z\partial_z - \partial_z^2 - \partial_1^2 - \partial_2^2)f = 0$

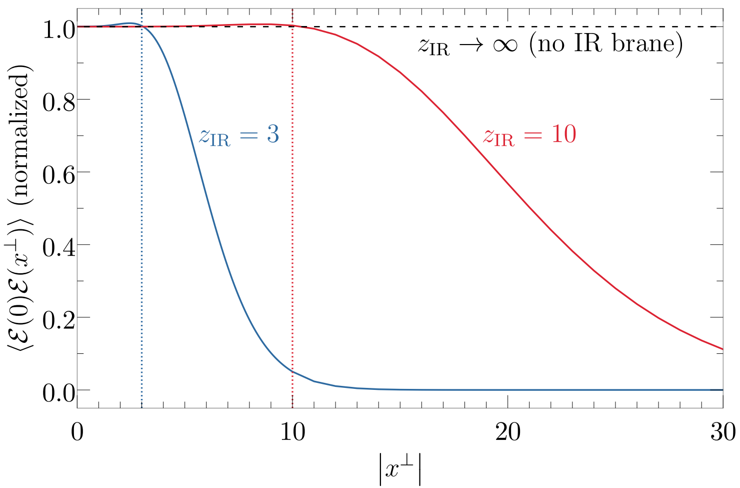
Brane modifies BC: $\partial_z f(x^\perp, z)\Big|_{z=z_{\mathrm{IR}}} = 0$

Solution

$$f(x^\perp, z) = \frac{1}{8}\int_0^\infty dk\, J_0(kr)k^3 z^2 \left[K_2(kz) + \frac{K_1(kz_{\mathrm{IR}})}{I_1(kz_{\mathrm{IR}})} I_2(kz)\right]$$

(where $r = |x^\perp|$)

Results



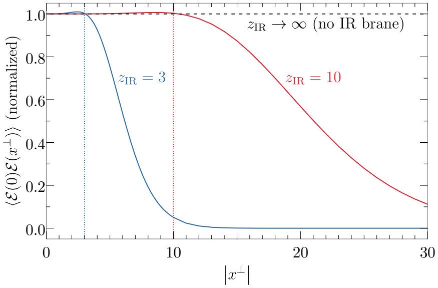
$\langle \mathcal{E}(0)\mathcal{E}(x^\perp) \rangle$ (normalized)
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
$|x^\perp|$
$z_{\rm IR} \to \infty$ (no IR brane)
$z_{\rm IR} = 3$
$z_{\rm IR} = 10$

Some comments

Manifest transition between regimes

Scales $\ll z_{\mathrm{IR}}$: constant, like strongly-coupled CFT

Scales $\gg z_{\mathrm{IR}}$: exponential decay



So what now?

First calculation of energy correlators in confining holographic model

Differences w/ QCD: no asymptotic freedom, no jets, etc.

Generalize to more sophisticated geometries:

- soft-wall confinement (WIP w/ S. Ferrante, stay tuned)
- asymptotic freedom (WIP w/ L. Kiriliuk)

[+ more speculative ideas (ask me later!)]

Thank you!

Photos: Jefferson Lab / Wikimedia Commons / CERN

more info:

arxiv.org/abs/2403.12123

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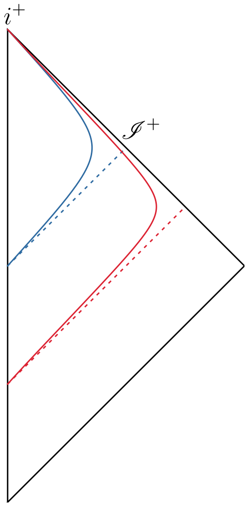
A caveat about limits

Be **careful** about the order of limits!

- $x^- \to \pm\infty$ (integrate over time)
- $x^+ \to \infty$ (calorimeter to boundary)

Only important for gapped theories



Inversion details

Avoid large-r limit with a conformal transformation:

$$x^+ \to -1/x^+, x^- \to x^- - \left|x^\perp\right|^2/x^+, x^\perp \to x^\perp/x^+$$

mapping $x^+ = \infty$ to $x^+ = 0$, $ds^2 \to ds^2/(x^+)^2$

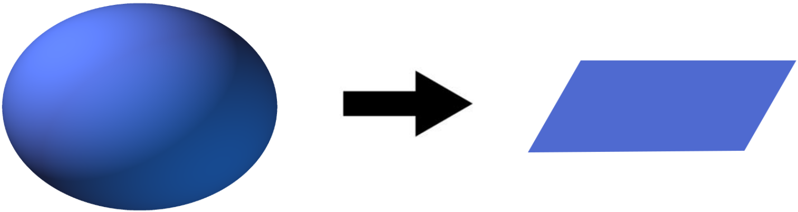
Energy flow calculated as

$$\mathcal{E}(\vec{n}) = \left(1 + \left|x^\perp\right|^2\right)^3 \int_{-\infty}^{\infty} dx^- \, T_{--}(x^+ = 0, x^-, x^\perp)$$

with mapping to celestial sphere: $x^1 + ix^2 = e^{i\phi} \tan\theta/2$

Mapping sphere to plane

How to generalize 4D transformation?



