Radioactivity

The Law of Radioactive Decay.

Radioactivity

The law of radioactive decay.

It turns out that the *probability per unit time* for any radioactive nucleus to decay is a *constant*, called the *decay constant*, *lambda*, λ.

If there are N radioactive nuclei, then the number that will decay in a time Δt is given by $\Delta N = -\lambda N \Delta t$.

The *rate of decay* is therefore $\frac{\Delta N}{\Delta t} = \frac{dN}{dt} = -\lambda N$

The minus sign indicates that N is *decreasing* with time.

Note: The rate of decay is also called the *radioactivity* of the sample containing the radioactive nuclei (or sometimes just the *activity*). The unit of activity is the Becquerel (Bq). [1 Bq is 1 disintegration per second (dps)]

Radioactivity

The law of radioactive decay.

So this is the *differential equation* for radioactive decay:

$$\frac{dN}{dt} = -\lambda N$$

The rate of radioactive decay is directly proportional to the number of nuclei present, N and to the decay constant λ.

This is a simple differential equation.

We solve it to find the *number of radioactive nuclei* and the *radioactivity* as a function of time.

Radioactivity

The law of radioactive decay.

This equation can be re-written as: $\frac{dN}{N} = -\lambda dt$

Integrating this expression gives: $\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$.

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

Where N_0 is the number of radioactive nuclei at a time $t = 0$.

Taking exponentials gives the well-known expression for exponential decay:

$$N = N_0 e^{-\lambda t}$$

Radioactivity

The law of radioactive decay.

$$N(t) = N_0 e^{-\lambda t}$$

Decay constant.

Number of radioactive nuclei left after a time *t*.

Number of radioactive nuclei present at time *t* = 0.

Note that, since the decay rate, $R(t) = \frac{dN}{dt} = \lambda N(t)$**,**

we can also write: $R(t) = R_0 e^{-\lambda t}$

Decay rate at a time *t*.

Decay rate at the time *t* = 0.

Radioactivity

The Half-life $T_{1/2}$

The concept of the *half-life,* $T_{½}$ **is often used to characterise radioactive nuclei.**

This is the time that it takes for half of the nuclei present to decay. That is the time for N **to equal** $\frac{N_0}{2}$ **.**

In other words, $\frac{\cancel{N_0}}{2} = \cancel{N_0} e^{-\lambda T_{½}}$ **by the definition of** $T_{½}$

$$e^{-\lambda T_{½}} = \frac{1}{2}$$

$$-\lambda T_{½} = \ln\left(\tfrac{1}{2}\right)$$

$$\lambda = \frac{\ln(2)}{T_{½}}$$

$$= \frac{0.693}{T_{½}}$$

Example

Age determination by carbon dating.

^{14}C has a half-life of 5730 years.

Cosmic-rays in the upper atmosphere produce ^{14}C at a roughly constant rate. As a result the ratio of ^{14}C to the stable ^{12}C is a constant (more or less) in materials that are in equilibrium with the atmosphere

As long as organisms are living they continuously exchange carbon with the environment (as CO_2) and they maintain this ratio.

Once they die this exchange with the environment is broken and the ^{14}C starts to decay. The age of a material, such as charcoal or bone can be determined by measuring its activity (radioactivity) per kg and applying the radioactive decay equation.

$$R(t) = R_0 e^{-\lambda t}$$

Radioactivity

Exercise 23

1. A living specimen in equilibrium with the atmosphere has an activity of 250 Bq of ^{14}C per kg of carbon .

   A sample of wood recovered from an archaeological site contains 200 g of carbon and is found to have an activity of 2.5 Bq. Assuming that the production rate of ^{14}C in the atmosphere is a constant and taking the half-life of ^{14}C as 5730 years, what is the age of the wood?

   If the activity of the sample were 1 Bq what would its age be?

