What is the Geometry of Effective Field Theories?

Zhengkang "Kevin" Zhang (University of Utah)

PIKIMO 17, University of Michigan, Nov 16, 2024

1st PIKIO meeting

Saturday Feb 27, 2016, 9:30 AM → 5:00 PM EST

Braunstein 301 (U. Cincinnati)

Description Particle physics in Indiana, Kentucky, Illinois and Ohio

Restaurants close t...

9:30 AM → 10:00 AM	**Coffee**		30m
10:00 AM → 11:15 AM	**Neutral Naturalness**	**Speaker**: Zackaria Chacko	1h 15m
11:15 AM → 12:30 PM	**Dark Photons from the Center of the Earth**	**Speaker**: Flip Tanedo	1h 15m
12:30 PM → 2:00 PM	**lunch**		1h 30m
2:00 PM → 2:25 PM	**Decay of high-energy astrophysical neutrinos: present and near future**	**Speaker**: Mauricio Bustamante	25m
2:25 PM → 2:50 PM	**Muon-induced backgrounds for MeV neutrinos**	**Speaker**: Shirley Li	25m
2:50 PM → 3:15 PM	**Sterile neutrino dark matter from freeze-in**	**Speaker**: Bibhushan Shakya	25m
3:15 PM → 3:45 PM	**coffee**		30m
3:45 PM → 4:10 PM	**Split Coupling SUSY**	**Speaker**: Yue Zhao	25m
4:10 PM → 4:35 PM	**Effective field theories vs. oblique parameters in precision analyses**	**Speaker**: Zhengkang Zhang	25m
4:35 PM → 5:00 PM	**Displaced Leptons at the LHC**	**Speaker**: Jared Evans	25m

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Effective field theories vs. oblique parameters in precision electroweak analyses

Zhengkang Zhang
University of Michigan

based on 1510.08462, 1512.03056 w/ J. Wells

Oblique parameters

Prototype of modern EFT analyses of precision data.

Milestone

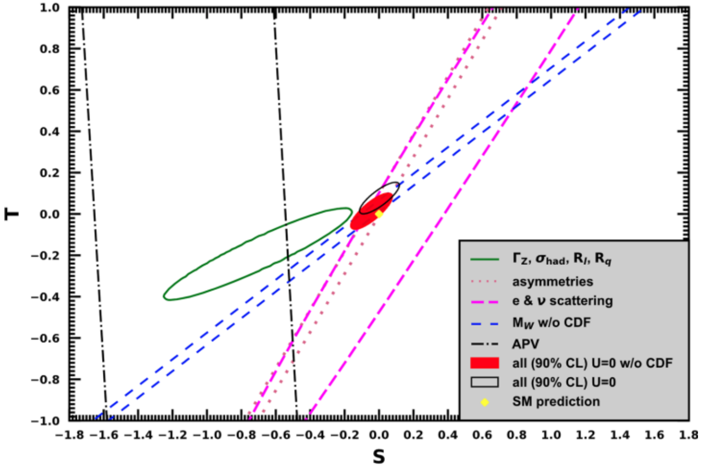
Estimation of oblique electroweak corrections

Michael E. Peskin and Tatsu Takeuchi
Phys. Rev. D **46**, 381 – Published 1 July 1992

An article within the collection: *Physical Review D* 50th Anniversary Milestones

$$\hat{S} = -\frac{c_\theta}{s_\theta}\,\Pi'_{3B}(0)$$

$$\hat{T} = \frac{1}{m_W^2}\left[\Pi_{WW}(0) - \Pi_{33}(0)\right]$$



[Particle Data Group]

Precision ⇒ virtual effects of BSM physics.

Let's look at a simple example...

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{c_B}{\Lambda^2}\frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right)\left(\partial^\nu B_{\mu\nu}\right) \qquad \Rightarrow \qquad \hat{S} = -\frac{c_\theta}{s_\theta}\,\Pi'_{3B}(0) = c_B\,\frac{m_W^2}{\Lambda^2}$$

However, if we redefine:

EOM: $-\partial^\nu B_{\mu\nu} + \frac{ig'}{2}H^\dagger \overleftrightarrow{D}^\mu H + g'\sum_f Y_f\,\bar{f}\gamma_\mu f$

$$B_\mu \to B_\mu + \frac{c_B}{\Lambda^2}\frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \qquad \Rightarrow \qquad \mathcal{L} \to \mathcal{L} + \left(\frac{\delta\mathcal{L}}{\delta B_\mu}\right)\frac{c_B}{\Lambda^2}\frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

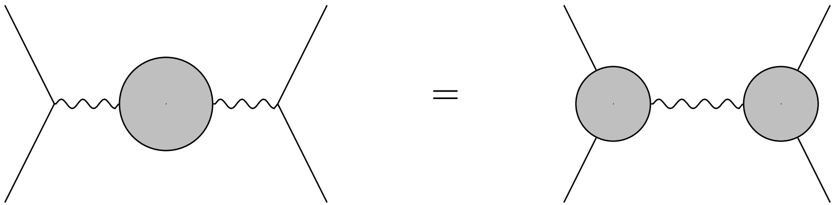
$$= \mathcal{L}_{\rm SM} - \frac{c_B}{\Lambda^2}\frac{g'^2}{4}\left(H^\dagger \overleftrightarrow{D}^\mu H\right)^2 + \frac{c_B}{\Lambda^2}\frac{ig'^2}{2}\sum_f Y_f\left(H^\dagger \overleftrightarrow{D}^\mu H\right)\left(\bar{f}\gamma_\mu f\right) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Neither of the new operators contribute to $\Pi'_{3B}(0) \quad \Rightarrow \quad \hat{S} = 0\,??$

In an EFT, **field redefinitions** relate different **operator bases**

Operators contributing to $\Pi'_{3B}(0)$

Operator	Warsaw	EGGM	SILH
$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W^a_{\mu\nu}$	$\times$		
$\mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}_\mu H) \partial^\nu B_{\mu\nu}$	$\times$		
$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$	$\times$	$\times$	
$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$\times$	$\times$	
$\mathcal{O}_{WW} = g^2 \|H\|^2 W^a_{\mu\nu} W^{a\mu\nu}$	$Q_{HW} = \|H\|^2 W^a_{\mu\nu} W^{a\mu\nu}$		$\times$
$\mathcal{O}_{WB} = gg' H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu}$	$Q_{HWB} = H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu}$		$\times$
$\mathcal{O}_{BB} = g'^2 \|H\|^2 B_{\mu\nu} B^{\mu\nu}$	$Q_{HB} = \|H\|^2 B_{\mu\nu} B^{\mu\nu}$		



