Review - the Particle Flux

The number of incident particles passing the nucleus per unit area per unit time is the particle flux, ϕ.

The units of flux, ϕ are $m^{-2}.s^{-1}$ (in other words particles per unit area per unit time).

The *reaction rate* can be calculated directly from these two quantities, the cross-section and the particle flux.

Review - the reaction cross-section and the reaction rate

Remember that the probability of a particular reaction occurring in a flux of incident particles is given by its cross-section, σ.

The dimension of the cross-section, σ, is area [L^2].

The nucleus presents an area equivalent to σ to the incoming flux of particles.

The units of the cross-section are either m^2 or barns (b).

$$1 \text{ b} = 10^{-28} \text{ m}^2$$

Remember that the number of interactions per second that take place when the flux interacts with a volume *V of material is*

$$\psi = Vn\sigma\phi$$

Reaction rate (ψ)

Volume of material exposed to the beam (V)

Number of nuclei per unit volume (density of nuclei) (n)

Cross-section (microscopic cross-section) (σ)

Particle flux (neutron flux) (ϕ)

The macroscopic cross-section

Another concept important in both reactor physics and the interaction of radiation with matter is the *macroscopic cross-section*.

$$\Sigma = n\sigma$$

Number of nuclei per unit volume

The units of Σ are m^{-1} ($m^{-3}.m^2$) and this represents the probability of an interaction per unit path length.

So the number of interactions per unit volume (per unit time) in the target is also given by:

$$\psi = n\sigma\phi$$
$$= \Sigma\phi$$

Reaction Rates again

In the case where the incident flux of particles is *mono-energetic*, the rate at which reactions occur is given by:

$$\psi = \sigma\phi$$ ***per target nucleus.***

This is the reaction rate *per target nucleus* in this case.

Remember that the units of the cross-section, σ, are m^2 while the units of flux, ϕ are $m^{-2}.s^{-1}$ (in other words particles per unit area per unit time).

Thus Ψ has the units: s^{-1}. It is the probability per unit time that a nucleus in the given flux will undergo that particular reaction.

If you have a total number *N* of nuclei in the particle flux, clearly the number of reactions that will occur per unit time is just:

$$\psi = N\sigma\phi$$

Reaction Rates

Exercise 33

1 If N nuclei are in a flux of ϕ neutrons. $m^{-2}s^{-1}$ and the cross-section for the reaction is σ, then the reaction rate is: $\psi = N\sigma\phi$.

Given the above equation derive an expression for the reaction rate per kilogram of a substance containing a concentration c of an element where the isotopic abundance of the target nucleus is f and its mass number is A.

Given that the thermal neutron cross-section for gold is 93 barns and its density is 19.7 kg.m-3, what is the macroscopic cross section for gold?

Neutron Flux *Distribution*

In most charged particle reactions (where the incident particle is a proton or a deuteron or an α particle, say) the incident particles are nearly mono-energetic.

This is not generally true in neutron-induced reactions.

If we plot the intensity of the particle flux as a function of the energy we get a *flux distribution*.

Reaction Rates in a neutron flux distribution

What happens when the particle flux is not mono-energetic, i.e when we have a neutron flux *distribution*?

Say that we have a neutron flux distribution $f(E)$.
The number of neutrons per unit area per unit time with energies between E and $E + dE$ is then $f(E)dE$.

The units of $f(E)$ are $m^{-2}.s^{-1}.[energy]^{-1}$
Often we talk about neutrons per square meter per second per MeV or per eV.

Clearly the reaction rate per target nucleus for particles (neutrons) with energies between E and $E + dE$ is:

$$d\psi = \sigma(E) f(E) dE$$

The total reaction rate per target nucleus in the flux distribution is:

$$\psi = \int \sigma(E) f(E) dE$$

Reaction rates in a neutron flux distribution

The total reaction rate per target nucleus in the flux distribution is:

$$\psi = \int \sigma(E) f(E) dE$$

This is the *convolution* of the excitation function and the flux distribution

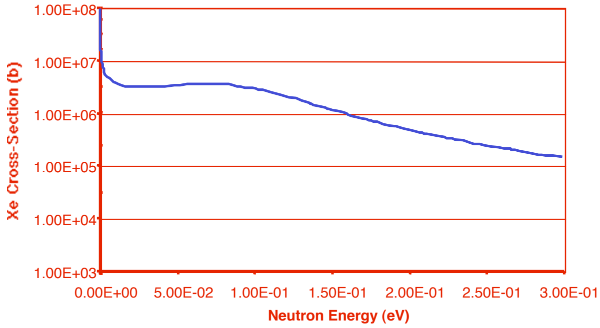
In other words this integral of the product $\sigma(E) f(E)$ is called the *convolution* of $\sigma(E)$ and $f(E)$.

Cross-sections and Excitation Functions

As you have learned the probability of a reaction occurring is characterised by its *cross-section*.

This probability depends on the energy of the incident particle.

When the cross section is plotted as a function of the energy of the incident particle it is known as the *excitation function*.



For example above we have the excitation function of the reaction $^{135}Xe(n,\gamma)^{136}Xe$ shown on a semi-log plot. The cross-section is plotted as a function of the energy of the incident neutron.