

International
Muon Collider
Collaboration



Status of the Muon Collider Lattice design

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- Reminder of the main requirements/challenges of the Muon Collider Ring
- Overview of the collider ring
 - Interaction region
 - Arcs
- Local chromatic correction section
 - General considerations
 - Previous versions: v0.6 – v0.7
 - New optics attempt: « v0.8 »
 - Comparison of the different versions
- Remaining issues and Next steps

Challenges of the Muon Collider Ring

10TeV Muon collider

$$\mathcal{L} = \frac{1}{4\pi} \frac{N_p^2}{\epsilon\beta^*} f_r f_{hg} \frac{\gamma T_\mu}{T_{rev}}$$

Parameter	Symbol	Value
Beam energy	E	5000 <i>GeV</i>
Luminosity per IP	\mathcal{L}	$\sim 20 * 10^{34} cm^{-2} s^{-1}$
Bunch population	N_p	$1.8 * 10^{12}$
Repetition rate	f_r	5 <i>Hz</i>
Normalized transverse rms emittance	$\epsilon_{nx} = \epsilon_{ny}$	25 μm
Geometric transverse rms emittance	$\epsilon_{gx} = \epsilon_{gy}$	0.528 <i>nm</i>
Longitudinal geometric rms emittance	ϵ_L	70 <i>mm</i>
Rms bunch length	σ_z	1.5 <i>mm</i>
Relative rms energy spread	$\delta = \frac{\sigma_E}{E}$	0.1 %
Beta function at IP	$\beta_x^* = \beta_y^*$	1.5 <i>mm</i>
Circumference	C	$\sim 10 km$

Muon collider: Challenges

Relative rms energy spread	δ	0.1 %
Beta function at IP	$\beta_x^* = \beta_y^*$	1.5 mm

- Very **small β^*** at IP $\rightarrow \beta$ of ~ 700 -800 km in the Final Focusing (FF) quadrupoles
- **Large relative energy spread**

FF quadrupoles introduce large chromaticity effects that must be corrected with sextupoles in a **local chromatic correction section**:

- W (Montague functions): describe variations of Twiss α and β for (small) momentum offsets.
- Q' (Chromaticity): Variation in tune with respect to momentum.

Large β in the FF quadrupoles:

- \rightarrow Significant magnet aperture and sensitivity to unwanted multipolar components.
- \rightarrow Very high magnetic field required (HTS technology) with good field quality.

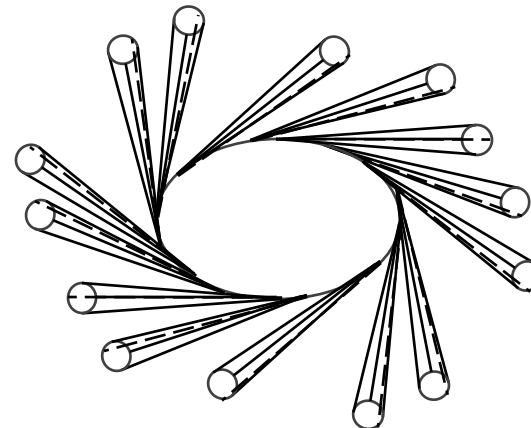
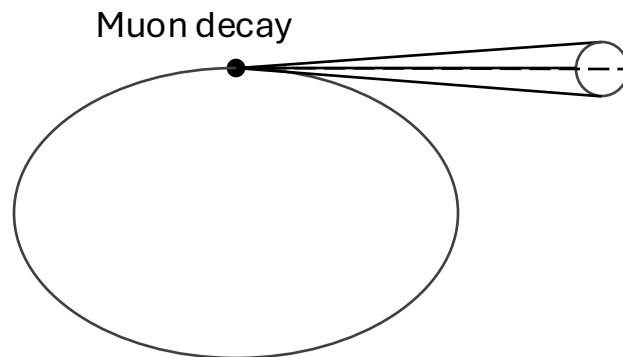
Muon collider: Challenges

Short bunch length (1.5mm) to be kept for $\sim > 1000$ turns: $\eta = 0$; $\alpha_c \sim 0$

→ **Flexible momentum compaction (FMC) arc cells**

Short muon lifetime → Muon decay

- Ring **circumference** should be as **small** as possible → High magnetic field
- **Neutrinos** emitted in a small cone tangential to the collider ring



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- Ring **circumference** should be as **small** as possible → High magnetic field
- **Neutrinos** emitted in a small cone tangential to the collider ring
 - **Fewest possible straight sections** in the lattice: **small drifts** for the element interconnections and **long straight sections** in the **Interaction Region (IR)**
 - **“Geoprofiler”**: Tool to place the ring close to CERN so that the **exit points** from the long straights in the IR are in the **Mediterranean** Sea and in sparsely populated areas of the **Jura**.

Muon collider: Challenges

Short bunch length (1.5mm) to be kept for $\sim > 1000$ turns: $\eta = 0$; $\alpha_c \sim 0$

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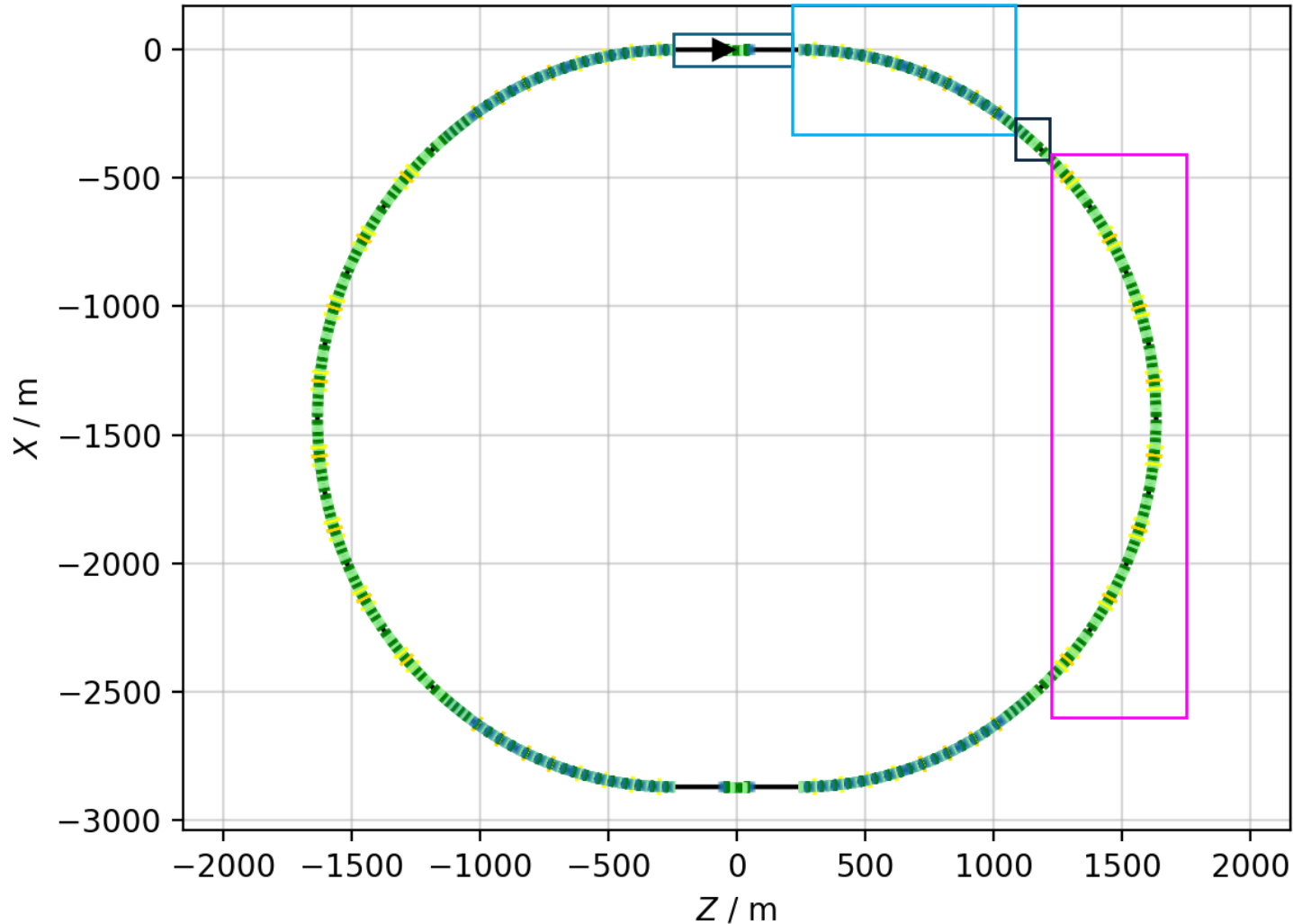
- Ring **circumference** should be as **small** as possible → High magnetic field
- **Neutrinos** emitted in a small cone tangential to the collider ring
 - **Fewest possible straight sections** in the lattice: **small drifts** for the element interconnections and **long straight sections** in the **Interaction Region (IR)**
 - “**Geoprofiler**”: Tool to place the ring close to CERN so that the **exit points** from the IR are in the **Mediterranean Sea** or in the **Jura**.
- **Beam-induced background** to be mitigated and **W shielding** (~ 30 - 40 mm) inside the magnets to absorb shower generated by e^+/e^-

Overview of the Collider Ring

Muon collider

Assumptions & aims

- ❖ 10 km collider ring
- ❖ Maximum 10 m long magnet
- ❖ Maximum field of 16 T for dipoles and 20 T for combined-function magnets
- ❖ 30 cm drift for interconnection

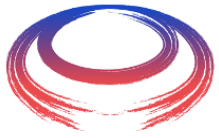


**Interaction region
(IR)**

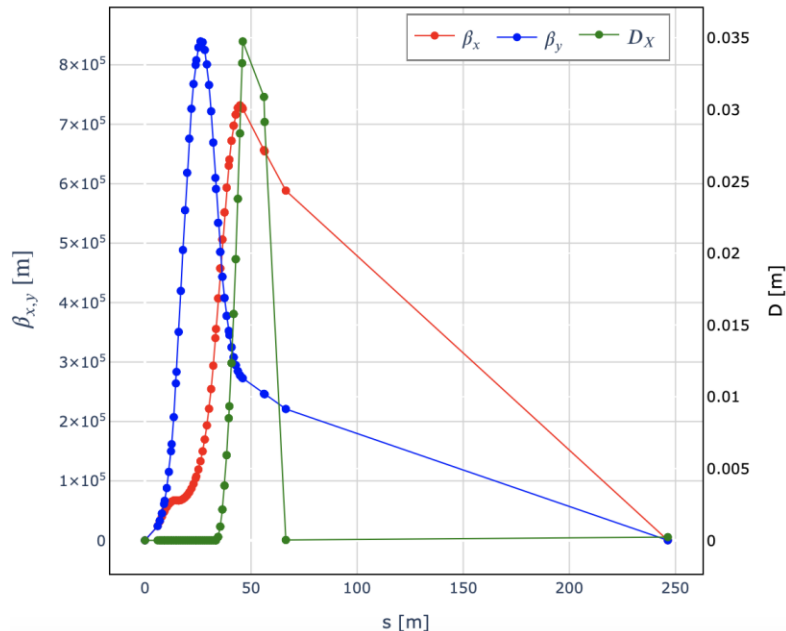
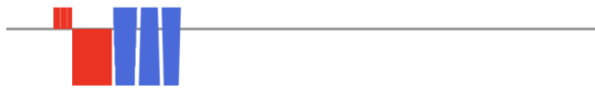
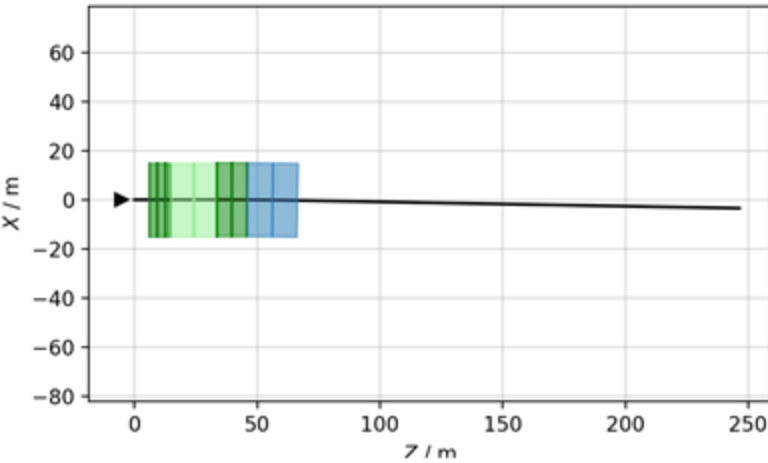
Local Chromatic
correction section
(CC)

Matching section
(MS)

Arcs (FMC cells)



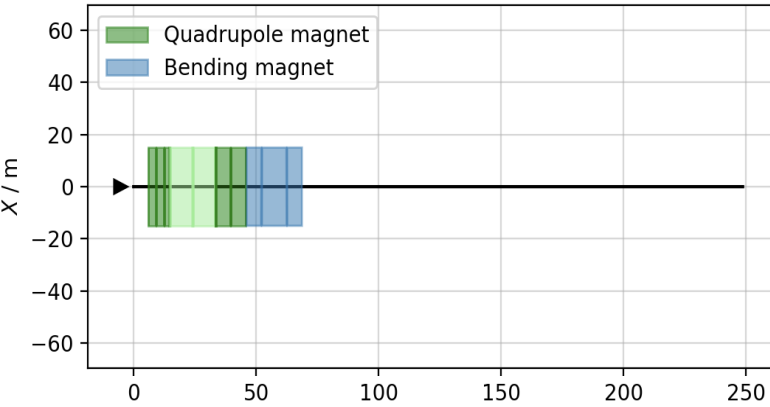
Interaction region - FF scheme v07



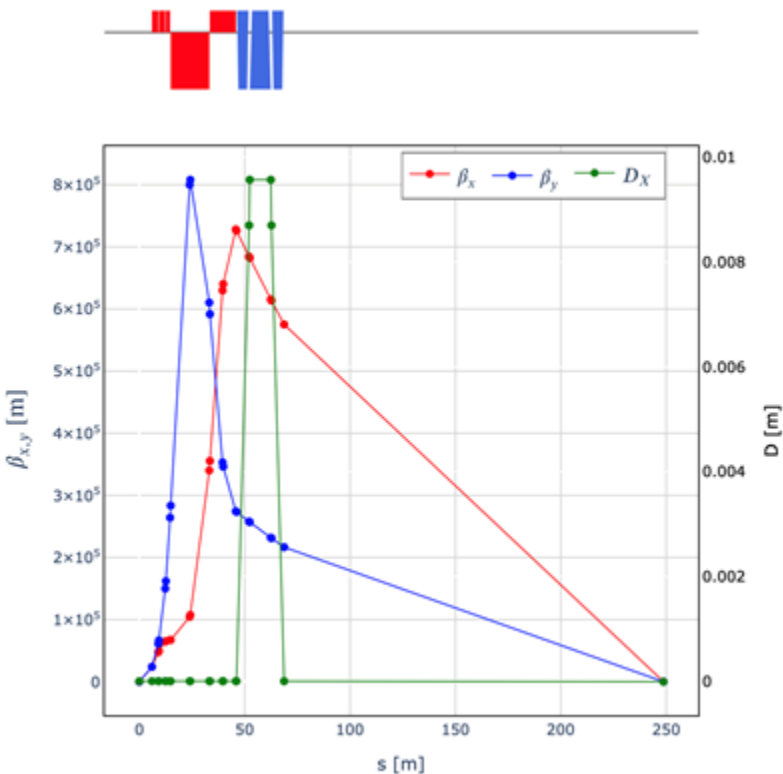
- **Long drift** for IP ($L^* = 6m$), **triplet** for the final focusing, **chicane** to reduce Beam-Induced Background (BIB), **long straight section** to smoothly reduce the beta functions without increasing W functions.
- The **first quadrupole** is divided into **three magnets** with different field gradients (maximum field set to 20 T at the magnet aperture).
- Last **FF quadrupole combined** with the first **bend of the chicane** (combined-function magnet) \rightarrow small residual deflection after the chicane (0.7 mrad).
- The IR **long straight sections** result in many secondary particles from muon decay that accumulate. A **chicane** before the FF helps remove these particles as much as possible before reaching the nozzle.



Interaction region - FF scheme v08



- **Long drift** for IP ($L^* = 6m$), **triplet** for the final focusing, **chicane** to reduce Beam-Induced Background (BIB), **long straight section** to smoothly reduce the beta functions without increasing W functions.
- The **first quadrupole** is divided into **three magnets** with different field gradients (maximum field set to 20 T at the magnet aperture).
- **No combined-function magnet** in FF triplet (the 3 dipoles of the chicane separated from the FF quads). **No residual deflection & dispersion** after the chicane.
- The IR **long straight sections** result in many secondary particles from muon decay that accumulate. A **chicane** before the FF helps remove these particles as much as possible before reaching the nozzle.

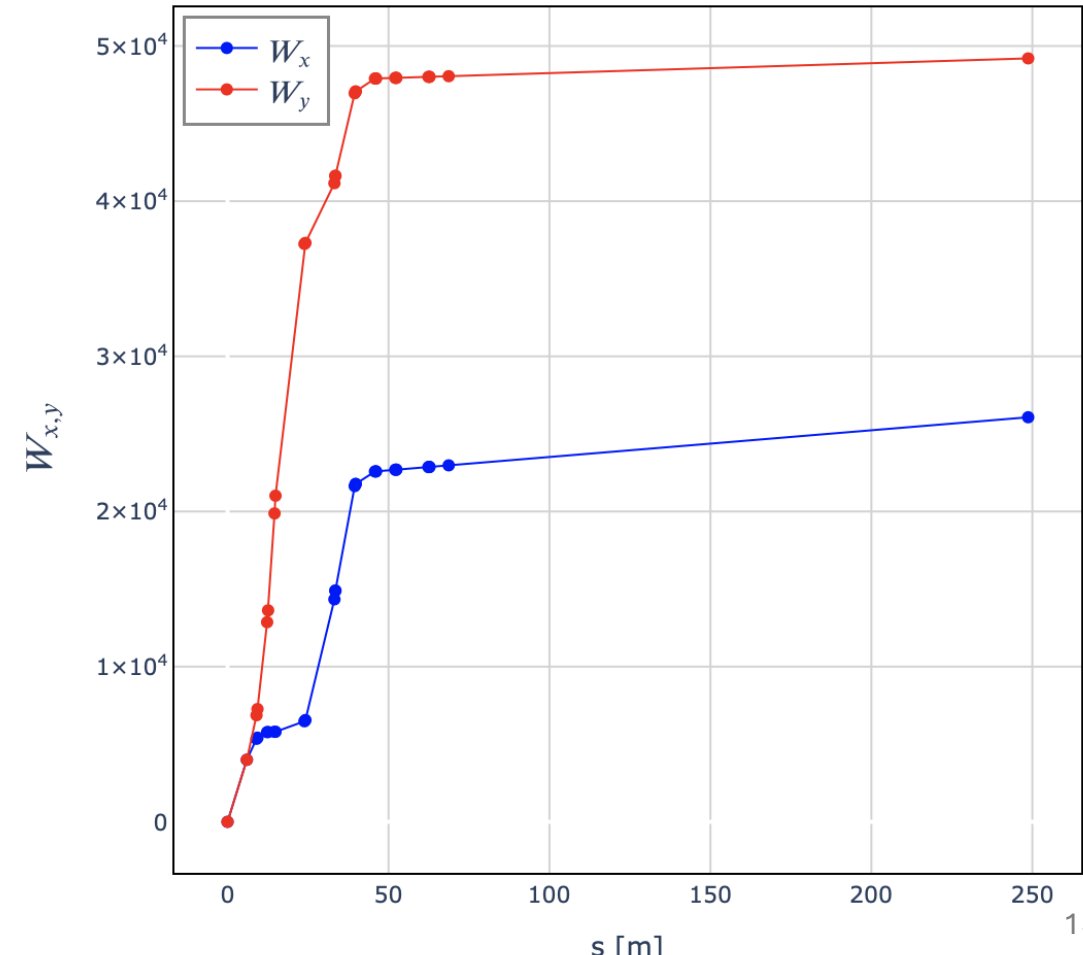


IR – Montague functions

Montague chromatic functions:

$$W = \sqrt{A^2 + B^2} \quad A = \frac{d\alpha}{d\delta} - \frac{\alpha}{\beta} \frac{d\beta}{d\delta} \quad B = \frac{1}{\beta} \frac{d\beta}{d\delta}$$

- The very small β^* at the IP induce very large β -function in the strong focusing FF quadrupoles, resulting in **significant chromatic effects**.
- **Very large W functions** at the end of the interaction region.
- Need for a **local chromatic correction section**

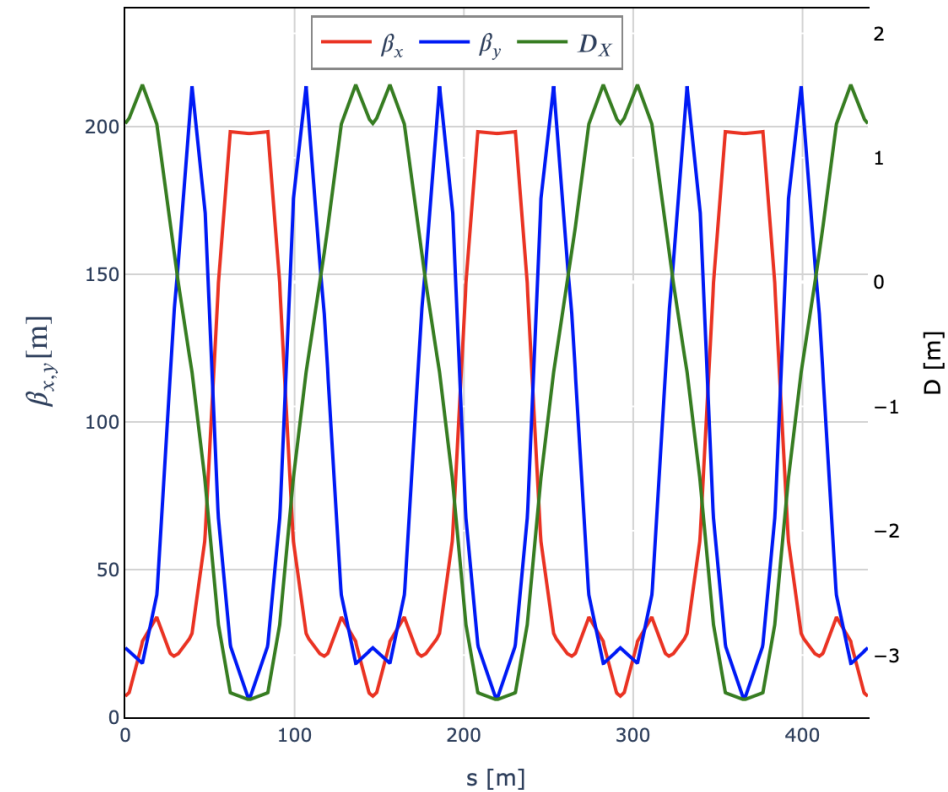
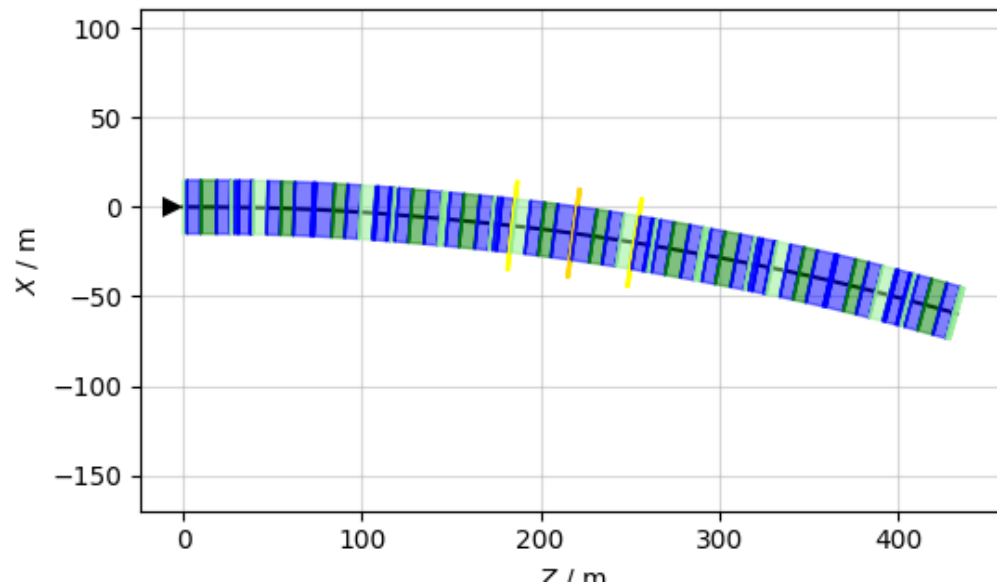




Muon collider: Arcs



- **Short bunch length (1.5mm)** to be kept for $\sim > 1000$ turns: $\alpha_c = 0 \rightarrow$ **Flexible momentum compaction (FMC) arcs.**
- The closing of the trajectory on the entire ring is controlled by the dipoles in the arcs.



