Alpha Decay

In 1928 George Gamow and, independently, Gurney and Condon used quantum mechanics and these ideas to explain α decay.

Classically the α particle could not penetrate the potential barrier.

But quantum mechanically tunnelling can occur, as we have seen in lectures 12 and 13.

Looking back we see that the probability of transmission was given by:

$$T \approx e^{-2KL}$$

L was the *width* of the (constant) potential barrier.

where

$$K = k_2 = \frac{\sqrt{2m(U-E)}}{\hbar}$$

Mass of the α particle (in this case).

This was the height of the (constant) potential barrier.

Energy of the α particle (4.85 MeV in this case).

This term must be changed because the height of the *Coulomb barrier* varies with distance.

Alpha Decay

As we have seen the height of the Coulomb barrier varies as:

$$U = k_e \frac{q_1 q_2}{r}$$

$$U(x) = k_e \frac{2Ze^2}{x}$$

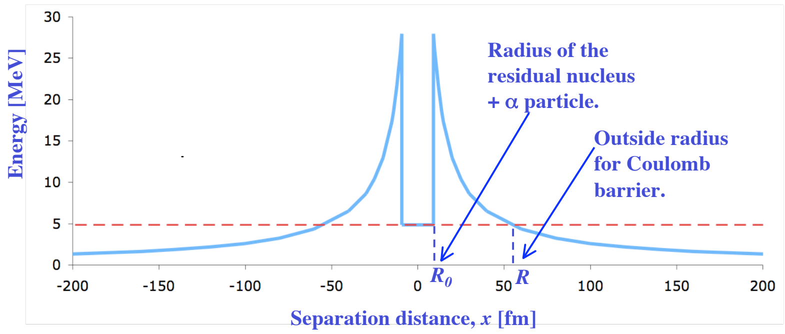
For alpha particle decay where *Z* is the atomic number of the residual nucleus and *x* is the separation distance.

This gives: $K(x) = k_2 = \dfrac{\sqrt{2m(U(x)-E)}}{\hbar} = \left(\dfrac{2m}{\hbar^2}\right)^{1/2}(U(x)-E) = \left(\dfrac{2m}{\hbar^2}\right)^{1/2}\left(k_e\dfrac{2Ze^2}{x} - E\right)$

and to get the equivalent for the term *KL* in $T \approx e^{-2KL}$ this must be integrated over the width of the Coulomb barrier from R_0, the radius of the nucleus, to *R* the outer radius of the Coulomb barrier.

Alpha Decay

Here is the Energy Diagram again.



So KL **must be replaced by:** $I = \int_{R_0}^{R} K(x)dx = \int_{R_0}^{R} \left(\frac{2m}{\hbar^2}\right)^{1/2} \left(k_e \frac{2Ze^2}{x} - E\right)^{1/2} dx$

At R **the Coulomb potential is equal to** E**, so:** $E = k_e \frac{2Ze^2}{R}$

The outside radius of the Coulomb barrier is directly related to the separation energy.

Alpha Decay

After doing the integral and using an approximation (See, for example, Concepts Of Modern Physics by A Beiser, 4th Edition, p 465) we find that:

$$I = \frac{e}{\hbar}(mk_e ZR_0)^{1/2} - \frac{2\pi e^2 Z}{\hbar}\left(\frac{m}{2E}\right)^{1/2}$$

and the transmission probability is $T = e^{-2I}$

so that once the separation energy is calculated, I **and hence** T **can be calculated for any possible α decay.**

Alpha Decay

Remember the transmission coefficient gives the probability that a particle moving towards the barrier will actually penetrate it.

The next idea that was needed was to predict how often the α particle interacts with the barrier (strikes the barrier).

In order to do that one could imagine that the α particle is bouncing backwards and forwards inside the nucleus with some speed v presumably related to its energy.

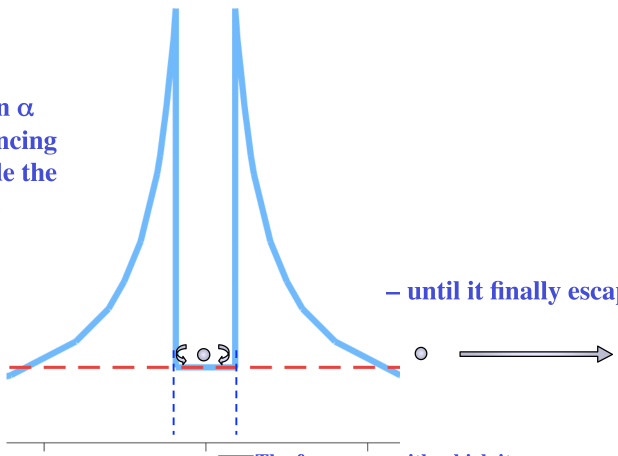
[This needs quite a leap of the imagination but it turns out that you don't need to know v accurately at all.]

