The Fission Spectrum

When a heavy nucleus (for example $^{236}_{92}$U, produced by $^{235}_{92}$U + n) fissions, a great deal of energy is released (something like 200 MeV).

This energy is mainly divided between the kinetic energy of the fission products and the fission neutrons (refer to a previous slide). As a result, the fission neutrons are energetic — they are fast neutrons.

(Remember that neutrons with energies in the MeV range are known as “fast” neutrons.)

These neutrons can be thought of as “evaporating” from the surface of the highly excited compound nucleus during the fission process.

The energy distribution of these fission spectrum neutrons is often represented by the so-called Watt Distribution.

The Fission Spectrum

Neutrons evaporating during fission.

Various fission products

$^{236}_{92}$U compound nucleus

Fission

Fission neutrons

These fast neutrons have different energies but with a definite spectrum, an energy distribution.

The energy distribution of these fission spectrum neutrons is often represented by the so-called Watt Distribution.
The Watt Distribution (Spectrum)

The distribution of the number of neutrons with energy in the fission spectrum is well represented by a mathematical function, the Watt function. The probability of a neutron from fission having an energy between \( E \) and \( E+dE \) is the function \( P(E)dE \).

This distribution is for neutrons from the fission of \(^{235}\text{U}\) with a slow neutron and is shown in the figure. Note: It only changes slightly for other types of fission.

\[
P(E) = 0.4865 \sinh(2E) e^{-E} \text{ MeV}^{-1}
\]

This is an empirical formula – it is just a convenient way of expressing the experimental distribution.

A better insight can be gained into what happens in a thermal reactor by plotting this using a log scale on the \( x \) axis.

Linear plot

Same data but a log plot on the \( x \) axis – showing low energy detail.

\[
P(E) = 0.4865 \sinh(\sqrt{2E}) e^{-E}
\]

Note that very few neutrons are in the eV range.

Next we turn this into a flux distribution …
The Watt neutron flux spectrum

The neutron flux is, of course, just the neutron density multiplied by the speed.

Why do we need to look at the flux distribution (flux spectrum)?

Well, remember that the reaction rate is calculated as the flux multiplied by the cross section.

\[ \Psi = \Phi \sigma \]

Or the convolution of the flux distribution and the excitation function.

\[ \Psi = \int \Phi(E)\sigma(E)dE \]

\[
\Phi(E) = N(E)v \propto P(E)v \quad \text{and} \quad E = \frac{1}{2}m_nv^2 \quad \text{so} \quad v = \sqrt{\frac{2E}{m_n}}
\]

therefore

\[ \Phi(E) = 0.4865 \sqrt{\frac{2 \times (3 \times 10^4)^3}{939.6}} \sqrt{E} \sinh \left( \sqrt{2E} \right) e^{-E} \]

for a Watt distribution.

As you can see, when the fission neutrons are formed, there are hardly any “slow” neutrons.
The Watt neutron flux spectrum

To repeat ... neutron flux from a Watt distribution.

\[ \Phi(E) = 0.4865 \sqrt{\frac{2 \times (3 \times 10^8)^2}{939.6}} \sqrt{E} \sinh(\sqrt{2E}) e^{-E}. \]

A good representation of the raw flux resulting from fission.

Comparison between the fission spectrum and the cross-section

Let’s remind ourselves what the excitation function (the cross-section as a function of energy) for \(^{235}\text{U}(n,f)\) looks like (on a linear-log plot).
Comparison between the fission spectrum and the cross-section

Here is the same thing, the excitation function (the cross-section as a function of energy) for $^{235}\text{U}(n,f)$ but on a log-log plot.

Fission Spectrum

Now, let’s compare this distribution with the cross section for the fission of uranium-235, $^{235}\text{U}(n,f)$ on a log-log plot.

Note that this is a log-log plot to show all the detail.

The next slide shows the same data on a linear log plot.
Fission Spectrum

Let’s compare this distribution with the cross section for the fission of uranium-235, $^{235}\text{U}(n,f)$.

Here you can see that there is very little overlap between the Watt spectrum and the large values of the $^{235}\text{U}(n,f)$ cross-section at low energies.