- Dispersion oscillations to obtain a negative momentum compaction factor in the arcs $\rightarrow \alpha_c \sim 0$ on the entire ring.
- Linear chromaticities controlled with sextupole pairs.

Local Chromatic Correction section (CC)

General considerations

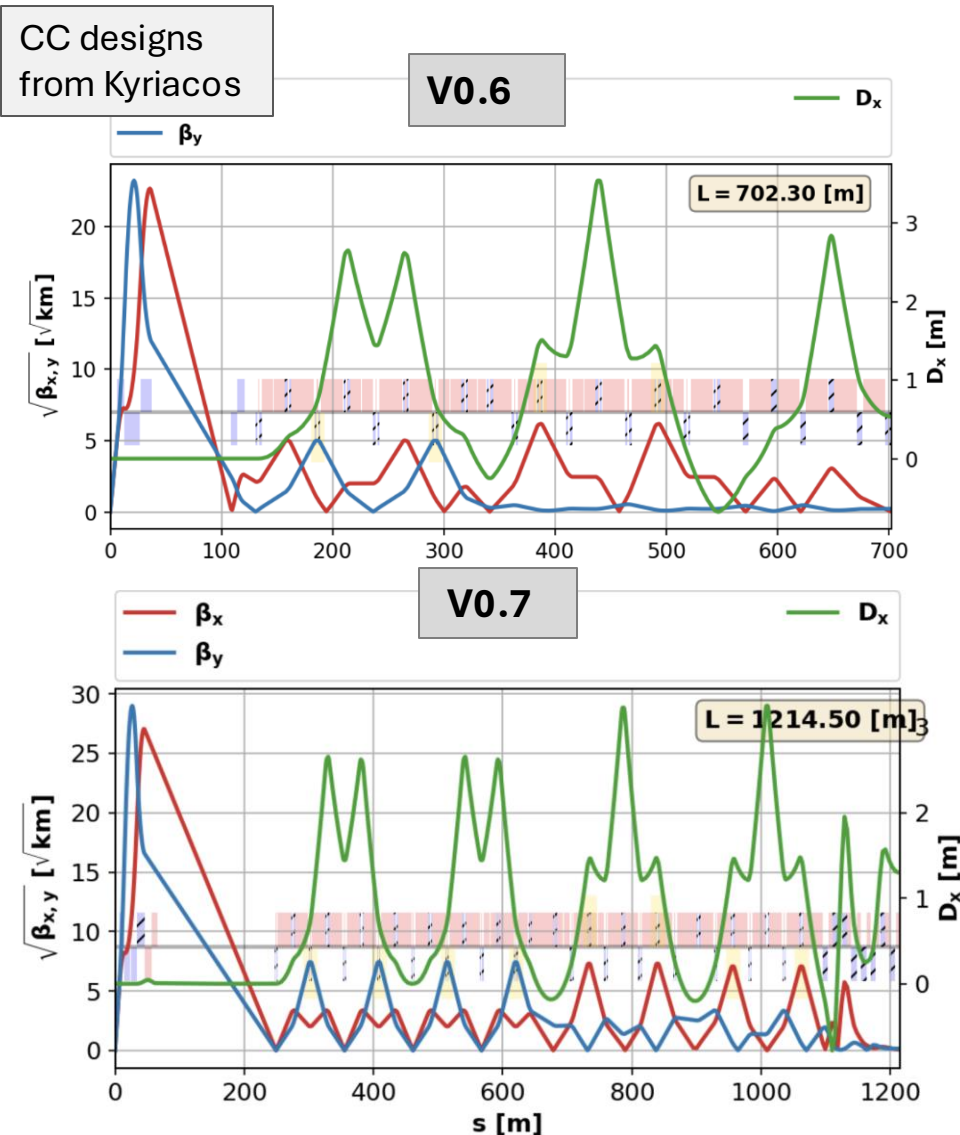
- Model used for the sextupoles: **Thin sextupoles** surrounded by dipoles.
- Code used: **XSUITE** based on initial MAD-X files from Kyriacos:
 - *Configure_bend_model* with sufficient number of **multipolar kicks** for long **combined-function magnets** (set the number of Yoshida steps).
 - Choose carefully the **finite difference step in energy** to ensure convergence of the (second order) chromaticities: *delta_chrom*.
 - Manually set **$h = k0$** in bends, otherwise change in the orbit and unwanted effects in the lattice.
 - **Footprint** plots: XSUITE does not yet make the difference between Q and 1-Q



Local Chromatic correction section



Multiple iterations: work in progress



- **Sextupoles at non-zero dispersion** locations to correct linear chromaticity: W_y is first corrected then W_x .
- The sextupoles come **in pairs** to cancel non-linearities, with the π phase advance controlled by the in-between quads.
- **Adjust the phase at the entrance of the first sextupole** in each pair to be in phase (π) with the main error sources (triplet) – $\mu_y = 0.75$ from IP to SD and $\mu_x = 2.25$ from IP to SF.
- **Reduction by an order of magnitude of the β functions** between the IR and the CC: trade-off between the sextupoles strength, β , and dispersion.
- Using the end of one section to match the desired optics in the following section (no periodic conditions).



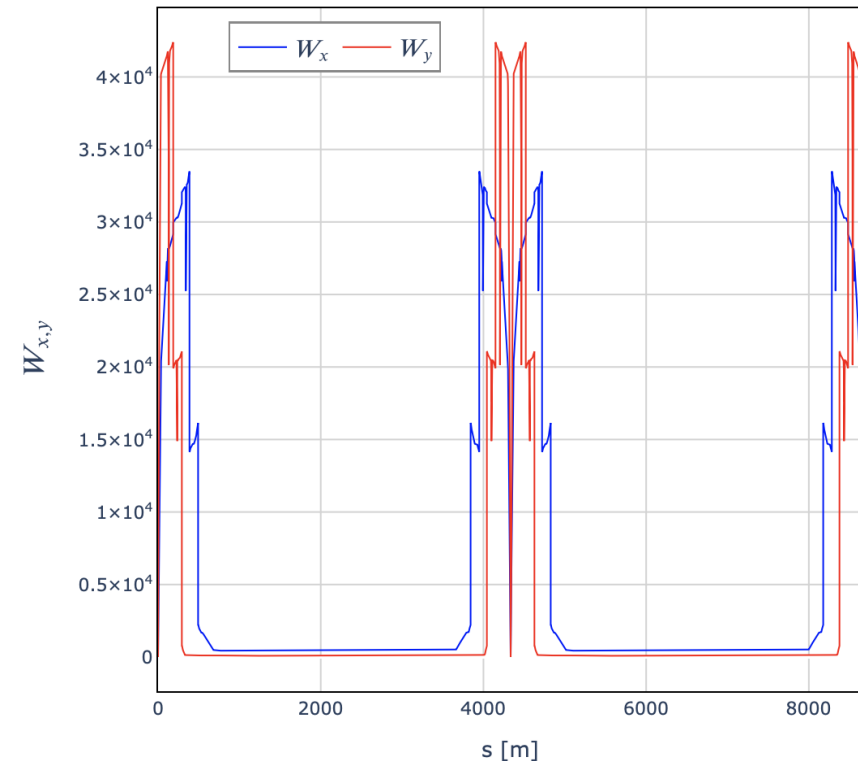
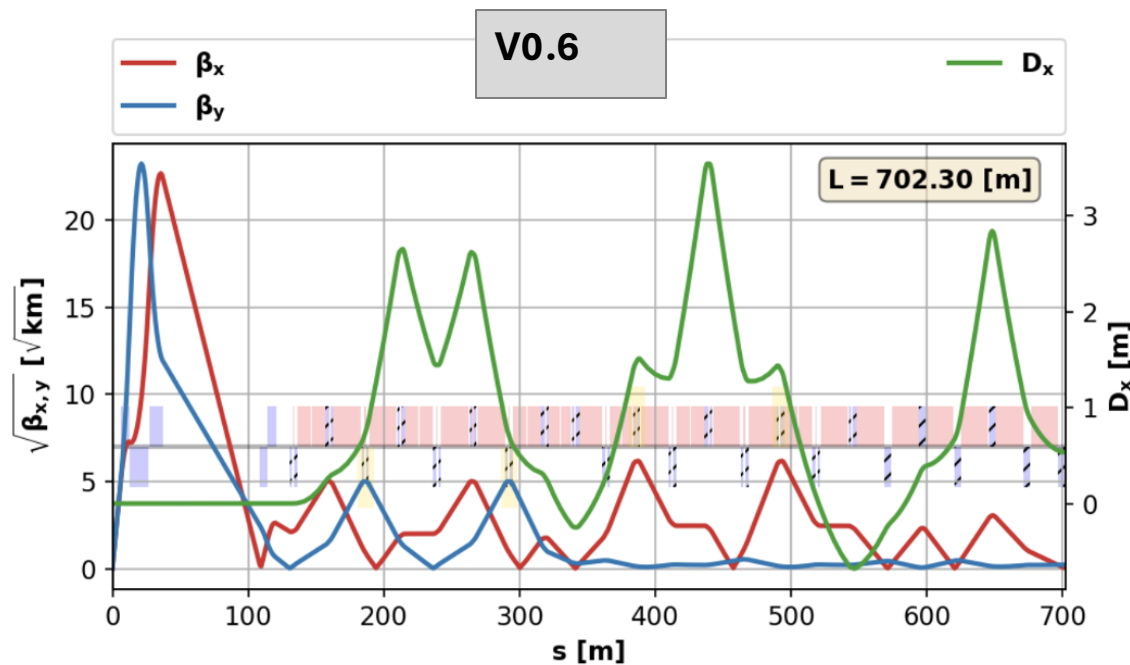
CC – v0.6



Chromatic correction section – v0.6



- Dependence between the CC section and the arcs, as there is **no Q' control in the CC**. Rematch of the entire ring required by imposing $W_x = W_y = 0$ and $Q'_x = Q'_y = 0 \rightarrow$ Check that W in the arcs are not too large (small $W * 10^{-3}$)



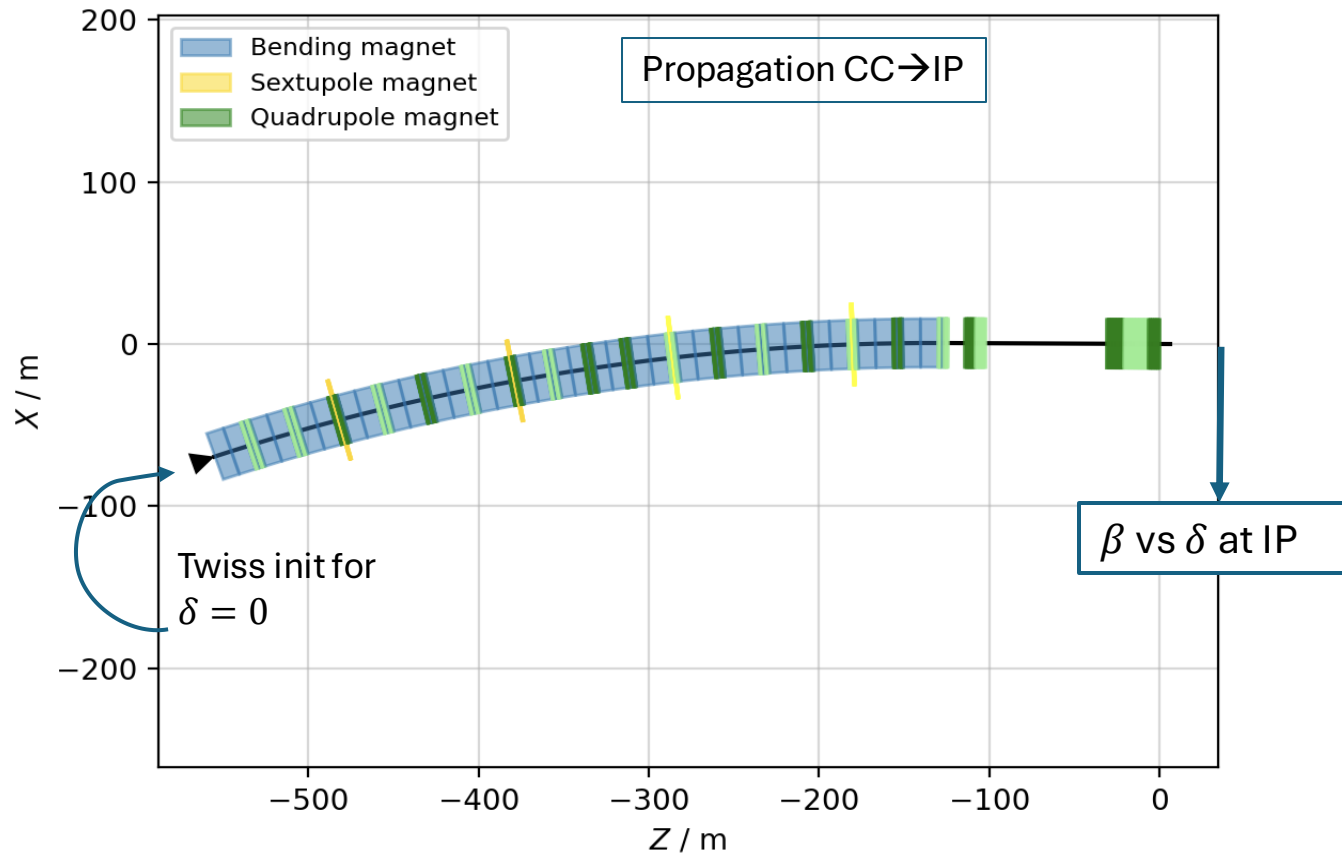


Chromatic correction section – v0.6

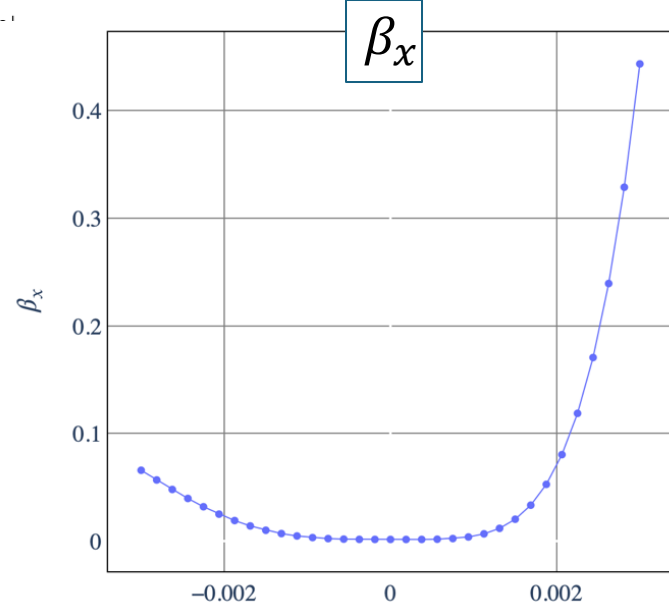


Slight phase shift at the sextupoles helps reduce the non-linearities in β vs. δ :

- Can be done manually or in a more automated way

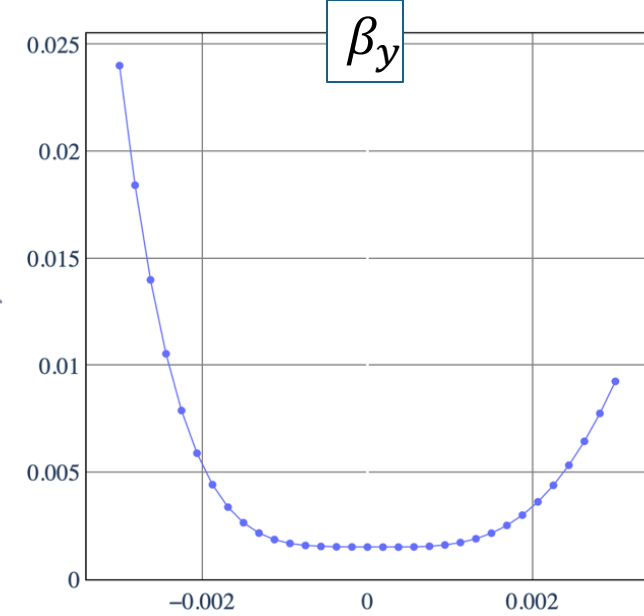


Chromatic correction section – v0.6

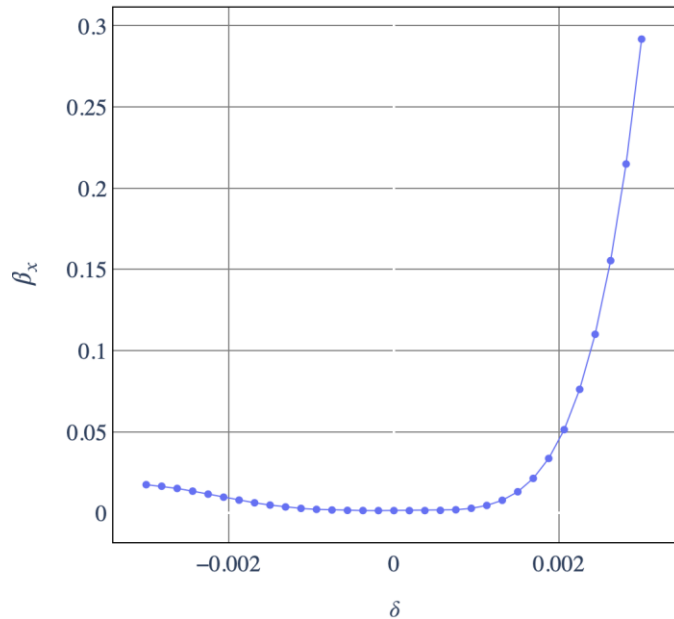


$$\mu_{y_{SD1}} = 0.75;$$

$$\mu_{x_{SF1}} = 2.25$$

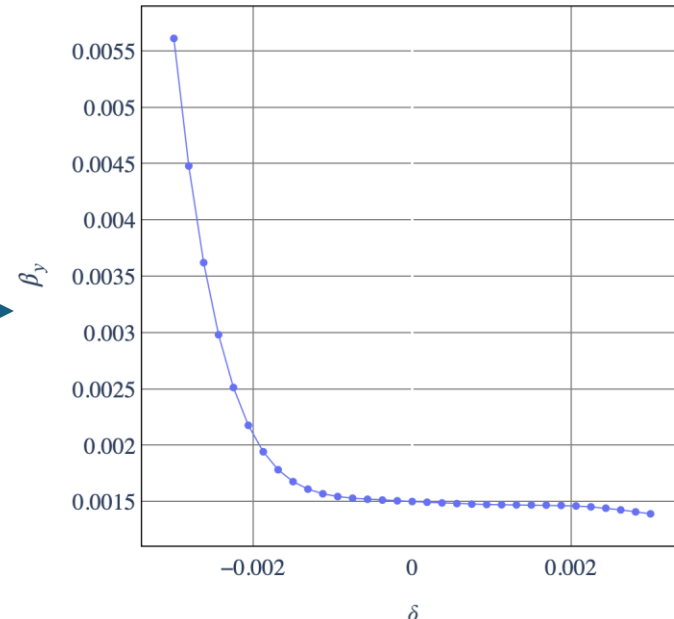


β vs δ

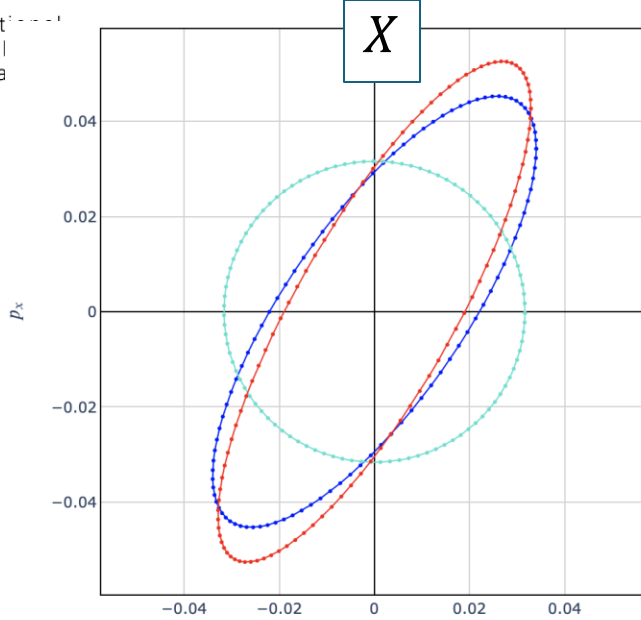


$$\mu_{y_{SD1}} = 0.75 - 0.0000225;$$

$$\mu_{x_{SF1}} = 2.25 + 0.000055$$



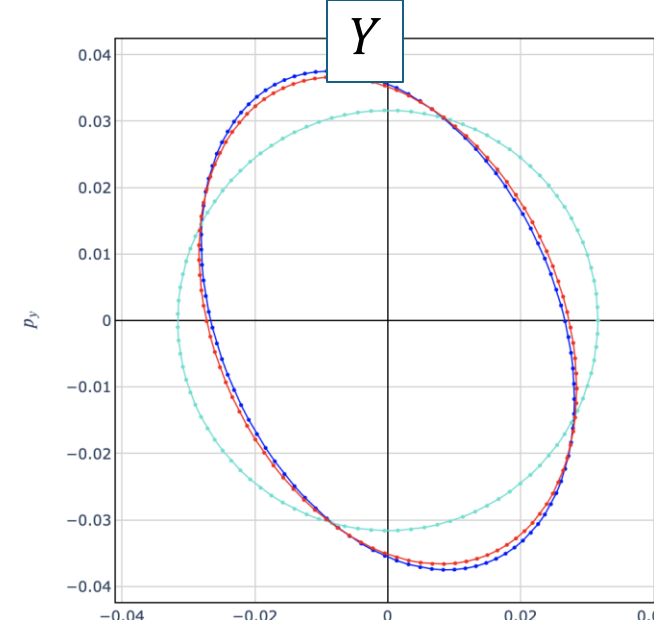
Chromatic correction section – v0.6



- 1.01e-3
- 0.81e-3
- 0.61e-3
- 0.41e-3
- 0.21e-3
- 0.01e-3
- 0.21e-3
- 0.41e-3
- 0.61e-3
- 0.81e-3
- 1.01e-3

$$\mu_{ySD1} = 0.75;$$

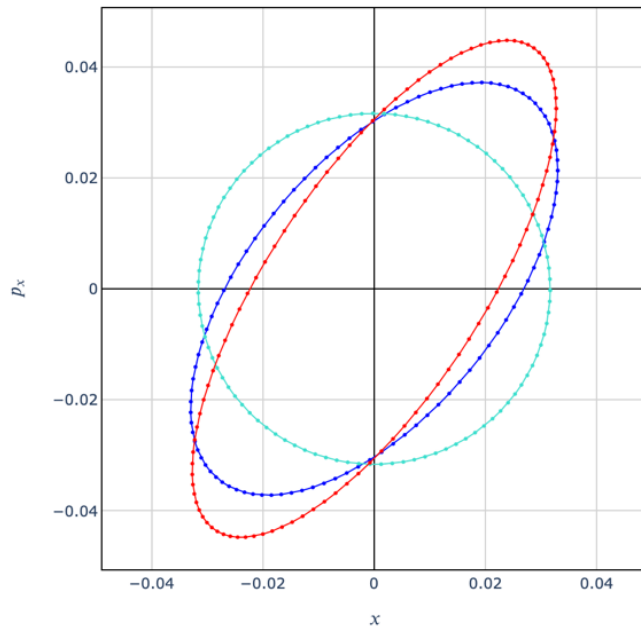
$$\mu_{xSF1} = 2.25$$



- 1.01e-3
- 0.81e-3
- 0.61e-3
- 0.41e-3
- 0.21e-3
- 0.01e-3
- 0.21e-3
- 0.41e-3
- 0.61e-3
- 0.81e-3
- 1.01e-3

