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Relativistic Quantum Particles and Fields
Some Theoretical Basics

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Exercises

1. Solve the center-of-mass kinematics of the $1 \to 2$ particle decay.

2. Solve the center-of-mass kinematics of the $2 \to 2$ particle scattering.

3. Establish the identity $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$.

4. Determine the values of the three Mandelstam variables in the case of 4 identical particles as a function of the scattering angle and the total energy in the center-of-mass frame.

5. Consider the evaluation of Heisenberg's uncertainty relation $\Delta x\, \Delta p \geq \frac{1}{2}\hbar$ for each of the Fock states of the quantum harmonic oscillator.

6. Through contour integration in the complex k^0 plane, confirm this manifest spacetime invariant expression of the Feynman propagator.

7. Establish the expressions for the 2-point functions, *i.e.*, the Feynman propagator of the complex scalar field. Explain the outcome of the analysis in terms of the conserved U(1) quantum number.

8. Consider a model with two species of neutral scalar particles ϕ and χ of masses m and M, respectively, such that $M > 2m$, with the following coupling,

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \chi\right)^2 - \frac{1}{2}M^2\chi^2 + \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}g\chi\phi^2$$

   the real coupling constant g having a dimension of mass in particle physics units. Compute to first order in perturbation theory the lifetime τ of the particle χ, $\tau = 1/\Gamma(\chi \to 2\phi)$.