NYU

High-Quality Axions from Higher-Form Symmetries in Extra Dimensions

Marius Kongsore (NYU) w/ *Nathaniel Craig* (UCSB/KITP)

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Outline

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- **PART I: Higher-form Symmetries**

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- **PART II: The Axion and the Quality Problem**

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- **PART IV: Breaking the Symmetries**

Part I

Higher-Form Symmetries

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$$S = \int \mathrm{d}^4x \left(-(\partial_\mu \phi)^\dagger \, \partial^\mu \phi - V(\phi^\dagger \phi) \right)$$

- Invariant under $\phi \to \phi e^{i\alpha}$. Usual global U(1) symmetry.

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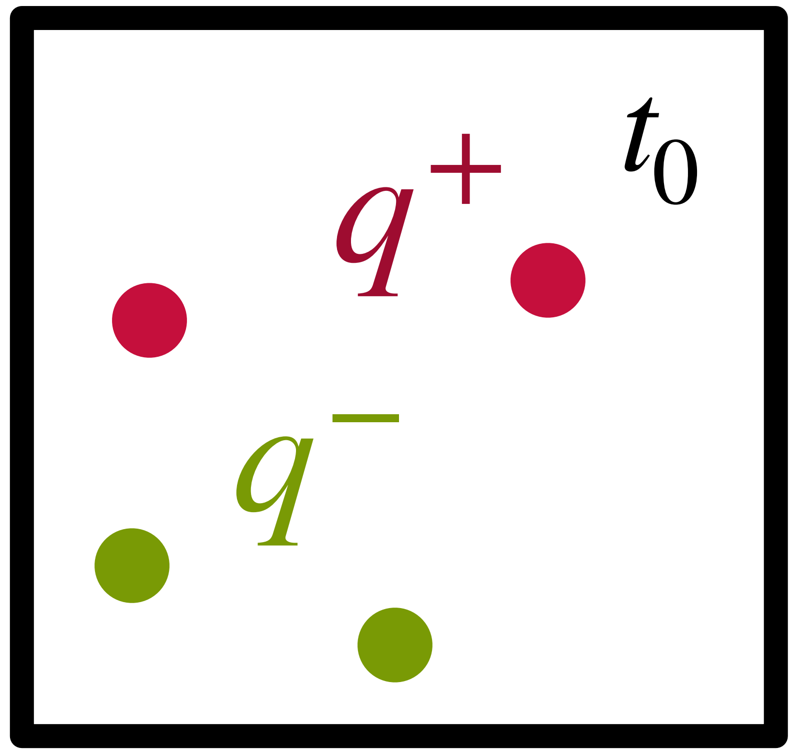

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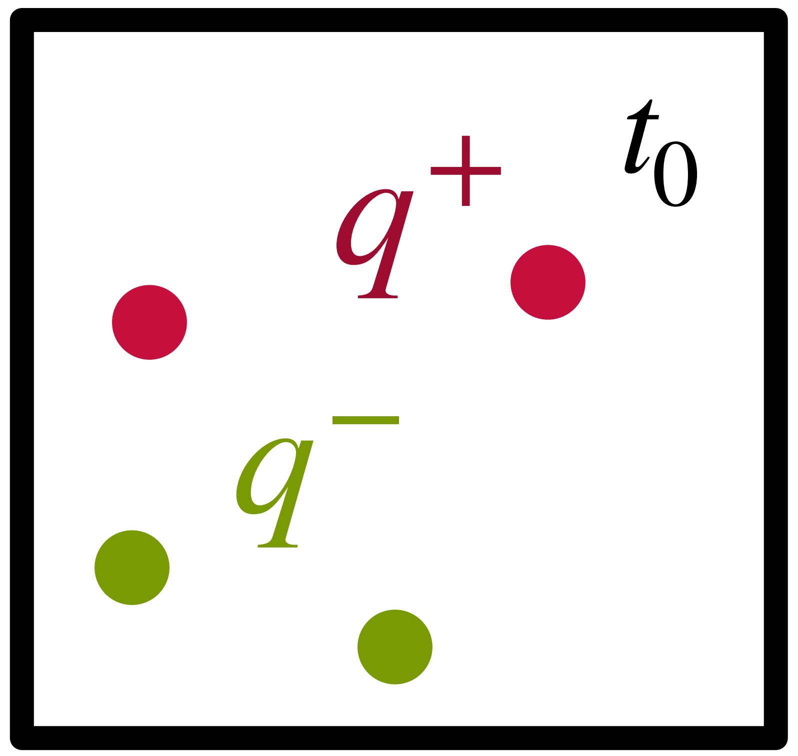

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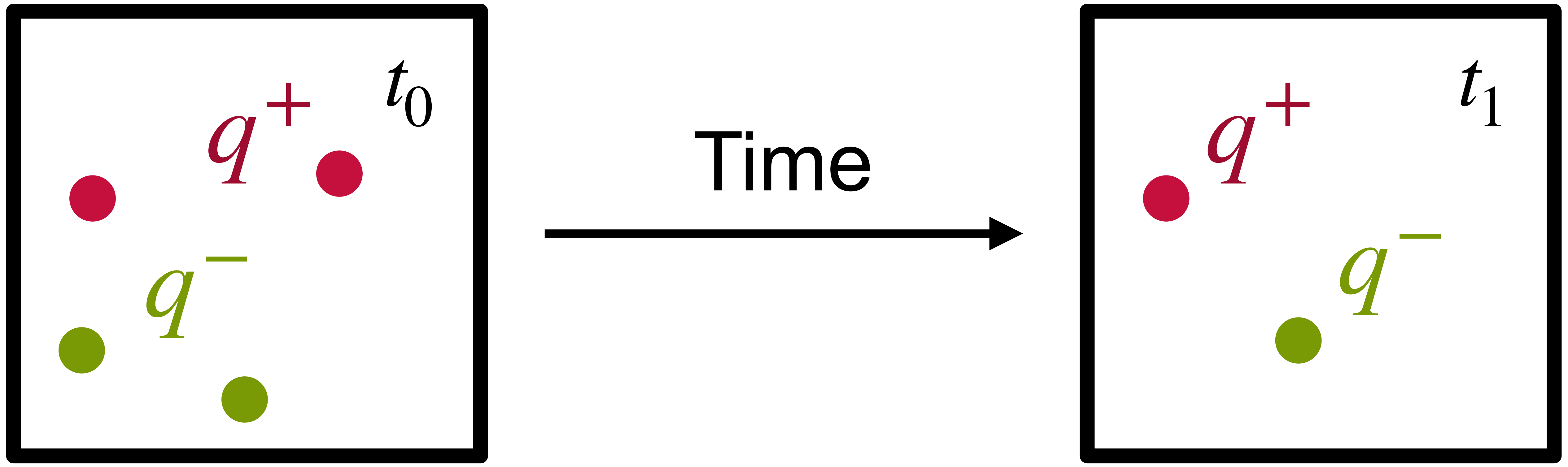

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$$U(\Sigma, \alpha) \equiv \exp\left(i\alpha \int_{\Sigma} J \right)$$

Symmetry Defect Operator (SDO)

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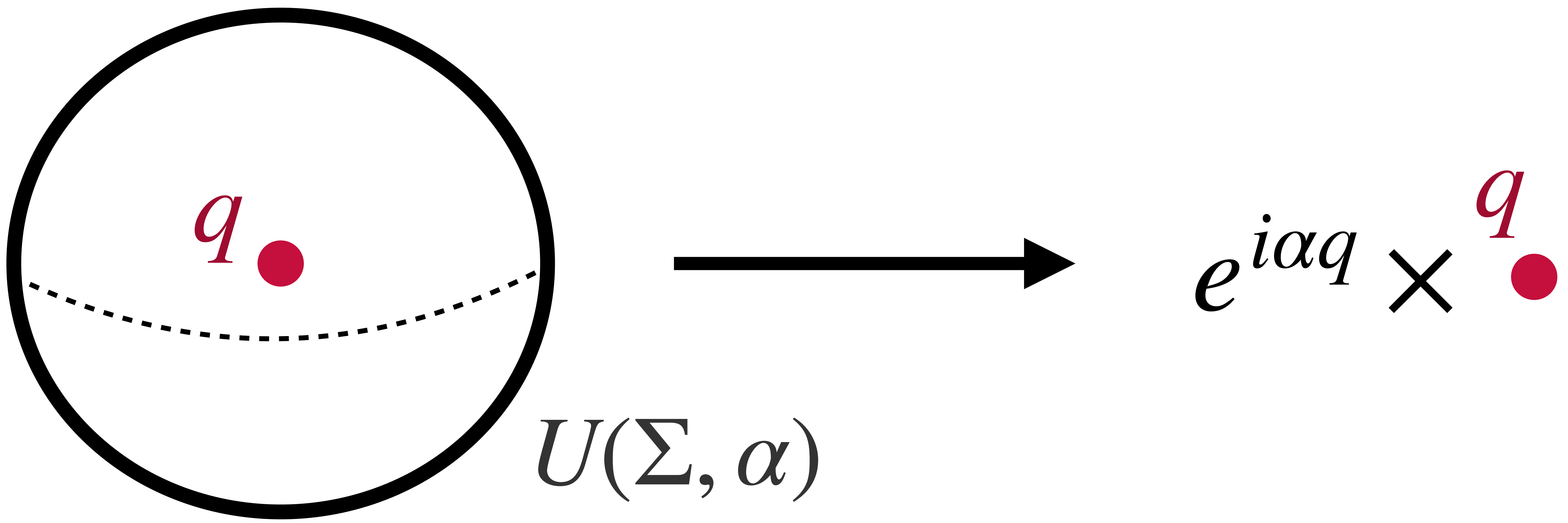

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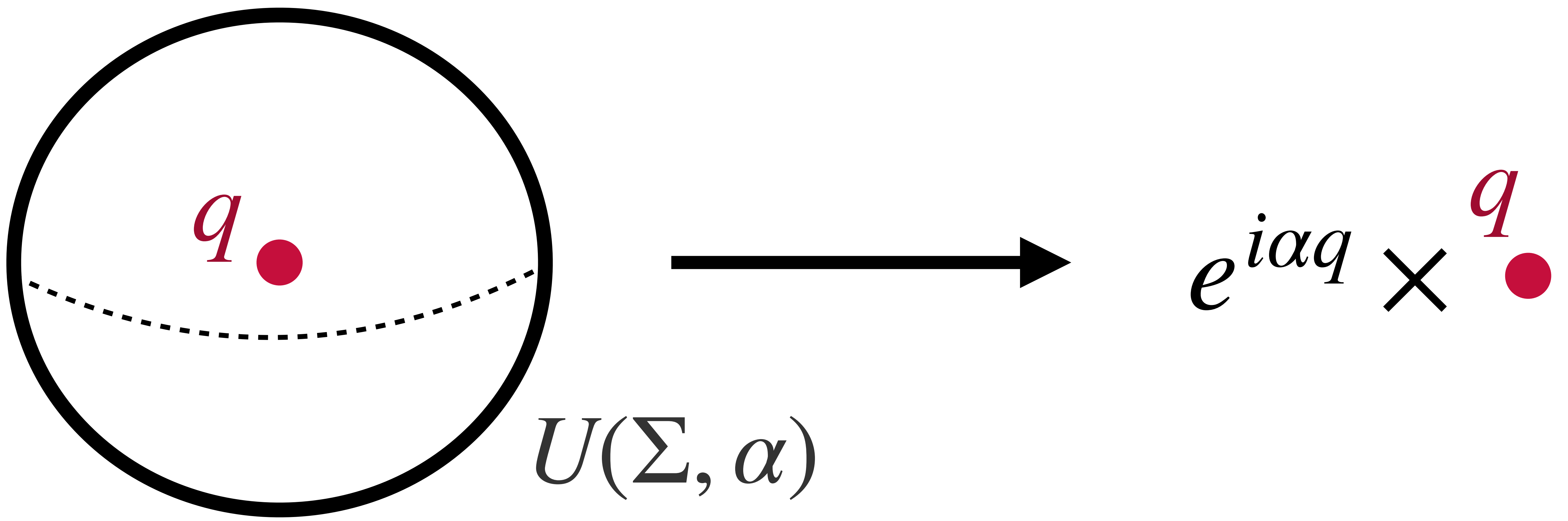

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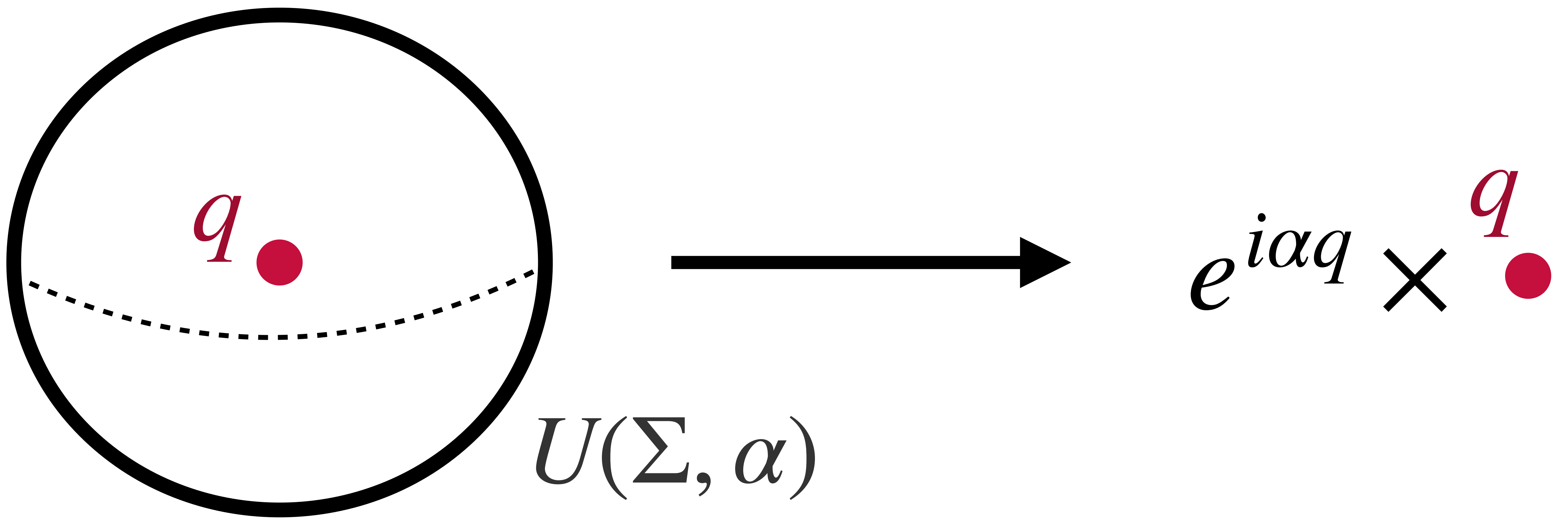

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- Follows **group multiplication rule**: $U(\Sigma, \alpha)U(\Sigma, \beta) = U(\Sigma, \alpha + \beta)$

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- The basic idea: Generalize!