SILH basis = Warsaw basis

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Effective theories of universal theories

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Abstract: It is well-known but sometimes overlooked that constraints on the oblique parameters (most notably S and T parameters) are generally speaking only applicable to a special class of new physics scenarios known as universal theories. In the effective field theory (EFT) framework, the oblique parameters should not be associated with Wilson coefficients in a particular operator basis, unless restrictions have been imposed on the EFT so that it describes universal theories. We work out these restrictions, and present a detailed EFT analysis of universal theories. We find that at the dimension-6 level, universal theories are completely characterized by 16 parameters. They are conveniently chosen to be: 5 oblique parameters that agree with the commonly-adopted ones, 4 anomalous triple-gauge couplings, 3 rescaling factors for the h^3, hff, hVV vertices, 3 parameters for hVV vertices absent in the Standard Model, and 1 four-fermion coupling of order y_f^2. All these parameters are defined in an unambiguous and basis-independent way, allowing for consistent constraints on the universal theories parameter space from precision electroweak and Higgs data.

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Effective field theories vs. oblique parameters in precision electroweak analyses

Zhengkang Zhang
University of Michigan

based on 1510.08462, 1512.03056 w/ J. Wells

Theory parameters depend on the **basis**...

Physical observables don't.

A QFT homework example [Srednicki] :

$$\mathcal{L} = -\tfrac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \tfrac{1}{2}m^2\varphi^2 - 2\lambda\varphi\partial^\mu\varphi\partial_\mu\varphi - \lambda m^2\varphi^3 - 2\lambda^2\varphi^2\partial^\mu\varphi\partial_\mu\varphi - \tfrac{1}{2}\lambda^2 m^2\varphi^4 \; .$$

$$\begin{aligned}\mathbf{V}_3 &= (-2i\lambda)(k_1^2+k_2^2+k_3^2) - 6i\lambda m^2 \\ &= (-2i\lambda)[(k_1^2+m^2)+(k_2^2+m^2)+(k_3^2+m^2)] \;,\end{aligned}$$

$$\begin{aligned}\mathbf{V}_4 &= (-2i\lambda^2)(2!)(k_1^2+k_2^2+k_3^2+k_4^2) - 12i\lambda m^2 \\ &= (-4i\lambda^2)[(k_1^2+m^2)+\ldots+(k_4^2+m^2)] + 4i\lambda^2 m^2 \;,\end{aligned}$$

$\varphi\varphi \to \varphi\varphi$ amplitude:

$$\begin{aligned}i\mathcal{T} &= [(-2i\lambda)(-s+m^2)]^2\,\frac{1}{i}\,\frac{1}{-s+m^2} + (s\to t) + (s\to u) + 4i\lambda^2 m^2 \\ &= 4i\lambda^2[(-s+m^2)+(-t+m^2)+(-u+m^2)+m^2] \\ &= 4i\lambda^2(-s-t-u+4m^2) \\ &= 0 \;.\end{aligned}$$

But this is in fact a free theory: $\widetilde{\varphi} = \varphi + \lambda\varphi^2 \quad \Rightarrow \quad \mathcal{L} = -\frac{1}{2}\partial^\mu\widetilde{\varphi}\partial_\mu\widetilde{\varphi} - \frac{1}{2}m^2\widetilde{\varphi}^2$

(Lagrangian formulation of) QFT has a huge amount of **redundancy**.

Not unique to QFT – true for **any** theory where we have to pick a coordinate system to calculate the physics.

A classical mechanics example: 2D harmonic oscillator.

$$\begin{aligned} L &= \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\omega^2\left(x^2 + y^2\right) \\ &= \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{1}{2}\omega^2 r^2 \end{aligned}$$

$$\begin{cases} x(r,\theta) = r\cos\theta \\ y(r,\theta) = r\sin\theta \end{cases}$$

We know this as a "coordinate transformation."

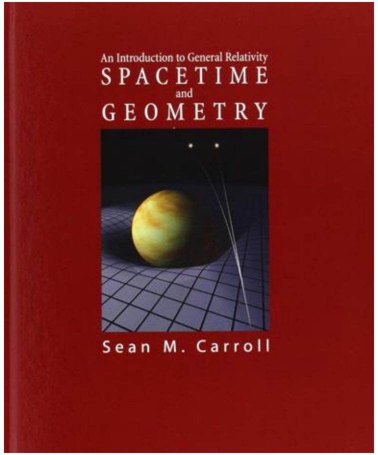
A more advanced example: general relativity.

$x^\mu \to \tilde{x}^\mu(x)$: coordinate transformation on the spacetime manifold.

Geometry of spacetime $\Rightarrow$ Physics is covariant.

$$R_{\mu\nu} = 8\pi G\, T_{\mu\nu} \quad \to \quad \tilde{R}_{\mu\nu} = 8\pi G\, \tilde{T}_{\mu\nu}$$

w/ $\tilde{R}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} R_{\alpha\beta}$, $\tilde{T}_{\mu\nu} = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} T_{\alpha\beta}$



Compare field redefinition in EFT.

$\phi \to \tilde{\phi}(\phi)$: coordinate transformation on the field manifold.

What is the geometry of EFTs?

Outline

- $\phi \to \tilde{\phi} = f(\phi)$: field space geometry.

- $\phi \to \tilde{\phi} = f\left(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, \ldots\right) = F[\phi]$: functional geometry.*

- Thoughts (and dreams).

* There is also a complementary approach to accommodate derivative field redefinitions utilizing Lagrange geometry and jet bundles: [Craig, Lee, Lu, Sutherland, 2305.09722] [Craig, Lee, 2307.15742] [Alminawi, Brivio, Davighi, 2308.00017] [Lee, 2410.21395].

Field manifold

[(Callan,) Coleman, Wess, Zumino] [Honerkamp]
[Volkov] [Tataru] [Alvarez-Gaume, Freedman(, Mukhi)]
[Vilkovisky] [DeWitt] [Gaillard] [Georgi]

[Alonso, Jenkins, Manohar, 1511.00724 + 1605.03602]

N_{flavor} dimensional, charted by ϕ^i (i = 1, … , N_{flavor}).

Coordinate transformation: $\phi^i = \phi^i(\tilde{\phi})$

$$\mathcal{L} = -V(\phi) + \frac{1}{2}\, g_{ij}(\phi)\left(\partial_\mu \phi^i\right)\left(\partial^\mu \phi^j\right) + \mathcal{O}(\partial^4) = -\tilde{V}(\tilde{\phi}) + \frac{1}{2}\, \tilde{g}_{ij}(\tilde{\phi})\left(\partial_\mu \tilde{\phi}^i\right)\left(\partial^\mu \tilde{\phi}^j\right) + \mathcal{O}(\partial^4)$$

$\tilde{V}(\tilde{\phi}) = V\big(\phi(\tilde{\phi})\big)$: (0,0) tensor, a.k.a. scalar.

