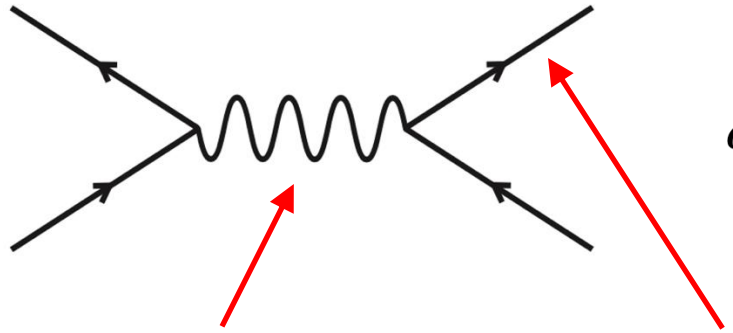
Precision physics at the future muon collider

Yang Ma
Centre for Cosmology, Particle Physics and Phenomenology (CP3)
Université catholique de Louvain (UCLouvain)

MadGraph5_aMC@NLO Meeting
February 04, 2025

Myths about lepton colliders

- **Lepton colliders are s-channel machines**



$$\sigma \sim \frac{1}{s} = \frac{1}{E^2}$$

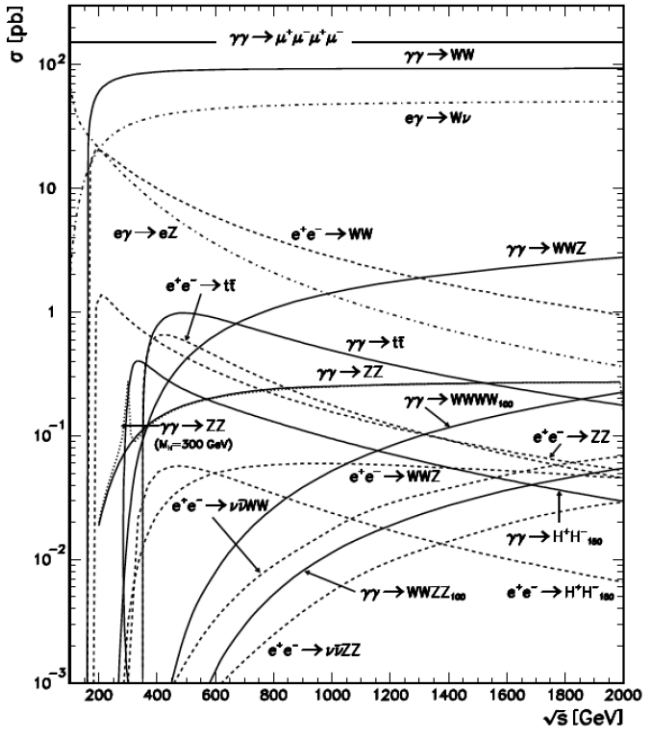
Resonance peaks:

- B Factories: $\Upsilon(4S)$ peak (10.6 GeV)
  e.g. Babar, Belle II
- Z Factories, Z peak (91.188 GeV)
  e.g. LEP

Pair production:
e.g. e+ e- Higgs Factory at 250 GeV
Look for a Z at 110 GeV, whatever on the other side, it is from Higgs decay!

Or Photon-Photon machines

$$f_{\gamma/\ell} = \frac{\alpha}{2\pi}\left(\frac{1+(1-x)^2}{x}\right)\log\left(\frac{Q^2}{m_\ell^2}\right)$$



S. Brodsky, SLAC-PUB-11581

- **Lepton colliders are clean**

Is this still the case for TeV lepton colliders?

A “dream machine” ?

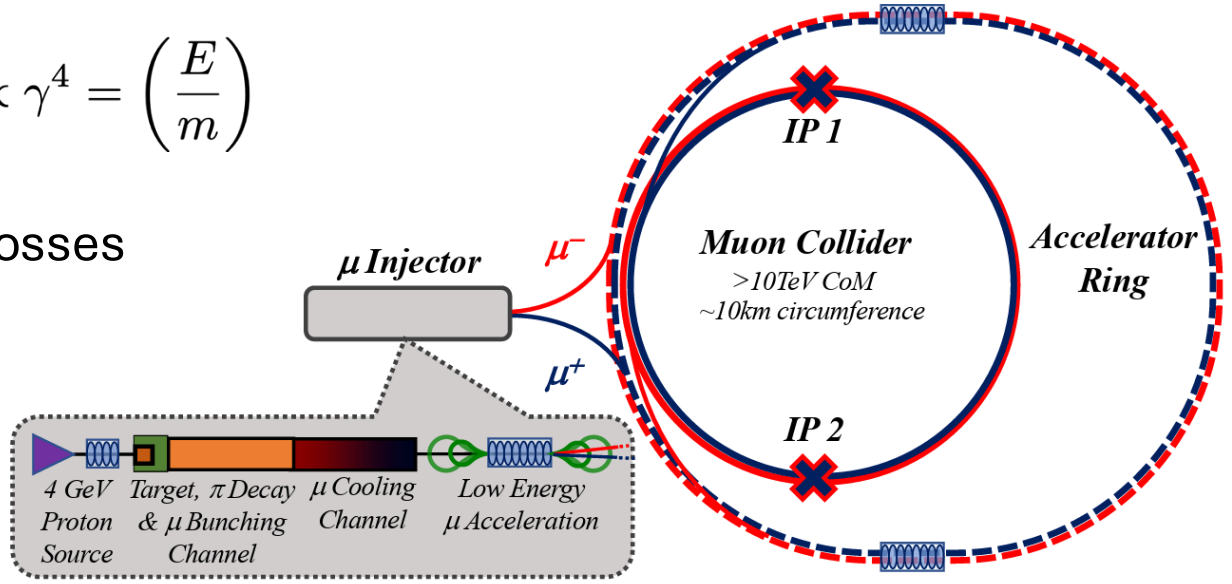
A new opportunity to go to the energy frontier

“Discovery machine”

Consider the power loss from synchrotron radiation $P \propto \gamma^4 = \left(\frac{E}{m}\right)$