Properties of symmetry operator	Ordinary symmetry	Higher-form symmetry	Non-invertible symmetry	Subsystem symmetry
Codimension in spacetime	1	> 1	≥ 1	> 1
Topological	yes	yes	yes	not completely but conserved in time
Fusion rule	group	group	category	group

**Cordova, Dumitrescu, Intriligator, Shao: Snowmass White Paper: Generalized Symmetries in Quantum Field Theory and Beyond*

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***p*-form symmetry current has *(p+1)* indices.**

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Can be broken by electrically/magnetically charged matter and new operators in action.

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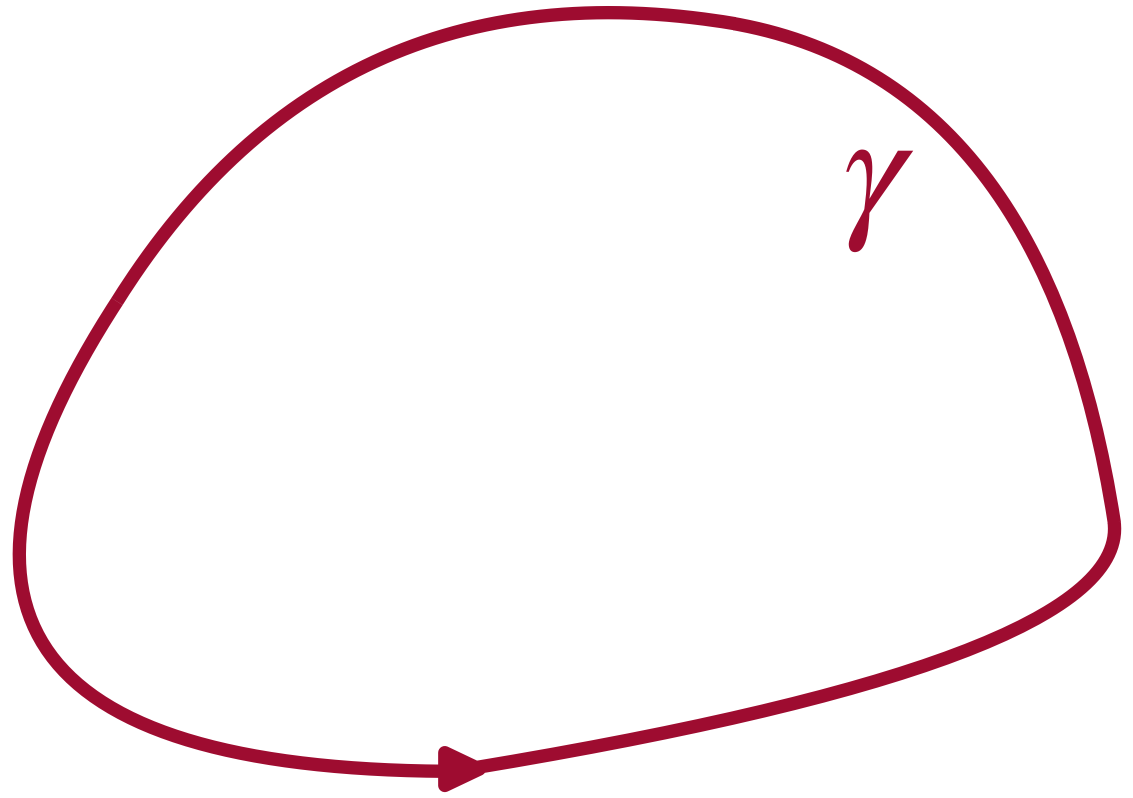

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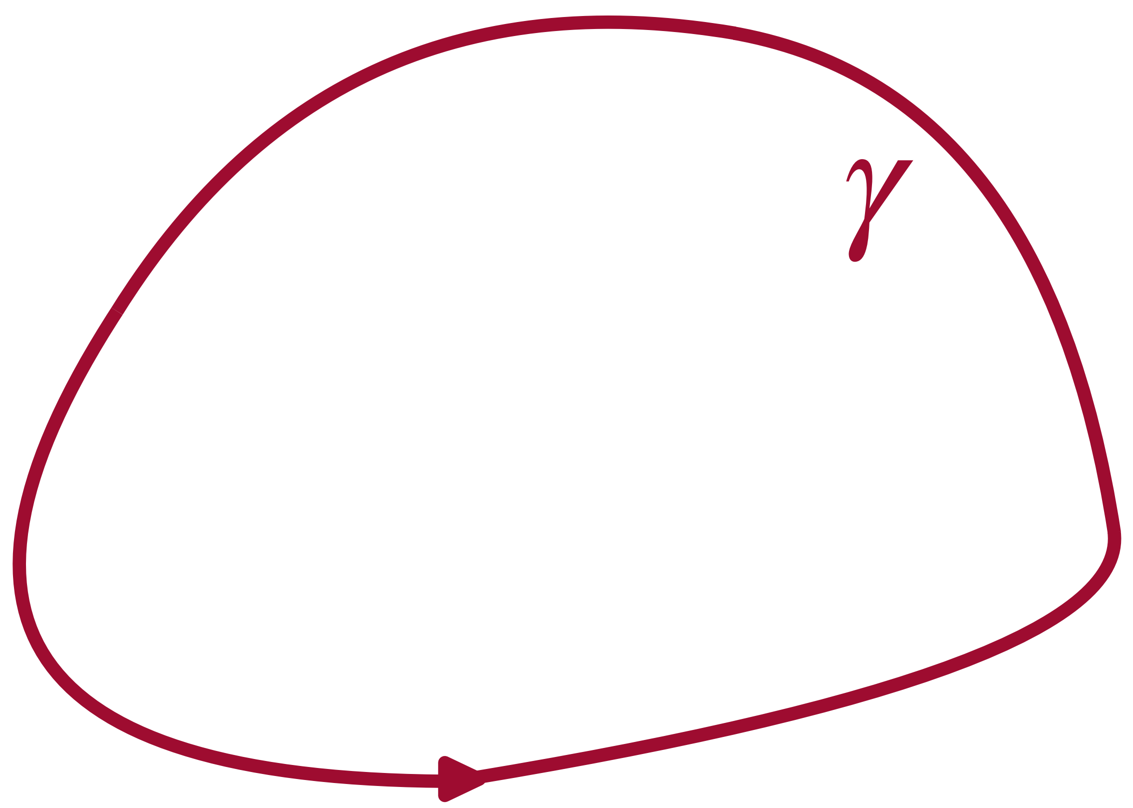

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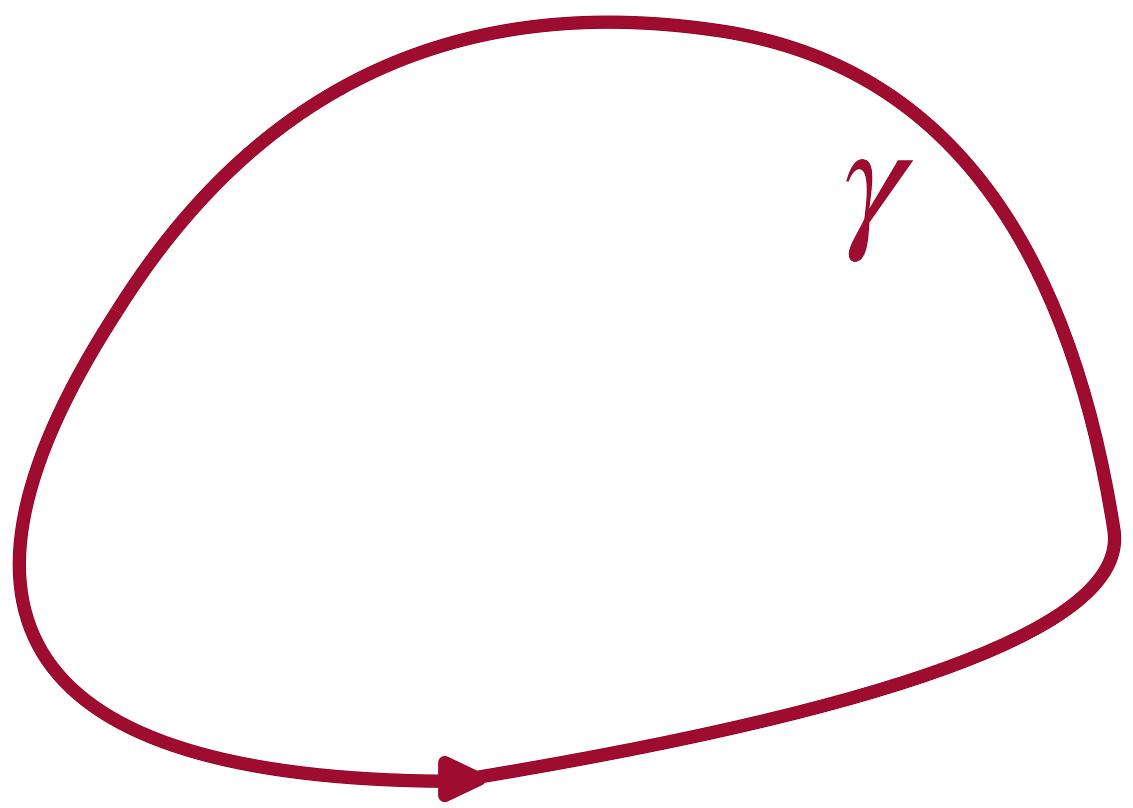
$$W[\gamma] = \exp(iq\int_\gamma \mathrm{d}\tau A_\tau) \to W[\gamma] \exp\left(iq2\pi\mathbb{R}\right)$$