Isometry:

$$x^+ \to -1/x^+, x^- \to x^- - (|x^\perp|^2 + z^2)/x^+, x^\perp \to x^\perp/x^+,$$
$$z \to z/x^+$$

Now we can evaluate $\mathcal{E}$ on the null plane

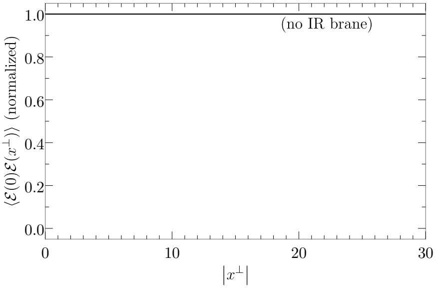
Holographic correlators

Scalar source $\Leftrightarrow$ bulk scalar ϕ

momentum $q^\mu = (q, \vec{0}) \Leftrightarrow$ BC $\phi(z=0) = e^{iqt}$

ϕ gets a "kick" at the shockwave(s):

$\Rightarrow \langle \mathcal{E} \rangle \sim 1$ (constant)
$\Rightarrow \langle \mathcal{E}\mathcal{E} \rangle \sim 1$



Scalar wavefunction

From AdS propagator: $\phi \sim z^2 J_2(qz)$

Conformal inversion + future null ∞:

Scalar source on the boundary

$$\phi|_{x^+=0} \sim e^{iqx^-/2}\delta^2(x^\perp)\delta(z-1)$$

(Can see by embedding AdS in $\mathbb{R}^{4,2}$ — ask me after)

Adding a shockwave

Scalar EOM modified to

$$\partial_- \partial_+ \phi + \epsilon \delta(x^+) f(x^\perp, z) \partial_-^2 \phi + \text{regular terms} = 0$$

Integrate over the δ $\Rightarrow$ ϕ gets a discontinuity:

$$\lim_{\delta \to 0} \partial_- \phi(x^+ = \delta) = e^{-\epsilon f(x^\perp, z) \partial_-} \partial_- \phi(x^+ = -\delta)$$

Recipe for correlators

One point: $\langle e^{\epsilon \mathcal{E}(y^\perp)} \rangle$

$$\sim \lim_{x^+ \to 0} \int \frac{dz}{z^3} d^2x^\perp dx^- \phi^* e^{-\epsilon \left(1+(y^\perp)^2\right)^3 f(x^\perp - y^\perp, z)\partial_-} \partial_- \phi$$

Two point (insert another shockwave, expand in ϵ):
$\langle \mathcal{E}(0)\mathcal{E}(y^\perp) \rangle$

$$\sim \lim_{x^+ \to 0} \int \frac{dz}{z^3} d^2x^\perp dx^- \phi^* f(x^\perp, z) \left(1 + (y^\perp)^2\right)^3 f(x^\perp - y^\perp, z) \partial_-^3 \phi$$

Scalar source

Scalar source, momentum $q^\mu = (q, \vec{0})$

$\Rightarrow$ bulk scalar ϕ with BC $\phi(z=0) = e^{iqt}$

ϕ gets a "kick" at the shockwave:

$$\lim_{\delta \to 0} \partial_- \phi(x^+ = \delta, x^-, x^\perp, z) = e^{-\epsilon f(x^\perp, z)\partial_-} \partial_- \phi(x^+ = -\delta, x^-, x^\perp, z)$$

(away from shockwave, usual AdS evolution)

Holographic correlators

One point: $\langle e^{\epsilon \mathcal{E}(y^\perp)} \rangle \sim$
$$\int \frac{dz}{z^3} d^2x^\perp dx^- \phi^* \exp\left[-\epsilon\left(1+(y^\perp)^2\right)^3 f(x^\perp - y^\perp, z)\partial_-\right]\partial_-\phi\Big|_{x^+=0}$$

Expanding in ϵ, find $\langle \mathcal{E} \rangle \sim 1$ (constant)

Higher-point functions from inserting more shockwaves, e.g. $\langle \mathcal{E}\mathcal{E} \rangle \sim 1$

Wavefunction at the shock

EOM:

$$\partial_-\partial_+\phi + \epsilon\delta(x^+) f(x^\perp, z)\partial_-^2\phi + \text{terms regular at shockwave} = 0,$$

