Cosmology

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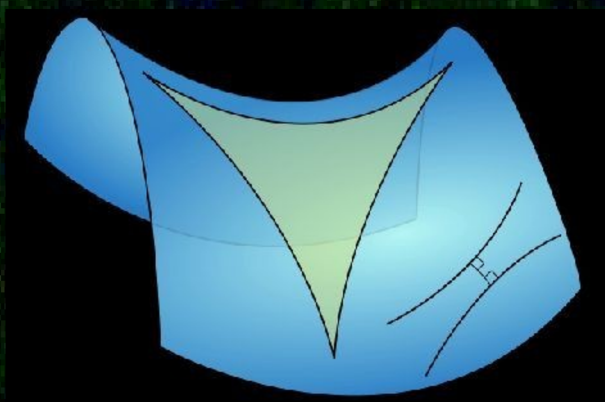
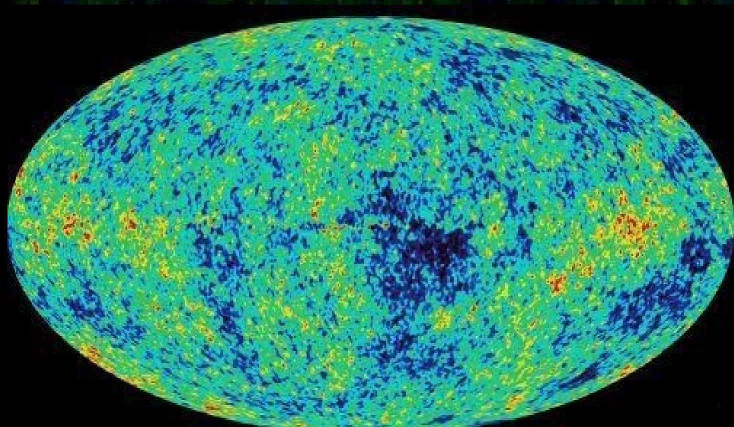
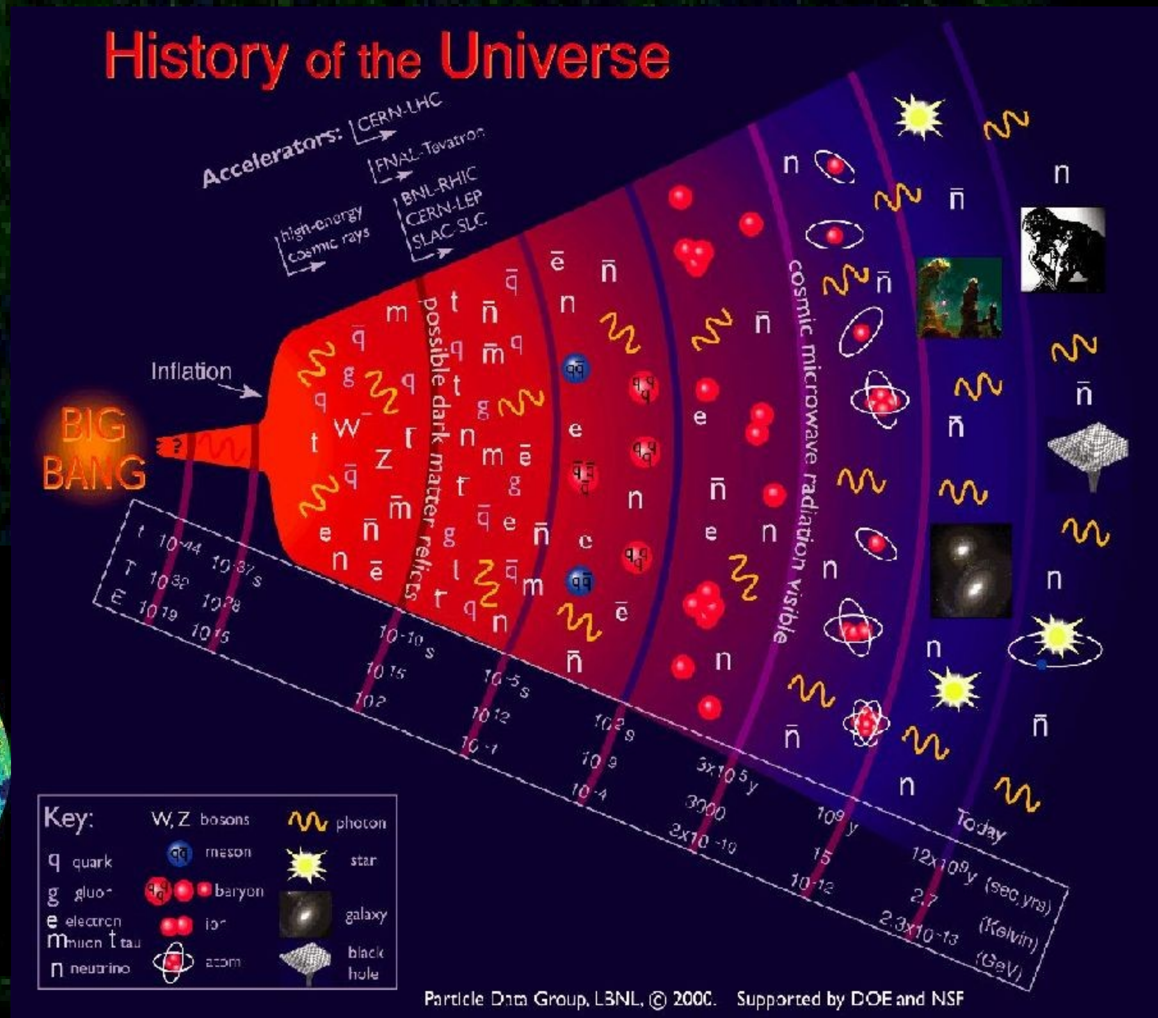
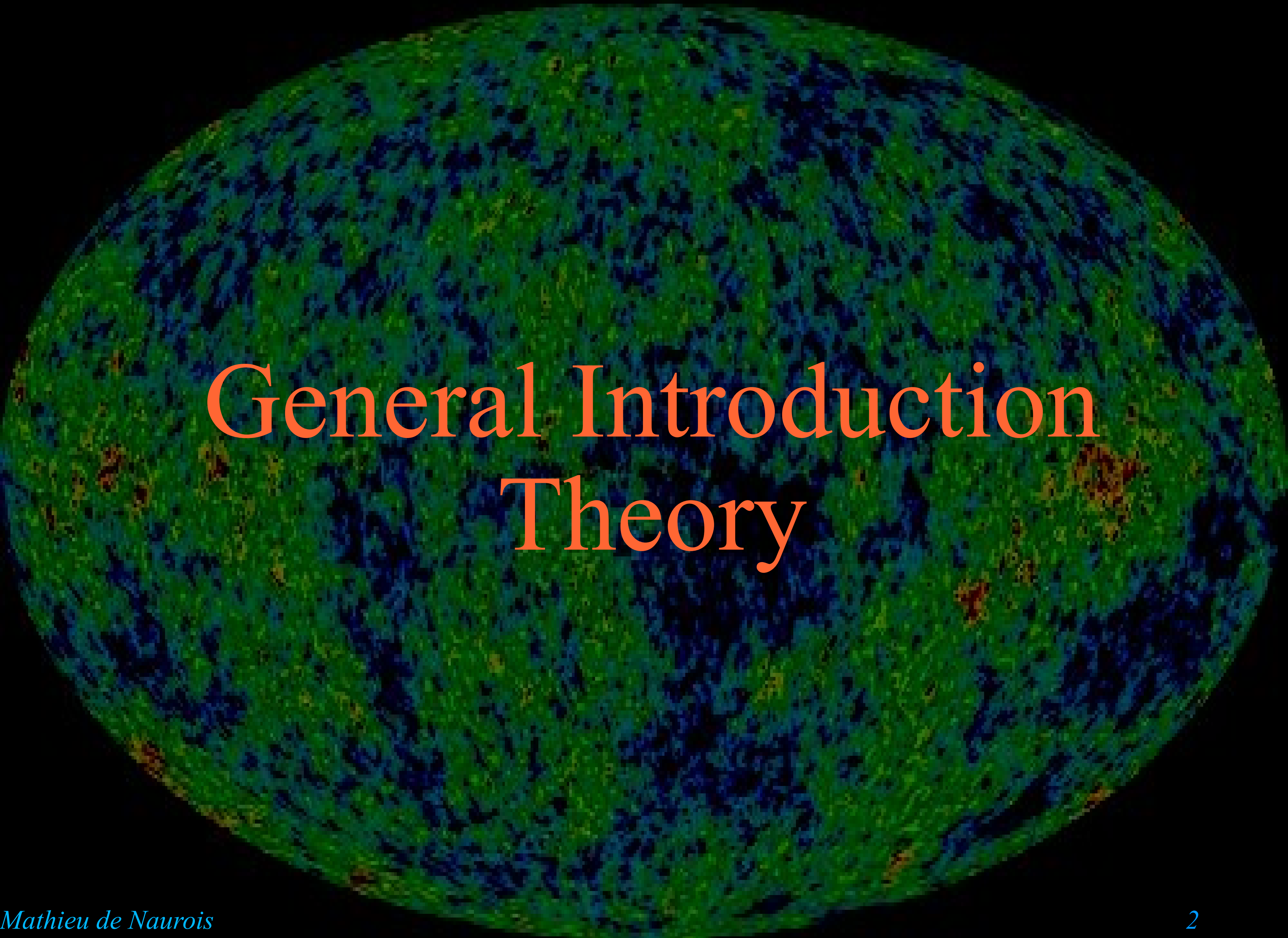








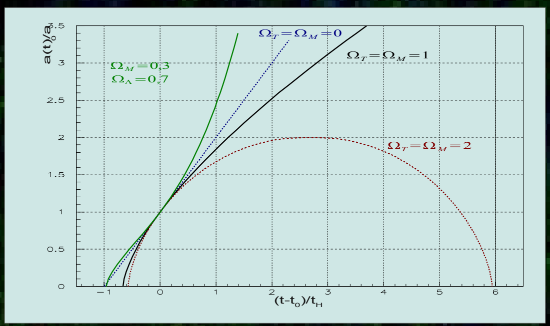
General Introduction
Theory

What is Cosmology?

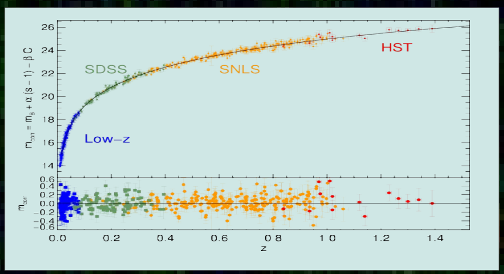
- Fundamental questions about the origin and destiny of the Universe:
  - What is the Universe made up of?
  - How did the matter and structures form in the Universe
  - Why is the Universe as we see it?
  - What is our place in the Universe?
  - Did the Universe always exists, and if not, what is its age
- Questions that appear in all cultures/religions

Open questions, observables

- ❑ Evolution of the Universe

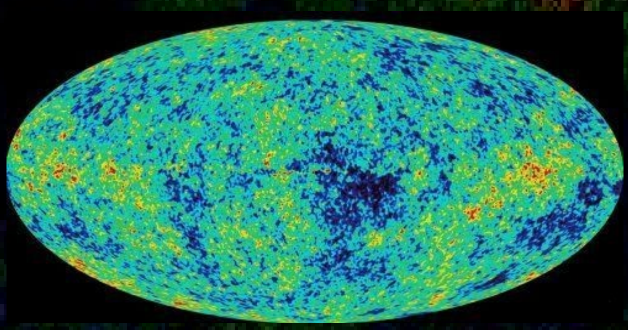


- ❑ Supernova 1a: distance versus recession velocity



- ❑ Formation of structures

- ❑ CMB



- ❑ Big bang Nucleosynthesis
- ❑ Abundances of light elements

Evolution of a matter Universe

- Radial force due to inner matter (Gauss theorem)
- Evolution of a "bubble":

$$\frac{d^2 R}{d t^2} = \frac{-G M(R)}{R^2}$$

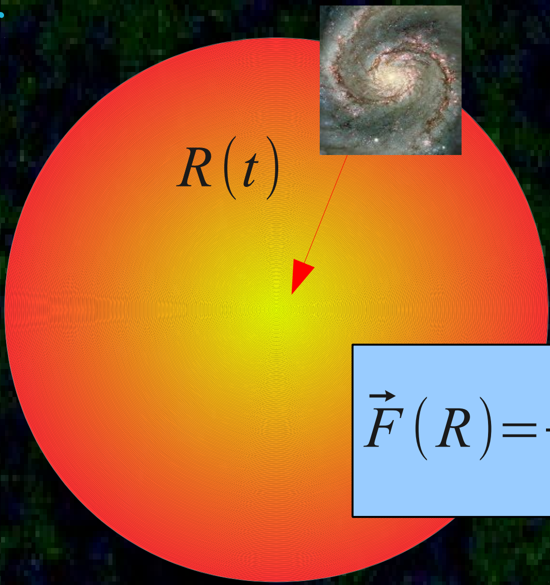
- Matter Universe:

$$M(R) = \frac{4}{3} \rho_m(t) R^3 = C_{\text{ste}}$$

- Evolution Equation

$$\left(\frac{\ddot{R}}{R}\right) = -\frac{4\pi}{3} \rho_m G \quad \Rightarrow \quad \dot{R}\ddot{R} = -\frac{4\pi}{3} (\rho_m R^3) G \frac{\dot{R}}{R^2}$$

$$\Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} (\rho_m R^3) \frac{G}{R^3} + \frac{C}{R^2}$$



$$\vec{F}(R) = \frac{-G M(R) m}{R^2} \vec{u}_R$$

Evolution of a matter Universe

- Evolution Equations:

Acceleration

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}(\rho_m R^3)\frac{G}{R^3} + \frac{C}{R^2} \quad \text{and} \quad \ddot{R} < 0$$

$$q = \left(\frac{\ddot{R}}{R}\right) = \frac{-8\pi G \rho_m}{3} = -H^2 \frac{\Omega_m}{2}$$

- Can be written as function of current values

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G \rho_m}{3} = H_0^2 \frac{R_0^2}{R^2}(1-\Omega_m), \quad \Omega_m = \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3 H_0^2}$$

- Solutions depend on the value of Ω_m, expansion of the Universe is decelerated by matter content

NO static Universe is possible

Evolution of a matter Universe

- Ω_m=0, monotonic expansion

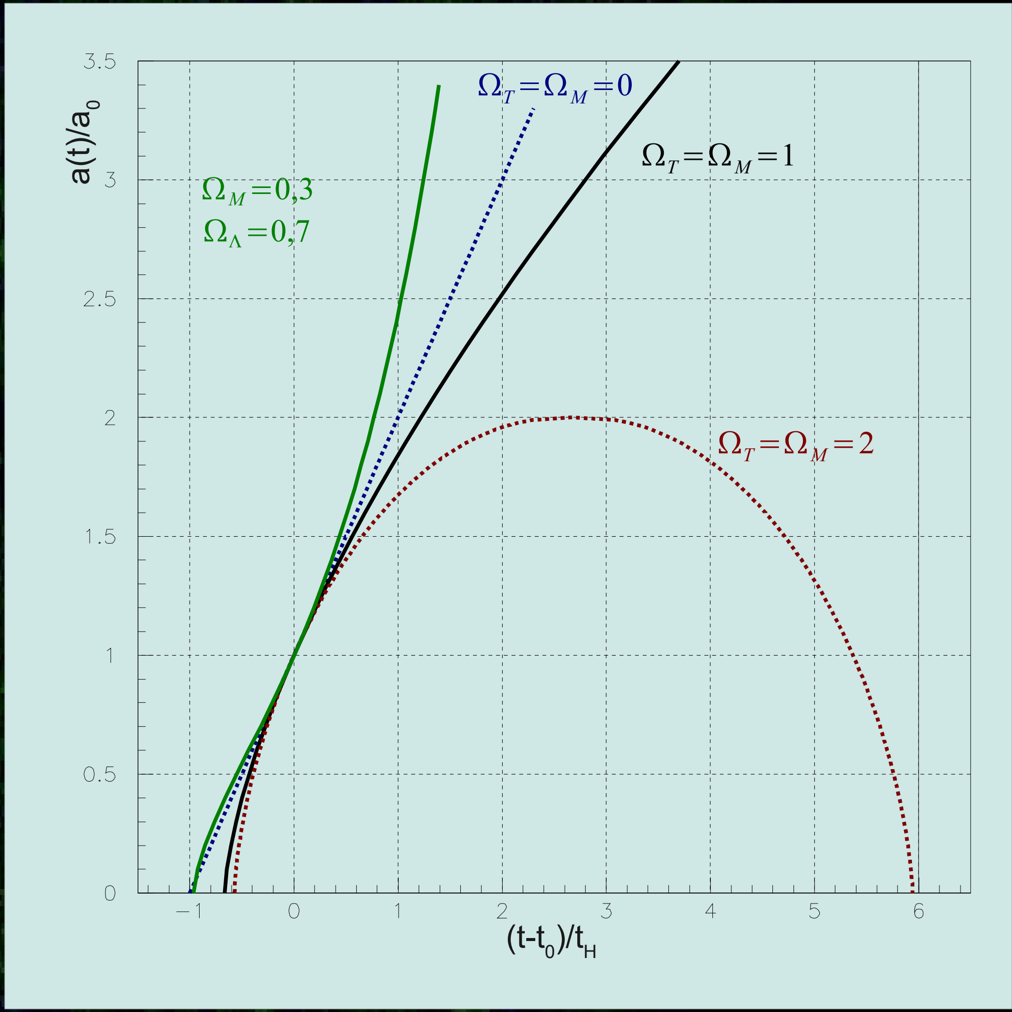
$$R(t)=R_0 H_0 \times t$$

- Ω_m=1 (critical Universe) Decelerating expansion

$$R(t)=R_0\left(\frac{3}{2}H_0\times t\right)^{2/3}$$

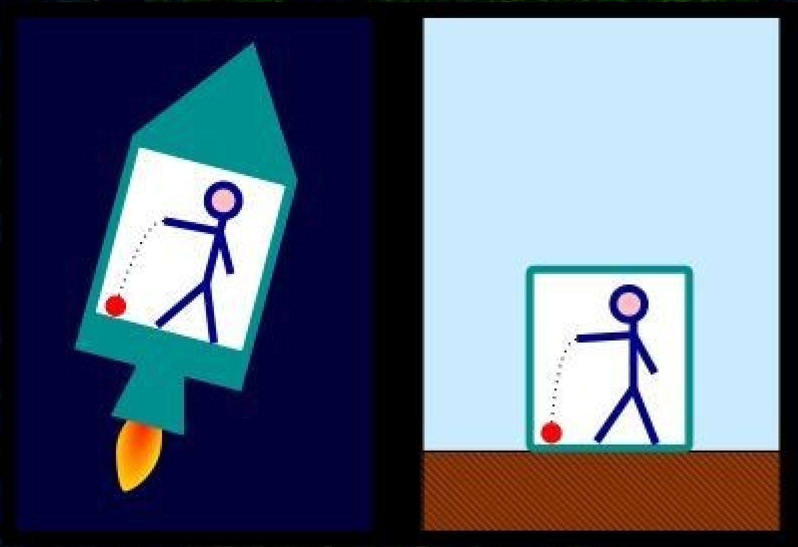
- Ω_m>1 (critical Universe) Collapsing Universe

$$R_{max}=R_0\frac{\Omega}{(\Omega-1)}$$



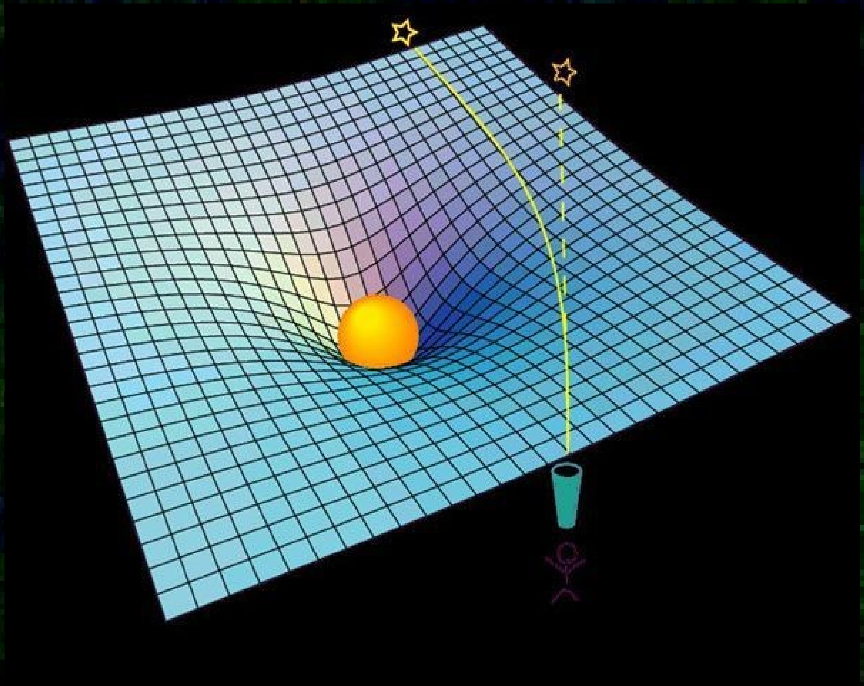
Equivalence Principle (A. Einstein)

- No difference could be found between inertial mass (in acceleration) and gravitation mass (in gravity forces)
- Implies that acceleration of a body in a gravitational field is independent of the nature of the body
- Thus there is no way to distinguish between a free-fall movement in gravity field from a accelerated movement in absence of field
- Implies that Gravity can be understood as a property of space and not of the falling body



General Relativity

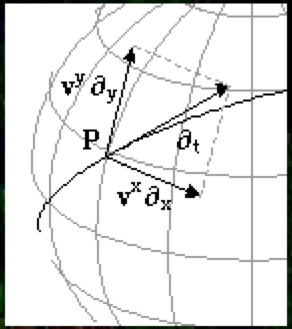
- Newtonian Gravity: Universe is flat, trajectories are curved due to a force (non-inertial movement)
- General relativity: Gravity is a geometric property of space, not a force. Trajectories are always inertial (geodesics) in a curved space
- Major conclusion: massless particles (light) are also affected, confirmed by measure of deflection of stars (Eddington, 1919)



Evolving Universe – Tensor Algebra

- We consider a space time, in which we have a base of vectors $\{\vec{e}_\mu\}$
- The metric is defined by the cross-product of vectors:

$$g_{\mu\nu} = \vec{e}_\mu \cdot \vec{e}_\nu$$



- Any vector can be decomposed on the base: $\vec{x} = x^\mu \vec{e}_\mu$

Covariant coordinates

- Several bases can describe the same Universe, transformation given by

$$d x^\mu = \frac{\partial x^\mu}{\partial y^\nu} d y^\nu = \Lambda^\mu{}_\nu d y^\nu, \quad \vec{e}_\mu = \Lambda_\mu{}^\nu \vec{f}_\nu$$

- Tensors are objects of higher rank (2, 3, ...) which transform in a similar manner

$$T^{\mu\nu} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta T'^{\alpha\beta}$$

Norm & Invariants

- Scalar are invariant by change of coordinate, for instance:

$$A = U^{\mu} \cdot V_{\mu} = g^{\mu\nu} U_{\mu} V_{\mu}$$

- The elementary distance, defining the metric, can be expressed as:

$$\mathrm{d}s^2 = \mathrm{d}x^{\mu} \cdot \mathrm{d}x_{\mu} = g^{\mu\nu} \mathrm{d}x_{\mu} \mathrm{d}x_{\mu}$$

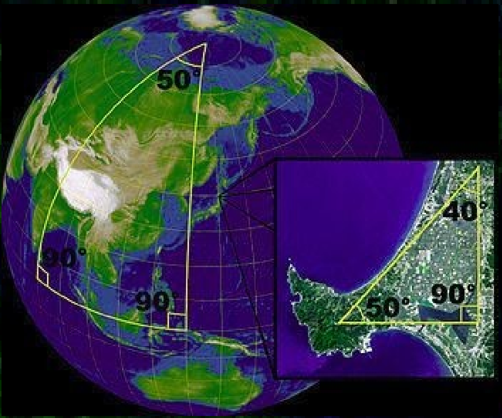
And is invariant by coordinate changes (such as the scalar product)

Curved Universe

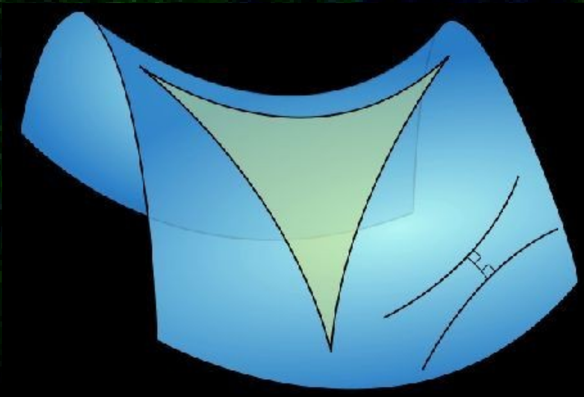
- In a flat Universe, the metric can be expressed in a diagonal form.

  e.g. Minkowski space (flat space-time):

$$g_{\mu\nu}=\begin{pmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$



- This is not the case any more in curved Universe
- The “curvature” is a mathematical concept that is obtained from derivatives of the metric:
  - Ricci tensor $R_{\mu\nu}$
  - Scalar curvature $R=g^{\mu\nu}R_{\mu\nu}$

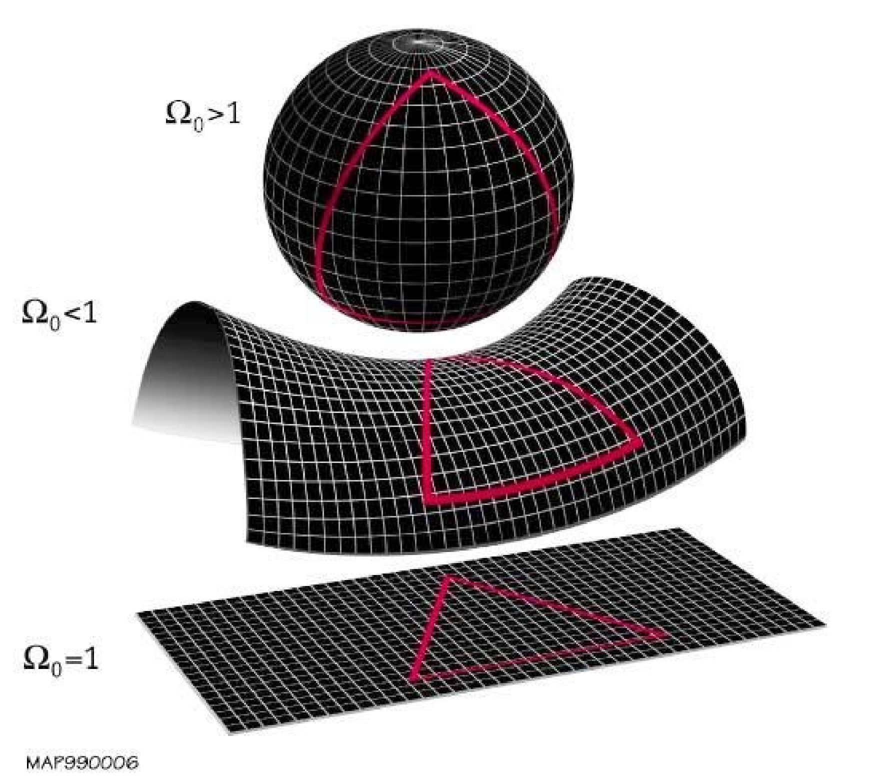


Uniform, Isotropic Universe

- Uniform, isotropic universes, can be described by the Friedman-Lemaitre-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right]$$

  - k = 1: Spherical space (Sum of angles > π)
  - k = -1: Hyperbolic space (Sum of angles < π)
  - k = 0: Euclidean space (Sum of angles = π)



Einstein Equation

- Start for the Poisson equation for gravitational potential

Field $$\nabla^2 \Phi_p = -4\pi\rho_g$$ **Matter Content**

- Construct a Lorentz-invariant (Covariant) version

Covariant Derivative $$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) A^\mu = 4\pi j^\mu$$ **Matter Quadri-current (Density is NOT Lorentz invariant)**

- Both Energy and Volume are affected by Lorentz transformation so covariant energy density must be a tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Curvature of Universe **Energy Content**

Energy Momentum Tensor?