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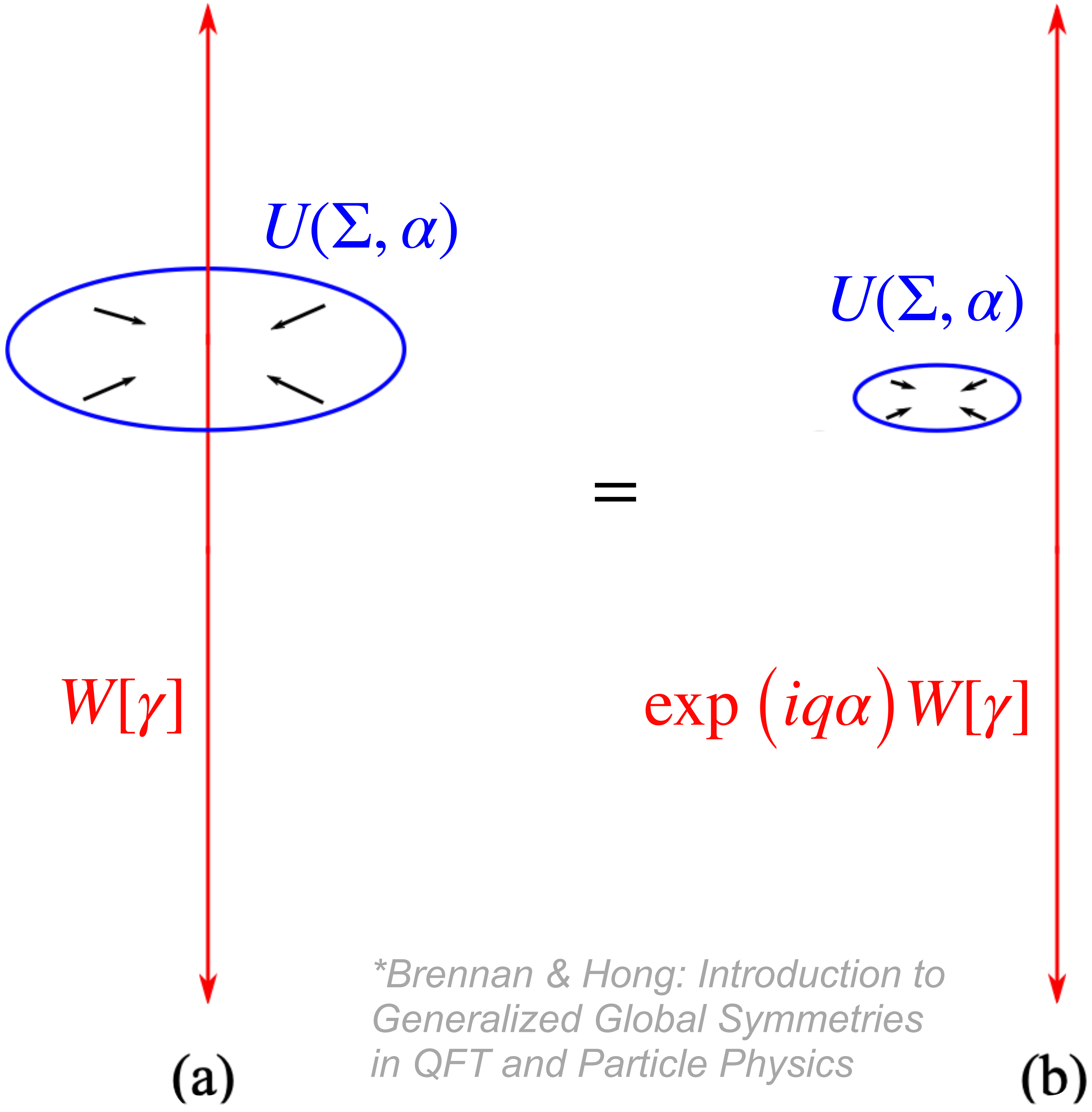

**Brennan & Hong: Introduction to Generalized Global Symmetries in QFT and Particle Physics*

What is measured?

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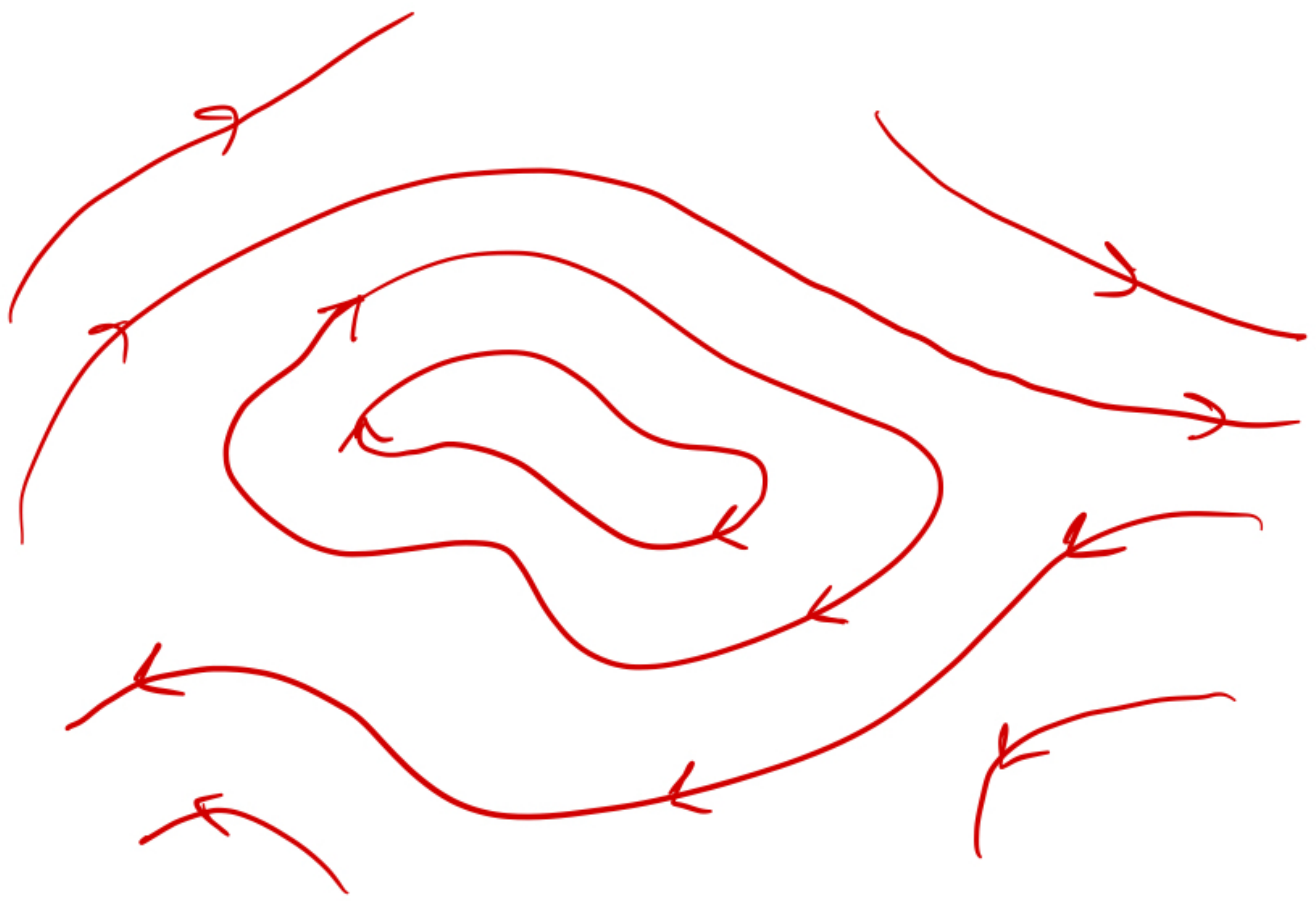
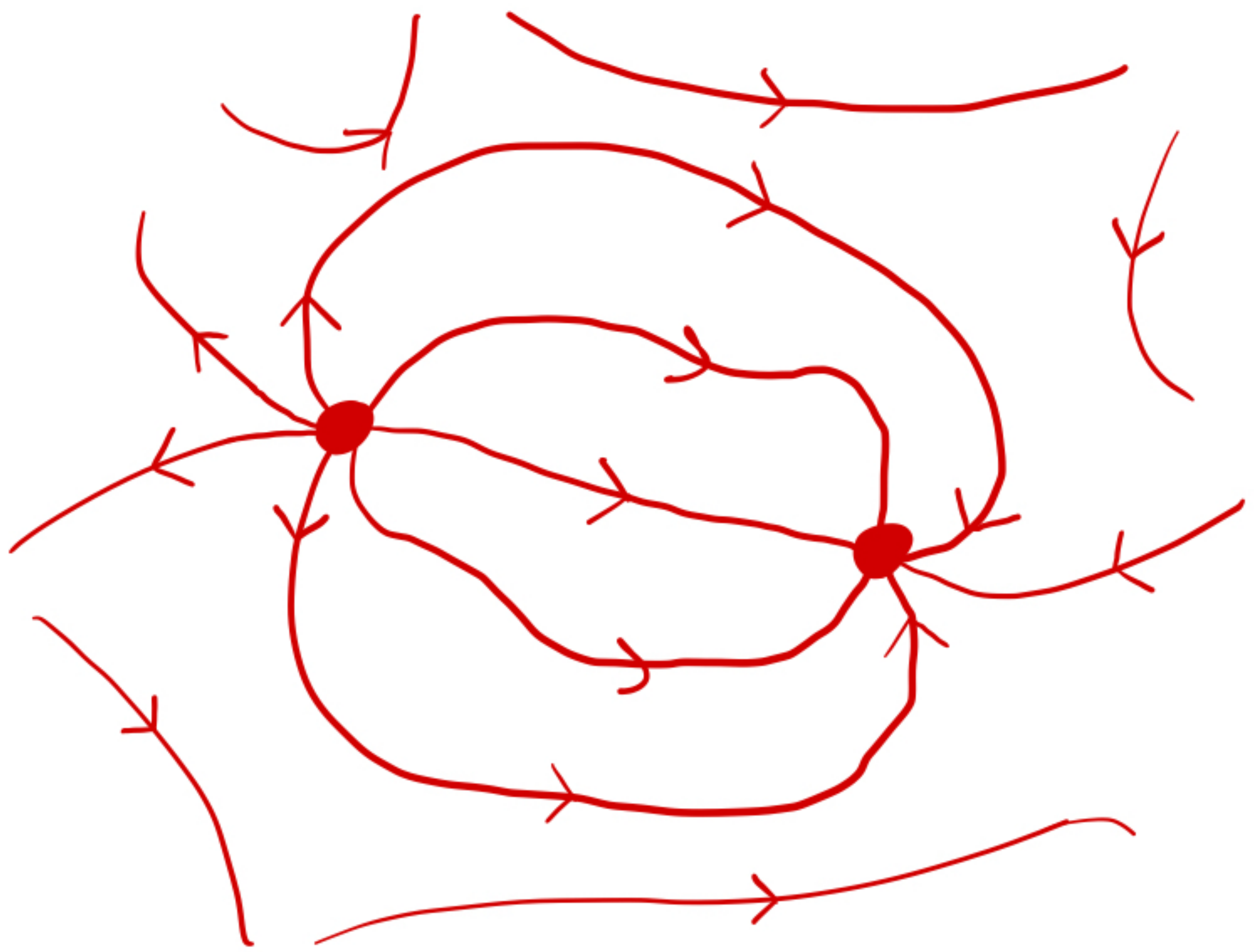
Symmetry Intact



v.s.

Symmetry Broken



The Open Question

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Are generalized symmetries useful for phenomenology/BSM physics?

Part II

Axions and the Quality Problem

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$$\mathscr{L}_{\rm IR} = -\frac{1}{2} f^2 (\partial_\mu \theta)^2 + f \frac{g^2}{32\pi^2} \theta G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} + \ldots$$

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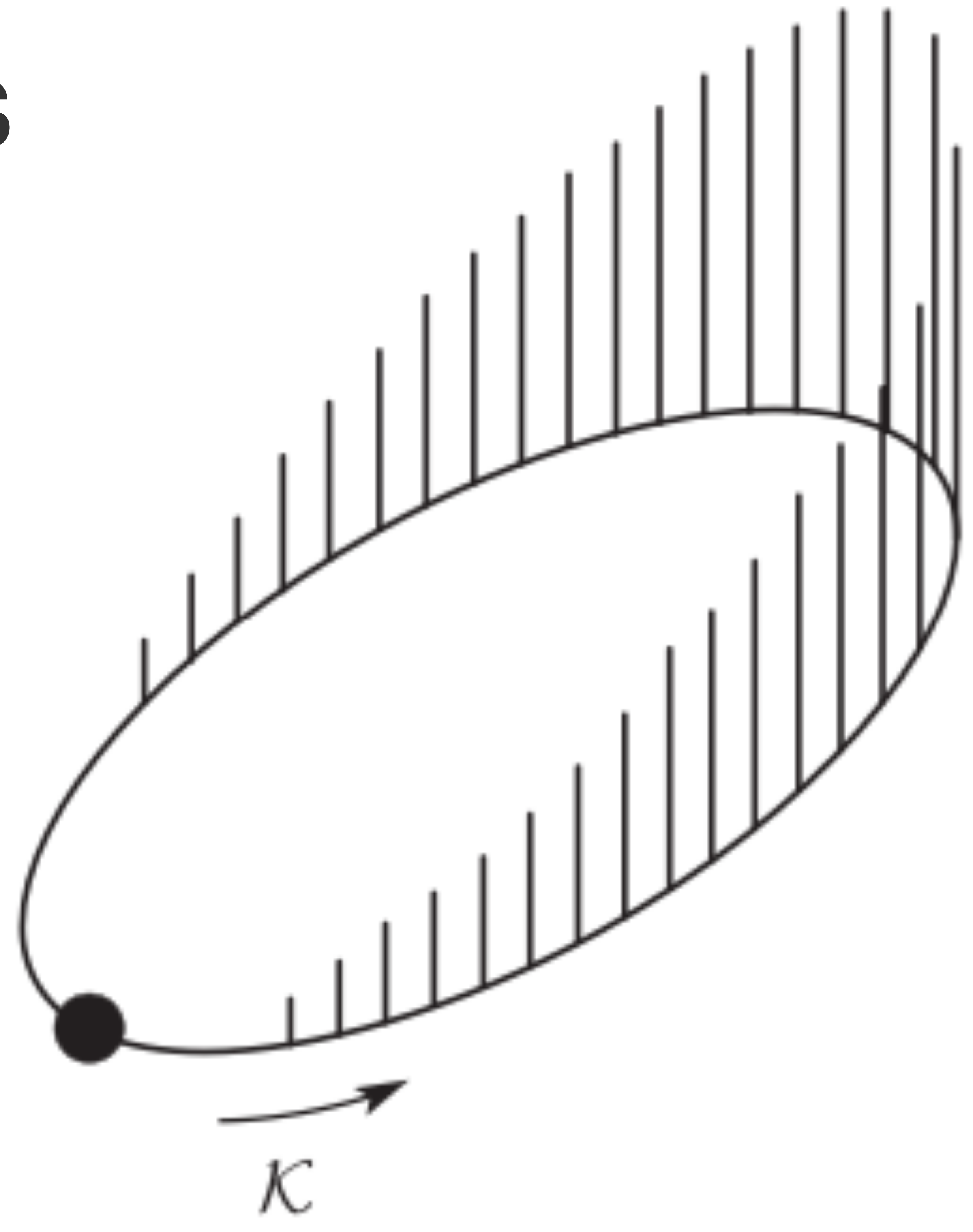