$\partial_\mu \tilde{\phi}^i = \dfrac{\partial \tilde{\phi}^i}{\partial \phi^k}\, \partial_\mu \phi^k$: (1,0) tensor, a.k.a. vector.

$\tilde{g}_{ij}(\tilde{\phi}) = \dfrac{\partial \phi^k}{\partial \tilde{\phi}^i} \dfrac{\partial \phi^l}{\partial \tilde{\phi}^j}\, g_{kl}\big(\phi(\tilde{\phi})\big)$: (0,2) tensor → identify as metric.

Riemannian geometry on the field manifold

$$\mathcal{L} = -V(\phi) + \frac{1}{2}\, g_{ij}(\phi)\left(\partial_\mu \phi^i\right)\left(\partial^\mu \phi^j\right) + \mathcal{O}(\partial^4)$$

Metric: g_{ij} .

Connection: $\Gamma^i_{jk} = \frac{1}{2} g^{il}\left(g_{lj,k} + g_{lk,j} - g_{jk,l}\right)$.

Riemann curvature: $R_{ijkl} = \frac{1}{2}\left(g_{il,jk} - g_{jl,ik}\right) - g_{mn}\,\Gamma^m_{ik}\,\Gamma^n_{jl} - (k \leftrightarrow l)$.

Covariant derivative: $\nabla_k T^{i\ldots}{}_{j\ldots} = \partial_k T^{i\ldots}{}_{j\ldots} + \left(\Gamma^i_{kl}\, T^{l\ldots}{}_{j\ldots} + \cdots\right) - \left(\Gamma^l_{kj}\, T^{i\ldots}{}_{l\ldots} + \cdots\right)$.

Physics is invariant under change of coordinates

On-shell amplitudes = scalar functions of geometric quantities.

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)(\partial_\mu \phi^i)(\partial^\mu \phi_j) - V(\phi)$$

[Cohen, Lu, Craig, Sutherland, 2108.03240]
[Cheung, Helset, Parra-Martinez, 2111.03045]

$\Downarrow$ standard Feynman rules

$$\mathcal{A}^{\alpha_1\alpha_2\alpha_3} = \left(\prod_{a=1}^{3} e^{\alpha_a i_a}\right) \overline{V}_{;(i_1 i_2 i_3)}$$

( ) = symmetrized

bar = evaluate at physical vacuum (point on field manifold)

$$\mathcal{A}^{\alpha_1\alpha_2\alpha_3\alpha_4} = \left(\prod_{a=1}^{4} e^{\alpha_a i_a}\right) \left[\overline{V}_{;(i_1 i_2 i_3 i_4)} + \frac{1}{3}\left(s_{12}\,\overline{R}_{i_1(i_3 i_4) i_2}\right)_{6\ \text{terms}} + \left(\overline{V}_{;(i_1 i_2 j)} \frac{1}{s_{12}\overline{g}_{jk} - \overline{V}_{;jk}} \overline{V}_{;(i_3 i_4 k)}\right)_{3\ \text{terms}} \right]$$

vielbeins (wavefunction renormalization factors in LSZ)

Many applications to theory & pheno

Geometric soft theorems. [Cheung, Helset, Parra-Martinez, 2111.03045] $\lim_{q\to 0} A_{n+1} \sim \left(\nabla + \frac{\nabla m^2}{p^2 - m^2}\right) A_n$

Resummation of $\left(\frac{v^2}{\Lambda^2}\right)^n$ effects in SMEFT. [Helset, Martin, Trott, 2001.01453]

Classification of SM extensions: SMEFT vs. HEFT. [Cohen, Lu, Craig, Sutherland, 2008.08597]

Figure from N. Craig @ HEFT 2021

Beyond tree level: effective actions

$$Z[J] = \int \mathcal{D}\varphi \sqrt{\det g}\, e^{i(S + J_i \varphi^i)} = e^{iW[J]}$$

Standard background field method:

$$\varphi^i = \varphi^i_{\rm b} + \eta^i \quad \text{w/} \quad \varphi^i_{\rm b} = \langle \varphi^i \rangle_J = \frac{\delta W}{\delta J_i} \qquad \Rightarrow \qquad S[\varphi] = S[\varphi_{\rm b}] + \eta^i \frac{\delta S}{\delta \varphi^i}[\varphi_{\rm b}] + \frac{1}{2}\eta^i \eta^j \frac{\delta^2 S}{\delta \varphi^j \delta \varphi^i}[\varphi_{\rm b}] + \cdots$$

Legendre transform $\Rightarrow$ 1PI effective action.

$$\Gamma[\varphi_{\rm b}] \equiv W\big[J[\varphi_{\rm b}]\big] - \varphi^i_{\rm b} J_i[\varphi_{\rm b}] = S[\varphi_{\rm b}] + \frac{i}{2}\,\mathrm{Tr}\log\left(-g^{ik}\frac{\delta^2 S}{\delta \varphi^j \delta \varphi^k}[\varphi_{\rm b}]\right) + \cdots$$

tree 1-loop

Beyond tree level: effective actions

Covariant version: $\xi^i = \frac{\mathrm{d}\gamma^i}{\mathrm{d}\lambda}(0)$

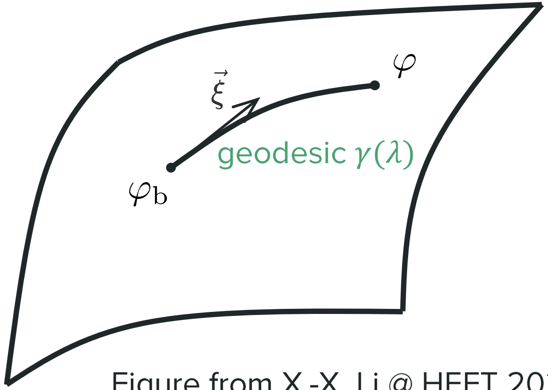


Figure from X.-X. Li @ HEFT 2024

$$S[\varphi] = S[\varphi_\mathrm{b}] + \xi^i\,\nabla_i S[\varphi_\mathrm{b}] + \frac{1}{2}\,\xi^i\xi^j\,\nabla_i\nabla_j S[\varphi_\mathrm{b}] + \cdots$$

$$Z_\mathrm{geo}[J] = \int \mathcal{D}\varphi\sqrt{\det g}\,e^{i(S+J_i(\varphi_\mathrm{b}^i+\xi^i))} = e^{iW_\mathrm{geo}[J]}$$

