The Liquid Drop Model

A simple model for the binding energy of a nucleus

The asymmetry energy

The term that describes this behaviour in the binding energy formula is known as the a*symmetry energy* term and it turns out to be:

$$-C_4 \frac{\left(Z - \frac{A}{2}\right)^2}{A}$$

It measures the energy produced by the asymmetry (the difference in the number of neutrons and protons in the nucleus) .

There is one more term, the *pairing energy*.

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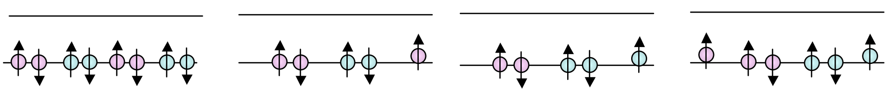
The Liquid Drop Model

A simple model for the binding energy of a nucleus

The pairing energy

So far we have only considered the average effect of the *nucleus* on an individual *nucleon*.

Nucleons also interact directly with each other, for example nucleons of the same type, but with opposite spin, pair to form a more stable grouping.



So even-even nuclei are the most stable

Even-odd (i.e. A_{odd}) nuclei are less stable

Odd-odd nuclei tend to be the least stable - other factors being equal.

Nuclei with even numbers of both neutrons and protons.

This term is written as:

$$\pm\delta\,(A_{even})$$

Nuclei with odd numbers of both neutrons and protons.

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The Liquid Drop Model

A simple model for the binding energy of a nucleus

Final form for the binding energy according to the charged liquid-drop model.

$$E_b = C_1 A - C_2 A^{2/3} - C_3 \frac{Z^2}{A^{1/3}} - C_4 \frac{\left(Z - \frac{A}{2}\right)^2}{A} \pm \delta \left(A_{\text{even}} \right)$$

Volume

Surface

Coulomb

Asymmetry

Pairing

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The Liquid Drop Model

A simple model for the binding energy of a nucleus

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Evans, The Atomic Nucleus gives the following values for these constants:

C_1 = 14 MeV **C_2 = 13 MeV**

C_3 = 0.6 MeV **C_4 = 4.8 MeV**

$$\delta = \frac{33.5}{A^{3/4}} \text{ MeV}$$

Exercise. **Use these values to calculate the binding energies for ^{12}C, ^{56}Fe, ^{140}Ba, ^{235}U and ^{238}U and compare these values with the experimental values (from your atomic mass tables).**

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The Liquid Drop Model

A simple model for the binding energy of a nucleus

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The Liquid Drop Model and Fission

Say that a nucleus were to split into two. How much energy would be released?

We can use the expression for the binding energy given by the liquid drop model to calculate this.

Let us suppose that a nucleus (Z,A) splits into two (Z_1,A_1) and (Z_2,A_2)

Clearly $Z = Z_1 + Z_2$ and $A = A_1 + A_2$

The energy released, E_R, is the difference between the final and the initial binding energies.

Here we ignore the asymmetry energy (or assume $Z = A/2$).

We also neglect the pairing energy.

$$E_R = C_1 (A_1 + A_2 - A)$$
$$- C_2 (A_1^{2/3} + A_2^{2/3} - A^{2/3})$$
$$- C_3 \left(\frac{Z_1^2}{A_1^{1/3}} + \frac{Z_2^2}{A_2^{1/3}} - \frac{Z^2}{A^{1/3}} \right)$$

Volume energies - cancel

Surface energies - different

Coulomb energies - different

The Liquid Drop Model and Fission

So the energy released in fission is given by:

$$E_R = -C_2\left(A_1^{2/3} + A_2^{2/3} - A^{2/3}\right) - C_3\left(\frac{Z_1^{\,2}}{A_1^{1/3}} + \frac{Z_2^{\,2}}{A_2^{1/3}} - \frac{Z^2}{A^{1/3}}\right)$$

This turns out to be a competition between the surface and the coulomb energies

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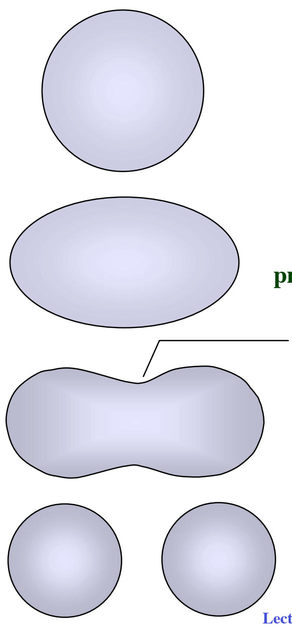
The Liquid Drop Model and Fission



Sphere — **Least surface energy. Greatest Coulomb energy**

Oval - prolate spheroid — **More surface energy Less Coulomb energy**

Scission

Coulomb energy wins over surface energy

Two nuclei - fission

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The Liquid Drop Model and Fission

$$E_R = -C_2(A_1^{2/3} + A_2^{2/3} - A^{2/3}) - C_3\left(\frac{Z_1^2}{A_1^{1/3}} + \frac{Z_2^2}{A_2^{1/3}} - \frac{Z^2}{A^{1/3}}\right)$$

We don't have enough time to do this in detail in this course, but this equation can be treated mathematically to show a number of very interesting results.