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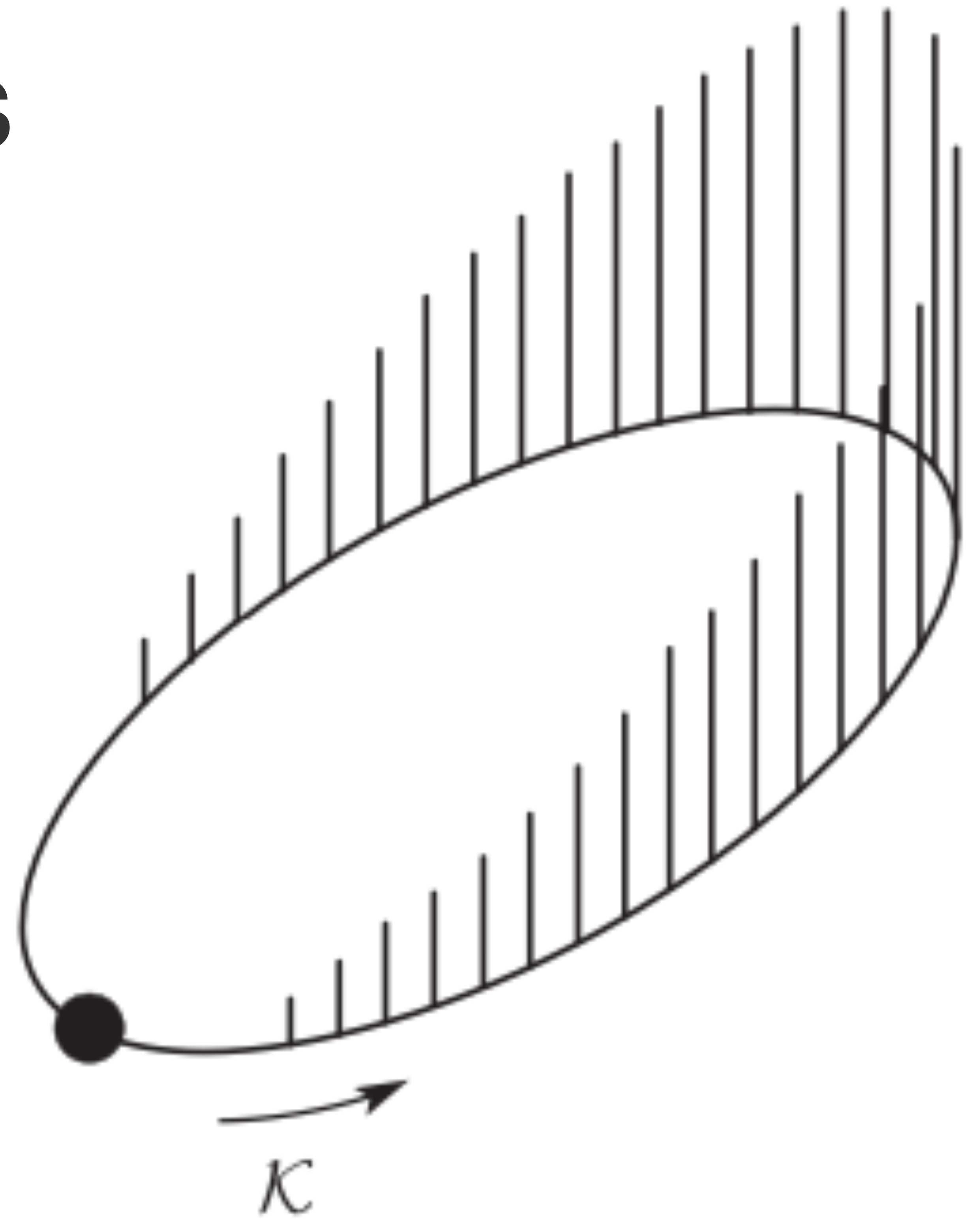

- Solves Strong CP Problem*

*Shifman: Advanced Topics in Quantum Field Theory

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- PQ symmetry could easily be accidental in the IR — by the No Global Symmetries conjecture, PQ must be broken in the UV.

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- Has been tested for a few specific UV completions; swampland makes clear that we should worry in general.

Part III

The Extra-Dimensional Axion and its Symmetries

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$\partial^\mu C^\nu - \partial^\nu C^\mu$

$$S^{5\mathrm{D}} = \int -\frac{1}{4g_{5\mathrm{D}}^2} F_{\mu\nu}F^{\mu\nu} + \frac{N}{32\pi^2}\epsilon^{\mu\nu\rho\sigma\gamma}C_\mu \mathrm{Tr}\left[G_{\nu\rho}G_{\sigma\gamma}\right]$$

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$$= \int -\frac{1}{2g_{5\mathrm{D}}^2} F \wedge \star F + \frac{N}{8\pi^2} C \wedge \mathrm{Tr}\left[G \wedge G\right]$$

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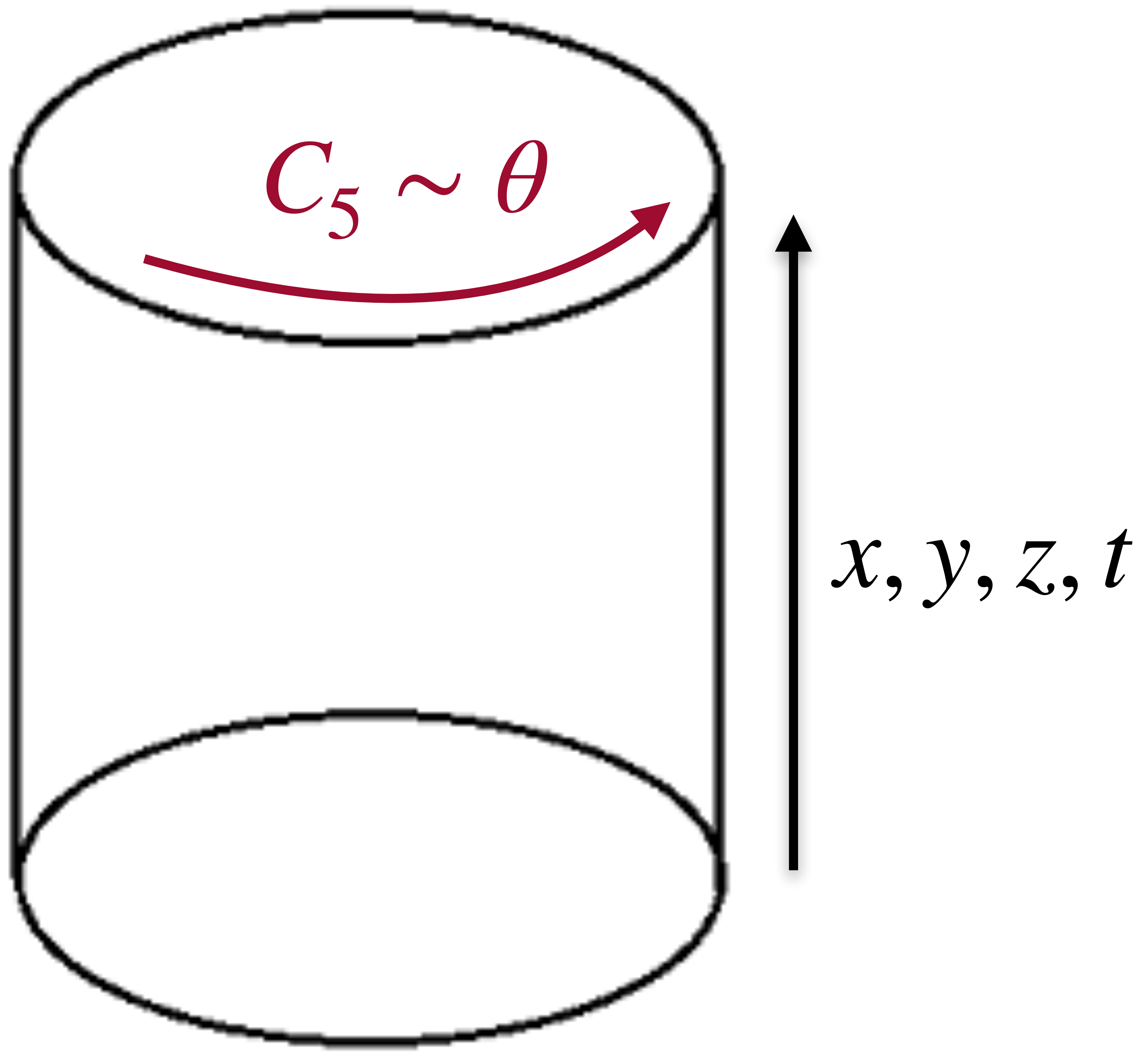

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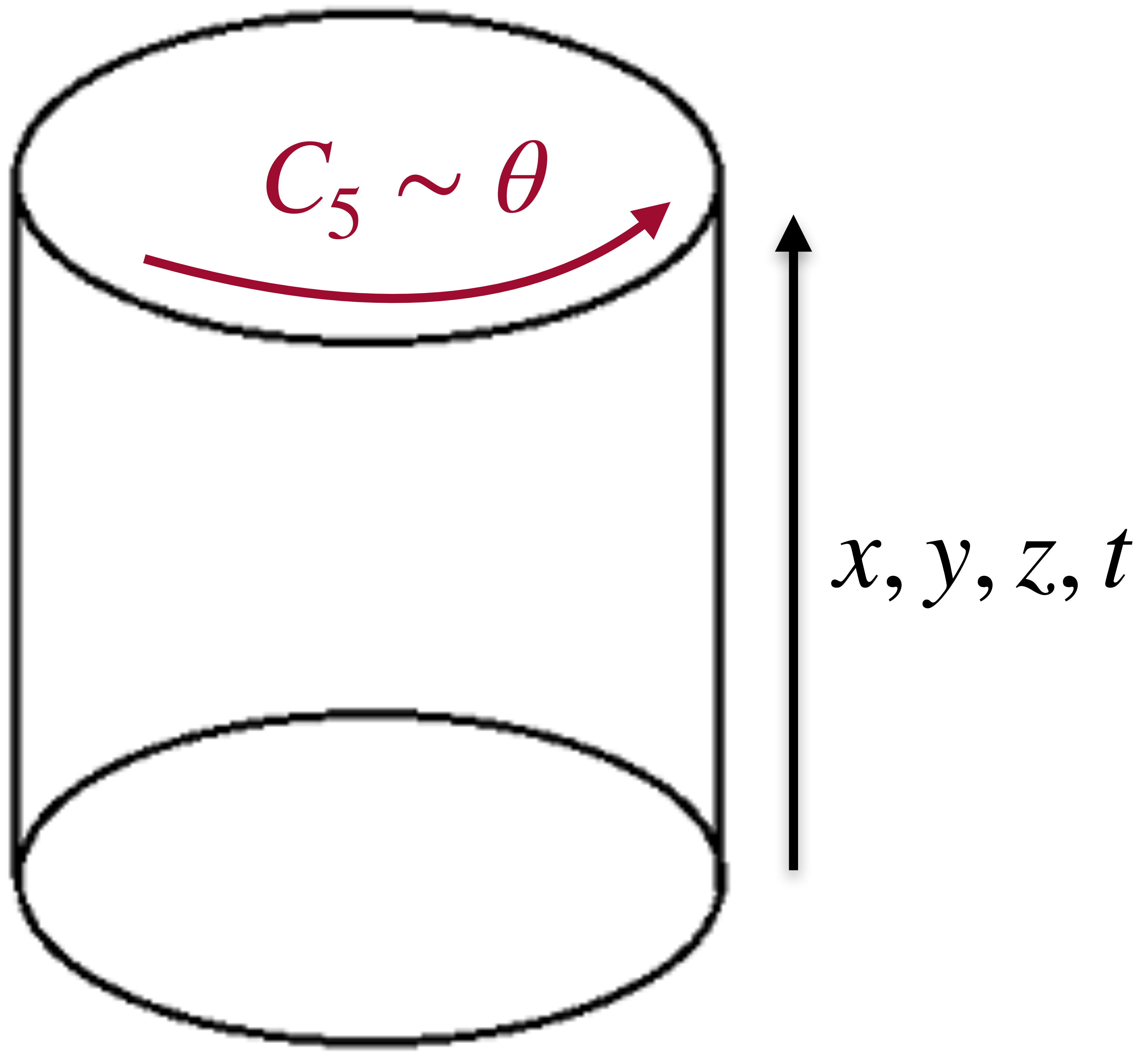