- Covariant (Lorentz invariant) formulation of energy conservation

$$\nabla_\mu T^\mu{}_\nu = 0$$

- Energy momentum tensor for a perfect fluid

$$T_{\mu\nu} = n(\tilde{x}) \frac{p_\mu p_\nu}{E} = \rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu)$$

u_μ is the four velocity

- In the rest frame of fluid, $u^\mu=(1,0,0,0)$ and thus:

$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & & & \\ & -P(t) & & \\ & & -P(t) & \\ & & & -P(t) \end{pmatrix}$$

Energy Momentum Tensor

Energy Density

Energy Flux

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Viscosity

Pressure

Momentum Density

Momentum Flux

Einstein Equation

- Minimum Covariant Equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Curvature of Universe **Energy Content**

- Energy Content:

$$T_{\mu\nu} = \sum_{\text{species}} \left(\rho\, u_\mu u_\nu + P \left(g_{\mu\nu} + u_\mu u_\nu \right) \right)$$

- One can add a Cosmological Constant to make the universe static (Compensates for matter)

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Matter Equation of State
Evolution of density

- Exercise:

  From the work of pressure $\delta W = -p \, \mathrm{d} V$

  and the expression of energy $E = \rho V$

  Show the evolution of density: $\frac{\mathrm{d}\rho}{\mathrm{d}t} = -3 \frac{\dot{a}}{a} (p + \rho)$

  In particular, $\frac{\mathrm{d}\rho}{\mathrm{d}t} = 0 \quad \Rightarrow p = -\rho$

- Using equation of state:

$$P = w\rho \quad \Rightarrow \quad \rho(t) = \rho_0 \left(\frac{a}{a_0} \right)^{-3(1+w)}$$

General relativity in Friedman-Lemaitre-Robertson-Walker metric

- Einstein Equation (Isotropic Uniform Universe)

$$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\sum_i \rho_i-\frac{k}{a^2}$$

- Acceleration

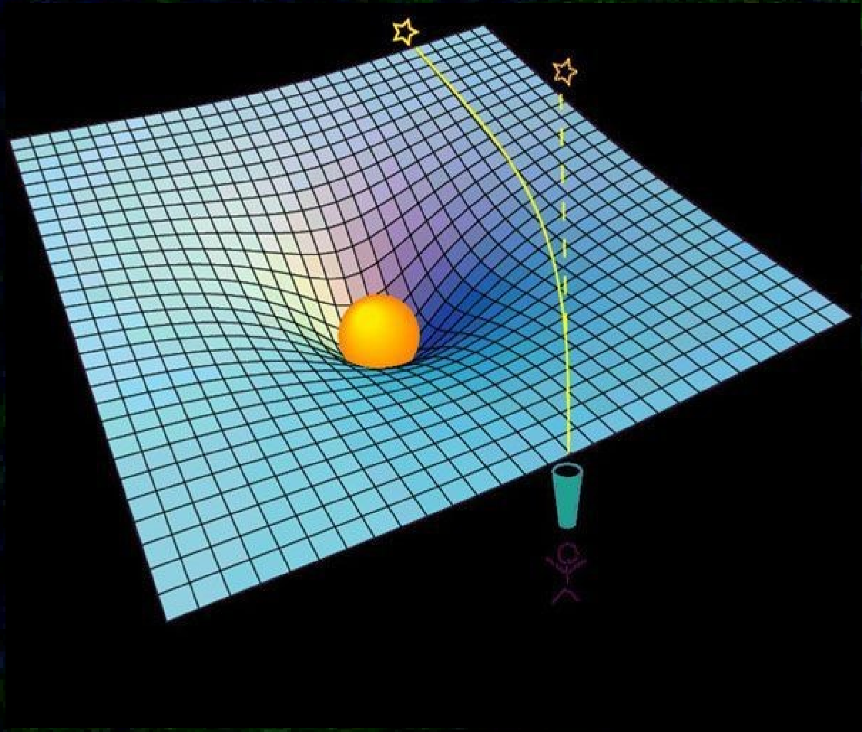
$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\sum_i (\rho_i+3p_i)$$

- Evolution of density

$$\frac{\partial\rho}{\partial t}+3H(\rho+p)=0$$

- Equation of state

$$p=w\rho$$



Matter, radiation, ...

Content	State Equation	Dilution Law	Evolution
Matter	$p \approx 0$	$\rho \propto a(t)^{-3}$	$a(t) \propto t^{2/3}$
Radiation	$p = \frac{\rho}{3}$	$\rho \propto a(t)^{-4}$	$a(t) \propto t^{1/2}$
Curvature		$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2}$	$a(t) \propto t$
Cosmological constant	$p = -\rho$	$\rho = C_{ste} = \frac{\Lambda}{8\pi G_N}$	$a(t) \propto e^{H \times t}$
Generic	$p = w\rho$	$\rho \propto a(t)^{-3(1+w)}$	$a(t) \propto t^{1/3(1+w)}$

Cosmological Constant

- Introduced by Einstein to allow for a static Universe (counteracting the mass)
- Positive energy density, independent of size, implying negative pressure, Kind of “vacuum energy”
- But in 1929 Edwin Hubble showed that the Universe is in expansion

Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life.

-- George Gamow, My World Line, 1970

Deceleration parameter

- One defines dimensionless densities using critical density:

$$\Omega_i^0 = \frac{\rho_i^0}{\rho_{\text{critic}}} = \frac{8\pi G}{3 H_0^2}\rho_i^{0}, \qquad \Omega_k^0 = \frac{-k}{a_0^2 H_0^2}$$

- Deceleration parameter

$$q = -\frac{1}{H^2}\left[\frac{\ddot{a}}{a}\right] = \frac{\Omega_m}{2} + \Omega_r - \Omega_\Lambda$$

  - Matter and radiation decelerates expansion
  - Cosmological constants accelerates expansion
  - Curvature is neutral
  - Null deceleration if $\Omega_m + 2\Omega_r = 2\Omega_\Lambda$

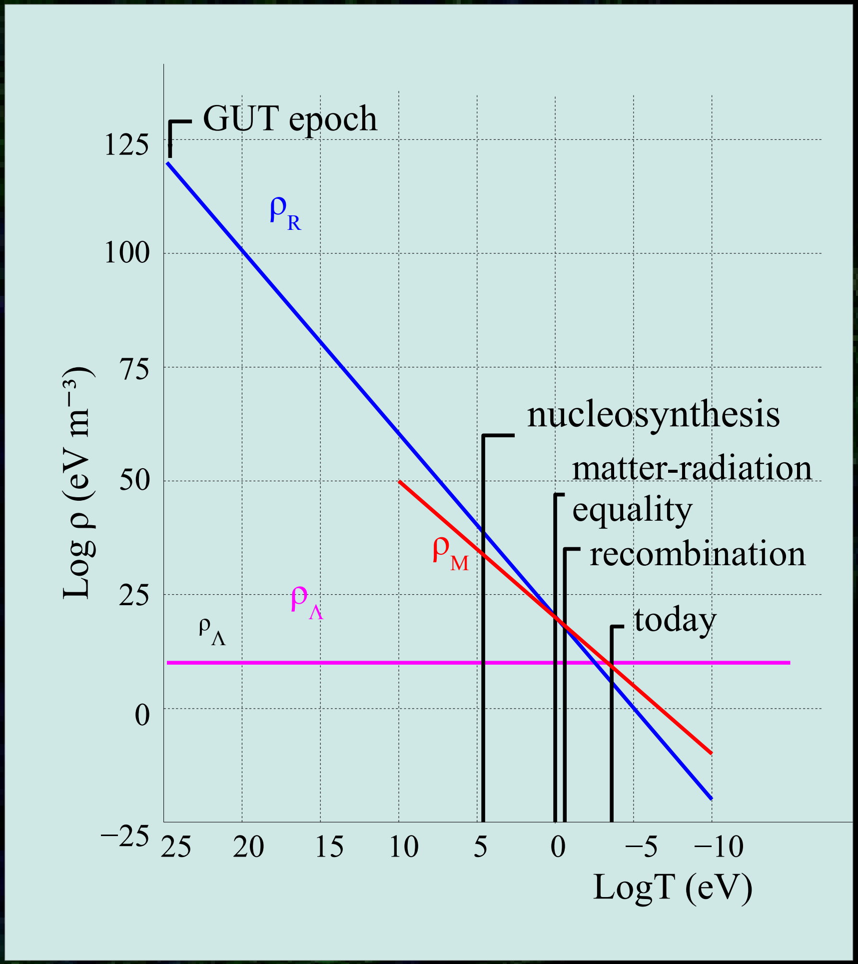
Epochs

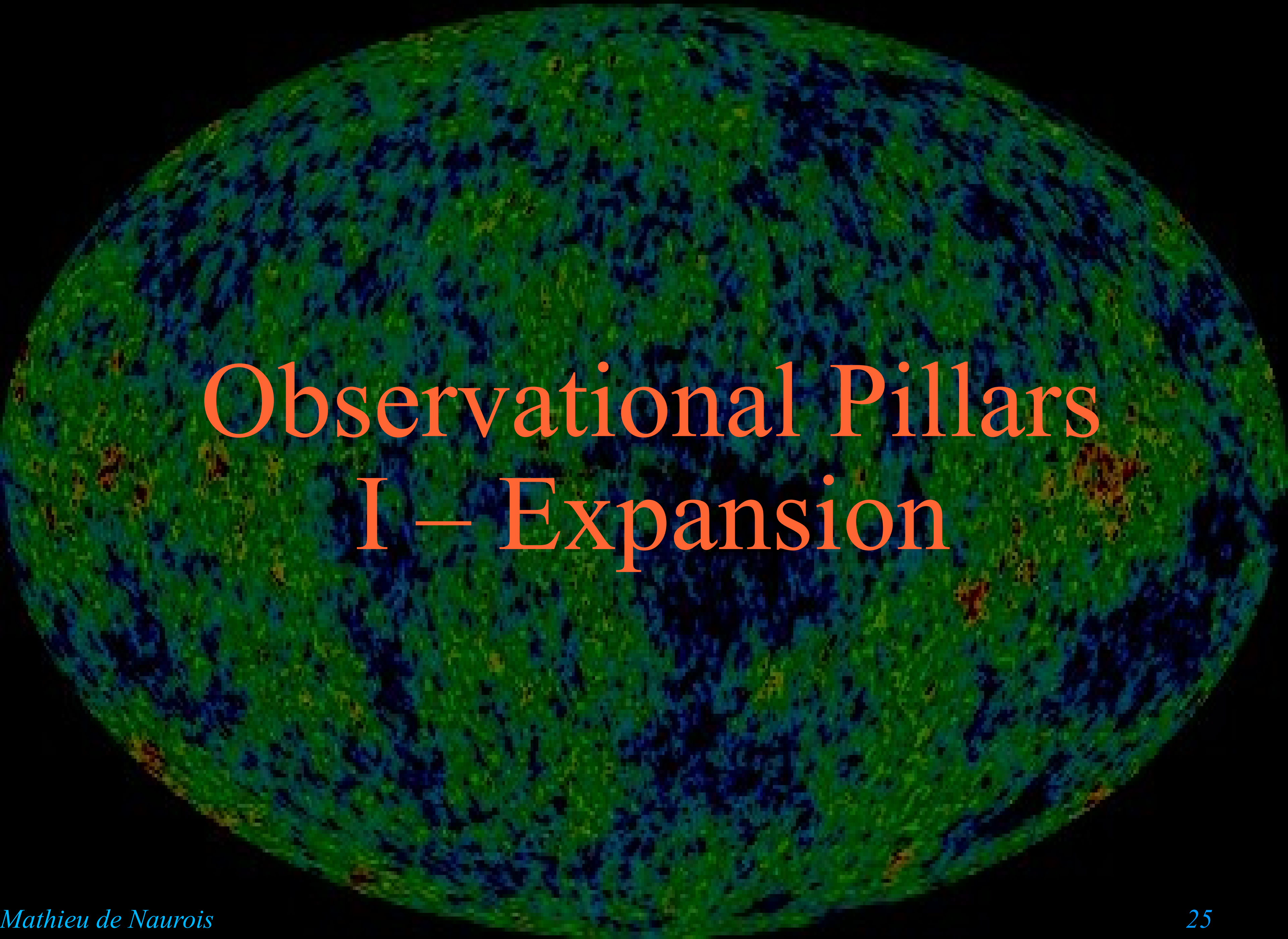
$$\left(\frac{H}{H_0}\right)^2 = \Omega_m^0\left(\frac{a_0}{a}\right)^3 + \Omega_r^0\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \left(1-\Omega_{tot}^0\right)\left(\frac{a_0}{a}\right)^2$$

- Matter: $\Omega_m^0\left(a_0/a\right)^3$
- Radiation: $\Omega_r^0\left(a_0/a\right)^4$ — Dominates in the early Universe
- Curvature: $\left(1-\Omega_{tot}\right)\left(a_0/a\right)^2$
- Cosmological Constant Ω_Λ

Epochs

- Universe starts by a radiation dominated era
- After some times, matters dominates over the radiation and expansion slows down
- If $\Omega_T > 1$ and $\Omega_\Lambda \sim 0$, the Universe re-collapses and radiation dominates again
- If $\Omega_T < 1$ and $\Omega_\Lambda \sim 0$, the Universe ends in free expansion governed by curvature
- If $\Omega_T < 1$ and $\Omega_\Lambda > 0$, the Universe ends in accelerated exponential expansion governed by cosmological constant



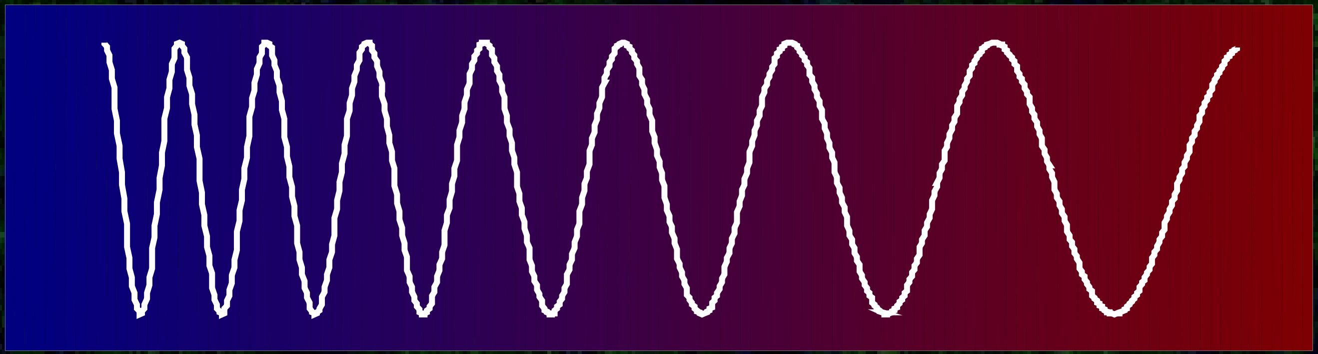


Observational Pillars
I – Expansion

Redshift

- During the propagation of a photon, the universe gets diluted and the wavelenght increases by the same amount:

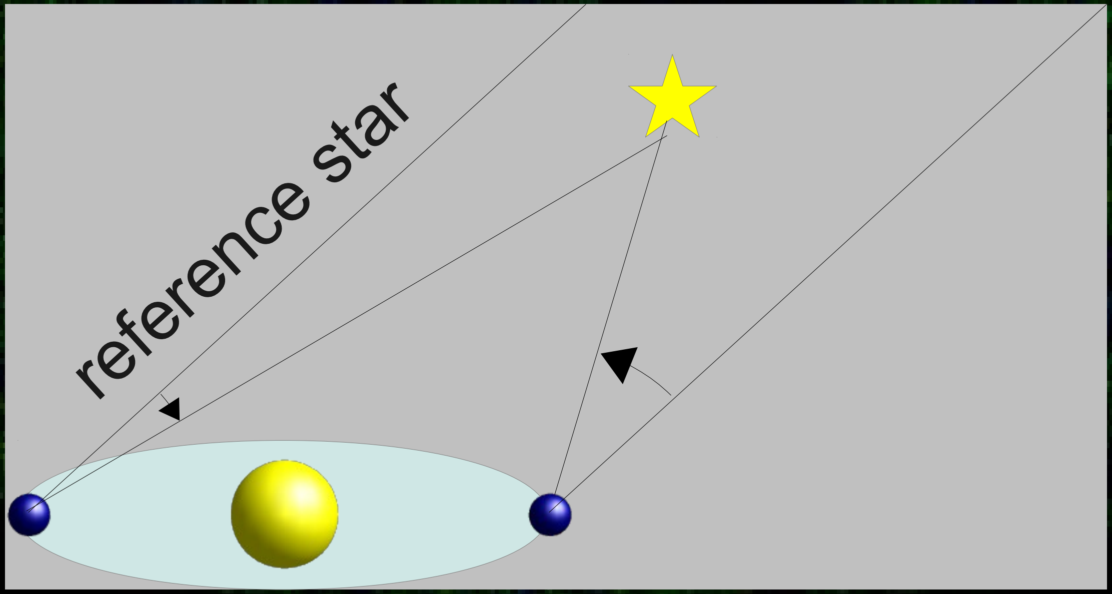
$$1+z=\frac{a_r}{a_e}=\frac{\lambda_r}{\lambda_e} \quad \text{where} \quad \begin{cases} \mathrm{e}=\text{emission} \\ \mathrm{r}=\text{reception} \end{cases}$$



emission Reception

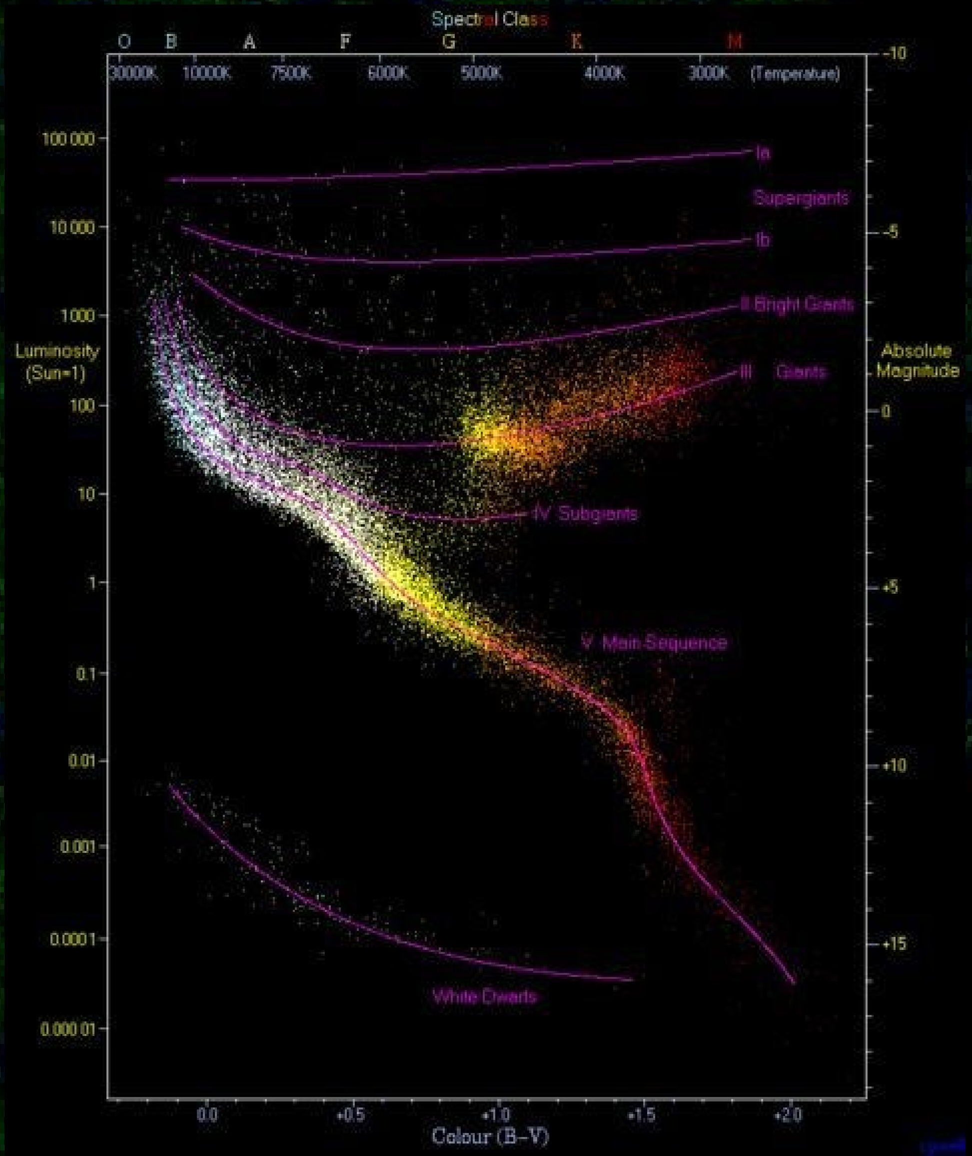
Distance Ladder – I

- Measuring the distance is not an easy task
- Several methods valid only in a given distance range
- Precise crosscalibration needed in overlapping range
- First level: parallax measurements (up to ~ 200 pc = 1/40 of the distance to the Galactic Centre). 10^5 stars measured by Hipparcos



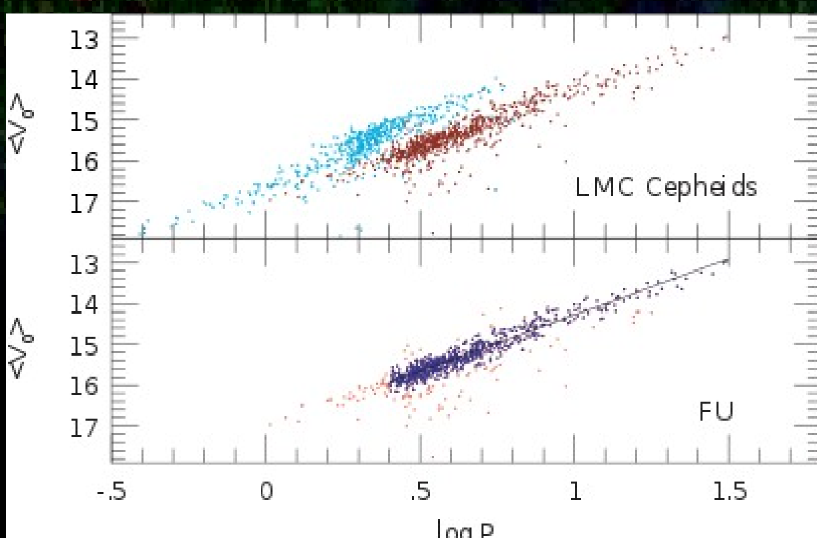
Distance Ladder – II

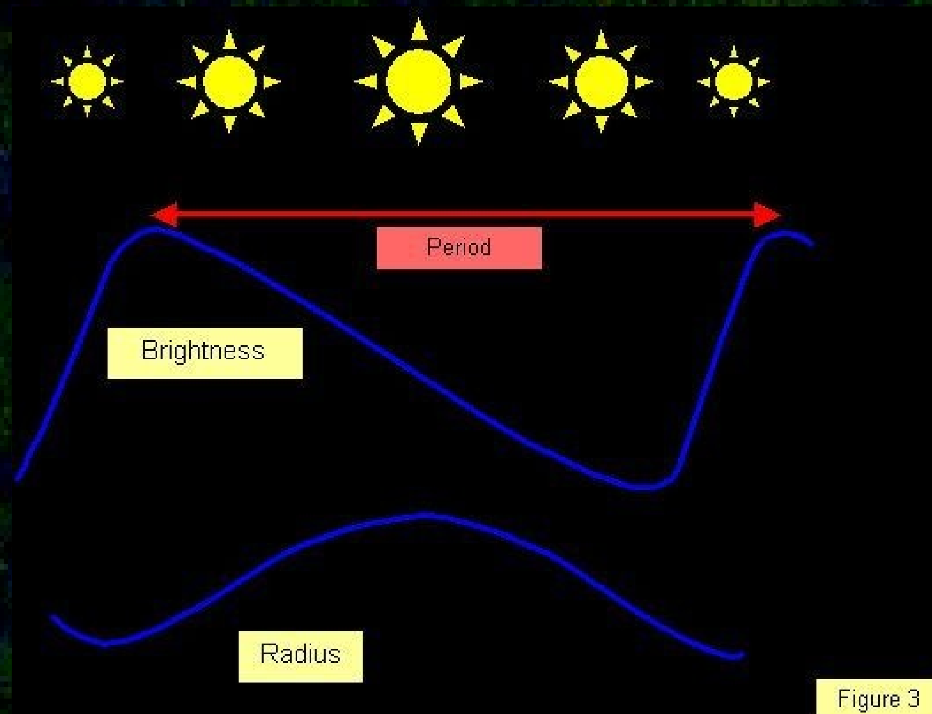
- Second level (in the Galaxy): luminosity of stars as function of spectral type(need correction for absorption)
- One can measure distance of a cluster by studying its star population



Distance Ladder – III

- Variables stars (Cepheids) with periodic luminosity behaviour related to the period (calibrated on those measured by Hipparcos)
- Helium heating ionizes it. It becomes more opaque, and ionized further, until it expands, cools and becomes transparent.
- Come in different flavours (fondamental, first harmonic)





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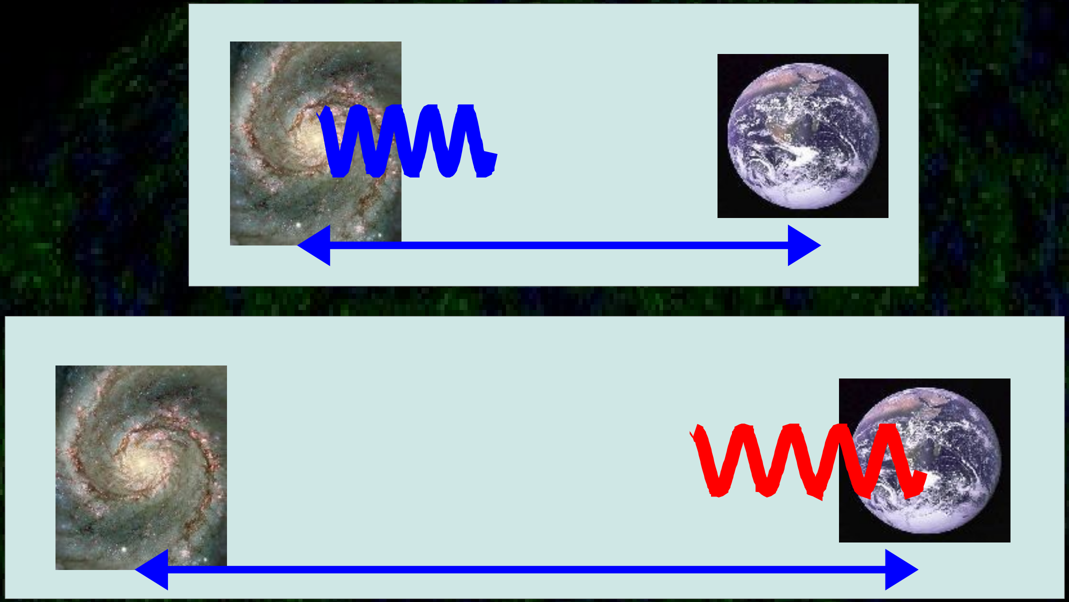
What distance?



