Elastic scattering

Now we have worked out how much energy is lost when a neutron is scattered through an angle, *θ*.

We would like to know how much energy, on average, is lost per collision.

In order to do that we need to know the *probability* that a neutron will be scattered through a particular angle *θ* in a particular collision.

This probability is expressed in terms of the differential scattering cross-section σ(*θ*).

The probability that a neutron will be scattered through angles between *θ* and *θ* + *dθ* is then σ(*θ*)*dθ*, by definition.

Elastic scattering

We won't derive this but it turns out that the average energy lost by the scattered neutrons in the case of light nuclei is given by:

$$\Delta\overline{E} = \frac{1}{2}\left[1-\left(\frac{A-1}{A+1}\right)^2\right]E$$

The average *fractional* energy loss is therefore:

$$\frac{\Delta\overline{E}}{E} = \frac{1}{2}\left[1-\left(\frac{A-1}{A+1}\right)^2\right]$$

The average *fractional* energy loss decreases from 1/2 in the case of hydrogen to almost zero for the heavier nuclei.

Slowing down neutrons.

We have seen that one of the important phenomena that one has to understand is the *slowing down* of neutrons in the reactor.

In a thermal reactor most of the slowing down takes place in the moderator by elastic scattering.

At higher energies some of it however does take place by *inelastic scattering* in heavier elements.

Slowing down neutrons.

In reactor engineering the concept of *lethargy* is often used instead of energy.

As we have seen in a simple treatment the average fractional energy loss is a constant for a particular material:

$$\frac{\Delta \overline{E}}{E} = \frac{1}{2}\left[1 - \left(\frac{A-1}{A+1}\right)^2\right]$$

If we define a variable *u* so that $du = -\frac{\Delta \overline{E}}{E}$

Then *du* will also remain constant under those conditions.

Integrating we get: $u = \ln\left(\frac{E_0}{E}\right)$

u* is called the *lethargy

Lethargy because *lethargy* is the opposite of *energy*.
As the *energy* decreases, the *lethargy* increases.

Slowing down neutrons.

In reactor engineering the concept of *lethargy* is often used instead of energy.

The average change in the lethargy when a neutron is scattered by a nucleus of mass A is given by the following expression:

$$\overline{\Delta u} = \xi = 1 - \frac{(A-1)^2}{2A}\ln\left(\frac{A+1}{A-1}\right)$$

Where ξ is a characteristic of the scatterer, called the logarithmic energy decrement or the logarithmic energy loss.

The average number of collisions needed for a neutron with an initial energy E_0 to be slowed down to a second energy, E_s, say is given by:

$$\overline{\langle \text{no. of collisions} \rangle} \approx \frac{\ln\left(\frac{E_0}{E_s}\right)}{\xi}$$

Slowing down neutrons.

There is one more parameter commonly used to characterise moderating materials.

This is called the *moderating ratio*. It takes into account the parameter, ξ together with the relative effectiveness of the material for scattering relative to absorption

This in turn is done in terms of the ratio of the macroscopic scattering cross-section to the macroscopic absorption cross-section.

$$\text{moderating ratio} = \xi\frac{\Sigma_s}{\Sigma_a}$$

Properties of Different Moderators.

The following table gives the properties of different moderators used in nuclear reactors.

The number of collisions is given for slowing down from 2 MeV to 1 eV

Moderator	ξ	Number of collisions	$\xi\Sigma_s/\Sigma_a$
H	1.0	14	
D	0.725	20	–
H_2O	0.920	16	71
D_2O	0.509	29	5670
He	0.425	43	83
Be	0.209	69	143
C	0.158	91	192
Na	0.084	171	1134
Fe	0.035	411	35
^{238}U	0.008	1730	0.0092

Slowing down neutrons.

So, the strategy in a thermal reactor is to slow down as many neutrons as possible to thermal energies so as to maximise the number of thermal-neutron induced fissions.

A number of things can happen on the way.

Resonance Capture

One example is *resonance capture.*

The neutron cross-section (excitation function) shows very strong peaks (and valleys).

These are called resonances because at particular values the energy of the incoming neutron *resonates* with levels in the compound nucleus.

In ^{235}U some of these resonances lead to fission which is fine, but some of them lead to radiative capture. As you know, this means that the neutron is absorbed producing ^{236}U and is no longer available to be slowed down and produce fission.