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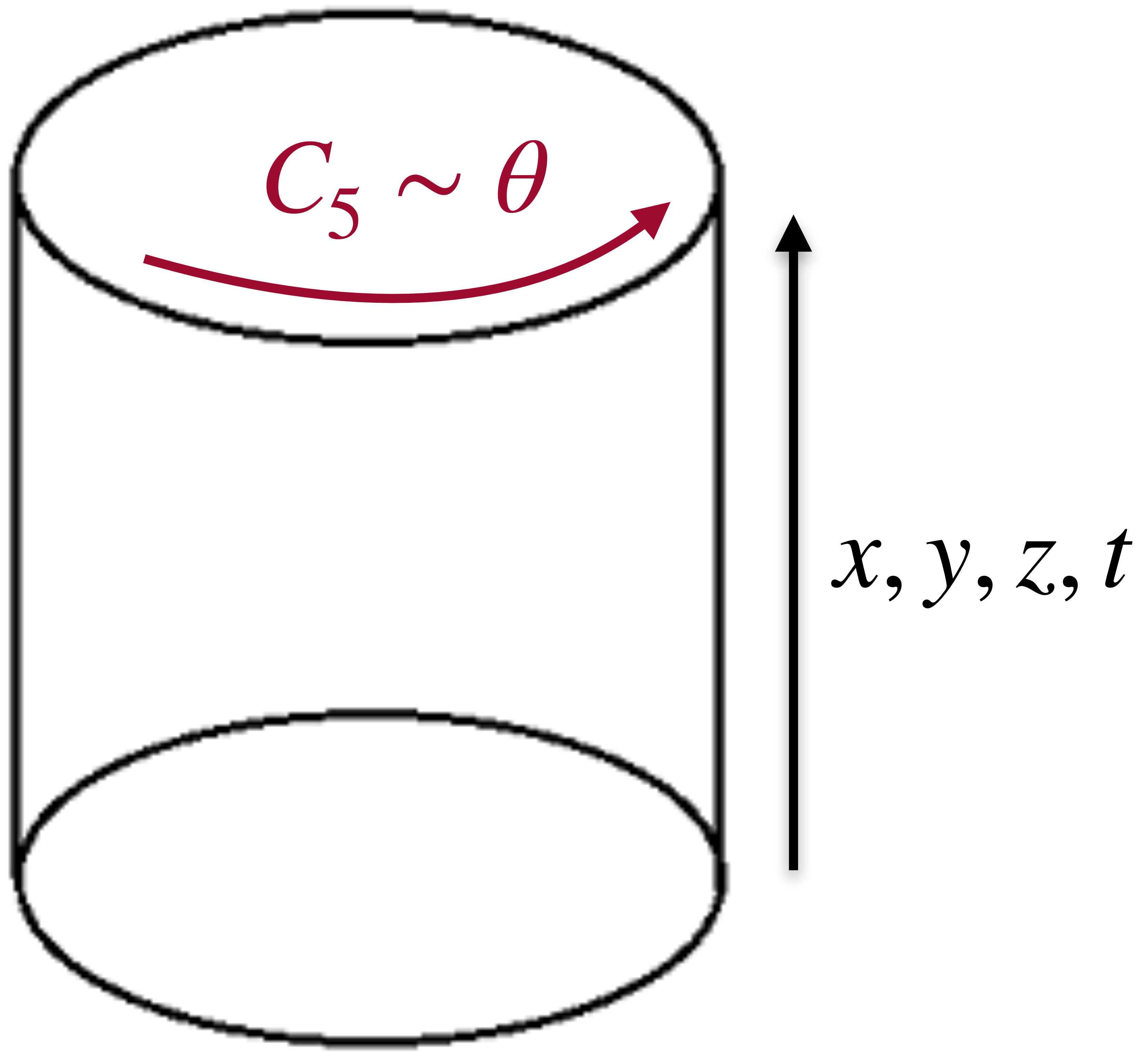
- Axion decay constant is

$$f \equiv \frac{1}{g_5\sqrt{2\pi R}}$$



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- **Idea:** Can (almost) only write operators in UV consisting of field strengths, so e.g. no ϕ^6, only F^3 and similar operators.

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- Does not tell you about where the potential protection fail: **has no instructive power**.

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5D		4D
Electric one-form symmetry of C	⟶	**Real Number Shift Symmetry of θ** + **Electric one-form symmetry of $C_{4\mathrm{D}}$**

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$$J_e^{\mu\nu} = \frac{1}{g_5^2} F^{\mu\nu} = \frac{1}{g_5^2} \partial^{[\mu} C^{\nu]} \qquad \partial_\mu J_e^{\mu\nu} \text{"="} \mathrm{d} J_e = 0 \longrightarrow \text{Symmetry Unbroken}$$

- Upon dimensional reduction

5D

Electric one-form symmetry of C ⟶

4D

Real Number Shift Symmetry of θ

+

Electric one-form symmetry of $C_{4\mathrm{D}}$

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$$\theta = \int_{S^1} \mathrm{d}x^5 C_5 \to \int_{S^1} \mathrm{d}x^5 \left(C_5 + \Lambda_5\right) = \int_{S^1} \mathrm{d}x^5 C_5 + \int_{S^1} \mathrm{d}x^5 \Lambda_5 = \theta + 2\pi\mathbb{R}$$

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- You will see this via practical examples.

Part IV

Breaking the One-Form Symmetry

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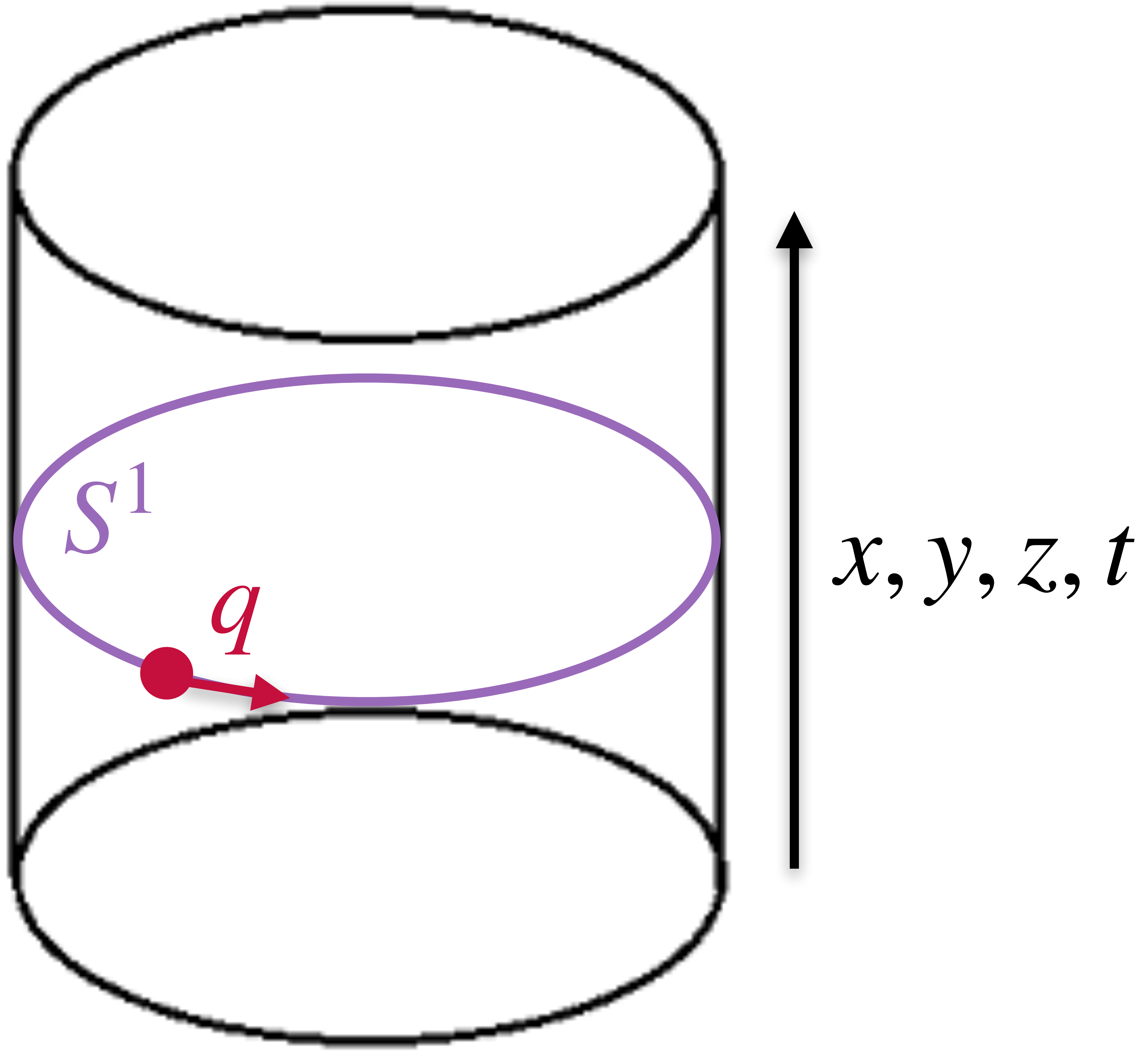

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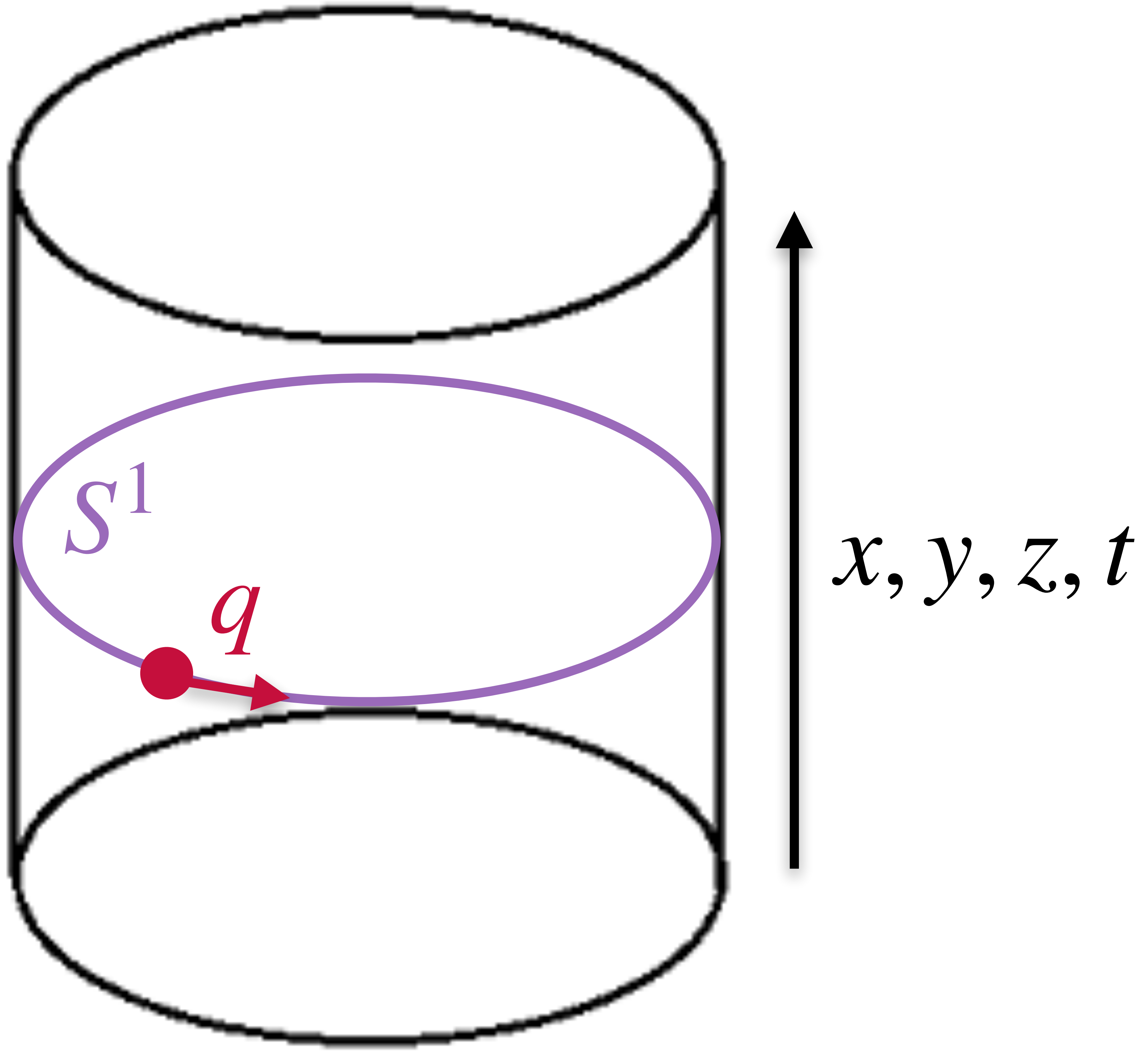

