Symbolica: modern computer algebra

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Symbolica

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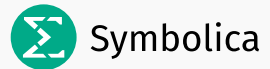
- Symbolica (https://symbolica.io) is a new computer algebra system
- Provides world-class performance
- Intuitive and easy to use
- Easy to integrate into existing software (Rust and Python API)
- Free for hobbyists, 1 core and 1 instance free for non-commercial use

Why switch?

- Easier to write fast programs
- Minimize daisy-chaining of tools
- New staff can be trained faster
- Support creation of novel algorithms

- Keep denominators of rational polynomials factorized
- No need for GCDs for multiplication and addition
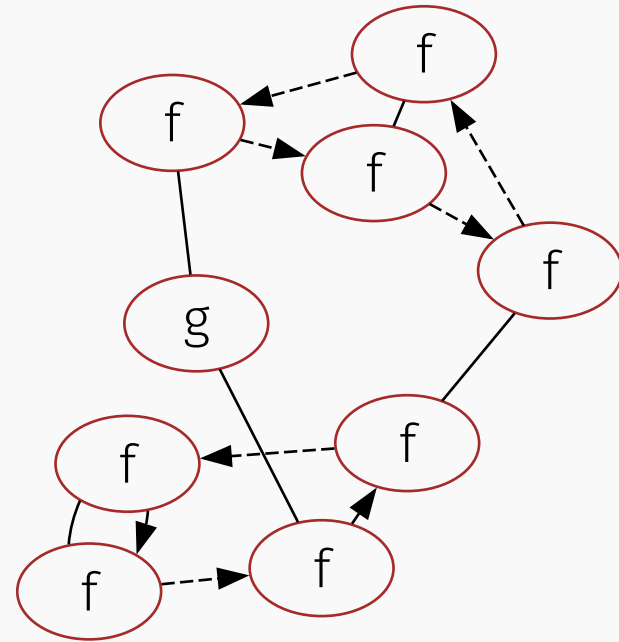- 3x improvement for some 5 variable IBP reductions

$$\frac{x^2 + y^2 + 5}{(x+6)(x+y+1)} \cdot \frac{x^2 + 8x + 12}{(x+1)(x+y+1)} = \frac{10 + 5x + 2x^2 + x^3 + 2y^2 + xy^2}{(x+1)(x+y+1)}$$

- Only two division tests needed:

$$\frac{x^2 + 8x + 12}{x+6}, \frac{x^2 + y^2 + 5}{x+1}$$

- $f_{\mathrm{cyc}}(\mu, \nu, \rho, \nu) f_{\mathrm{cyc}}(\rho, \sigma, \sigma, \delta) g(\mu, \delta) = f_{\mathrm{cyc}}(\alpha, \gamma, \alpha, \mu) f_{\mathrm{cyc}}(\varepsilon, \varepsilon, \delta, \gamma) g(\mu, \delta)$?
- Canonize (cycle)symmetric tensors using graphs

- Generate high quality ASM code for expression evaluation
- Bypasses long C++ compilation

```cpp
void sigma(const std::complex<double> *params, std::complex<double> *out) {
    __asm__(
        "movupd 208(%2), %%xmm1\n\t"
        "movupd 224(%2), %%xmm2\n\t"
        "movapd %%xmm1, %%xmm0\n\t"
        "unpckhpd %%xmm0, %%xmm0\n\t"
        "unpcklpd %%xmm1, %%xmm1\n\t"
        "mulpd %%xmm2, %%xmm0\n\t"
        "mulpd %%xmm2, %%xmm1\n\t"
```

Benchmarks

- Polynomial arithmetic 1000x faster than Sympy
- User reported a 60000x speedup over Sympy for evaluations
- Symbolica is used in production for large calculations

Set. No. Simplifier	1	2	3
CoCoA	70.6	127.5	84.5
CoCoA (lib)	56.2	100.3	83.3
Fermat	13.9	15.6	1.8
FORM	62.0	107.2	1967.8
GiNaC	21.7	43.3	4.6
GiNaC (lib)	11.9	27.2	4.5
Macaulay2	81.9	244.7	-
Maple	192.1	276.0	85.4
Maxima	106.6	182.8	10.9
Nemo	19.6	35.5	3.4
PARI / GP	10.3	18.3	736.2
Symbolica	10.3	16.5	1.8
Symbolica (lib)	1.9	4.2	1.8
Wolfram Mathematica	349.4	882.0	44.2

arxiv:2304.13418

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Computer algebra is hard

$$\frac{a}{b} = \frac{x^8 + x^6 - 3x^4 - 3x^3 + 8x^2 + 2x - 5}{3x^6 + 5x^4 - 4x^2 - 9x + 21}$$

Does the fraction simplify?

$$\frac{a}{b} = \frac{x^8 + x^6 - 3x^4 - 3x^3 + 8x^2 + 2x - 5}{3x^6 + 5x^4 - 4x^2 - 9x + 21}$$

$$a \bmod b = r_1 = -15x^4 + 3x^2 - 9$$

$$b \bmod r_1 = r_2 = 15795x^2 + 30375x - 5953$$

$$r_1 \bmod r_2 = r_3 = 254542875143750x - 1654608338437500$$

$$r_2 \bmod r_3 = r_4 = 12593338795500743100931141992187500$$

- Coefficients blow-up when doing exact arithmetic