At the same energy, e^+e^- collider losses

$(m_\mu/m_e)^4 \sim 10^9\times$ power

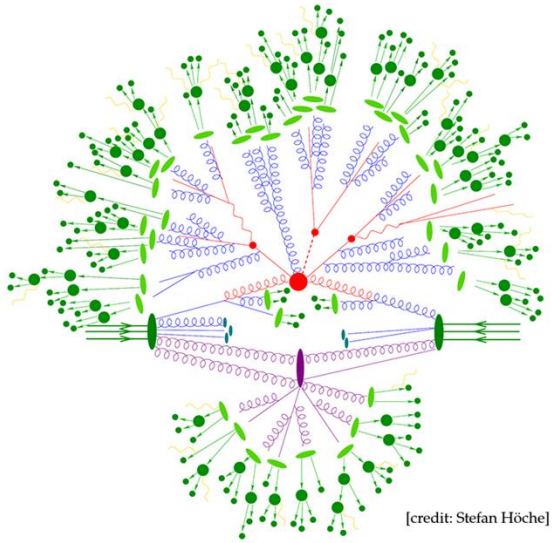


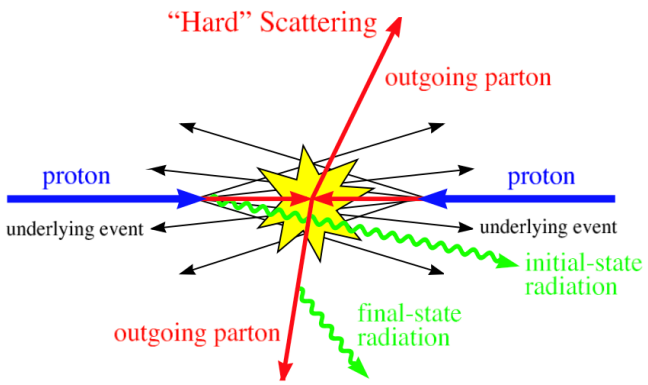
- A smaller ring is possible
- More power efficient for the same luminosity
- Multi-TeV partonic c.m. energy is possible

$$\mathcal{L}_{\rm int} = 10{\rm ab}^{-1} \times \left(\frac{E_{\rm cm}}{10\,{\rm TeV}}\right)$$

Read more in *Towards a Muon Collider*, 2303.08533

An EW version of LHC !





What happens at Tevatron/LHC?

- Hard interaction: fixed-order calculation (in QCD $\equiv 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \dots$)
- All-order radiative corrections: parton shower emissions
- Hadronisation from confinement property (quarks and gluons cannot live individually for long)
- Multiple parton interactions (MPI)
- Hadron decays

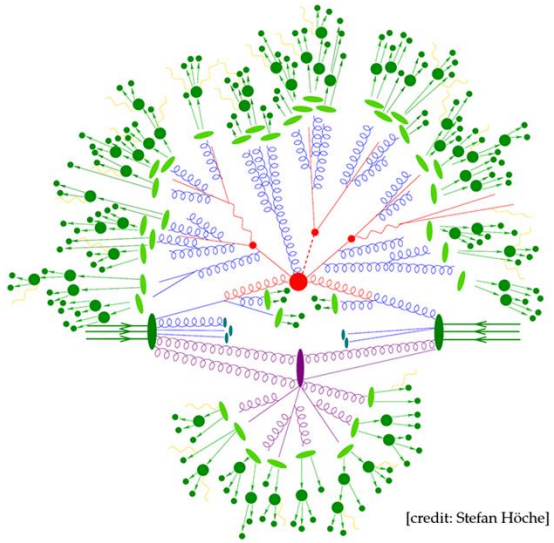
The QCD factorization formalism

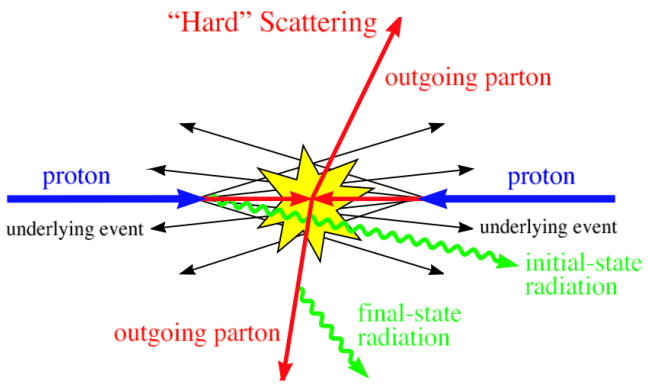
- Hadrons are composite and contains "partons" inside them
- The parton distributon functions (PDF) describes the probability to find a parton with a particular momentum from the beam particle.

$$\sigma(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X)$$

An EW version of LHC !

Copy Needed for the EW Version





What happens at Tevatron/LHC?

- Hard interaction: fixed-order calculation (in QCD $\equiv 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \ldots$)
- All-order radiative corrections: parton shower emissions
- Hadronisation from confinement property (quarks and gluons cannot live individually for long)
- Multiple parton interactions (MPI)
- Hadron decays

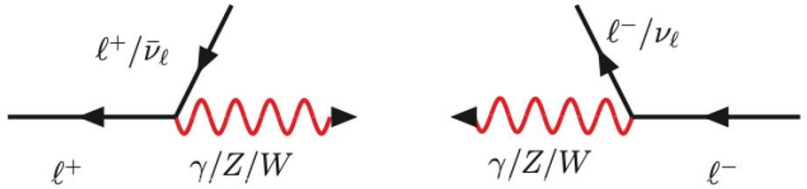
The QCD factorization formalism

- Hadrons are composite and contains "partons" inside them
- The parton distributon functions (PDF) describes the probability to find a parton with a particular momentum from the beam particle.

$$\sigma(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X)$$

The “Effective W approx.”