Ellipses at the
end of CC

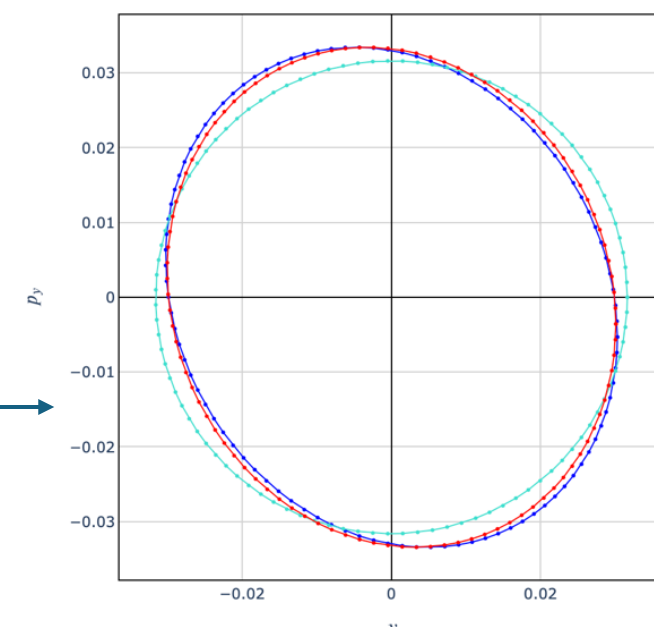
Propagation IP→CC



- 1.01e-3
- 0.81e-3
- 0.61e-3
- 0.41e-3
- 0.21e-3
- 0.01e-3
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- 0.61e-3
- 0.81e-3
- 1.01e-3

$$\mu_{ySD1} = 0.75 - 0.0000225;$$

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- 0.01e-3
- 0.21e-3
- 0.41e-3
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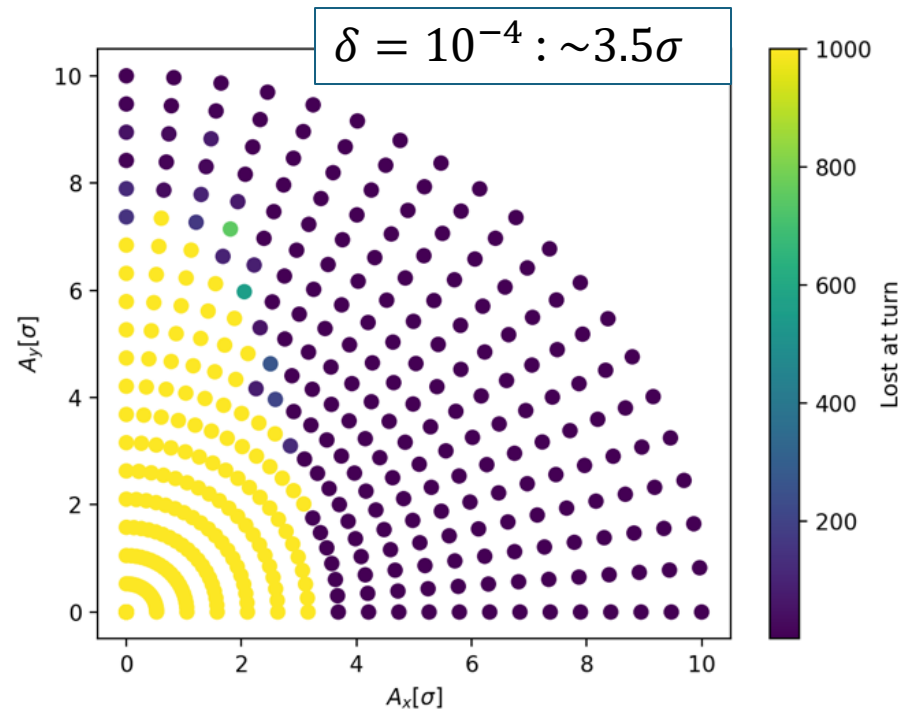
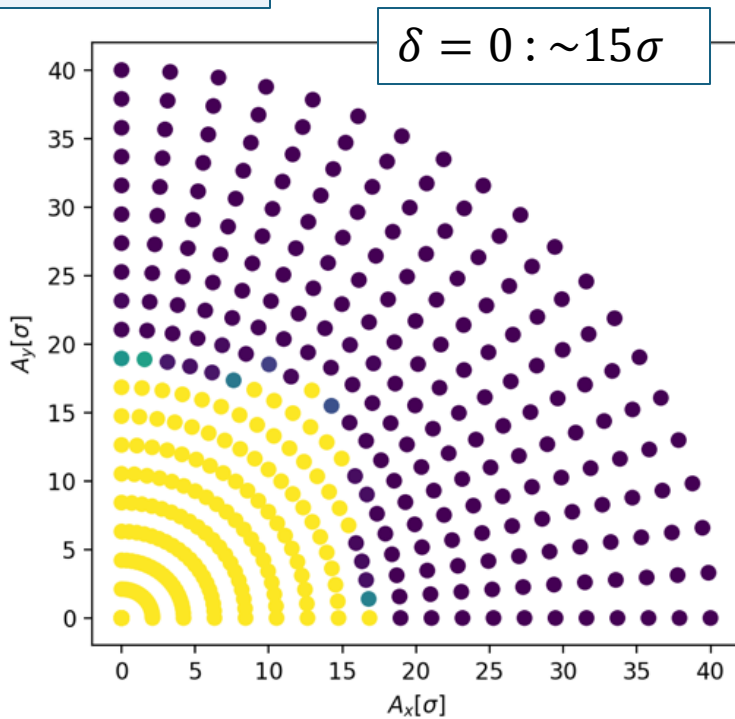


Chromatic correction section – v0.6



- Slight phase shift at the sextupoles helps reduce the non-linearities in β vs. δ :
 - Can be done manually or in a more automated manner
 - Increase the DA for several deltas

$\mu_{ySD1} = 0.75;$
 $\mu_{xSF1} = 2.25$



$\delta = 10^{-3} : 0\sigma$



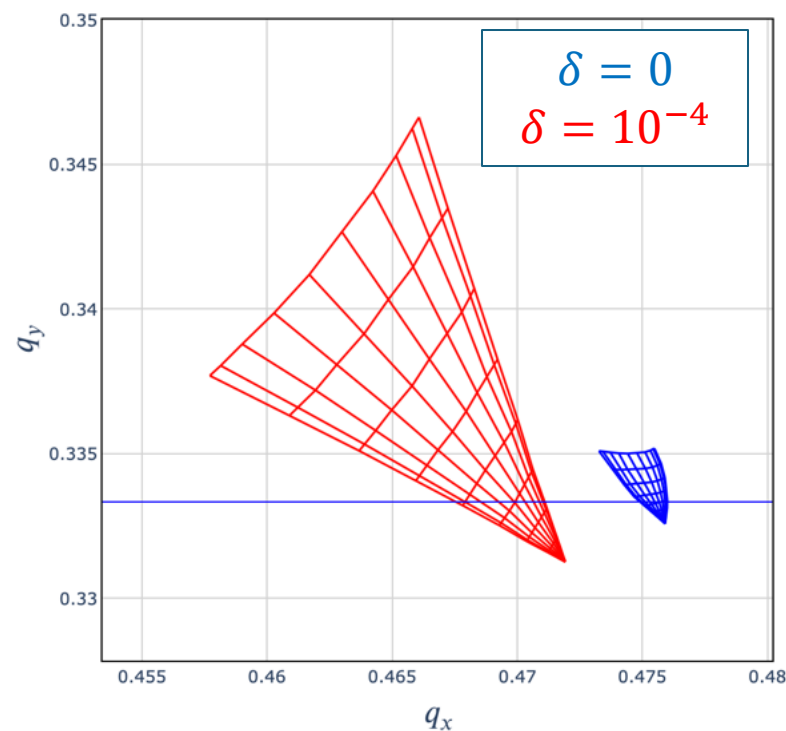
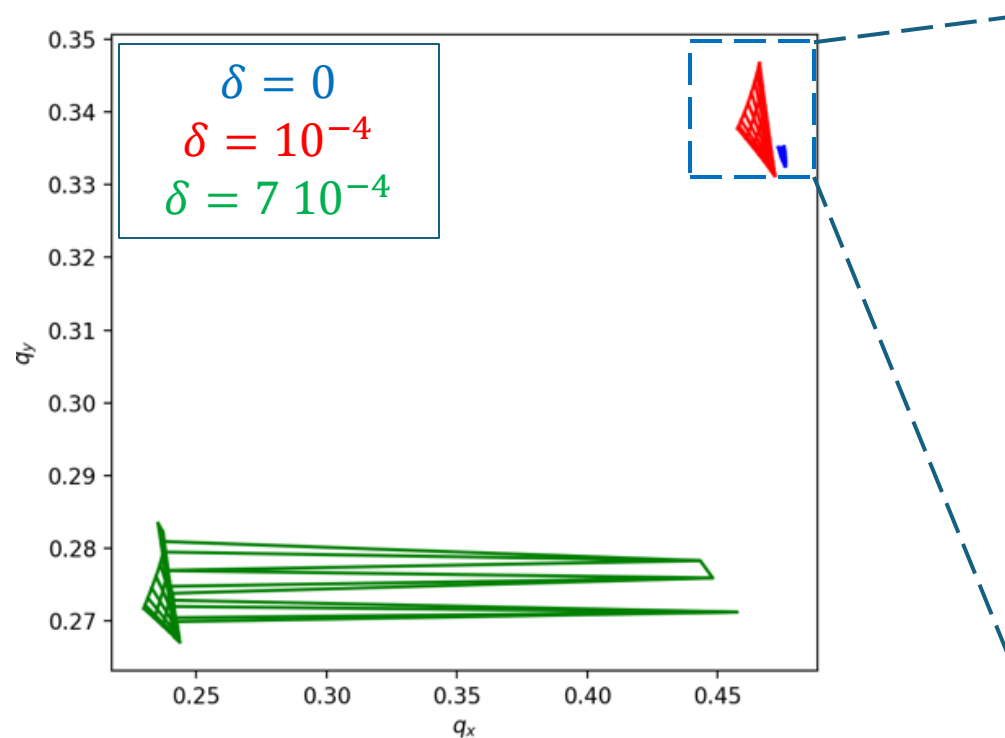


Chromatic correction section – v0.6



- Reduced DA for off-momentum particles: Footprint
 - XSUITE does not yet make the difference between nu and 1-nu.
 - Large **tune shifts** in tune space **for off-momentum particles**.
 - Difference in detuning with amplitude for off-momentum particles.

$$\begin{aligned}\mu_{ySD1} &= 0.75; \\ \mu_{xSF1} &= 2.25\end{aligned}$$



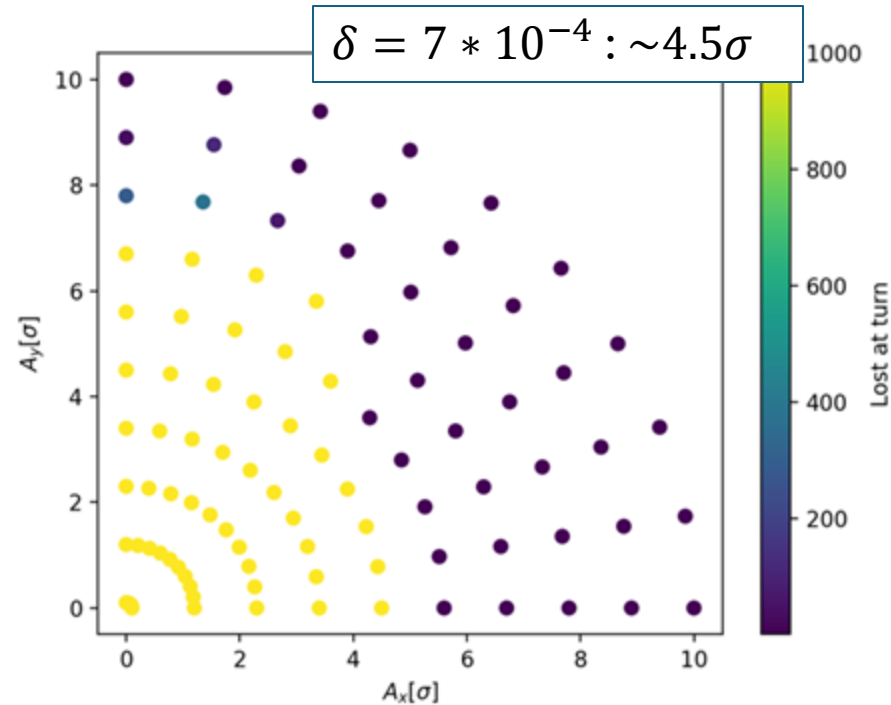
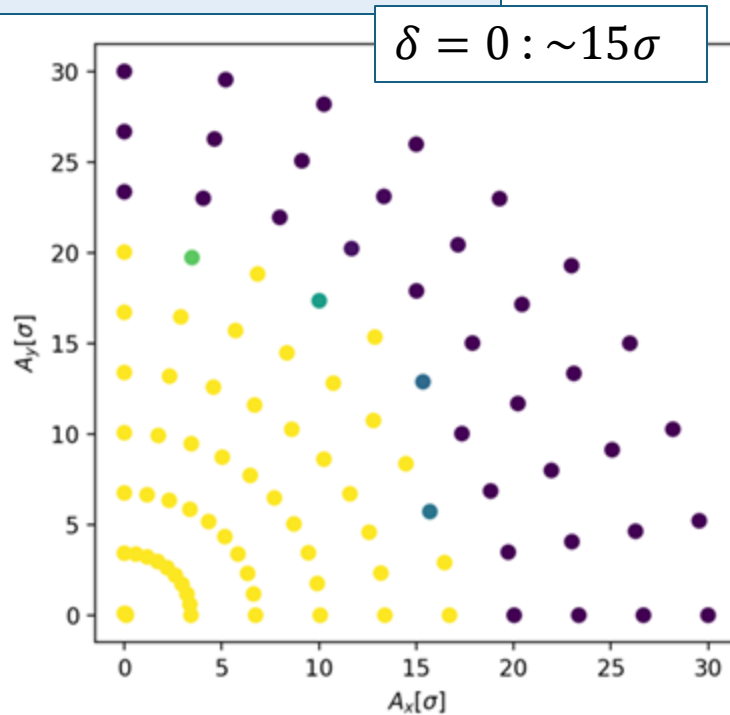


Chromatic correction section – v0.6



- Slight phase shift at the sextupoles helps reduce the non-linearities in β vs. δ :
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$$\begin{aligned}\mu_{y_{SD1}} &= 0.75 - 0.0000225; \\ \mu_{x_{SF1}} &= 2.25 + 0.000055\end{aligned}$$



$\delta = 10^{-3} : 0\sigma$



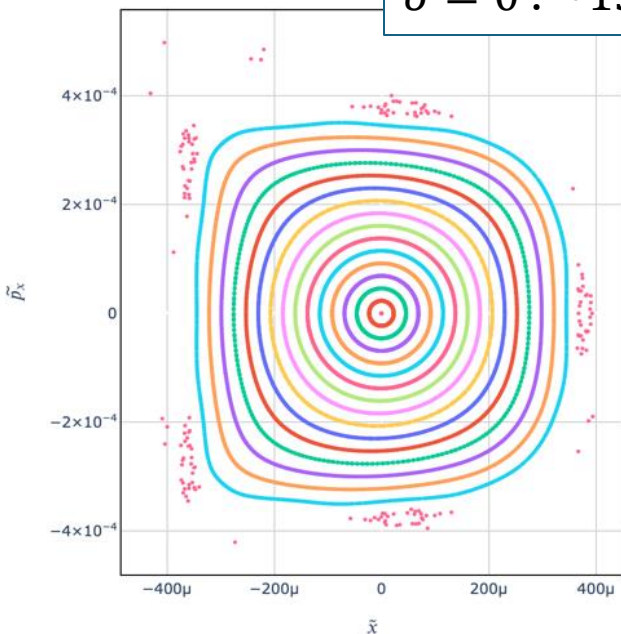


Chromatic correction section – v0.6

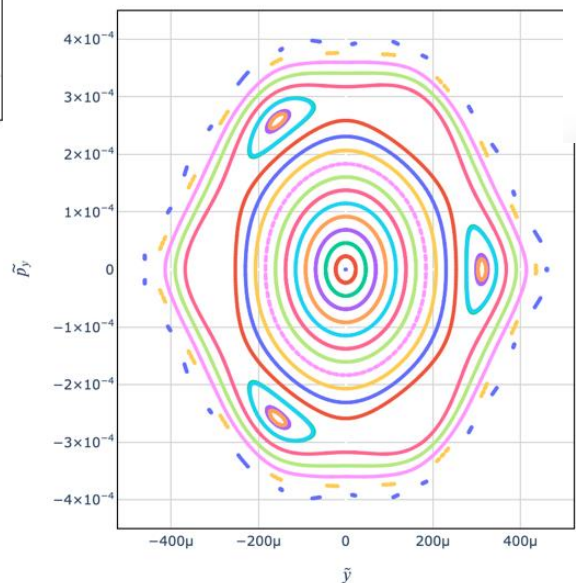


- Reduced DA for off-momentum particles: phase space plots

$\delta = 0 : \sim 15\sigma$

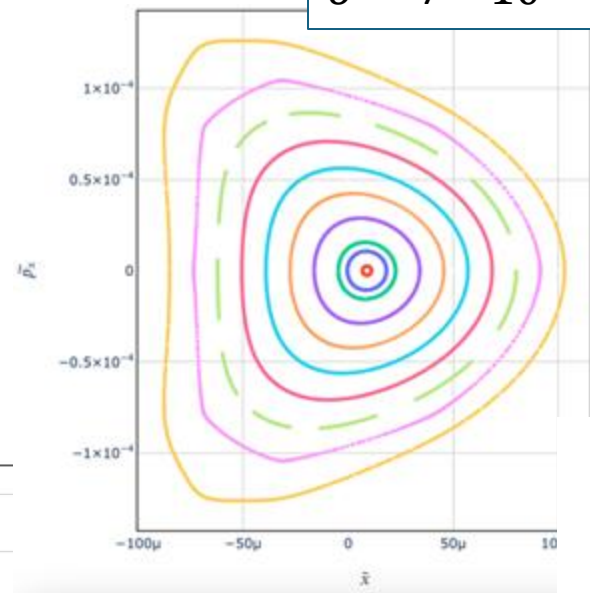


- $x_{init} = 0.0[\sigma]$
- $x_{init} = 1.0[\sigma]$
- $x_{init} = 2.0[\sigma]$
- $x_{init} = 3.0[\sigma]$
- $x_{init} = 4.0[\sigma]$
- $x_{init} = 5.0[\sigma]$
- $x_{init} = 6.0[\sigma]$
- $x_{init} = 7.0[\sigma]$
- $x_{init} = 8.0[\sigma]$
- $x_{init} = 9.0[\sigma]$
- $x_{init} = 10.0[\sigma]$
- $x_{init} = 11.0[\sigma]$
- $x_{init} = 12.0[\sigma]$
- $x_{init} = 13.0[\sigma]$
- $x_{init} = 14.0[\sigma]$
- $x_{init} = 15.0[\sigma]$
- $x_{init} = 16.0[\sigma]$



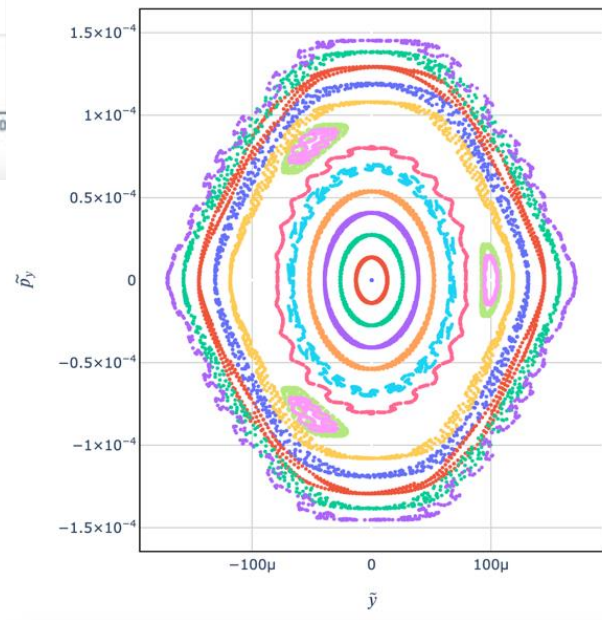
- $y_{init} = 5.0[\sigma]$
- $y_{init} = 6.0[\sigma]$
- $y_{init} = 7.0[\sigma]$
- $y_{init} = 8.0[\sigma]$
- $y_{init} = 9.0[\sigma]$
- $y_{init} = 10.0[\sigma]$
- $y_{init} = 11.0[\sigma]$
- $y_{init} = 12.0[\sigma]$
- $y_{init} = 13.0[\sigma]$
- $y_{init} = 14.0[\sigma]$
- $y_{init} = 15.0[\sigma]$
- $y_{init} = 16.0[\sigma]$
- $y_{init} = 17.0[\sigma]$
- $y_{init} = 18.0[\sigma]$
- $y_{init} = 19.0[\sigma]$
- $y_{init} = 20.0[\sigma]$

$\delta = 7 * 10^{-4} : \sim 4.5\sigma$



- $x_{init} = 0.5[\sigma]$
- $x_{init} = 1.0[\sigma]$
- $x_{init} = 1.5[\sigma]$
- $x_{init} = 2.0[\sigma]$
- $x_{init} = 2.5[\sigma]$
- $x_{init} = 3.0[\sigma]$
- $x_{init} = 3.5[\sigma]$
- $x_{init} = 4.0[\sigma]$
- $x_{init} = 4.5[\sigma]$
- $x_{init} = 5.0[\sigma]$
- $x_{init} = 5.5[\sigma]$
- $x_{init} = 6.0[\sigma]$

