Simulation tools for polarized bosons

#COMETA Polarization Party – Vienna, AT

Richard Ruiz

Institute of Nuclear Physics – Polish Academy of Science (IFJ PAN)

21 February 2025

thank you for the invitation!

brief motivation

(meeting is short but intense)

Broad motivation for exploring high-energy scattering of helicity-polarized states (req. lots of data!)

VBSCan review (Rev.Phys.'22) [2106.01393]

Wishlist for MC tools

– loop-induced in QCD and/or EW

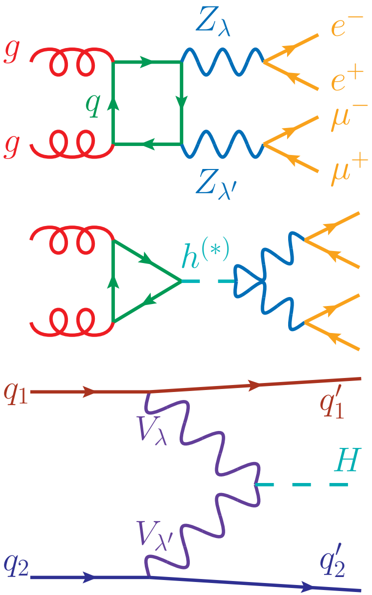
– full NLO in QCD and/or EW

– off-shell/finite-width

– interference **(int.)** between resonant and non-res. diagrams

– int. between polarization configurations

– *s*- and *t*-channel configurations



many state-of-the-art tools can do many things...

many state-of-the-art tools can do many things...
but not everything, not yet

this talk: a 15ish min summary by a theorist (me!) to many ex'ers on what (some) MC tools can do today re: polarization

scope: **COMETA Monte Carlo Olympics**

not the offical name. just sounds nice!

COMETA Olympics: polarized ZZ production and decay

COMETA-2025-XYZ

Precise Standard-Model predictions for polarised Z-boson pair production and decay at the LHC

Costanza Carrivale[a], Roberto Covarelli[a], Ansgar Denner[a], Christoph Haitz[a], Mareen Hoppe[a], Martina Javurkova[a], Duc Ninh Le[a], Jakob Linder[a], Rafael Coelho Lopes de Sa[a], Olivier Mattelaer[a], Susmita Mondal[a], Giovanni Pellicciol[a], Rene Poncelet[a], Richard Ruiz[a], Marek Schönherr[a], Frank Siegert[a], Lailin Xu[a], Giulia Zanderighi[a]

COMETA Olympics: polarized ZZ production and decay

COMETA-2025-XYZ

Precise Standard-Model predictions for polarised Z-boson pair production and decay at the LHC

PRELIMINARY

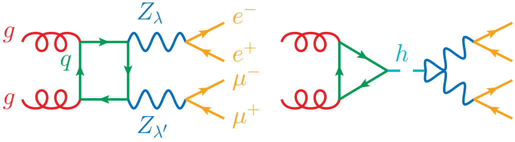
Costanza Carrivale[a], Roberto Covarelli[a], Ansgar Denner[a], Christoph Haitz[a], Mareen Hoppe[a], Martina Javurkova[a], Duc Ninh Le[a], Jakob Linder[a], Rafael Coelho Lopes de Sa[a], Olivier Mattelaer[a], Susmita Mondal[a], Giovanni Pellicciolі[a], Rene Poncelet[a], Richard Ruiz[a], Marek Schönherr[a], Frank Siegert[a], Lailin Xu[a], Giulia Zanderighi[a]

simple idea: are predictions among tools
(a) consistent and/or (b) correct?

COMETA Olympics: polarized ZZ production and decay

compare predictions for $pp \rightarrow Z_\lambda Z_\lambda \rightarrow e^+e^-\mu^+\mu^-$ at various orders

code	OS appr.	full	unpol.	LL	LT	TL	TT
Tree level $q\bar{q} \to \mathrm{ZZ} \to 4\ell$							
MoCaNLO	DPA	11.336(1)	11.242(1)	0.6574(1)	1.3332(2)	1.3370(2)	7.7874(8)
STRIPPER	DPA	11.3357(4)	11.2451(2)	0.6560(0)	1.3326(0)	1.3365(0)	7.7925(1)
MulBos	DPA	11.3563(7)	11.2393(3)	0.6572(0)	1.3329(1)	1.3366(1)	7.7846(2)
BBMC	DPA	11.3372(4)	11.2424(3)	0.6574(0)	1.3333(1)	1.3372(1)	7.7872(2)
PowHeg-Box	DPA	11.335(1)	11.245(1)	0.6575(1)	1.3333(1)	1.3374(1)	7.7885(8)
Sherpa	NWA	11.363(6)	11.513(4)	0.6767(4)	1.3538(6)	1.3734(6)	7.952(3)
MG5aMC	BW	11.38(2)	11.29(2)	0.660(1)	1.335(2)	1.338(2)	7.81(1)
Loop induced $\mathrm{gg} \to \mathrm{ZZ} \to 4\ell$							
MoCaNLO	DPA	1.6968(6)	1.6978(6)	0.0914(0)	0.0360(0)	0.0356(0)	1.5360(5)
STRIPPER	DPA	1.682(7)	1.700(2)	0.0912(1)	0.0360(0)	0.0357(0)	1.538(2)
MulBos	DPA	–	1.6981(9)	0.0913(1)	0.0360(0)	0.0357(0)	1.5363(8)
MG5aMC+VPolar	BW	1.699(6)	1.697(6)	0.0902(3)	0.0355(1)	0.0359(1)	1.539(6)