Emission

Reception

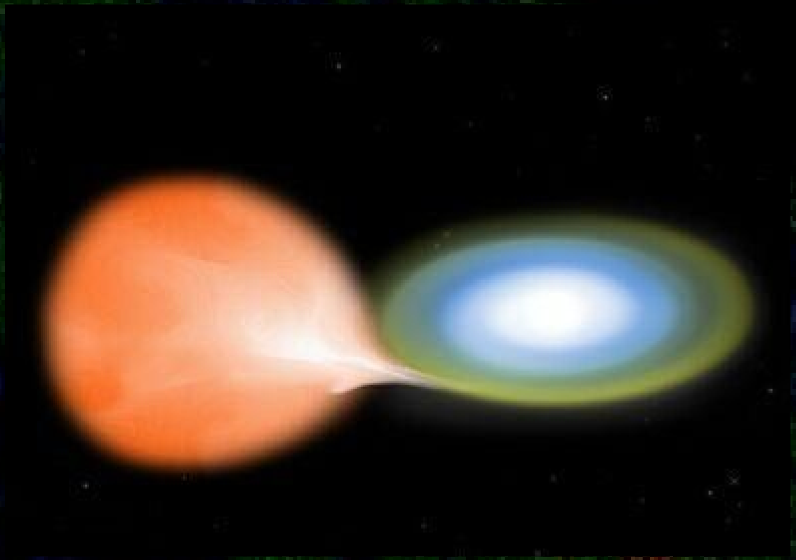
- Distance at emission time? At reception time?
- Best definition: distance travelled by photon as Universes expands (Comoving distance)

$$d = a_0 \int_{t_e}^{t_e} \frac{\mathrm{d}t}{a(t)} = \int_{a_0/(1+z)}^{a_0} \frac{a_0 \,\mathrm{d}a}{a \dot{a}} \propto \int \frac{\mathrm{d}z}{H(z)}$$

Depends on expansion evolution of the Universe
=> Probe for expansion

Distance Ladder – IV

- Standard Candles:
  - Type 1a supernova (Accreting white dwarf exploding when reaching the Chandrasekhar mass)
  - Light-curves have to be calibrated (different composition, ...)
- Larger distance: redshift is the only information, but relies on a cosmological model



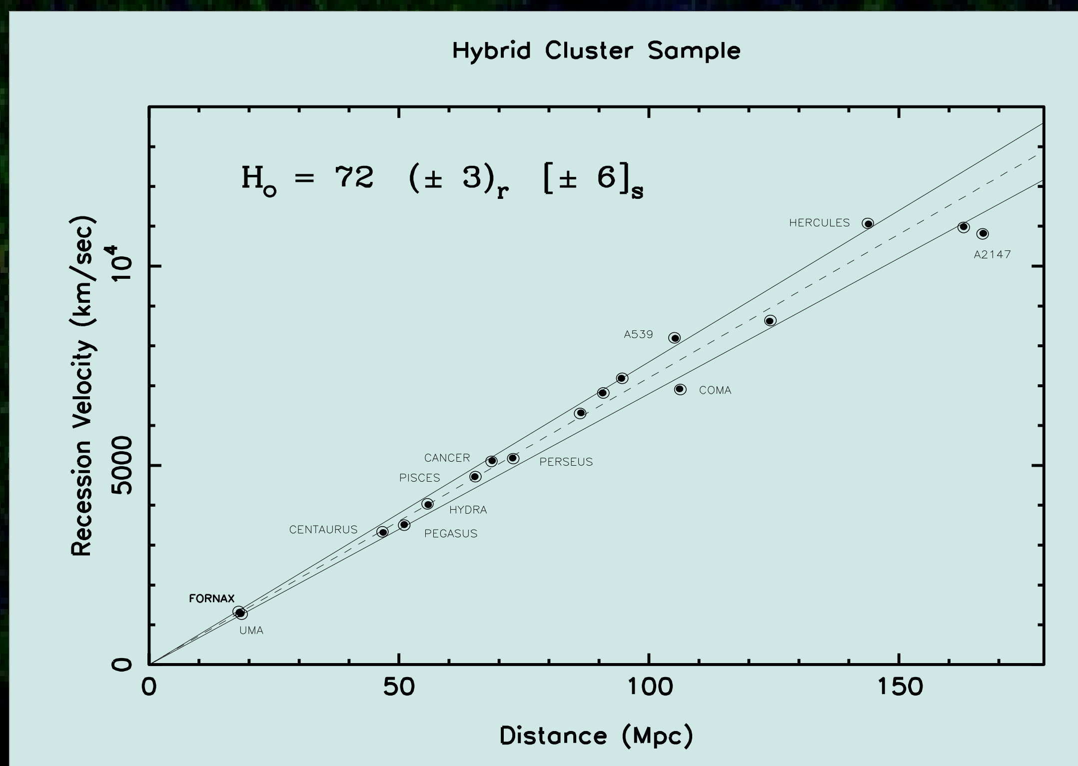
Hubble Law

- Galaxies are separating apart at a speed proportional to their distance

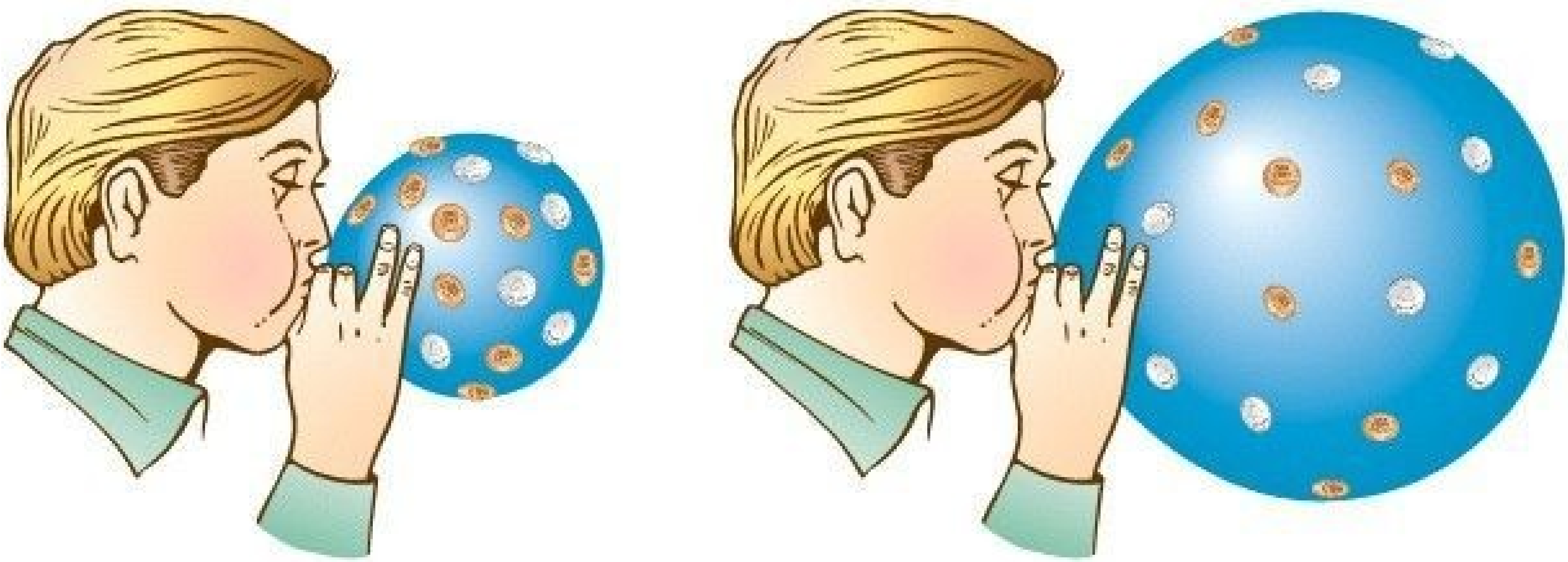
$$\frac{\mathrm{d}R}{\mathrm{d}t} = H_0 R + v_p$$

Hubble flow

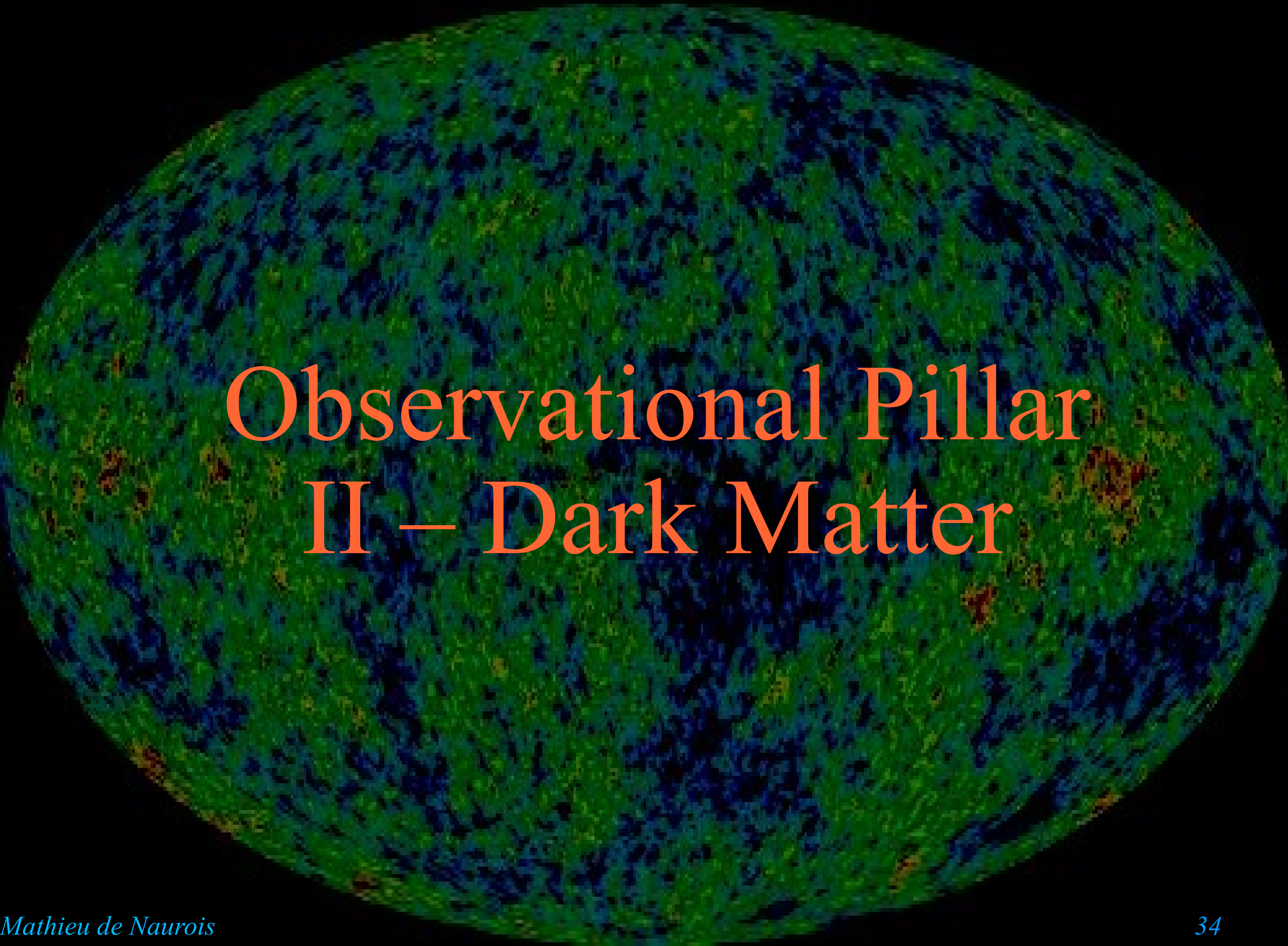
Proper Motion



Expansion



- It's the space itself that gets diluted, not the galaxies that are moving in the space!



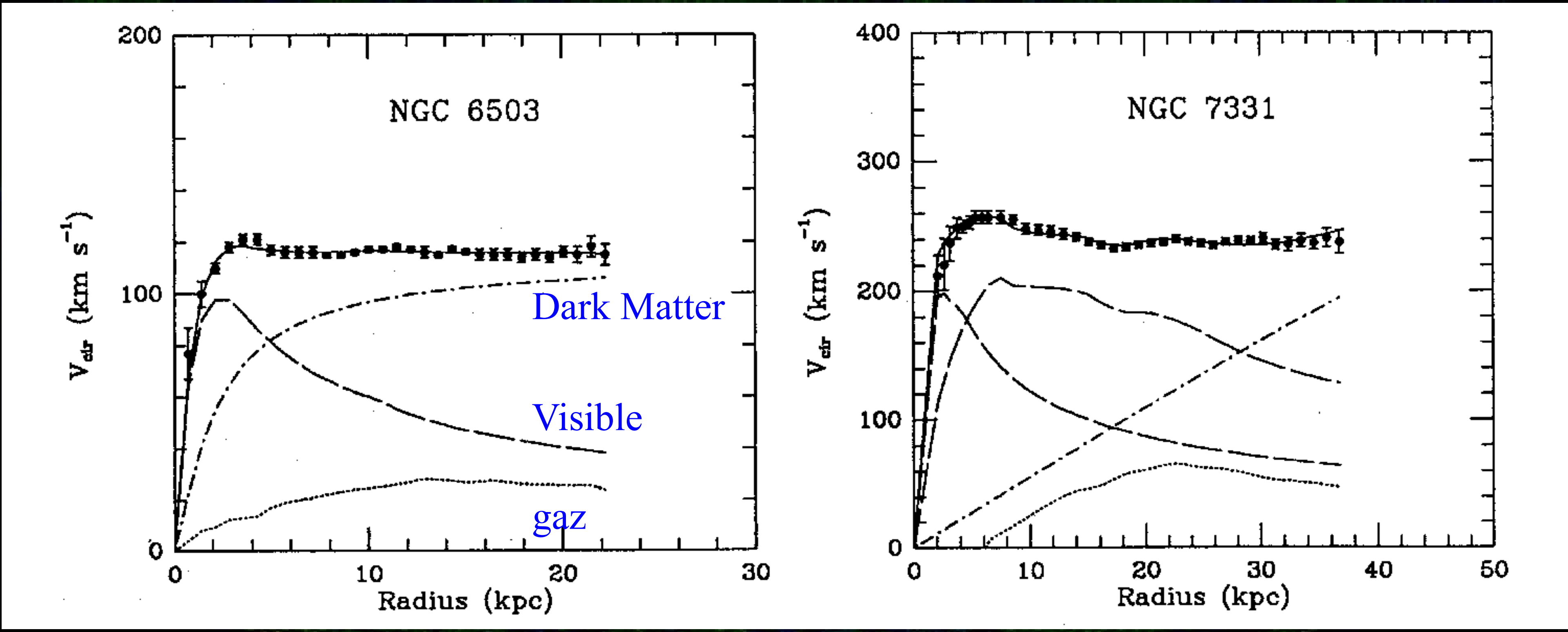
Observational Pillar II – Dark Matter

Rotation Curve

- For Kepler Motion

$$V(R)=\sqrt{\frac{G\,M(R)}{R}}$$

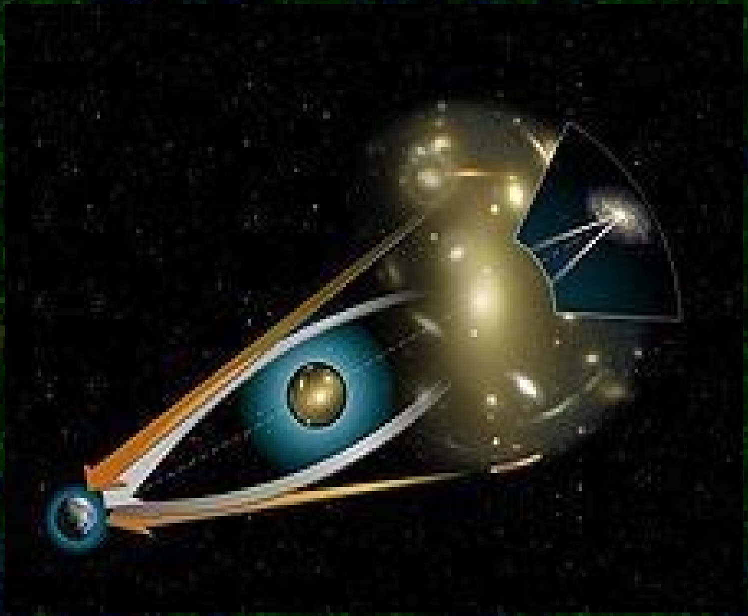
Exercise!



- Dark Matter represents ~ 85% of matter, and ~ 25% of total energy

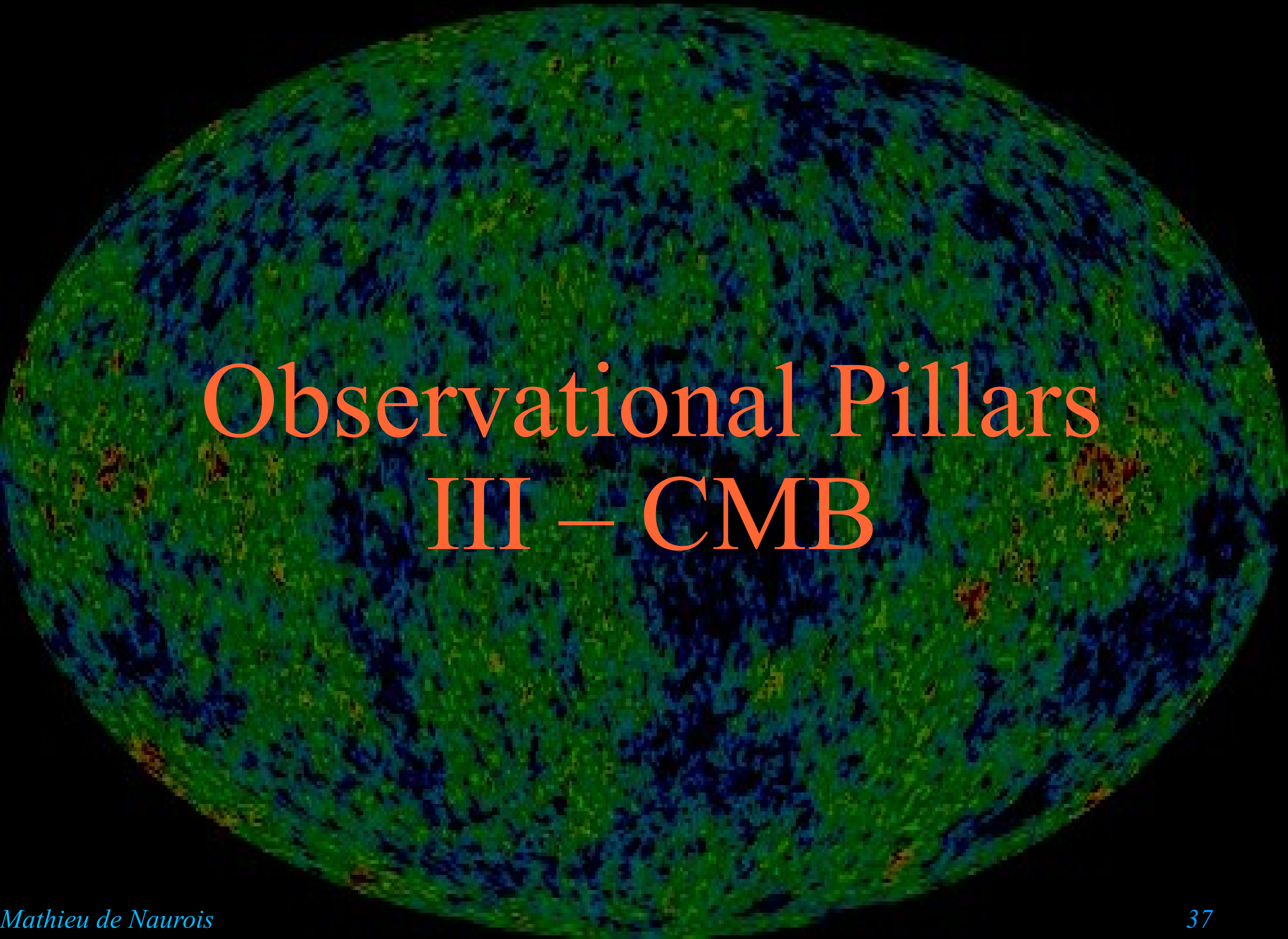
Other evidences

- Gravitational Lensing



- Temperature distribution of hot gas in galaxies and clusters of galaxies
- CMB

Observational Pillars
III – CMB

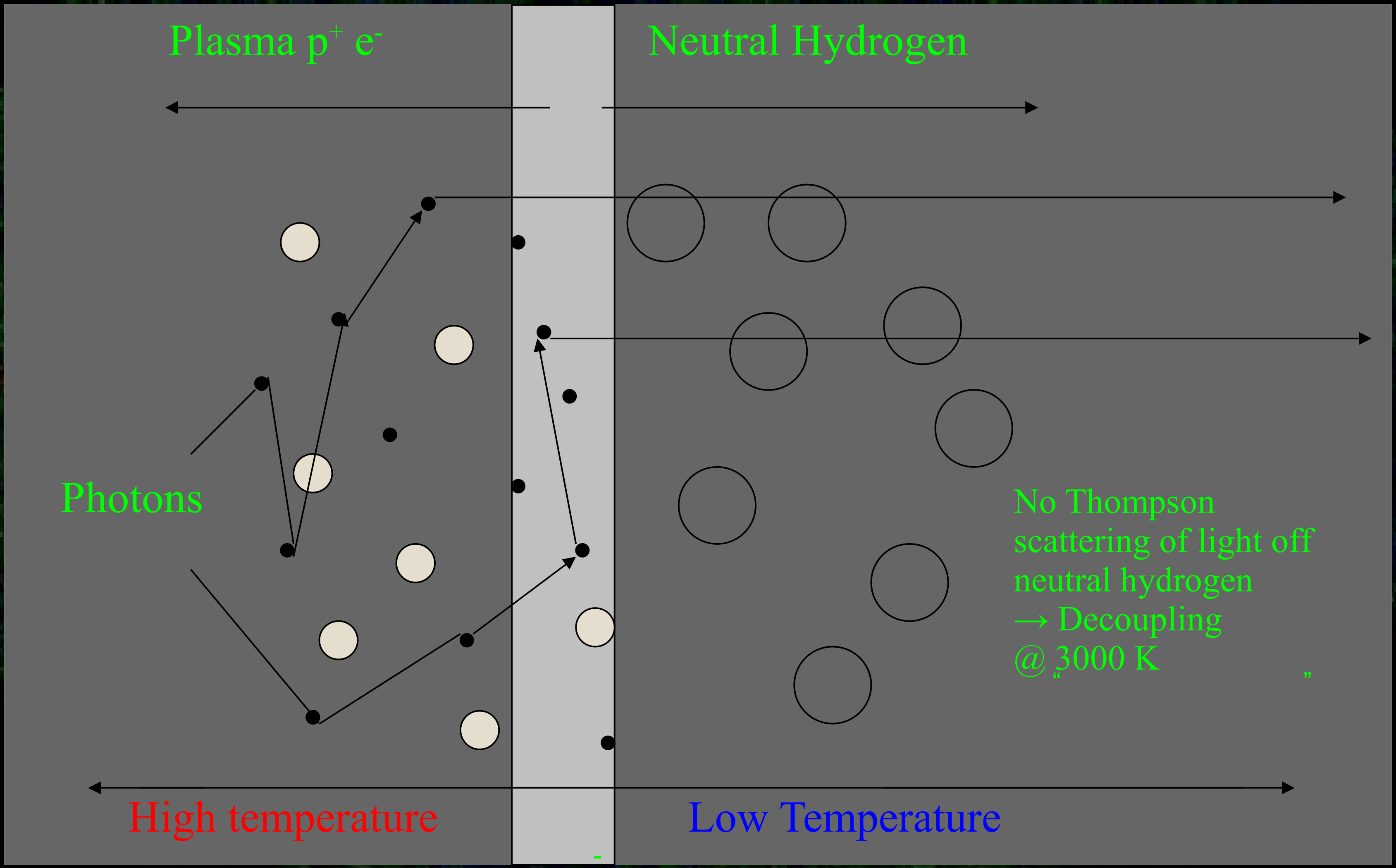


Predicted in the 1950 s, detected in 1964

- Thermal emission emitted at the time of decoupling (transition from an nuclei-electron plasma to neutral atoms)
- Diluted and redshifted by the expansion of the Universe

