

# Current and expected precision in the determination of the CKM triangle

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# Flavor in the quark sector

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(Gauge couplings to fermions)

$$\mathcal{L}_{SM} \sim -\frac{1}{4}(F_{\mu\nu})^2 + i\bar{\psi}\not{D}\psi$$

(Higgs self-interaction)

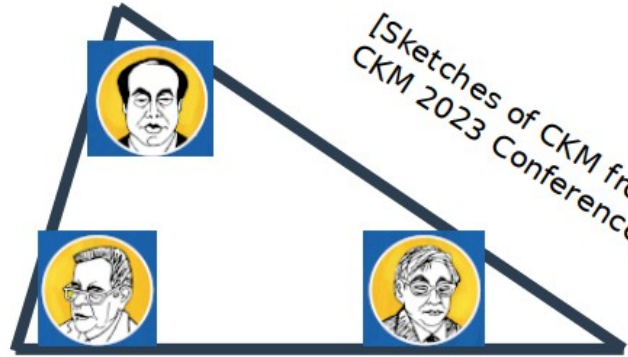
$$+ |D_{\mu}H|^2 - V(H)$$

(short-range weak interactions)

$$+ YH\bar{\psi}\psi + \text{h.c.}$$

(spectrum of fermion masses, CKM matrix)

Particle physics  
framework

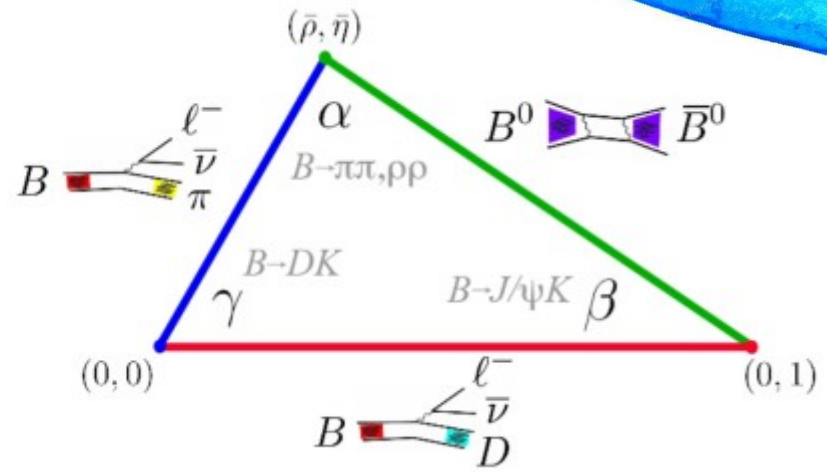
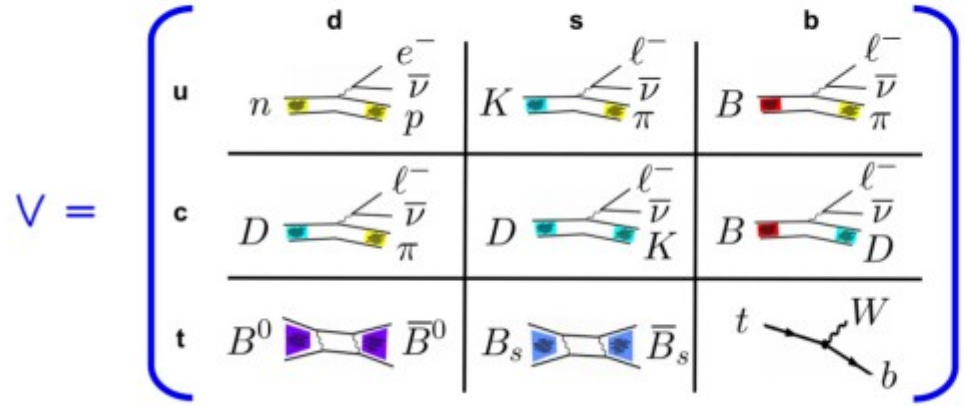


Experimental  
data

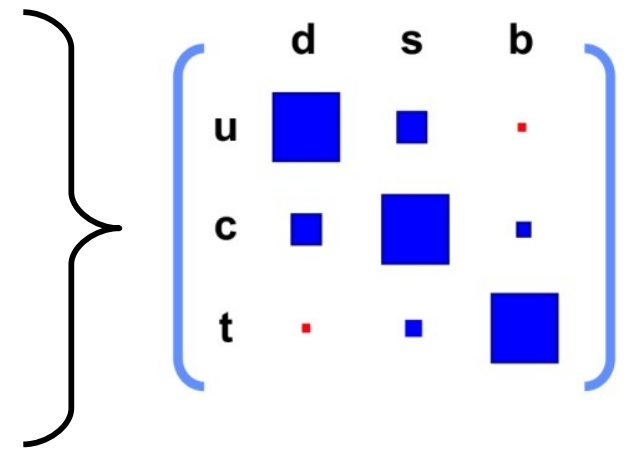
Theoretical  
inputs

- Goal is **testing the SM**, and **possibly point out tensions**
- Many flavour observables enjoy the status of **precision physics**, thanks to progress in **different fronts** (e.g. QCD inputs)
- Flavor transitions pattern is likely to change in the presence of NP

# Measuring the CKM matrix



- Tree: e.g.,  $|V_{ub}|$
- Loop: e.g.,  $\Delta m_d$ ,  $\Delta m_s$ ,  $\epsilon_K$ ,  $\sin(2\beta)$
- CP-conserving: e.g.,  $|V_{ub}|$ ,  $\Delta m_d$ ,  $\Delta m_s$
- CP-violating: e.g.,  $\gamma$ ,  $\epsilon_K$ ,  $\sin(2\beta)$
- Exp. uncs.: e.g.,  $\alpha$ ,  $\sin(2\beta)$ ,  $\gamma$
- Syst. uncs.: e.g.,  $|V_{ub}|$ ,  $|V_{cb}|$ ,  $\epsilon_K$ ,  $\Delta m_d$ ,  $\Delta m_s$



# Theo. inputs: hadronic effects

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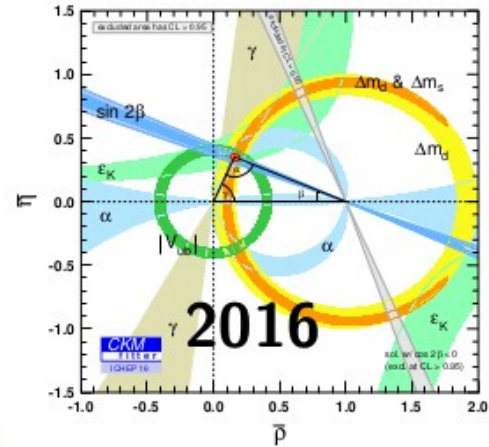
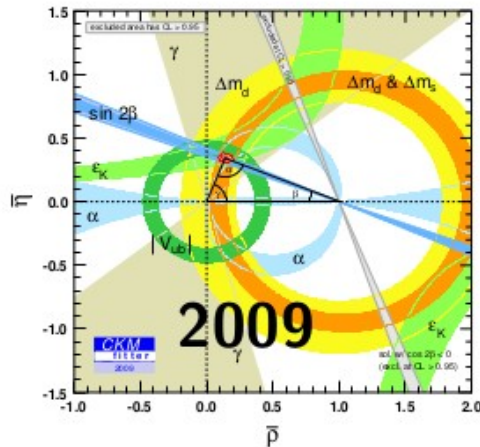
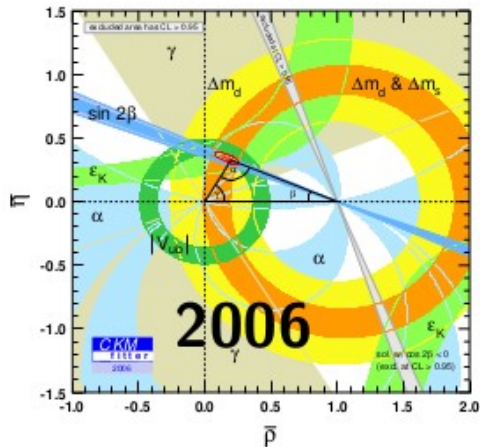
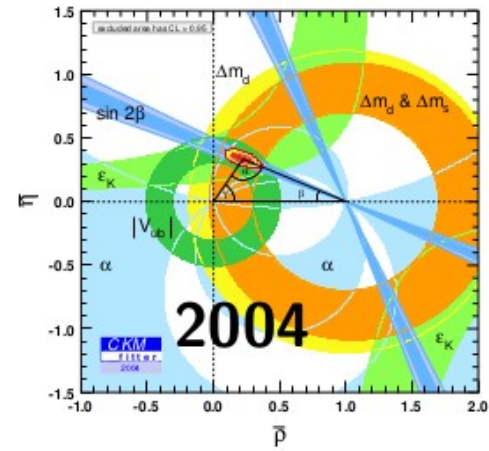
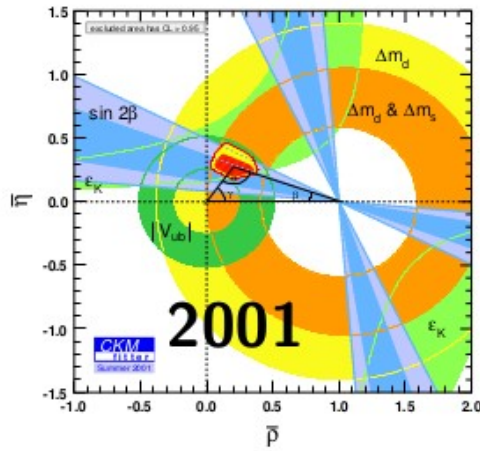
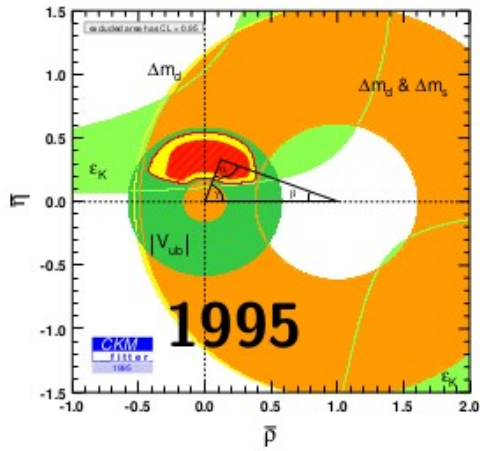
Meson-mixing	$B_{(s)}\bar{B}_{(s)}, K\bar{K}$ : bag-parameters $\hat{B}_{B_s}, \hat{B}_{B_s}/\hat{B}_{B_d}, \hat{B}_K$ $\frac{2}{3} m_K^2 f_K^2 B_K = \langle \bar{K}   (\bar{s}\gamma^\mu P_L d) (\bar{s}\gamma_\mu P_L d)   K \rangle$
(semi-)leptonic decays	$\pi \rightarrow \ell\nu, K \rightarrow \pi\ell\nu$ , etc.: decay constants, form factors Ex.: $f_\pi, f_+^{K\rightarrow\pi}(0)$ $-p_\mu f_\pi = \langle 0   (\bar{d}\gamma_\mu \gamma_5 u)   \pi(p) \rangle,$ $f_+^{K\rightarrow\pi}(q^2)(p+p')_\mu + f_-^{K\rightarrow\pi}(q^2)(p-p')_\mu = \langle \pi(p')   (\bar{s}\gamma_\mu P_L u)   K(p) \rangle$