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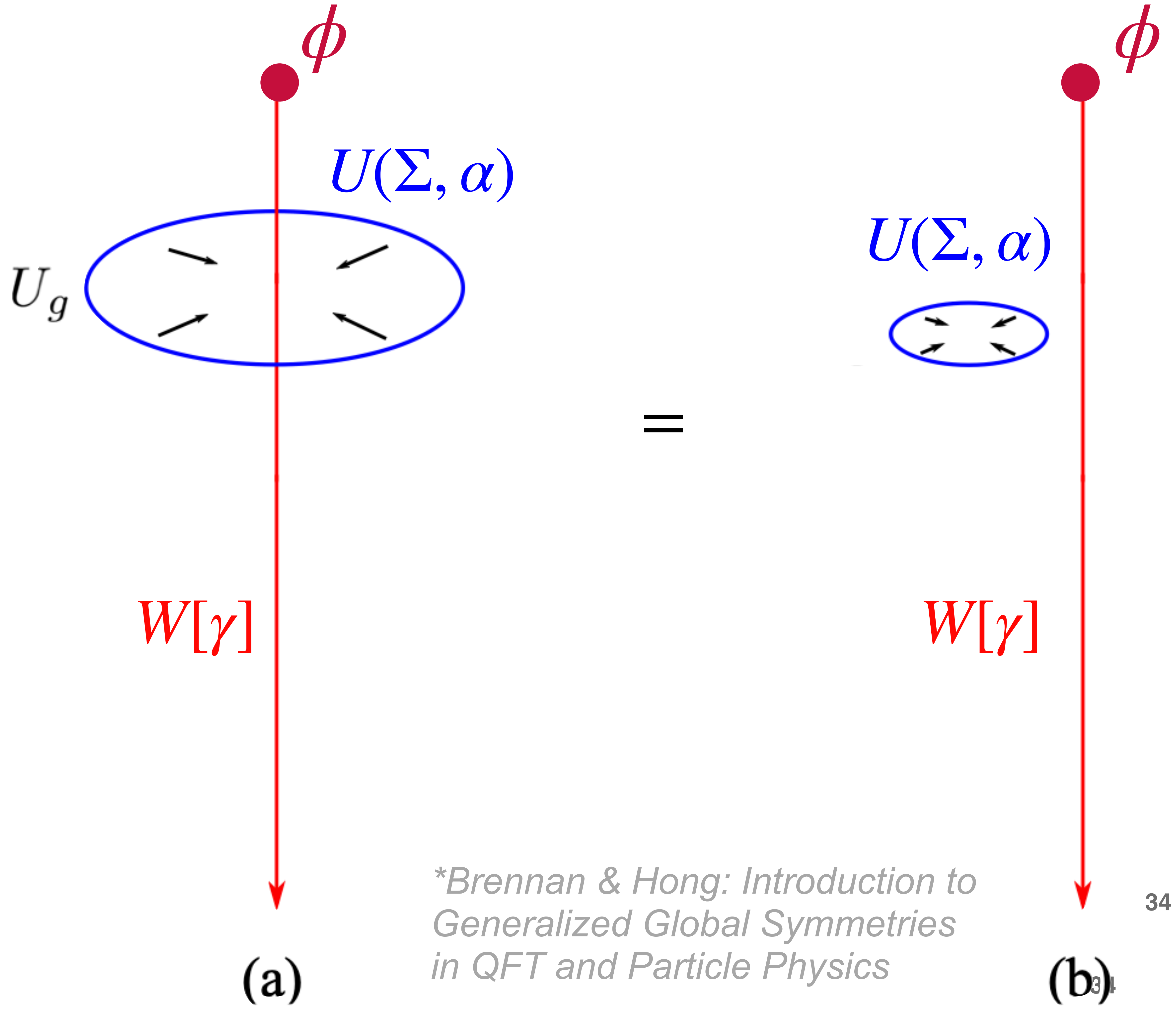

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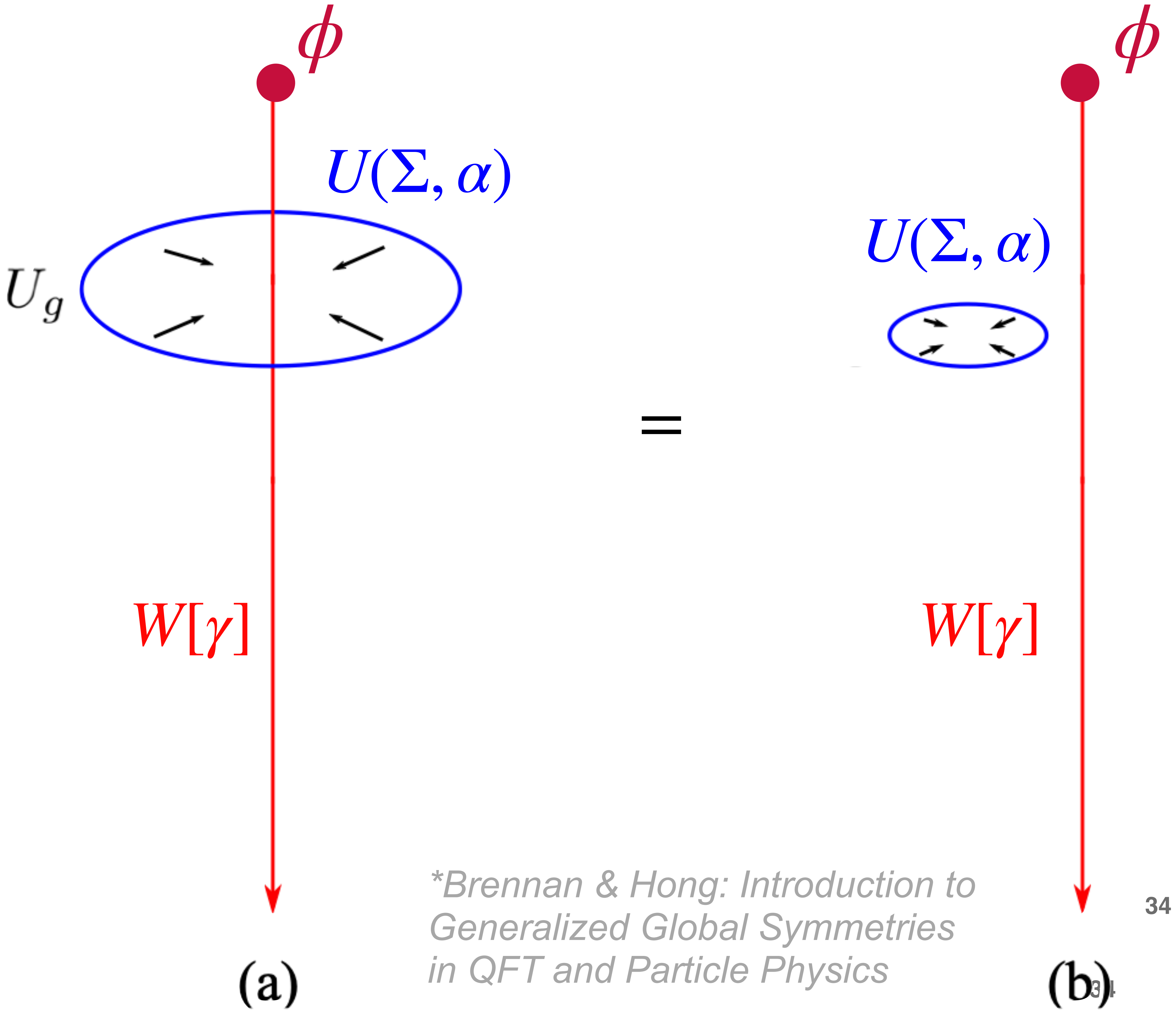
- Breaks $U(1)$ one-form symmetry down to $\mathbb{Z}_q$ one-form symmetry.



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- Monodromatic: technically invariant under $\theta \to \theta + 2\pi/M$.

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- This is just the usual gluon potential after dimensional reduction.

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Potential generation for the extra-dimensional axion can be understood systematically via all the ways of breaking/gauging the electric one-form symmetry

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- It has been shown that monopole loops in 4D yield axion potential contributions. ***"Axion Mass From Magnetic Monopole Loops" Fan, Fraser, Reece, Stout.***

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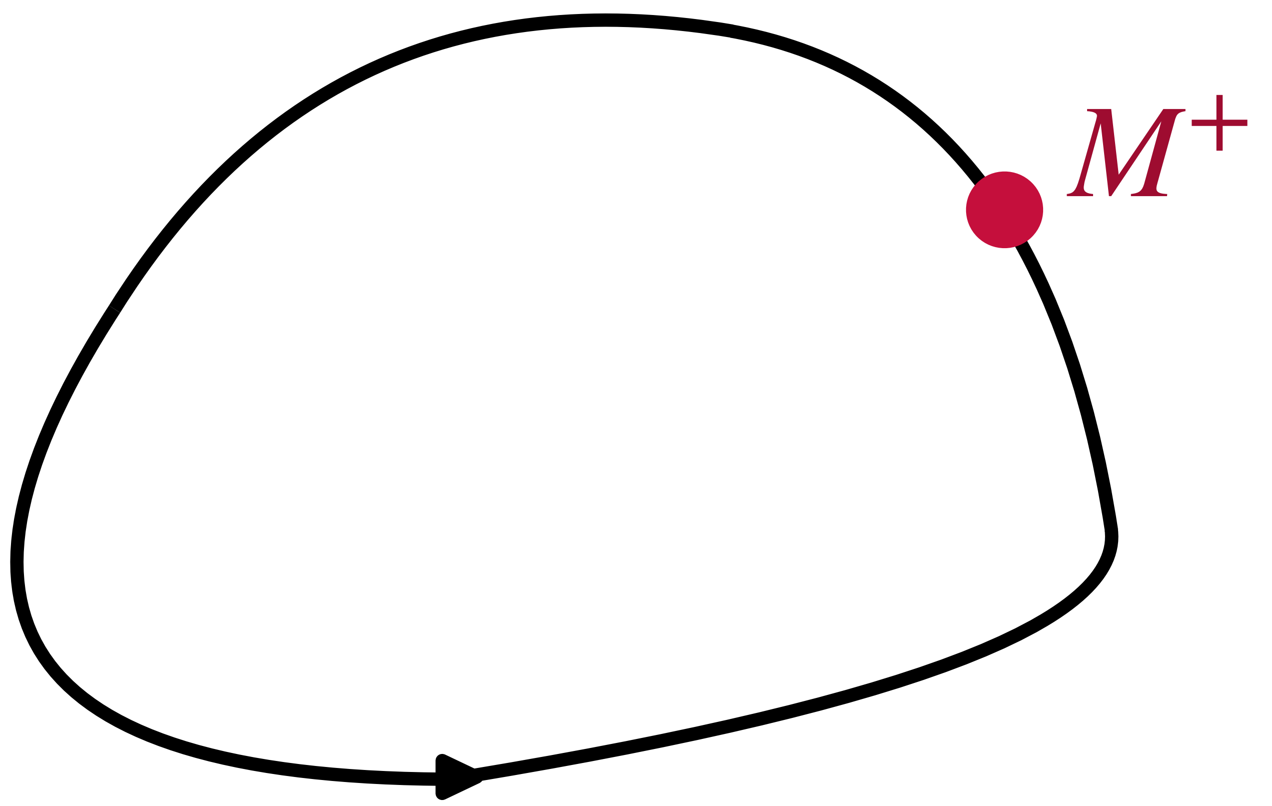

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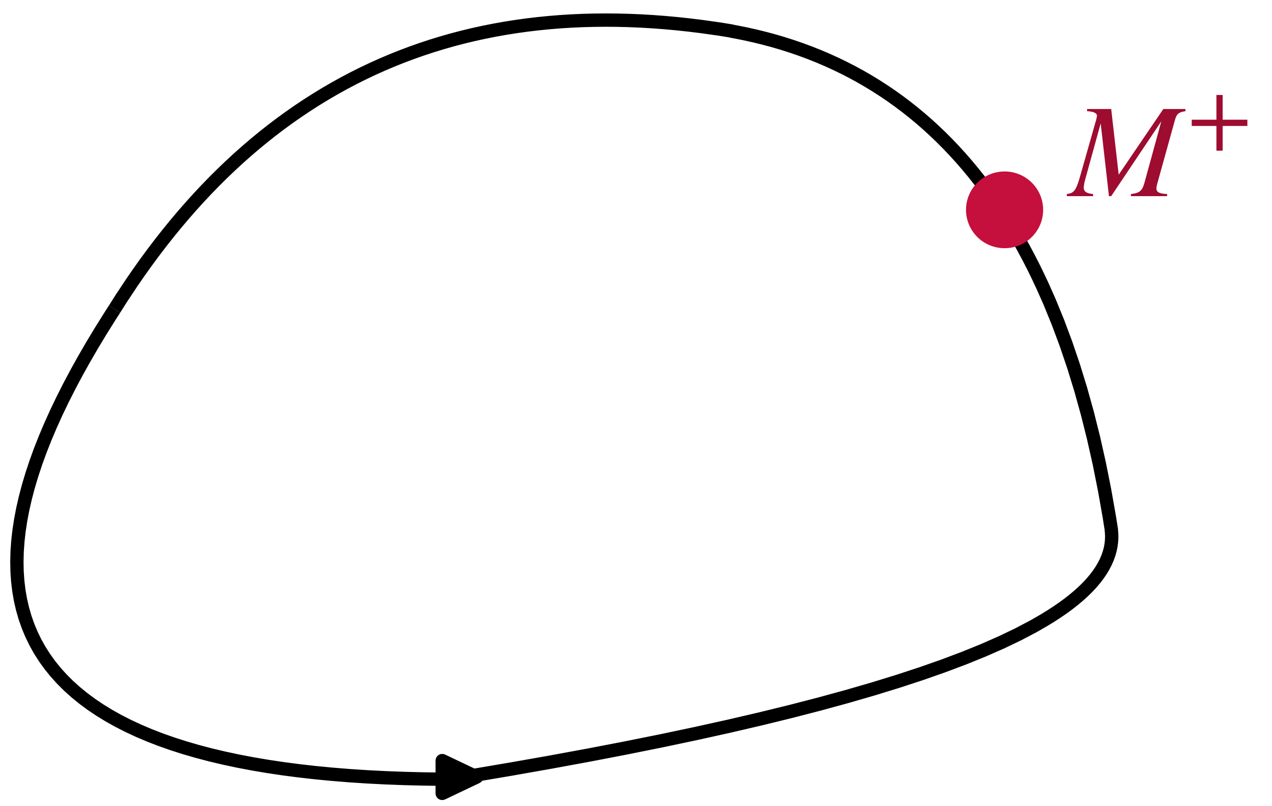