$\mu_{ySD1} = 0.75 - 0.0000225;$
 $\mu_{xSF1} = 2.25 + 0.000055$



- $y_{init} = 0.0[\sigma]$
- $y_{init} = 0.57[\sigma]$
- $y_{init} = 1.14[\sigma]$
- $y_{init} = 1.71[\sigma]$
- $y_{init} = 2.29[\sigma]$
- $y_{init} = 2.86[\sigma]$
- $y_{init} = 3.43[\sigma]$
- $y_{init} = 4.0[\sigma]$
- $y_{init} = 4.57[\sigma]$
- $y_{init} = 5.14[\sigma]$
- $y_{init} = 5.71[\sigma]$
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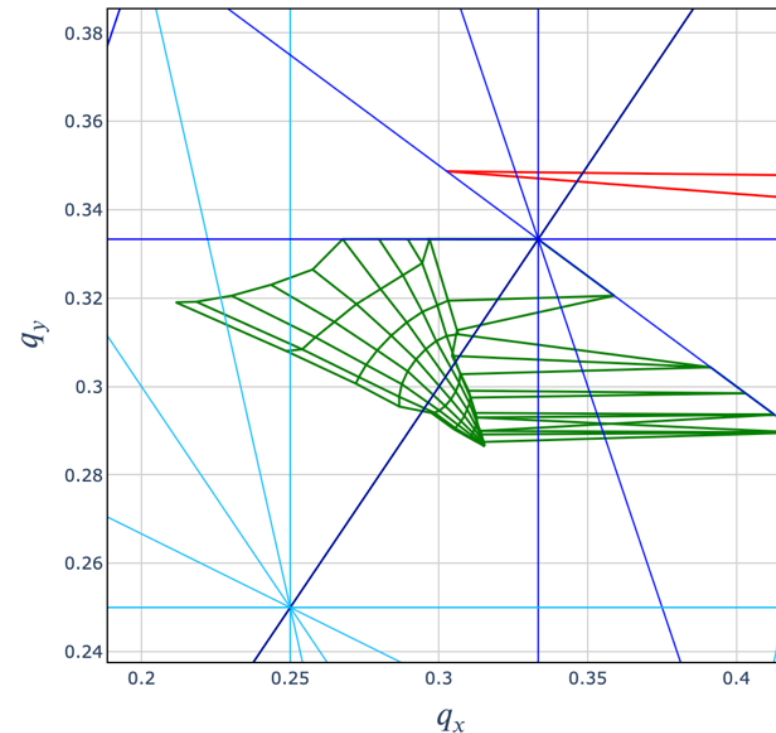
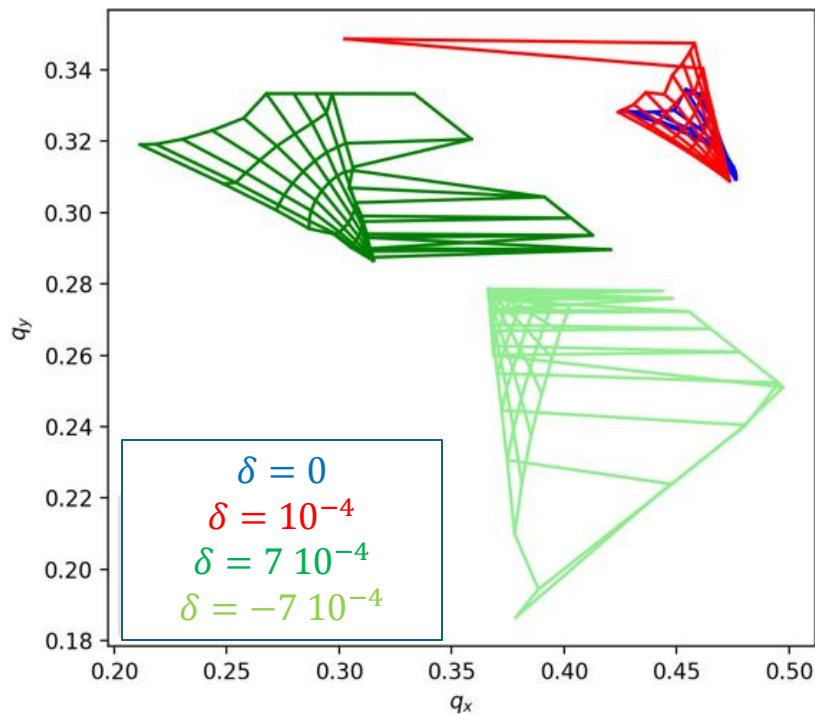


Chromatic correction section – v0.6



- Reduced DA for off-momentum particles: Footprint
 - XSUITE does not yet make the difference between nu and 1-nu
 - Tune shifts for off-momentum particles **reduced thanks to the optimized phase** at sextupoles (but **still large tune shifts**).
 - Larger amplitude detuning for off-momentum particles.

$$\begin{aligned}\mu_{y_{SD1}} &= 0.75 - 0.0000225; \\ \mu_{x_{SF1}} &= 2.25 + 0.000055\end{aligned}$$



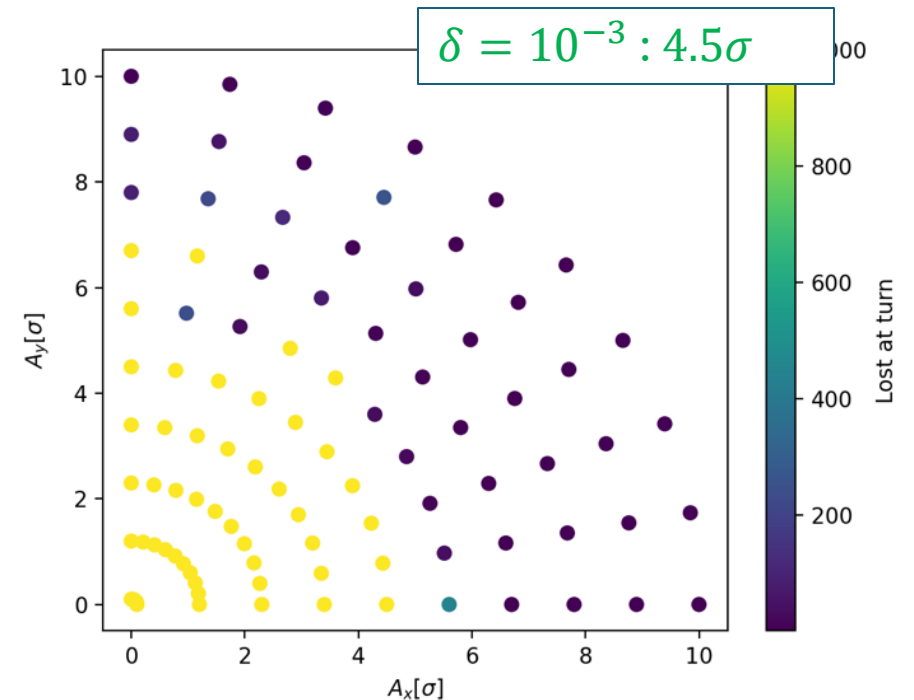
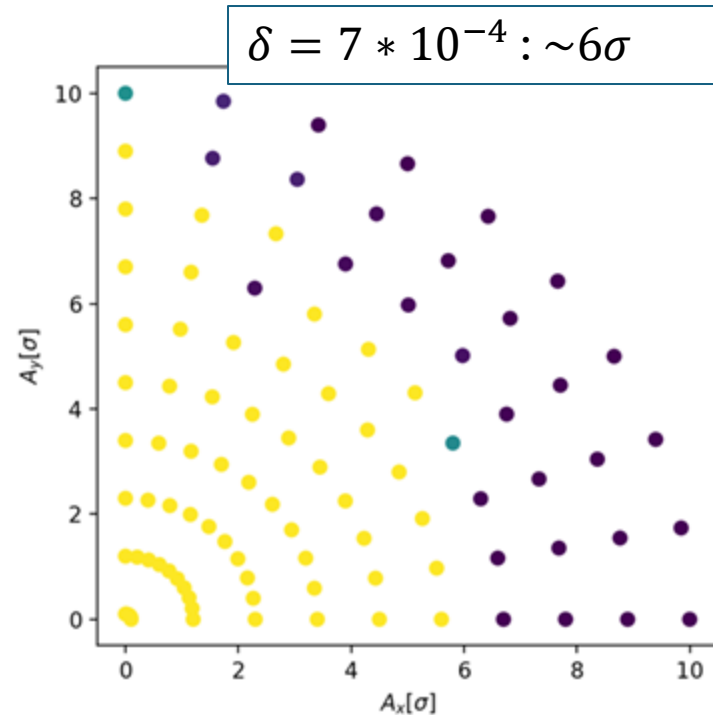
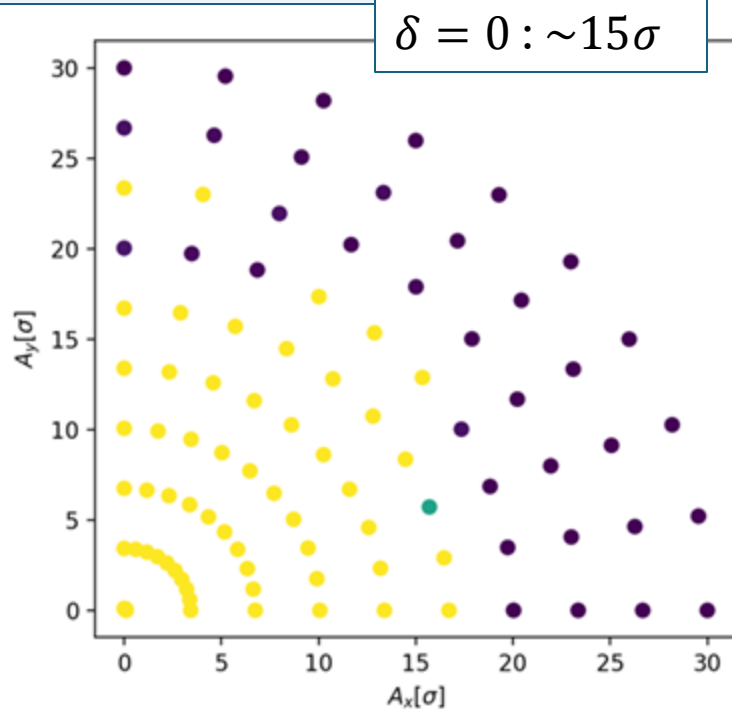


Chromatic correction section – v0.6



- Slight phase shift at the sextupoles helps reduce the non-linearities in β vs. δ :
 - Can be done manually or in a more automated manner
 - Increase the DA for several deltas

Optimized phase with automated cost

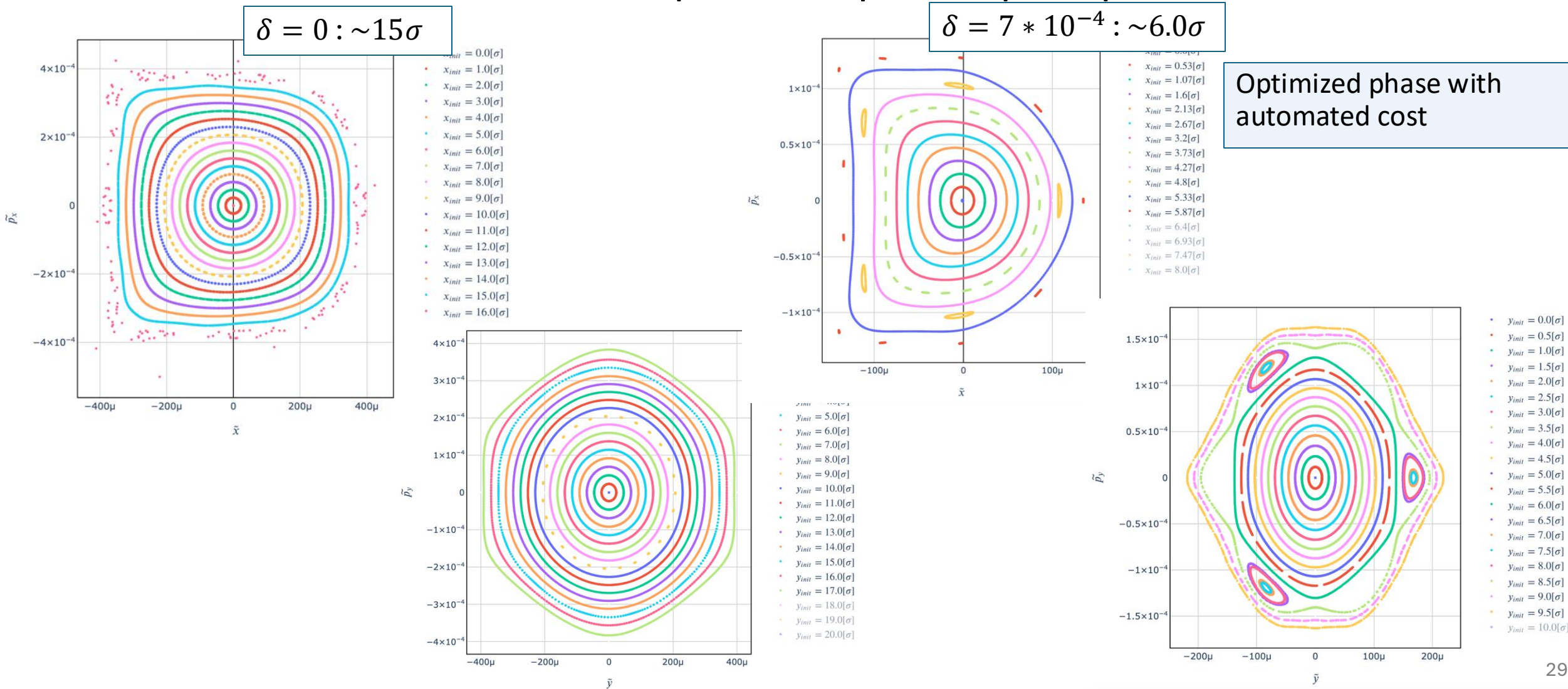




Chromatic correction section – v0.6



- Reduced DA for off-momentum particles: phase space plots



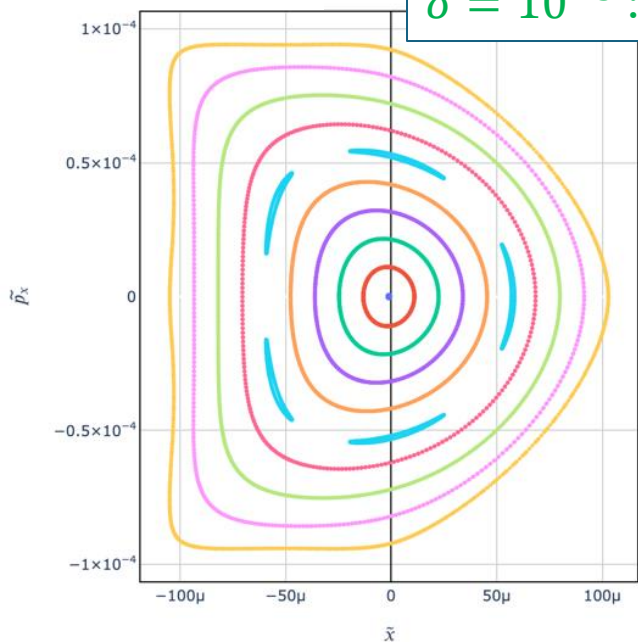


Chromatic correction section – v0.6



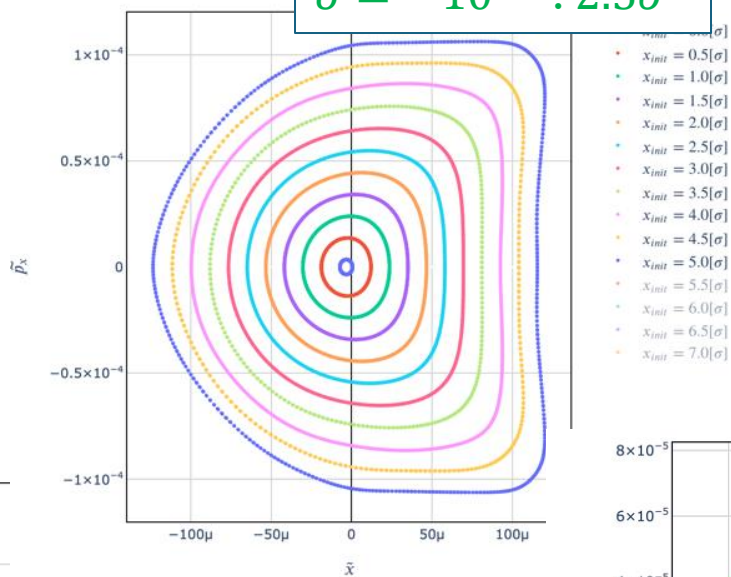
- Reduced DA for off-momentum particles: phase space plots

$\delta = 10^{-3} : 4.5\sigma$



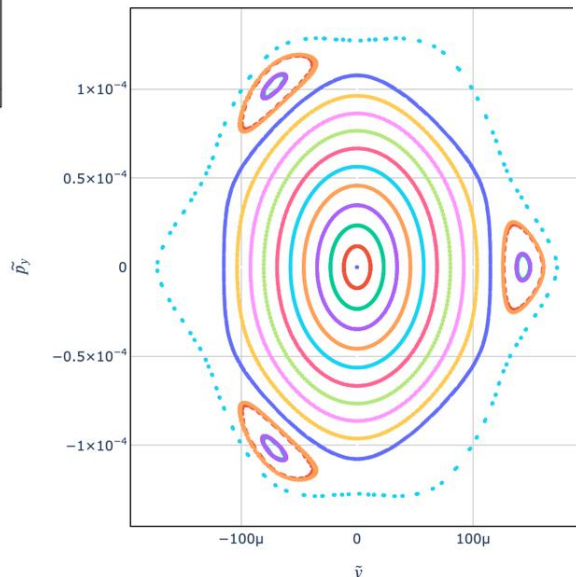
- $x_{init} = 0.5[\sigma]$
- $x_{init} = 1.0[\sigma]$
- $x_{init} = 1.5[\sigma]$
- $x_{init} = 2.0[\sigma]$
- $x_{init} = 2.5[\sigma]$
- $x_{init} = 3.0[\sigma]$
- $x_{init} = 3.5[\sigma]$
- $x_{init} = 4.0[\sigma]$
- $x_{init} = 4.5[\sigma]$
- $x_{init} = 5.0[\sigma]$
- $x_{init} = 5.5[\sigma]$
- $x_{init} = 6.0[\sigma]$

$\delta = -10^{-3} : 2.5\sigma$

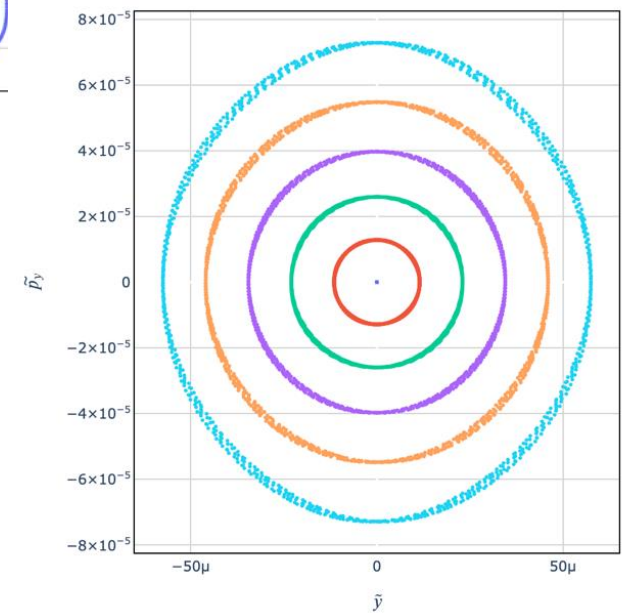


- $x_{init} = 0.5[\sigma]$
- $x_{init} = 1.0[\sigma]$
- $x_{init} = 1.5[\sigma]$
- $x_{init} = 2.0[\sigma]$
- $x_{init} = 2.5[\sigma]$
- $x_{init} = 3.0[\sigma]$
- $x_{init} = 3.5[\sigma]$
- $x_{init} = 4.0[\sigma]$
- $x_{init} = 4.5[\sigma]$
- $x_{init} = 5.0[\sigma]$
- $x_{init} = 5.5[\sigma]$
- $x_{init} = 6.0[\sigma]$
- $x_{init} = 6.5[\sigma]$
- $x_{init} = 7.0[\sigma]$

Optimized phase with automated cost



- $y_{init} = 2.0[\sigma]$
- $y_{init} = 2.5[\sigma]$
- $y_{init} = 3.0[\sigma]$
- $y_{init} = 3.5[\sigma]$
- $y_{init} = 4.0[\sigma]$
- $y_{init} = 4.5[\sigma]$
- $y_{init} = 5.0[\sigma]$
- $y_{init} = 5.5[\sigma]$
- $y_{init} = 6.0[\sigma]$
- $y_{init} = 6.5[\sigma]$
- $y_{init} = 7.0[\sigma]$
- $y_{init} = 7.5[\sigma]$
- $y_{init} = 8.0[\sigma]$



- $y_{init} = 0.0[\sigma]$
- $y_{init} = 0.5[\sigma]$
- $y_{init} = 1.0[\sigma]$
- $y_{init} = 1.5[\sigma]$
- $y_{init} = 2.0[\sigma]$
- $y_{init} = 2.5[\sigma]$
- $y_{init} = 3.0[\sigma]$
- $y_{init} = 3.5[\sigma]$
- $y_{init} = 4.0[\sigma]$

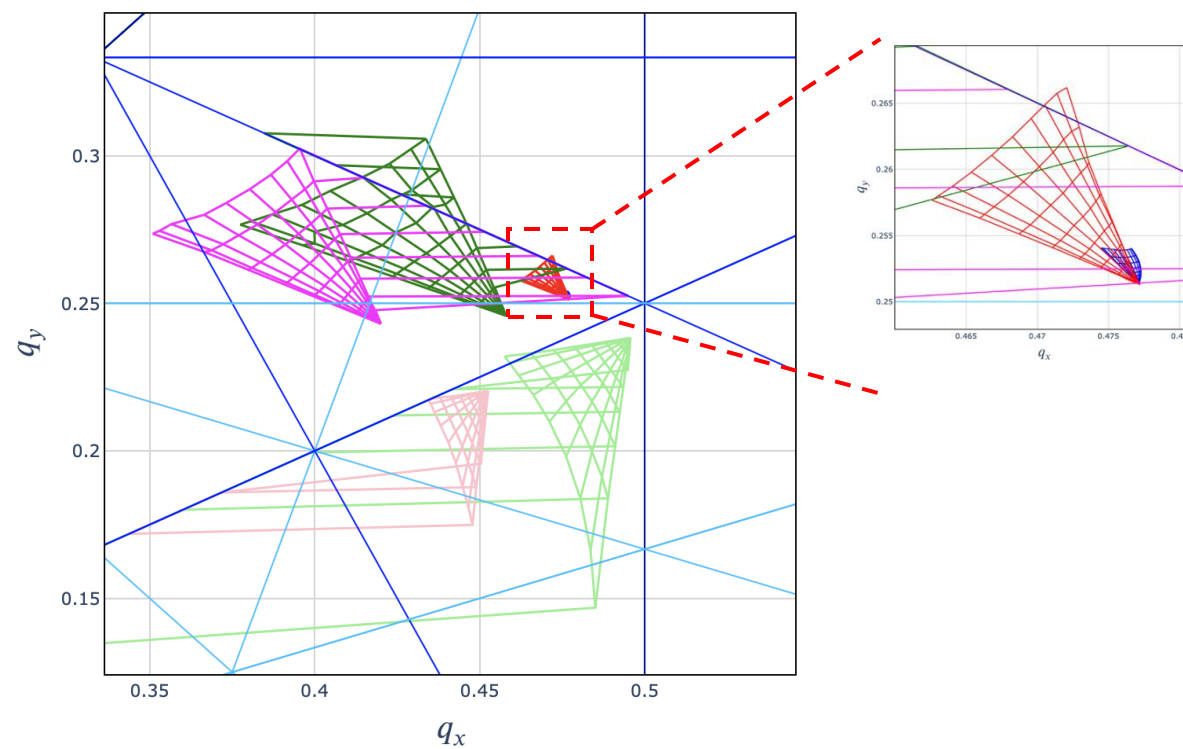
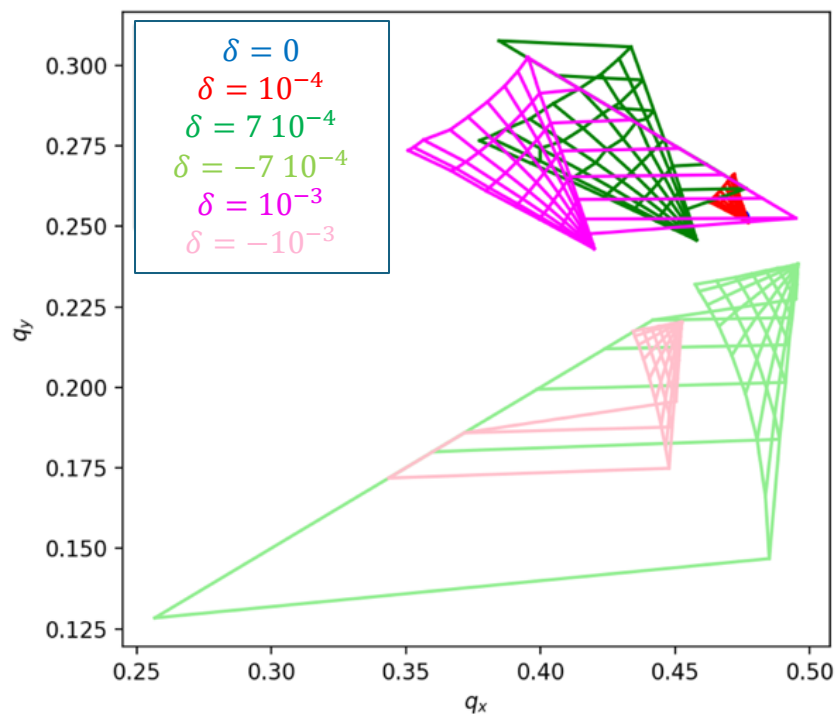


Chromatic correction section – v0.6



- Reduced DA for off-momentum particles: Footprint
 - XSUITE does not yet make the difference between nu and 1-nu
 - Tune shifts for off-momentum particles **reduced thanks to the optimized phase** at sextupoles (but still large tune shifts).
 - Larger amplitude detuning for off-momentum particles.