→ Determine  $\mathcal{L}_{SM(NP)}^{eff} \sim \sum_i C_i(\mu) \times O_i(\mu)$ , where  $\mu \sim \mathcal{O}(\text{few})$  GeV:  
 $C_i$  collects *short*-distance physics;  $O_i$  collects *long*-distance physics

→ Lattice QCD: extractions of non-perturbative parameters;  
 averages dominated by **systematic uncertainties**

# Progress over the years

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# Inputs



[CKMfitter]

$\mathcal{L} \stackrel{\text{Rfit}}{=} \mathcal{L}_{\text{stat}} \times \mathcal{L}_{\text{theo}}$   
 $\chi^2 = -2 \ln \mathcal{L}$   
 $\mathcal{L}_{\text{stat}}$ : exp. data  
 $\mathcal{L}_{\text{theo}}$ : had. inputs

CKM	Process	Observable	Non-perturbative inputs
$ V_{ud} $	$0^+ \rightarrow 0^+ \beta$	$ V_{ud} _{\text{nucl}} = 0.97373 \pm 0.00009 \pm 0.00053$ [14]	Nuclear matrix elements
$ V_{us} $	$K \rightarrow \pi \ell \nu_\ell$	$ V_{us} _{\text{SL}} f_+^{K \rightarrow \pi}(0) = 0.21635 \pm 0.00038$ [38]	$f_+^{K \rightarrow \pi}(0) = 0.9675 \pm 0.0011 \pm 0.0023$
	$K \rightarrow e \nu_e$	$\mathcal{B}(K \rightarrow e \nu_e) = (1.582 \pm 0.007) \cdot 10^{-5}$ [39]	
	$K \rightarrow \mu \nu_\mu$	$\mathcal{B}(K \rightarrow \mu \nu_\mu) = 0.6356 \pm 0.0011$ [39]	$f_K = 155.57 \pm 0.17 \pm 0.57 \text{ MeV}$
	$\tau \rightarrow K \nu_\tau$	$\mathcal{B}(\tau \rightarrow K \nu_\tau) = (0.6986 \pm 0.0085) \cdot 10^{-2}$ [19]	
$ V_{us}/V_{ud} $	$K \rightarrow \mu \nu_\mu / \pi \rightarrow \mu \nu_\mu$	$\frac{\mathcal{B}(K \rightarrow \mu \nu_\mu)}{\mathcal{B}(\pi \rightarrow \mu \nu_\mu)} = 1.3367 \pm 0.0028$ [39]	$f_K/f_\pi = 1.1973 \pm 0.0007 \pm 0.0014$
	$\tau \rightarrow K \nu_\tau / \tau \rightarrow \pi \nu_\tau$	$\frac{\mathcal{B}(\tau \rightarrow K \nu_\tau)}{\mathcal{B}(\tau \rightarrow \pi \nu_\tau)} = (6.437 \pm 0.092) \cdot 10^{-2}$ [19]	
$ V_{cd} $	$\nu N$	$ V_{cd} _{\text{not lattice}} = 0.230 \pm 0.011$ [39]	
	$D \rightarrow \tau \nu_\tau$	$\mathcal{B}(D \rightarrow \tau \nu_\tau) = (1.20 \pm 0.27) \cdot 10^{-3}$ [19]	$f_{D_s}/f_D = 1.1782 \pm 0.0006 \pm 0.0033$
	$D \rightarrow \mu \nu_\mu$	$\mathcal{B}(D \rightarrow \mu \nu_\mu) = (3.77 \pm 0.17) \cdot 10^{-4}$ [19]	
	$D \rightarrow \pi \ell \nu_\ell$	$ V_{cd} _{\text{SL}} f_+^{D \rightarrow \pi}(0) = 0.1426 \pm 0.0018$ [19]	$f_+^{D \rightarrow \pi}(0) = 0.624 \pm 0.004 \pm 0.006$
$ V_{cs} $	$W \rightarrow c \bar{s}$	$ V_{cs} _{\text{not lattice}} = 0.967 \pm 0.011$ [40]	
	$D_s \rightarrow \tau \nu_\tau$	$\mathcal{B}(D_s \rightarrow \tau \nu_\tau) = (5.32 \pm 0.10) \cdot 10^{-2}$ [18]	$f_{D_s} = 249.23 \pm 0.27 \pm 0.65 \text{ MeV}$
	$D_s \rightarrow \mu \nu_\mu$	$\mathcal{B}(D_s \rightarrow \mu \nu_\mu) = (5.43 \pm 0.16) \cdot 10^{-3}$ [19]	
	$D \rightarrow K \ell \nu_\ell$	$ V_{cs} _{\text{SL}} f_+^{D \rightarrow K}(0) = 0.7180 \pm 0.0033$ [19]	$f_+^{D \rightarrow K}(0) = 0.742 \pm 0.002 \pm 0.004$
$ V_{ub} $	semileptonic $B$	$ V_{ub} _{\text{SL}} = (3.86 \pm 0.07 \pm 0.12) \cdot 10^{-3}$ see text	form factors, shape functions
	$B \rightarrow \tau \nu_\tau$	$\mathcal{B}(B \rightarrow \tau \nu_\tau) = (1.09 \pm 0.24) \cdot 10^{-4}$ [19]	$f_{B_s}/f_B = 1.2118 \pm 0.0020 \pm 0.0058$
$ V_{cb} $	semileptonic $B$	$ V_{cb} _{\text{SL}} = (41.22 \pm 0.24 \pm 0.37) \cdot 10^{-3}$ see text	form factors, OPE matrix elements
$ V_{ub}/V_{cb} $	semileptonic $\Lambda_b$	$\frac{\mathcal{B}(\Lambda_b \rightarrow p \mu \nu_\mu)_{q^2 > 15}}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \mu \nu_\mu)_{q^2 > 7}} = (0.918 \pm 0.083) \cdot 10^{-2}$ [27]	$\frac{\zeta(\Lambda_b \rightarrow p \mu \nu_\mu)_{q^2 > 15}}{\zeta(\Lambda_b \rightarrow \Lambda_c \mu \nu_\mu)_{q^2 > 7}} = 1.471 \pm 0.096 \pm 0.290$
	semileptonic $B_s$	$\frac{\mathcal{B}(B_s \rightarrow K \mu \nu_\mu)_{q^2 > 7}}{\mathcal{B}(B_s \rightarrow D_s \mu \nu_\mu)} = (3.25 \pm 0.28) \cdot 10^{-3}$ [41]	$\frac{\zeta(B_s \rightarrow K \mu \nu_\mu)_{q^2 > 7}}{\zeta(B_s \rightarrow D_s \mu \nu_\mu)} = 0.363 \pm 0 \pm 0.065$
	inclusive	$ V_{ub}/V_{cb} _{\text{incl}} = 0.100 \pm 0.006 \pm 0.003$ [29]	
$\alpha$	$B \rightarrow \pi \pi, \rho \pi, \rho \rho$	branching ratios, $CP$ asymmetries	see text
$\beta$	$B \rightarrow (c \bar{c}) K$	$\sin(2\beta)_{[c \bar{c}]} = 0.708 \pm 0.011$ [19]	subleading penguins neglected
$\gamma$	$B \rightarrow D^{(*)} K^{(*)}$	$\gamma = (65.9_{-3.5}^{+3.3})^\circ$ [19]	GGSZ, GLW, ADS methods
$\phi_s$	$B_s \rightarrow J/\psi(KK, \pi\pi)$	$(\phi_s)_{b \rightarrow c \bar{c} s} = -0.039 \pm 0.016$ [19]	
$V_{tq}^* V_{tb}$	$\Delta m_d$	$\Delta m_d = 0.5065 \pm 0.0019 \text{ ps}^{-1}$ [19]	$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.007 \pm 0.010 \pm 0.014$
	$\Delta m_s$	$\Delta m_s = 17.765 \pm 0.006 \text{ ps}^{-1}$ [42]	$\hat{B}_{B_s} = 1.313 \pm 0.012 \pm 0.030$
	$B_s \rightarrow \mu \mu$	$\mathcal{B}(B_s \rightarrow \mu \mu) = (3.45 \pm 0.29) \cdot 10^{-9} [\times (1 - 0.063)]$ [19]	$f_{B_s} = 228.75 \pm 0.69 \pm 1.87 \text{ MeV}$
$V_{td}^* V_{ts}$ and $V_{cd}^* V_{cs}$	$\varepsilon_K$	$ \varepsilon_K  = (2.228 \pm 0.011) \cdot 10^{-3}$ [39]	$\hat{B}_K = 0.7567 \pm 0.0020 \pm 0.0123$ $\kappa_\varepsilon = 0.940 \pm 0.013 \pm 0.023$