$\Rightarrow$ geometrized 1PI effective action:

$$\Gamma_\mathrm{geo}[\varphi_\mathrm{b}] = S[\varphi_\mathrm{b}] + \frac{i}{2}\,\mathrm{Tr}\log\big(-g^{ik}\,\nabla_j\nabla_k S[\varphi_\mathrm{b}]\big) + \cdots$$

$$-(\mathcal{D}^2)^i{}_j - R^i{}_{kjl}(\partial_\mu\varphi^k)(\partial^\mu\varphi^l) - V_;{}^i{}_j$$

Beyond tree level: effective actions

$$\Gamma_{\text{geo}}[\varphi_{\text{b}}] = S[\varphi_{\text{b}}] + \frac{i}{2}\,\text{Tr}\log\big(-g^{ik}\,\nabla_j\nabla_k S[\varphi_{\text{b}}]\big) + \cdots$$

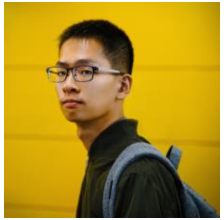
$$-(\mathcal{D}^2)^i{}_j - R^i{}_{kjl}(\partial_\mu\varphi^k)(\partial^\mu\varphi^l) - V_{;}{}^i{}_j$$

$\frac{1}{\epsilon_{UV}}$ terms $\Rightarrow$ RGE in terms of geometric objects (new insights on selection rules).

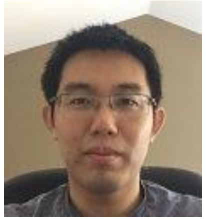
[Helset, Jenkins, Manohar, 2212.03253] [Assi, Helset, Manohar, Pagès, Shen, 2307.03187]
[Jenkins, Manohar, Naterop, Pagès, 2308.06315 + 2310.19883]

+ finite terms $\Rightarrow$ matching UV theory to EFT.

[Li, Lu, **ZZ**, 2411.04173]



Xu-Xiang Li (U. Utah)



Xiaochuan Lu (UCSD)

Geometrizing EFT matching

$\varphi^i = \{\Phi^A, \phi^a\}$ with $M_\Phi \gg m_\phi$.

Integrate out a subset of coordinates $\{\Phi^A\}$

$\Rightarrow$ EFT on the submanifold spanned by $\{\phi^a\}$.

Tree level: EOM. $\mathcal{L}^{[0]}_{\rm EFT}[\phi] = \mathcal{L}\big[\Phi_{\rm c}[\phi], \phi\big]$

scalar on EFT submanifold

$$\Phi^A_{\rm c} = -\frac{1}{M^2}\,\delta^{AB}\left[W_{,B} + g_{Bj}\big(\mathcal{D}^\mu(\partial_\mu\varphi)\big)^j\right]\Big|_{\Phi=\Phi_{\rm c}}$$

$$W(\varphi) = V(\varphi) - \frac{1}{2}\,M^2\,\delta_{AB}\,\Phi^A\Phi^B$$

Geometrizing EFT matching

1-loop level: use covariant derivative expansion (CDE).

[Gaillard] [Chan] [Cheyette] [Henning, Lu, Murayama, 1412.1837]

~ intrinsic curvature of EFT submanifold

$$\mathcal{L}_{\text{EFT}}^{[1]} = -\frac{i}{2}\int \mathrm{d}M^2 \int \frac{\mathrm{d}^d q}{(2\pi)^d}\, \mathrm{tr}\left[\sum_{m=0}^{\infty} \frac{1}{q^2-\mathbf{M}^2}\left\{\left[\{q^\mu, \widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu\} - (\widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu)^2\right]\frac{1}{q^2-\mathbf{M}^2}\right\}^m\right]\Bigg|_{\Phi=\Phi_c[\phi]}$$

$$-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int \frac{\mathrm{d}^d q}{(2\pi)^d}\, \mathrm{tr}\Bigg[\sum_{m=0}^{\infty} \frac{1}{q^2-\mathbf{M}^2}\left\{\left[\{q^\mu, \widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu\} - (\widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu)^2\right]\frac{1}{q^2-\mathbf{M}^2}\right\}^m$$

$$\times\left(\widetilde{\mathcal{U}} - \widetilde{\mathcal{K}}_\mu\widetilde{\mathcal{K}}^\mu + \left\{q_\mu - \widetilde{\mathcal{G}}'_{\mu\nu}\partial_q^\nu, \widetilde{\mathcal{K}}^\mu\right\}\right)\Bigg]^n\Bigg|_{\Phi=\Phi_c[\phi]}$$

~ (potential + intrinsic curvature)

~ extrinsic curvature

Universal One-Loop Effective Action (UOLEA)

"For ***any*** UV theory of the form $\mathcal{L}_{UV} = \cdots$, the 1-loop EFT is $\mathcal{L}_{EFT}^{[1]} = \cdots$"

1st version: [Henning, Lu, Murayama, 1412.1837]

$$\mathcal{L}_{UV}[\Phi,\phi] = \mathcal{L}[\phi] + \frac{1}{2}\left(D_\mu\Phi\right)^2 - \frac{1}{2}\left(M^2 + U[\phi]\right)\Phi^2 + \mathcal{O}(\Phi^3) \quad \Rightarrow$$

non-derivative interactions

$$\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\text{tr}\Bigg\{
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2}-\frac{3}{2}\right)\right]\\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2}-1\right)U\right]\\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2}-1\right)G'^2_{\mu\nu}-\frac{1}{2}\log\frac{m^2}{\mu^2}U^2\right]\\
&+ \frac{1}{m^2}\left[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2-\frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu}-\frac{1}{12}(P_\mu U)^2-\frac{1}{6}U^3-\frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right]\\
&+ \frac{1}{m^4}\Bigg[\frac{1}{24}U^4+\frac{1}{12}U\left(P_\mu U\right)^2+\frac{1}{120}\left(P^2U\right)^2+\frac{1}{24}\left(U^2G'_{\mu\nu}G'_{\mu\nu}\right)\\
&\qquad -\frac{1}{120}\left[(P_\mu U),(P_\nu U)\right]G'_{\mu\nu}-\frac{1}{120}\left[U[U,G'_{\mu\nu}]\right]G'_{\mu\nu}\Bigg]\\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5-\frac{1}{20}U^2\left(P_\mu U\right)^2-\frac{1}{30}\left(UP_\mu U\right)^2\right]\\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned} \tag{2.54}$$

Universal One-Loop Effective Action (UOLEA)

