Spin-density matrix in MadSpin

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MadGraph developers meeting
4 February 2025

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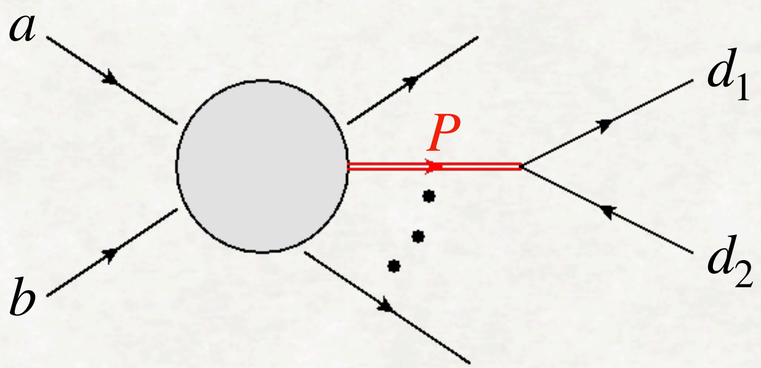
Introduction: spin correlations



Consider decay $ab \to d_1 d_2 + X$ with d_1, d_2 produced via the decay of an intermediate heavy resonance.

In general if spin(P) $\neq 0$

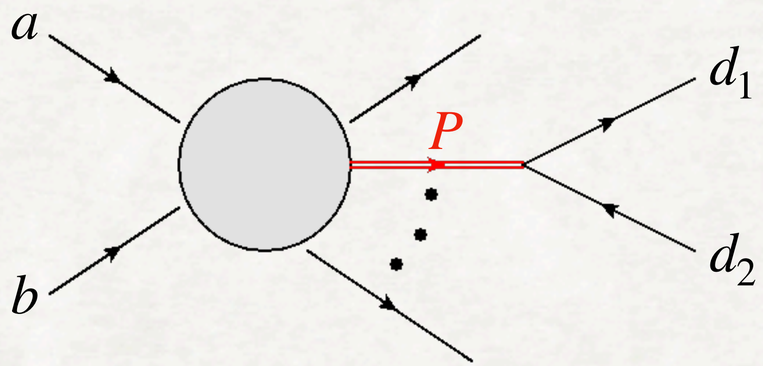
if $\mathscr{M}$ depends on $d_1 \cdot d_2$ momenta → decay spin correlations

if $\mathscr{M}$ depends on $a \cdot d_i, b \cdot d_i$, momenta → production spin correlations

How can we evaluate $\mathscr{M}(ab \to d_1 d_2 + X)$ with the best possible accuracy?

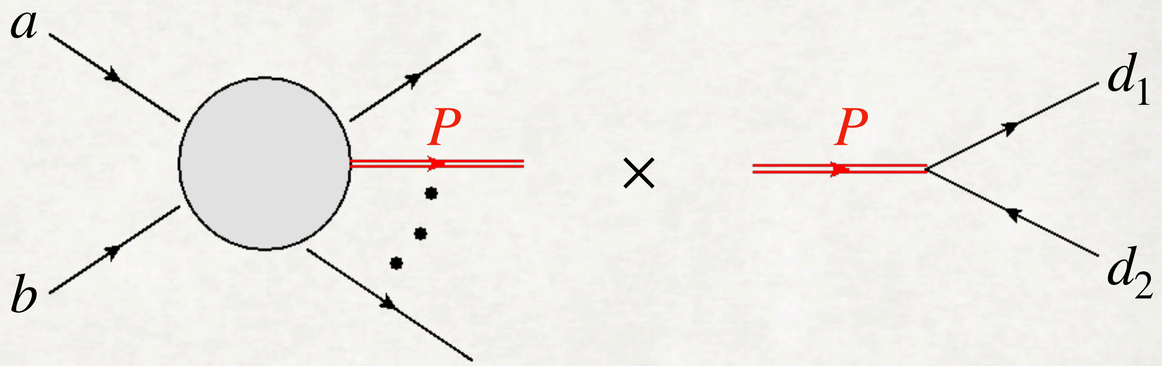
The two extremes

Full ME - complete final state in MG5aMC



- Most accurate: $\mathscr{M}_{full} = \mathscr{M}(ab \to d_1 d_2 + X)$
- ✓ full set of NLO corrections (productions & decay)
- ✓ spin correlations in production & decay
- ✓ off-shell effects
- ◉ might be practically impossible for high final-state multiplicities
- e.g. generate p p > l+ l- l+ l- [QCD]