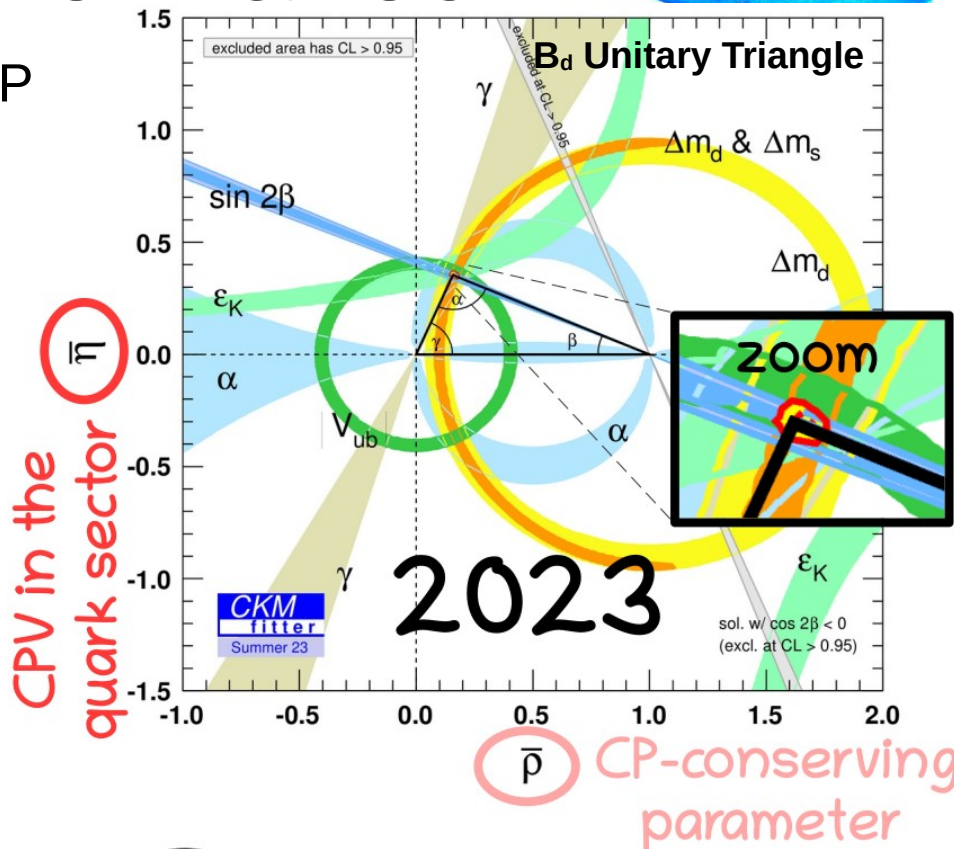
[LVS, 2405.08046]

# Current status of flavour

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- A **single** phase must be responsible for CP violation across distinct flavour sectors
- Observables of very different natures

$$\begin{aligned}
 A &= 0.8215^{+0.0047}_{-0.0082} \quad (0.8\% \text{ unc.}) \\
 \lambda &= 0.22498^{+0.00023}_{-0.00021} \quad (0.1\% \text{ unc.}) \\
 \bar{\rho} &= 0.1562^{+0.0112}_{-0.0040} \quad (4.9\% \text{ unc.}) \\
 \bar{\eta} &= 0.3551^{+0.0051}_{-0.0057} \quad (1.5\% \text{ unc.}) \\
 &68\% \text{ C.L. intervals}
 \end{aligned}$$



Rephasing invariant:

$$\frac{|V_{us}|}{(|V_{ud}|^2 + |V_{us}|^2)^{1/2}} = \lambda, \quad \frac{|V_{cb}|}{(|V_{ud}|^2 + |V_{us}|^2)^{1/2}} = A\lambda^2, \quad -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = \bar{\rho} + i\bar{\eta}$$

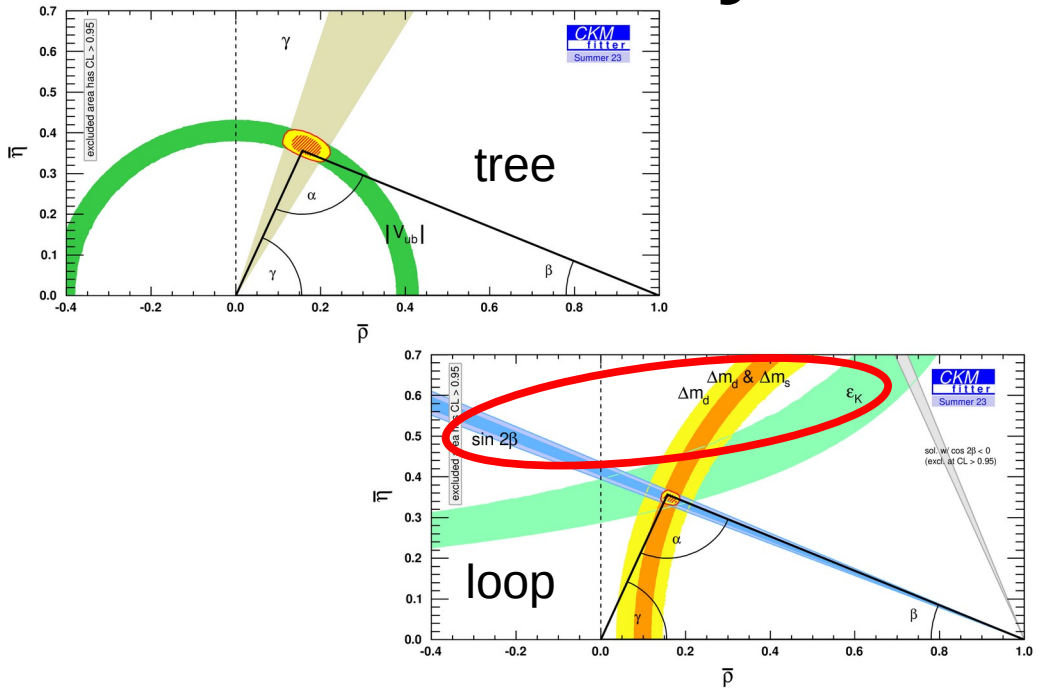
**b**

$|V_{xb}|$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  
 $\Delta m_d$ ,  $\Delta m_s$