"For ***any*** UV theory of the form $\mathcal{L}_{UV} = \cdots$, the 1-loop EFT is $\mathcal{L}_{EFT}^{[1]} = \cdots$ "

Geometric UOLEA: [Li, Lu, **ZZ**, 2411.04173]

$$\mathcal{L}_{UV}[\Phi, \phi] = \frac{1}{2}\, g_{ij}(\varphi)\big(\partial_\mu \varphi^i\big)\big(\partial^\mu \phi^j\big) - V(\varphi)$$

$$\Rightarrow \mathcal{L}_{EFT}^{[1]} = \frac{1}{16\pi^2} \sum_i c_i \operatorname{tr} \mathcal{O}_i$$

derivative + non-derivative interactions

built from tensors on EFT submanifold (manifestly covariant)

$\mathcal{O}(\mathcal{K}^2\mathcal{G}')$ terms	
$\frac{1}{18}(11-6L)$	$(\mathcal{K}^\mu)_{HL}(\mathcal{G}'_{\mu\nu})_{LL}(\mathcal{K}^\nu)_{LH}$
$\mathcal{O}(\mathcal{K}^2\mathcal{P}'^2)$ terms	
$-\frac{1}{18}(4-3L)$	$[\mathcal{P}'_\mu, \mathcal{K}^\mu]_{HL}[\mathcal{P}'_\nu, \mathcal{K}^\nu]_{LH}$
$\frac{1}{36}(5-6L)$	$[\mathcal{P}'^\nu, \mathcal{K}^\mu]_{HL}[\mathcal{P}'_\nu, \mathcal{K}_\mu]_{LH}$
$\mathcal{O}(\mathcal{U}\mathcal{K}\mathcal{G}'\mathcal{P}')$ terms	
$-\frac{1}{36M^2}(17-6L)$	$(\mathcal{K}_\nu)_{HL}(\mathcal{G}'^{\mu\nu})_{LL}[\mathcal{P}'_\mu,\mathcal{U}]_{LH} + [\mathcal{P}'_\mu,\mathcal{U}]_{HL}(\mathcal{G}'^{\mu\nu})_{LL}(\mathcal{K}_\nu)_{LH}$
$-\frac{1}{36M^2}(5-6L)$	$[\mathcal{P}'_\mu, \mathcal{K}_\nu]_{HL}(\mathcal{G}'^{\mu\nu})_{LL}\mathcal{U}_{LH} + \mathcal{U}_{HL}(\mathcal{G}'^{\mu\nu})_{LL}[\mathcal{P}'_\mu, \mathcal{K}_\nu]_{LH}$
$\mathcal{O}(\mathcal{U}\mathcal{K}\mathcal{P}'^3)$ terms	
$-\frac{1}{12M^2}$	$[\mathcal{P}'_\mu, \mathcal{K}^\mu]_{HL}\big[\mathcal{P}'_\nu, [\mathcal{P}'^\nu, \mathcal{U}]\big]_{LH} - \big[[\mathcal{P}'_\nu, [\mathcal{P}'^\nu, \mathcal{U}]\big]_{HL}[\mathcal{P}'_\mu, \mathcal{K}^\mu]_{LH}$
$\mathcal{O}(\mathcal{U}^2\mathcal{G}'^2)$ terms	
$\frac{1}{72M^4}(5-6L)$	$\mathcal{U}_{HL}(\mathcal{G}'_{\mu\nu})_{LL}(\mathcal{G}'^{\mu\nu})_{LL}\mathcal{U}_{LH}$
$\mathcal{O}(\mathcal{U}^2\mathcal{G}'\mathcal{P}'^2)$ terms	
$-\frac{1}{36M^4}(17-6L)$	$[\mathcal{P}'_\mu,\mathcal{U}]_{HL}(\mathcal{G}'^{\mu\nu})_{LL}[\mathcal{P}'_\nu,\mathcal{U}]_{LH}$
$\mathcal{O}(\mathcal{U}^2\mathcal{P}'^4)$ terms	
$\frac{1}{12M^4}$	$\big[\mathcal{P}'_\mu, [\mathcal{P}'^\mu, \mathcal{U}]\big]_{HL}\big[\mathcal{P}'_\nu, [\mathcal{P}'^\nu, \mathcal{U}]\big]_{LH}$

(+ a few more tables)

Outline

- $\phi \to \tilde{\phi} = f(\phi)$: field space geometry.

  Geometry on the field manifold; geometrizing EFT matching.

- $\phi \to \tilde{\phi} = f\left(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, \dots\right) = F[\phi]$: functional geometry.

- Thoughts (and dreams).

Upgrade field manifold → functional manifold

Charted by: $\{\phi^i\}$ → $\{\phi^i(x)\}$ or $\{\phi^i(p)\}$

N_{flavor} → $N_{\text{flavor}} \times N_{\text{site}}$ $(= \infty)$ dimensional

Coordinate transformations are:

nonderivative field redefinitions → general field redefinitions

$\phi = f(\tilde{\phi})$ $\qquad$ $\phi = f(\tilde{\phi}, \partial_\mu \tilde{\phi}, \partial_\mu \partial_\nu \tilde{\phi}, \dots) = F[\tilde{\phi}]$

[Cohen, Craig, Lu, Sutherland, 2202.06965] [Cohen, Lu, Sutherland, 2312.06748]

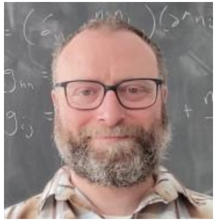
Geometry on the functional manifold

Starting point in field space geometry: $\partial_\mu \phi^i$ is a vector on the field manifold.

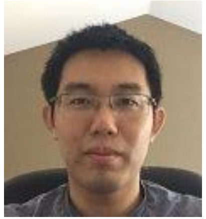
$$\mathcal{L} \supset \frac{1}{2}\, g_{ij}(\phi)\big(\partial_\mu \phi^i\big)\big(\partial^\mu \phi^j\big) \quad \Rightarrow \quad g_{ij}(\phi) \text{ is a (0,2) tensor } \rightarrow \text{ metric.}$$

It turns out the same logic extends to the functional manifold.

[Cohen, Lu, **ZZ**, 2410.21378]



Tim Cohen (CERN, EPFL & U. Oregon)



Xiaochuan Lu (UCSD)

Geometry on the functional manifold

"Field redefinitions $\phi = F[\tilde{\phi}]$ are the origin of operator redundancies in EFTs."