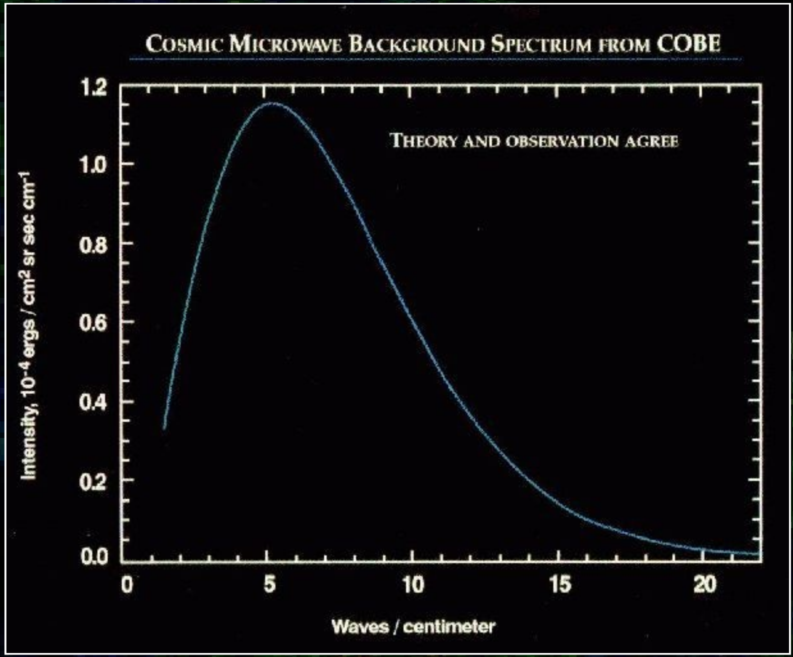
Penzias and Wilson, 1964

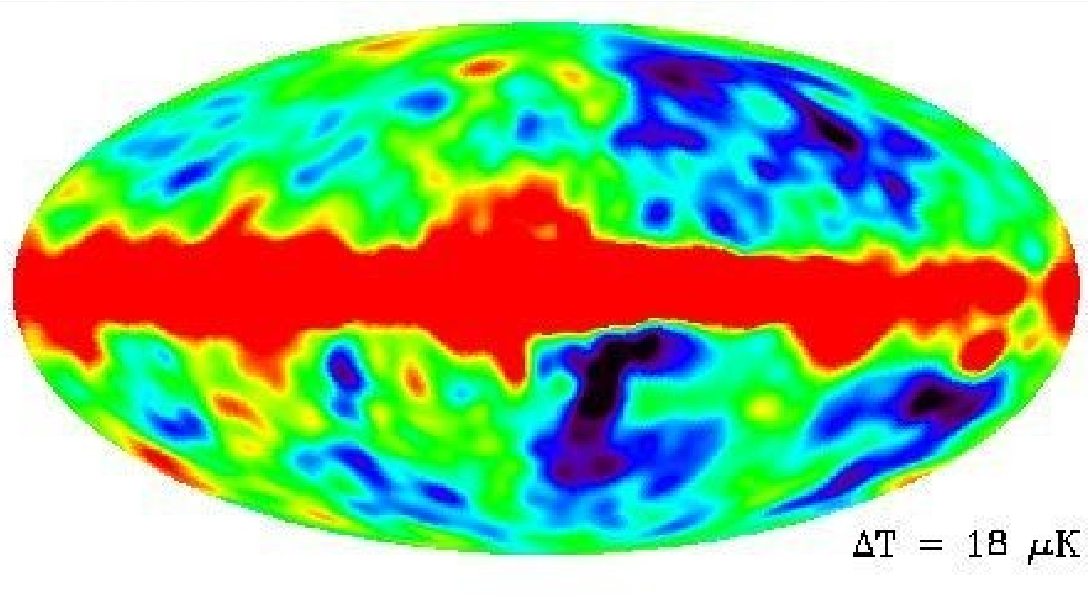
Recombination & Decoupling (z = 1100)



The Cosmic Microwave Background

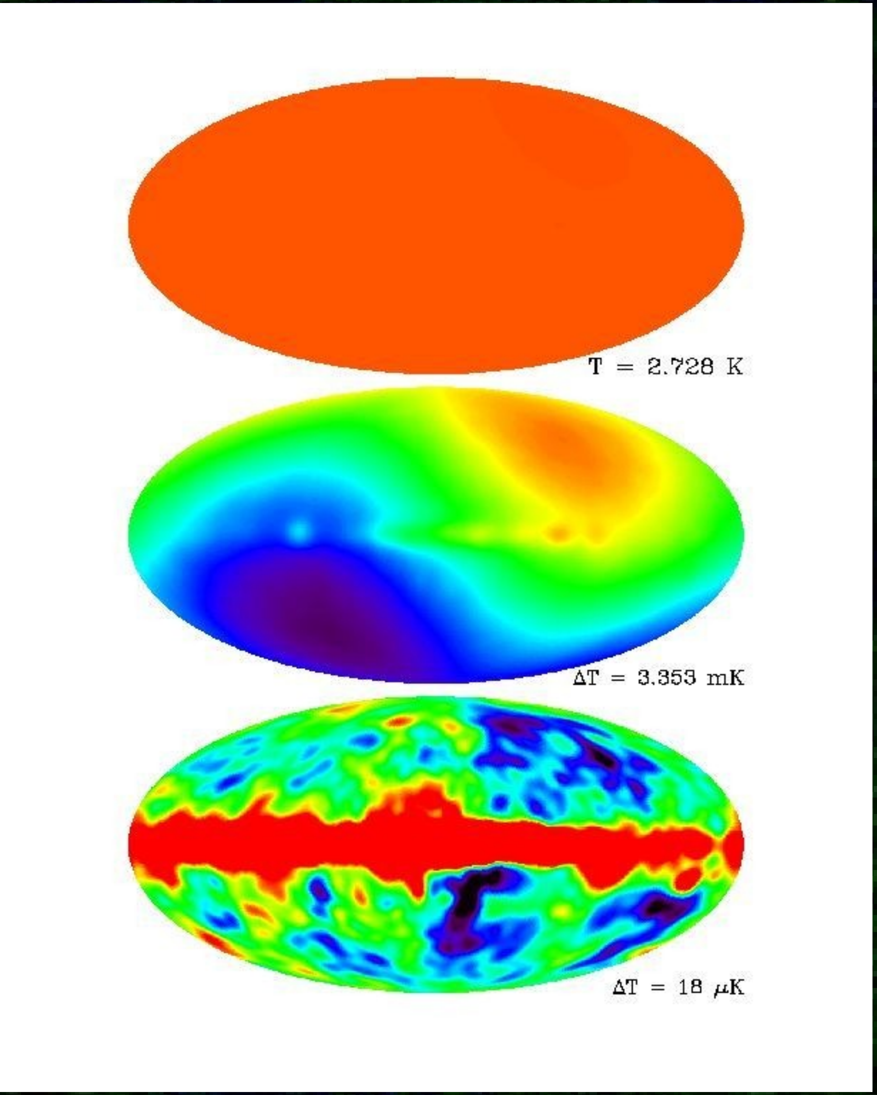
- Discovered 1965 (Penzias & Wilson)
  - 2.7 K blackbody
  - Isotropic (<1%)
  - Relic of hot "big bang"
- 1970's and 1980's
  - 3 mK dipole (local Doppler)
  - $\delta T/T < 10^{-5}$ on arcminute scales



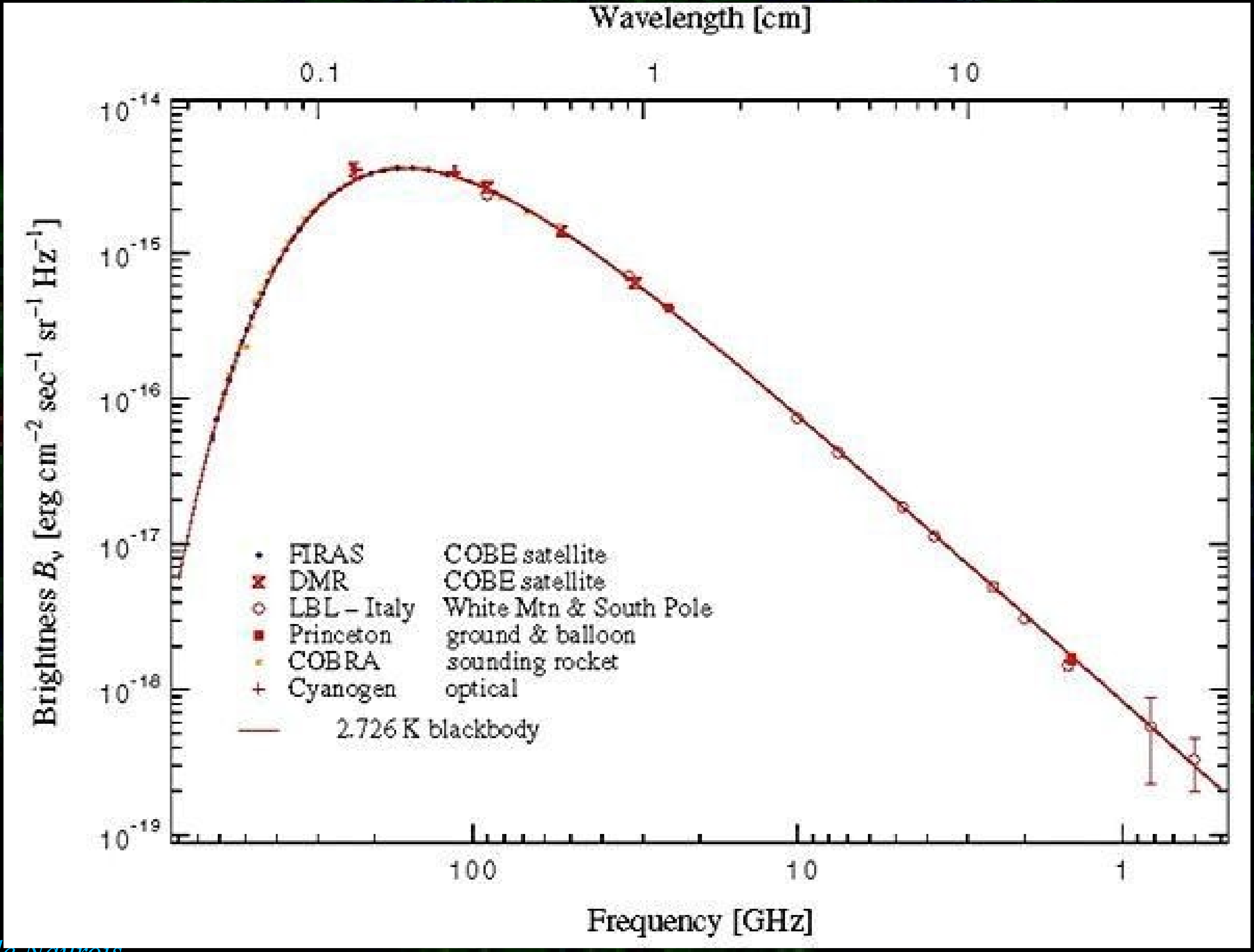


- COBE 1992
- Blackbody 2.728 K
- $\ell < 30 : \delta T/T \approx 10^{-5}$

COBE



CMB Spectrum



From COBE to WMAP

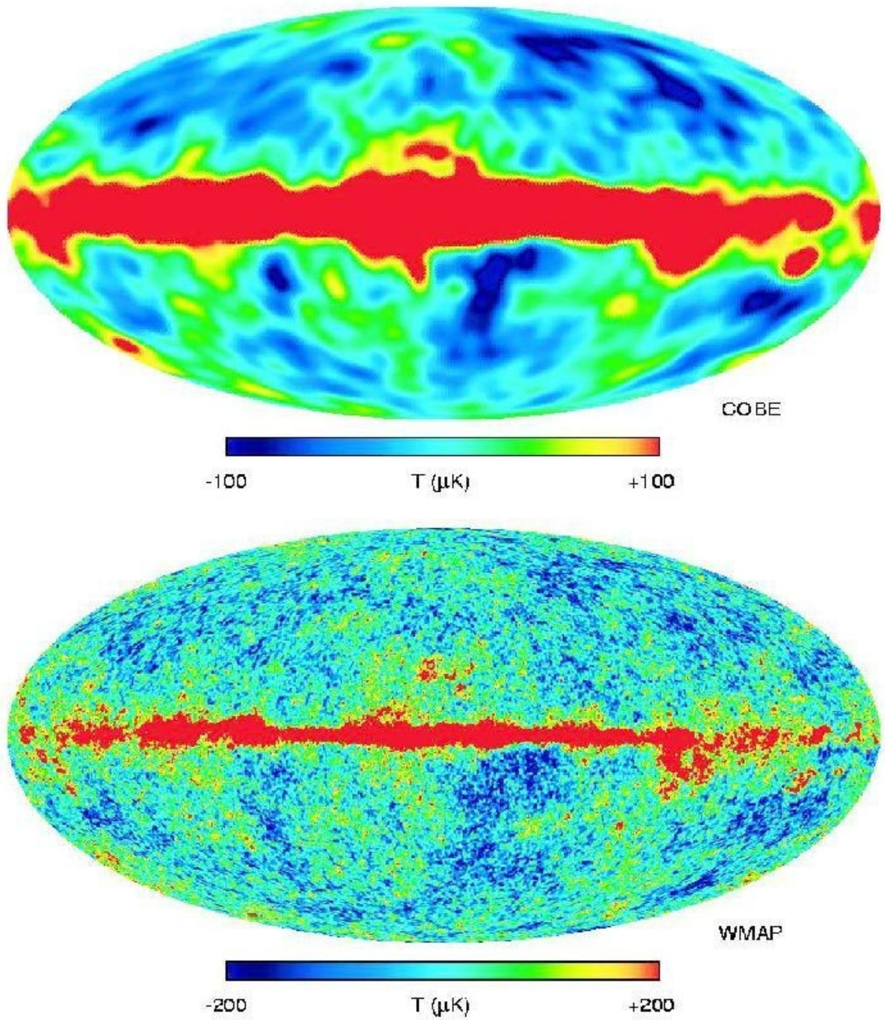
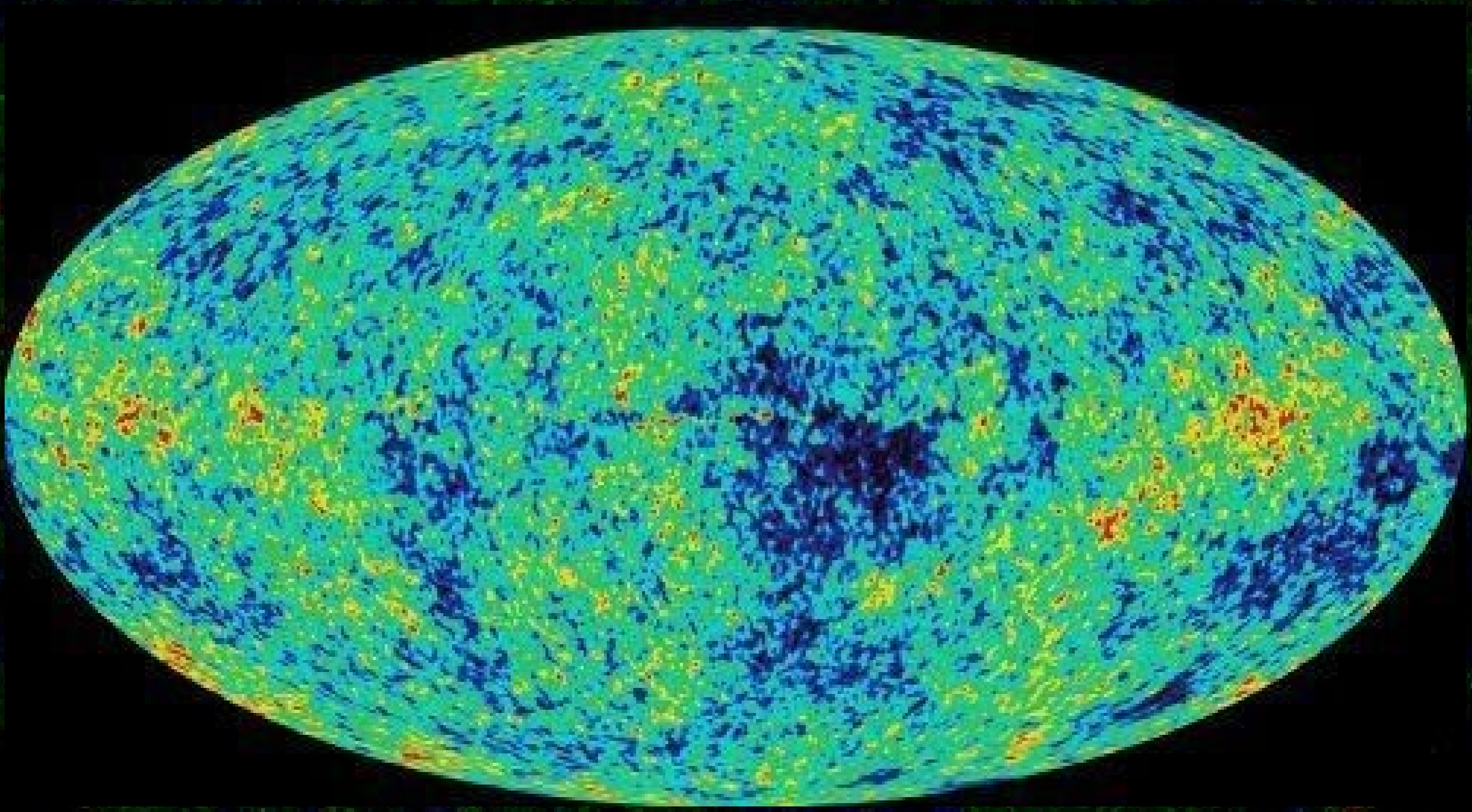


Fig. 7.— A comparison of the *COBE* 90 GHz map (Bennett et al. 1996) with the W-band *WMAP* map. The *WMAP* map has 30 times finer resolution than the *COBE* map.

Is CMB Homogeneous?

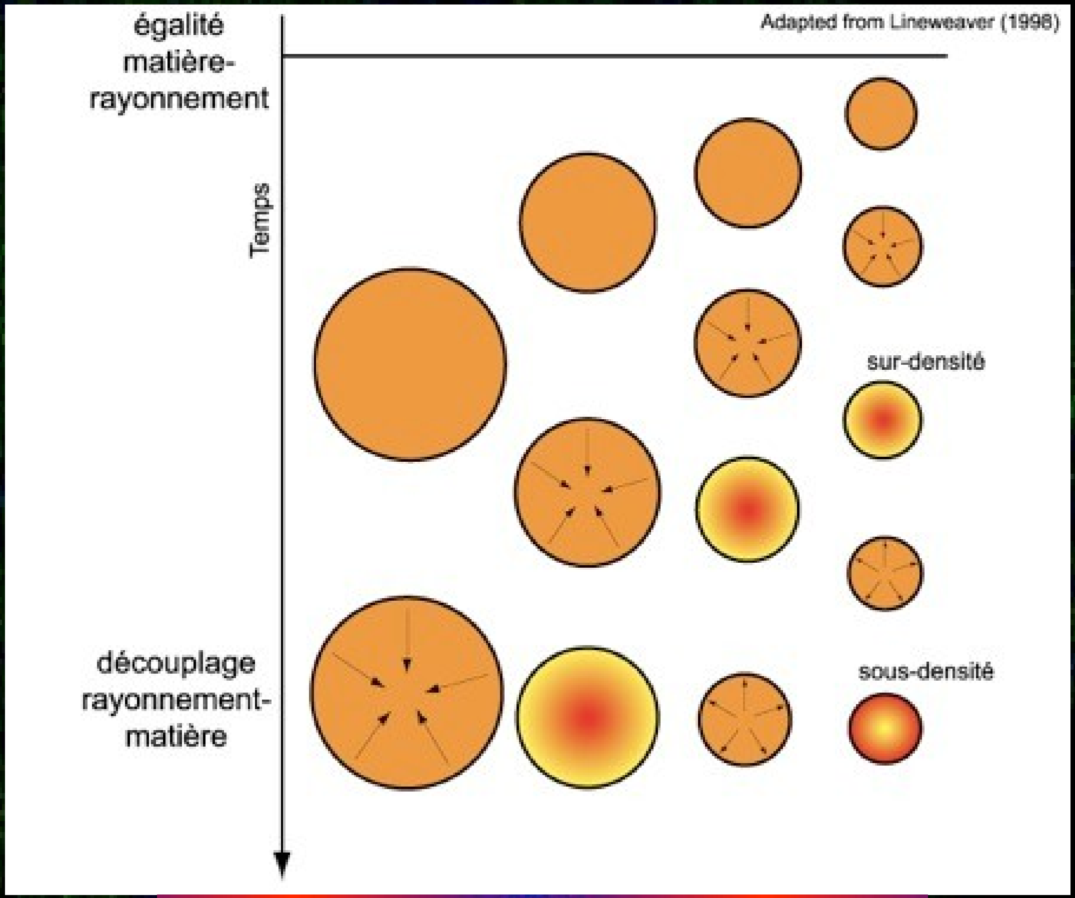
- Wmap temperature Map

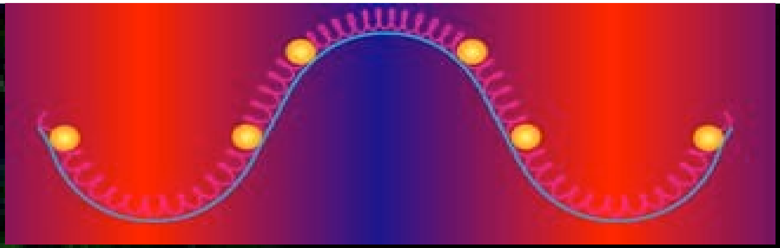


CMB

- Oscillations due to coupling between matter and radiation (radiation pressure)
- Wave travelling at

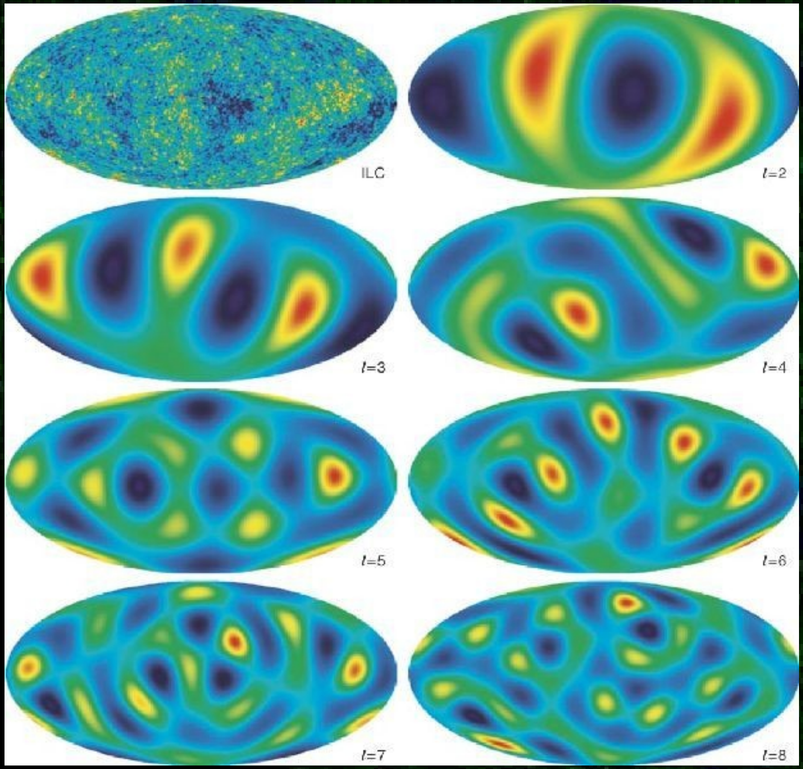
  $c/\sqrt{3}$
- Small fluctuations oscillate faster
- At the time of decoupling, situation is frozen => characteristic angular scale appear
- Density fluctuation translate into temperature variations