**s**

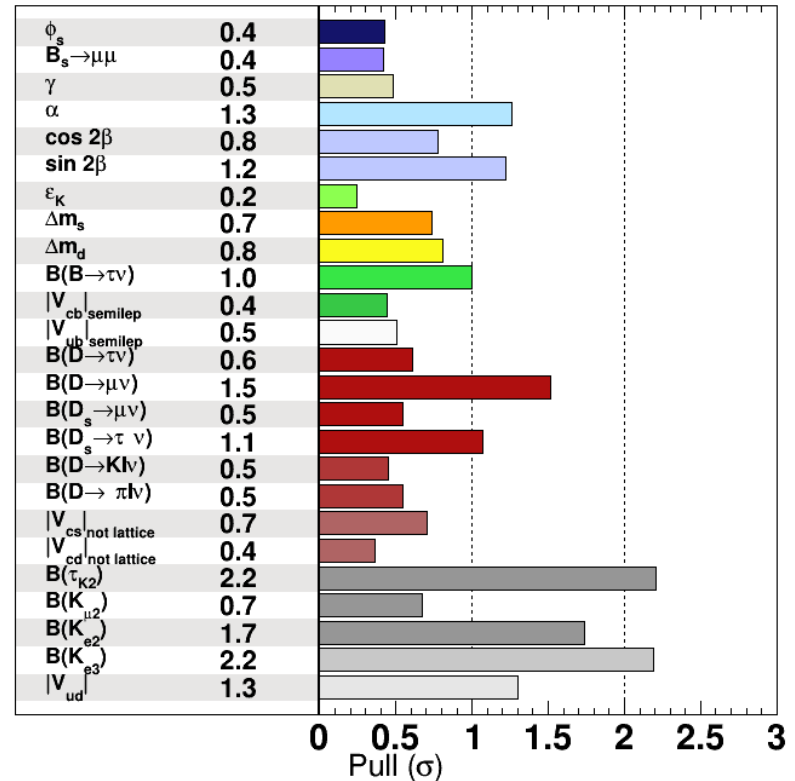
$\epsilon_K$

# Consistency among observables



$$pull_{O_{exp}} = \sqrt{\chi_{min}^2 - \chi_{min, !O_{exp}}^2}$$

!O<sub>exp</sub>:  $\chi_{min}^2$  w/o  $O_{exp}$

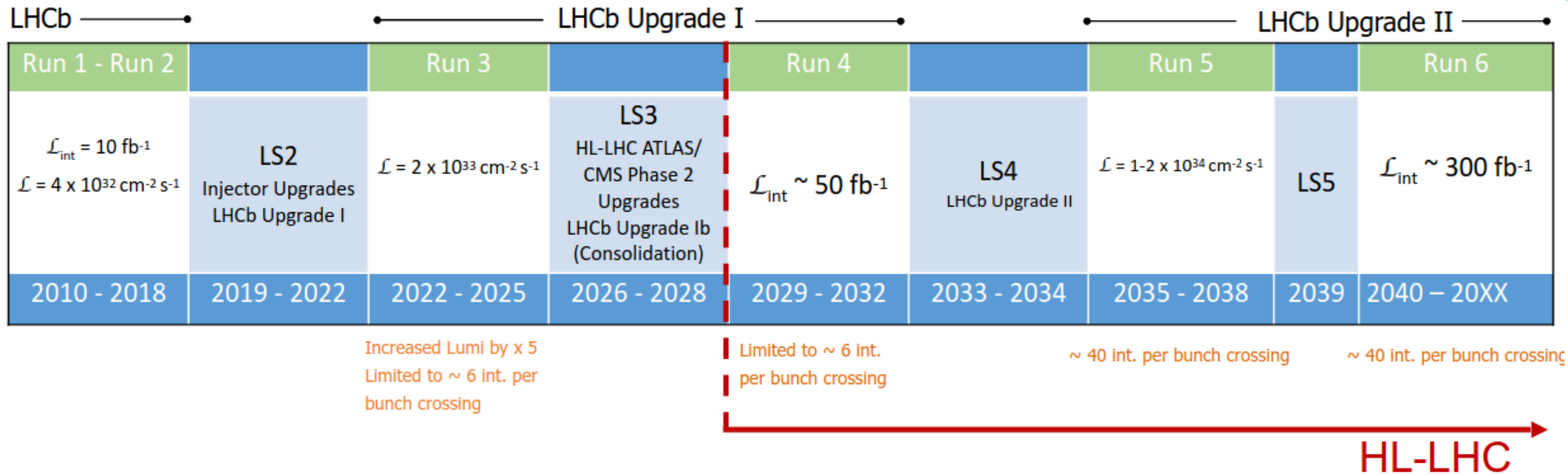


- If NP in loop: extraction of  $\rho$ ,  $\eta$  degraded
- Overall agreement w/ the SM, but some existing tensions



# New era of flavour ahead

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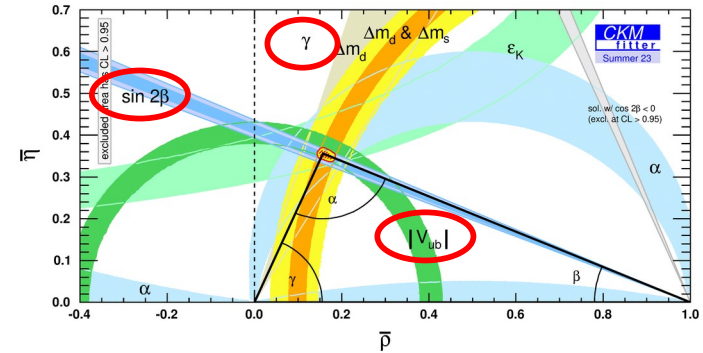


- **LHCb Upgrades [1812.07638]; Belle II [1808.10567]**
- Phase I: **LHCb-upgrade I 50/fb**, & **Belle II 50/ab**
- Other later phases (more uncertain) left in back up

# Experimental inputs

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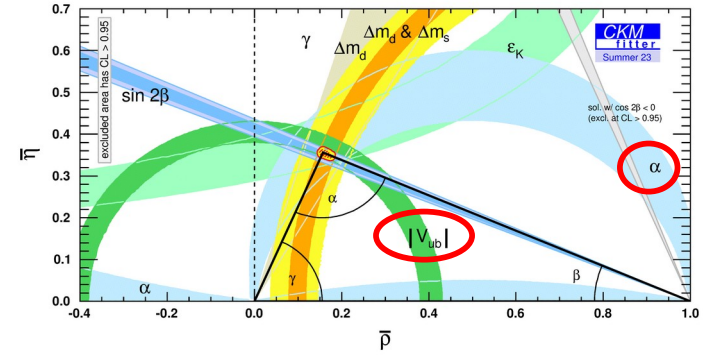
Quantity	Current (Rfit)	Unc. Phase I
$\sin(2\beta)_{[c\bar{c}]}$	0.011	0.005
$\gamma$	$3.4^\circ$	$1^\circ$
$ V_{ub} / V_{cb} _{\Lambda_b}$	10 %	3 %
$(\phi_s)_{[b \rightarrow c\bar{c}s]}$	21 mrad	14 mrad



- Sizes of  $1\sigma$  relative uncertainties [HL-LHC 1812.07638; Belle II 1808.10567]
- **Important role played by LHC in extracting quantities above**
- $|V_{ub}/V_{cb}|$ : lattice determination of  $\Lambda_b$  form factors [Detmold, Lehner, Meinel '15]; extraction from  $B_s$  carries currently similar uncertainty (but larger)
- $\beta$  and  $\phi_s$ : **penguin pollution**, various estimates (QCDF, SU(3), etc.) w/ small central values and uncs. in the range  $|\Delta\beta| \sim < 1^\circ$  [Boos, Mannel, Reuter '04; Ciuchini et al. '05; De Bruyn, Fleischer '14; Frings, Nierste, Wiebusch '15; ...]

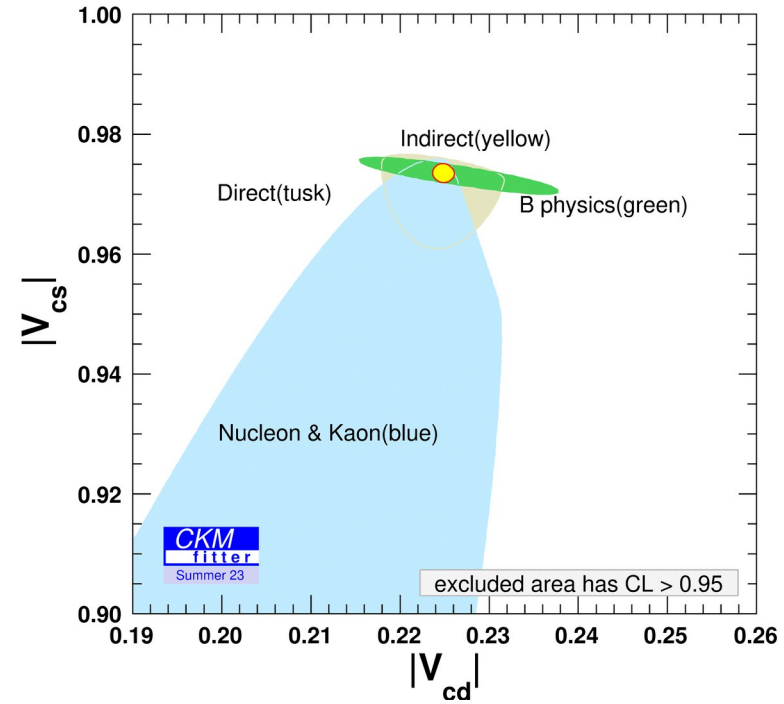
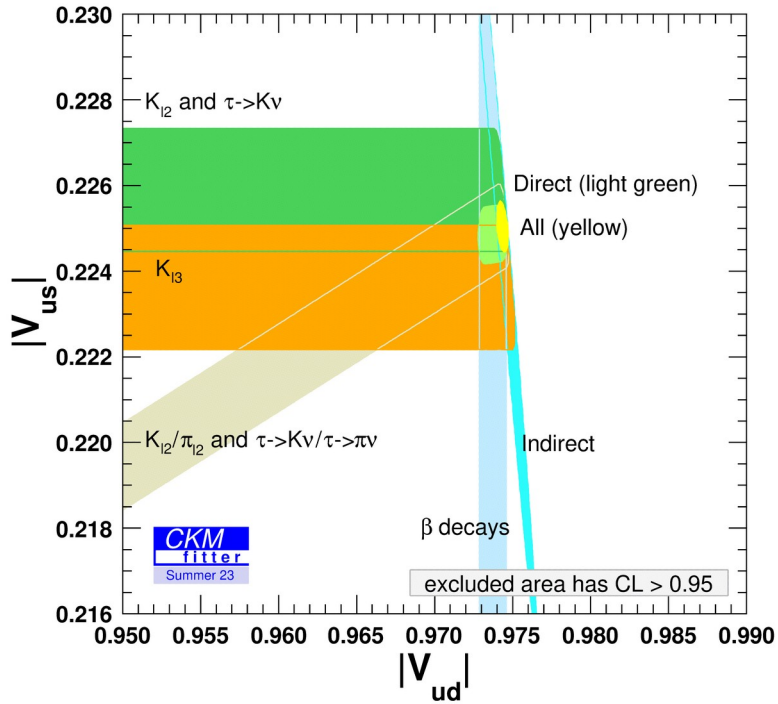
# Other experimental inputs

Quantity	Current (Rfit)	Unc. Phase I
$ V_{ub} _{SL}$	0.19	0.042
$ V_{cb} _{SL}$	0.61	0.60
$\alpha$	$3.7^\circ$	$0.6^\circ$
$\mathcal{B}(B \rightarrow \tau\nu)$	22 %	5 %



- Sizes of  $1\sigma$  relative uncertainties [**Belle II** 1808.10567]
- $|V_{ub}|$  and  $|V_{cb}|$  **exclusive**: assumptions about lattice improvements; only exclusive extractions considered for projections (“Current” includes inclusive)
- $\alpha$ : limitation from neutral modes, main experimental improvement expected from Belle II; need to account for **isospin breaking corrections** [Charles, Deschamps, Descotes-G., Niess ‘17]
- **Strangeness**: presently small role in extracting  $\rho$  and  $\eta$

