Radioactivity
The Radioactive Decay Chain

So far we have examined the equation governing the decay of a radioactive nuclide when the *product of the decay*, the *daughter*, is stable.

But, what happens if the daughter itself is also radioactive?

Let's suppose that $N_1(t)$ is the number of nuclei of the original radioactive nuclide (the mother) as a function of the time and that λ_1 is its decay constant.

Let's also suppose that: $N_2(t)$ is the number of nuclei of the radioactive product (the daughter) as a function of time and that λ_2 is its decay constant.

We now write down the *differential equations* governing this situation.

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For the original radioactive nuclide we have, just as before that:

$$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t)$$ **with the solution** $N_1(t) = N_0 e^{-\lambda_1 t}$ **as before.**

But in the case of the daughter we have:

$$\frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + \lambda_1 N_1(t)$$

Total *rate of change* of the number of nuclei N_2, with time.

Rate of *radioactive decay* of the daughter, (N_2).

***Rate of formation* of the daughter (from the radioactive decay of the mother).**

Make sure that you understand this – ask a question!

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How do we solve this equation?

First use $N_1(t) = N_0 e^{-\lambda_1 t}$ **to get:**

$$\frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + \lambda_1 N_0 e^{-\lambda_1 t}$$

This looks complicated but it can be solved. Let's see how.

Multiply through by $e^{\lambda_2 t}$ **and collect the terms with** N_2 **on the left hand side (lhs).**

$$e^{\lambda_2 t}\frac{dN_2(t)}{dt} + e^{\lambda_2 t}\lambda_2 N_2(t) = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

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We have:

$$e^{\lambda_2 t}\frac{dN_2(t)}{dt} + e^{\lambda_2 t}\lambda_2 N_2(t) = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} \qquad \textbf{(I)}$$

Now notice that $\frac{d\left[e^{\lambda_2 t}N_2(t)\right]}{dt} = e^{\lambda_2 t}\frac{dN_2(t)}{dt} + e^{\lambda_2 t}\lambda_2 N_2(t)$ **which is the lhs of our equation I.**

So (I) gives $\frac{d\left[e^{\lambda_2 t}N_2(t)\right]}{dt} = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$

and integrating both sides gives – $e^{\lambda_2 t}N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 e^{(\lambda_2 - \lambda_1)t} + C$ **where** C **is an integration constant.**

In order to find C **we have to know how much of nuclide 2, i.e.** N_2**, there was at time** $t = 0$**.**

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Usually (but not always) $N_2(0) = 0$

In general $e^{\lambda_2 0} N_2(0) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 e^{(\lambda_2 - \lambda_1)0} + C$

i.e. $N_2(0) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 1 + C$

so $C = N_2(0) - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0$

In the case where $N_2(0) = 0$ $C = -\frac{\lambda_1}{\lambda_2 - \lambda_1} N_0$

and we have $e^{\lambda_2 t} N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 e^{(\lambda_2 - \lambda_1)t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0$

This simplifies to give: $N_2(t) = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$

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Finally, we have: $N_2(t) = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$

The *activity* from the parent is found by multiplying $N_1(t)$ by λ_1 and the *activity* from the daughter is found by multiplying the above expression for $N_2(t)$ by λ_2 (as usual).

You should play with the above equation using a spreadsheet to see what happens in different cases.

If the half life of the parent is long in comparison with that of the daughter, then the activity of the daughter will tend to the activity of the parent. (It will approach it asymptotically.)

If the half life of the parent is short in comparison with that of the daughter, then the activity of the daughter will grow to a maximum and then decay approaching an exponential decay with its own decay constant.

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As an example of a longer lived parent with a shorter lived daughter we take the following:

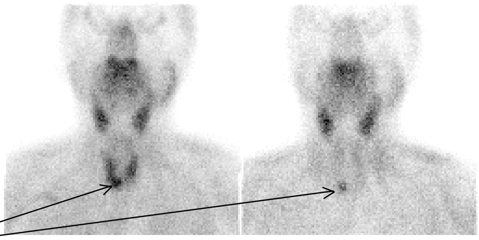
$$^{99}\text{Mo} \xrightarrow[T_{1/2}=67\,\text{hrs}]{} {}^{99m}\text{Tc} \xrightarrow[T_{1/2}=6\,\text{hrs}]{} {}^{99}\text{Tc}$$

In its decay ^{99m}Tc emits a low energy 140 keV gamma-ray which is used in nuclear medicine to image the function of the heart (myocardium), thyroid, lungs and other organs.

The 140 keV is easy to detect and image because of its relatively low energy.

In addition it has the advantage of a short half-life so that good images can be obtained without delivering a large integrated radiation dose to the patient.

Use of ^{99m}Tc in Nuclear Medicine



Tumour in parathyroid gland

Gamma-camera image of ^{99m}Tc compound absorbed in the parathyroid gland. (Image of patients head and neck at two decay times.)

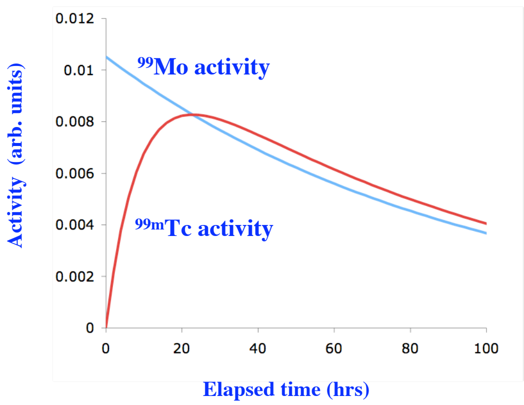
The image shows the presence of a (benign) tumour affecting the operation of this particular endocrine gland.