Decay Chain Approximation



- Least accurate: $|\mathscr{M}_{dca}|^2 = |\mathscr{M}_{prod}|^2 |\mathscr{M}_{dec}|^2$
- $\mathscr{M}_{prod} = \mathscr{M}(ab \to P), \mathscr{M}_{dec} = \mathscr{M}(P \to d_1 d_2)$
- ◉ $\mathscr{M}_{dec}$ only LO accurate
- ◉ production spin correlations lost
- ◉ no off-shell effects
- e.g. generate p p > z z [QCD] + Pythia

Production spin correlations via FLMW

To keep production spin correlations one can simply

- generate events (@ NLO) keeping intermediate resonances on-shell
- generate resonance decays separately
- unweighting step re-introduces production spin correlations at LO

$$\text{keep event if: } \frac{|\mathscr{M}_{prod+dec}|^2}{|\mathscr{M}_{prod}|^2} > r \max\left(\frac{|\mathscr{M}_{prod+dec}|^2}{|\mathscr{M}_{prod}|^2}\right)$$

- FLMW [hep-ph/0702198]: analytical calculation of maximum weight
- MadSpin [1212.3460]: max weight calculated by sampling $|\mathscr{M}|^2$ ratios

MadSpin modes

spinmode = full

- Use $\frac{|\mathscr{M}_{full}|^2}{|\mathscr{M}_{prod}|^2}$, full = prod+dec

✓ spin correlations in production & decay

✓ off-shell effects via BW smearing

- Most accurate mode in MadSpin

◉ cannot compute 3-body decays

◉ cannot do loop-induced/coupling order restriction

spinmode = onshell

- Use $\frac{|\mathscr{M}_{full}|^2}{|\mathscr{M}_{prod}|^2 |\mathscr{M}_{dec}|^2}$, full = prod+dec

✓ spin correlations in production & decay

✓ can compute 3-body decays

◉ no off-shell effects (but they can be added)

◉ cannot do loop-induced/coupling order restriction

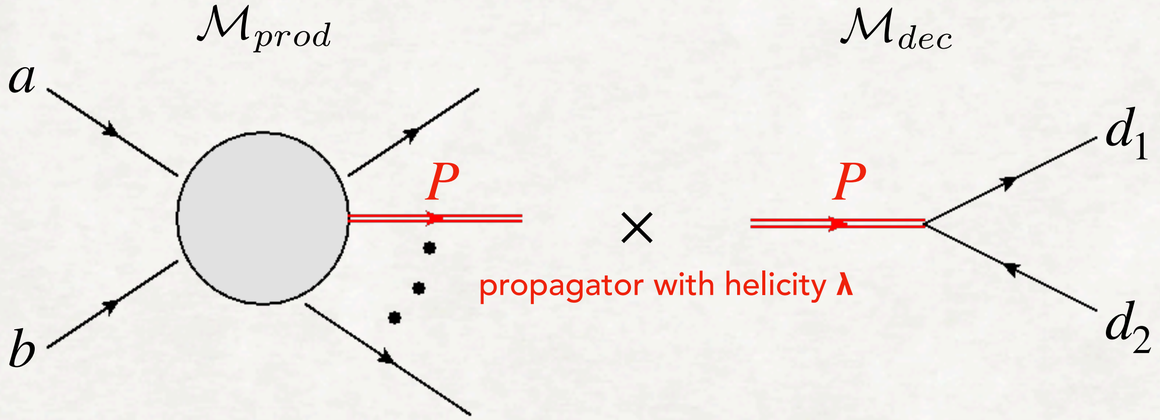
Different ratios used because decay events are weighted (“full”) or unweighted (“onshell”)

In both cases $\mathscr{M}$ calculated by MG5aMC

➡ MadSpin fails if it cannot compute $\mathscr{M}_{full}$ (e.g. loop-induced, amplitude restrictions (for interference terms or BSM samples))