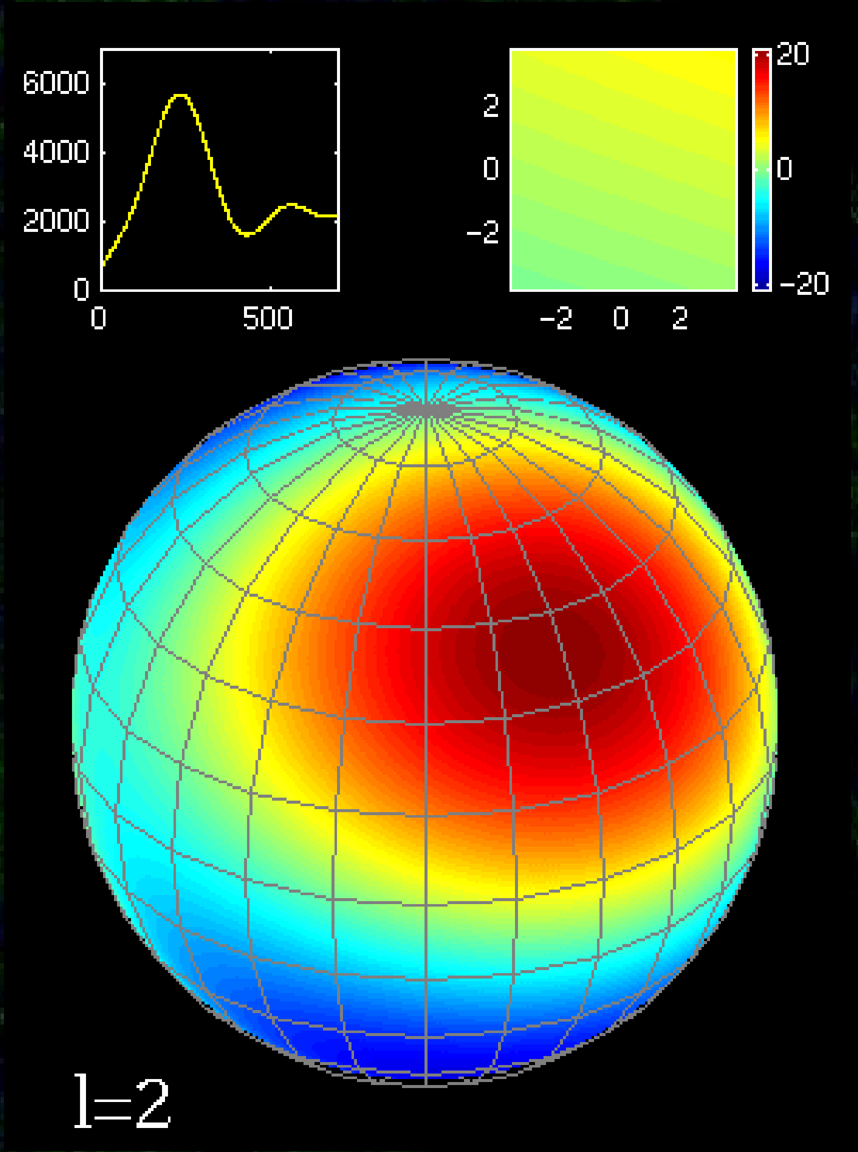




Spherical Harmonics

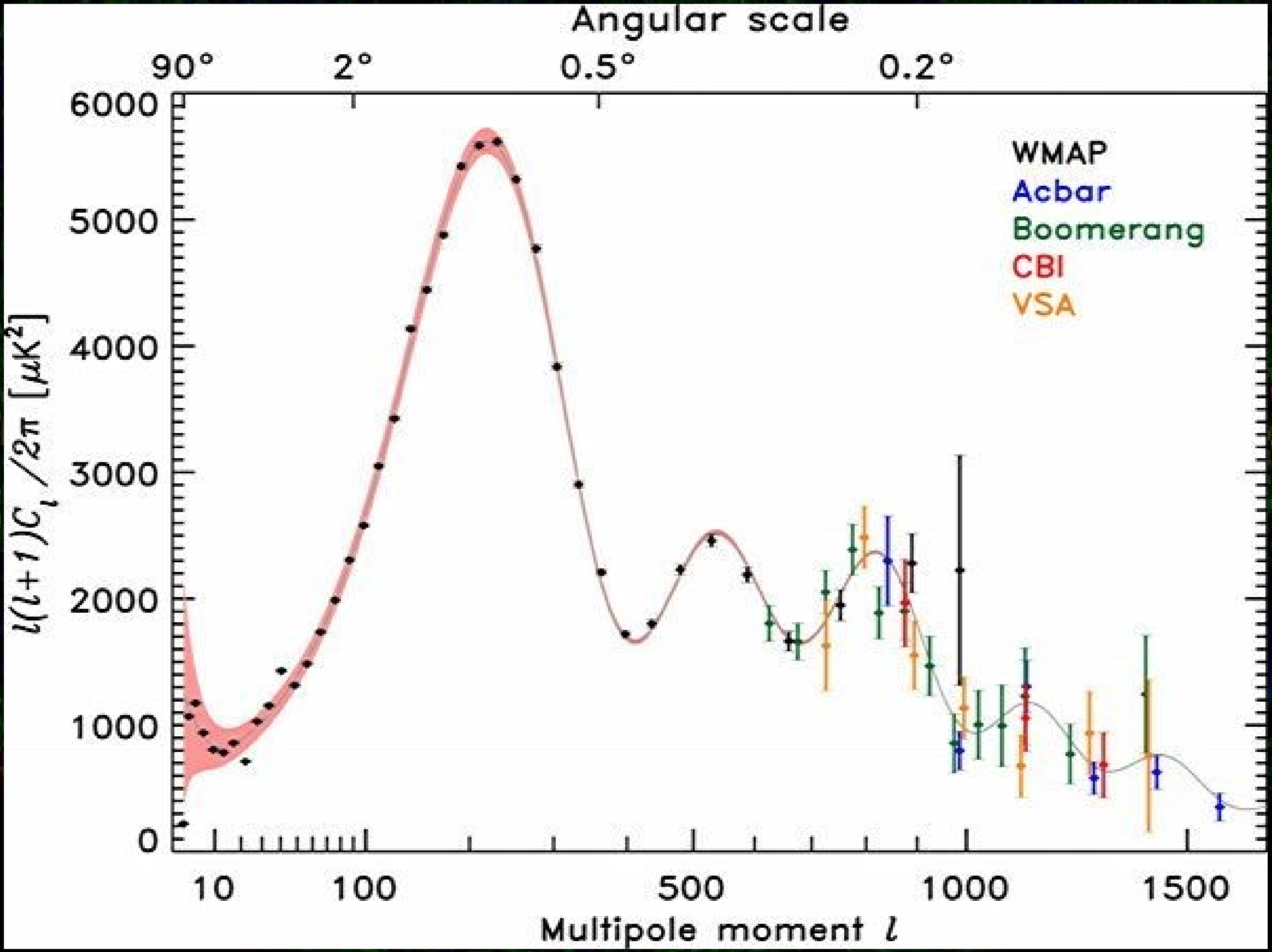
- l correspond to angular scale
- Large values correspond to small scales





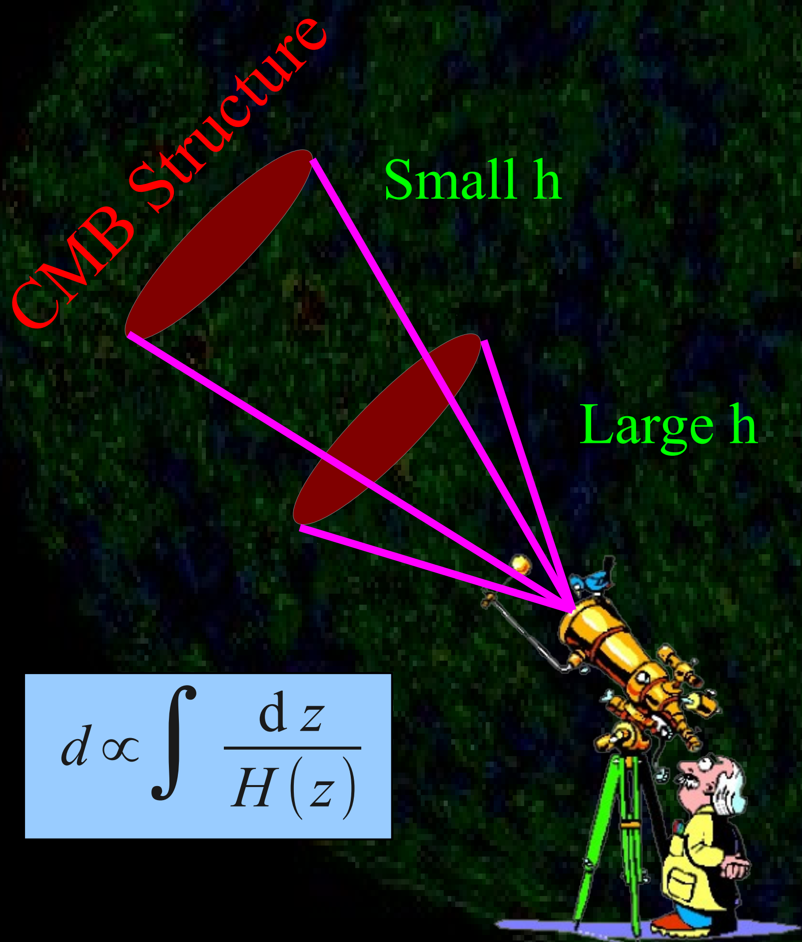
The lower WMAP harmonics...

CMB Angular Spectrum

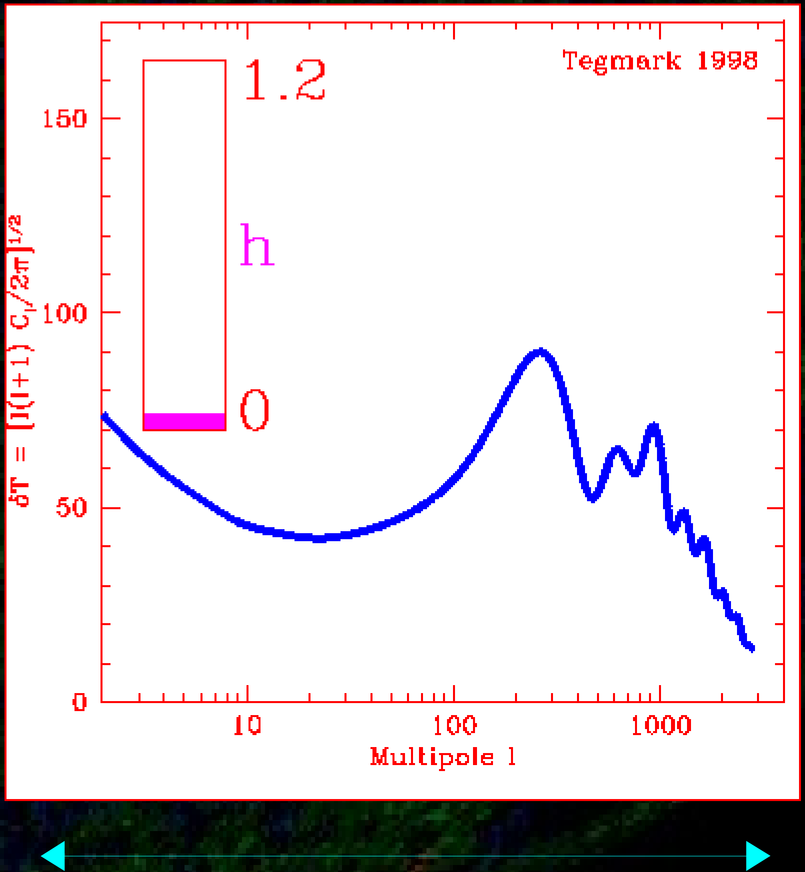


Hubble Constant

- Large expansion speed makes larger redshifts correspond to smaller distance. Structures appear larger



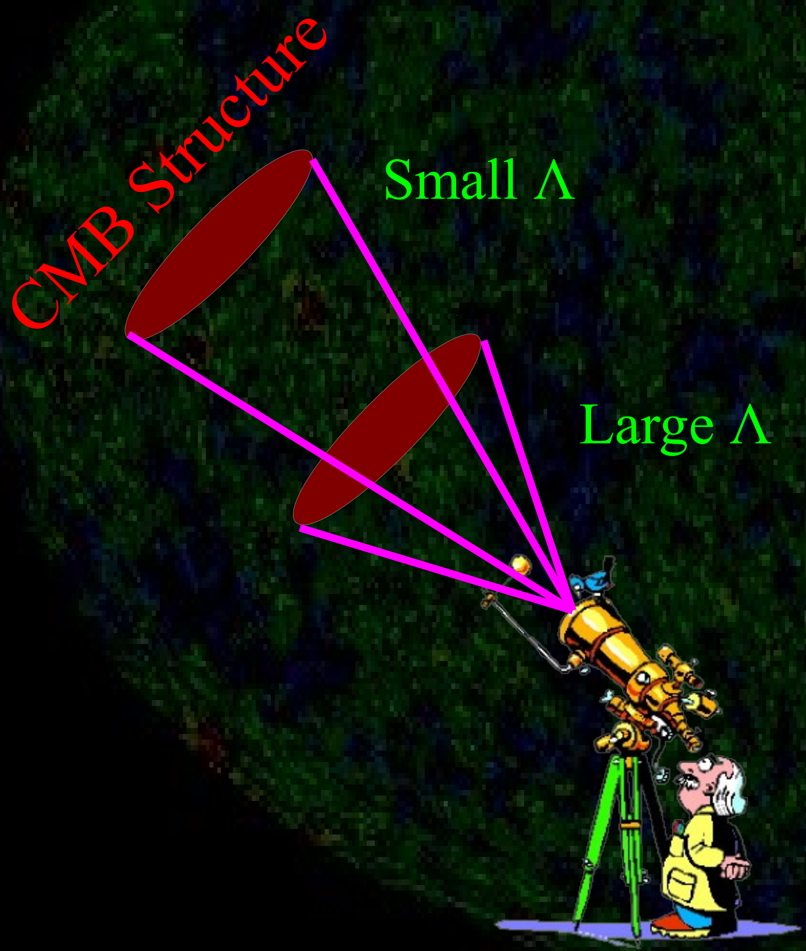
$$d \propto \int \frac{\mathrm{d}\,z}{H\left(z\right)}$$

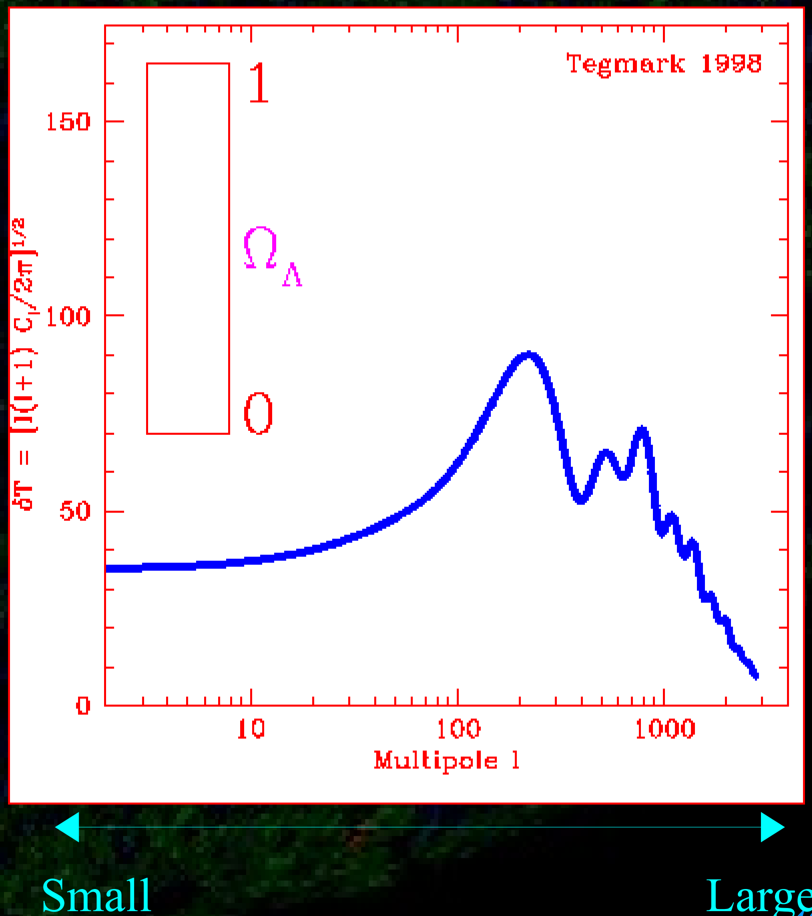


Small Large

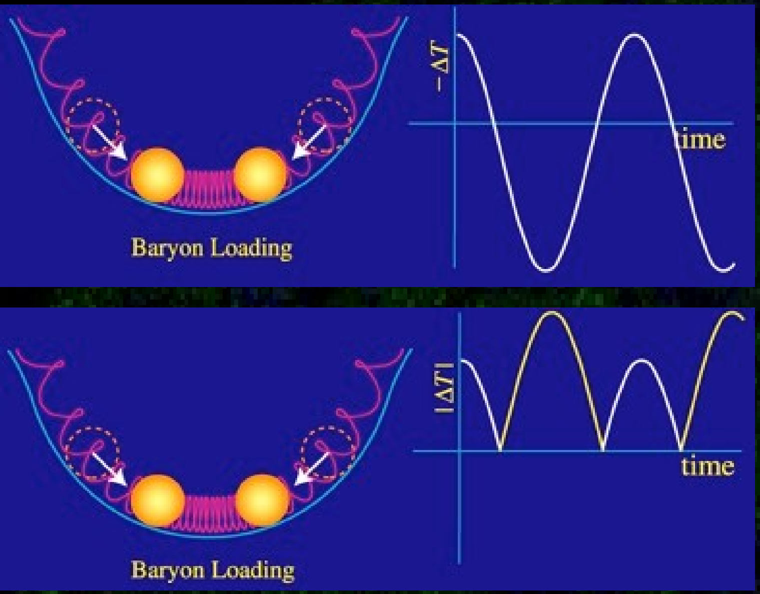
Cosmological Constant

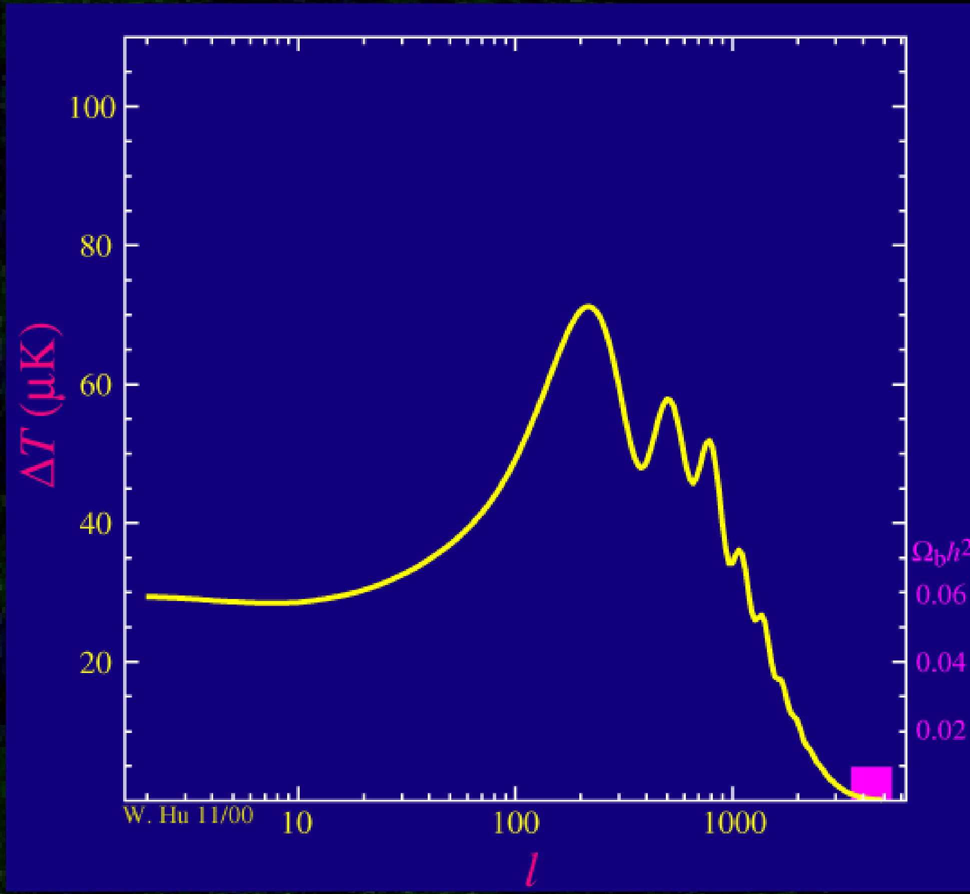
- Cosmological constant increases expansion speed, structures appear larger





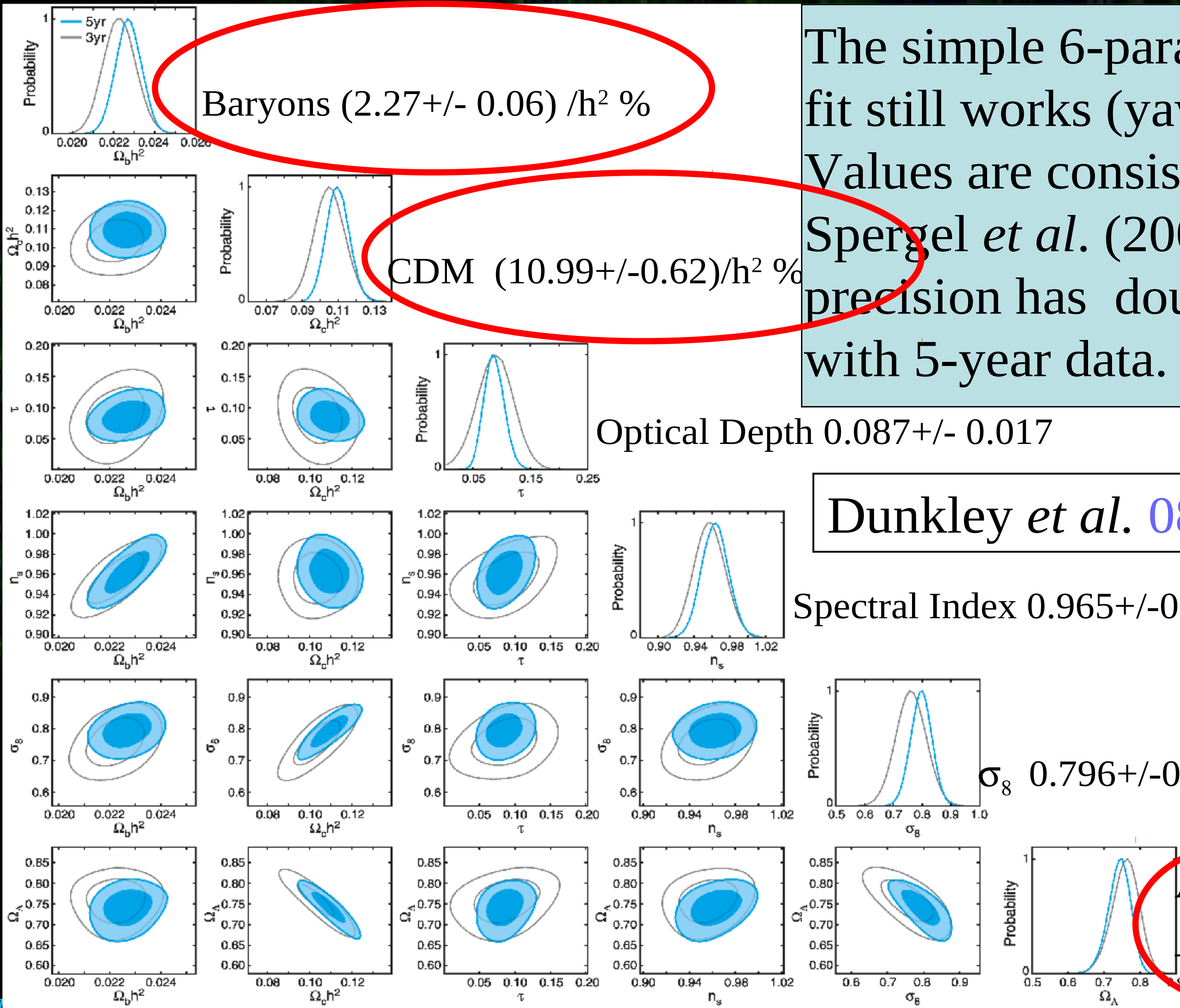
Matter Content





- Coupling between matter and radiation affects oscillation pattern

WMAP Parameters



Baryons (2.27+/- 0.06) /h² %

CDM (10.99+/-0.62)/h² %

Optical Depth 0.087+/- 0.017

Spectral Index 0.965+/-0.014

σ_8 0.796+/-0.036

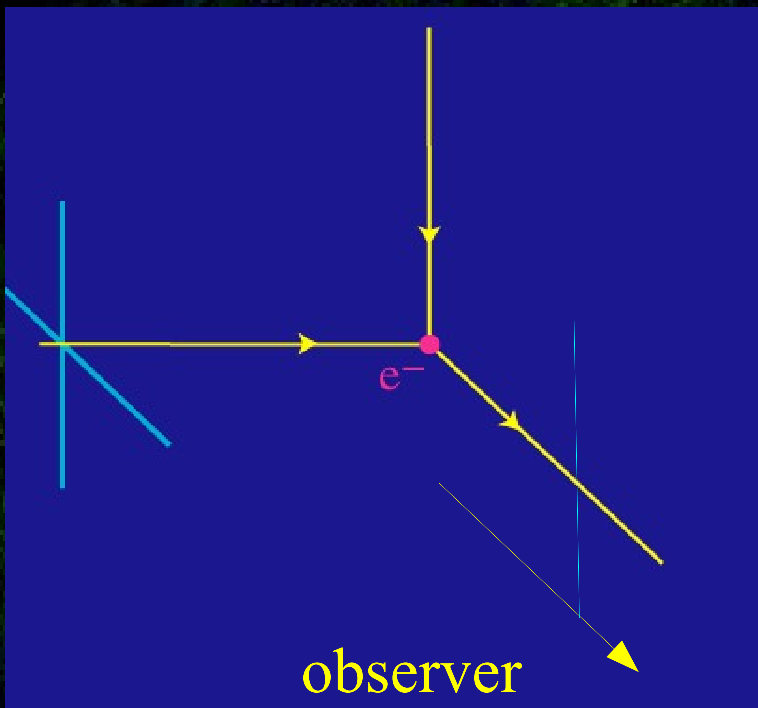
Λ 0.742 +/– 0.036

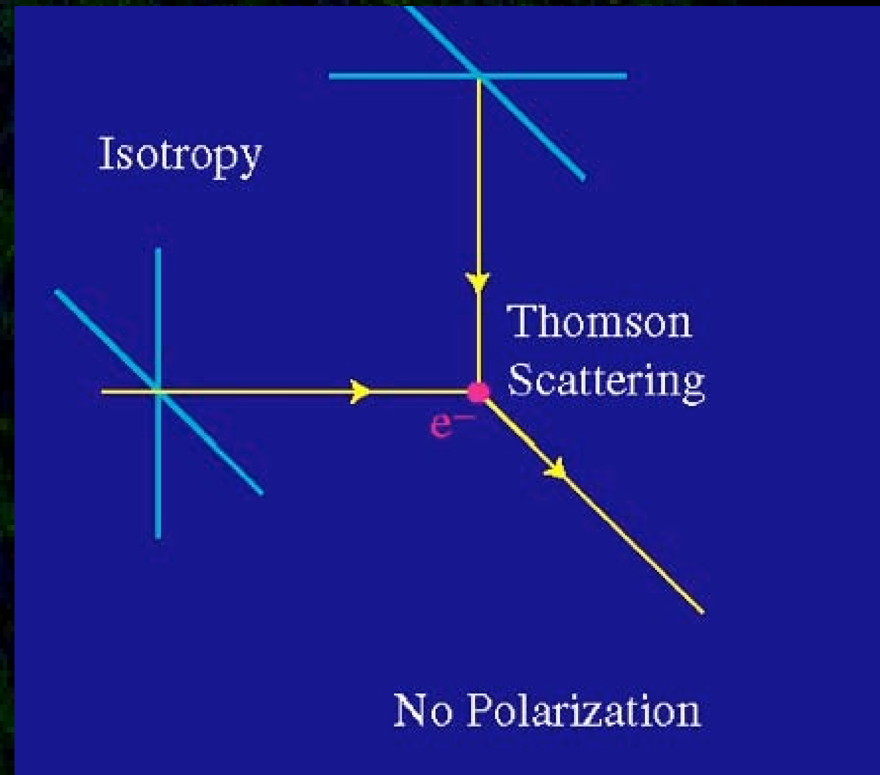
The simple 6-parameter fit still works (yawn). Values are consistent with Spergel *et al*. (2007) and precision has doubled with 5-year data.