Local, translation-invariant

[Cohen, Lu, **ZZ**, 2410.21378]

$\mathcal{T}_\epsilon F[\phi] = F[\mathcal{T}_\epsilon \phi]$ where $\mathcal{T}_\epsilon \phi(x) = \phi(x+\epsilon) = \phi(x) + \epsilon^\mu \partial_\mu \phi(x) + \mathcal{O}(\epsilon^2)$

$\Rightarrow \quad \partial_\mu \phi^i(x) = \int \mathrm{d}^4 y \, \frac{\delta \phi^i(x)}{\delta \tilde{\phi}^j(y)} \, \partial_\mu \tilde{\phi}^j(y)$ again transforms as a vector

$$S = -\frac{1}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{d}^4 q}{(2\pi)^4} \boxed{g_{ij}(p,q)} p_\mu \phi^i(p) \, q^\mu \phi^j(q)$$

metric on the functional manifold

[Cheung, Helset, Parra-Martinez, 2202.06972]

Example: $\phi^3 + \phi^4$ theory

$$S = \int \mathrm{d}^4x \left[\frac{1}{2}\,(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}\,m^2\phi^2 - \frac{1}{6}\,\mu\,\phi^3 - \frac{1}{24}\,\lambda\,\phi^4\right] = -\frac{1}{2}\int \frac{\mathrm{d}^4p_1}{(2\pi)^4}\,\frac{\mathrm{d}^4p_2}{(2\pi)^4}\,g_{\phi\phi}(p_1,p_2)\,(p_1\cdot p_2)\,\phi(p_1)\,\phi(p_2)$$

Metric: $$g_{\phi\phi}(p_1,p_2) = (2\pi)^4\delta^4(p_{12})\left(1+\frac{m^2}{p_1p_2}\right) + \frac{1}{3}\mu\,\frac{1}{p_1p_2}\,\phi(-p_{12}) + \frac{1}{12}\lambda\,\frac{1}{p_1p_2}\int \frac{\mathrm{d}^4q_1}{(2\pi)^4}\,\frac{\mathrm{d}^4q_2}{(2\pi)^4}\,(2\pi)^4\delta^4(p_{12}+q_{12})\,\phi(q_1)\phi(q_2)$$

Connection: $$\overline{\Gamma}^a_{12} = -(2\pi)^4\delta^4(p_a - p_{12})\,\frac{p_a^2}{p_a^2-m^2}\,\frac{1}{6}\mu\left(\frac{1}{p_ap_1}+\frac{1}{p_ap_2}+\frac{1}{p_1p_2}\right)$$

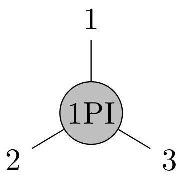
Riemann curvature: $$\overline{R}_{1234} = (2\pi)^4\delta^4(p_{1234})\Bigg[\frac{1}{12}\lambda\left(\frac{1}{p_1p_4}-\frac{1}{p_2p_4}\right) - \frac{1}{36}\mu^2\,\frac{p_{13}^2}{p_{13}^2-m^2}\left(\frac{1}{p_1p_{13}}+\frac{1}{p_3p_{13}}+\frac{1}{p_1p_3}\right)\left(\frac{1}{p_2p_{24}}+\frac{1}{p_4p_{24}}+\frac{1}{p_2p_4}\right)\Bigg] - (p_3\leftrightarrow p_4)$$

Amplitudes on the functional manifold

… can be built from the 1PI effective action $\Gamma[\phi]$.

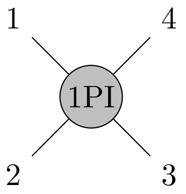
scalar on the functional manifold

$\mathcal{M}_{123} =$

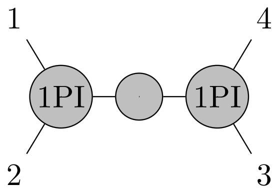


$= \dfrac{\delta^3\Gamma}{\delta\phi^1\delta\phi^2\delta\phi^3} = V_{123}$

$\mathcal{M}_{1234} =$



$+$



$+ \ (\text{crosses})$

$$= V_{1234} - \left(\Delta^{ab}\, V_{a12}\, V_{b34}\right)_{3\ \text{terms}} = \mathcal{M}_{123,4} - \left(\Delta^{ab}\, V_{b34}\, \mathcal{M}_{12a}\right)_{3\ \text{terms}}$$

functional derivative (differentiation on the functional manifold)

Amplitudes on the functional manifold

Can show recursively: [Cohen, Craig, Lu, Sutherland, 2202.06965] [Cohen, Lu, Sutherland, 2312.06748]

$$\mathcal{M}_{1\cdots n(n+1)} = \mathcal{M}_{1\cdots n,n+1} - \sum_{k=1}^{n} \boxed{\Delta^{ab}\, V_{b(n+1)k}}\, \mathcal{M}_{1\cdots \cancel{k} a \cdots n}$$

looks like a connection?

$$\cancel{\stackrel{?}{=}} \mathcal{M}_{1\cdots n;n+1}$$

But it's not!

Quick way to see this can't be right: $\mathcal{M}_{1\cdots n;\, ij} \stackrel{?}{=} \mathcal{M}_{1\cdots n;\, ji}$ (crossing symmetry).

All EFTs have vanishing curvature??

The catch is that $\mathcal{M}_{1\cdots n}$ are **off-shell** amplitudes

… defined on the entire functional manifold & for arbitrary 4-momenta.

They are not covariant, but only “**on-shell covariant**.”

[Cohen, Craig, Lu, Sutherland, 2202.06965] [Cohen, Lu, Sutherland, 2312.06748]

$$\phi = F[\tilde{\phi}] \quad \Rightarrow \quad \widetilde{\mathcal{M}}_{a_1\cdots a_n} = \left(\frac{\delta\phi^{b_1}}{\delta\tilde{\phi}^{a_1}} \cdots \frac{\delta\phi^{b_n}}{\delta\tilde{\phi}^{a_n}}\right)\mathcal{M}_{b_1\cdots b_n} + X_{a_1\cdots a_n}$$

bar = evaluate at physical vacuum (point on functional manifold)

with $X_{a_1\cdots a_n} \neq 0$ but $\overline{X}_{a_1\cdots a_n}\big|_{\text{on-shell}} = 0$

$p_i^2 = m_i^2$

Amplitudes in terms of geometry

[Cohen, Lu, **ZZ**, 2410.21378]

Here's the upshot:

$$\mathcal{M}_{1\cdots n} = \mathcal{F}_{1\cdots n}\left(\Delta^{ab}\,,\ \{V_{a_1\cdots a_k}\}\right)$$

function recursively defined from standard Feynman rules

e.g. $\mathcal{M}_{123} = V_{123}\,,$

$$\mathcal{M}_{1234} = V_{1234} - \Delta^{ab}\big(V_{b41}V_{a23} + V_{b42}V_{1a3} + V_{b43}V_{12a}\big)$$