***good* agreement**

Polarization in MC tools

apologies: many authors/contributors and papers not listed for brevity

- MoCaNLO (Denner, Pellicciolli, et al) - diboson + VBS @ NLO QCD+EW
- BBMC (Biedermann, Billoni, Denner, et al) - diboson @ NLO QCD+EW
- PowHeg-Box (Pellicciolli, Zanderighi, et al) - diboson @ NLO QCD+EW
- MulBos (Le, Dao, et al) - diboson @ NLO QCD+EW
- STRIPPER (Poncelet, Popescu, et al) - single, diboson @ NNLO QCD
- Sherpa (Hoppe, Siegert, et al) -arbitrary res. processes @ NLO-partial QCD
- MG5aMC (w/ Mattelaer, et al) - arbitrary res. processes (incl. BSM) @ LO
  +VPolar (w/ Javurkova, et al) - + non-res. (incl. BSM) @ 1-loop QCD

different approaches to treating intermediate weak boson

(let us be honest, this is what I want to talk about)

common bit #1: helicity-polarized propagator

Popular (and successful) paradigm: decompose numerator of propagator via completeness relationship

care is need at this step!

$$-g_{\mu\nu} + q_\mu q_\nu / M_V^2 = \sum_{\lambda=\pm,0,S} \eta_\lambda \, \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda),$$

note: $\eta_+ = \eta_- = \eta_0 = -\eta_S = +1$

Popular (and successful) paradigm: decompose numerator of propagator via completeness relationship

care is need at this step!

$$-g_{\mu\nu} \;+\; q_\mu q_\nu/M_V^2 \;=\; \sum_{\lambda=\pm,0,S} \eta_\lambda \; \varepsilon_\mu(q,\lambda)\varepsilon_\nu^*(q,\lambda),$$

note: $\eta_+ = \eta_- = \eta_0 = -\eta_S = +1$

vector boson propagator becomes sum over **truncated propagators**

similar result for massive fermions

$$\Pi_{\mu\nu}^V(q) \;=\; \frac{-i\left(g_{\mu\nu} - q_\mu q_\nu/M_V^2\right)}{q^2 - M_V^2 + iM_V\Gamma_V}$$

$$= \sum_{\lambda\in\{0,\pm1,S\}} \underbrace{\eta_\lambda}_{\pm1} \underbrace{\left(\frac{i\varepsilon_\mu(q,\lambda)\;\varepsilon_\nu^*(q,\lambda)}{q^2 - M_V^2 + iM_V\Gamma_V}\right)}_{\equiv\Pi_{\mu\nu}^{V\lambda}\ \text{truncated prop.}}$$

Popular (and successful) paradigm: decompose numerator of propagator via completeness relationship

care is need at this step!

$$-g_{\mu\nu} + q_\mu q_\nu / M_V^2 = \sum_{\lambda=\pm,0,S} \eta_\lambda \, \varepsilon_\mu(q,\lambda)\varepsilon_\nu^*(q,\lambda),$$

note: $\eta_+ = \eta_- = \eta_0 = -\eta_S = +1$

vector boson propagator becomes sum over **truncated propagators**

similar result for massive fermions

$$\Pi_{\mu\nu}^V(q) = \frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M_V^2\right)}{q^2 - M_V^2 + iM_V\Gamma_V}$$

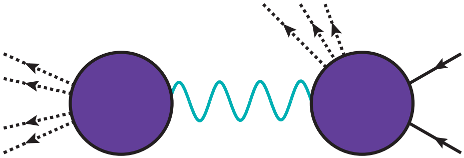
$$= \sum_{\lambda\in\{0,\pm1,S\}} \underbrace{\eta_\lambda}_{\pm1} \underbrace{\left(\frac{i\varepsilon_\mu(q,\lambda)\ \varepsilon_\nu^*(q,\lambda)}{q^2 - M_V^2 + iM_V\Gamma_V}\right)}_{\equiv \Pi_{\mu\nu}^{V\lambda} \text{ truncated prop.}}$$

note: what happens after this decomposition differs between groups!