Optimized phase with automated cost



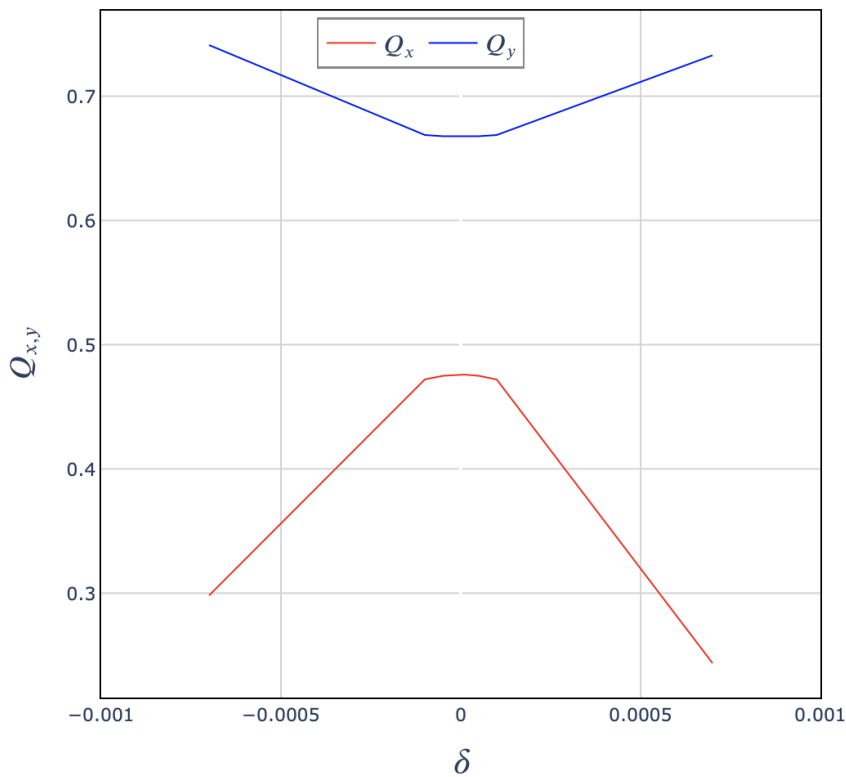


Chromatic correction section – v0.6

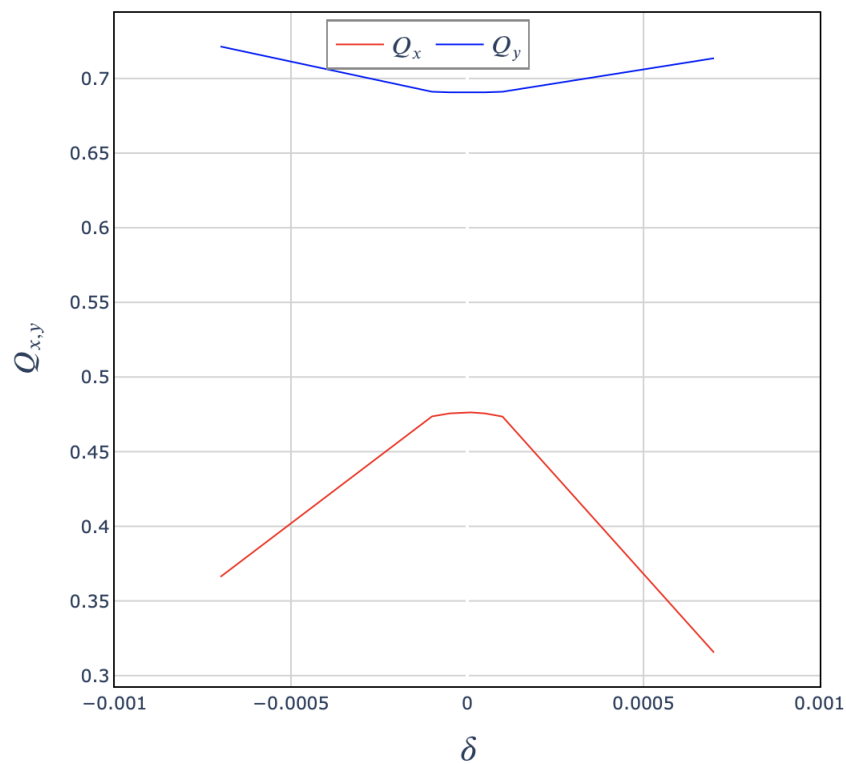


Dependence of the tunes on δ

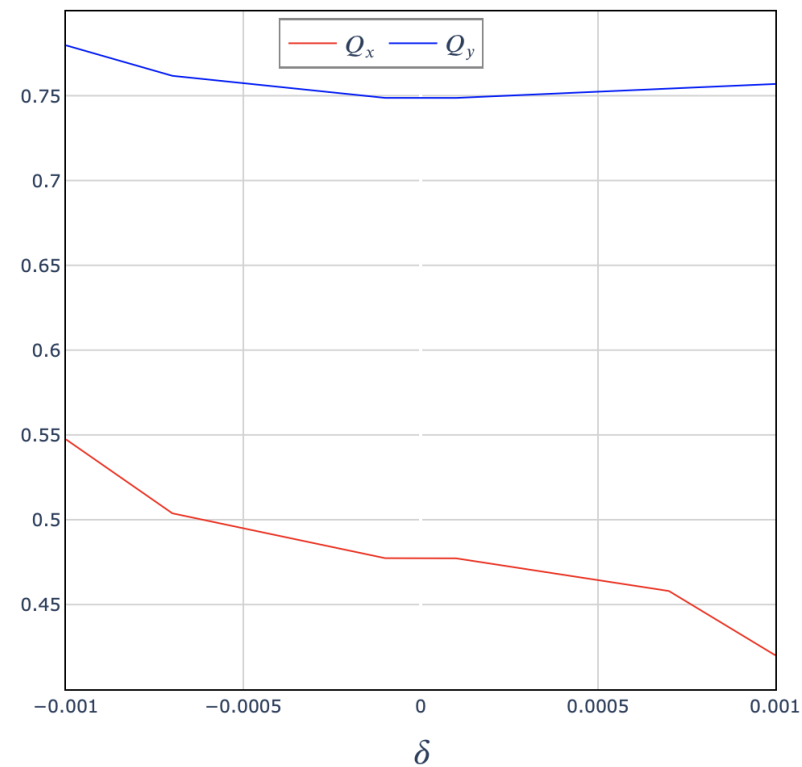
$$\begin{aligned}\mu_{y_{SD1}} &= 0.75; \\ \mu_{x_{SF1}} &= 2.25\end{aligned}$$



$$\begin{aligned}\mu_{y_{SD1}} &= 0.75 - 0.0000225; \\ \mu_{x_{SF1}} &= 2.25 + 0.000055\end{aligned}$$



Optimized phase with automated cost





CC – v07



Chromatic correction section – v0.7

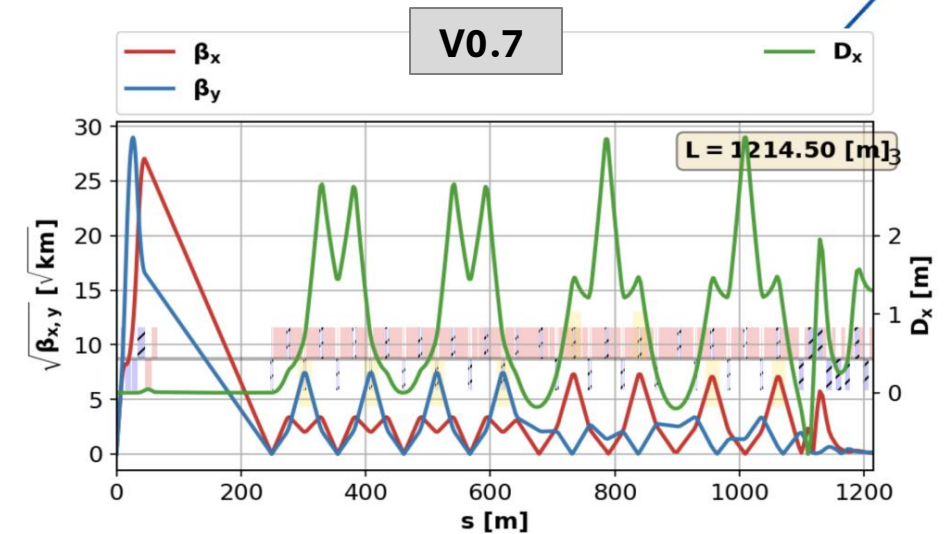


Main differences with v0.6:

Double pair of sextupoles to try to control the chromaticities and the matching of $W = \sqrt{A^2 + B^2}$ in the CC (independence of the CC and the arcs)

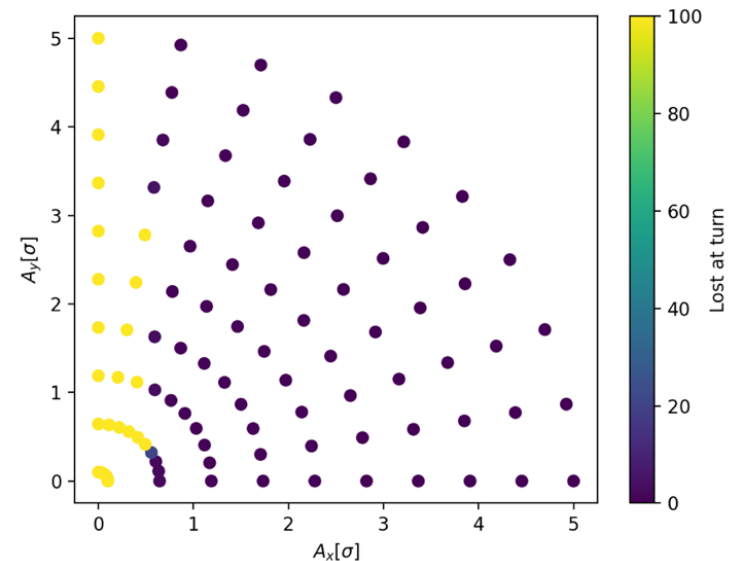
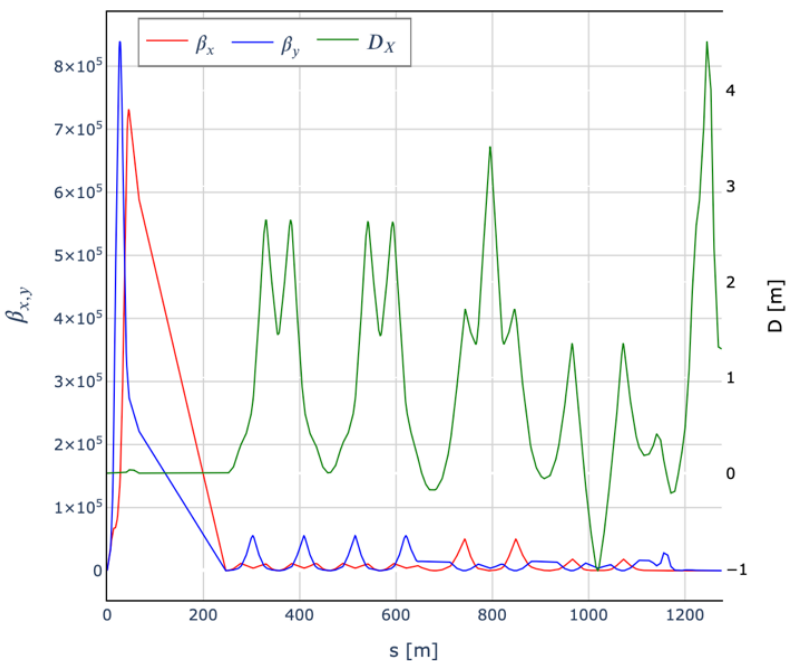
Main issues/problems in Kyriacos version:

- **Magnetic field of 50T** in the matching section
 - Better results by **limiting the quadrupole strength** in the matching of the MS ($B < 20T$ with shielding of 3cm).
- **Matching of A&B and chromaticities** not possible at the same time
 - Need 4 variables (double pair of sextupoles with slightly different phase advances at sextupoles) to correctly match $(A_x, B_x), (A_y, B_y)$ but **impossible to match W AND Q'**
 - Dependence between the CC section and the arcs (**no Q' control in the CC**)
- **D and β not the same in both sextupoles of each pair**





Chromatic correction section – v0.7



D and β not the same in both sextupoles of each pair

- Adapt the quadrupole and dipole strengths to find a suitable solution
- Lattice strongly constrained (almost all quad strengths used for matching purposes).
- The DA is very small, even for $\delta = 0$
 - The limitation came from a 3rd order resonance ($\nu_1 + 2\nu_2 = 2$), which should have been canceled out by the second sextupole of the pair.
 - **HUGE sensitivity of the lattice to the phase advance** (max error of 5^{-5} on the π phase advance between two sextupoles of a pair).

→ Doesn't seem well for future error studies!

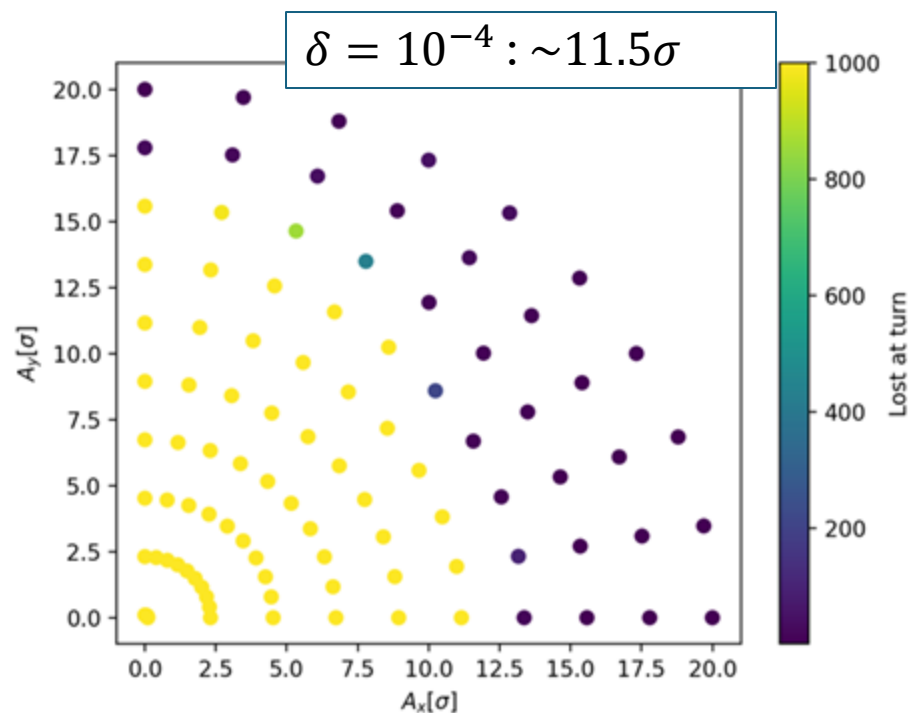
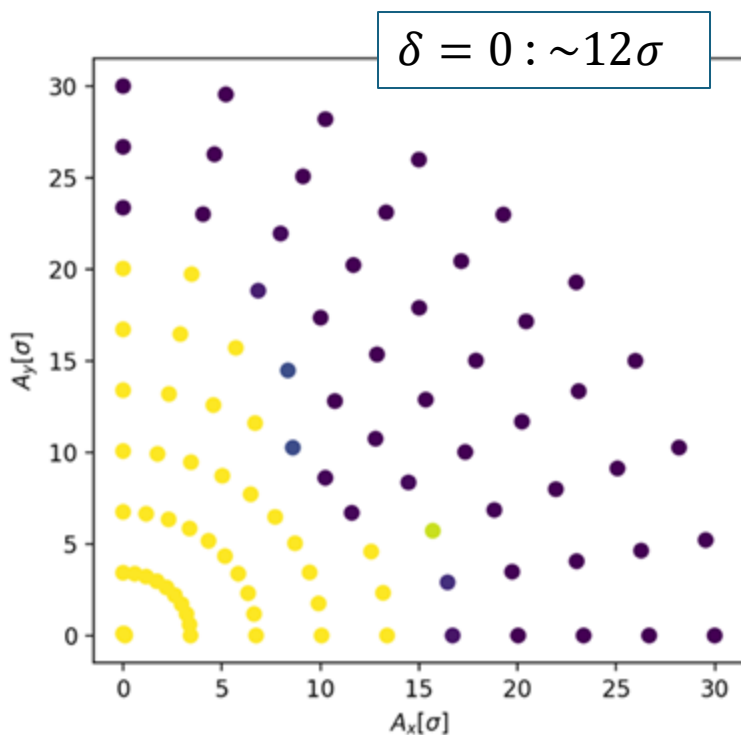


Chromatic correction section – v0.7



DA on the v0.7 with the correct MS and best chromaticity achieved without re-matching of the entire ring:

- DA **much smaller than for v0.6** (and probably the previous version of v7 as well?)
- The off-momentum DA is not sufficiently large \rightarrow **Really bad DA** starting from $\delta = 7 \cdot 10^{-4}$; even for the « baseline lattice », without any changes.



$\delta = 7 * 10^{-4} : 0\sigma$

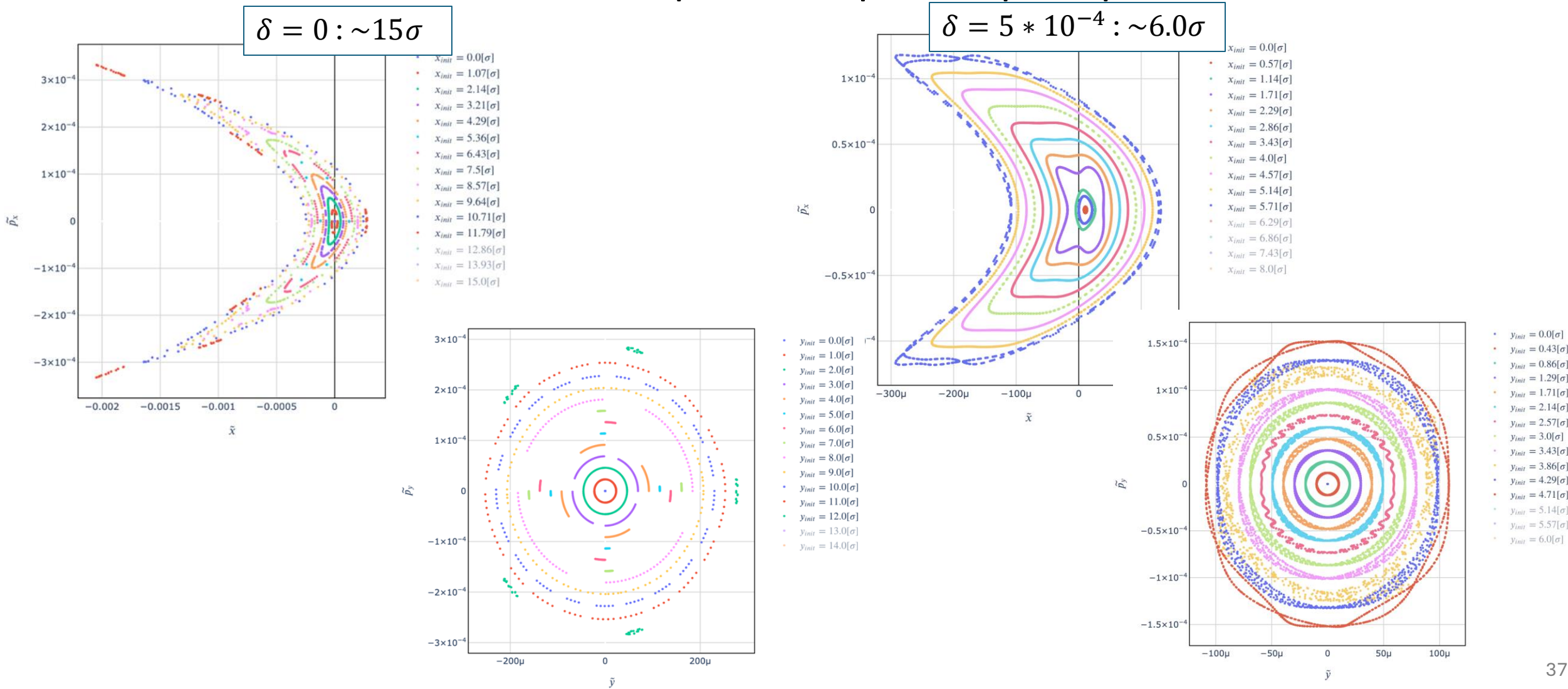




Chromatic correction section – v0.7



- Reduced DA for off-momentum particles: phase space plots

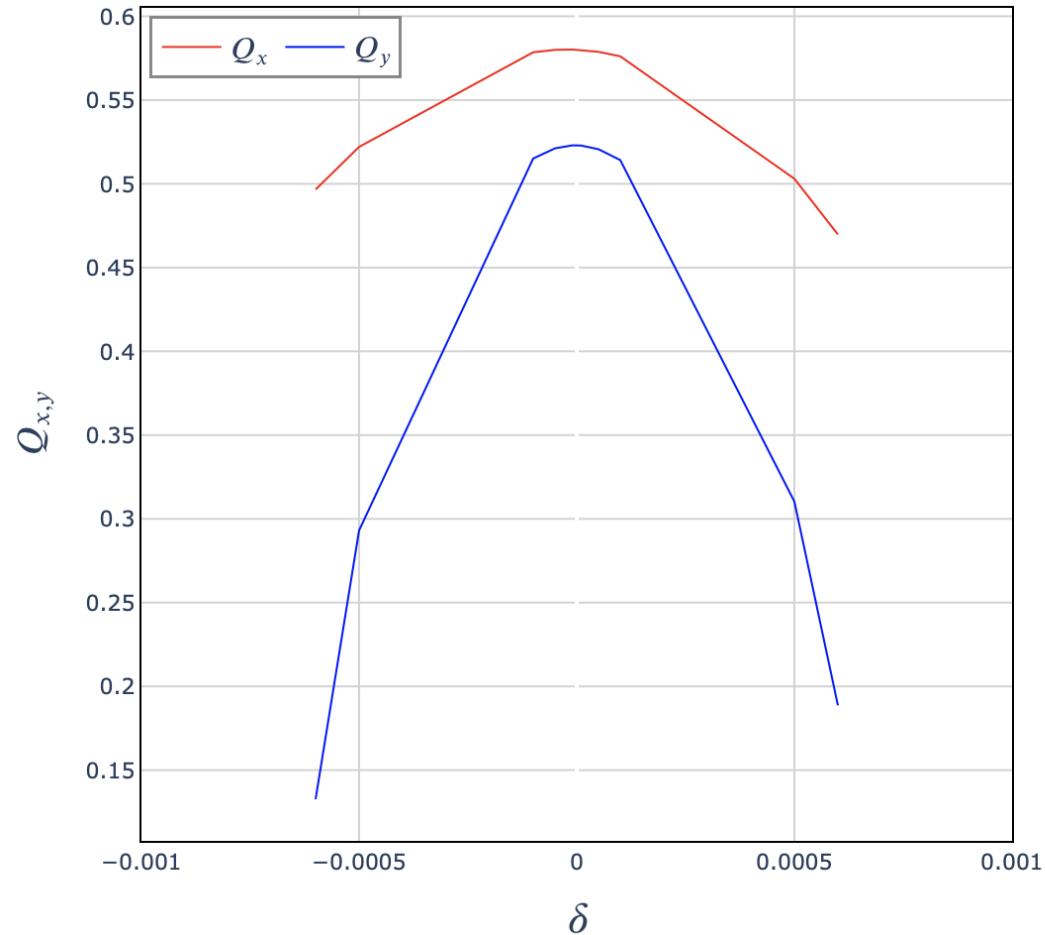




Chromatic correction section – v0.7



Dependence of the tunes on δ :