“When the energy scale is large enough, W/Z can be radiated off the beam collinearly, just like the photon”



S. Dawson Nucl. Phys. B 249 (1985) 42, G. Kane, W. Repko, W. Rolnick Phys. Lett. B 148 (1984) 367

Cross section

$$\sigma_{\rm VBF} \simeq \sum_{V_1V_2} \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2 f_{V_1/\ell}(x_1,Q^2) f_{V_2/\ell}(x_2,Q^2) \hat{\sigma}(V_2V_2 \to \mathcal{F}),$$

EVA PDFs

$$f_{V_-/\ell_L} \simeq \frac{g_L^2}{8\pi^2}\frac{1}{x}\log\frac{Q^2}{M_V^2}, \quad f_{V_+/\ell_L} \simeq \frac{g_L^2}{8\pi^2}\frac{(1-x)^2}{x}\log\frac{Q^2}{M_V^2}, \quad f_{V_0/\ell_L} \simeq \frac{g_L^2}{4\pi^2}\frac{1-x}{x},$$

$$f_{V_+/\ell_R} \simeq \frac{g_R^2}{8\pi^2}\frac{1}{x}\log\frac{Q^2}{M_V^2}, \quad f_{V_+/\ell_R} \simeq \frac{g_R^2}{8\pi^2}\frac{(1-x)^2}{x}\log\frac{Q^2}{M_V^2}, \quad f_{V_0/\ell_R} \simeq \frac{g_R^2}{4\pi^2}\frac{1-x}{x},$$

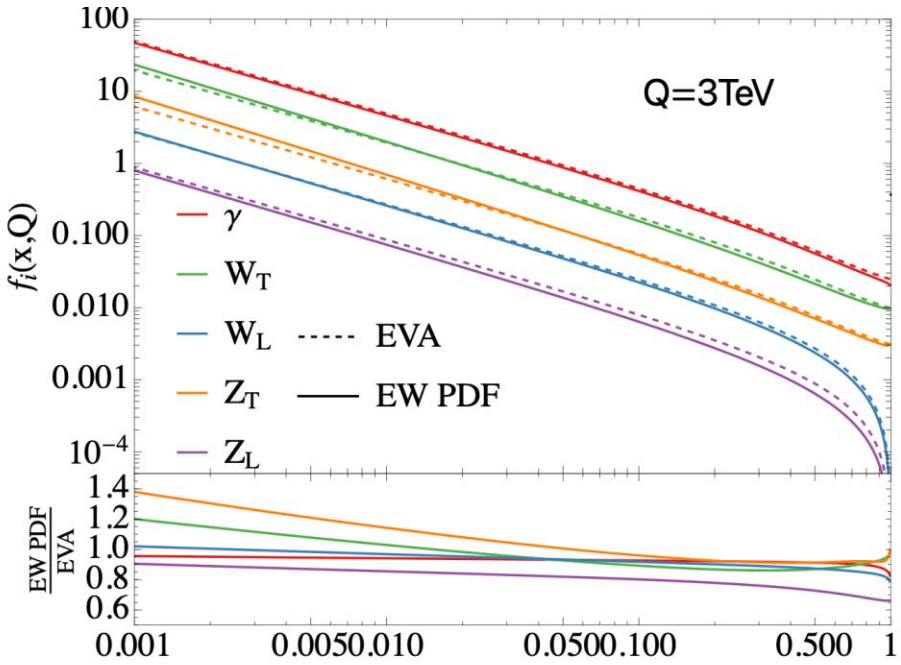
Implemented in MadGraph 5 in 2021

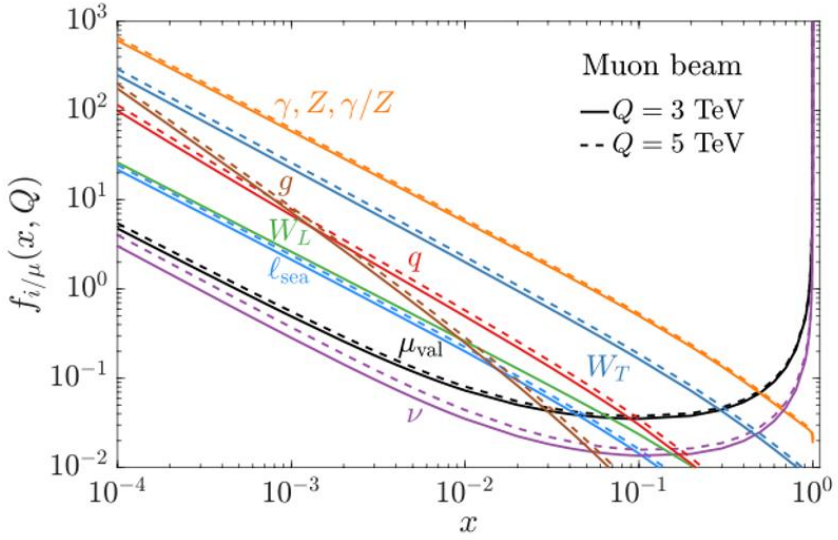
R. Ruiz, A. Costantini, F. Maltoni, and O. Mattelaer, 2111.02442

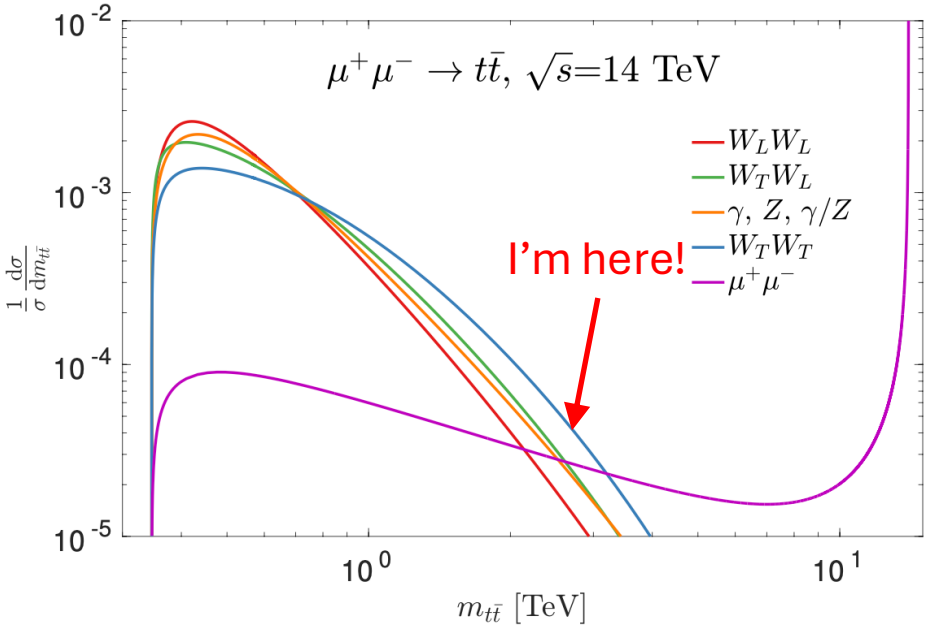
EW PDF

"Beyond the EW scale, all SM particles can be parton, and the PDF is controlled by the DGLAP evolution"

T. Han, YM and K. Xie, 2007.14300, 2103.09844



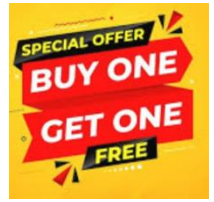




Similar PDF: F. Garosi, D. Marzocca and S. Trifinopoulos, 2303.16964

NLO EW correction is necessary

The muon colliders are also at the precision frontier



NLO EW corrections at muon colliders can be as large as the NLO QCD corrections at the LHC.