Discontinuity:

$$\lim_{\delta\to 0} \partial_-\phi(x^+ = \delta, x^-, x^\perp, z) = e^{-\epsilon f(x^\perp, z)\partial_-}\partial_-\phi(x^+ = -\delta, x^-, x^\perp, z)$$

Inversion with a brane

Mapping of celestial sphere to null plane:

$$x^+ \to -1/x^+, x^- \to x^- - \left(\left|x^\perp\right|^2 + z^2\right)/x^+, x^\perp \to x^\perp/x^+,$$
$$z \to z/x^+$$

This moves the brane to $z = x^+ z_{\mathrm{IR}}$

Brane location behaves covariantly under transformation

Two-point correlator, done wrong

Recall $\langle \mathcal{E}(0)\mathcal{E}(y^\perp)\rangle$

$$\sim \lim_{x^+\to 0} \int \frac{dz}{z^3} d^2x^\perp dx^- \phi^* f(x^\perp, z) \left(1 + (y^\perp)^2\right)^3 f(x^\perp - y^\perp, z) \partial_-^3 \phi$$

For source, high-energy limit $q z_{\mathrm{IR}} \gg 1$:

$$\phi|_{x^+=0} \sim e^{iqx^-/2} \delta^2(x^\perp)\delta(z-1)$$

But $z \in (0, x^+ z_{\mathrm{IR}})$, so no support for ϕ

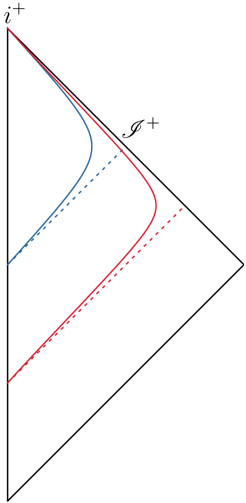
$\Rightarrow \langle \mathcal{E}\mathcal{E}\rangle$ vanishes?!

I warned you!

We sent $x^+ \to 0$ before doing x^- integral

At small x^+:
$\phi \sim \delta^2(x^\perp)\delta(z - \sqrt{1 + x^+x^-})e^{iq(z-1)/x^+}$

Has support for
$x^- \in (-1/x^+, -1/x^+ + x^+ z_{\rm IR}^2)$



Two-point correlator, done right

$x^\perp, z$ integrals are trivial now: $\langle \mathcal{E}(0)\mathcal{E}(y^\perp)\rangle$

$$\sim \lim_{x^+\to 0} \int_{-1/x^+}^{-1/x^+ + x^+ z_{\mathrm{IR}}^2} \frac{dx^-}{z^3} f(0,z)\left(1+(y^\perp)^2\right)^3 f(y^\perp, z)$$

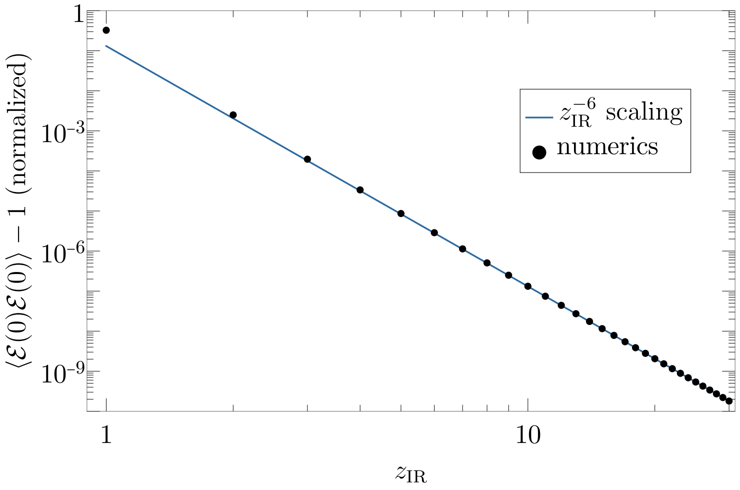
$$\times\, e^{-iq(z-1)/x^+}\partial_-^3\left(e^{-iq(z-1)/x^+}\right)\Big|_{z=\sqrt{1+x^+x^-}}.$$

Now we can go to null infinity:

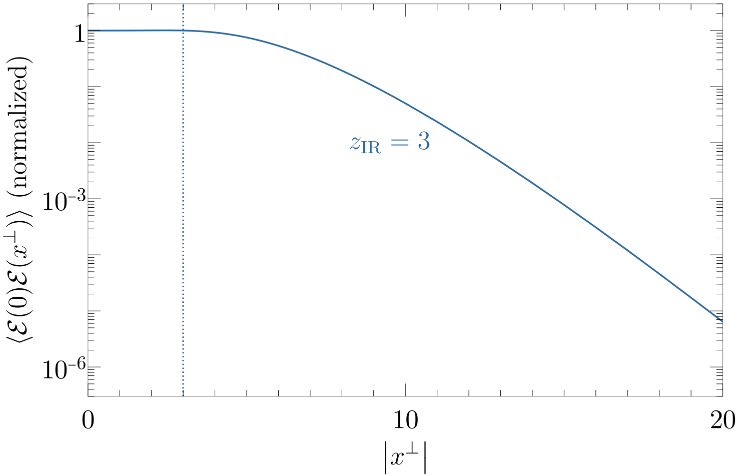
Two-point correlator

$$\langle \mathcal{E}(0)\mathcal{E}(y^\perp)\rangle \sim \left(1+(y^\perp)^2\right)^3 f(y^\perp, z=1)$$

More figures 1



More figures 2



General warped background

Metric $ds^2 = e^{-2A(z)}(dx^\mu dx_\mu - dz^2)$

Include bulk scalar to stabilize the metric (solve EFEs)

Warp factor

$$A = \log z + w(z/z_{\rm IR})$$

Asymptotically AdS → $\log z$

IR deformation (confinement at $1/z_{\rm IR}$) → $w(z/z_{\rm IR})$

From the sphere to the plane

Recall mapping:

$$x^+ \to -1/x^+, x^- \to x^- - (\left|x^\perp\right|^2 + z^2)/x^+, x^\perp \to x^\perp/x^+,$$
$$z \to z/x^+$$

Still an isometry if $z_{\text{IR}} \to z_{\text{IR}}/x^+$

(Same idea as hard-wall: confinement scale transforms covariantly)

Shockwaves

Modify metric, $\delta ds^2 = \epsilon e^{-2A(z)} f(x^\perp, z)(dx^+)^2$

Remarkably, field equations are linear

Shockwave equation

$$\left(3A'(z)\partial_z - \partial_z^2 - \partial_1^2 - \partial_2^2\right) f = 0$$

Everything carries over from RS case

(shockwave [+ source] $\rightarrow$ correlators)

Example 1: linear dilaton / gapped continuum

Warp factor: $A(z) = \log z + \mu z$ (confinement scale $\sim \mu$)

IR spectrum is gapped continuum

$$f(z,k) =$$

$$\frac{3k^2 z^{3/2}\left(k^2 - 18\mu^2\right) e^{\frac{3\mu z}{2}} \Gamma\left(\frac{9\mu}{2\sqrt{4k^2+9\mu^2}} - \frac{3}{2}\right) W_{-\frac{9\mu}{2\sqrt{4k^2+9\mu^2}},2}\left(z\sqrt{4k^2+9\mu^2}\right)}{\left(4k^2+9\mu^2\right)^{5/4}}$$

Example 2: quadratic dilaton / linear confinement

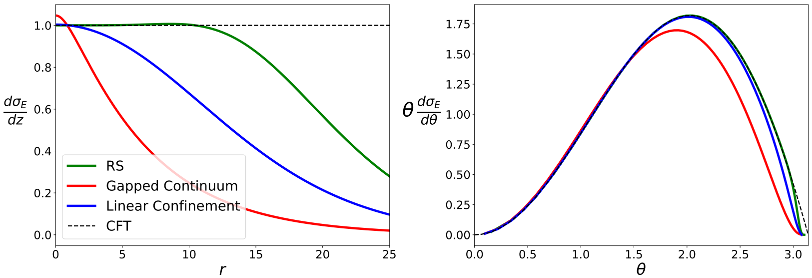
Warp factor: $A(z) = \log z + \mu^2 z^2/2$

IR spectrum has linear Regge trajectories $m_n^2 \sim n$

$$f(z,k) =$$

$$\frac{k^2\left(k^2+6\mu^2\right)\Gamma\left(\frac{k^2}{6\mu^2}\right)U\left(\frac{k^2}{6\mu^2},-1,\frac{3z^2\mu^2}{2}\right)}{18\mu^4}$$

Results



1.0
0.8
$\frac{d\sigma_E}{dz}$ 0.6
0.4
0.2
0.0
0
5
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25
r
RS
Gapped Continuum
Linear Confinement
CFT
1.75
1.50
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$\theta \frac{d\sigma_E}{d\theta}$ 1.00
0.75
0.50
0.25
0.00
0.0
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2.0
2.5
3.0
θ

Comments and outlook

Not obvious *a priori* this calculation was possible!

Models distinguished in back-to-back limit

Open problems:

- Why back-to-back (and not collinear)?
- Incorporate asymptotic freedom (working on it)