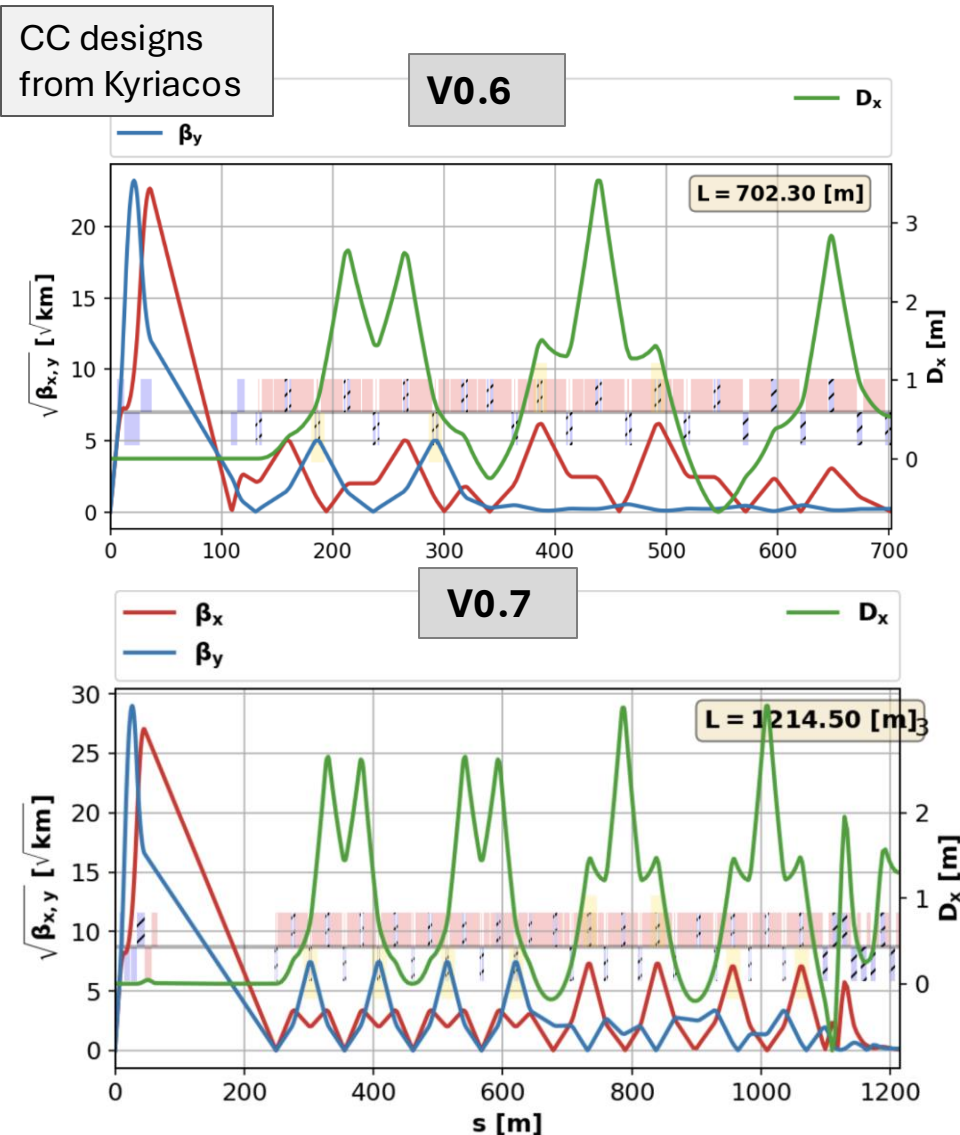




Local Chromatic correction section



Remaining issues of v0.6 & v0.7



- Sufficient dynamic aperture for on-momentum particles, but **requirements not yet met for the entire momentum range.**
- **Huge sensitivity to phase advance** error between the sextupoles of a pair \rightarrow poor DA for small errors.
- Beta remains very high in the CC (compared to a circumference of 10 km) \rightarrow The lattice will probably be very sensitive to alignments/field errors.
- **No** clear knobs to **control the working point** in the current lattice.
- The sextupoles are placed on the slope of the dispersion. Ideally, we would like to place them at a maximum of the dispersion ($D' = 0$).
- Matching section between CC and arcs not easy to match

New optics attempt: « v0.8 »

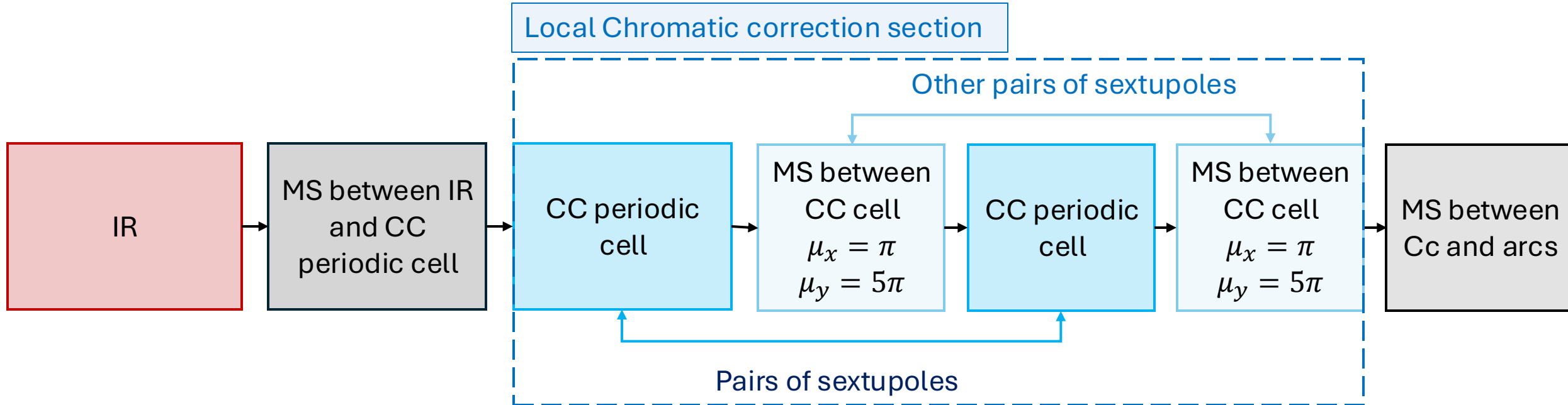
Goals of the "new optics"

Change the optics in order to have:

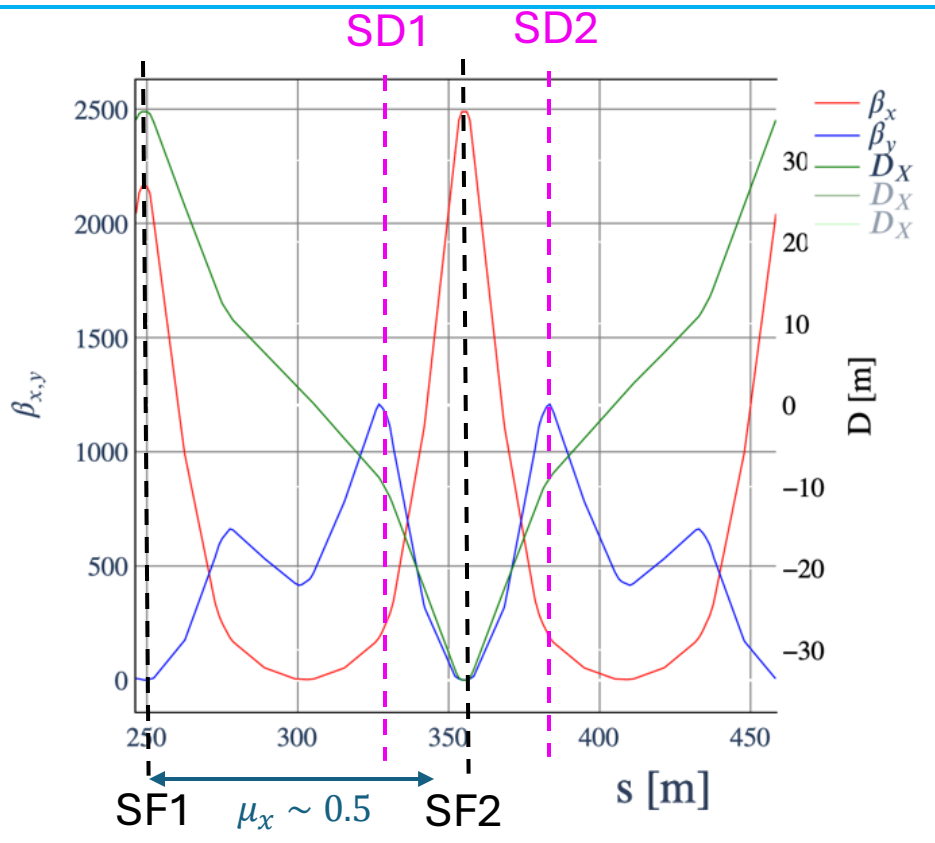
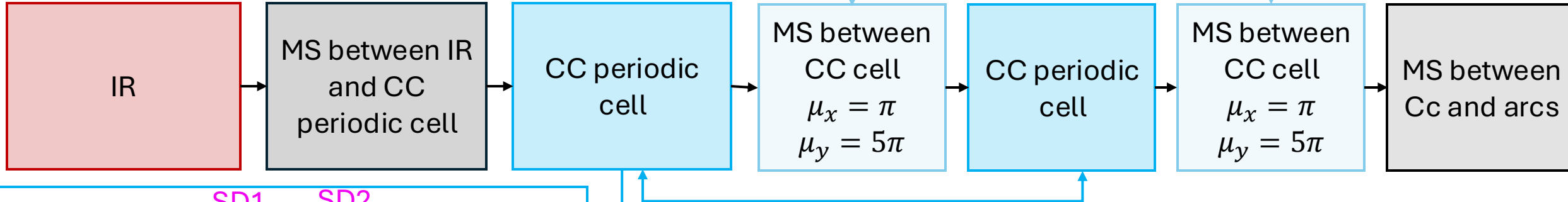
- **Lower sextupole strengths** and smaller β functions in CC (less sensitive to small errors).
- A **periodic and symmetric cell** for the CC that could be **repeated** as needed, without having to adjust all the quad strengths in each part (less constrained lattice); We automatically have the **same β and D in the sextupoles of a pair**.
- **Maximum dispersion** at the **sextupoles** ($\tilde{D}' = 0$).
- **More sextupoles** to have more variables to match all the required quantities (W_x, W_y, Q'_x, Q'_y); all sextupoles in a same cell.

→ CC with **dispersion oscillation** to take advantage of a **larger dispersion** and reduce the beta functions and sextupole strengths.

V0.8 - Structure of the CC



V0.8 - Structure of the CC



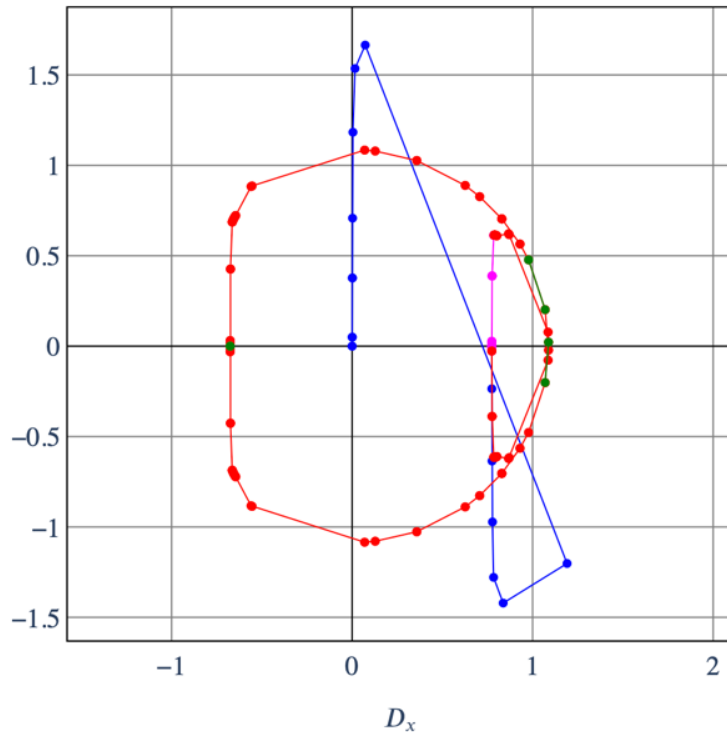
Pairs of sextupoles

- The **maximum of β_x** corresponds to the **maximum of dispersion** \rightarrow Sextupoles to correct chromaticity errors in X.
 - The **maximum of β_y** corresponds to $D \sim -8m \rightarrow$ Sextupoles to correct chromaticity errors in Y.
 - Periodic cell: $\mu_x \sim \mu_y \sim 2\pi$ ($q_1 = 0.985, q_2 = 0.994$).
 - **Need for a second cell**, shifted by π in μ_x and μ_y to have sextupole pairs and **cancel the non-linear kicks**.
- \rightarrow Need for a **matching section between 2 CC cells** with $\mu_x = (2n + 1)\pi, \mu_y = (2n + 1)\pi$.

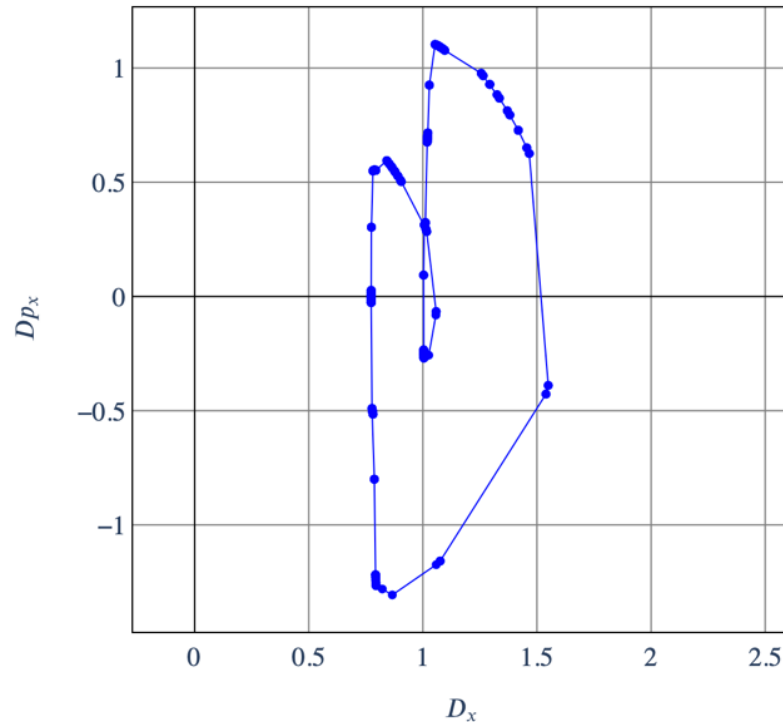
V0.8 - Matching section between 2 CC cells

- Matching section done manually based on the plots of the normalized dispersion: $\mu_x = \pi$, $\mu_y = 5\pi$.

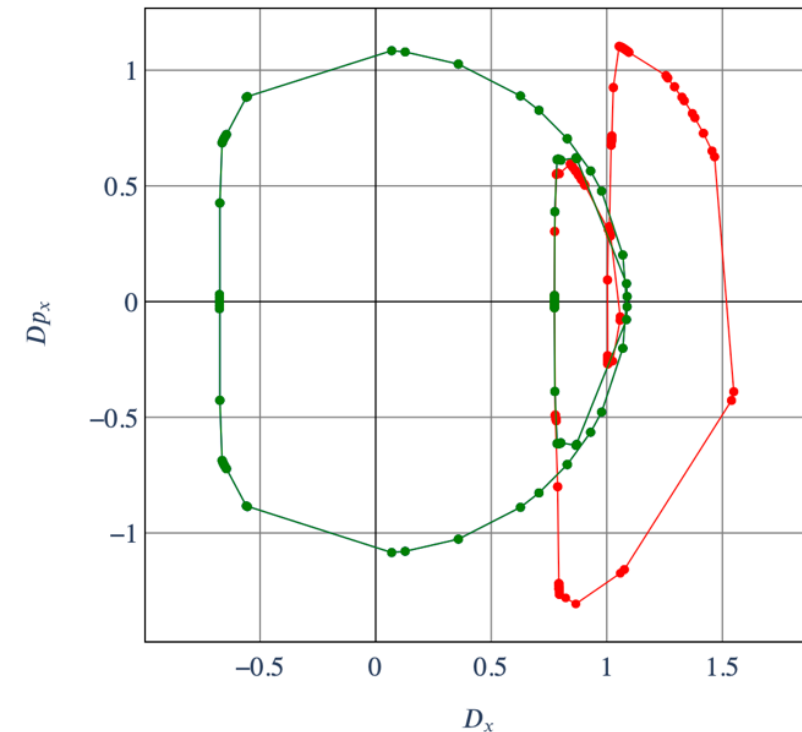
From ID \rightarrow end of CC cell



From ID \rightarrow end of CC cell



Start of 1st CC cell
 \rightarrow End of 2nd CC cell

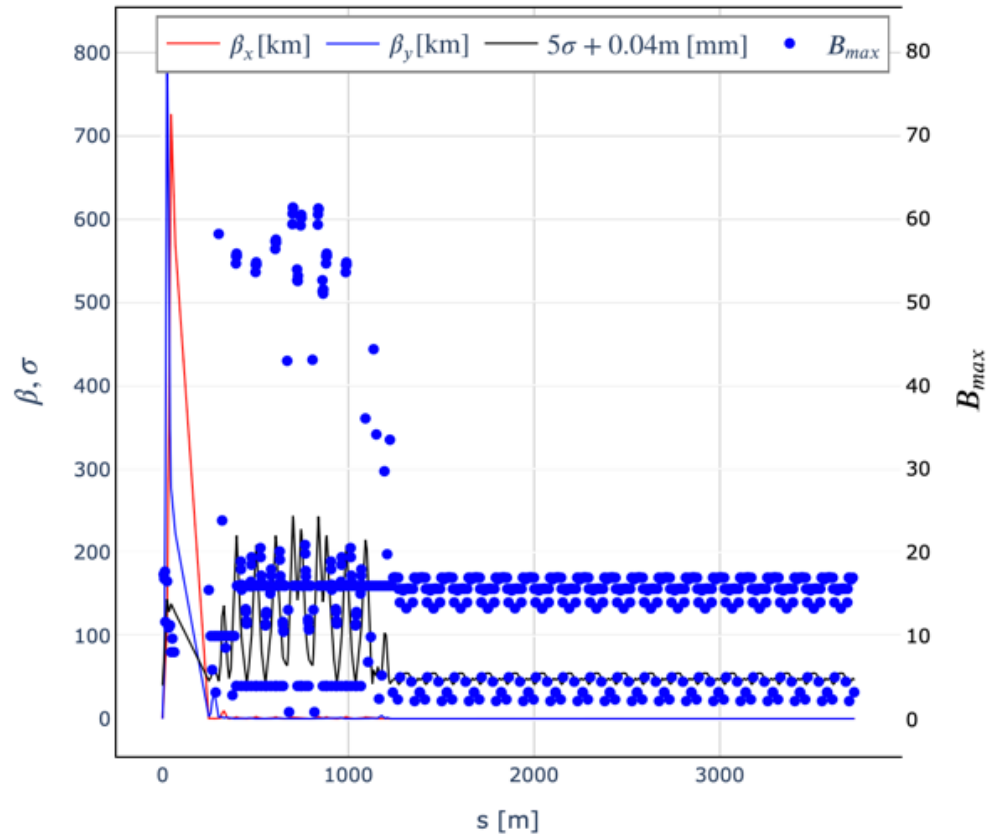
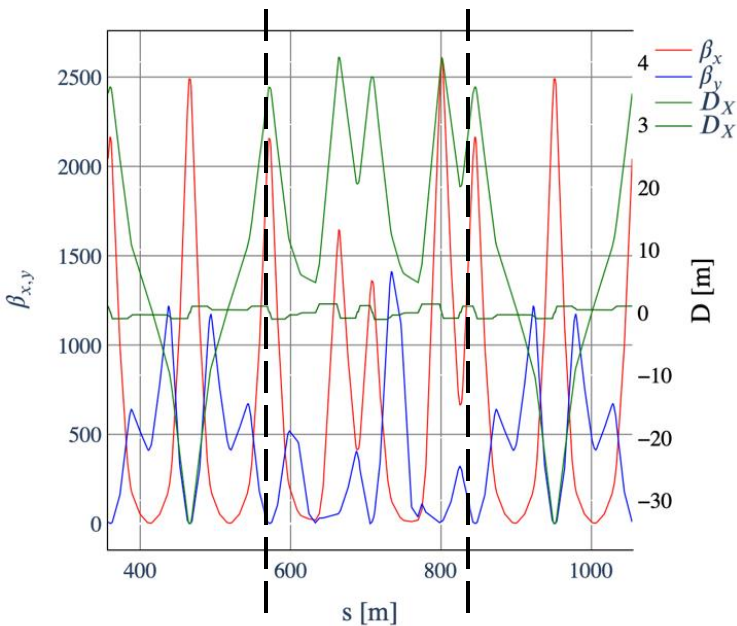
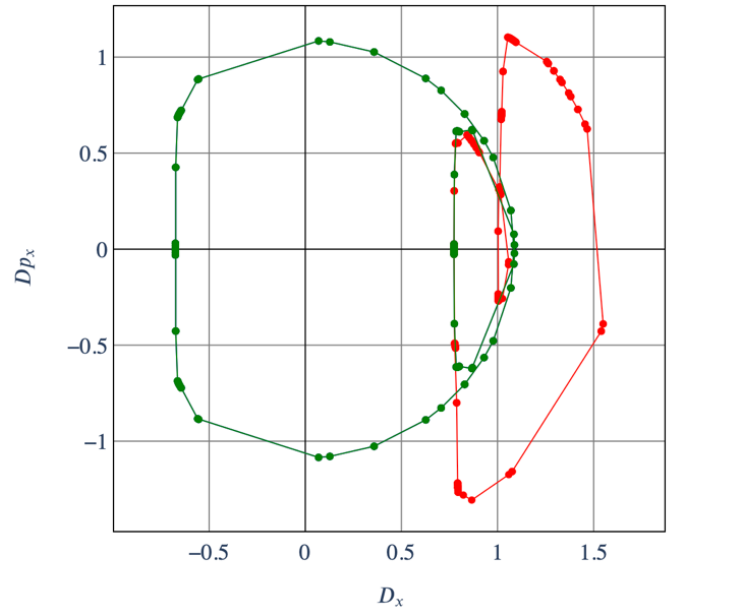