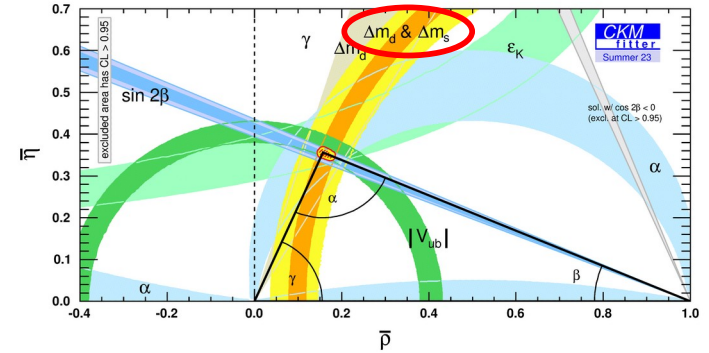
# Other experimental inputs



- $|V_{ud}|, |V_{us}|$ : no expected exp. change from (semi-)leptonic decays, only from lattice inputs
- $|V_{cd}|, |V_{cs}|$ : would need substantial exp (and theo) improvement from the charm sector

# Lattice inputs

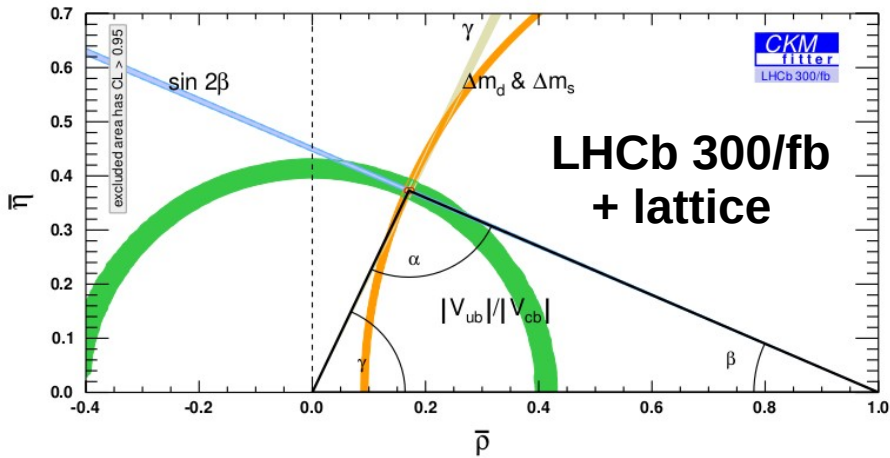
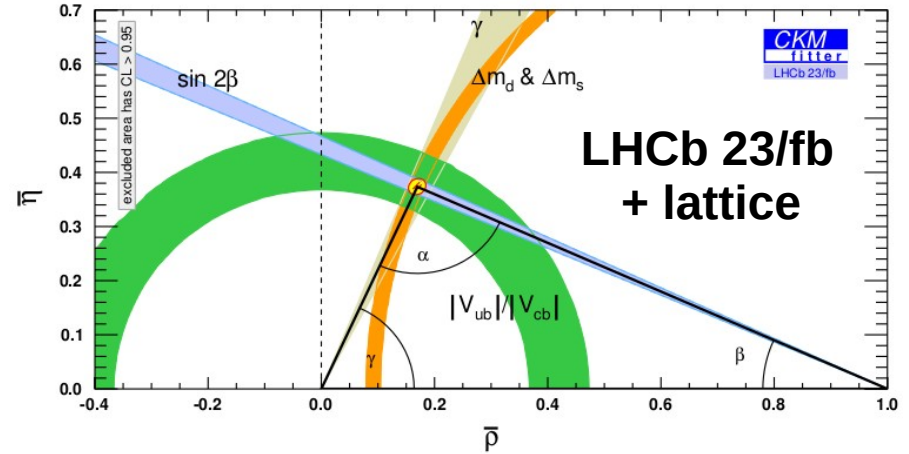
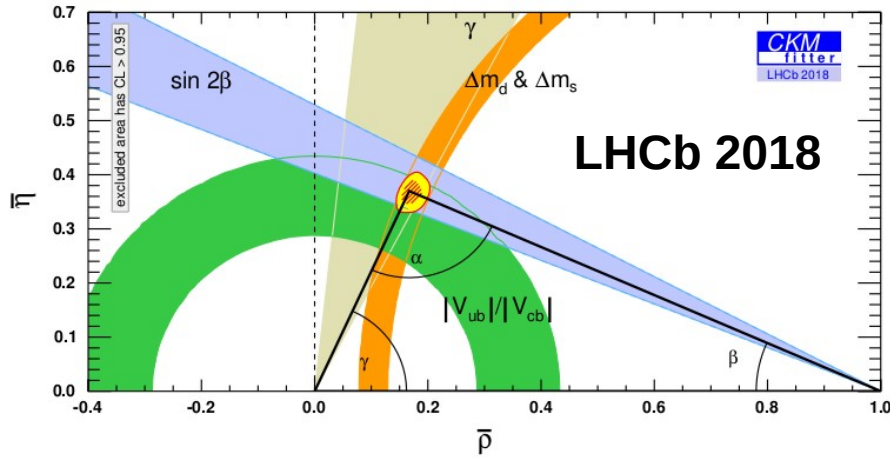
Quantity	Current (Rfit)	Unc. Phase I
$f_K$	0.5 %	0.3 %
$f_+^{K \rightarrow \pi}(0)$	0.4 %	0.1 %
$f_{B_s}$	1.1 %	0.5 %
$B_{B_s}$	3.2 %	0.8 %
$f_{B_s}/f_{B_d}$	0.6 %	0.4 %
$B_{B_s}/B_{B_d}$	2.4 %	0.5 %



- Sizes of  $1\sigma$  relative uncertainties [1812.07638]
- **Ratios**: better control of chiral extrapolations to light quark masses
- **Isospin and QED corrections** for leptonic and semileptonic decays: known and around 1% effect for K; will be needed for B (and D)

# Impact of LHCb + lattice

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[1808.08865]

Illustrative purpose: only including LHCb

# CKM metrology [Preliminary!]

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Quantity	Current (Rfit)	Full agree	Unc. Phase I
$A$	1.8 %	1.9 %	1.4 %
$\lambda$	0.2 %	0.3 %	0.3 %
$\bar{\rho}$	10 %	9.3 %	3.9 %
$\bar{\eta}$	3.7 %	3.6 %	1.4 %

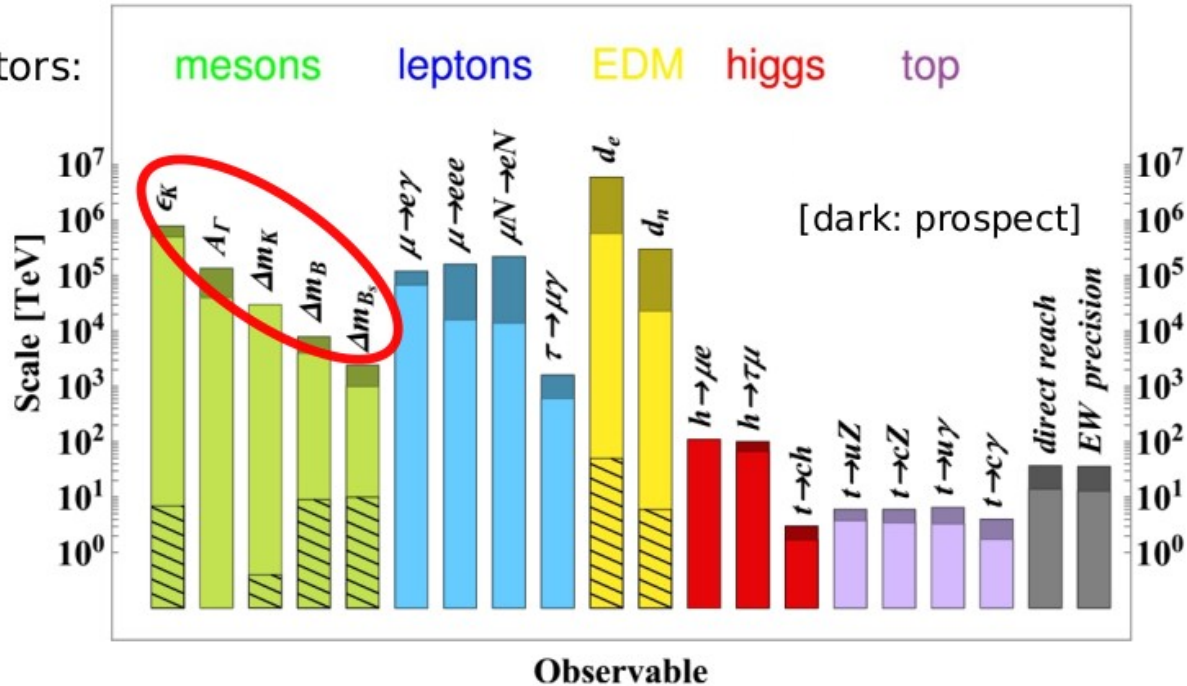
- Sizes of  $2\sigma$  relative uncertainties
  - “Current” uncertainties follow Rfit scheme
  - “Full agree”: central values adjusted to have **perfect agreement** among the constraints, and total theo. unc. treated as **Gaussian**
  - Future uncertainties treated as Gaussian
- $A$  and  $\lambda$  extractions driven by  $|V_{ud}|$ ,  $|V_{us}|$ ,  $|V_{cb}|$  inputs
- **Improvement of  $\rho$ ,  $\eta$  by factor of 2-3**

# Reach to New Physics (NP)

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- **Low-energy observables**: probe energies much beyond the reach of direct searches
- The bounds on non-SM contributions **shape NP candidates**
- If deviation seen, possible **NP manifestation!**

Different sectors:





# NP in B meson mixing

- **NP in  $|\Delta B|=2$ :**

**$h_d$  and  $h_s$  set sizes**

- **Assumptions:**

- No NP in  $|\Delta F|=1$ :

tree level in SM ( $\gamma$ ,  $|V_{ub}|$ ,  $|V_{cb}|$ , ...) free of NP

- NP is short-distance

[B-mesogenesis: Miró, Escudero, Nebot '24]