Unpolarized matrix elements (MEs) become sum over polarized MEs

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm 1, 0, S\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$

$$= \sum_{\lambda \in \{\pm 1, 0, S\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

$-i\mathcal{M}_\lambda =$ 

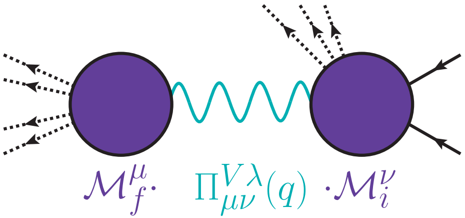
$\mathcal{M}_f^{\mu} \cdot \quad \Pi_{\mu\nu}^{V\lambda}(q) \quad \cdot \mathcal{M}_i^{\nu}$

Unpolarized matrix elements (MEs) become sum over polarized MEs

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm 1, 0, S\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$

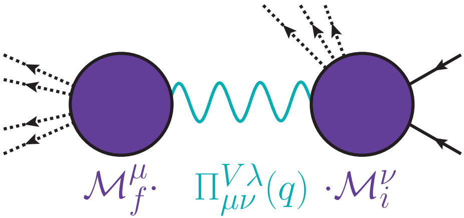
$$= \sum_{\lambda \in \{\pm 1, 0, S\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

$-i\mathcal{M}_\lambda =$



Unpolarized matrix elements (MEs) become sum over **polarized MEs**

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm 1, 0, S\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$
$$= \sum_{\lambda \in \{\pm 1, 0, S\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

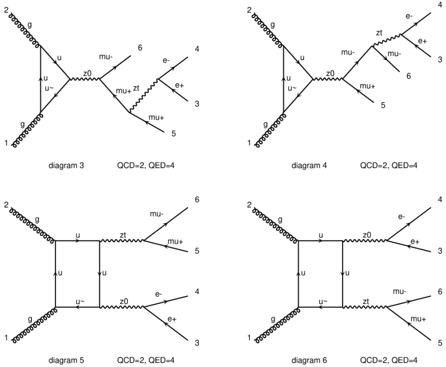
$-i\mathcal{M}_\lambda =$ 

Polarized cross sections are then built from **polarized MEs**

$$d\sigma_\lambda = \frac{1}{\text{flux}} \frac{1}{\text{spin/color avg.}} \sum_{\text{dof.}} \int dPS \, |\mathcal{M}_\lambda|^2$$

common bit #2: diagram selection

In practice non-(doubly)resonant diagrams **(top row)** are neglected



– **polarization** in MC tools is typically **hard-coded** at level of $|\mathcal{M}|^2$

exception is `MG5aMC+VPolar` w/ Javurkova, et al (PLB'24) [2401.17365]

– impact of **gauge dependence**, **interference** suppressed through strict kinematical/analysis requirements, e.g., $|m_{\ell\ell} - M_Z| < 10$ GeV

difference #1: narrow width approximation

NWA and spin-correlated NWA (1/2)

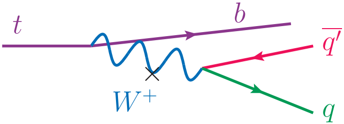
Narrow Width Approximation (NWA):

$$\text{at } |\mathcal{M}|^2 \text{ level: } \frac{1}{(q^2-M_V^2)^2+(M_V\Gamma_V)^2} \rightarrow \frac{\pi}{M_V\Gamma_V}\delta(q^2-M_V^2)$$

$$\sigma(A\to B\to C) \approx \sigma(A\to B) \times \mathrm{BR}(B\to C)$$

Procedure:

1. Generate n $A\to B$ events
2. Generate n $B\to C$ events
3. For each event $k\in\{n\}$, boost C event to $A's$ frame, concatenate



NWA and spin-correlated NWA (2/2)

Spin-Correlated NWA:

SHERPA : $\frac{1}{(q^2-M_V^2)^2+(M_V\Gamma_V)^2} \rightarrow \frac{\pi}{M_V\Gamma_V}\delta(q^2-M_V^2)$ [no finite width]

MADSPIN : $d\sigma(A \rightarrow B \rightarrow C) \approx \frac{|\mathcal{M}(A\rightarrow B\rightarrow C)|^2}{|\mathcal{M}(A\rightarrow B)|^2} \times d\sigma(A \rightarrow B)$

Procedure:

1. Generate $A \rightarrow B$ [MADSPIN]
2. Generate $A \rightarrow B \rightarrow C$ with $A \rightarrow B$ momenta
3. Reweight [both]

COMETA Olympics:

– SHERPA: spin-correlated NWA

– MGAMC: no NWA, only diagram selection for $A \rightarrow B \rightarrow C$

Double Pole Approximation

DPA aka On-Shell Projection (OSP): Denner, Dittmaier, et al (NPB'00, NPB'05)

0. diagram selection [implicit because **cannot** be done without it]