Use of ^{99m}Tc in Nuclear Medicine

45	97Rh 30.7 M ε	98Rh 8.72 M ε	99Rh 16.1 D ε	100Rh 20.8 H ε	101Rh 3.3 Y ε	102Rh 207 D ε	103Rh STABLE 100%	104Rh 42.3 S β-	105Rh 35.36 H β-
	96Ru STABLE 5.54%	97Ru 2.791 D ε	98Ru STABLE 1.87%	99Ru STABLE 12.76%	100Ru STABLE 12.60%	101Ru STABLE 17.06%	102Ru STABLE 31.55%	103Ru 39.26 D β-	104Ru STABLE 18.62%
43	95Tc 20.0 H ε	96Tc 4.28 D ε	97Tc 4.21E+6 Y ε	98Tc 4.2E+6 Y β-	99Tc 2.111E+5 Y β-	100Tc 15.8 S β-	101Tc 14.22 M β-	102Tc 5.28 S β-	103Tc 54.2 S β-
	94Mo STABLE 9.25%	95Mo STABLE 15.92%	96Mo STABLE 16.68%	97Mo STABLE 9.55%	98Mo STABLE 24.13%	99Mo 2.7489 D β-	100Mo 0.78E+19 Y 9.63% 2β-	101Mo 14.61 M β-	102Mo 11.3 M β-
41	93Nb STABLE 100%	94Nb 2.03E+4 Y β-	95Nb 34.991 D β-	96Nb 23.35 H β-	97Nb 72.1 M β-	98Nb 2.86 S β-	99Nb 15.0 S β-	100Nb 1.5 S β-	101Nb 7.1 S β-
	52		54		56		58		60

1. A molybdenum target in a reactor is used to make ^{99}Mo through the reaction ^{98}Mo(n,γ)^{99}Mo.
2. The longer-lived ^{99}Mo is supplied to the nuclear physician as a "cow", that is in a convenient form so that the ^{99m}Tc can be "milked off" when needed.
3. The activity of the ^{99m}Tc is high but dies off fast so that the total dose to the patient is low.
4. The ^{99}Tc which is left has a very long half-life so that the dose to the patient from this source is extremely low.

Use of ^{99m}Tc in Nuclear Medicine



Radioactivity

The radioactive decay chain – the xenon effect.

Another example is the growth of ^{135}Xe, a neutron "poison", after reactor shutdown

^{135}Xe is the daughter of ^{135}I in one chain of fission products (see later).

$T_{1/2}(^{135}I) = 6.7$ h

$T_{1/2}(^{135}Xe) = 9.2$ h

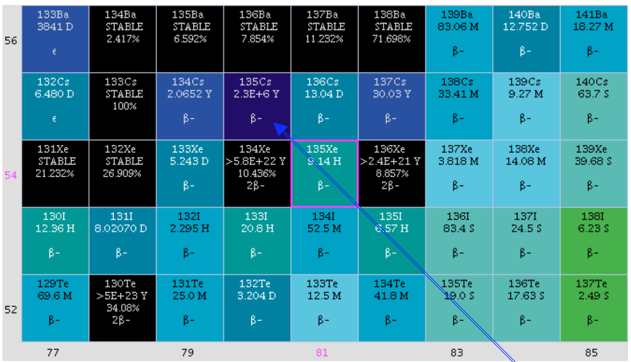
The neutron capture reaction $^{135}Xe + n \Rightarrow {}^{136}Xe + \gamma$ has a very high cross-section. (This is why it is a poison).

$$\sigma[^{135}Xe(n,\gamma)] = 2.636 \times 10^{6} \text{ b (ENDF B/6)}$$

$$\text{While } \sigma[^{135}I(n,\gamma)] = \text{about } 0.02 \text{ b}$$

Radioactivity

The radioactive decay chain – the xenon effect.



Fission product decay chain (one of many)

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Because of its high cross-section the ^{135}Xe is burned up during the reactor operation (It is transformed by the flux)

While the reactor is operating the ^{135}Xe is at a relatively low level

But what happens once the reactor stops?

For argument's sake let's suppose that the level of ^{135}Xe is 1/10th of the ^{135}I during the steady state operation of a reactor.

(This is a purely arbitrary number)

The iodine starts to decay into xenon and the amount can be predicted from our equation.

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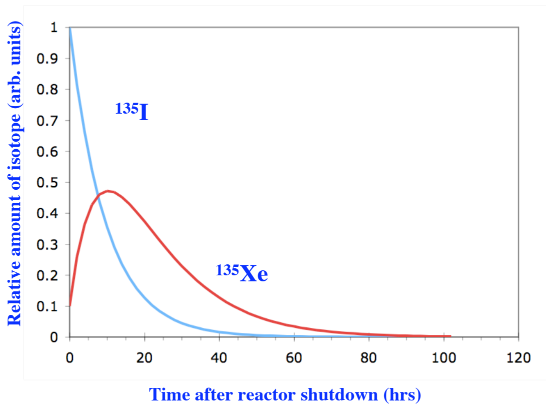
It will look like this:



The xenon will grow and after 5 hours there will be almost five times as much as during normal operation – the reactor will be difficult to start

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Exercise 24

1. In a hypothetical reactor the ^{135}Xe level is 10% of the ^{135}I level during normal operation.

   The reactor trips.

   In this hypothetical reactor it cannot be restarted if the ^{135}Xe level is more than twice the equilibrium level during normal operation.

   How long will the reactor be off before it can start putting power back into the national grid?

   Give two answers.