Binding energy

In the special theory of relativity, Einstein showed that energy and mass are equivalent.

If a body has a mass *m*, then it contains an amount of energy:

$$E = mc^2$$

probably the most famous equation in physics.

This means that *if* the mass of a nucleus is *less* than the mass of its constituents, then those constituents are in a *lower energy state* when they are bound together inside the nucleus.

This difference in mass, expressed as energy (normally MeV), is the *binding energy* of those constituents inside the nucleus.

Binding energy

For example consider the carbon nucleus, ${}^{12}_{6}\mathrm{C}$ shown in the previous table.

Let's calculate its binding energy.

It contains 6 protons and 6 neutrons so *A* = 12.

From the table the mass of a carbon *atom* is 12u (by definition of the *atomic mass unit*, u).

This is the mass of the carbon atom, i.e. the mass of the nucleus *plus the six electrons in the neutral carbon atom.*

From the table, the mass of the ${}^{12}_{6}\mathrm{C}$ *nucleus* is therefore:

$$12 - 6 \times 5.486 \times 10^{-4} = 11.9967 \text{ u}$$

Binding energy

The binding energy of $^{12}_{6}\mathrm{C}$ is:

$$6 \times 1.007276 + 6 \times 1.008665 - 11.9967 = 0.0989\mathrm{u}$$

$$\times 931.494\,\mathrm{MeV}/c^2$$

Mass of a proton

Mass of a neutron

Mass of ^{12}C nucleus

This is 92.2 MeV in total

Normally we express this as the binding energy per nucleon.

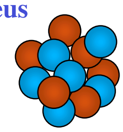
In this case it is 7.68 MeV per nucleon.

Let's look at this again …

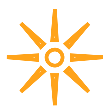
Start with 6 protons and 6 neutrons far away from each other –

end up with a C-12 nucleus + 92.2 MeV.

nucleus



$$^{12}_{6}\mathrm{C} + 92.2\ \mathrm{MeV}$$



energy

Binding Energy

It is found that the mass of a nucleus is always *less than* the sum of the masses of its constituent neutrons and protons (nucleons).

What is the reason for this? Well Einstein showed that mass and energy are equivalent. The lower mass shows that the nucleons in the nucleus are in a *lower energy state* than if they were all separate, isolated particles.

It means that there must be some force between the nucleons that binds them together in the nucleus. This is the *strong force*.

This decrease in mass (known as the mass decrement) gives the *binding energy* of the particular nucleus in terms of the equivalent *mass*.

Working in terms of the actual binding energy, we calculate as follows.

Say for example if we have a nucleus with Z protons and N neutrons and mass M_A, where $A = Z + N$ then its binding energy in MeV is given by:

$E_b(MeV) = (Zm_p + Nm_n - M_A)$ x 931.494 MeV/u

All in atomic mass units, u | or *all* in nuclide masses, u

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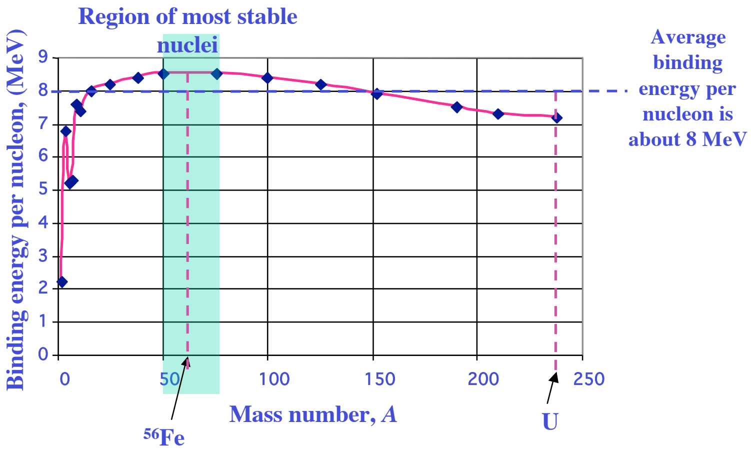
Binding Energy and Nuclear Forces

Binding Energy per Nucleon



The binding energy per nucleon as a function of the mass number.

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Energy from fission and fusion

We can see from this diagram that the most stable form of matter (nuclear matter) is ^{56}Fe.

In fact the "white dwarf" star that is left at the end of the main sequence of stellar evolution is essentially iron.

It is clear from this diagram that energy can be obtained from heavier nuclei (such as uranium) by the process of *fission* – i.e. when the heavy nucleus is divided up into two lighter nuclei.

It is also clear that energy can be obtained from the lighter nuclei by the process of *fusion* – i.e. when two light nuclei fuse two form a heavier nucleus.

Exercise 17

1. A deuteron consists of a neutron and a proton bound together. Its mass is found to be 2.013553 u.

   Calculate its binding energy in MeV.

2. A ^{226}Ra nucleus undergoes α particle decay (into ^{222}Rn). Given that the atomic mass of ^{226}Ra is 226.025406 u, that of ^{222}Rn is 222.017574 u and that of ^{4}He is 4.002603 u, calculate the energy of the α particle that is emitted. The recoil energy of the ^{222}Rn is negligible.