1. usual **Breit-Wigner pole** with $q^2 \neq M_V^2$ dictated by dPS integration

$$|\mathcal{M}|^2 \quad \propto \quad \frac{1}{(q^2-M_V^2)^2+(M_V\Gamma_V)^2}$$

Double Pole Approximation

DPA aka On-Shell Projection (OSP): Denner, Dittmaier, et al (NPB'00, NPB'05)

0. diagram selection [implicit because **cannot** be done without it]

1. usual **Breit-Wigner pole** with $q^2 \neq M_V^2$ dictated by dPS integration

$$|\mathcal{M}|^2 \quad \propto \quad \frac{1}{(q^2-M_V^2)^2+(M_V\Gamma_V)^2}$$

2. usual **"helicity-truncated" tensor structure**

$$-i\left(g_{\mu\nu} - q_\mu q_\nu / M_V^2\right) \quad \to \quad i\varepsilon_\mu(q,\lambda)\ \varepsilon_\nu^*(q,\lambda)$$

Double Pole Approximation

DPA aka On-Shell Projection (OSP): Denner, Dittmaier, et al (NPB'00, NPB'05)

0. diagram selection [implicit because **cannot** be done without it]

1. usual **Breit-Wigner pole** with $q^2 \neq M_V^2$ dictated by dPS integration

$$|\mathcal{M}|^2 \quad \propto \quad \frac{1}{(q^2-M_V^2)^2+(M_V\Gamma_V)^2}$$

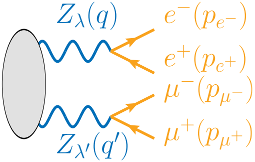
2. usual **"helicity-truncated" tensor structure**

$$-i\left(g_{\mu\nu} - q_\mu q_\nu / M_V^2\right) \quad \rightarrow \quad i\varepsilon_\mu(q,\lambda)\ \varepsilon^*_\nu(q,\lambda)$$

3. **important!** momentum mapping / shuffling

reduced gauge dependency vs energy conservation

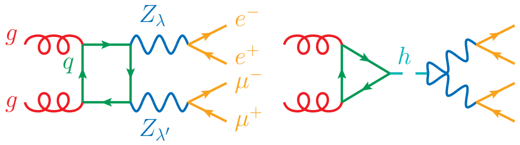
$$q + q' = \tilde{q} + \tilde{q}' \text{ where } \tilde{q}^2, \tilde{q}'^2 = M_Z^2$$



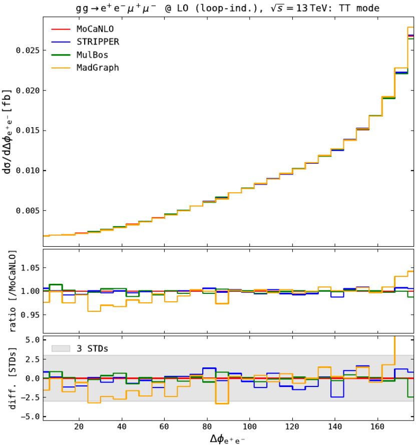
COMETA Olympics: polarized ZZ production and decay

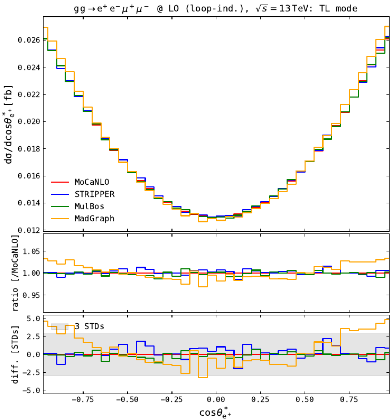
compare predictions for $pp \rightarrow Z_\lambda Z_\lambda \rightarrow e^+e^-\mu^+\mu^-$ at various orders

code	OS appr.	full	unpol.	LL	LT	TL	TT
			Tree level $q\bar{q} \rightarrow$ ZZ $\rightarrow 4\ell$				
MoCaNLO	DPA	11.336(1)	11.242(1)	0.6574(1)	1.3332(2)	1.3370(2)	7.7874(8)
STRIPPER	DPA	11.3357(4)	11.2451(2)	0.6560(0)	1.3326(0)	1.3365(0)	7.7925(1)
MulBos	DPA	11.3563(7)	11.2393(3)	0.6572(0)	1.3329(1)	1.3366(1)	7.7846(2)
BBMC	DPA	11.3372(4)	11.2424(3)	0.6574(0)	1.3333(1)	1.3372(1)	7.7872(2)
PowHeg-Box	DPA	11.335(1)	11.245(1)	0.6575(1)	1.3333(1)	1.3374(1)	7.7885(8)
Sherpa	NWA	11.363(6)	11.513(4)	0.6767(4)	1.3538(6)	1.3734(6)	7.952(3)
MG5aMC	BW	11.38(2)	11.29(2)	0.660(1)	1.335(2)	1.338(2)	7.81(1)
			Loop induced gg $\rightarrow$ ZZ $\rightarrow 4\ell$				
MoCaNLO	DPA	1.6968(6)	1.6978(6)	0.0914(0)	0.0360(0)	0.0356(0)	1.5360(5)
STRIPPER	DPA	1.682(7)	1.700(2)	0.0912(1)	0.0360(0)	0.0357(0)	1.538(2)
MulBos	DPA	–	1.6981(9)	0.0913(1)	0.0360(0)	0.0357(0)	1.5363(8)
MG5aMC+VPolar	BW	1.699(6)	1.697(6)	0.0902(3)	0.0355(1)	0.0359(1)	1.539(6)