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma_{\rm LO}^{\rm incl}$ [fb]	$\sigma_{\rm NLO}^{\rm incl}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$4.6591(2) \cdot 10^2$	$4.847(7) \cdot 10^2$	$+4.0(2)$
ZZ	$2.5988(1) \cdot 10^1$	$2.656(2) \cdot 10^1$	$+2.19(6)$
HZ	$1.3719(1) \cdot 10^0$	$1.3512(5) \cdot 10^0$	$-1.51(4)$
HH	$1.60216(7) \cdot 10^{-7}$	$5.66(1) \cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2) \cdot 10^1$	$2.568(8) \cdot 10^1$	$-22.9(2)$
W^+W^-H	$1.1253(5) \cdot 10^0$	$0.895(2) \cdot 10^0$	$-20.5(2)$
ZZZ	$3.598(2) \cdot 10^{-1}$	$2.68(1) \cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4) \cdot 10^{-2}$	$6.60(3) \cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1) \cdot 10^{-2}$	$2.451(5) \cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6) \cdot 10^{-8}$	$0.86(7) \cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$0.993(6) \cdot 10^0$	$-33.1(4)$
W^+W^-ZZ	$1.209(1) \cdot 10^0$	$0.699(7) \cdot 10^0$	$-42.2(6)$
W^+W^-HZ	$8.754(8) \cdot 10^{-2}$	$6.05(4) \cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1) \cdot 10^{-2}$	$0.655(5) \cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$1.799(7) \cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$1.766(6) \cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	$-25.7(2)$

$\mu^+\mu^- \to X, \sqrt{s} = 10$ TeV	$\sigma_{\rm LO}^{\rm incl}$ [fb]	$\sigma_{\rm NLO}^{\rm incl}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$5.8820(2) \cdot 10^1$	$6.11(1) \cdot 10^1$	$+3.9(2)$
ZZ	$3.2730(4) \cdot 10^0$	$3.401(4) \cdot 10^0$	$+3.9(1)$
HZ	$1.22929(8) \cdot 10^{-1}$	$1.0557(8) \cdot 10^{-1}$	$-14.12(7)$
HH	$1.31569(5) \cdot 10^{-9}$	$42.9(4) \cdot 10^{-9}$ *	
W^+W^-Z	$9.609(5) \cdot 10^0$	$5.86(4) \cdot 10^0$	$-39.0(2)$
W^+W^-H	$2.1263(9) \cdot 10^{-1}$	$1.31(1) \cdot 10^{-1}$	$-38.4(5)$
ZZZ	$8.565(4) \cdot 10^{-2}$	$5.27(8) \cdot 10^{-2}$	$-38.5(9)$
HZZ	$1.4631(6) \cdot 10^{-2}$	$0.952(6) \cdot 10^{-2}$	$-34.9(4)$
HHZ	$6.083(2) \cdot 10^{-3}$	$2.95(3) \cdot 10^{-3}$	$-51.6(5)$
HHH	$2.3202(4) \cdot 10^{-9}$	$-1.0(2) \cdot 10^{-9}$ *	

P. Bredt, W. Kilian, J. Reuter and P. Stienemeier, 2208.09438

$\mathcal{O}(10 - 100\%)$ in the differential distributions

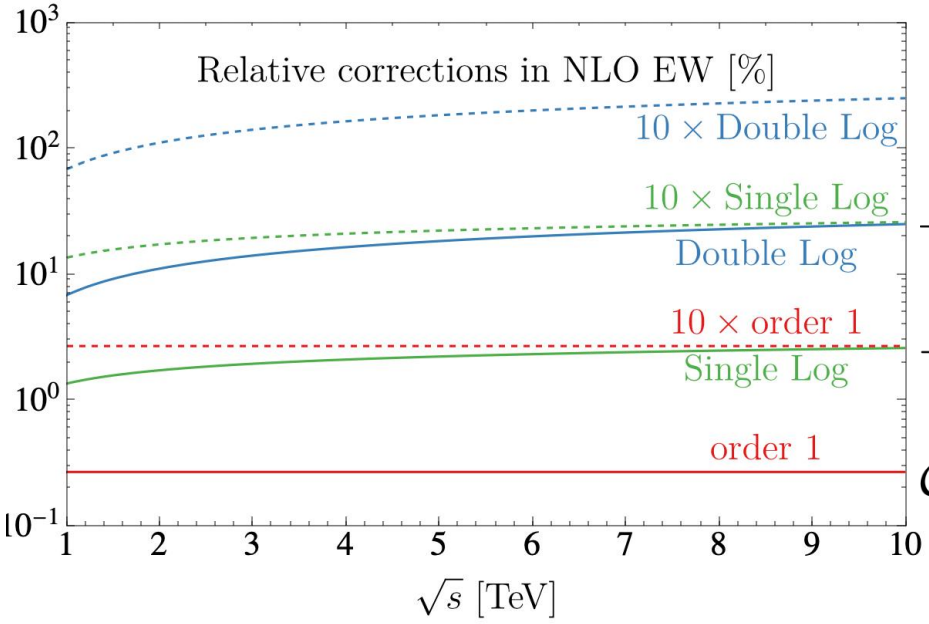
What is inside the NLO EW correction?