The distance across the nucleus is just $2R_0$

So, on this rather simple model the a particle strikes one of the barriers a total of: $f = \dfrac{v}{2R_0}$ times per second

Alpha Decay

Imagine an α particle bouncing around inside the nucleus



– until it finally escapes

The decay constant (this is the probability per unit time for a decay to occur)

$$\lambda = fT$$

The frequency with which it strikes the barrier

The probability of transmission each time it strikes

$$= \frac{v}{2R_0} e^{-2I}$$

Alpha Decay

$$\lambda = fT = \frac{v}{2R_0} e^{-2I}$$

We don't actually know v, but it doesn't matter because e^{-2I} varies over some 25 orders of magnitude for different nuclei that undergo α decay and dominates the behaviour of this expression.

For example ^{238}U and ^{210}Po are both a emitters.

The half-life of ^{210}Po is about 0.35 μs.

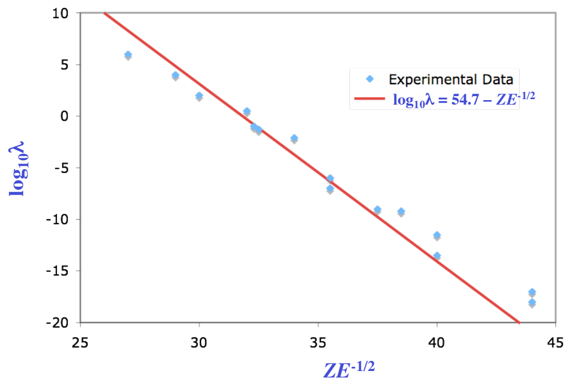
The half-life of ^{238}U is 4.47×10^{9} years, i.e.a factor of about 4×10^{23} longer.

This huge difference is explained by the different separation energies and the exponential effect of the energy in determining the probability of decay.

This was an early and remarkable triumph of quantum mechanics in explaining nuclear behaviour.

Alpha Decay

The following graph is adapted from Concepts of Modern Physics by Arthur Beiser, 4th Edition. It illustrates how the enormous variation in the half-life for α decay depends on the separation energy (and the atomic number) and is explained in terms of quantum mechanics.



Exercise 22

1. Continue with the example of the α decay of ^{226}Ra

   Calculate the width of the Coulomb barrier.

   Sketch the potential seen by the α particle in this case.

   Use the formulae in the previous lecture to estimate the decay constant for this decay.

Beta Decay

The most common form of radioactive decay is beta decay.

This changes the proton number (i.e. the atomic number of the radioactive nucleus.

An additional particle, the neutrino (little neutral one) is emitted in all beta decays.

In β^- decay (beta minus decay) one of the neutrons in the nucleus changes into a proton, a β^- particle and an anti-neutrino. The β^- is emitted from the nucleus which therefore increases it atomic number by one.

$$n \rightarrow p + e^- + \bar{\nu}$$

or β^-

$$^A_Z X \rightarrow {}^A_{Z+1}X + e^- + \bar{\nu}$$

Beta Decay

In one kind of β⁺ decay a positron is emitted and one of the protons in the nucleus changes into a neutron, a β⁻ particle and a neutrino.

$$p \rightarrow n + e^{+} + \nu$$

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X + e^{+} + \nu$$

Electron capture involves the absorption of one of the innermost atomic electrons into the nucleus.

$$p + e^{-} \rightarrow n + \nu$$

$$^{A}_{Z}X + e^{-} \rightarrow {}^{A}_{Z-1}X + \nu$$

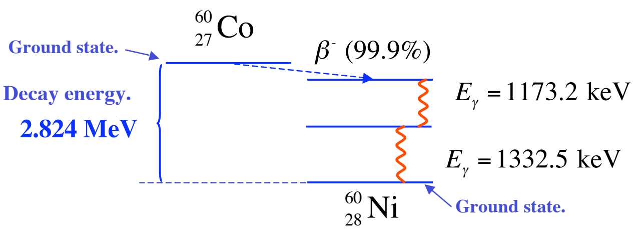
As in positron emission a proton within the nucleus changes into a neutron and the atomic number is reduced by one.

Beta decays involve the weak interaction.

Gamma Emission (Decay)

After α or β decay the residual nucleus is normally left in an excited state and it de-excites down to the ground state by emitting gamma-rays.

For example here is a simplified energy level diagram for the decay of ^{60}Co.



Beta decays involve the weak interaction.