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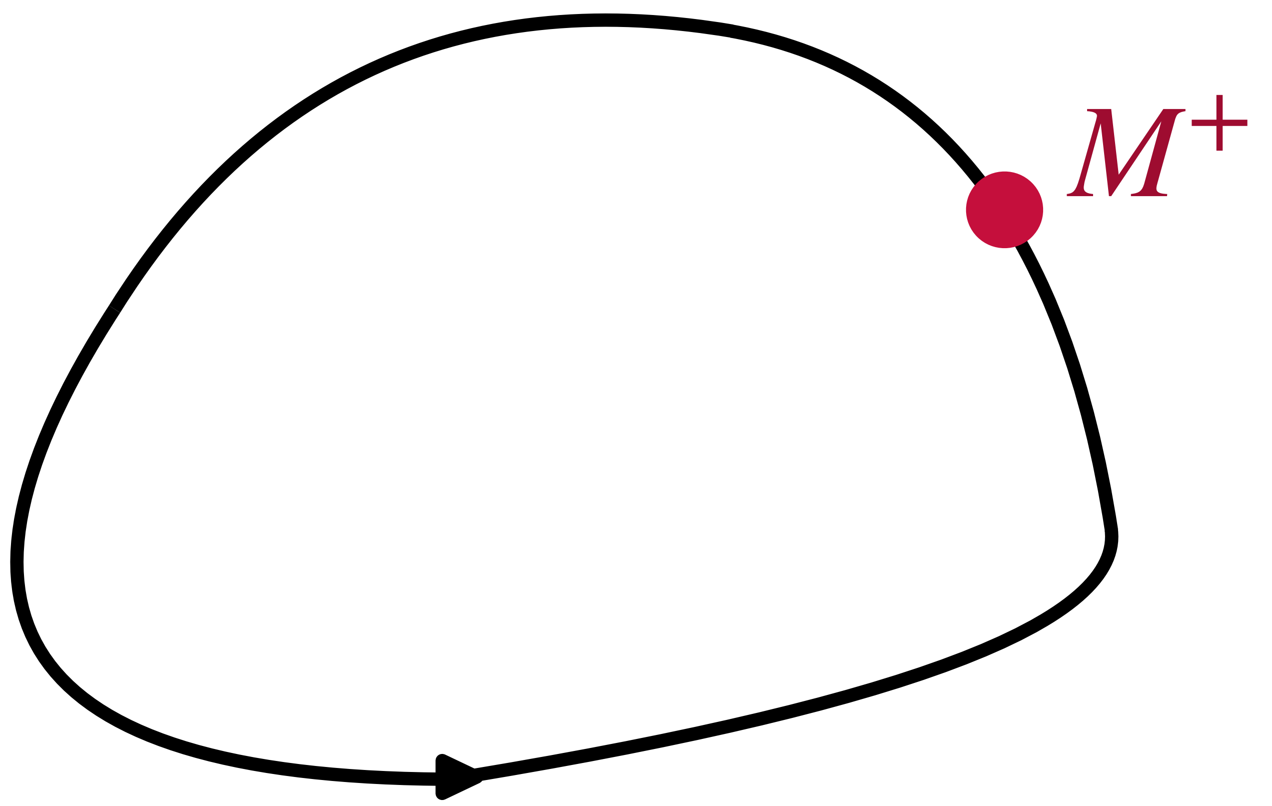

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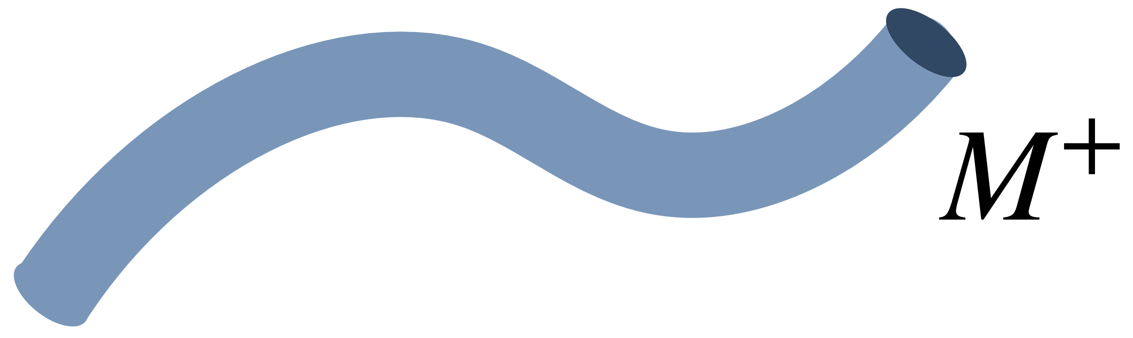

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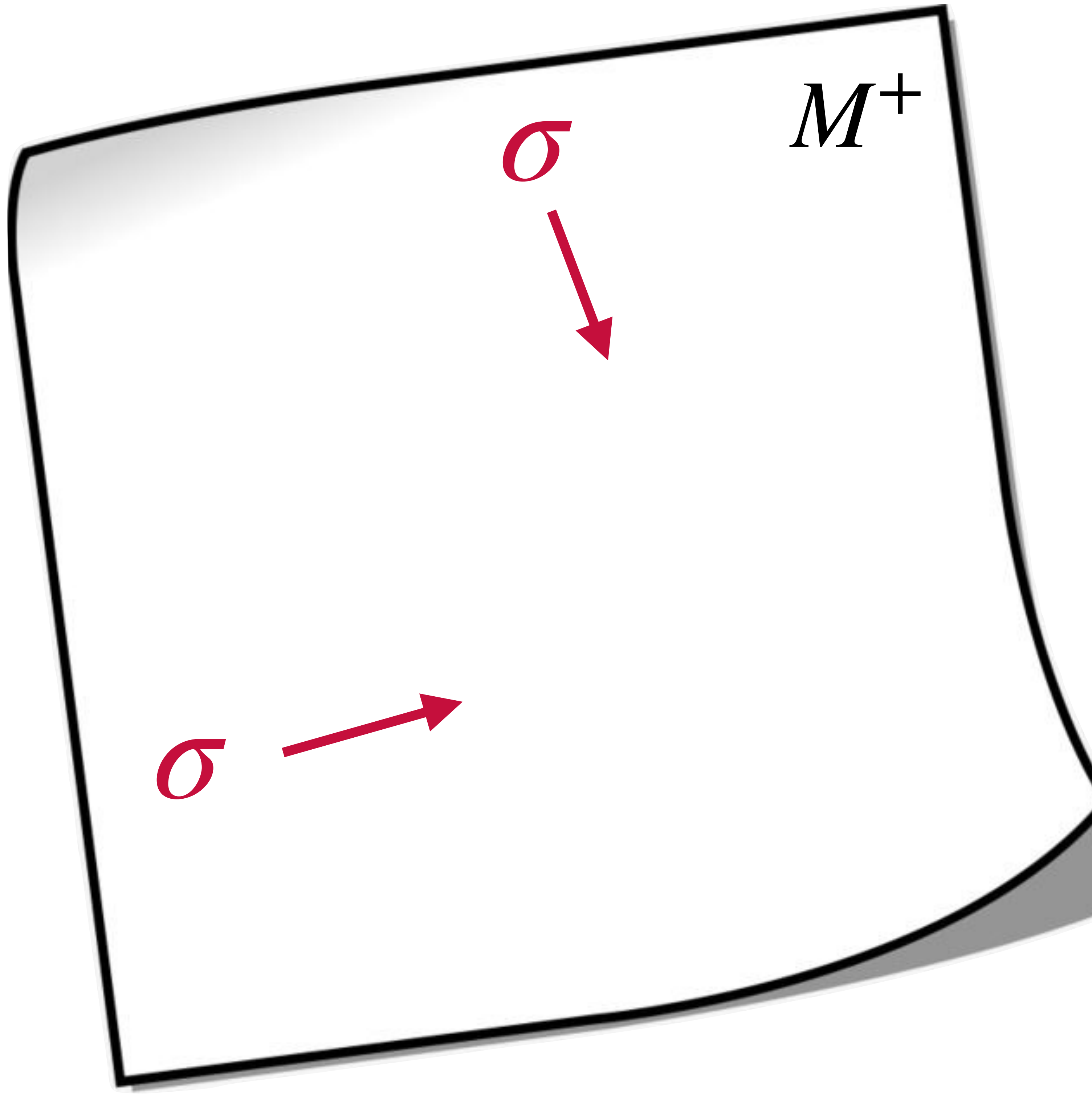

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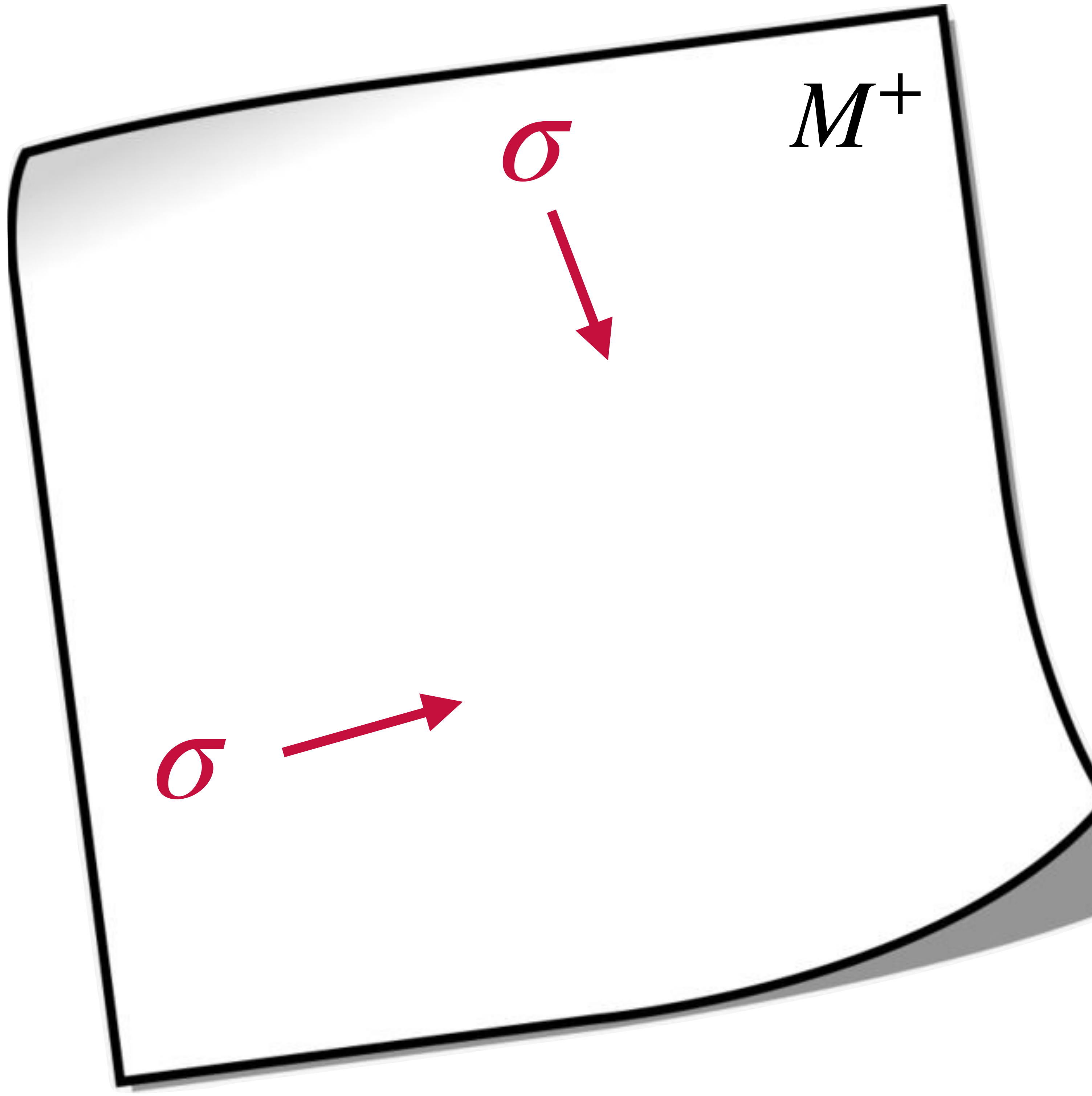