One loop virtual correction = Sudakov log terms $\mathcal{O}(-\alpha \log^{1,2}(s/m_W^2))$

+ const. term $\mathcal{O}(1)$

+ mass-suppressed terms $\mathcal{O}(m_W^2/s)$

$$\mathcal{O}(\alpha)$$



$$\rightarrow \frac{\alpha}{4\pi s_W^2} \log^2\left(s/m_W^2\right)$$

$$\rightarrow \frac{\alpha}{4\pi s_W^2} \log\left(s/m_W^2\right)$$

$$\mathcal{O}(1) \rightarrow \frac{\alpha}{4\pi s_W^2} \sim 0.3\%$$

EW Sudakov Logs

Master formula (Denner&Pozzorini)

Born amplitude: $\mathcal{M}_0^{i_1\dots i_n}(p_1,\dots,p_n)$ *Denner Pozzorini '01*

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1\dots i_n}(p_1,\dots,p_n) = \mathcal{M}_0^{i'_1\dots i'_n}(p_1,\dots,p_n)\,\delta_{i'_1 i_1\dots i'_n i_n}$

other tree-level amplitudes — the logs

$$\delta = \delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}} + \delta^{\mathrm{C}} + \delta^{\mathrm{PR}}$$

eikonal approximation of soft EW boson exchange

Leading Soft-Collinear	Subleading Soft-Collinear	Collinear	Parameter renormalis.
It depends only on s and it is the only term involving double logarithms.	The only one involving ratios of s with other invariants and also angular dependences.	In an on-shell scheme, the dependence on the UV regularisation scale cancels. No μ_r dependence is left.	

The logs inside the δ^i have always the form.

$$L(|r_{kl}|,M^2) \equiv \frac{\alpha}{4\pi}\log^2\frac{|r_{kl}|}{M^2}$$

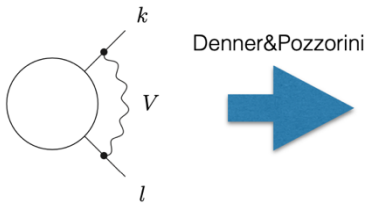
$$l(|r_{kl}|,M^2) \equiv \frac{\alpha}{4\pi}\log\frac{|r_{kl}|}{M^2}$$

$$M = M_W, M_Z, m_f, \lambda, \dots \qquad r_{kl} \equiv (p_k+p_l)^2$$

ASSUMPTIONS:

$r_{kl} \equiv (p_k+p_l)^2 \simeq 2p_kp_l \gg M_W^2 \simeq M_H^2, m_t^2, M_W^2, M_Z^2$ — the high-energy limit

$r_{kl}/r_{k'l'} \simeq 1$ — All invariants $\simeq s$. Reasonable, but $r_{kl} = s$ is impossible.



Denner&Pozzorini

$$L(|r_{kl}|,M^2) = L(s,M^2) + 2l(s,M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|,s)$$

$$= L(s) + 2l(s)\log\frac{M_W^2}{M^2} + 2l(s)\log\frac{|r_{kl}|}{s} + \cdots$$

LSC SSC

$L(s) \equiv L(s,M_W^2)$ and $l(s) \equiv l(s,M_W^2)$

SDK_0

Remove some QED logs that invlove MW and IR scale to remove the IR singularities

$\mathrm{SDK}_{\mathrm{weak}}$

Cluster the photons with the charged particles. A pure version of EWSL.

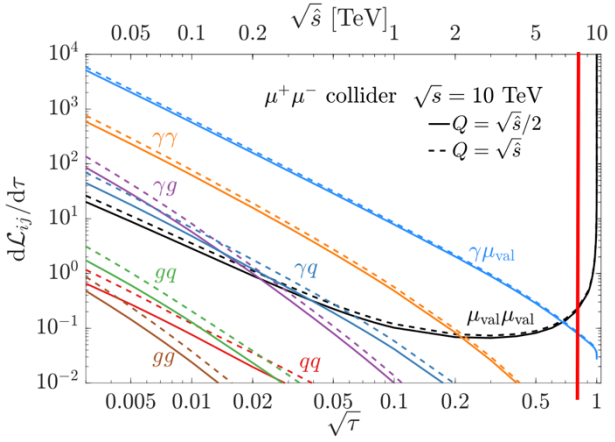
$\Delta^{s\to r_{kl}}$

To include the $\log(|r_{kl}|/|r_{k'l'}|)$

Calculation set-up

Kinematic cuts:

For annihilation $m(F) \geq 0.8\sqrt{S}$



T. Han, YM and K. Xie, 2103.09844

Other cuts:

$$\eta(X) < 2.44\,, \qquad p_T(X) > 100\ \text{GeV}\,, \qquad \Delta R(X,Y) > 0.4$$

Cluster photon with charged particles

$$\Delta R(X,\gamma) < 0.2$$

Initial-state-radiation (ISR)

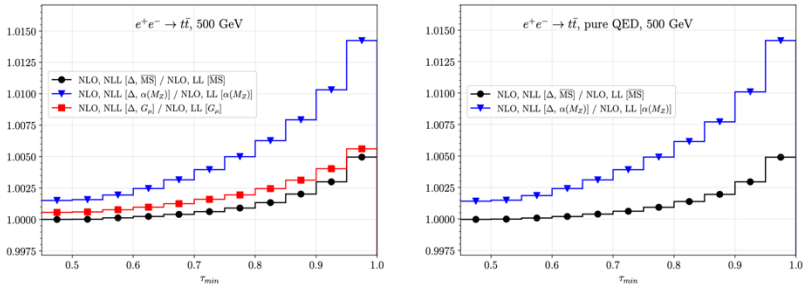
We used leading-log (LL) muon PDF

$$\Gamma_{\rm LO}(z) = \frac{\exp\left(3\beta_S/4 - \gamma_E\beta_E\right)}{\Gamma\left(1+\beta_E\right)}\,\beta_E(1-z)^{\beta_E-1} - \frac{1}{2}\beta_H(1+z) + \mathcal{O}(\alpha^2)$$

Beta scheme $\beta_E = \beta_S = \beta_H = e_e^2\beta$

Eta scheme $\beta_E = \beta_S = e_e^2\beta\,, \qquad \beta_H = e_e^2\eta$

$$\eta = \frac{\alpha}{\pi}\log\frac{\mu^2}{m^2}\,, \qquad \beta = \frac{\alpha}{\pi}\left(\log\frac{\mu^2}{m^2} - 1\right)$$