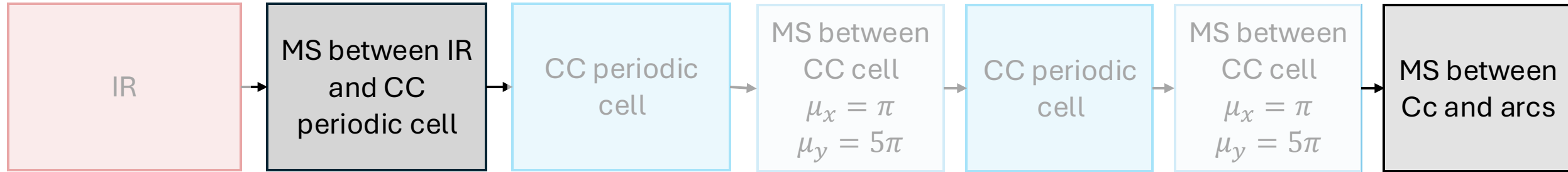
V0.8 - Matching section between 2 CC cells



- Large dispersion \rightarrow Large magnet apertures and magnetic field



Matching sections IR-CC; CC-arcs



MS between IR and CC:

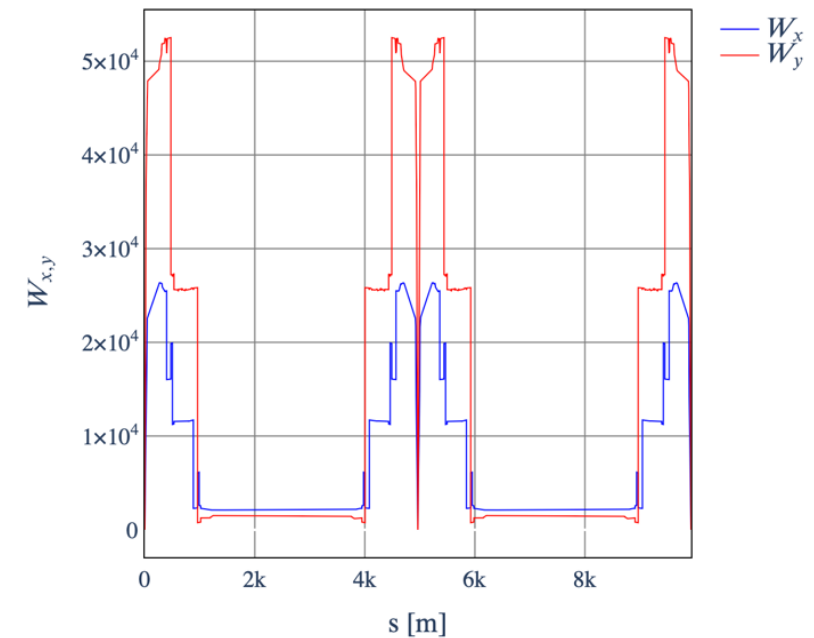
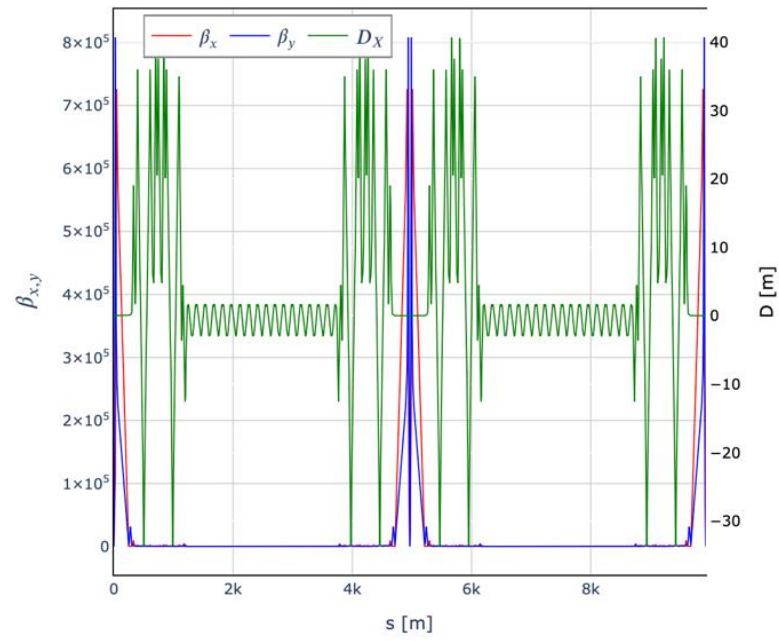
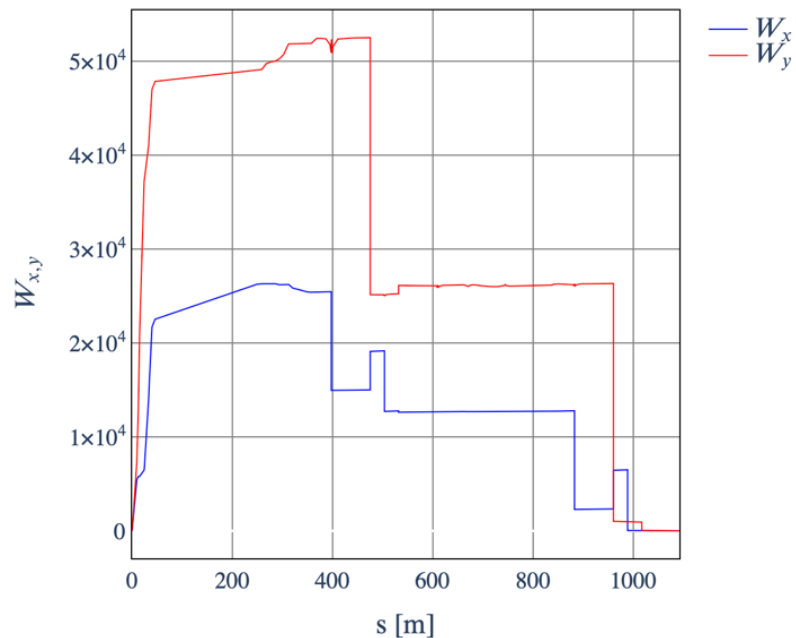
- **Difficult** matching section because it requires significantly **increasing the dispersion** with not too high beta functions, while respecting the **phase constraints** : $\mu_x = 1.25$ at first SF, $\mu_y = 0.75$ at first SD.
- Magnetic field too high.

MS between CC and arcs:

- Fairly **easy** to find a working solution, **limiting the quad strengths** to reduce the magnetic field.
- May be used later to **adjust the working point**.

Matching of W_x , W_y

- The **sextupoles strengths** have been adjusted to correctly **match W_x and W_y** at the entrance of the arcs.
 - **No control of chromaticities** → Rematch of the entire ring to achieve $W_x = W_y = 0$, and zero chromaticities → **Large W** in the arcs!
- **Need to control the chromaticities in the CC to have $W < 1000$ in the arcs!**



- **Quads in the MS** between IR and CC that **drive linear chromaticity** at $\mu_x \sim 1$
- Place SF sextupole at a phase multiple of $1[2\pi]$ to correct Q'_x not coming from the FF triplet.

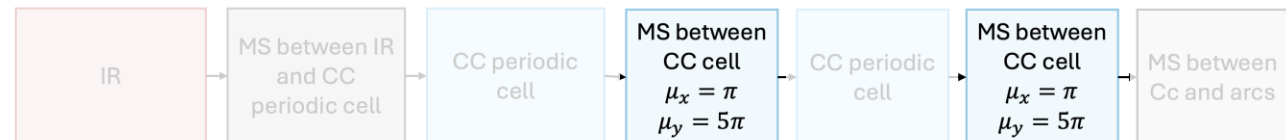
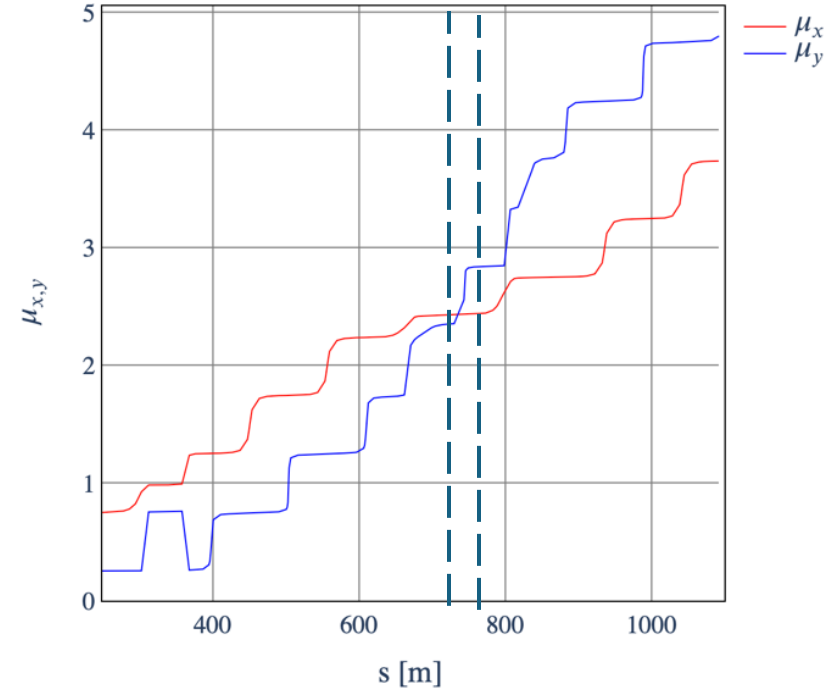
There are two terms that are independent of the phase variable

$$\begin{aligned} h_{11001} &= \frac{1}{4} \sum_{i=1}^N [(b_2L)_i - 2(b_3L)_i \eta_{xi}^{(1)}] \beta_{xi} + O(\delta^2), \\ h_{00111} &= -\frac{1}{4} \sum_{i=1}^N [(b_2L)_i - 2(b_3L)_i \eta_{xi}^{(1)}] \beta_{yi} + O(\delta^2) \end{aligned} \quad (95)$$

which drive the linear chromaticity, the initial reason for introducing sextupoles into the lattice. The remaining are

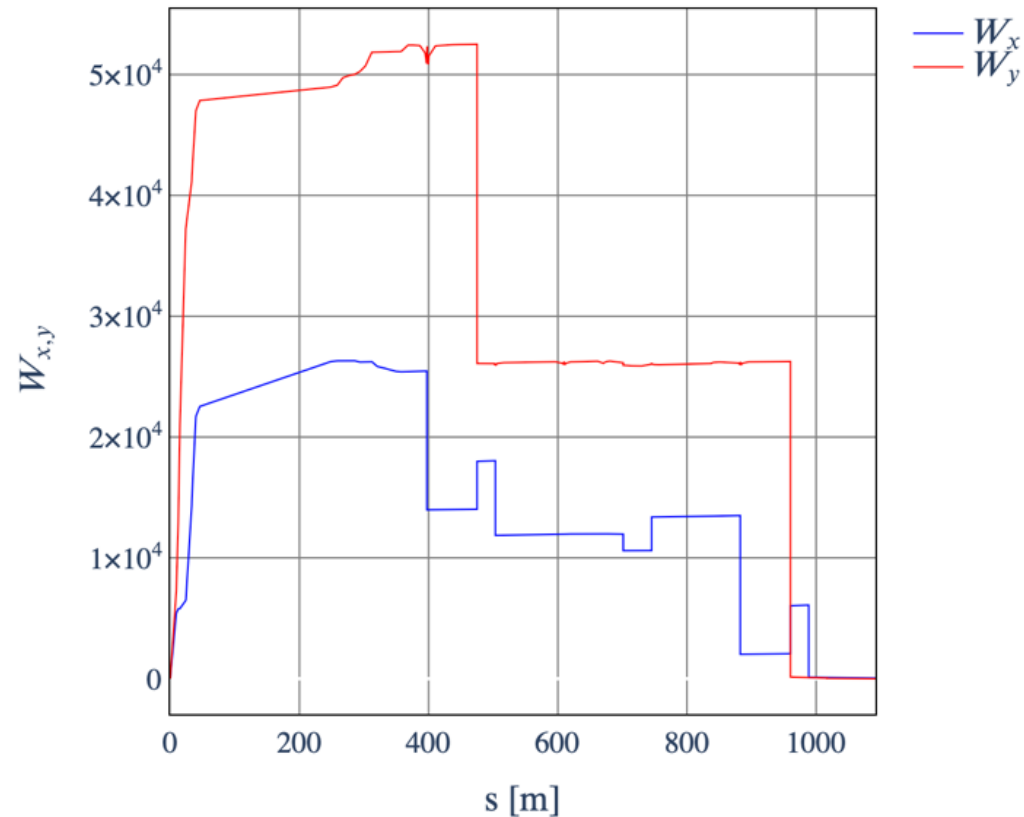
$$\begin{aligned} h_{20001} &= h_{02001}^* = \frac{1}{8} \sum_{i=1}^N [(b_2L)_i - 2(b_3L)_i \eta_{xi}^{(1)}] \beta_{xi} e^{i2\mu_{xi}} + O(\delta^2), \\ h_{00201} &= h_{00021}^* = -\frac{1}{8} \sum_{i=1}^N [(b_2L)_i - 2(b_3L)_i \eta_{xi}^{(1)}] \beta_{yi} e^{i2\mu_{yi}} + O(\delta^2), \\ h_{10002} &= h_{01002}^* = \frac{1}{2} \sum_{i=1}^N [(b_2L)_i - (b_3L)_i \eta_{xi}^{(1)}] \eta_{xi}^{(1)} \sqrt{\beta_{xi}} e^{i\mu_{xi}} + O(\delta^3) \end{aligned} \quad (96)$$

- Add sextupoles in the MS between 2 CC cells, at $\mu_x = 2.42$ and $\mu_x = 2.43$; need a second MS at the end of the CC to have sextupole pairs.



V0.8 – Q' control in CC

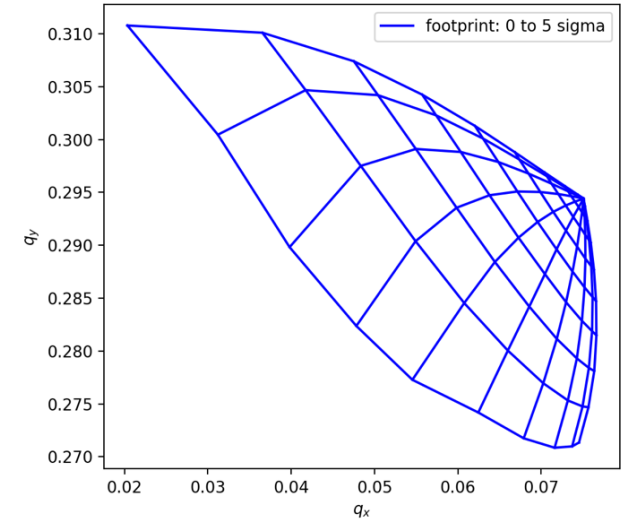
- Possible to **control of W and Q' in the CC**
- Independence of the CC and the arcs.



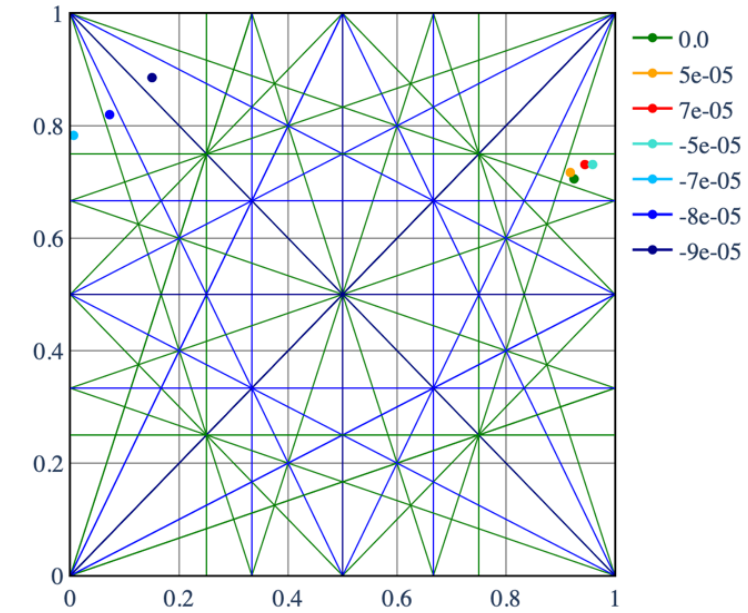
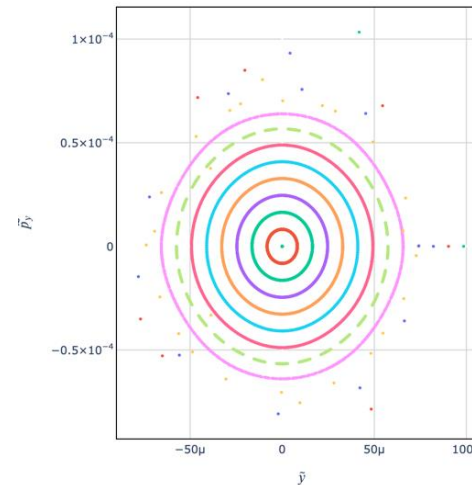
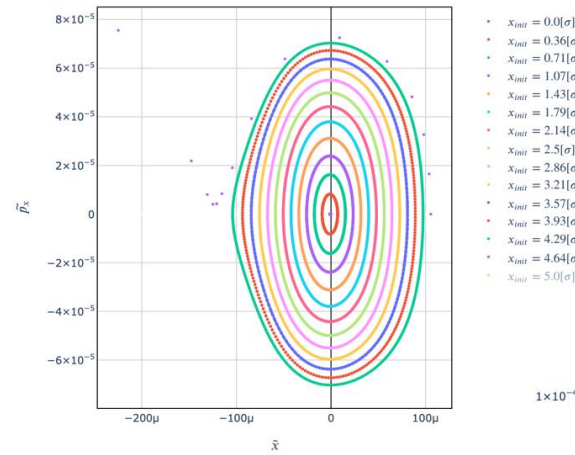
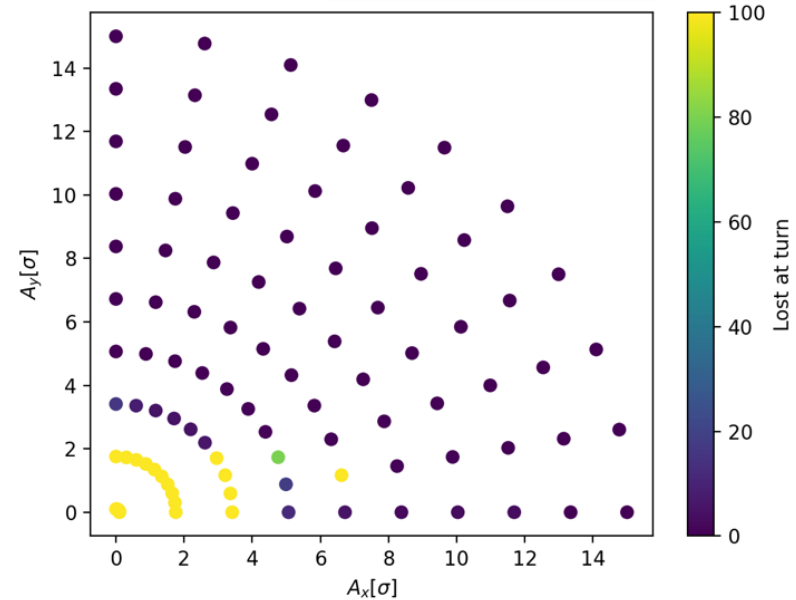
V0.8 – Dynamic apertures

- Poor DA and momentum acceptance
 - Due to higher order chromaticities ?

$$Q_x'', Q_y'' = 1.9 e^5, 1.2 e^7$$



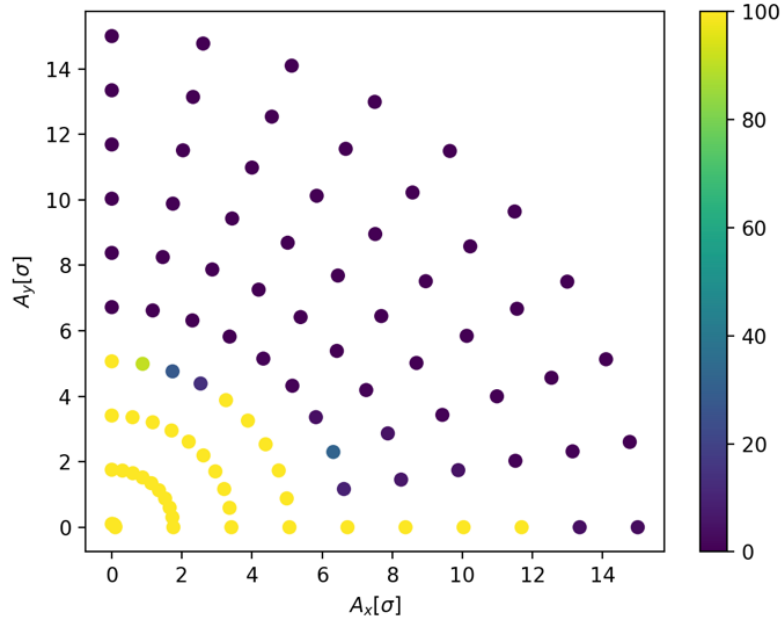
$\delta = 0 : \sim 2\sigma$



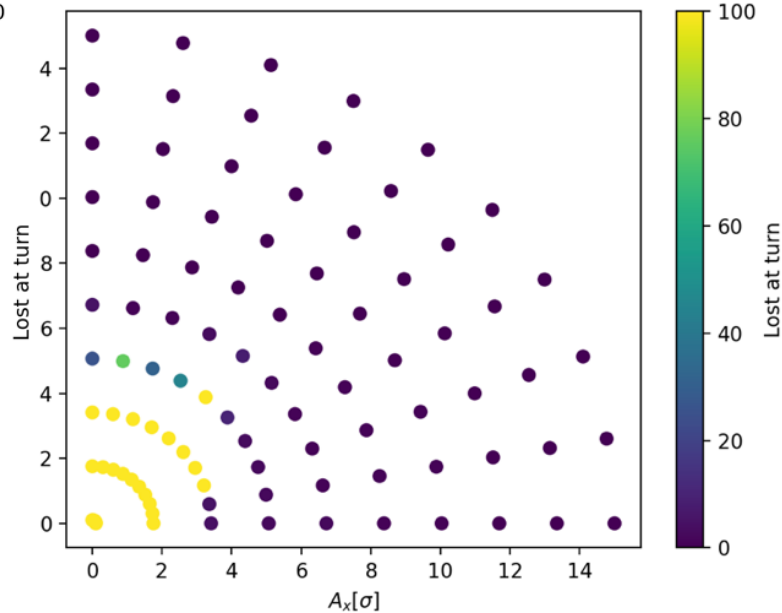
V0.8 – Dynamic apertures

- Poor DA and momentum acceptance
 - Match the second order chromaticities on the entire ring with all sets of sextupoles
 - Increased but still small DA even for $\delta = 0$; DA increased to 8σ if we divide all sextupole strengths by 2 → Sextupole strengths too high ?