- Unitarity of the CKM 3x3 matrix

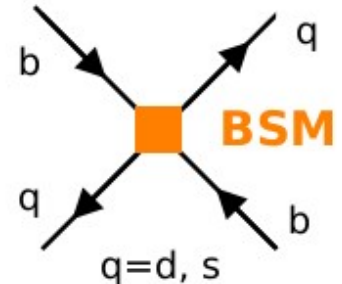
- Unrelated NP in  $B_d$  and  $B_s$  systems

[See: PRD 89, 033016 (2014),  
arxiv:1309.2293]

CKM (in presence of NP),  
bag parameters,  
↓ decay constants

$$M_{12} = M_{12}^{\text{SM}} \times (1 + h e^{2i\sigma})$$

NP parameters



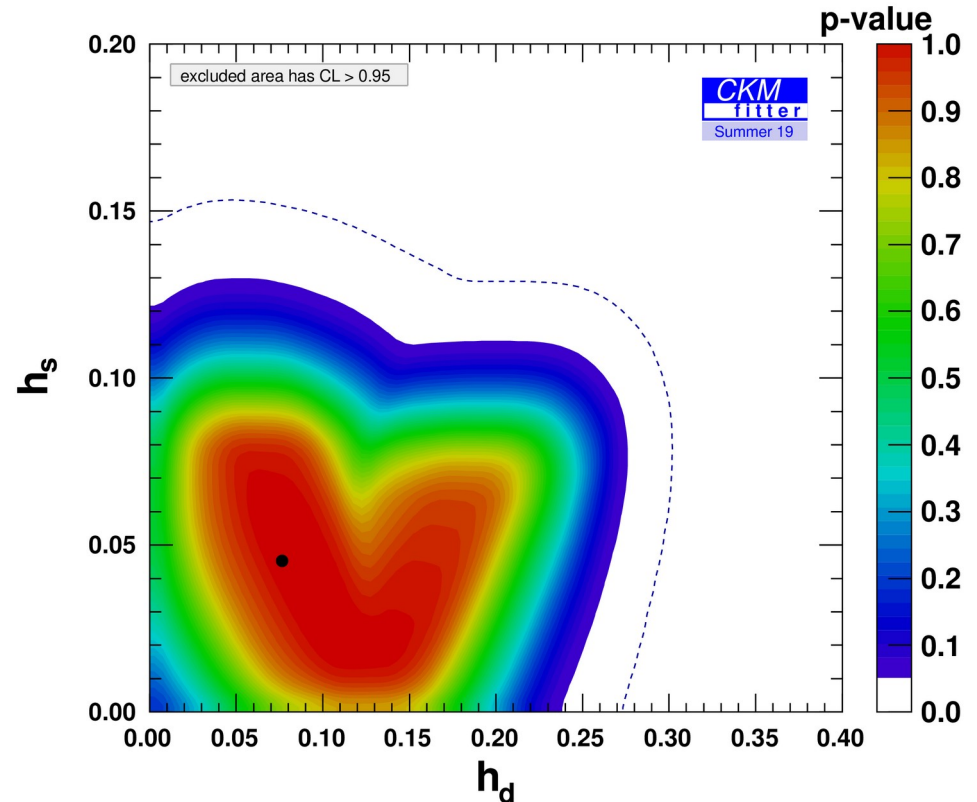
# Status of NP in B meson mixing

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- Status as of Summer '19: outdated, but conclusions should be similar
- Agreement with the SM ( $h_d=h_s=0$ ) at  $\sim 1\sigma$
- Allowed size for NP at the level of O(20%)!
- Extractions of  $\rho$  and  $\eta$  (Wolfenstein parm.) **degrade by factor  $\sim 3$**

[Charles, Descotes-G., Ligeti, Monteil, Papucci, Trabelsi, LVS '20]

Black dot: best fit point;  $\sigma_d$  and  $\sigma_s$  are unconstrained

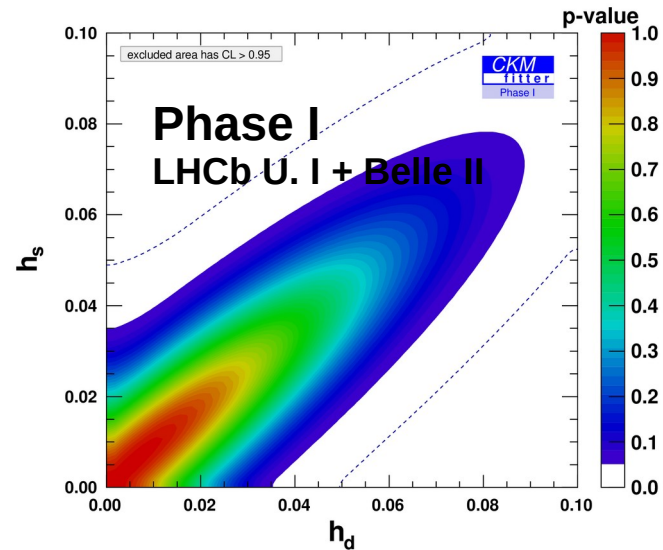
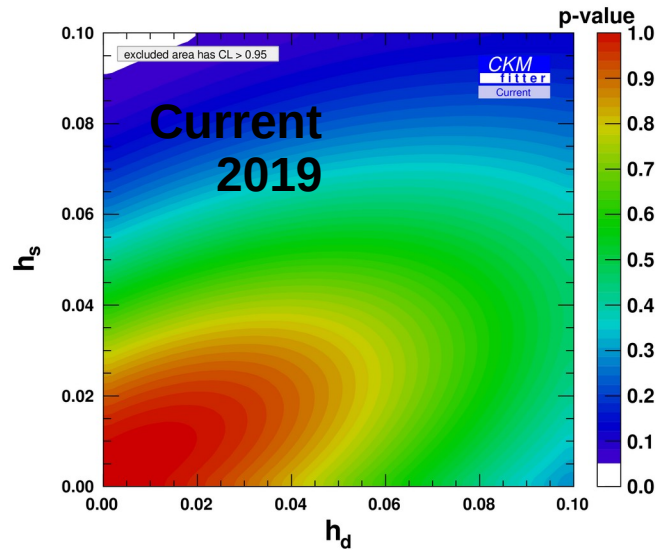


[See also: De Bruyn, Fleischer, Malami, van Vliet '23]

# Future improvements

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- SM reference: shift the central values
- Compared to Current, **improvement by factor >3**



# Conclusions

- Global fit of a rich variety of processes sensitive to CP Violation and SM predictions in agreement
- We are then able to extract accurate values for the fundamental parameters describing the CKM matrix
- The mechanism of CP Violation in the SM (still) gives an accurate picture of nature: no clear indication of NP
- One example of the impact of NP: B meson mixing
- Exciting future prospects for Belle II, LHC, NA62,...

# CKMfitter Collaboration

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MORE DETAILS @ [CKMfitter](#)

Jérôme Charles, Theory  
Olivier Deschamps, LHCb  
Sébastien Descotes-Genon, Theory  
Stéphane Monteil, LHCb  
Jean Orloff, Theory  
Wenbin Qian, LHCb/BESIII  
Vincent Tisserand, LHCb/BABAR  
Karim Trabelsi, Belle/Belle II  
Philip Urquijo, Belle/Belle II  
Luiz Vale Silva, Theory

THANKS!



The screenshot shows the CKMfitter website homepage. The header features the CKMfitter logo and navigation links: Home, Plots & Results, Specific Studies, Talks & Writeups, Publications, and CKMfitter Group. The main content area is titled "The CKMfitter group provides:" and lists three main areas:

- Plots & Results**: Preliminary results as of Moriond 2021 (updated Jan 2022)
- Talks**: Workshop on High Energy Physics Phenomenology (WHEPP) XVI, Dec 1-10, Guwahati, India. A featured talk is "Beyond 1st and 3rd generation unitarity triangle: what can we learn from the others?" (pdf).
- Tools**: Perform your own flavour analyses online with CKMlive.
- Specific Studies**: Prospective studies for Opportunities in Flavour Physics at the HL-LHC and HE-LHC.

BACK UP

# Benchmarks for the future

- **Phase I:** LHCb-upgrade I 50/fb, & Belle II 50/ab
- **Phase II:** LHCb-upgrade II 300/fb, & Belle II upgrade 250/ab
- **Phase III:** Phase II + FCC-ee

**FCC-ee:** initial phase of FCC; operates at different EW thresholds:  
 **$5 \times 10^{12}$  Z bosons,  $10^8$  WW pairs,  $>10^6$  Higgses,  $>10^6$  tt pairs**

Attribute	$\Upsilon(4S)$	$pp$	$Z^0$	Particle species	$B^0$	$B^+$	$B_s^0$	$\Lambda_b$	$B_c^+$	$c\bar{c}$	$\tau^-\tau^+$
All hadron species		✓	✓	Yield ( $\times 10^9$ )	310	310	75	65	1.5	600	170
High boost		✓	✓								
Enormous production cross-section		✓									
Negligible trigger losses	✓		✓								
Low backgrounds	✓		✓								
Initial energy constraint	✓		(✓)								

[FCC Physics Opportunities, Conceptual Design Report]

[Flavour cases: EPJPlus 136, 837 [arXiv:2106.01259](https://arxiv.org/abs/2106.01259),  
and EPJPlus 136, 912 [arXiv:2106.12168](https://arxiv.org/abs/2106.12168)]