Dunkley *et al*. 0803.0586

The CMB is polarized (~10%)

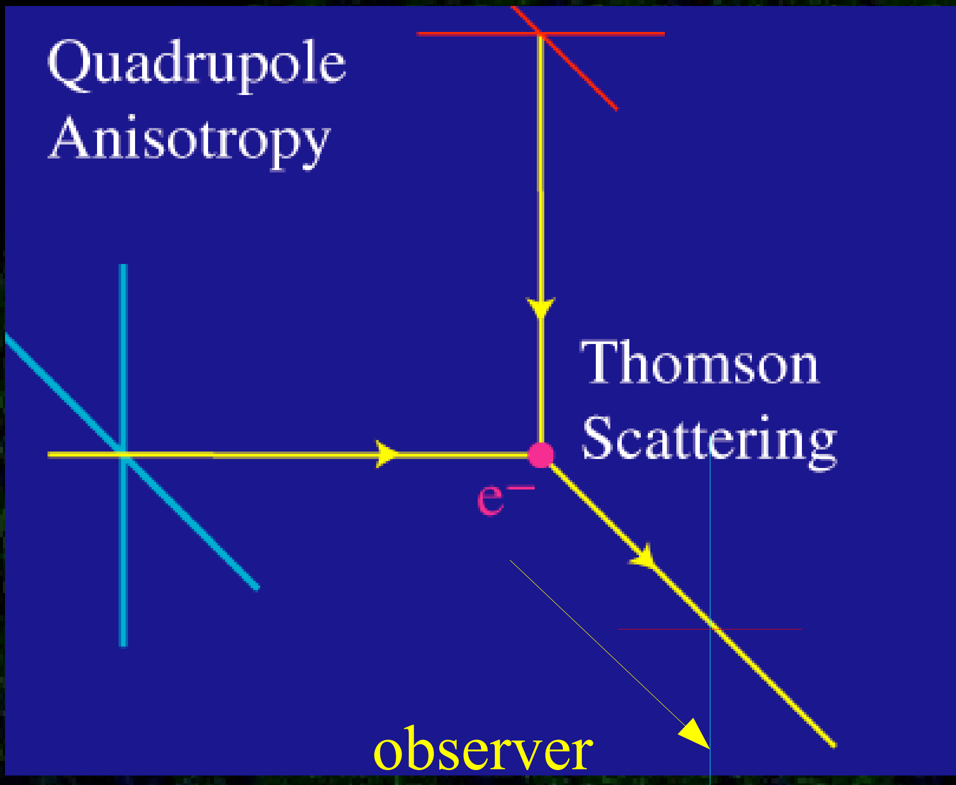
W. Hu, N. Ponthieu

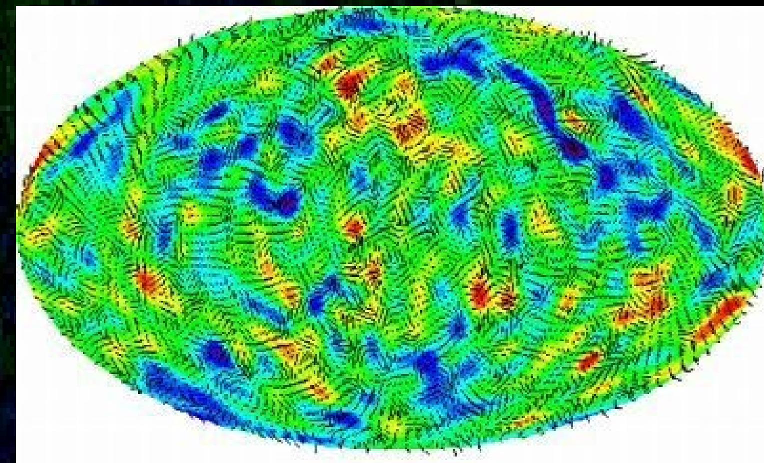


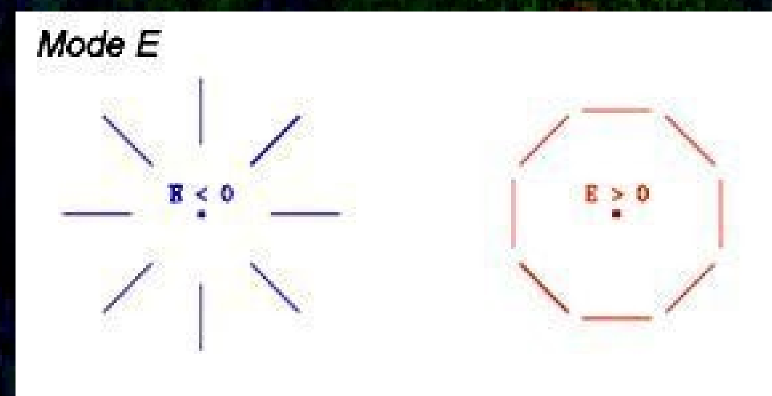


- Since photons only have two polarization states, the Compton scattering results into linear polarisation for orthogonal scattering.
- Isotropic distribution of seed photons results in NO polarisation

The CMB is polarized

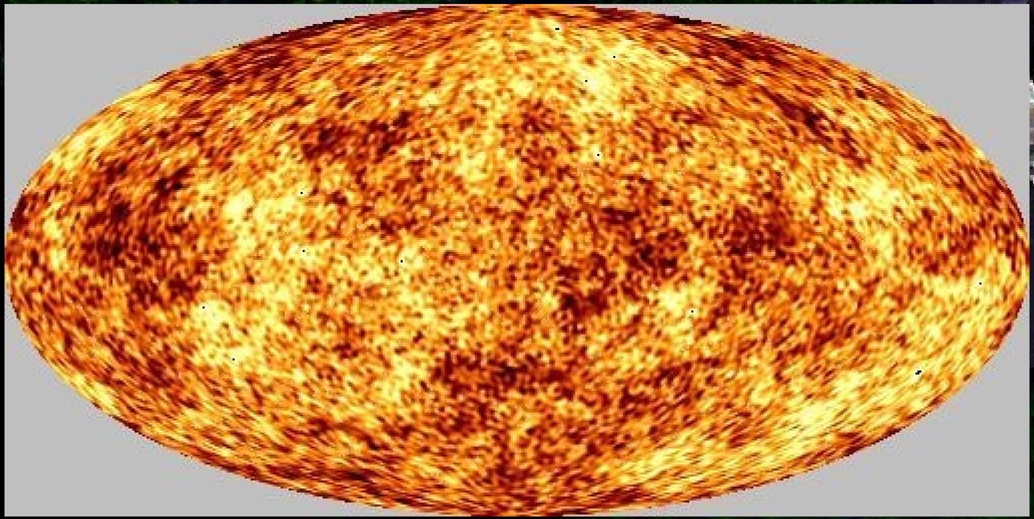




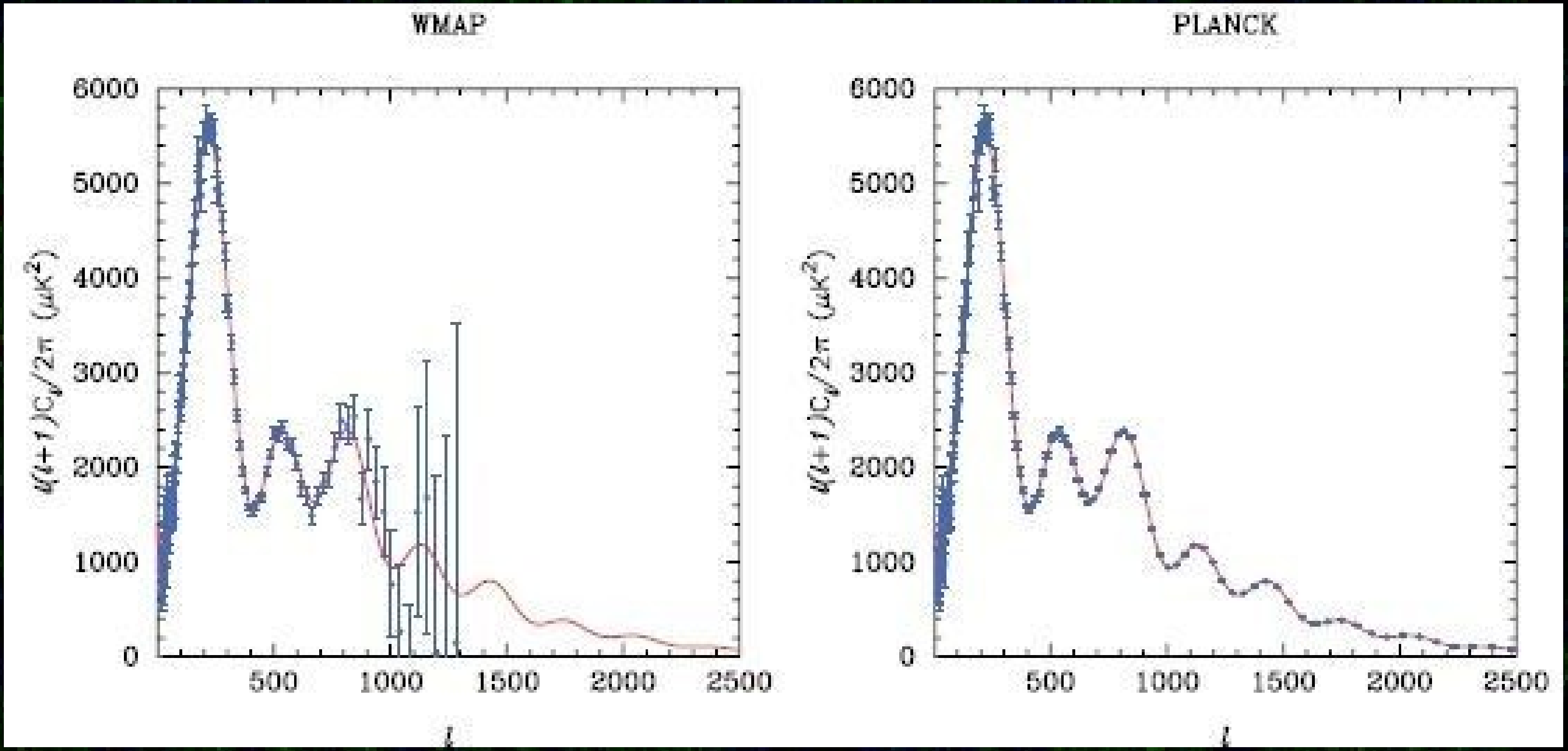


- Temperature anisotropies at the last scattering surface are induce partial polarisation
- Polarisation map is correlated with temperature map

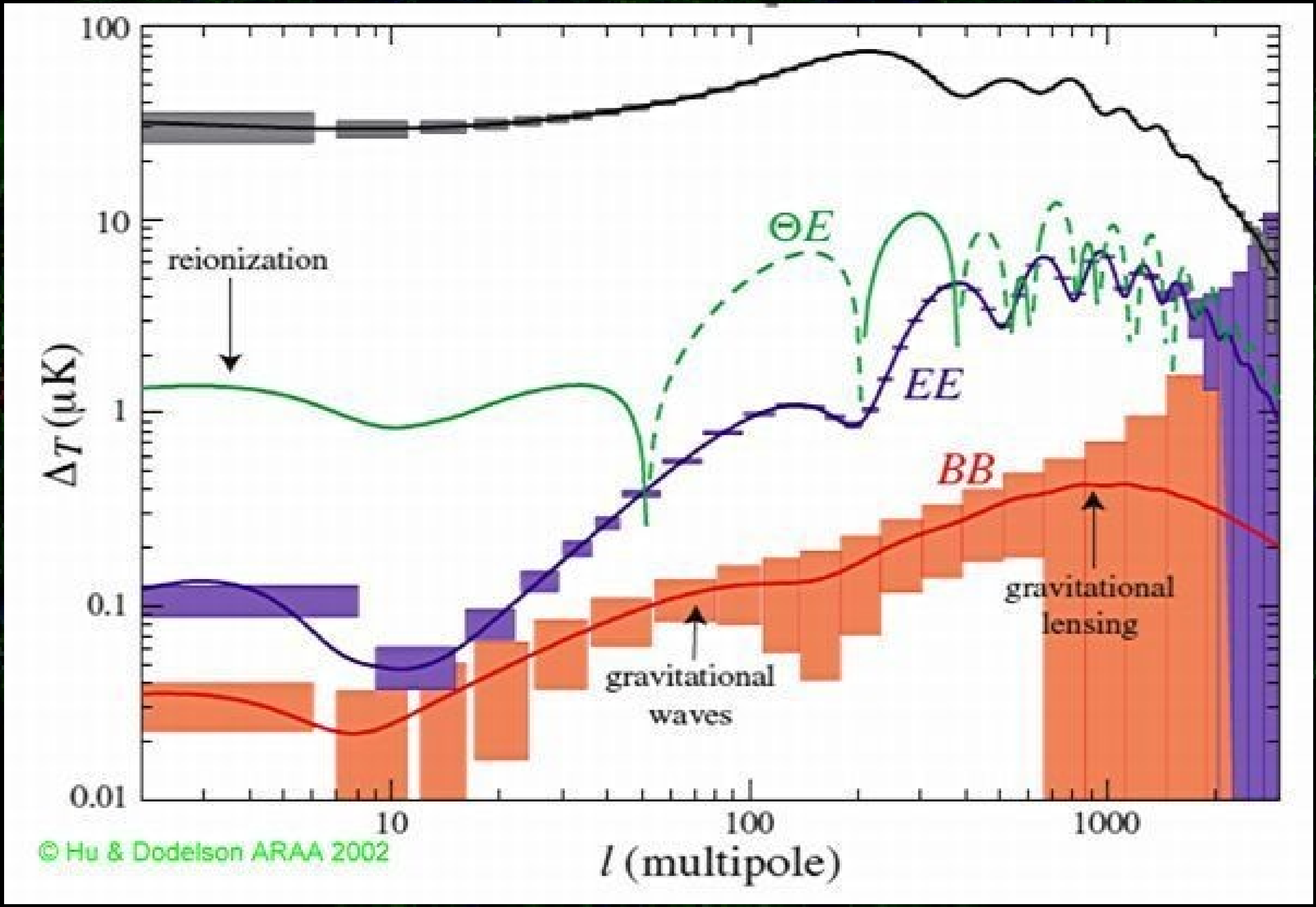
After WMAP: Planck

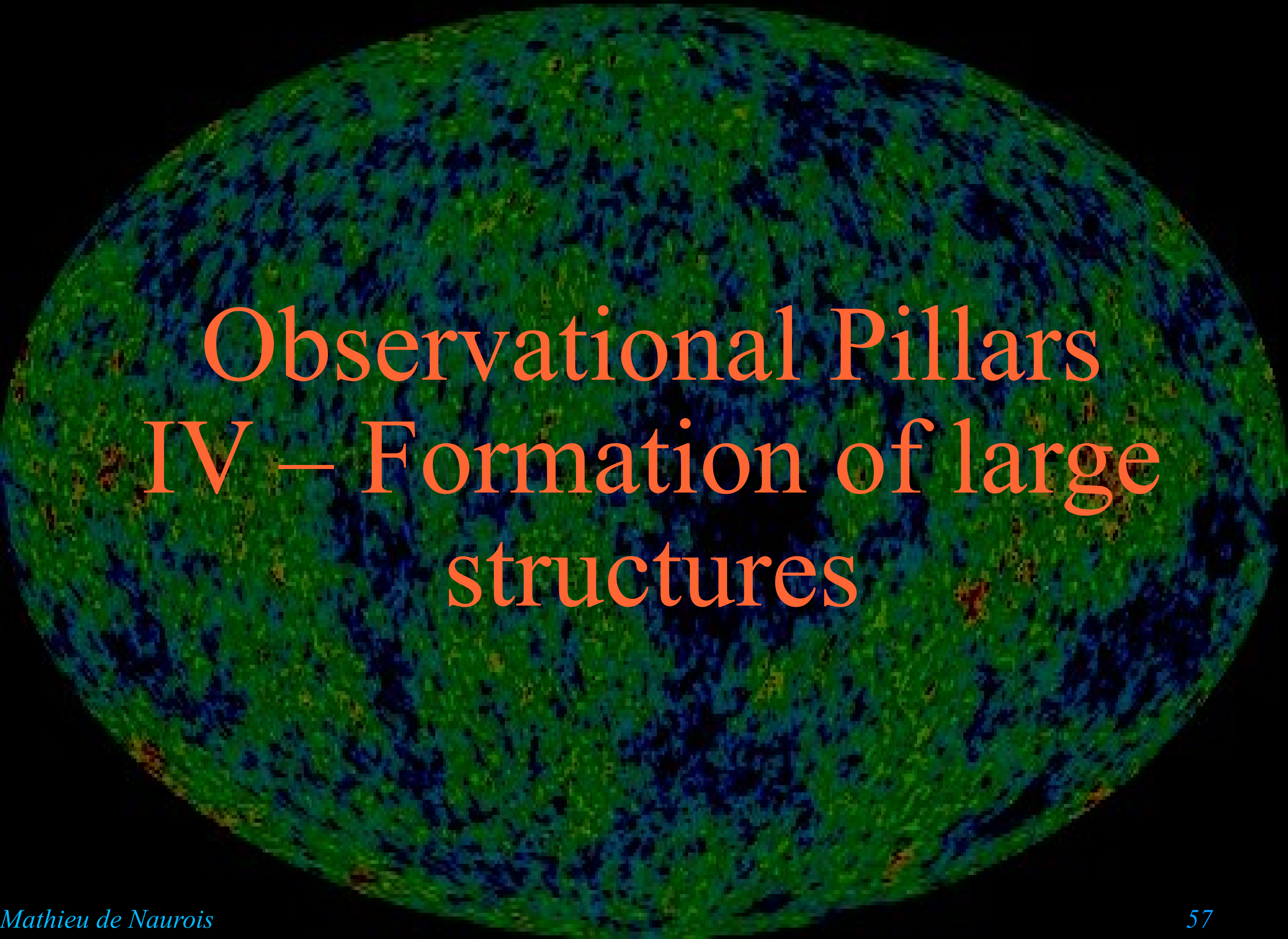


Planck Capabilities



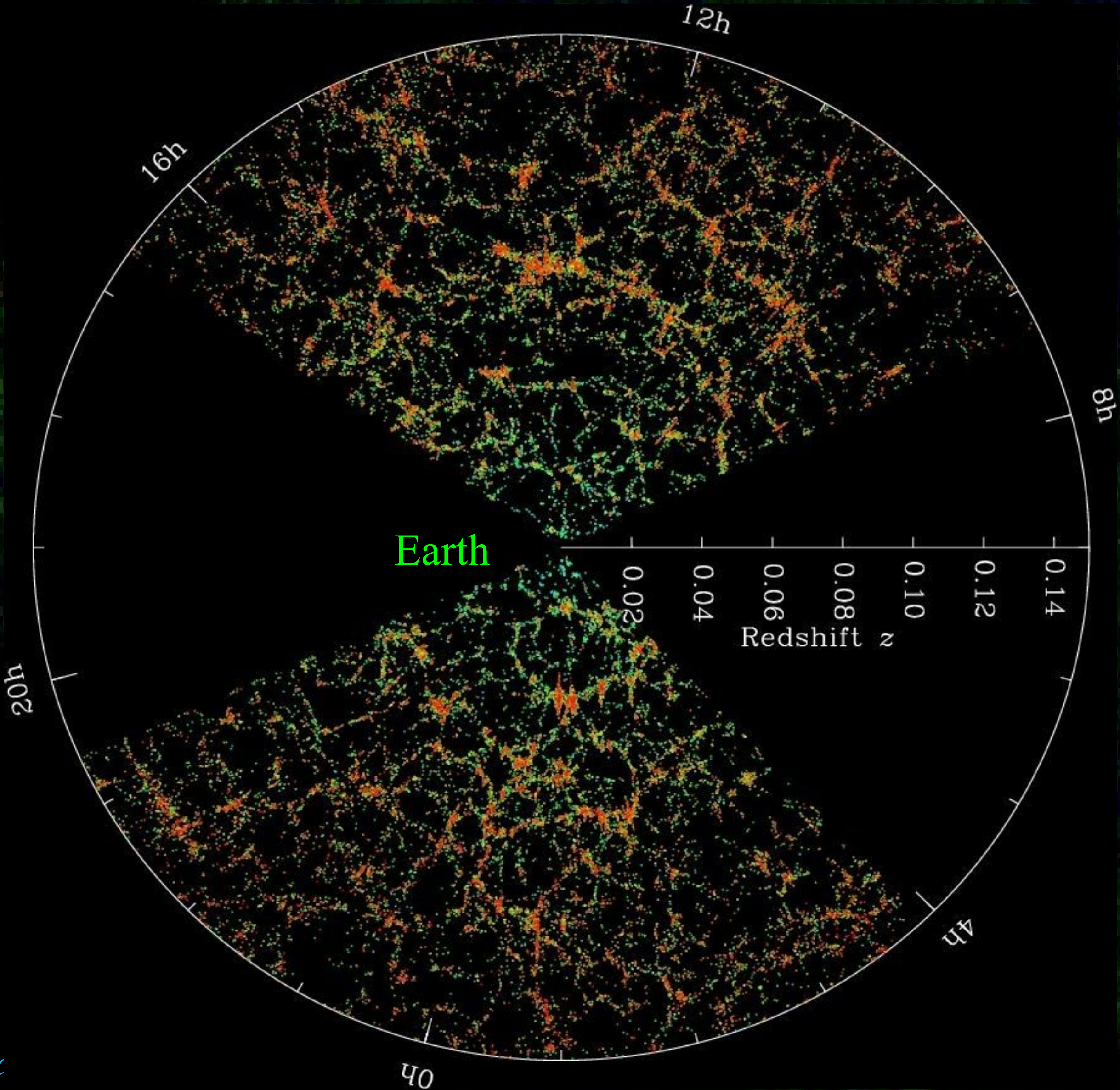
Polarisation @ Planck





Observational Pillars IV – Formation of large structures

Distribution of matter

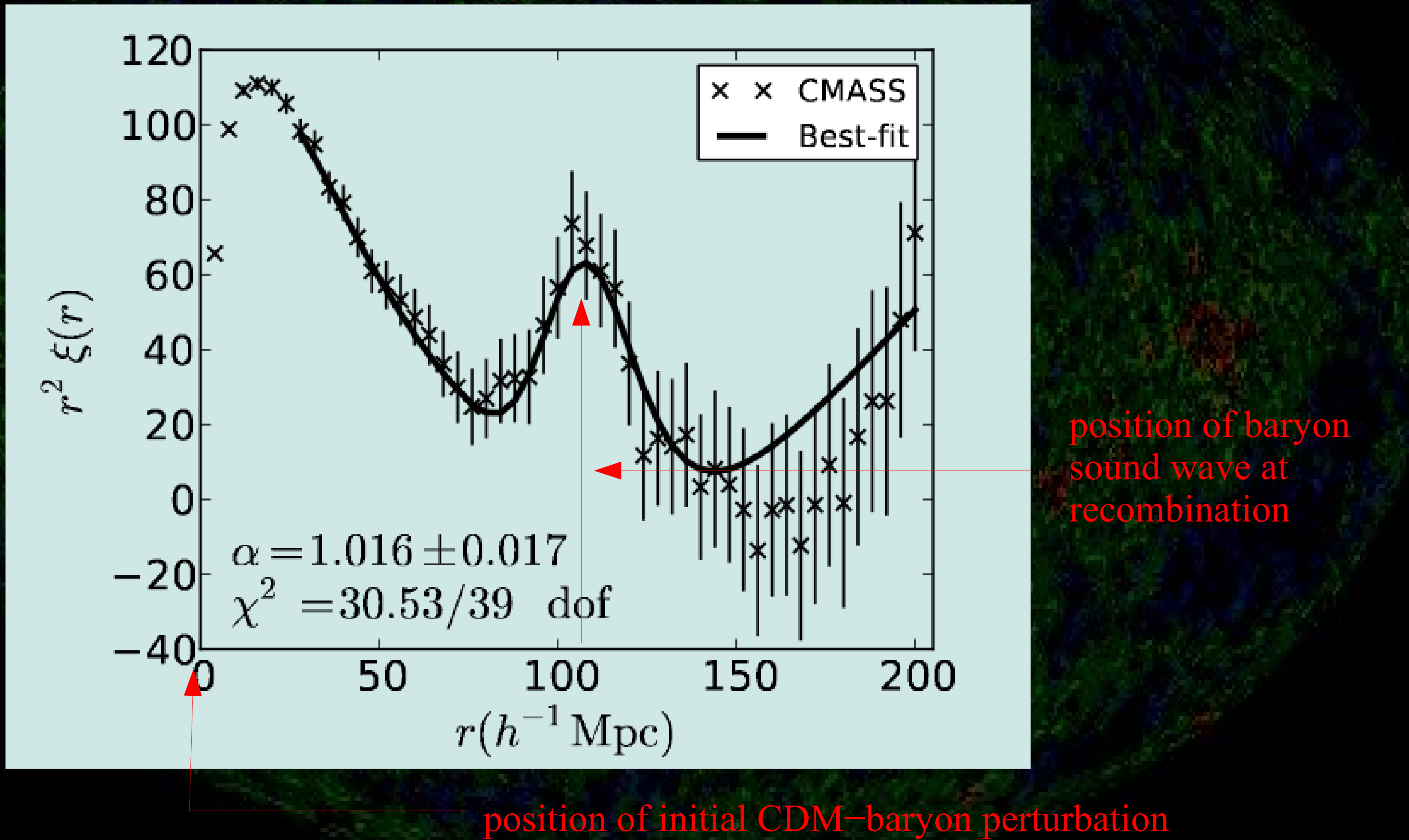


SDSS 2D Map,
of galaxies

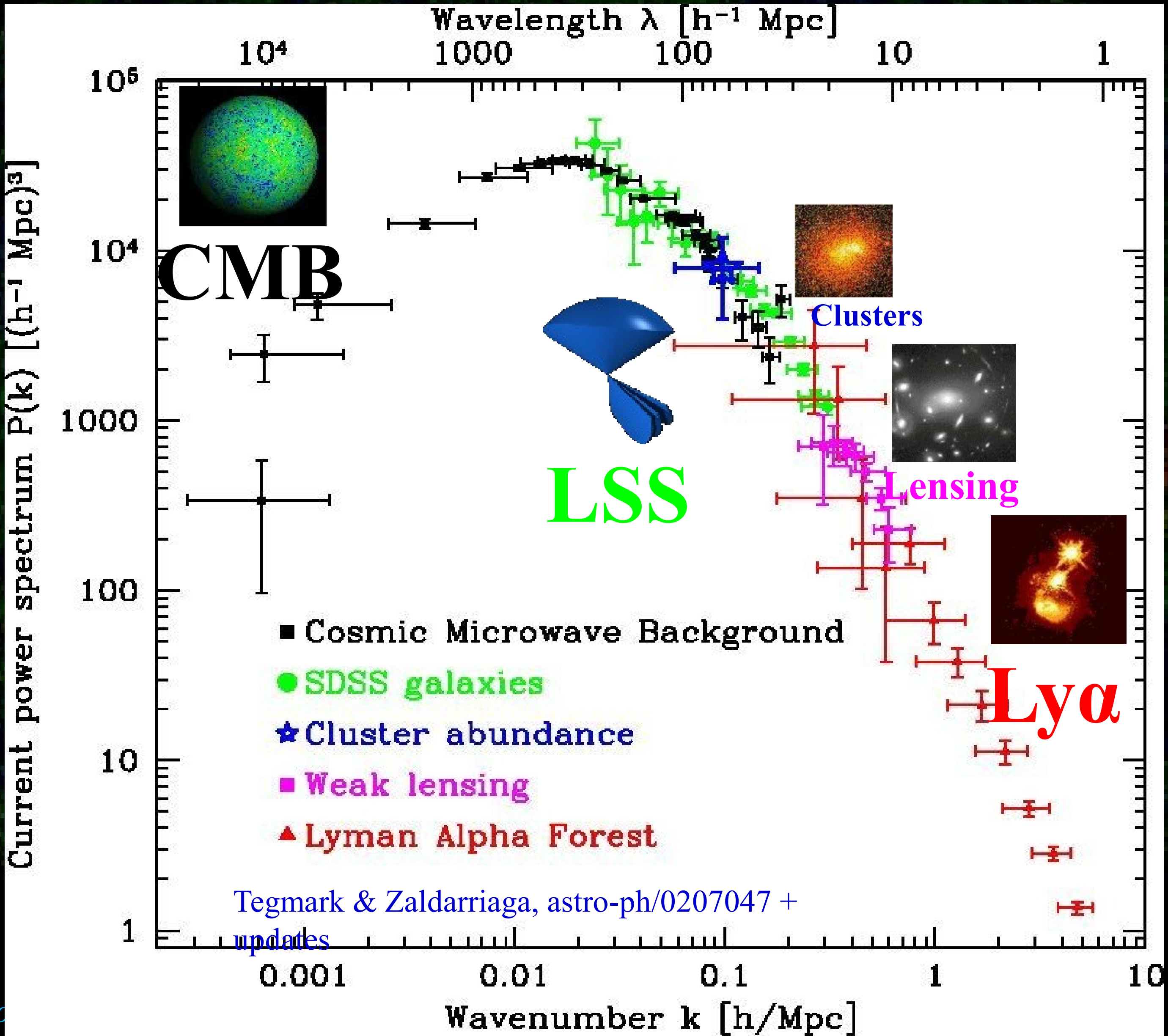
3D Map contains
930 000 Galaxies

Baryonic Oscillations

- The acoustic peak of the CMB is also visible in the Galaxy distribution



Clustering….



Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
10
1
Current power spectrum P(k) [(h⁻¹ Mpc)³]
CMB
LSS
Clusters
Lensing
Lyα
Cosmic Microwave Background
SDSS galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
Tegmark & Zaldarriaga, astro-ph/0207047 + updates
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]

Formation of structures

- Massive simulations try to reproduce the distribution of matter in Universe

http://cosmicweb.uchicago.edu

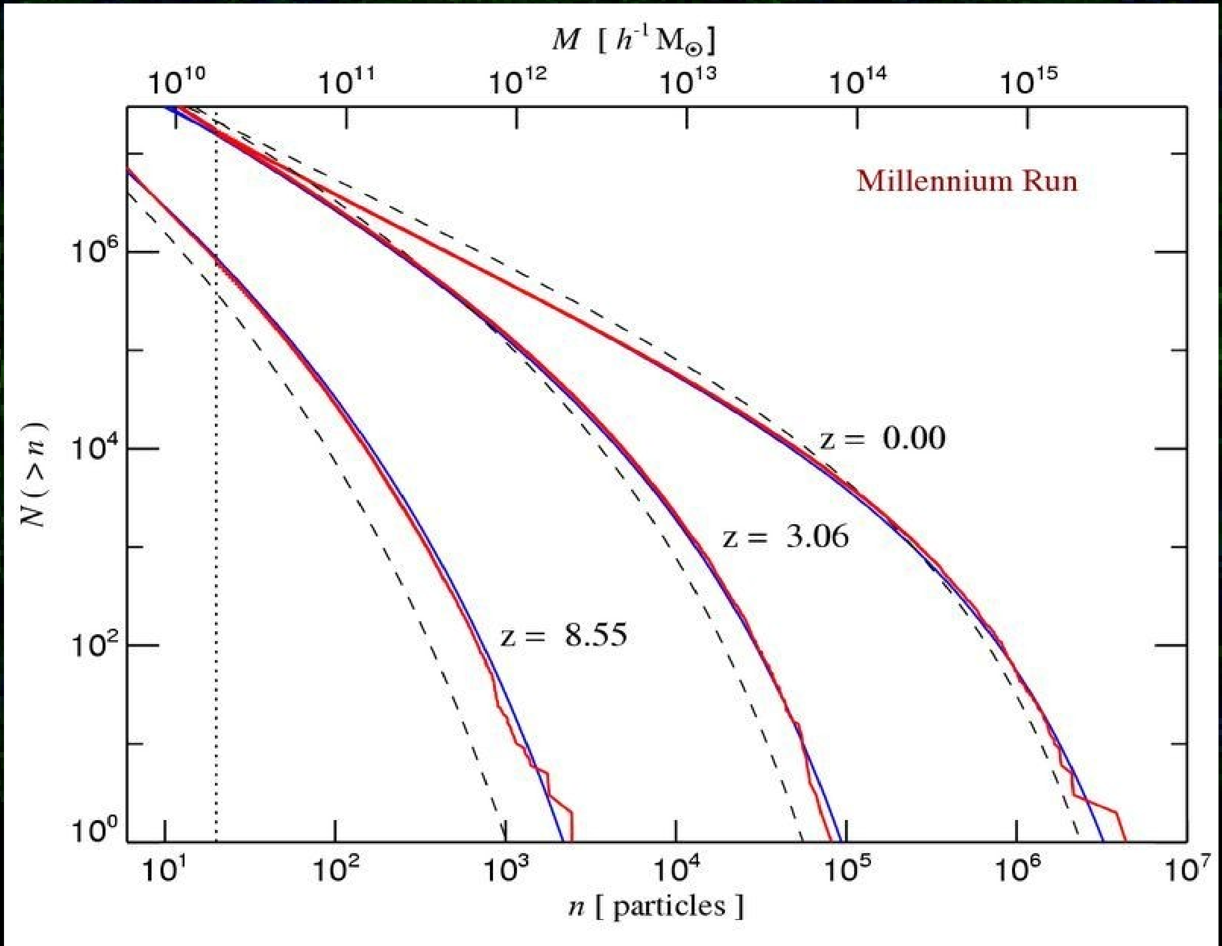
Simulated Universe

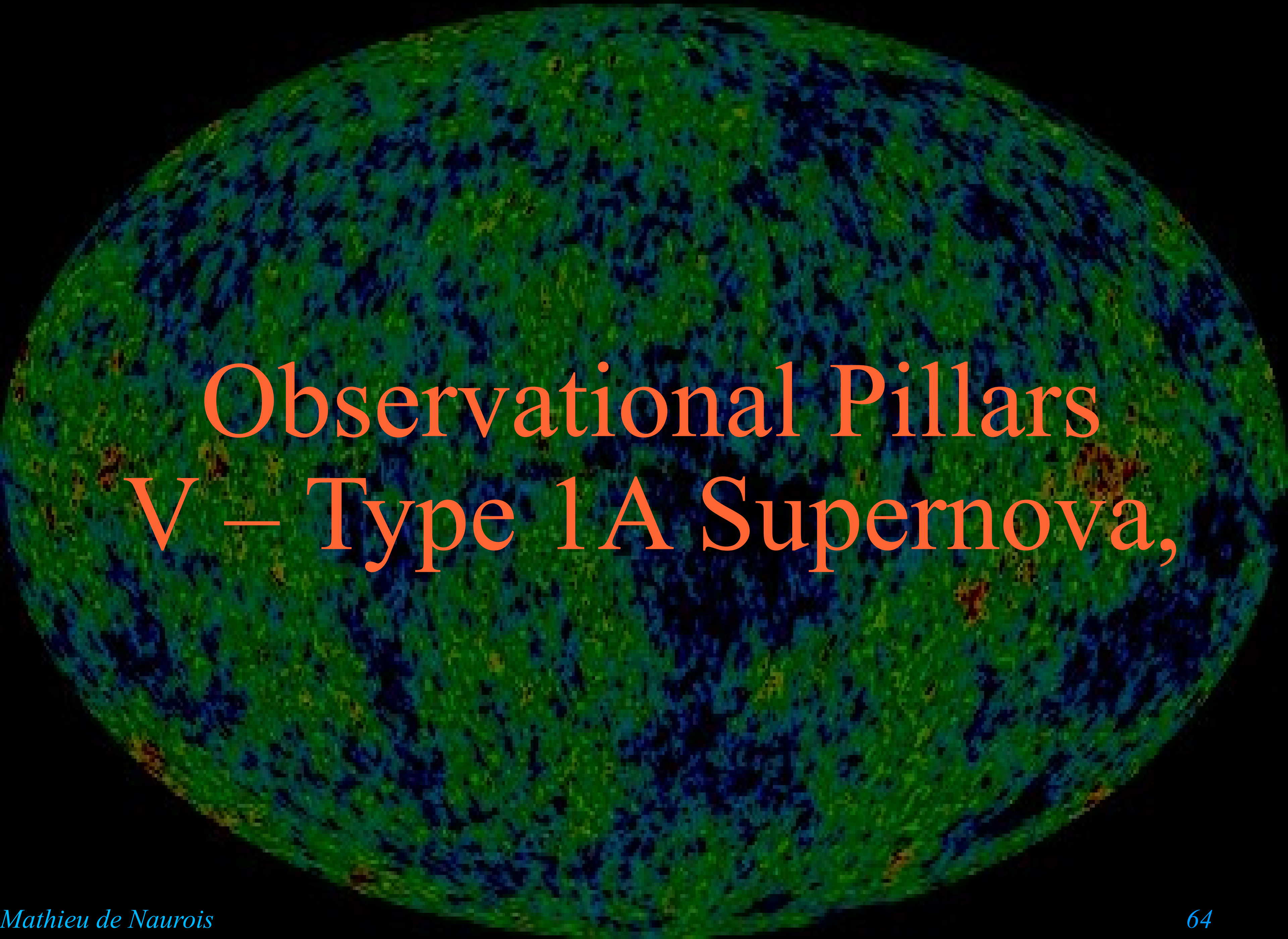
- Dark matter is the driver for structure formatione

Millennium Simulation, Springel et al. (2005) ,

Mass Function

- Observed mass distributions well reproduced when incorporating dark matter
- In the absence of dark matter, predicted structures are too small



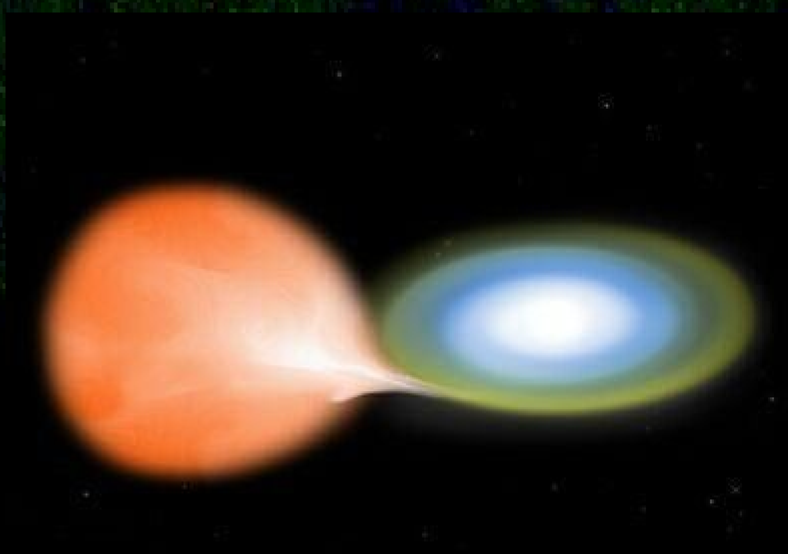


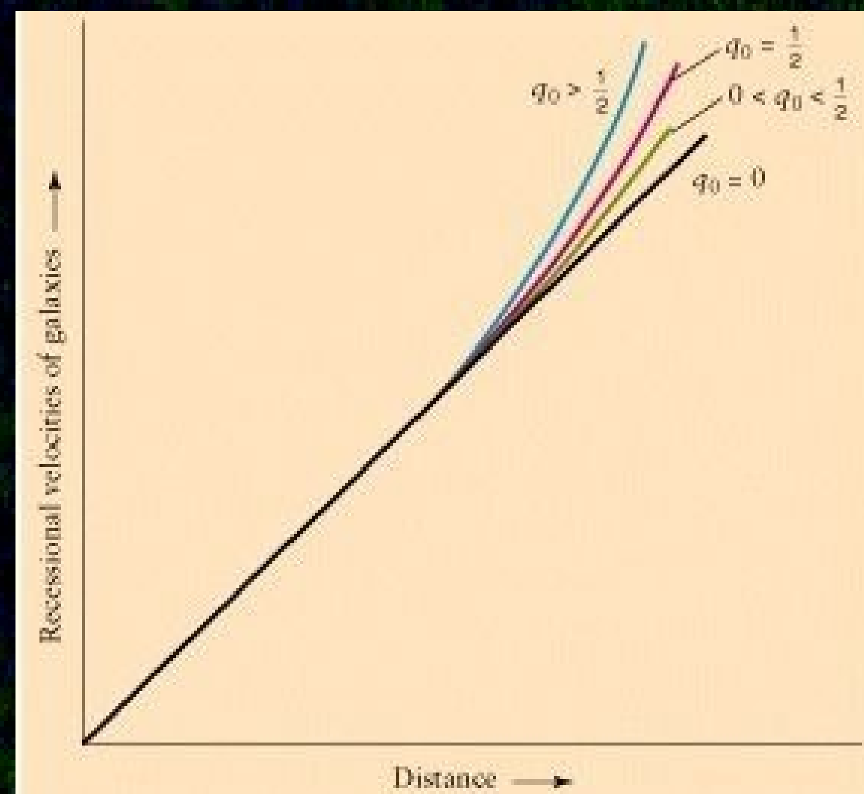
Observational Pillars
V – Type 1A Supernova,

Type Ia Supvernova

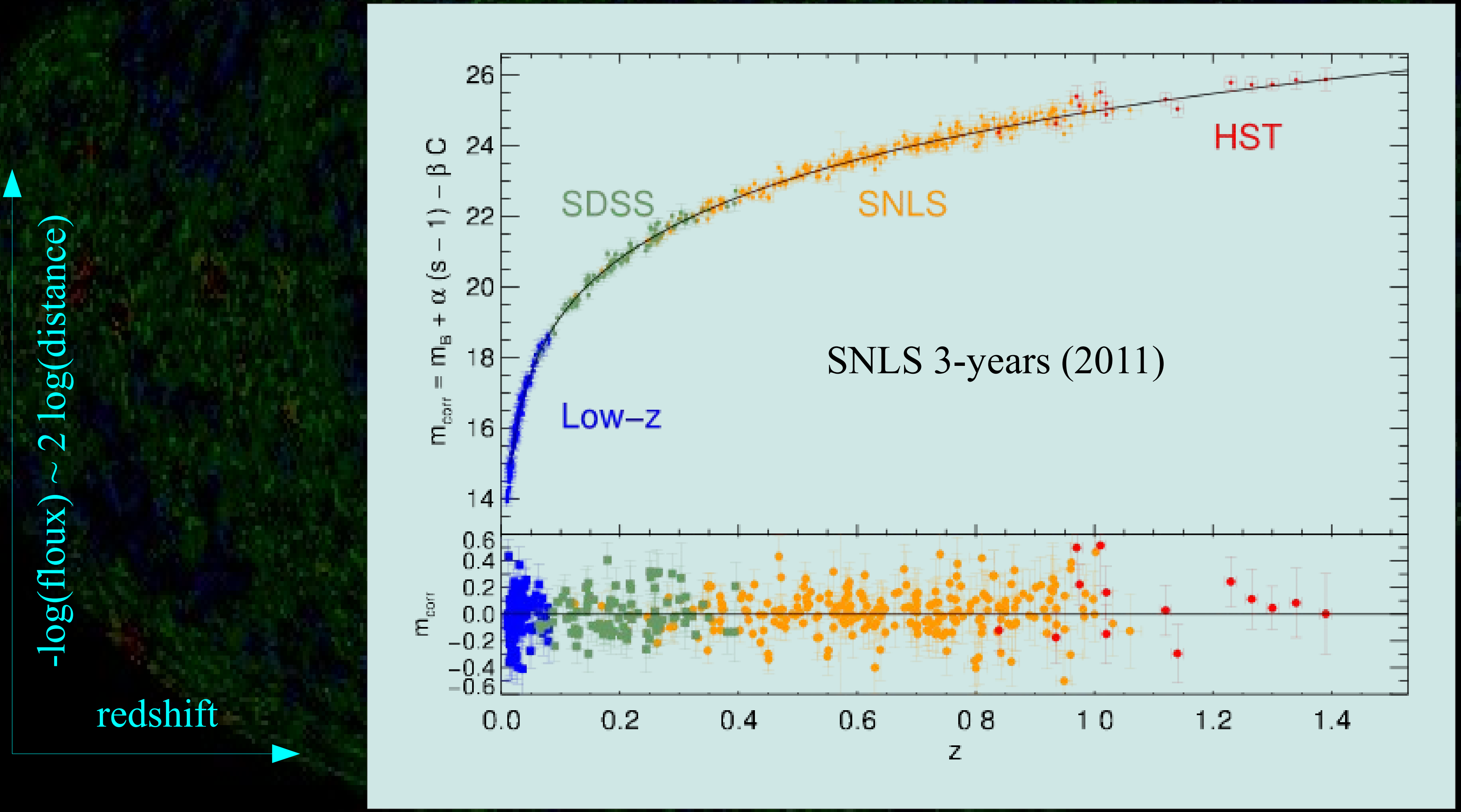
- Accreting white dwarf exploding when reaching the Chandrasekhar mass
- Almost Standard Candles
- Luminosity – redshift relation is related to history of the Universe

$$d \propto \int \frac{\mathrm{d}\, z}{H(z)}$$

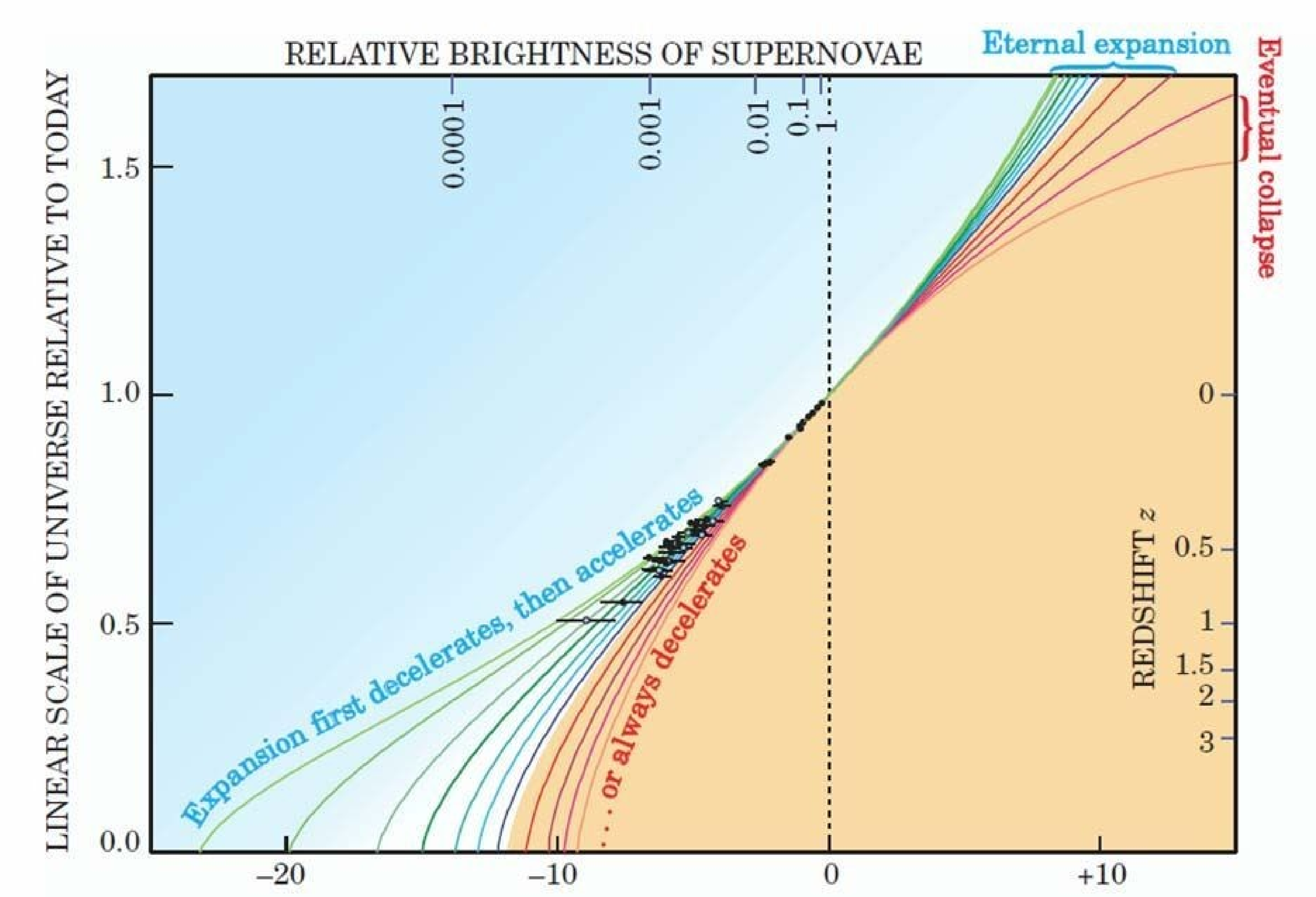




Hubble Diagram

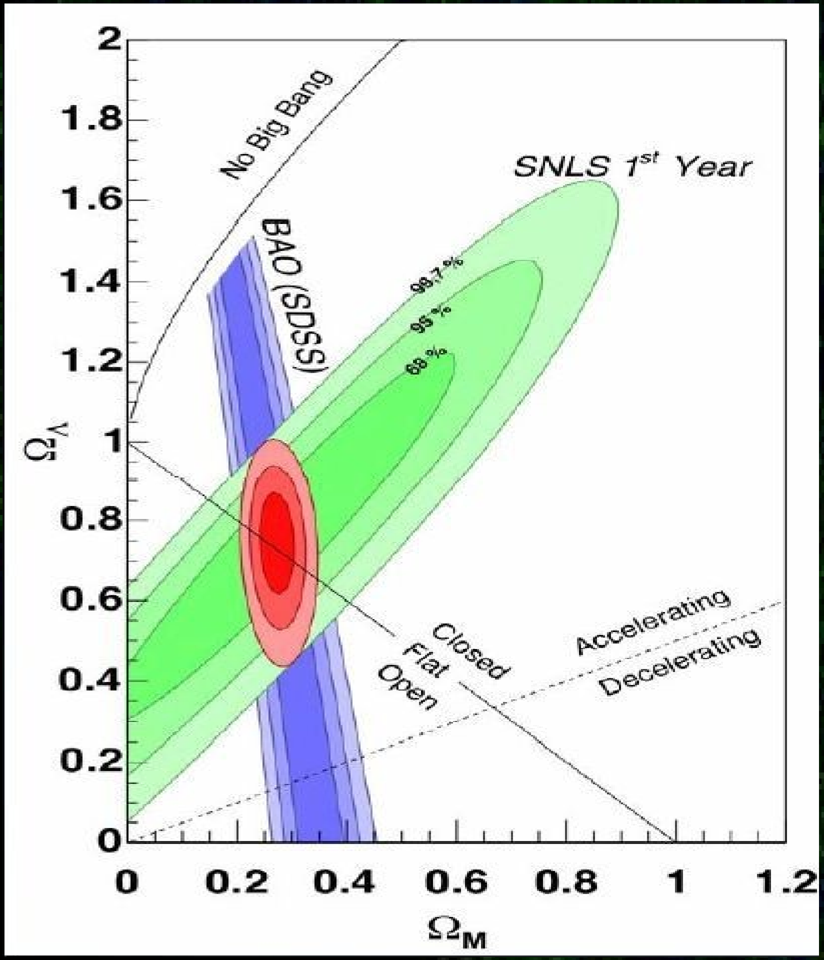


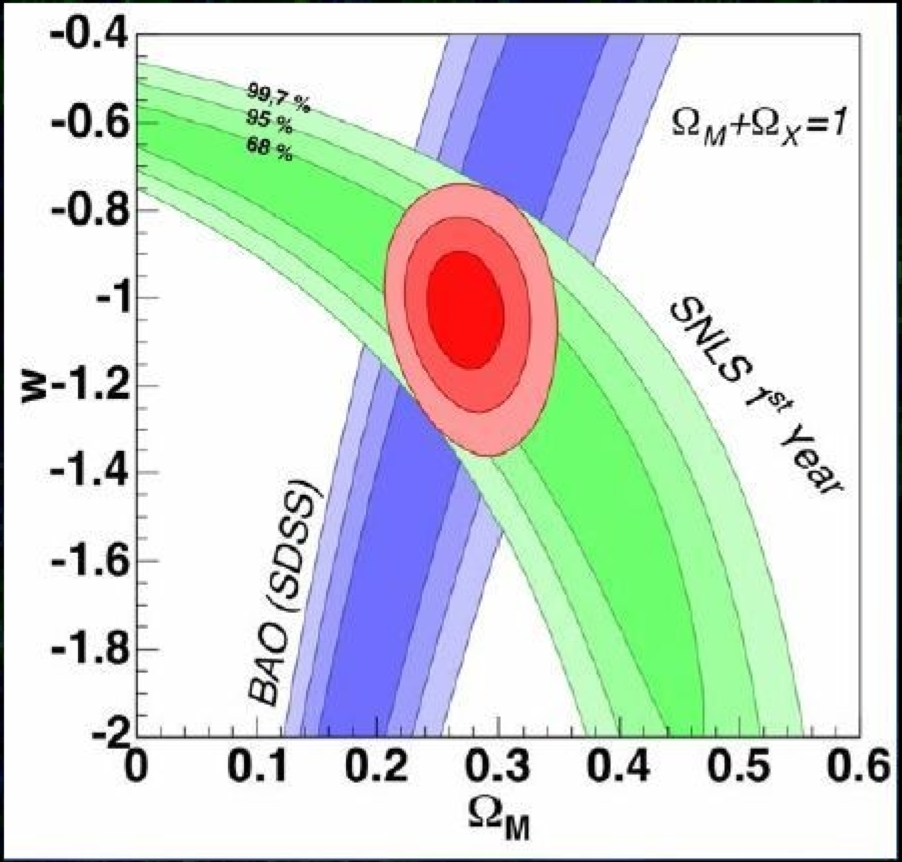
SNI1a: Universe in accelerated Expansion