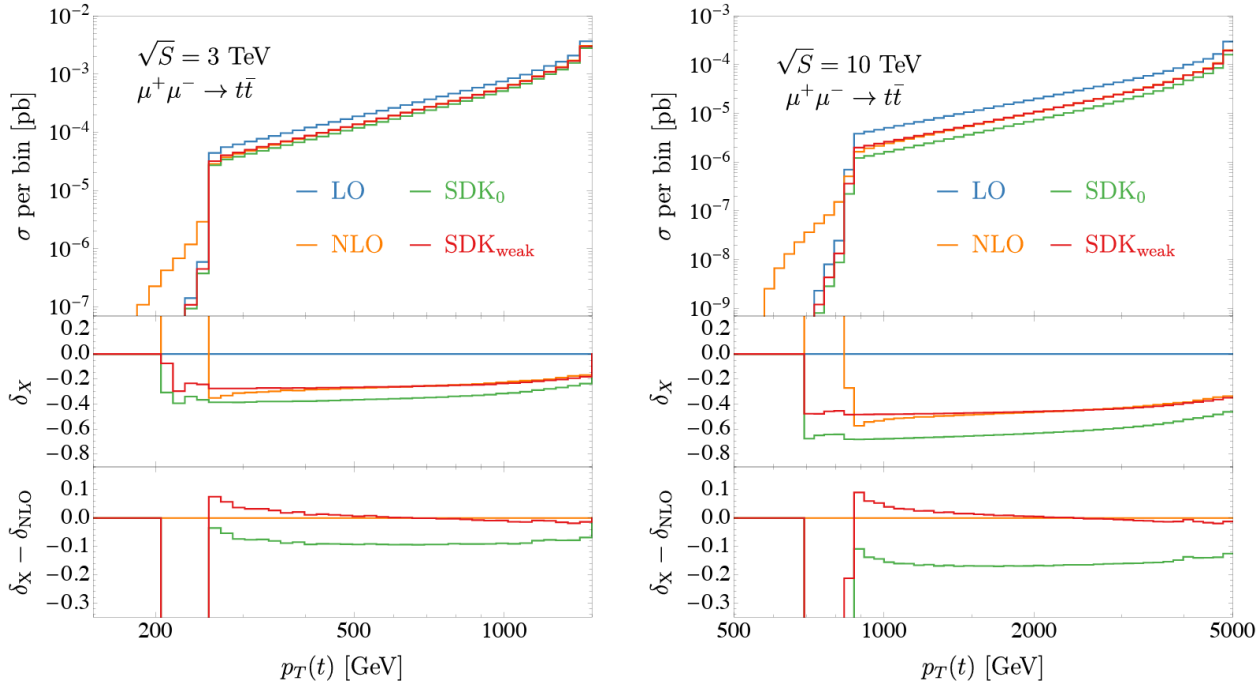


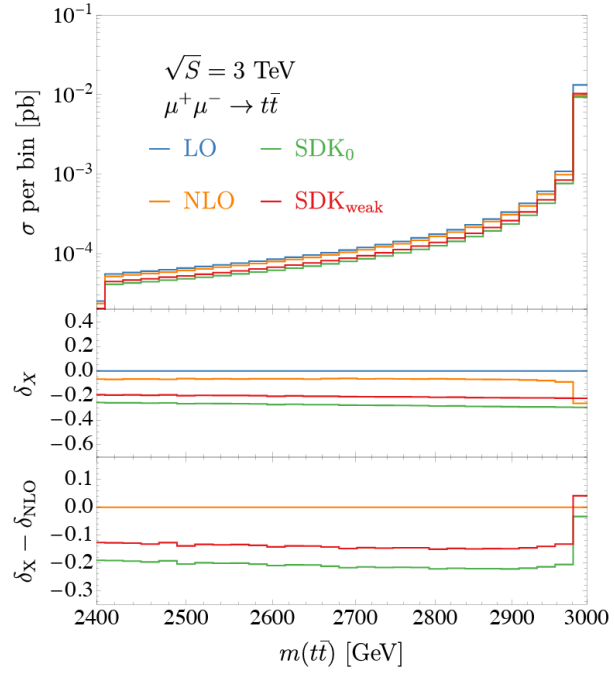
Better PDFs are needed for better accuracy, but is beyond the scope of this talk

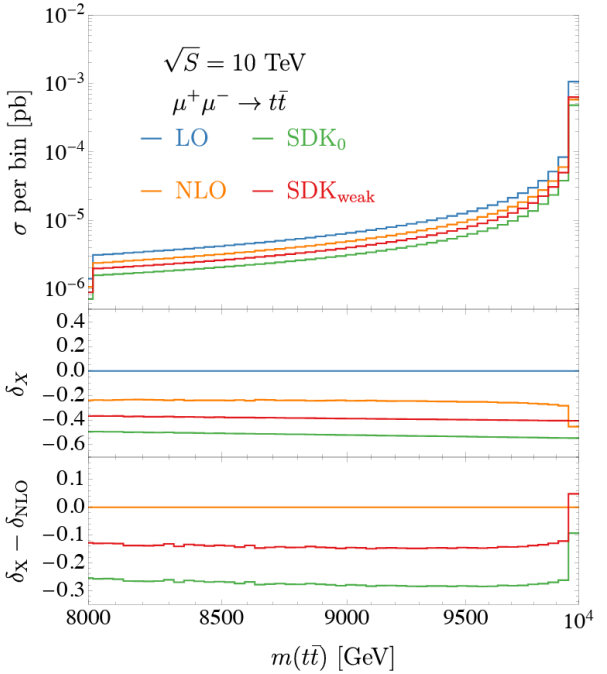
V. Bertone, M. Cacciari, S. Frixione, G. Stagnitto, M. Zaro and X. Zhao, 2207.03265