COMETA Olympics: polarized ZZ production and decay





(e) TL

– **TT** shows impact of momentum mapping in leptons
– **TL** show impacts of momentum mapping in ZZ and leptons

COMETA Olympics: polarized ZZ production and decay

COMETA-2025-XYZ

Precise Standard-Model predictions for polarised Z-boson pair production and decay at the LHC

PRELIMINARY

Costanza Carrivale[(a)], Roberto Covarelli[(a)], Ansgar Denner[(a)], Christoph Haitz[(a)], Mareen Hoppe[(a)], Martina Javurkova[(a)], Duc Ninh Le[(a)], Jakob Linder[(a)], Rafael Coelho Lopes de Sa[(a)], Olivier Mattelaer[(a)], Susmita Mondal[(a)], Giovanni Pellicciolì[(a)], Rene Poncelet[(a)], Richard Ruiz[(a)], Marek Schönherr[(a)], Frank Siegert[(a)], Lailin Xu[(a)], Giulia Zanderighi[(a)]

what is on the horizon?

polarization vectors are special in gauge theories

decomposing propagators

For **generic momenta** $q = (E, \vec{q})$ with $q^2 \neq M_V$ in R_ξ gauge:

transverse polarizations:

$$\sum_{\lambda=\pm 1} \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda) \ = \ -g_{\mu\nu} \ - \ \underbrace{\Theta_{\mu\nu}(\theta_V,\phi_V)}_{\text{cos and sin}}$$

longitudinal polarization:

$$\varepsilon_\mu(q,\lambda=0)\varepsilon_\nu(q,\lambda=0) \ = \ \frac{q_\mu q_\nu}{q^2} \ + \ \Theta_{\mu\nu}$$

scalar/auxiliary polarization (unitary gauge when $\xi \to \infty$):

$$\varepsilon_\mu(q,\lambda=S)\varepsilon_\nu(q,\lambda=S) \ = \ q_\mu q_\nu \ \left(\frac{1}{q^2} + \frac{(\xi-1)}{q^2-\xi M_V^2}\right)$$

sum recovers propagator **but** polarized xsec feature miscancellations:

$$\mathcal{M}\text{-level miscancellations} \sim \ \mathcal{O}(\Theta) \text{ and } \mathcal{O}(q^2 - M_V^2)$$

how large are these miscancellations?

work in progress

how large ~~are these miscancellations~~ is the interference? ☺

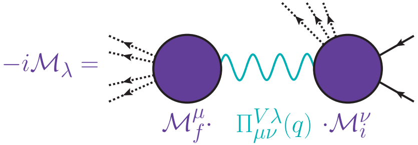
work in progress

the road map

Unpolarized ME becomes sum over **polarized MEs**

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm 1, 0, A\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$

$$= \sum_{\lambda \in \{\pm 1, 0, A\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

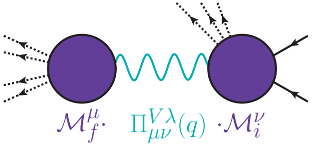


Unpolarized ME becomes sum over **polarized MEs**

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm 1, 0, A\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$

$$= \sum_{\lambda \in \{\pm 1, 0, A\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{= \mathcal{M}_\lambda \text{ polarized ME}}$$

$$-i\mathcal{M}_\lambda =$$

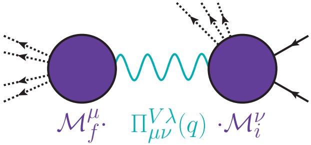


Question: what do these two lines mean?

Unpolarized ME becomes sum over polarized MEs

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda\in\{\pm1,0,A\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$

$$= \sum_{\lambda\in\{\pm1,0,A\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

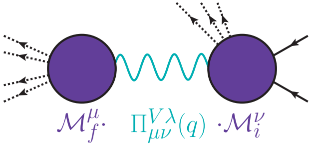
$-i\mathcal{M}_\lambda =$ 

Question: what do these two lines mean?

top: full **matrix element** is sum over **helicity polarizations**

Unpolarized ME becomes sum over polarized MEs

$$\mathcal{M} = \mathcal{M}_f^{\mu} \left(\sum_{\lambda \in \{\pm1, 0, A\}} \eta_\lambda \times \Pi_{\mu\nu}^{V\lambda} \right) \mathcal{M}_i^{\nu}$$
$$= \sum_{\lambda \in \{\pm1, 0, A\}} \eta_\lambda \times \underbrace{\mathcal{M}_f^{\mu} \cdot \Pi_{\mu\nu}^{V\lambda} \cdot \mathcal{M}_i^{\nu}}_{=\mathcal{M}_\lambda \text{ polarized ME}}$$

$-i\mathcal{M}_\lambda =$ 

Question: what do these two lines mean?