Radioactivity

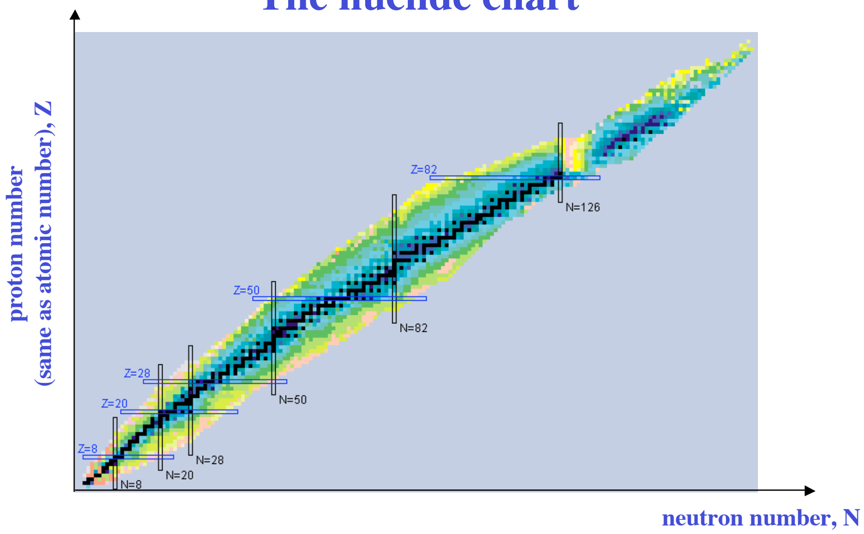
Radioactive nuclei can be produced through nuclear reactions.

- Some of are “neutron rich” and generally decay by emitting a negative beta particle, β^-.
- Some of are “proton rich” and generally decay by emitting a positive beta particle, β^+, or positron.

In each case the decay brings them back to the line of stable isotopes.

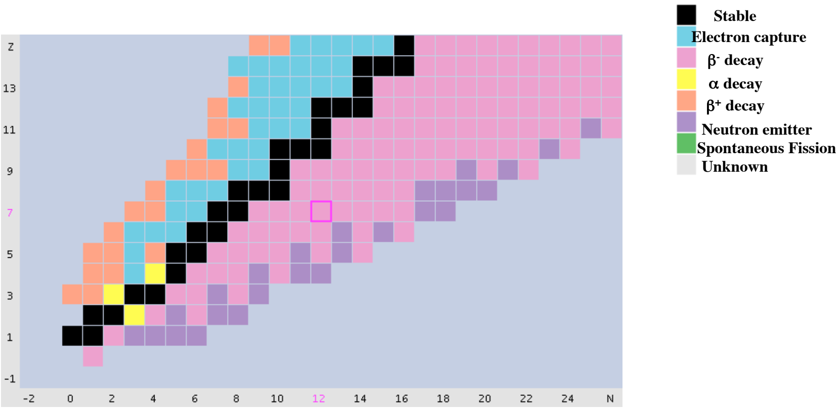
The details are shown graphically on a *nuclide chart* – see the next slide.

The nuclide chart

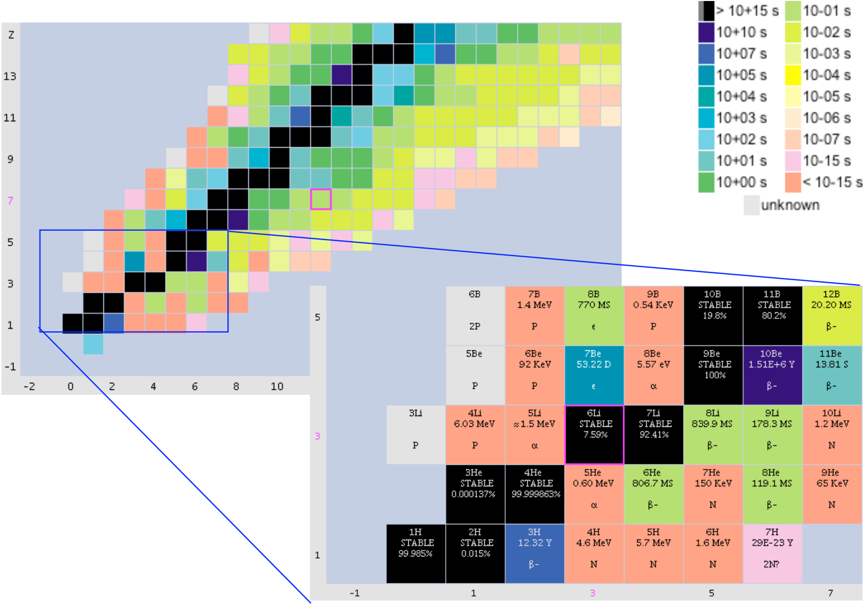


The stable isotopes as well as the radioactive ones can be seen on a nuclide chart. This is the nuclide chart shown on the IAEA web-site - http://www-nds.iaea.org/

Light isotope end of the nuclide chart, colour coded for the *mode* of decay.