Top pair production

EWSL works well in reproducing the full NLO EW





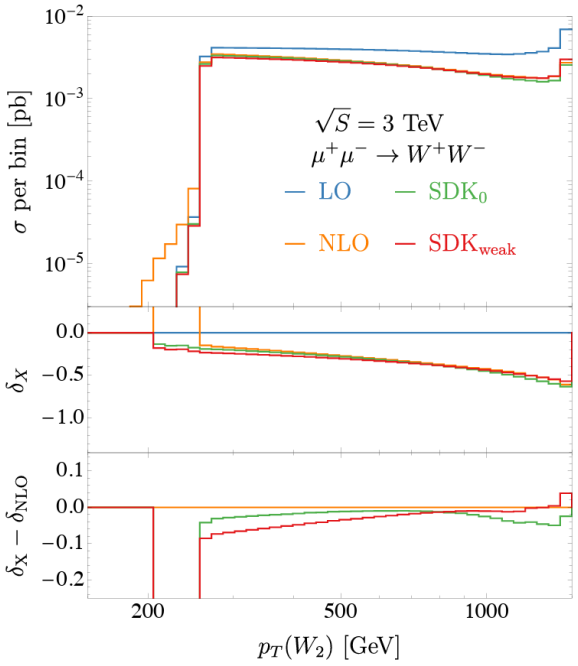


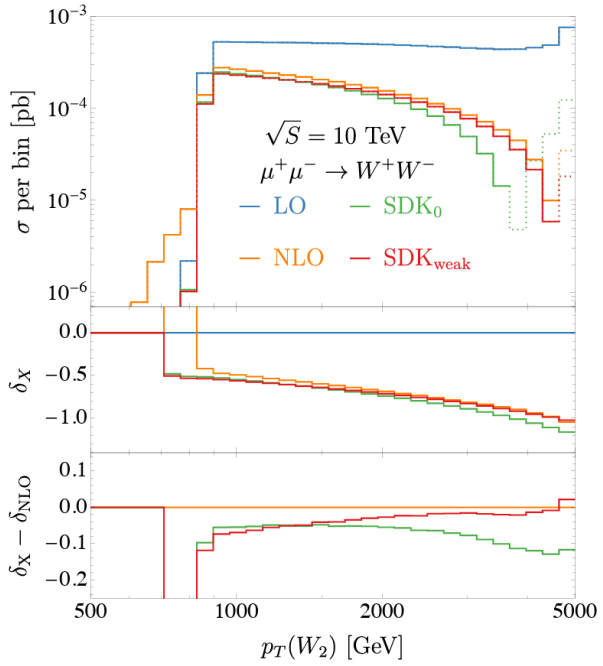
$$\delta_X \equiv \frac{\sigma_X - \sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}}$$