# Experimental inputs: Phases II & III

Quantity	Current (Rfit)	Unc. Phase I	Unc. Phase II	Unc. Phase III
$\sin(2\beta)_{[c\bar{c}]}$	0.011	0.005	0.002	0.0008
$\gamma$	$3.4^\circ$	$1^\circ$	$0.25^\circ$	$0.20^\circ$
$ V_{ub} / V_{cb} $	9 %	3 %	1 %	id
$(\phi_s)_{[b \rightarrow c\bar{c}s]}$	21 mrad	14 mrad	4 mrad	2 mrad
$ V_{ub} _{SL}$	0.19	0.042	0.032	id
$ V_{cb} $	0.61	0.60	0.44	0.17
$\alpha$	$1.8^\circ$	$0.6^\circ$	id	id
$\mathcal{B}(B \rightarrow \tau\nu)$	22 %	5 %	2 %	1 %



# Lattice inputs: Phases II & III

Quantity	Current (Rfit)	Unc. Phase I	Unc. Phases II & III
$f_K$	0.5 %	0.3 %	id
$f_+^{K \rightarrow \pi}(0)$	0.4 %	0.1 %	id
$f_{B_s}$	1.1 %	0.5 %	id
$B_{B_s}$	3.2 %	0.8 %	0.6 %
$f_{B_s}/f_{B_d}$	0.6 %	0.4 %	id
$B_{B_s}/B_{B_d}$	2.4 %	0.5 %	0.3 %

# CKM metrology [Preliminary!]

Quantity	Current (Rfit)	Full agree	Unc. Phase I	Unc. Phase II	Unc. Phase III
$A$	0.8 %	0.9 %	0.7 %	0.7 %	0.5 %
$\lambda$	0.1 %	0.2 %	0.1 %	0.1 %	0.1 %
$\bar{\rho}$	4.9 %	4.6 %	1.9 %	0.9 %	0.7 %
$\bar{\eta}$	1.5 %	1.7 %	0.7 %	0.3 %	0.2 %

$1\sigma$

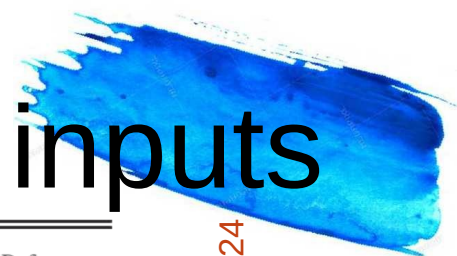
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$2\sigma$

Quantity	Current (Rfit)	Full agree	Unc. Phase I	Unc. Phase II	Unc. Phase III
$A$	1.8 %	1.9 %	1.4 %	1.2 %	0.9 %
$\lambda$	0.2 %	0.3 %	0.3 %	0.3 %	0.3 %
$\bar{\rho}$	10 %	9.3 %	3.9 %	1.8 %	1.5 %
$\bar{\eta}$	3.7 %	3.6 %	1.4 %	0.7 %	0.4 %

Other projections are found in the CKMfitter webpage; see also [CERN courier](#)

# Experimental and theoretical inputs



“Current”: 2019,  
used only for NP  
in  $|\Delta B|=2$

	Central values	Uncertainties				Reference Phases I–III
		Current [28]	Phase I	Phase II	Phase III	
$ V_{ud} $	0.97437	$\pm 0.00021$	id	id	id	[28]
$ V_{us}  f_+^{K \rightarrow \pi}(0)$	0.2177	$\pm 0.0004$	id	id	id	[28]
$ V_{cd} $	0.2248	$\pm 0.0043$	$\pm 0.003$	id	id	[40,41]
$ V_{cs} $	0.9735	$\pm 0.0094$	id	id	id	[28,40,41]
$\Delta m_d$ [ps <sup>-1</sup> ]	0.5065	$\pm 0.0019$	id	id	id	[17]
$\Delta m_s$ [ps <sup>-1</sup> ]	17.757	$\pm 0.021$	id	id	id	[17]
$ V_{cb} _{\text{SL}} \times 10^3$	42.26	$\pm 0.58$	$\pm 0.60$	$\pm 0.44$	id	[29]
$ V_{cb} _{W \rightarrow cb} \times 10^3$	...	...	...	...	$\pm 0.17$	[34–36]
$ V_{ub} _{\text{SL}} \times 10^3$	3.56	$\pm 0.22$	$\pm 0.042$	$\pm 0.032$	id	[29]
$ V_{ub}/V_{cb} $ (from $\Lambda_b$ )	0.0842	$\pm 0.0050$	$\pm 0.0025$	$\pm 0.0008$	id	[30]
$\mathcal{B}(B \rightarrow \tau \nu) \times 10^4$	0.83	$\pm 0.24$	$\pm 0.04$	$\pm 0.02$	$\pm 0.009$	[29,34]
$\mathcal{B}(B \rightarrow \mu \nu) \times 10^6$	0.37	...	$\pm 0.03$	$\pm 0.02$	id	[29]
$\sin 2\beta$	0.680	$\pm 0.017$	$\pm 0.005$	$\pm 0.002$	$\pm 0.0008$	[29,30,34]
$\alpha^\circ$ (mod 180°)	91.9	$\pm 4.4$	$\pm 0.6$	id	id	[29]
$\gamma^\circ$ (mod 180°)	66.7	$\pm 5.6$	$\pm 1$	$\pm 0.25$	$\pm 0.20$	[29,30,34]
$\beta_s$ [rad] x(-2)	-0.035	$\pm 0.021$	$\pm 0.014$	$\pm 0.004$	$\pm 0.002$	[30,34]
$A_{\text{SL}}^d \times 10^4$	-6	$\pm 19$	$\pm 5$	$\pm 2$	$\pm 0.25$	[14,17,34,37]
$A_{\text{SL}}^s \times 10^5$	3	$\pm 300$	$\pm 70$	$\pm 30$	$\pm 2.5$	[14,17,34,37]
$\bar{m}_t$ [GeV]	165.30	$\pm 0.32$	id	id	$\pm 0.020$	[28,34]
$\alpha_s(m_Z)$	0.1185	$\pm 0.0011$	id	id	$\pm 0.00003$	[28,34]
$f_+^{K \rightarrow \pi}(0)$	0.9681	$\pm 0.0026$	$\pm 0.0012$	id	id	[30]
$f_K$ [GeV]	0.1552	$\pm 0.0006$	$\pm 0.0005$	id	id	[30]
$f_{B_s}$ [GeV]	0.2315	$\pm 0.0020$	$\pm 0.0011$	id	id	[30]
$B_{B_s}$	1.219	$\pm 0.034$	$\pm 0.010$	$\pm 0.007$	id	[30]
$f_{B_s}/f_{B_d}$	1.204	$\pm 0.007$	$\pm 0.005$	id	id	[30]
$B_{B_s}/B_{B_d}$	1.054	$\pm 0.019$	$\pm 0.005$	$\pm 0.003$	id	[30]
$\tilde{B}_{B_s}/\tilde{B}_{B_d}$	1.02	$\pm 0.05$	$\pm 0.013$	id	id	[30,42,43]
$\tilde{B}_{B_s}$	0.98	$\pm 0.12$	$\pm 0.035$	id	id	[30,42,43]
$\eta_B$	0.5522	$\pm 0.0022$	id	id	id	[44]

experimental

theory

[See for refs.: PRD 102, 056023 (2020), arXiv:2006.04824  
and also EPJPlus 136, 837 arXiv:2106.01259,  
and EPJPlus 136, 912 arXiv:2106.12168]

- **CKMfitter**: Frequentist statistics based on a  $\chi^2$  analysis
- $\chi^2_{min}$ : **goodness-of-fit** under SM (or NP), **estimators** for  $V_{CKM}$
- $\Delta\chi^2$  ( $\chi^2$ -distributed): **Confidence Level (CL)** intervals
- *Range* fit scheme (*Rfit*) incorporates **theoretical uncertainties**
- Theo. inputs: published Lattice papers, **with error budgets**, different sources of syst. uncertainty are **combined linearly**, using FLAG reports as a guide to sort results

$$\mathcal{L} \stackrel{Rfit}{=} \mathcal{L}_{stat} \times \mathcal{L}_{theo},$$

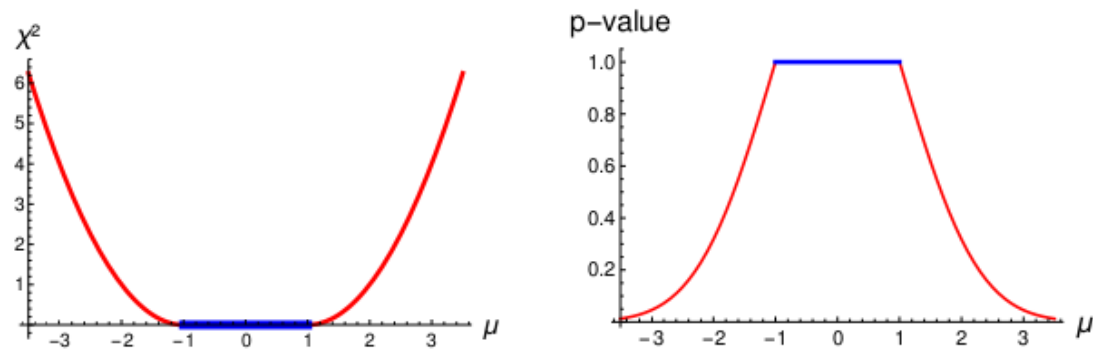
$$\chi^2 = -2 \ln \mathcal{L}$$

$\mathcal{L}_{stat}$ : exp. data  
 $\mathcal{L}_{theo}$ : had. inputs

[cf. Charles, Descotes-G., Niess, LVS '17]