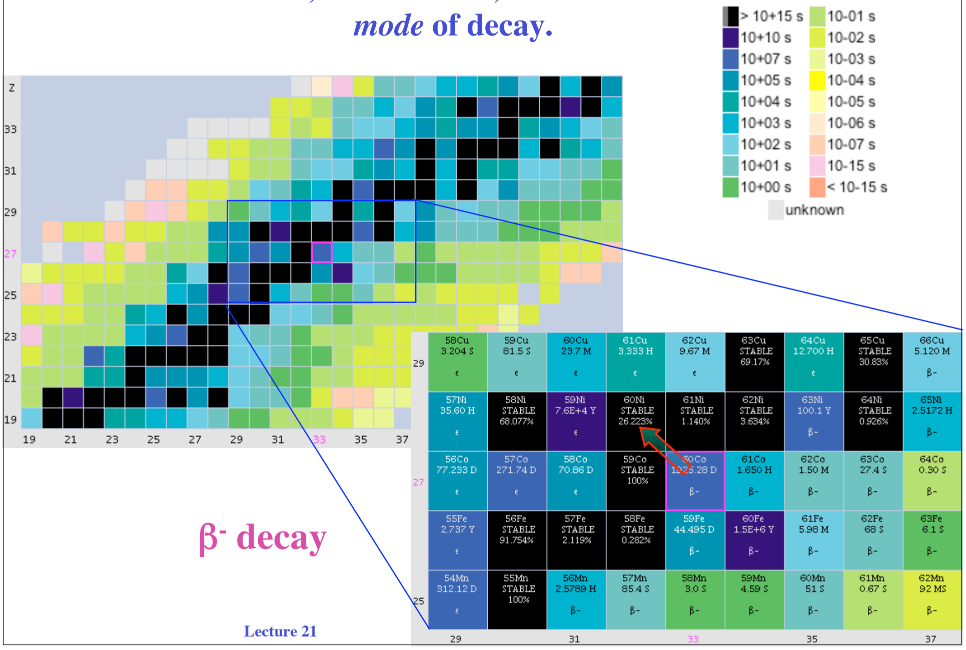


Light isotope end of the nuclide chart, colour coded for the *half-life*.



	-1		1		3		5		7
5			6B 2P	7B 1.4 MeV P	8B 770 MS ϵ	9B 0.54 KeV P	10B STABLE 19.8%	11B STABLE 80.2%	12B 20.20 MS β-
			5Be P	6Be 92 KeV P	7Be 53.22 D ϵ	8Be 5.57 eV α	9Be STABLE 100%	10Be 1.51E+6 Y β-	11Be 13.81 S β-
3		3Li P	4Li 6.03 MeV P	5Li ≈1.5 MeV α	6Li STABLE 7.59%	7Li STABLE 92.41%	8Li 839.9 MS β-	9Li 178.3 MS β-	10Li 1.2 MeV N
			3He STABLE 0.000137%	4He STABLE 99.999863%	5He 0.60 MeV α	6He 806.7 MS β-	7He 150 KeV N	8He 119.1 MS β-	9He 65 KeV N
1		1H STABLE 99.985%	2H STABLE 0.015%	3H 12.32 Y β-	4H 4.6 MeV N	5H 5.7 MeV N	6H 1.6 MeV N	7H 29E-23 Y 2N?	

Nuclide chart, around ^{60}Co, colour coded for the *mode* of decay.



29	58Cu 3.204 S ϵ	59Cu 81.5 S ϵ	60Cu 23.7 M ϵ	61Cu 3.333 H ϵ	62Cu 9.67 M ϵ	63Cu STABLE 69.17%	64Cu 12.700 H ϵ	65Cu STABLE 30.83%	66Cu 5.120 M β-
	57Ni 35.60 H ϵ	58Ni STABLE 68.077%	59Ni 7.6E+4 Y ϵ	60Ni STABLE 26.223%	61Ni STABLE 1.140%	62Ni STABLE 3.634%	63Ni 100.1 Y β-	64Ni STABLE 0.926%	65Ni 2.5172 H β-
27	56Co 77.233 D ϵ	57Co 271.74 D ϵ	58Co 70.86 D ϵ	59Co STABLE 100%	60Co 1925.28 D β-	61Co 1.650 H β-	62Co 1.50 M β-	63Co 27.4 S β-	64Co 0.30 S β-
	55Fe 2.737 Y ϵ	56Fe STABLE 91.754%	57Fe STABLE 2.119%	58Fe STABLE 0.282%	59Fe 44.495 D β-	60Fe 1.5E+6 Y β-	61Fe 5.98 M β-	62Fe 68 S β-	63Fe 6.1 S β-
25	54Mn 312.12 D ϵ	55Mn STABLE 100%	56Mn 2.5789 H β-	57Mn 85.4 S β-	58Mn 3.0 S β-	59Mn 4.59 S β-	60Mn 51 S β-	61Mn 0.67 S β-	62Mn 92 MS β-
	29		31		33		35		37