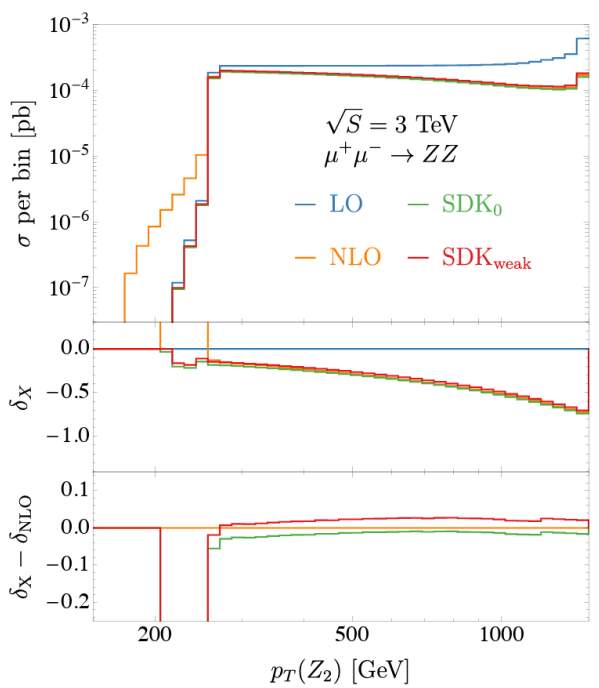
YM, D. Pagani, and M. Zaro, 2409.09129

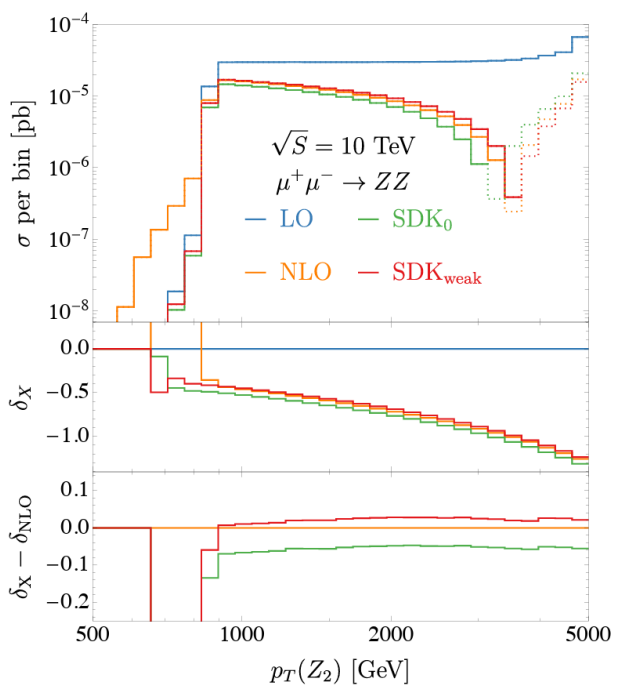
Di-V production

$\mathrm{SDK_{weak}}$ has better behaviors





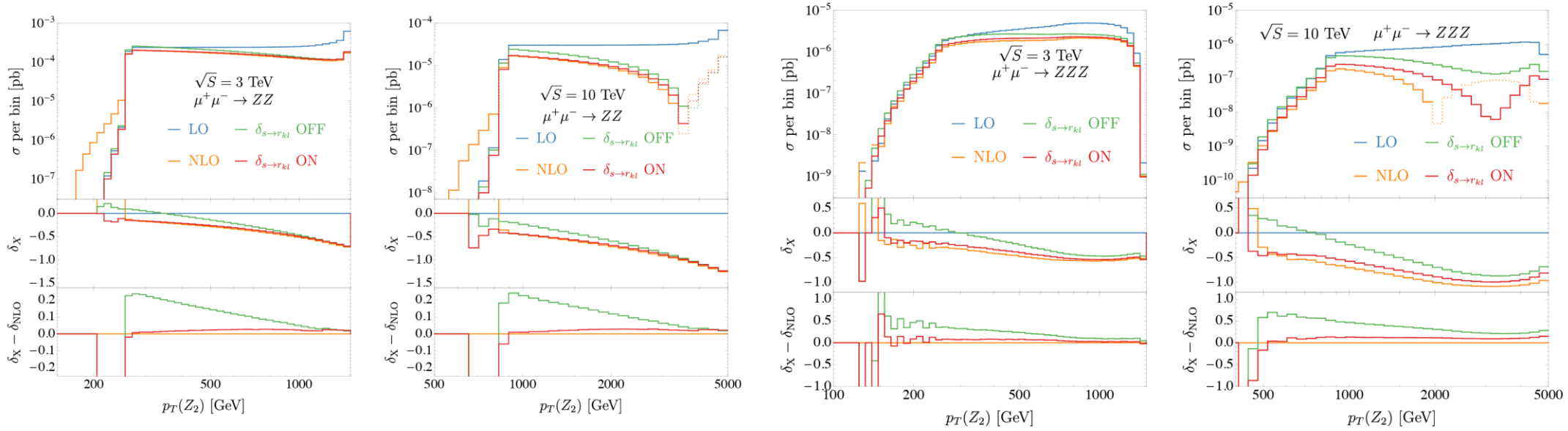




YM, D. Pagani, and M. Zaro, 2409.09129

Go beyond the DP EWSL

$\Delta^{s\to r_{kl}}$: To include the $\log(|r_{kl}|/|r_{k'l'}|)$



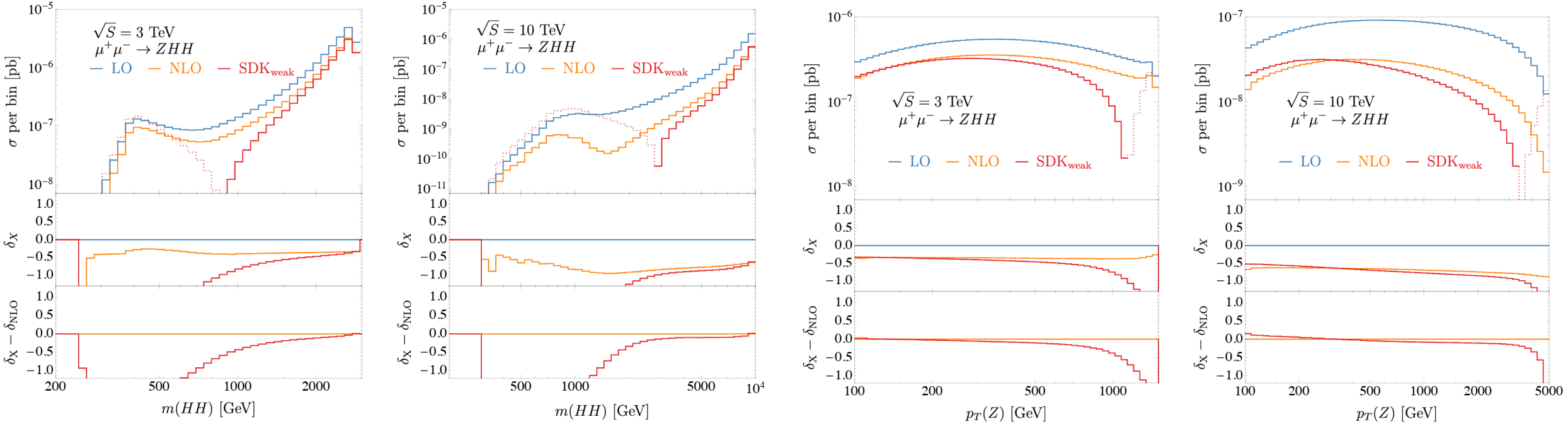
YM, D. Pagani, and M. Zaro, 2409.09129

The red lines better mimic the full NLO EW calculation

When EW Sudakov fails?

ZHH production

mass-suppressed term $\frac{v}{m(HH)}$ dominates in the large $p_T(Z)$ and small m(HH) range

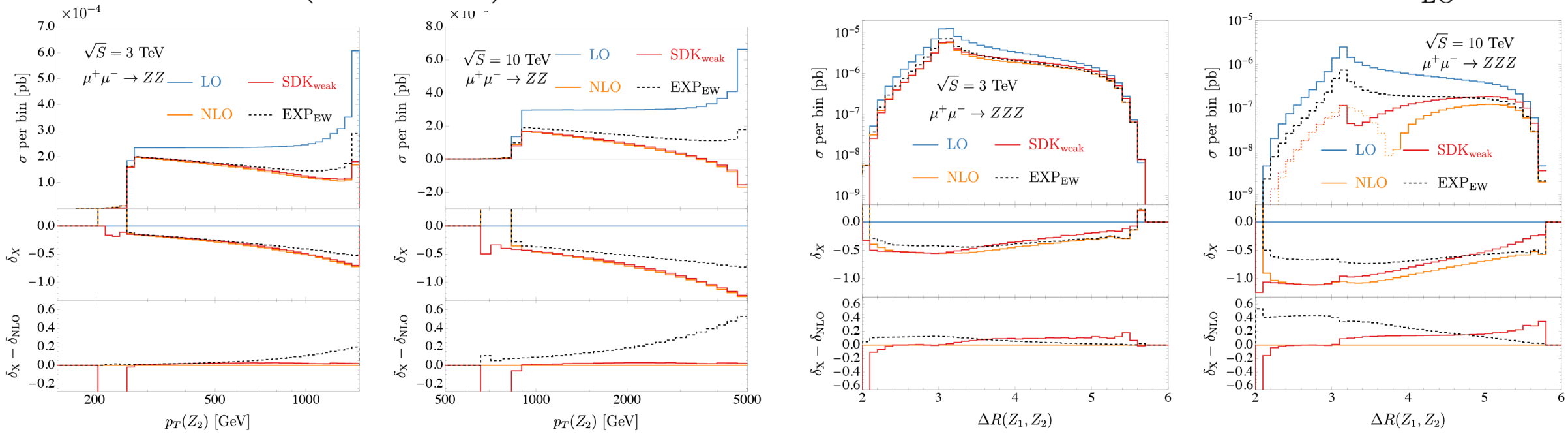


It also happens to VBF single Higgs production

YM, D. Pagani, and M. Zaro, 2409.09129

Resuming the EW Sudakov Logs

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}. \qquad \delta_X \equiv \frac{\sigma_X - \sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}}$$



The negative cross sections are fixed

An automated resummation is carried out in
A. Denner, S. Rode, 2402.10503

YM, D. Pagani, and M. Zaro, 2409.09129

Conclusions

- Multi-TeV muon colliders bring us more physics opportunities at the new frontiers, as a discovery machine/precision machine
- For precision physics, the EW PDF is needed to improve the existing EVA implementation in MadGraph
- The NLO EW is mandatory for the pheno studies at the muon colliders
- The EW Sudakov Logs can give good approximate for the NLO EW correction, but not always
- To cure the negative differential cross sections, resuming the EWSL is essential