Spin-density formalism

- Spin-density: avoid calculating $\mathscr{M}_{full}$ to lift MadSpin restrictions



$$\mathcal{M}_{full} = \mathcal{M}^{\mu}_{prod} P_{\mu\nu} \mathcal{M}^{\nu}_{dec}$$

rewrite propagator with the help of completeness relation

$$P_{\mu\nu} = \frac{\sum_{\lambda} (\epsilon^{\lambda}_{\mu})^{*} \epsilon^{\lambda}_{\nu}}{(q^2 - m^2) + im\Gamma}$$

$$\mathscr{M}^{\lambda} \equiv \mathscr{M}^{\mu} \epsilon^{\lambda}_{\mu}$$

- "onshell" mode: $|M_{full}|^2$ calculated by MG
- "density" mode: $\mathscr{M}_{prod}, \mathscr{M}_{dec}$ calculated by MG & convolution performed in MadSpin
- note that currently the on-shell approximation is taken but this need not be the case

$$|\mathcal{M}_{full}|^2 = \frac{1}{m^2\Gamma^2} \sum_{\lambda_1,\lambda_2} (\mathcal{M}^{\lambda_1}_{prod})^{*} \mathcal{M}^{\lambda_2}_{prod} (\mathcal{M}^{\lambda_1}_{dec})^{*} \mathcal{M}^{\lambda_2}_{dec}$$

provided by MG5aMC
(technically: colour-ordered amplitude * colour factor)

Convolution performed in MadSpin

The new "density" mode

- Use $\frac{|\mathscr{M}_{full}|^2}{|\mathscr{M}_{prod}|^2 |\mathscr{M}_{dec}|^2}$, full calculated via density matrix

✓ spin correlations in production & decay

✓ can compute 3-body decays

✓ can do loop-induced/coupling order restriction

◉ no off-shell effects (but they can be added)

In principle should be the best mode to use in all cases, unless resonance width is large.

Status

Initially prepared by Quentin Heirebaudt [master thesis]

- for single decay only
- validated for decay of colourless (W), coloured (t) and BSM resonance ($\tilde{g}$)

Ongoing work

- ✓code consolidation to handle more complicated cases (new DensityMatrix class)
- ✓extended to any number of decays
- performance optimisation
- illustration with examples that were previously not possible e.g.
  - loop-induced processes
  - processes with coupling restrictions

Performance - time for calculation steps (10k events)

Preliminary

$t\bar{t}$ events

t-decay

Step	onshell	density	density/onshell
Decay event generation	21.1	20.2	0.96
ME calculation	11.7	9.9	0.85
max(w) calculation	6.98	11.9	1.70
Decay	3.76	6.18	1.64

$t\bar{t}$ events

t-decay & $\bar{t}$-decay

Step	onshell	density	density/onshell
Decay event generation	40.5	40.8	1.01
ME calculation	10.1	10.5	1.04
max(w) calculation	11.3	46.4	4.11
Decay	6.44	25.3	3.93

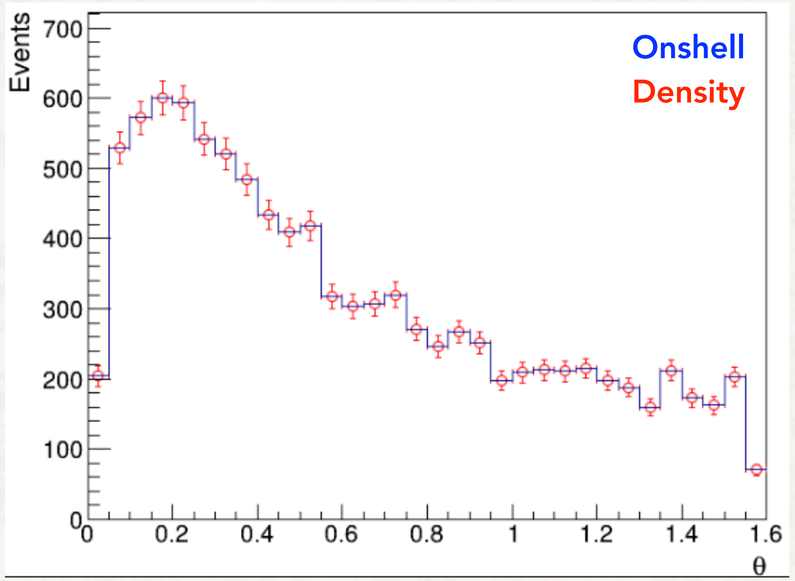
At the moment “density” mode substantially slower than “onshell”, however:

- extra operations are still kept for debugging
- we expect the situation to be reversed after optimisation