For example fission becomes energetically possible as E_R changes from negative to positive values with increasing A. In other words when $E_R = 0$.

If it is assumed that $A_1 = A_2 = A/2$, then this happens when: $\frac{Z^2}{A} \approx 0.7\frac{C_2}{C_3}$

However fission doesn't happen at this value, although it becomes energetically possible, because of the so-called "Coulomb Barrier".

127

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The Liquid Drop Model and Fission

As A increases, fission becomes energetically possible when: $\frac{Z^2}{A} \approx 0.7\frac{C_2}{C_3}$

Working this out, substituting values for C_1 and C_2 and setting $Z \sim A/2$, shows that nuclei heavier than $A \sim 72$ are unstable against fission

But this doesn't actually happen.

Nuclei only start to fission spontaneously when A reaches ~240

Why?

To understand this we need the concept of the Coulomb barrier again.

128

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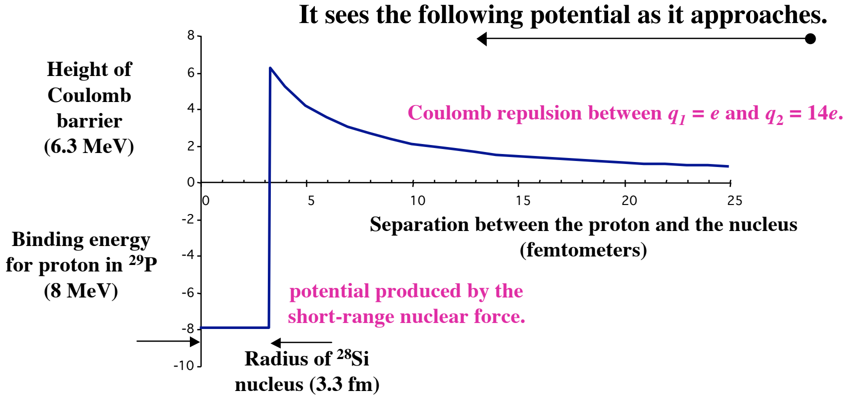
The Liquid Drop Model and Fission

The Coulomb barrier

Say that a proton approaches a ^{28}Si nucleus and forms ^{29}P.

It sees the following potential as it approaches.



The Liquid Drop Model and Fission

Fission and the Coulomb barrier

A nucleus will only fission *spontaneously* if its separation energy is near the top of the Coulomb barrier.

It can be calculated that this happens when:

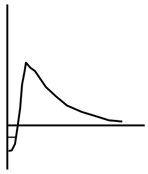
$$\frac{Z^2}{A} \approx 2.1 \frac{C_2}{C_3}$$

The Liquid Drop Model and Fission

Fission and the Coulomb barrier

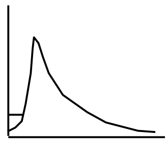
$$\frac{Z^2}{A} << 0.7\frac{C_2}{C_3}$$

Stable against fission



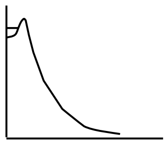
$$\frac{Z^2}{A} >\approx 0.7\frac{C_2}{C_3}$$

Unstable against fission but cannot penetrate the Coulomb barrier



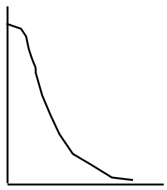
$$\frac{Z^2}{A} \approx 2.1\frac{C_2}{C_3}$$

Unstable against fission - close to the top of the Coulomb barrier



$$\frac{Z^2}{A} > 2.1\frac{C_2}{C_3}$$

Fissions spontaneously



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The Liquid Drop Model and Fission

Fission and the Coulomb barrier

Exercise 26

1. **Examine the possible fission of tin into manganese. Do this by considering the fission of 112Sn (Z = 50) into two symmetric fractions 56Mn (Z = 25).**

   The height of the Coulomb barrier is 107MeV, while the separation energy is 24 MeV

   Draw a sketch of the energy of these two fission fragments as a function of their separation. Hence explain why ^{112}Sn does not undergo spontaneous fission.

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Fission

All nuclear reactors depend on fission.

This is the physical process that takes place when, for example, a ^{235}U nucleus absorbs a thermal neutron and then splits into two parts, as we have seen.

Today this process seems obvious.

But as human beings explored the world of the nucleus just before the Second World War, it was far from being obvious.

In the next section we will review the history of how this discovery happened.

133

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