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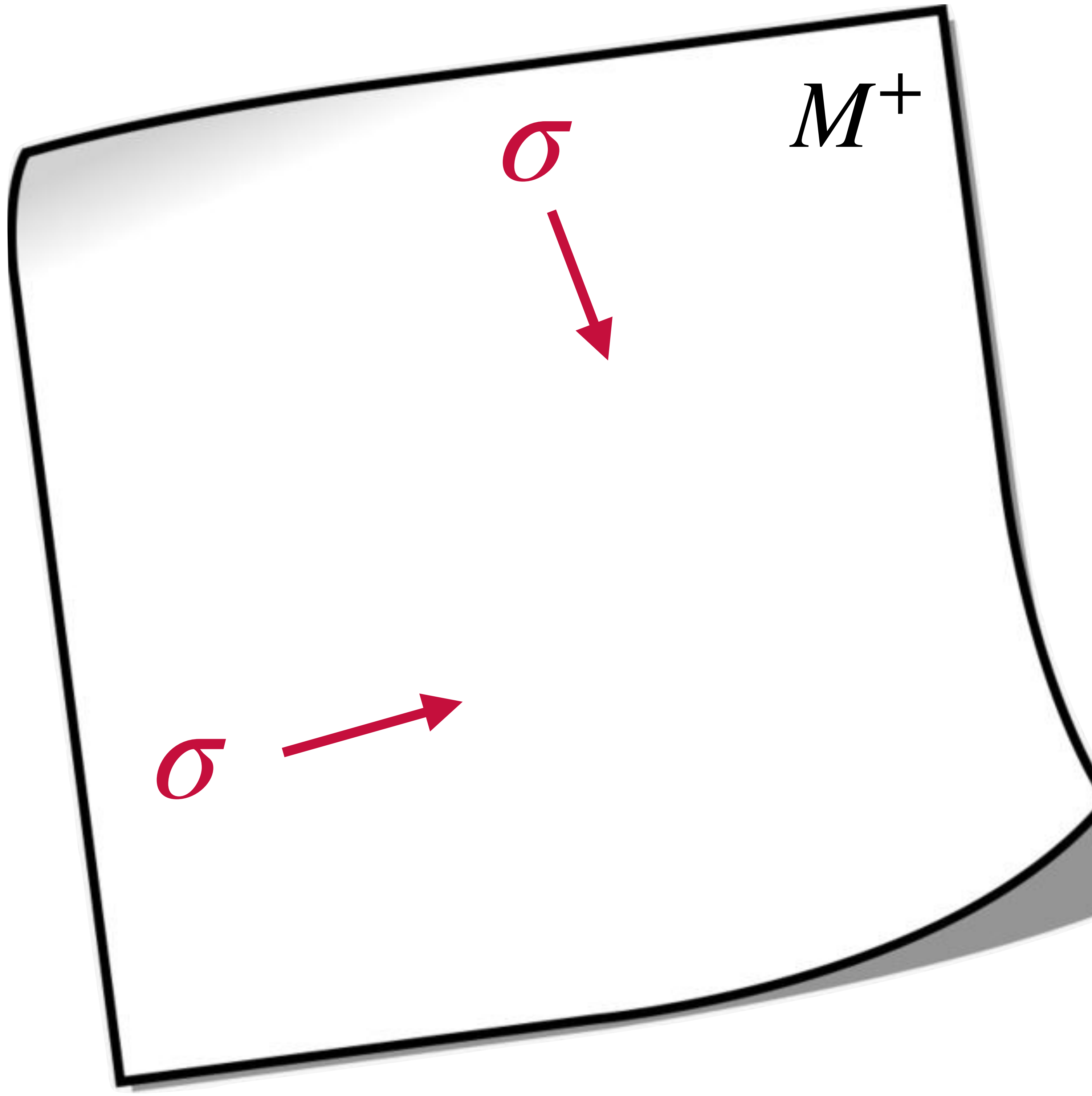

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$$V(\theta) = -\sum_{\ell=1}^{\infty}\frac{R^2T_M^2m_W^2}{8\pi^2\ell^3}e^{-4\pi^2\ell RT_M/m_W}\cos(\ell\theta)\times\left(1+\frac{3m_W}{4\pi^2\ell RT_M}+\frac{3m_W^2}{16\pi^4\ell^2R^2T_M^2}\right)$$

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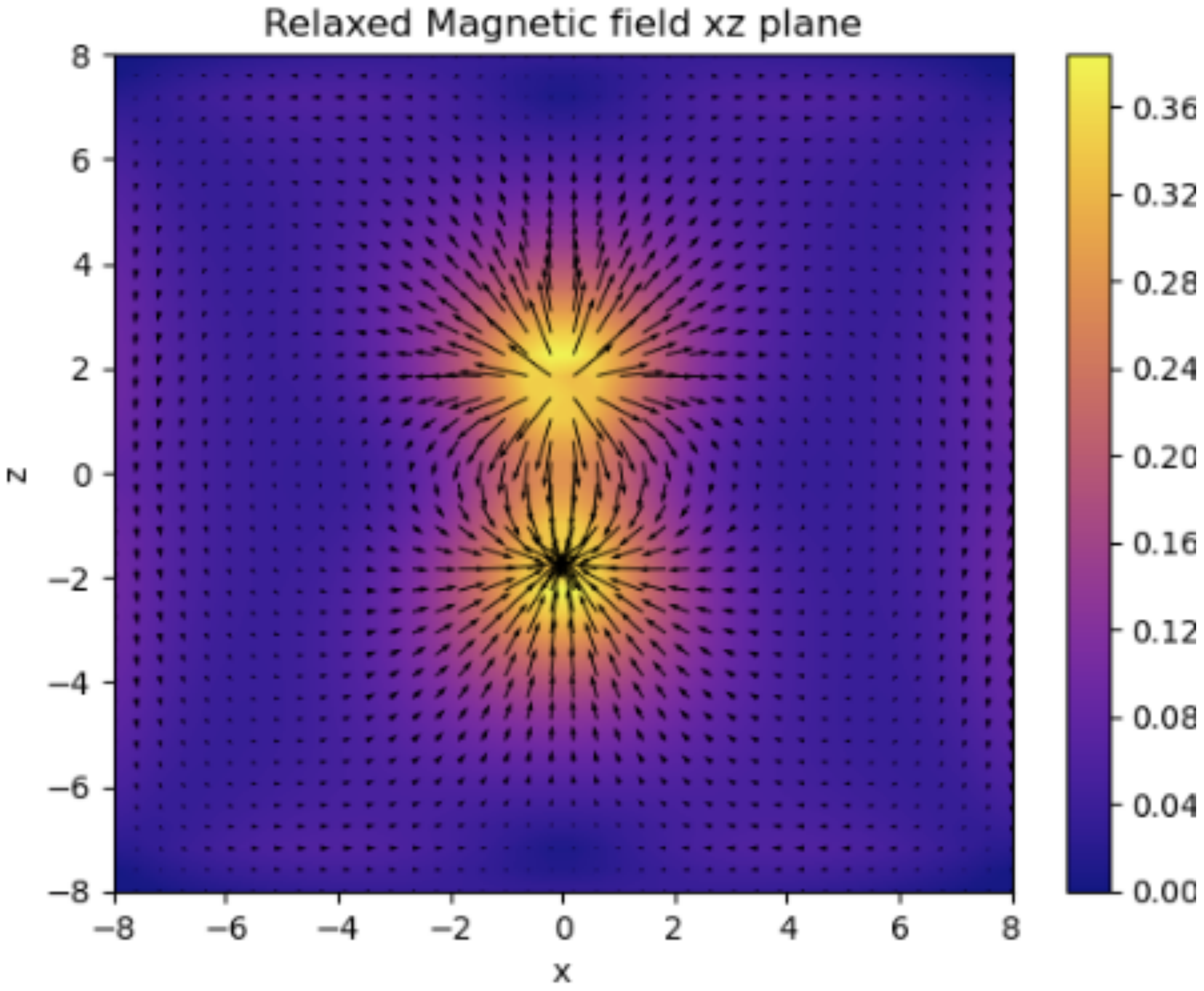

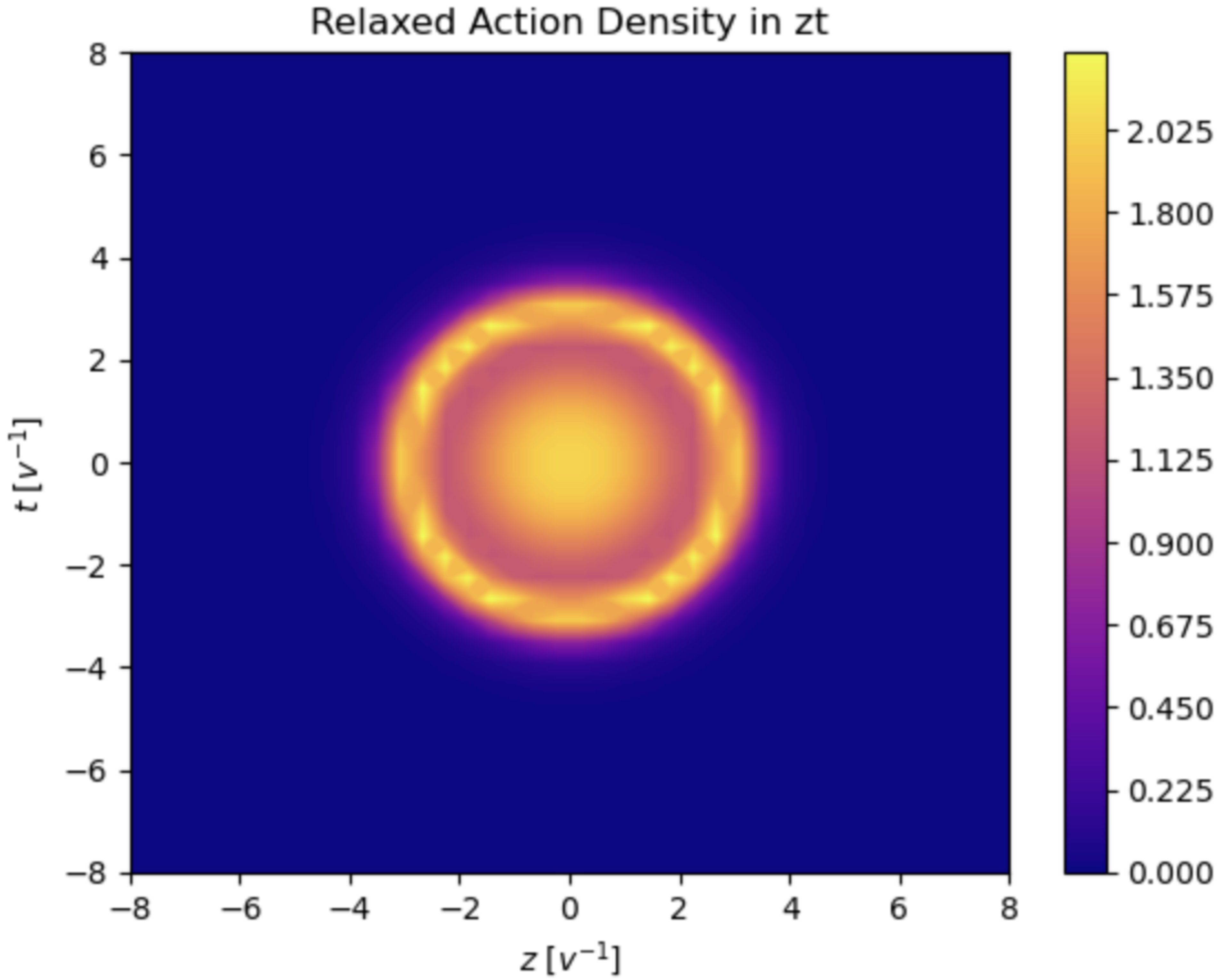

A Symmetry Breaking Scorecard

	Current Equation	Remnant Symmetry	Potential
Electrically Charged Matter	$\mathrm{d}J_e = j_{\text{matter}}$	$\mathbb{Z}_q^{(1)}$	$V \simeq \frac{(m_{5D}R)^2}{(2\pi R)^4} e^{-m_{5d}R} \cos(q\theta)$
Gauging Two-Form Magnetic	$\mathrm{d}J_e = \frac{M}{2\pi}\mathrm{d}K$	$\mathbb{Z}_M^{(1)}$	$m_\theta = \frac{M}{2\pi} g_4\, e_{K,4}$
Gauging One-Form Electric	$kJ_e' = \frac{1}{e_B^2}\mathrm{d} \star \mathrm{d}B$	$U(1)^{(1)}$	$m_{\tilde{A}} = k\, e_{B,4}\, f$
ABJ Term	$\mathrm{d}J_e = \frac{N}{8\pi^2}\mathrm{Tr}\left[G \wedge G\right]$	$\mathbb{Z}_N^{(1)}$	$V \simeq -\Lambda_{\text{monopole}}^4 \cos(N\theta)$

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- **All ways of generating a potential $\leftrightarrow$ all ways of breaking $U(1)_e^{(1)}$.**

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- **Interesting interplay between electric and magnetic sector: stay tuned for future work!**