Amplitudes in terms of geometry

[Cohen, Lu, **ZZ**, 2410.21378]

Here's the upshot:

same function — **geometrized** vertices

$$\mathcal{M}_{1\cdots n} = \mathcal{F}_{1\cdots n}\left(\Delta^{ab}\,,\ \{V_{a_1\cdots a_k}\}\right) = \mathcal{F}_{1\cdots n}\underbrace{\left(\Delta^{ab}\,,\ \{\mathcal{V}_{a_1\cdots a_k}\}\right)}_{\text{on-shell-covariant building blocks}}$$

on-shell-covariant building blocks

where $\quad \mathcal{V}_{1\cdots k} \equiv S_{;1\cdots k} + \Gamma^{a}_{1\cdots k} S_{,a} + \sum_{b\in\text{external}} \Gamma^{a}_{1\cdots \cancel{b}\cdots k} S_{,ab}$

e.g. $\quad \mathcal{M}_{123} = \mathcal{V}_{123}\,,$

$$\mathcal{M}_{1234} = \mathcal{V}_{1234} - \Delta^{ab}\left(\mathcal{V}_{b41}\mathcal{V}_{a23} + \mathcal{V}_{b42}\mathcal{V}_{1a3} + \mathcal{V}_{b43}\mathcal{V}_{12a}\right)$$

} manifestly on-shell covariant!

Sketch of the proof

[Cohen, Lu, **ZZ**, in preparation]

A family of equivalent recursions for off-shell amplitudes:

$$\mathcal{M}_{1\cdots n(n+1)} = \mathcal{D}^{(\gamma)}_{n+1}\,\mathcal{M}_{1\cdots n}$$

linear differential operator parameterized by γ^a_{bc}

$$\begin{cases} \gamma^a_{bc} = 0 : \quad \mathcal{V}^{(0)}_{1\cdots k} = V_{1\cdots k} \\ \gamma^a_{bc} = \Gamma^a_{bc} : \quad \mathcal{V}^{(\Gamma)}_{1\cdots k} = \mathcal{V}_{1\cdots k} \end{cases}$$

$$\begin{cases} \mathcal{D}^{(\gamma)}_s \mathcal{V}^{(\gamma)}_{1\cdots k} = \mathcal{V}^{(\gamma)}_{1\cdots ks} - \sum\limits_{b\in\text{external}} \Delta^{ac}\,\mathcal{V}^{(\gamma)}_{cbs}\,\mathcal{V}^{(\gamma)}_{1\cdots \cancel{b}a\cdots k} \\ \mathcal{D}^{(\gamma)}_s \Delta^{ab} = -\Delta^{ac}\,\mathcal{V}^{(\gamma)}_{cds}\,\Delta^{db} \end{cases}$$

interpolates between standard and geometrized recursions.

Outline

✅ $\phi \to \tilde{\phi} = f(\phi)$: field space geometry.

Geometry on the field manifold; geometrizing EFT matching.

✅ $\phi \to \tilde{\phi} = f\left(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, \ldots\right) = F[\phi]$: functional geometry.

Geometry on the functional manifold; amplitudes in terms of on-shell-covariant building blocks.

- Thoughts (and dreams).

Mysteries of small numbers

The universe as we know it:

$$\mathcal{L} = -\Lambda^4\,\mathbf{1}$$

dim-0: cosmological constant problem

$$+\, m^2\,|H|^2$$

dim-2: hierarchy problem

$$+\, \frac{\theta}{16\pi^2}\,\mathrm{tr}\,G\wedge G + \cdots$$

dim-4: strong CP problem

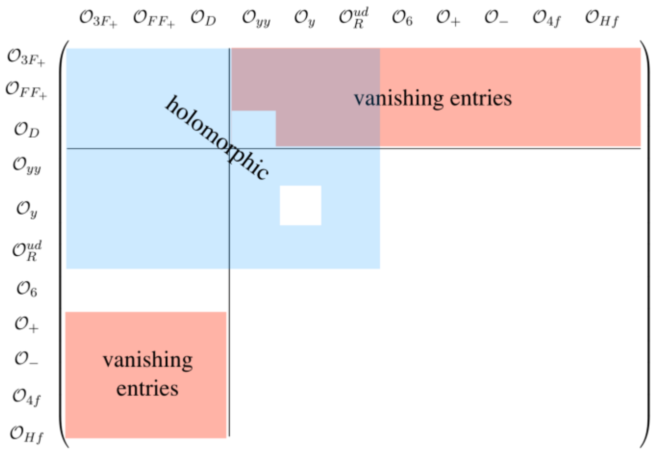
$$+\, (\text{irrelevant operators})$$

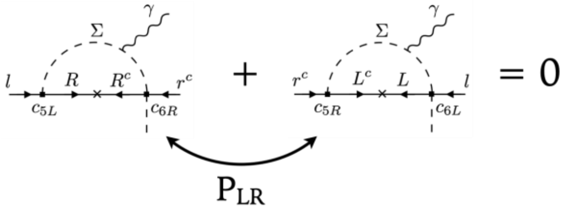
many more mysteries here!

Magic zeroes?

Hidden symmetries and selection rules in EFT matching and running.

[Alonso, Jenkins, Manohar, 1409.0868] [Elias-Miro, Espinosa, Pomarol, 1412.7151] [Cheung, Shen, 1505.01844] [Bern, Parra-Martinez, Sawyer, 1910.05831] [Jiang, Shu, Xiao, Zheng, 2001.04481] [Arkani-Hamed, Harigaya, 2106.01373] [Delle Rose, Harling, Pomarol, 2201.10572] [Bao, Gu, Liu, Shu, Wang, 2408.08948] **[Craig, Garcia Garcia, Vainshtein, ZZ, 2112.05770]**





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What can EFT geometry buy us?

Freedom to change basis $\Rightarrow$ clarify structures of EFTs? [Li, Lu, **ZZ**, 2411.04173]

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 \underbrace{- A|H|^2 S - \frac{1}{2}\kappa |H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4}_{\text{relevant/marginal nonderivative interactions}}$$

26 terms to compute for 1-loop matching up to dim-6

$$S = f_0\big(|H|^2\big) + f_1\big(|H|^2\big)\,\widetilde{S} + f_2(|H|^2)\,\widetilde{S}^2 + f_3\big(|H|^2\big)\,\widetilde{S}^3 + \cdots$$

$$\mathcal{L} = \underbrace{\frac{1}{2}\left(\partial_\mu \widetilde{S} \;\; \partial_\mu \tilde{h} \;\; \partial_\mu \pi^a\right) \begin{pmatrix} g_{SS} & g_{Sh} & 0 \\ g_{hS} & g_{hh} & 0 \\ 0 & 0 & g_{ab} \end{pmatrix} \begin{pmatrix} \partial^\mu \widetilde{S} \\ \partial^\mu \tilde{h} \\ \partial^\mu \pi^b \end{pmatrix}}_{\text{irrelevant derivative interactions}} - \frac{1}{2} M^2\,\widetilde{S}^2 - W(\tilde{h})$$

14 terms to compute for 1-loop matching up to dim-6

Same story in the symmetry explanation of magic zero [Craig, Garcia Garcia, Vainshtein, ZZ, 2112.05770]

Marginal nonderivative $\rightarrow$ irrelevant derivative $\Rightarrow$ unique spurion combination.