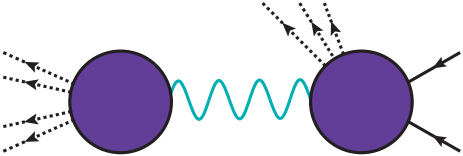
top: full **matrix element** is sum over **helicity polarizations**

btm: full **matrix element** is sum over ***sub*amplitudes**

treat $\mathcal{M}_\lambda$ as a full subamplitude, not “just” as a component of a subamplitude

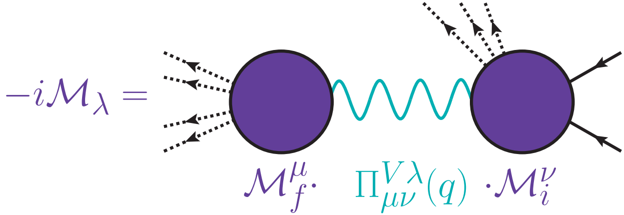
treat $\mathcal{M}_\lambda$ as a full subamplitude, not "just" as a component of a subamplitude, i.e., put on same footing as any other interferring diagram

$$-i\mathcal{M}_\lambda =$$



$$\mathcal{M}_f^{\mu} \cdot \quad \Pi^{V\lambda}_{\mu\nu}(q) \quad \cdot \mathcal{M}_i^{\nu}$$

treat $\mathcal{M}_\lambda$ as a full subamplitude, not "just" as a component of a subamplitude, i.e., put on same footing as any other interferring diagram

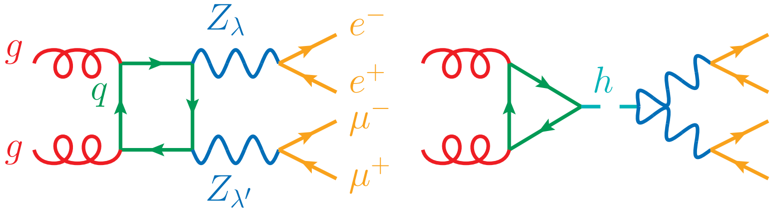


$\implies$ **promoting truncated propagator to a Feynman rule**

w/ Javurkova, et al (PLB'24) [2401.17365]

$$\frac{-i\ \varepsilon_\mu(q,\lambda)\ \varepsilon^*_\nu(q,\lambda)}{q^2-M_V^2} = \quad V_\lambda(q)$$

an example



Example: consider full $2 \to 4$ process $gg \to e^+e^-\mu^+\mu^-$ at $\mathcal{O}(\alpha_s^2\alpha^4)$

Example: consider full $2 \to 4$ process $gg \to e^+e^-\mu^+\mu^-$ at $\mathcal{O}(\alpha_s^2\alpha^4)$

– replace every instance of Z by Z_0, Z_T, Z_A **16 diag** $\to$ $3^2 \times 16$ **diag** $=$ 144 **diag**

Example: consider full $2 \to 4$ process $gg \to e^+e^-\mu^+\mu^-$ at $\mathcal{O}(\alpha_s^2\alpha^4)$

– replace every instance of Z by Z_0, Z_T, Z_A **16 diag** $\to$ $3^2 \times 16$ **diag** $=$ 144 **diag**

– full **ME** given by sum of ***all*** diagrams (subamplitudes)

Example: consider full $2 \to 4$ process $gg \to e^+e^-\mu^+\mu^-$ at $\mathcal{O}(\alpha_s^2\alpha^4)$

– replace every instance of Z by Z_0, Z_T, Z_A **16 diag** $\to$ $3^2 \times 16$ **diag** $=$ 144 **diag**

– full **ME** given by sum of ***all*** diagrams (subamplitudes)

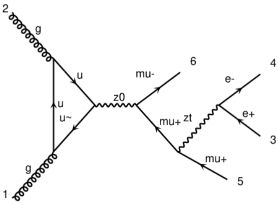


diagram 3 QCD=2, QED=4

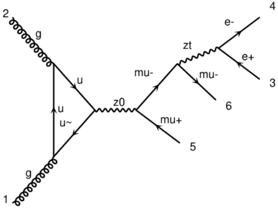


diagram 4 QCD=2, QED=4

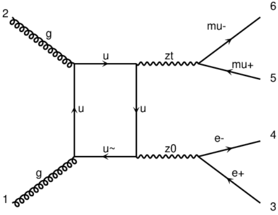


diagram 5 QCD=2, QED=4

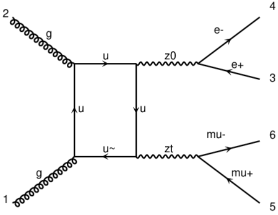


diagram 6 QCD=2, QED=4

Example: consider full $2 \to 4$ process $gg \to e^+e^-\mu^+\mu^-$ at $\mathcal{O}(\alpha_s^2\alpha^4)$

– replace every instance of Z by Z_0, Z_T, Z_A **16 diag** $\to$ $3^2 \times 16$ **diag** $=$ 144 **diag**

– full **ME** given by sum of ***all*** diagrams (subamplitudes)

– **diagram filtering** then gives desired subset of diagrams (subamps.)