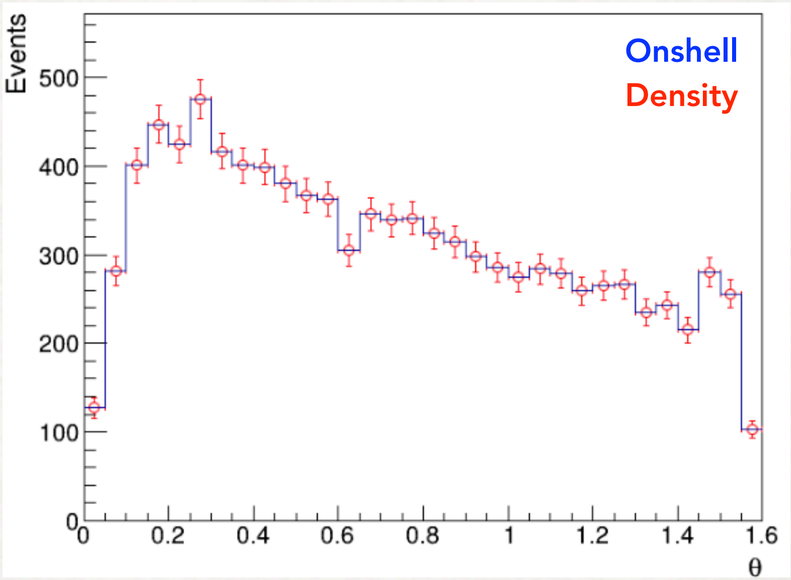
Validation $t\bar{t}$

- 4x4 square density matrix
- $pp \rightarrow t\bar{t}$ LO events
- decay t > w+ b
- decay t~ > w- b~
- plotting θ angle of W boson/b-quark from LHE file