$\delta = 0 : \sim 4\sigma$



$\delta = 10^{-4} : \sim 3.1\sigma$



$\delta > 10^{-4} : \sim 0\sigma$



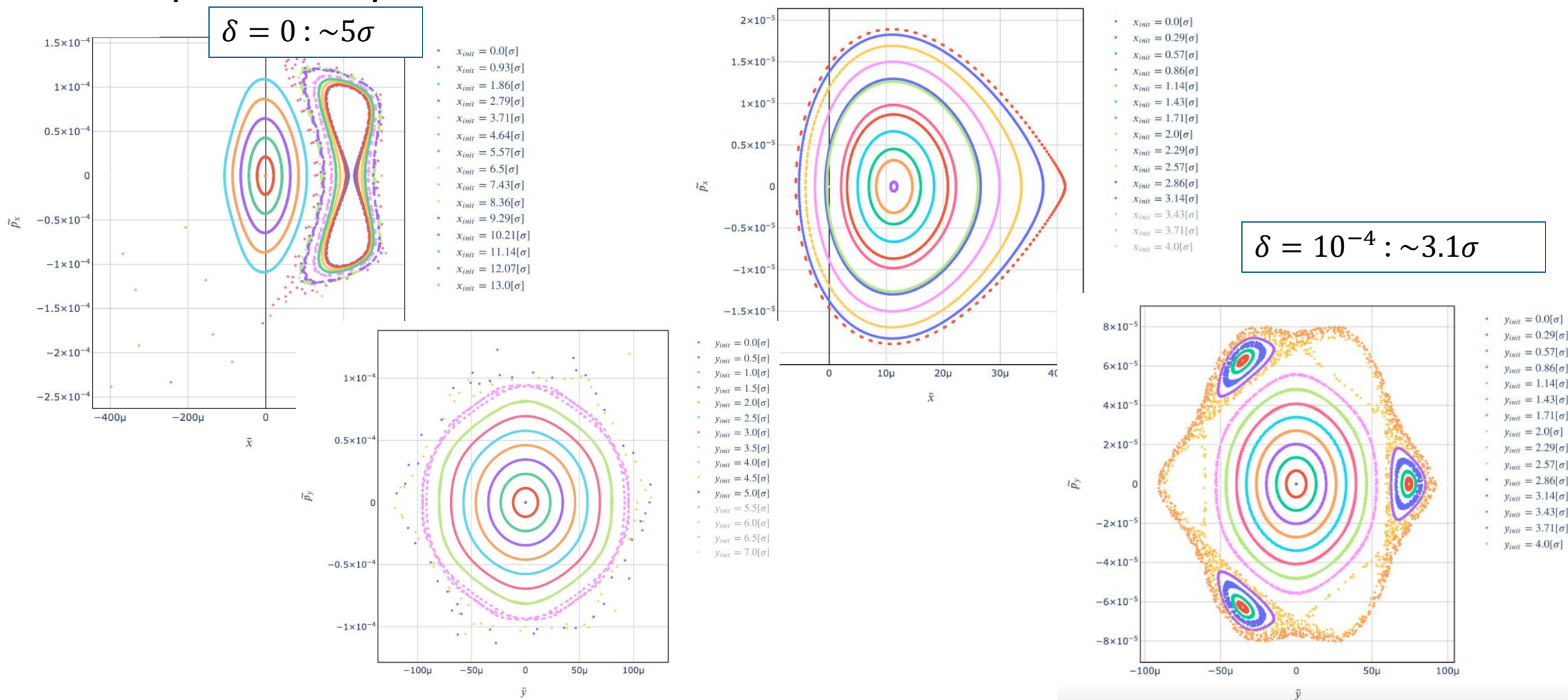
0	ON	False	0.00500132	0.00500132	0	'dqx', val=0,
1	ON	True	9.9476e-07	9.9476e-07	0	'dqy', val=0,
2	ON	False	0.792507	0.792507	0	('wx_chrom',
3	ON	False	0.0361833	0.0361833	0	('wy_chrom',
4	ON	False	-7.86571	-7.86571	0	'ddqx', val=0,
5	ON	False	6.62226	6.62226	0	'ddqy', val=0,



V0.8 – phase spaces

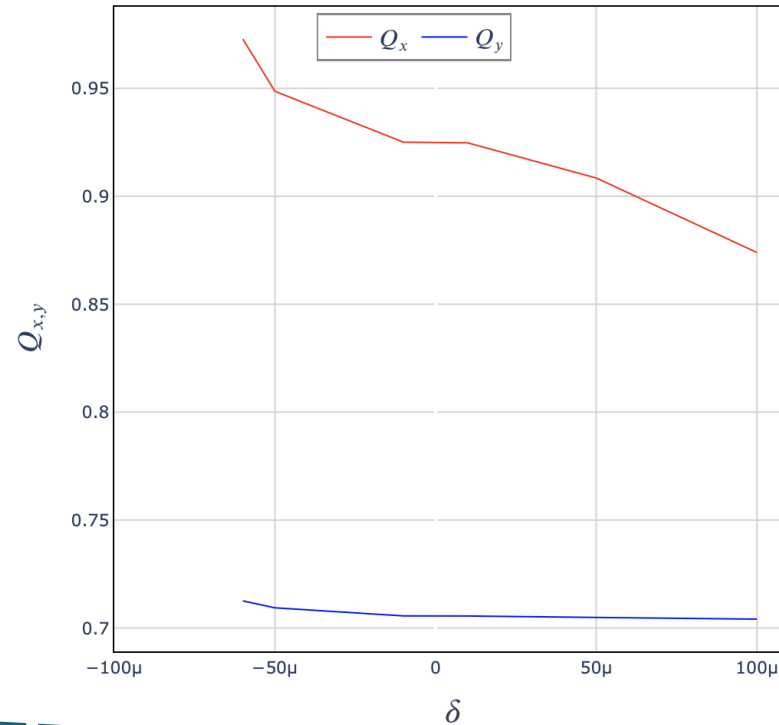
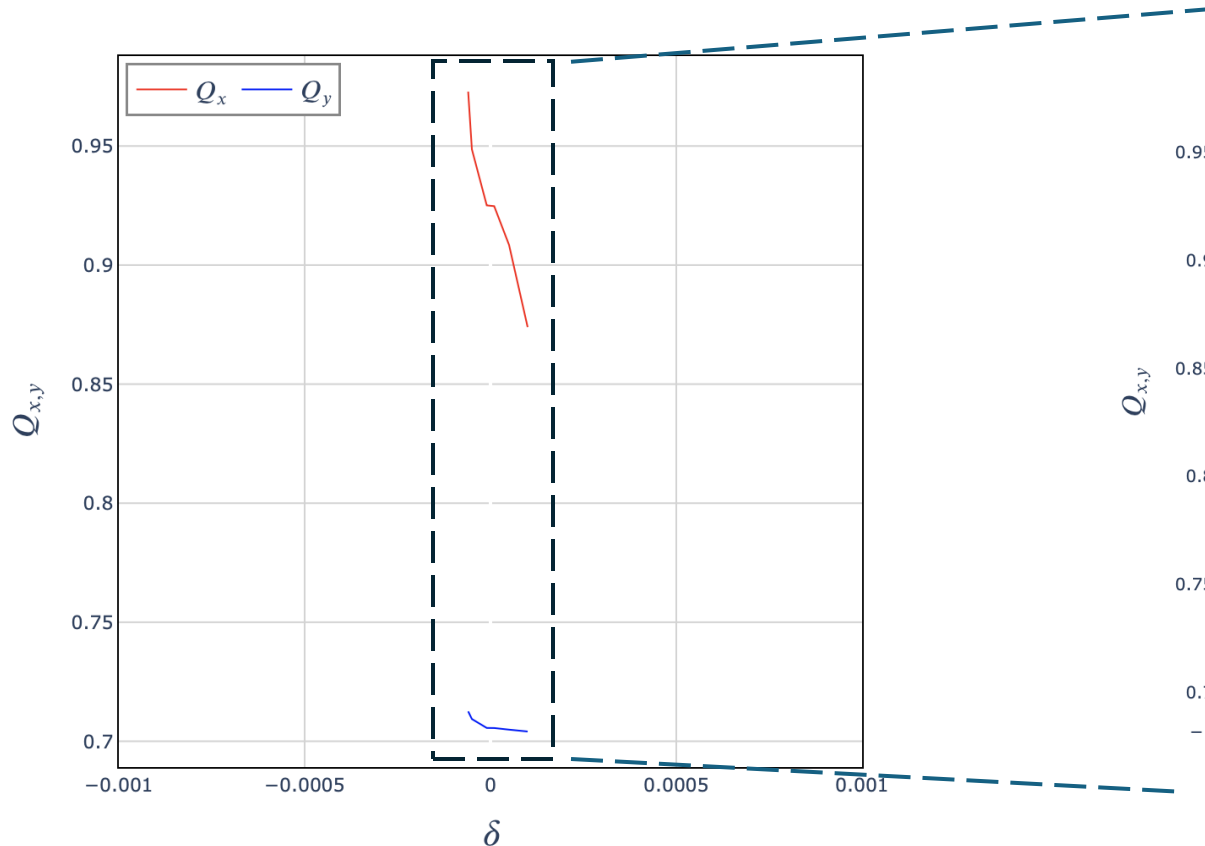
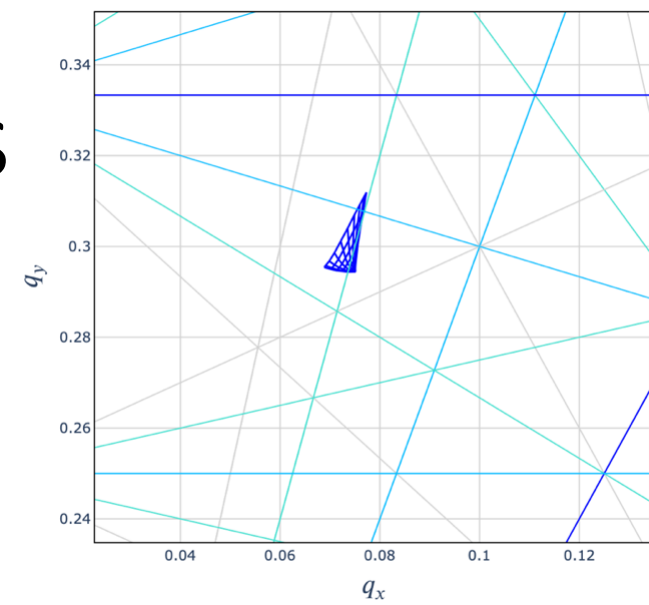


- Phase space in Y quite similar to what we had in v0.6!



V0.8 – Footprint & tune variations

- Limitations not clear from the footprint
- **Huge tune shift** with δ even if $Q' \rightarrow 0$ and $Q'' \rightarrow 0(10)$



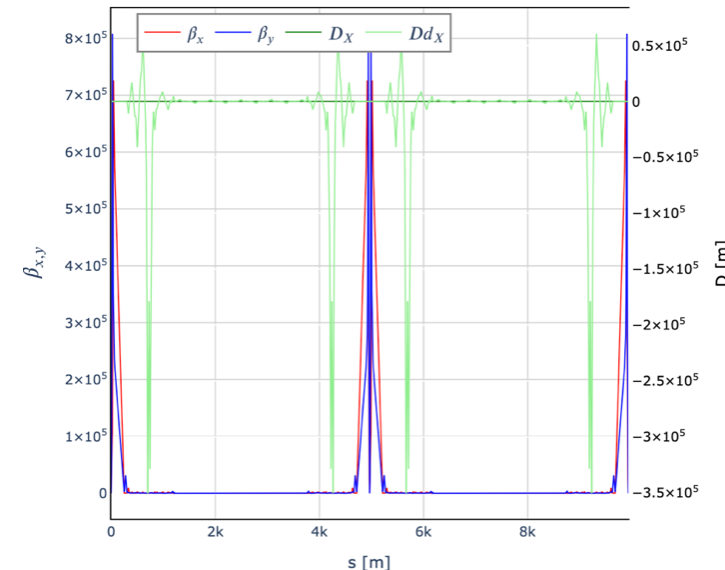
Main Advantages/issue of v0.8

Advantages:

- Play on dispersion rather than beta and sextupole strengths → The lattice will be likely **less sensitive to errors**.
- **W and Q' control in the CC !** Seems possible to control of Q_x'', Q_y'' on the entire ring
- **Reducing β** also helps **avoid adding chromatic errors from other CC quads** that may not be in phase with the FF triplet errors.
- The **structure of the lattice** allows for **adding as many sextupoles as necessary**, with the non-linear kicks always being canceled out by the sextupoles in the second cell.
- Reducing the β in the CC helps to **minimize path length differences** due to betatron oscillations (to be compared to the approximately 10 km circumference of the ring).

Issues:

- Huge **second-order dispersion**
- Huge **magnetic field** due to the large dispersion oscillation amplitude
- Tunes not (yet) adjusted; no clear knob for **working point control**.
- SD strength still really large !
- Low DA but lattice design much less mature than the other versions.



Comparaison of v0.6, v0.7, v0.8

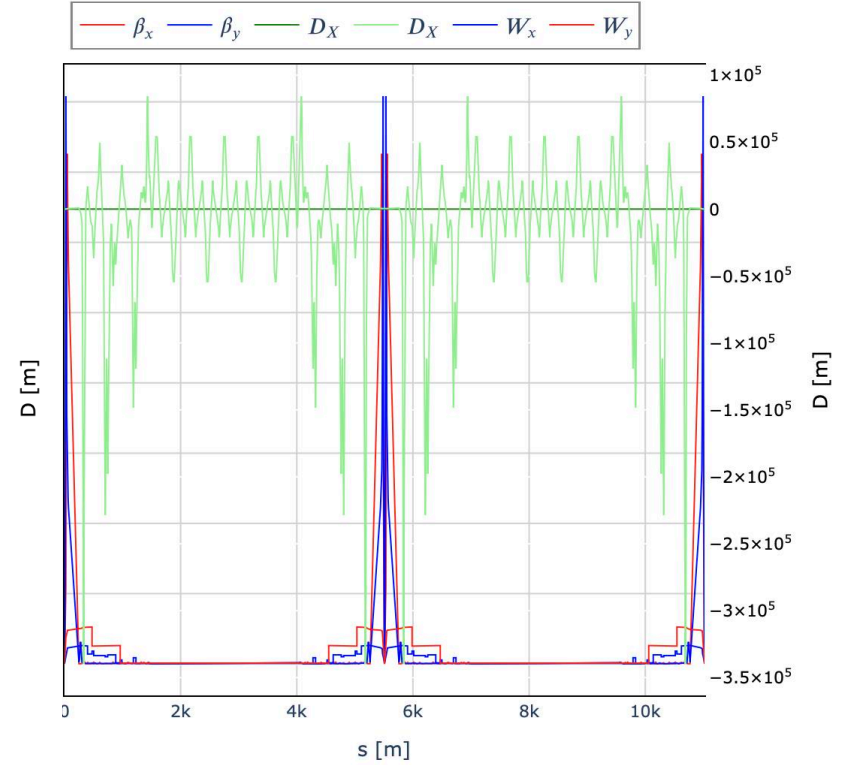
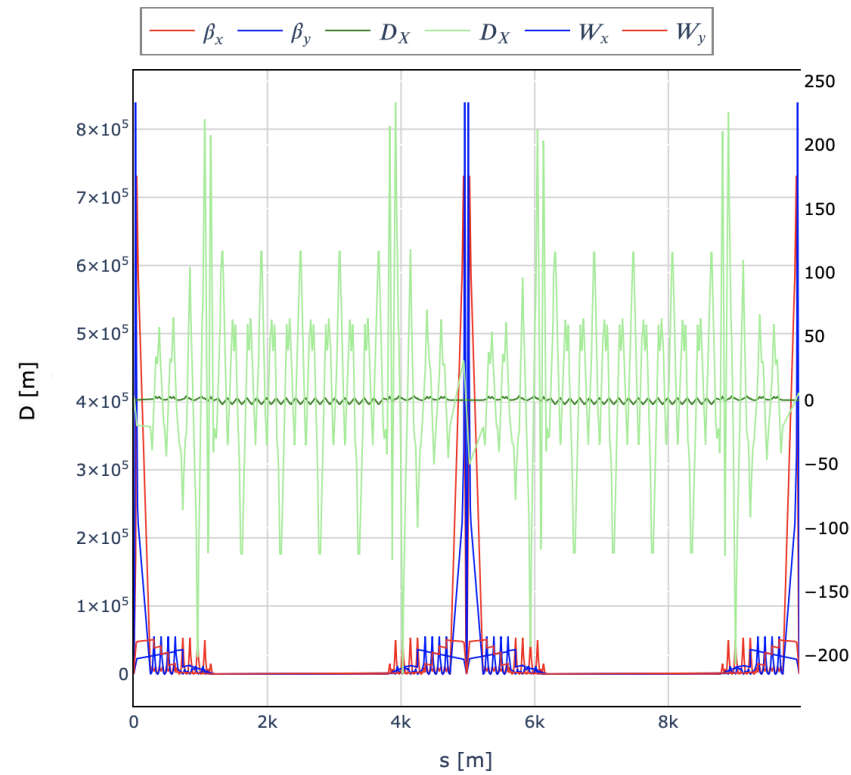
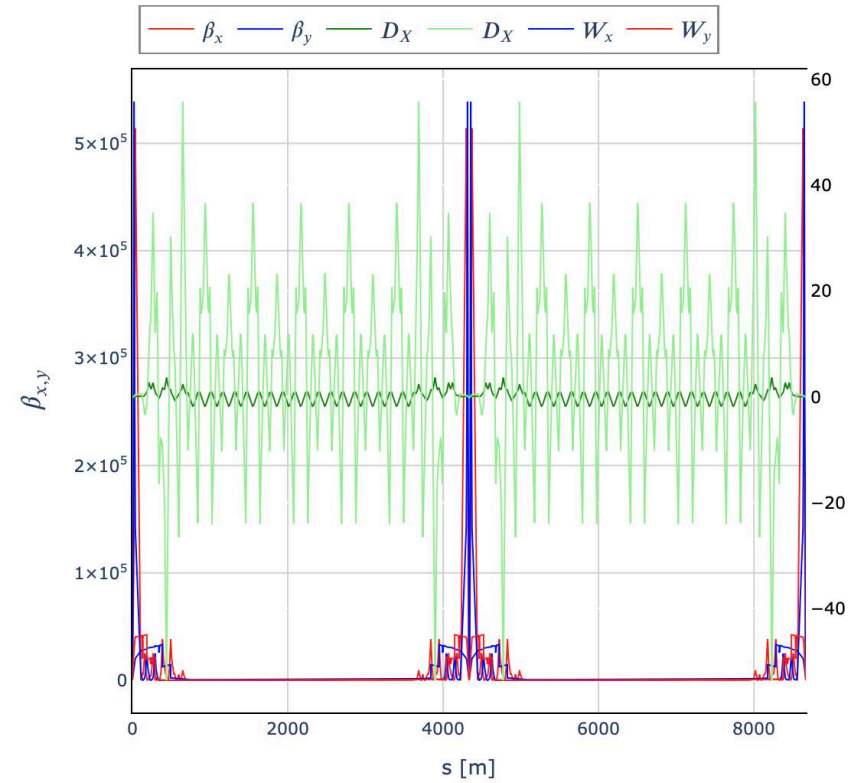
Comparison of v06, v07 and v08

	V0.6	V0.7	V0.8
<u>Sextupoles strengths</u>			
SF_arcs	0.064294	-0.062492	-0.101590
SD_arcs	0.117603	0.223955	0.219194
SF_CC_1	0.345776	0.327754	0.147995
SF_CC_2	/	-0.02236	-0.076368
SD_CC_1	-1.227299	-0.300536	2.516348
SD_CC_2	/	-0.470982	-0.060867
MS_SF_1	/	/	0.163888
MS_SF_2	/	/	-0.120175
<u>β - Functions</u>			
β_x max in CC	~25km	~50km	~2.5 km
β_y max in CC	~38km	~50km	~1.2 km
<u>Max Dispersion in CC</u>	~3.5	~5.77	~40.6
Dispersion in sextupoles	~0.77 - ~1.4	0.77 – 1.44	~35.9



Comparison of v06, v07 and v08

	V0.6	V0.7	V0.8
DA (Best)			
$\delta = 0$	$\sim 15\sigma$	$\sim 10\sigma$	$\sim 4\sigma$
$\delta = 10^{-4}$	$\sim 12.4\sigma$	$\sim 12\sigma$	$\sim 3.7\sigma$
$\delta = 7 * 10^{-4}$	$\sim 6\sigma$	X	X
$\delta = 10^{-3}$	$\sim 2.5\sigma - 4.5\sigma$	X	X
$\delta = 2 * 10^{-3}$	X	X	X
Q_x	44.4759 – 44.4763 – 44.4772	49.5801	45.924904
Q_y	39.6674 – 39.6905 – 39.7485	47.523	62.705569
Q'_x	$3.8e^{-4} - 3.4e^{-6} - 1.4e^{-4}$	-11.7408	$5e^{-3}$
Q'_y	$8.1e^{-8} - 1.5e^{-8} - 2.8e^{-8}$	-5.4623	$9.9e^{-7}$
Q''_x	$-7.9e^5 - 5.5e^5 - 1.7e^4$	$-5.5e^5$	-7.87
Q''_y	$2.6e^5 - 1.1e^5 - 3.8e^4$	$-1.6e^6$	6.62
Circumference	8.668km	9.959km	~ 11 km

Comparison of v06, v07 and v08



Comparison of v06, v07 and v08

	V0.6	V0.7	V0.8
	<ul style="list-style-type: none"> - DA up to 4.5σ for $\delta = 1 * 10^{-3}$ with phases optimized at the sextupoles 	<ul style="list-style-type: none"> - More compact arcs and slightly changed IR but no advantages in the CC 	<ul style="list-style-type: none"> - W and Q' control in the CC ! - Smaller β in CC (less sensitive to errors?) - Control of Q''_x, Q''_y on the entire ring - All sextupole pairs automatically separated by $\mu_x = \pi, \mu_y = \pi$
		<ul style="list-style-type: none"> - Smaller DA than v06 and poor momentum acceptance (Bad DA from $\delta = 7 * 10^{-4}$) - Everything is constrained \rightarrow Not enough degrees of freedom to optimize anything else - Huge sensitivity to phase advance error between the sextupoles of a pair 	<ul style="list-style-type: none"> - Huge B due to large dispersion oscillations. - Huge second order dispersion - Particle loss for $\delta > 7 * 10^{-4}$ - Larger circumference of the ring - Poor DA but also less mature lattice
	<ul style="list-style-type: none"> - Beta remains very high in the CC - Dependence between the CC section and the arcs (no Q' control in the CC). - The sextupoles are placed on the slope of the dispersion 		

Remaining issues and Next steps

Main design choices to discuss & issues

- **Which version** should we keep ?
 - V0.6 with new IR ? But what are the remaining possibilities for improvements in this version?
 - If v0.7: Adding a third pair could help (3 sextupole variables for 3 parameters to match in each plane)?
 - What could be the limitations for the DA for v0.8 ?
- How can we handle the remaining **tune shift with momentum** and the dependence of footprint with momentum ?

Remaining issues (to keep in mind)

- High **magnetic field value** → probably not possible to have magnets of 20T (FF) or 16T (arcs). We should try to reduce the field or to relax the collider requirements (increase the β^*).
- The **distance for element interconnection** still very small (30cm). We can try to increase it, at the expense of the dose due to neutrinos.
- **No** clear knobs to **control the working point** (transverse tunes) in the current lattice.

Next steps

- Sensitivity study of the momentum acceptance as a function of β^*
- **Implement** the capability to have a combined function **dipole-sextupole magnet in Xsuite**; Evaluate the effects of thick sextupoles.
- Improve the lattice design for momentum acceptance:
 - Any ideas ?
 - Look more closely at the RDTs
 - Second order dispersion ?
 - Add other multipolar components
- ...