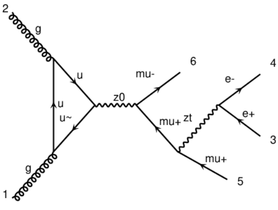


diagram 3 QCD=2, QED=4

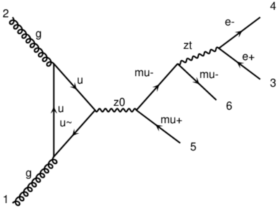


diagram 4 QCD=2, QED=4

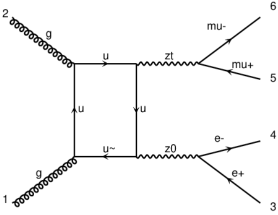


diagram 5 QCD=2, QED=4

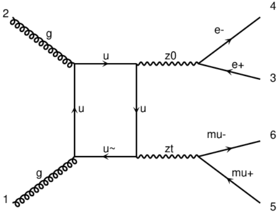


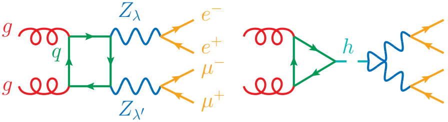
diagram 6 QCD=2, QED=4

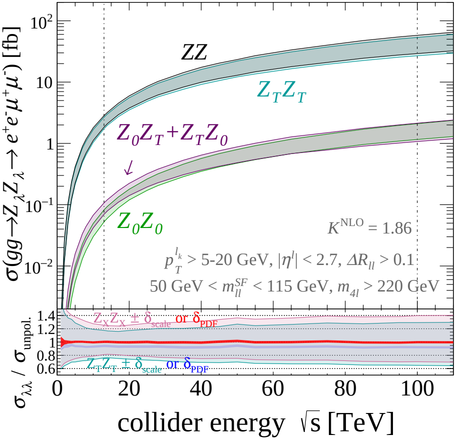
Treating **helicity polarization** as a **Feynman rule** provides a new method for computing polarized xsec

Javurkova, Ruiz, et al (PLB'24) [2401.17365]

- loop-induced processes ✓
- interference between different polarizations configurations ✓
- non-resonant diagrams ✓
- off-shell/finite-width effects ✓

$$\frac{-i\ \varepsilon_\mu(q,\lambda)\ \varepsilon^*_\nu(q,\lambda)}{q^2-M_V^2} =$$ 
$V_\lambda(q)$





w/ Javurkova, et al (PLB'24) [2401.17365] in agreement with Denner & Pelliccioli (JHEP'21) [2107.06579]

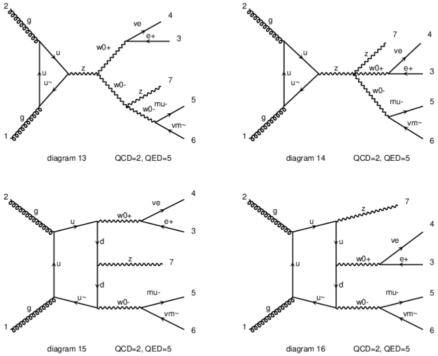
Treating **helicity polarization** as a **Feynman rule** provides a new method for computing polarized xsec

Javurkova, Ruiz, et al (PLB'24) [2401.17365]

- loop-induced processes ✓
- interference between different polarizations configurations ✓
- non-resonant diagrams ✓
- off-shell/finite-width effects ✓

$$\frac{-i\ \varepsilon_\mu(q,\lambda)\ \varepsilon^*_\nu(q,\lambda)}{q^2-M_V^2} =$$  $V_\lambda(q)$

works for $gg \to VVV$



(resources limited!)