W boson



b-quark



✓ identical results for onshell and density as expected

Validation tW

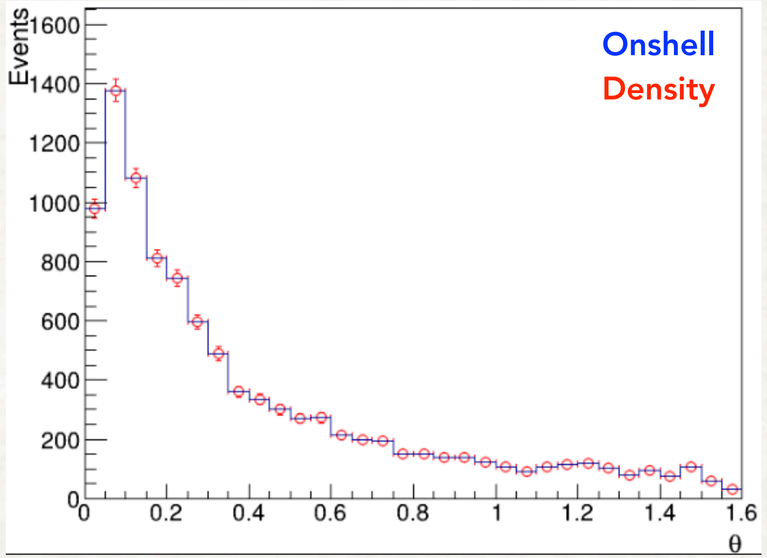
- 4x9 non-square density matrix

- $pp \rightarrow tW^-$ LO events

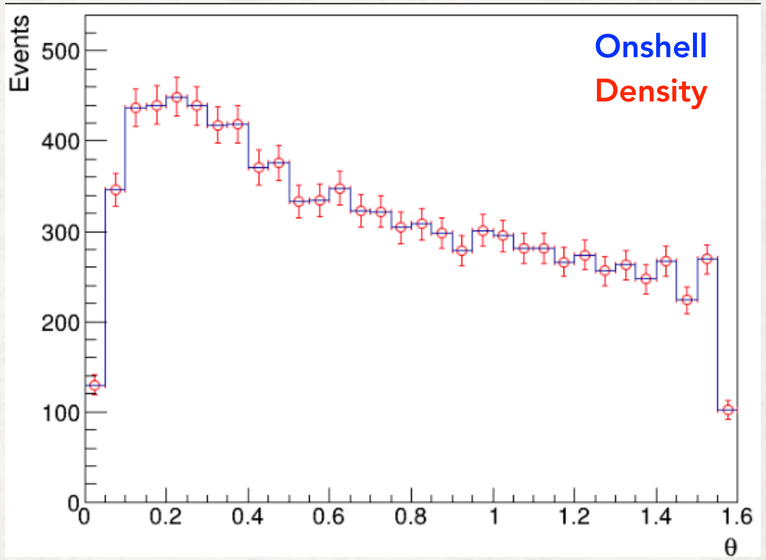
- decay t > w+ b
- decay w- > e- ve~

- plotting θ angle of b-quark/electron from LHE file

b-quark



electron



✓ identical results for onshell and density as expected

Issue with decay chains

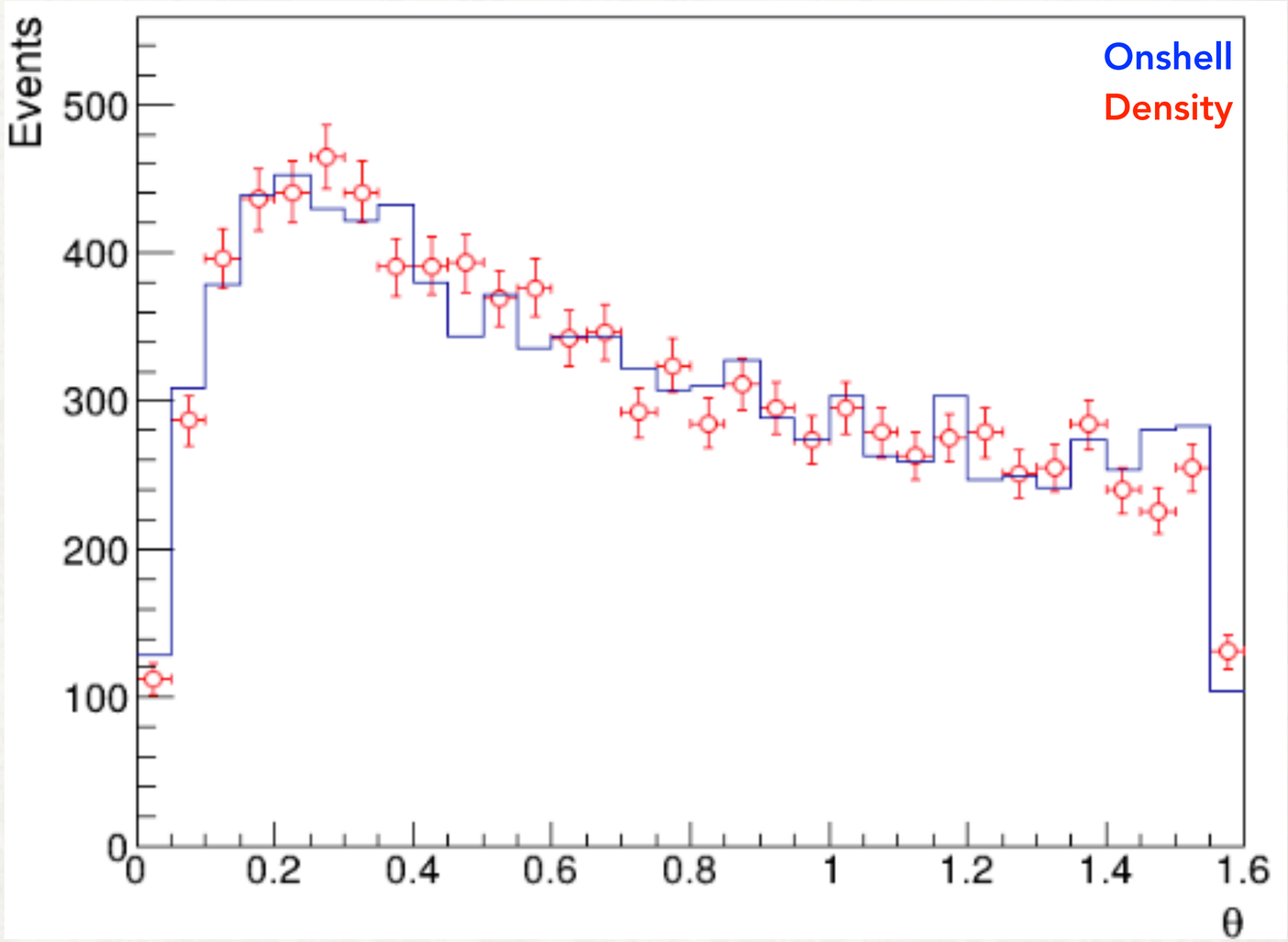
- distributions differ for all particles when using decay chains

- decay t > w+ b , (w+ > e+ ve)
- decay t- > w- b~ , (w- > e- ve~)

- matrix elements are identical

- issue unclear at the moment - needs further investigation

electron



Summary and outlook

✓ New decay mode being developed based on spin-density-matrix formalism

✓ Will allow lifting current restrictions (loop-induced, coupling restrictions, …)

✓ Code validated for up to 2 decays for particles with different spin & colour

Next steps

- solving issue with decay chains
- performance optimisation
- testing on loop-induced processes & coupling restriction
- access to density matrix allows entanglement studies (see talk by Eleni)