What can EFT geometry buy us?

A geometry-kinematics duality:

$$\text{Nonlinear sigma model amplitudes} \xrightarrow{\bar{\mathrm{g}} \to \bar{g}} \text{general scalar field theory amplitudes.}$$

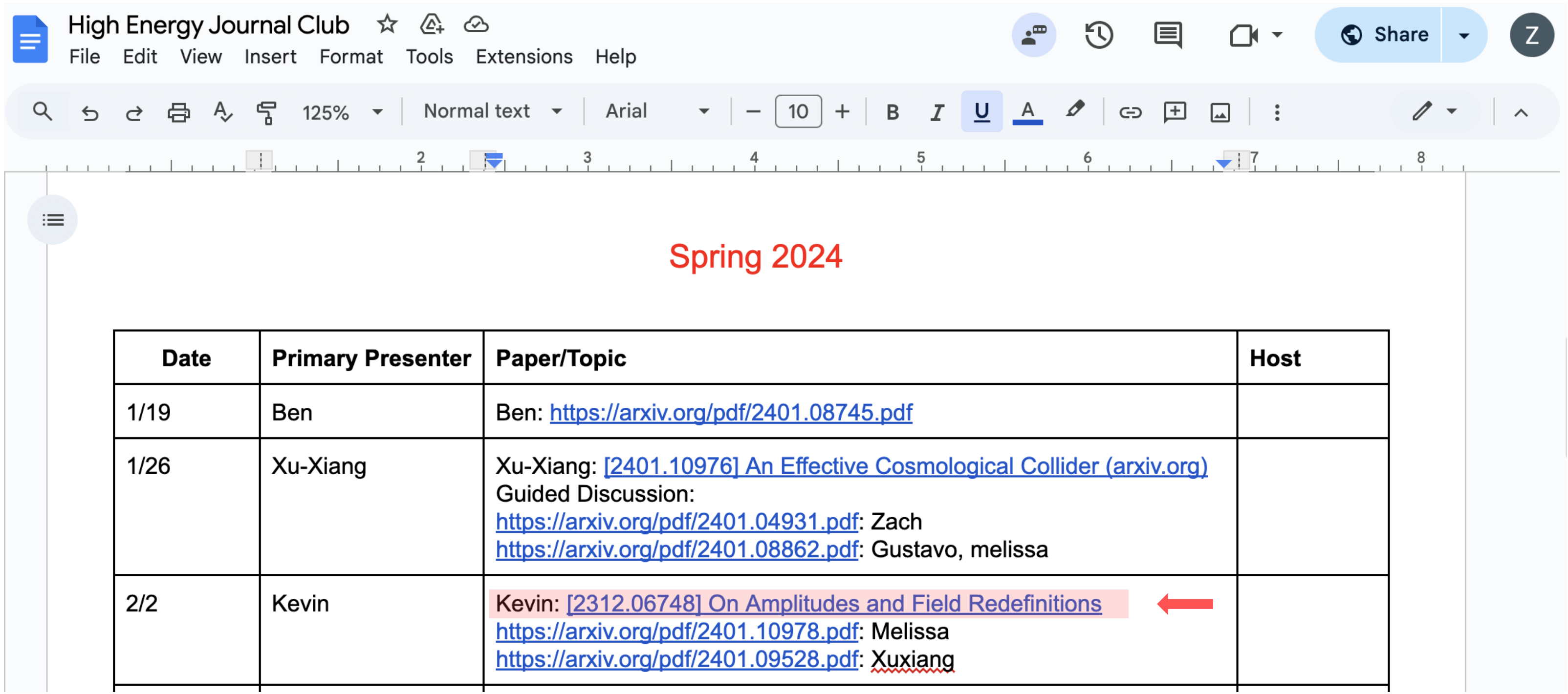
[Cheung, Helset, Parra-Martinez, 2202.06972] [Cohen, Lu, **ZZ**, 2410.21378]

Relations between different EFTs' amplitudes?

Classification of EFTs?

Before it ends...

I'd like to share the story of how it started:



High Energy Journal Club

Spring 2024

Date	Primary Presenter	Paper/Topic	Host
1/19	Ben	Ben: https://arxiv.org/pdf/2401.08745.pdf	
1/26	Xu-Xiang	Xu-Xiang: [2401.10976] An Effective Cosmological Collider (arxiv.org) Guided Discussion: https://arxiv.org/pdf/2401.04931.pdf: Zach https://arxiv.org/pdf/2401.08862.pdf: Gustavo, melissa	
2/2	Kevin	Kevin: [2312.06748] On Amplitudes and Field Redefinitions https://arxiv.org/pdf/2401.10978.pdf: Melissa https://arxiv.org/pdf/2401.09528.pdf: Xuxiang	

DEPARTMENT OF PHYSICS & ASTRONOMY
COLLEGE OF SCIENCE

No Stone Unturned 2025

The Search for New Physics: Leaving No Stone Unturned

March 10-14, 2025
University of Utah
Salt Lake City, UT

This is the second in a series of workshops that gather leading experts from around the world to discuss recent developments and new directions related to the search for physics beyond the Standard Model. No Stone Unturned workshops cover important frontiers in particle physics and their interface with astrophysics and cosmology. At No Stone Unturned 2025, a broad range of topics will be addressed, including search strategies for terrestrial and space-based detectors, complementarity, and new directions for theoretical, experimental, and observational approaches. We hope you can join us in Salt Lake City!

Organizers

Gustavo Marques Tavares

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Summary

$\phi \to \tilde{\phi} = f(\phi):$ field space geometry.

Geometry on the field manifold; geometrizing EFT matching.

$\phi \to \tilde{\phi} = f\big(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, \dots\big) = F[\phi]:$ functional geometry.

Geometry on the functional manifold; amplitudes in terms of on-shell-covariant building blocks.

Thoughts (and dreams).

Magic zeroes; hidden structures of EFT amplitudes?