somewhere in Krakow a PhD student is computing sums and differences of many propagators

summary

many state-of-the-art tools can do many things...
but not everything, not yet

Wishlist for MC tools

– loop-induced in QCD and/or EW

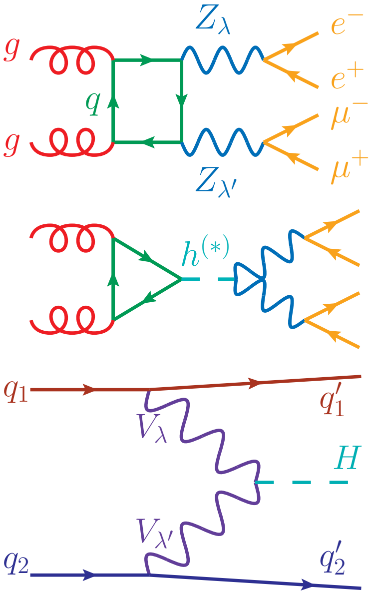
– full NLO in QCD and/or EW

– off-shell/finite-width

– interference **(int.)** between resonant and non-res. diagrams

– int. between polarization configurations

– *s*- and *t*-channel configurations



lots of progress

lots of progress

horizon: subtle theory uncertainties will soon be known

thank you!

backup

Decomposing Propagators

Completeness relationships between **propagators** & **polarization vectors** in gauge theories are subtle. Example: **QED** in Feynman gauge

$\implies \xi = 1$ so $(1-\xi)q_\mu q_\nu/q^2 \to 0$:

$$-g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix} = \sum_{\lambda=\pm,0,S} \eta_\lambda \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda)$$

For $q = (q^0, 0, 0, q^3)$ and **transverse** pols $\varepsilon_\mu(\lambda = \pm) = (0, \mp 1, -i, 0)/\sqrt{2}$

$$\sum_{\lambda=\pm} \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda) = \begin{pmatrix} 0 & & & \\ & +1 & 0 & \\ & 0 & +1 & \\ & & & 0 \end{pmatrix}$$

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

$$\varepsilon_\mu(q, \lambda = 0)\varepsilon_\nu(q, \lambda = 0) \;=\; \frac{q^2}{q^2}\begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & +1 \end{pmatrix} \;+\; \frac{q_\mu q_\nu}{q^2}$$

For "**auxiliary**" (A) or "**scalar**" (S) polarization $\varepsilon_\mu(\lambda = S) = q_\mu \sqrt{\xi/q^2}$

$$\varepsilon_\mu(q, \lambda = S)\varepsilon_\nu(q, \lambda = S) \;=\; q_\mu q_\nu \left(\frac{\xi}{q^2}\right)$$

Precise form for $\lambda = 0, S$ depends on several factors:
- broken (massive) or unbroken (massless) gauge symmetry
- gauge (Feynman vs Landau vs Unitary vs Axial)
- gauge fixing ($\xi = 1$ or $n^2 = -1$)

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

$$\varepsilon_\mu(q, \lambda = 0)\varepsilon_\nu(q, \lambda = 0) \ = \ \frac{q^2}{q^2} \begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & +1 \end{pmatrix} \ + \ \frac{q_\mu q_\nu}{q^2}$$

For "**auxiliary**" (A) or "**scalar**" (S) polarization $\varepsilon_\mu(\lambda = S) = q_\mu \sqrt{\xi/q^2}$

$$\varepsilon_\mu(q, \lambda = S)\varepsilon_\nu(q, \lambda = S) \ = \ q_\mu q_\nu \ \left(\frac{\xi}{q^2}\right)$$

Example: for W/Z in R_ξ gauge, $\varepsilon_\mu^{W/Z}(\lambda = S) = q_\mu \sqrt{\frac{1}{q^2} + \frac{(\xi - 1)}{q^2 - \xi M_V^2}}$

$$\varepsilon_\mu(q, \lambda)\varepsilon_\nu(q, \lambda) \ = \ \left(\frac{1}{q^2} + \frac{(\xi - 1)}{q^2 - \xi M_V^2}\right) \ q_\mu q_\nu$$

utilizing frameworks built for *new physics*

Tools for BSM@LHC

Monte Carlo Tool chains have long been adapted for new physics:

– new particles with spin 0, 1, 1/2, 2

– new vertices

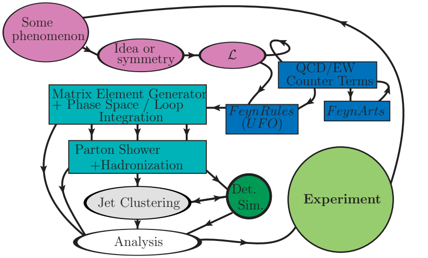
– alternative propagators

FeynRules

A Mathematica package to calculate Feynman rules

Universal FeynRules Object (UFO) libraries encode (.py) Feynman rules (incl. UV and R2 count terms) for MadGraph5, SHERPA, . . .

Monte Carlo / Event Simulation Chain



We wrote a UFO with $W^\pm_\lambda$ and Z_λ

VPolar feynrules.irmp.ucl.ac.be/wiki/VPolarization

```
...
Definitions ->
    {Z[mu_]->Z0[mu]+ZT[mu]+ZA[mu]+ZX[mu]}
...
Definitions ->
    {W[mu_]->W0[mu]+WT[mu]+WA[mu]+WX[mu]}
```