β^- decay

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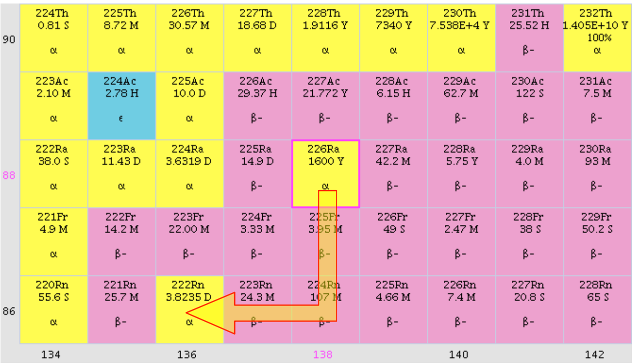
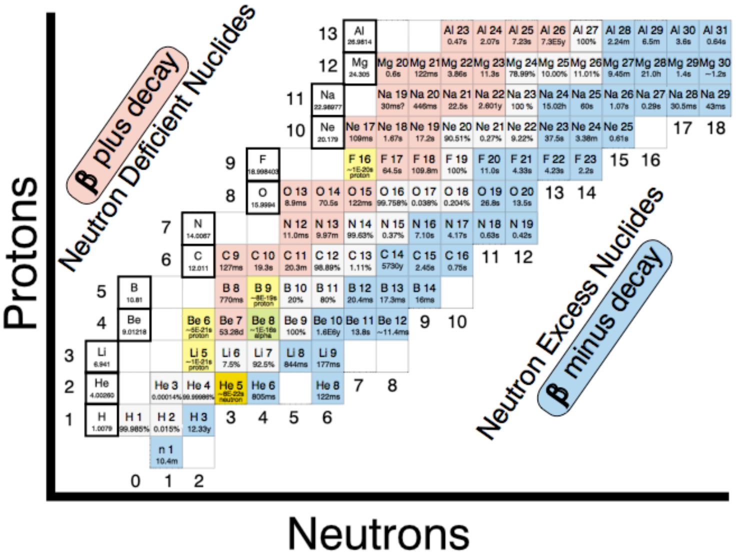
^{226}Ra decays to ^{222}Rn by emitting an alpha particle



90	224Th 0.81 S α	225Th 8.72 M α	226Th 30.57 M α	227Th 18.68 D α	228Th 1.9116 Y α	229Th 7340 Y α	230Th 7.538E+4 Y α	231Th 25.52 H β-	232Th 1.405E+10 Y 100% α
	223Ac 2.10 M α	224Ac 2.78 H ϵ	225Ac 10.0 D α	226Ac 29.37 H β-	227Ac 21.772 Y β-	228Ac 6.15 H β-	229Ac 62.7 M β-	230Ac 122 S β-	231Ac 7.5 M β-
88	222Ra 38.0 S α	223Ra 11.43 D α	224Ra 3.6319 D α	225Ra 14.9 D β-	226Ra 1600 Y α	227Ra 42.2 M β-	228Ra 5.75 Y β-	229Ra 4.0 M β-	230Ra 93 M β-
	221Fr 4.9 M α	222Fr 14.2 M β-	223Fr 22.00 M β-	224Fr 3.33 M β-	225Fr 3.95 M β-	226Fr 49 S β-	227Fr 2.47 M β-	228Fr 38 S β-	229Fr 50.2 S β-
86	220Rn 55.6 S α	221Rn 25.7 M β-	222Rn 3.8235 D α	223Rn 24.3 M β-	224Rn 107 M β-	225Rn 4.66 M β-	226Rn 7.4 M β-	227Rn 20.8 S β-	228Rn 65 S β-
	134		136		138		140		142

CHART OF THE NUCLIDES



Radioactivity

Example: Fission products from a nuclear reactor.

Another example of radioactive nuclides is the fission products that are formed during the operation of a reactor.

These form high level radioactive waste and the disposal of this waste is one of the principal issues in the widespread utilisation of nuclear power.

Note

Some fission products, such as ^{135}Xe and ^{149}Sm have very large (n,γ) cross-sections for thermal neutrons.

They act as “poisons” because they absorb neutrons and, for example, make it more difficult to start a reactor after shut down.