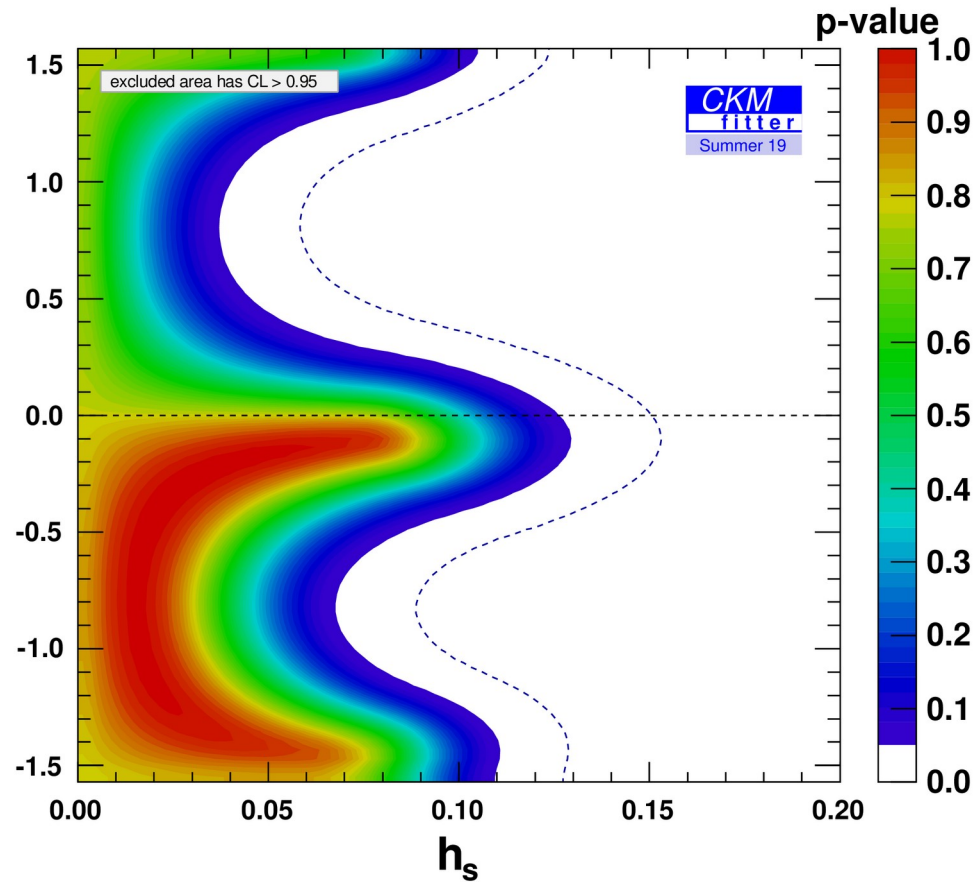
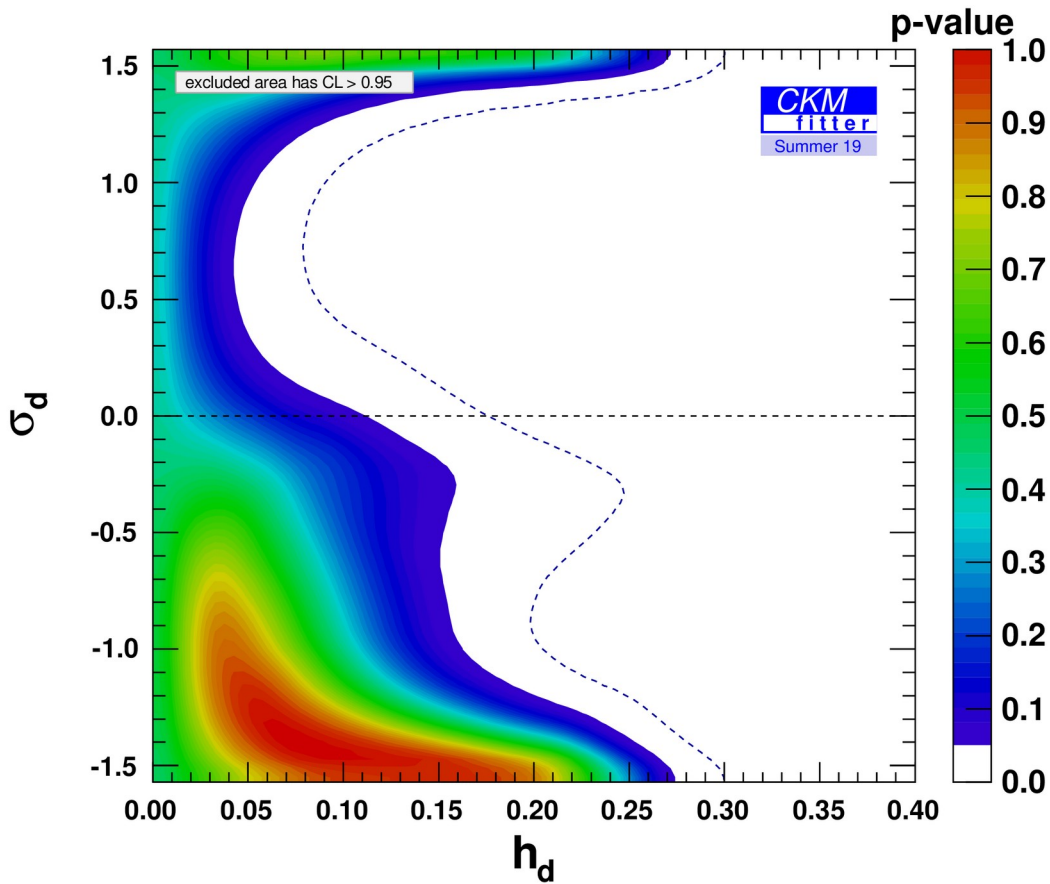
[Hoecker et al. '01, Charles et al. '04]

Example in 1D,  $0 \pm 1_{stat} \pm 1_{theo}$  ( $N_{dof} = 1$ )



$\chi^2$ : flat bottom, quadratic walls

# 2019 status plots



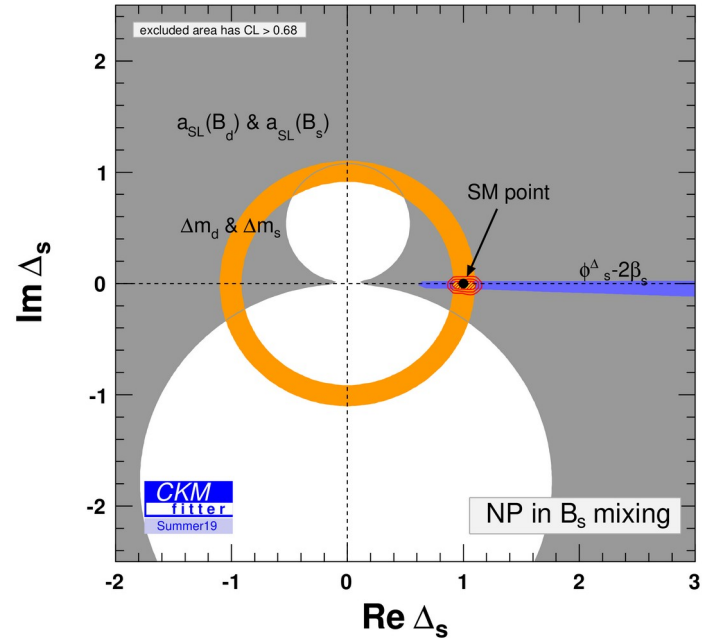
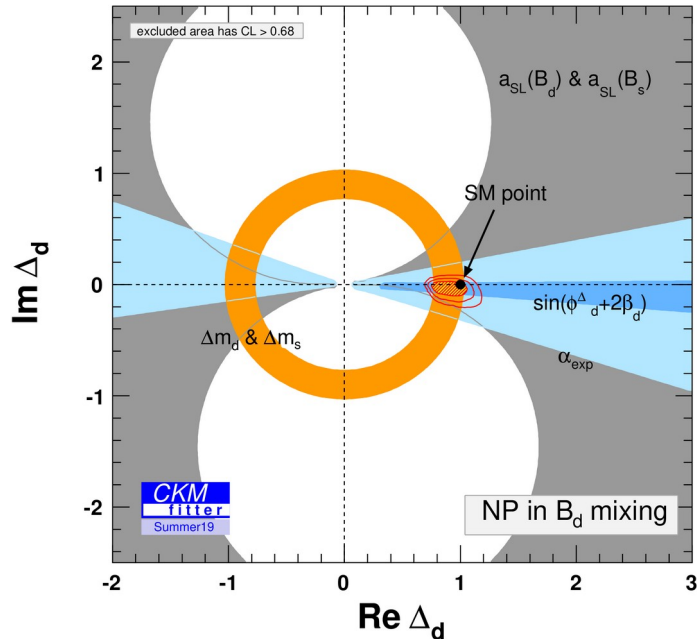
# Different representation



$$\langle B_q | \mathcal{H}_{\Delta B=2}^{\text{SM+NP}} | \bar{B}_q \rangle \equiv \langle B_q | \mathcal{H}_{\Delta B=2}^{\text{SM}} | \bar{B}_q \rangle \times (\text{Re}(\Delta_q) + i \text{Im}(\Delta_q))$$

$$\text{Re}(\Delta_q) + i \text{Im}(\Delta_q) = r_q^2 e^{i2\theta_q} = 1 + h_q e^{i\sigma_q}$$

Soares & Wolfenstein, PRD 47, 1021 (1993)  
 Deshpande, Dutta & Oh, PRL77, 4499 (1996)  
 Silva & Wolfenstein, PRD 55, 5331 (1997)  
 Cohen et al., PRL78, 2300 (1997)  
 Grossman, Nir & Worah, PLB 407, 307 (1997)



# Future improvements

Sensitivities	Summer 2019	Phase I	Phase II	Phase III
$h_d$	0.26	0.073	0.049	0.038
$h_s$	0.12	0.065	0.044	0.031