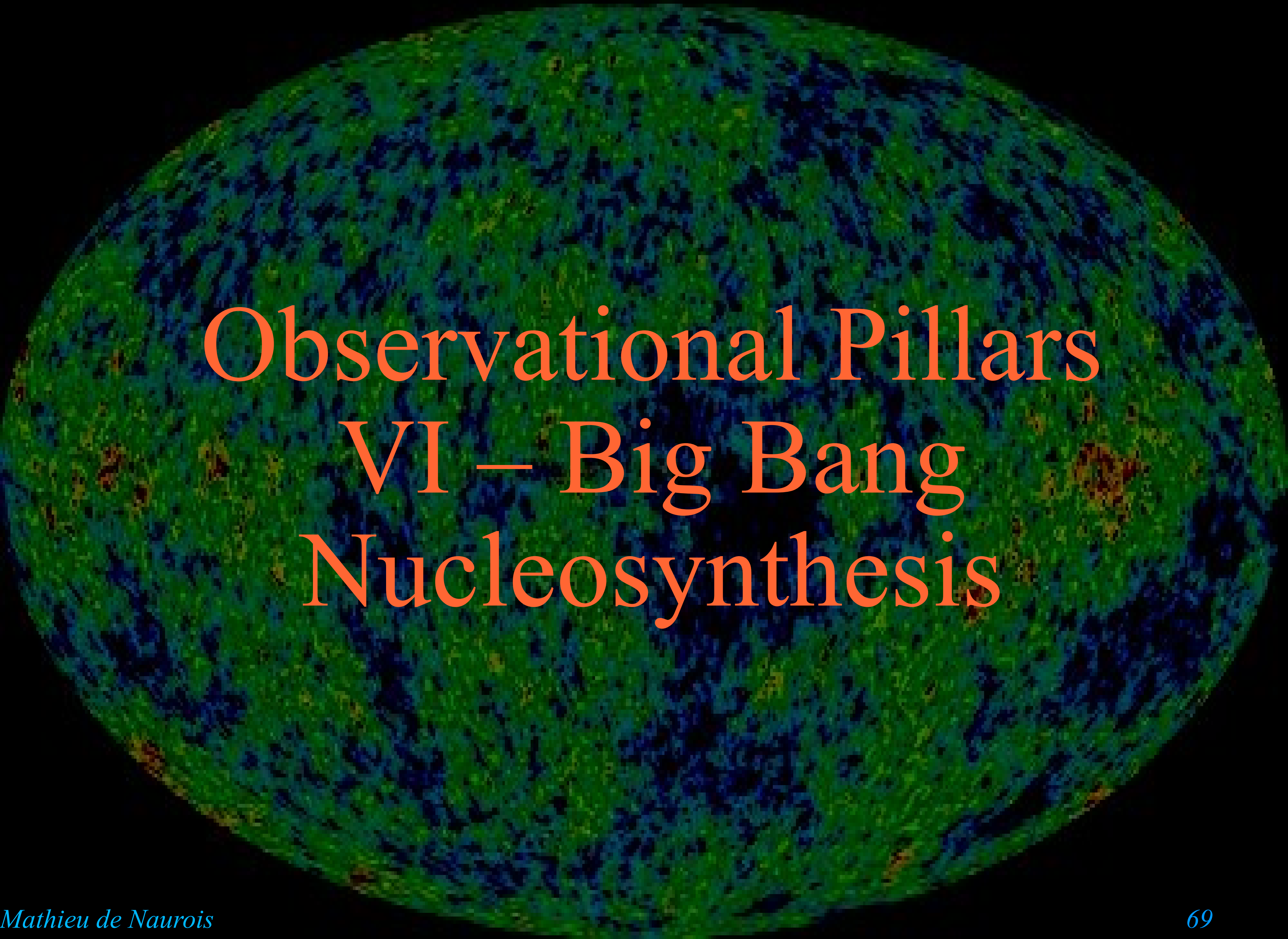
CFHTLS / SNLS

- Toward a FLAT universe, with cosmological constant (only usng supernova and baryonic oscillations)





Observational Pillars VI – Big Bang Nucleosynthesis



Baryogenesis

- The observed abundances of light elements according to mass fraction are:
  - Hydrogen 75%
  - Helium 24%
  - Metals ~1%
- Why?
- BBN happens on small scales at energies below 10 MeV, hence we should have complete control over the physics (unlike the very early Universe).
- BBN predictions are very sensitive to ambient conditions at t ~ 1 sec (T~ 1 MeV). Hence the constraints on new physics are some of the best available...

Baryogenesis

- Relative abundances:

$$M_i \propto (m_i T)^{3/2} \exp\left(\frac{-m_i}{T}\right)$$

- Equilibrium ratio of neutrons to protons

$$\frac{n}{p} \approx \exp\left(-\frac{Q}{T}\right) \quad \text{where} \quad Q = m_n - m_p \approx 1{,}29\,\text{MeV}$$

- At high T, n ~ p, whereas at low T, n/p → 0
- In competition with expansion of the Universe
- In competition with neutron decay

Equilibrium ?

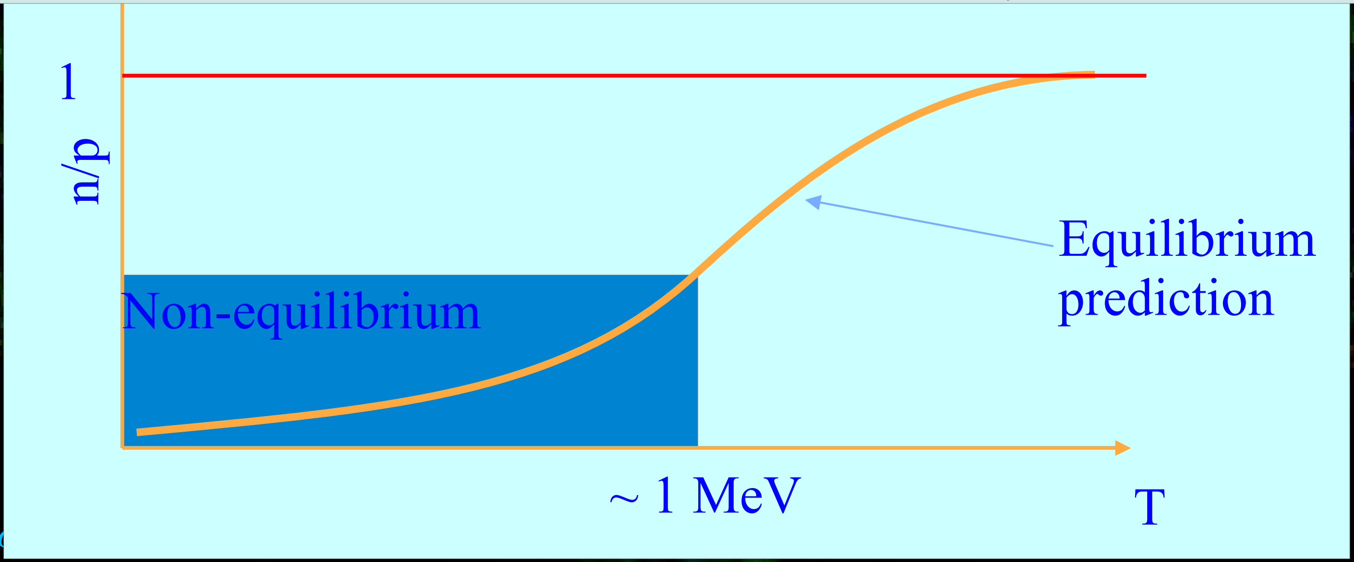
- Equilibrium condition valid only when reaction rate is large enough

$$\Gamma > H = \left(\frac{\dot{a}}{a}\right)$$

$$n + \nu_e \Leftrightarrow p + e^-$$

- Freeze-out temperature

$$\left.\begin{array}{l}\Gamma = G_F^2 T^5 \sim 10^{-10}\,\mathrm{GeV}^4 T^5 \\ H \sim T^2 / M_{pl} \quad \text{where} \quad M_{pl} = 10^{19}\,\mathrm{GeV}\end{array}\right\} \Rightarrow T_c = 0{,}8\,\mathrm{MeV}$$



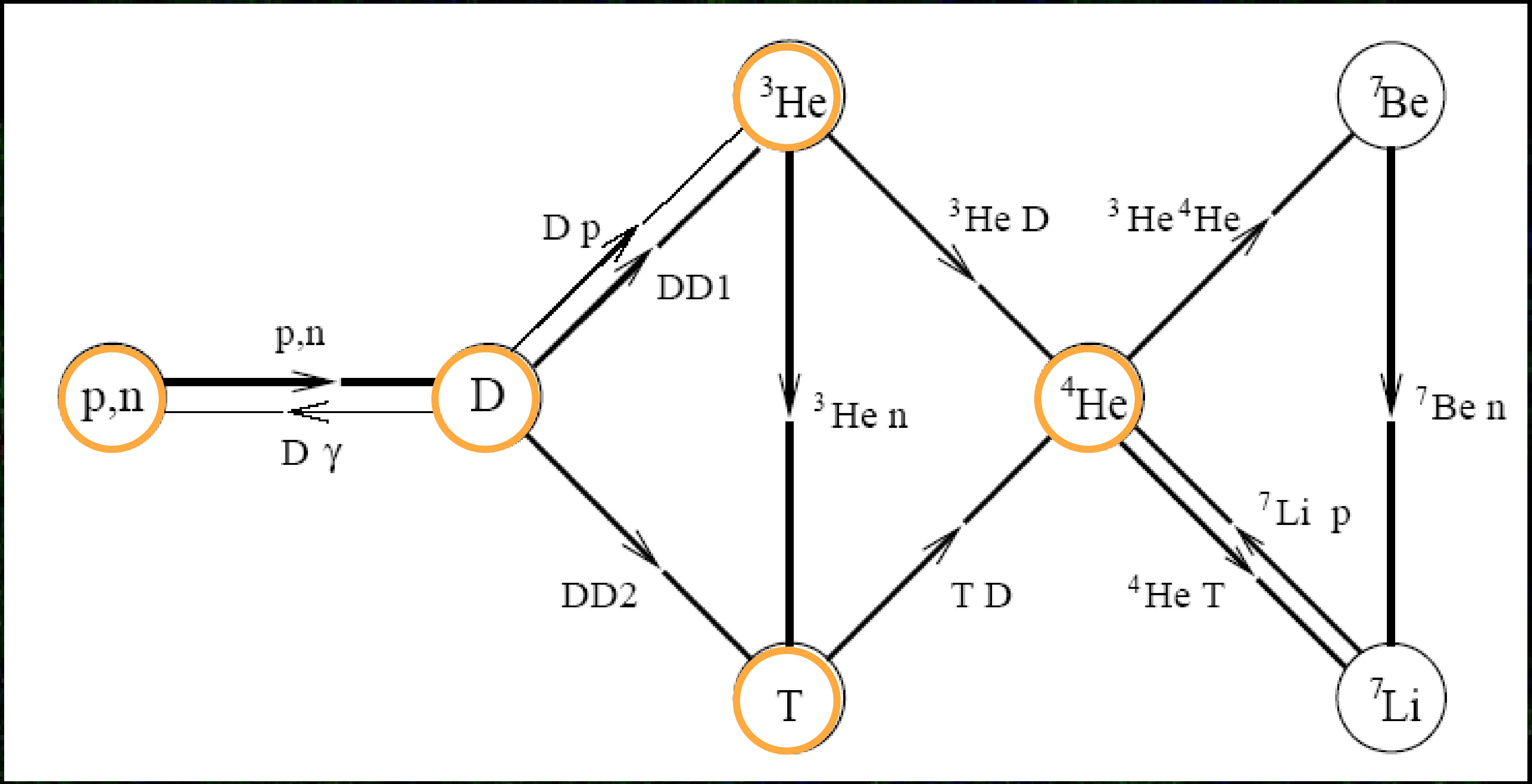
Predicted Ratio

- At freeze-out

$$\frac{n}{p} \approx \exp\left(-\frac{Q}{T_c}\right) = \exp\left(-\frac{1{,}29}{0{,}8}\right) \approx 1/5$$

- With time this decreases slightly to ~ 1/6. due to neutron decay:
- Hence, at most we could form 33% of ^{4}He by mass(using all available neutrons) which is significantly larger than the observed 24%. Why is there only 24% helium?

Going to Heavy Elements



- No through $2n + 2p \rightarrow {}^4\text{He}$
- But through Deuterium

Deuterium Bottleneck

- Production of Deuterium is at equilibrium at ~ 1 MeV

$$p+n \Leftrightarrow D+\gamma$$

- Equilibrium depends on the photon to baryon ration. This is the only free parameter in the model:

$$\eta = n_B / n_\gamma \sim 10^{-8} \Omega_B h^2$$

- The theory then predicts, at T ~ 1 MeV, the following abundances:

$$X_D \sim \eta X_p X_n \quad 10^{-12} \quad \text{where} \quad X_D = 2 n_D / n_B$$

- The Universe is still too hot, Deuterium is immediately destroyed by encounter with high energy photons

Deuterium

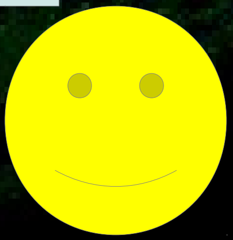
- Helium production starts later in time, when the number of photons above Deuterium binding energy (2.2 MeV) becomes small.
- This happens at T = 0.06 MeV
- Question: if T = 1 MeV at t = 1s, at what age the Universe temperature reaches T = 0.06 MeV?

Deuterium formation

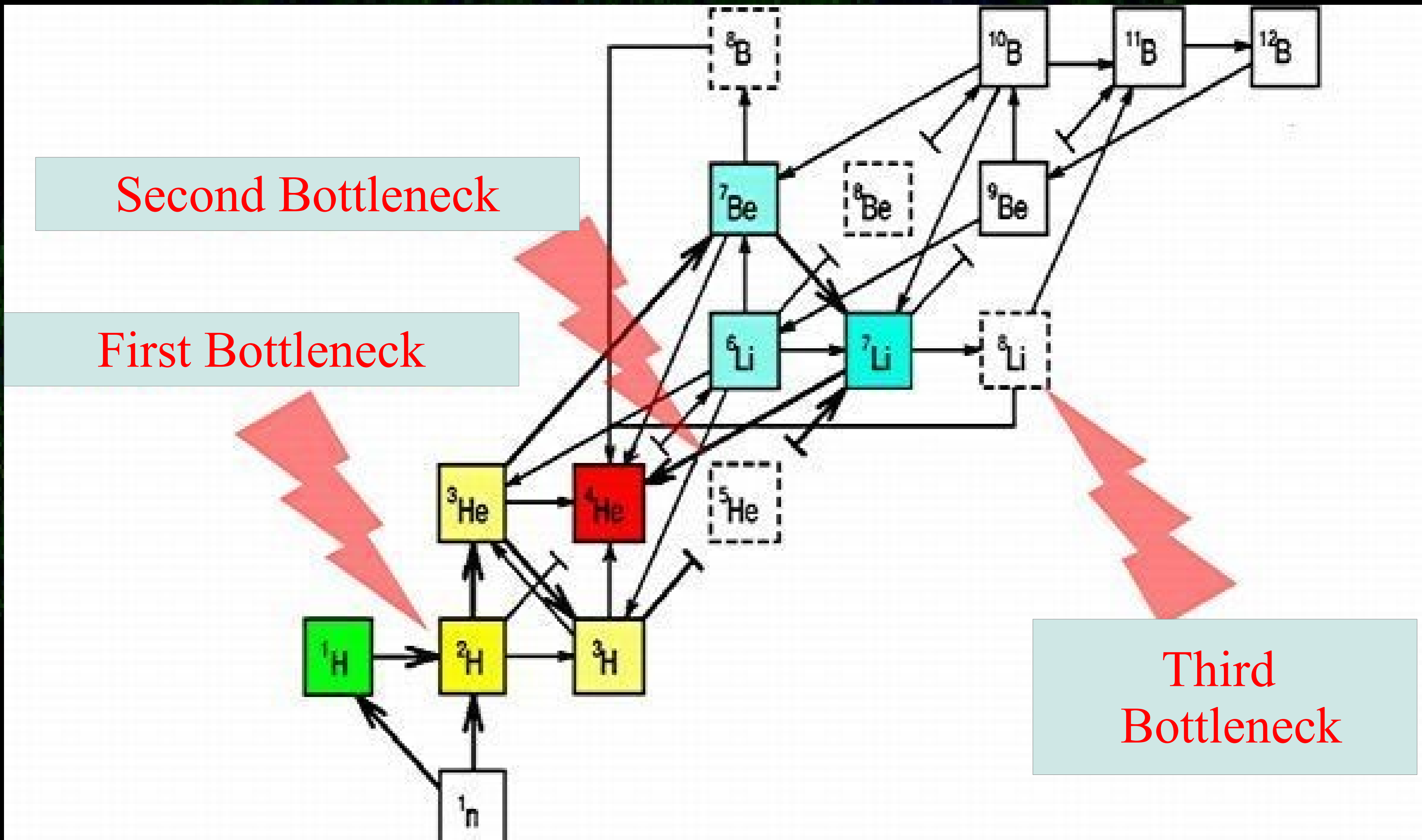
- So... D formation starts later, at t ~ 156 s
- At that time, all neutrons are used to produce Deuterium
- What is the neutron fraction at this time?
- Neutron decay decreased the number: $\tau_n = 885{,}7 \pm 0{,}8\,\mathrm{s}$
- At t = 200 s, the neutron ration decreased to

$$\frac{n}{p} = \frac{n_0}{p_0} \times \exp\left(-\frac{t}{\tau}\right) \approx \frac{1}{6}\exp\left(-\frac{200}{886}\right) = 0.125$$

- So we expect $H_{^4\mathrm{He}} \approx 2 \times (n/p) = 0.25$

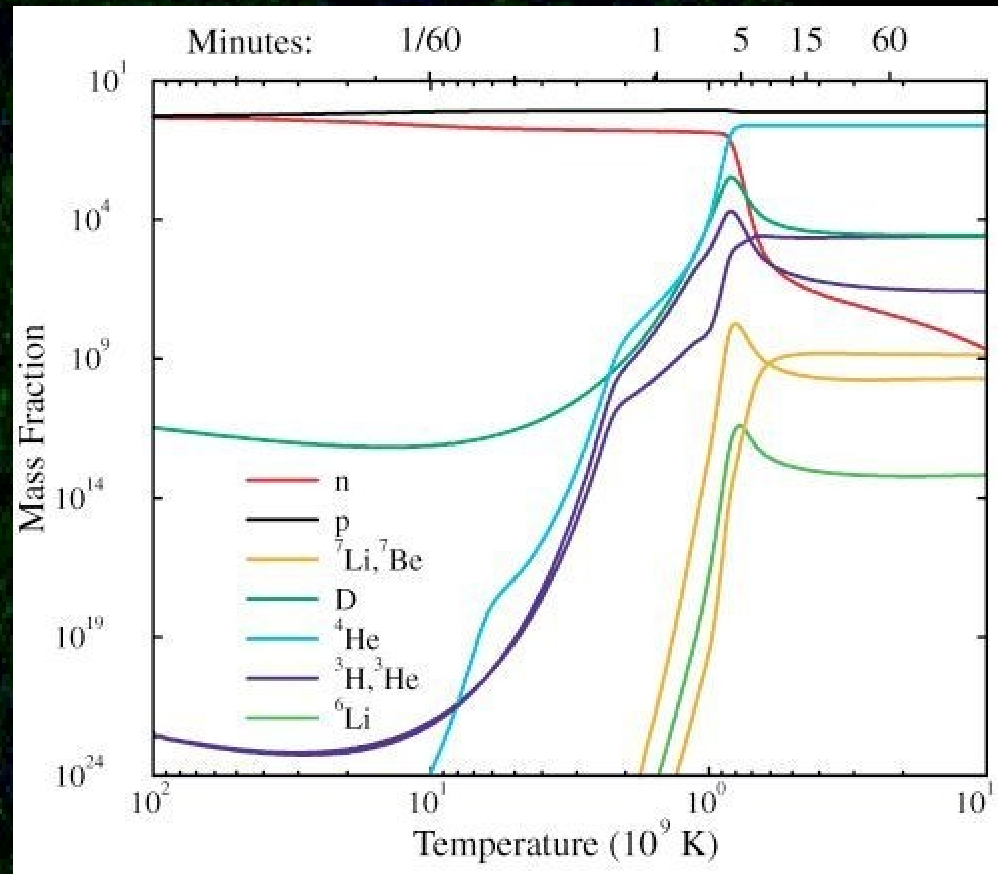


Baryogenesis caveats



Formation of elements

- At 1 MeV, feeeze out of neutron – proton equilibrium
- Neutrons fraction decrease due to decay up to T = 0.06 MeV
- D forms at T~0.08 MeV;
- nuclear chain produced heaver elements



Z \ N	0	1	2	3	4	5	6	7	8
0		n							
1	H	^{2}H	^{3}H						
2		^{3}He	^{4}He						
3				^{6}Li	^{7}Li	^{8}Li			
4				^{7}Be		^{9}Be			
5				^{8}B		^{10}B	^{11}B	^{12}B	
6						^{11}C	^{12}C	^{13}C	^{14}C
7						^{12}N	^{13}N	^{14}N	^{15}N
8							^{14}O	^{15}O	^{16}O

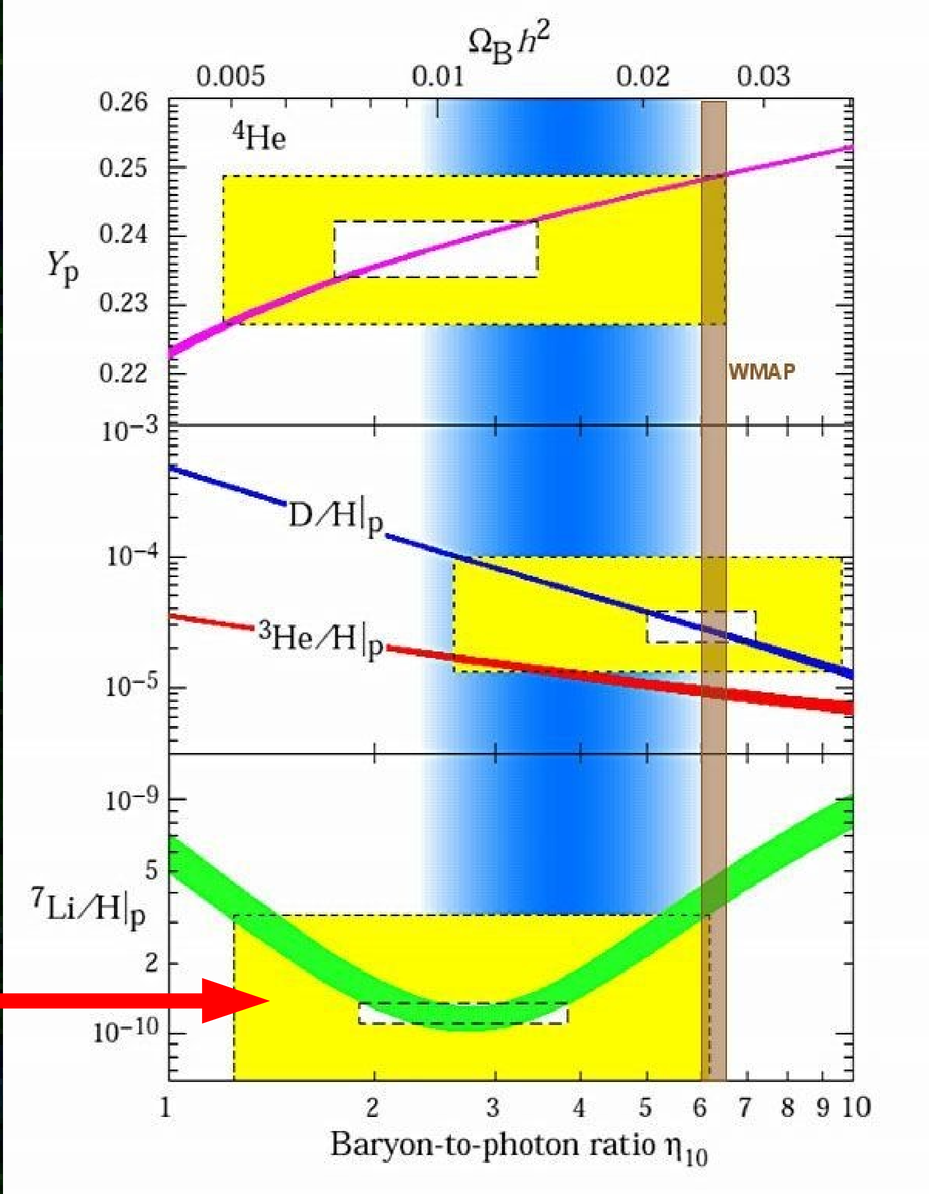
The Miracle

- The abundance of light elements is very sensitive to two things:
  - The age of the universe when the temperature drops to 0.08 MeV (why?)
  - The expansion rate of the cosmos at T ~ 1MeV (why?)
- Why does the expansion rate permits freeze-out at T = 1 MeV? Later freeze out would result in no neutrons at all, thus no life
- Why is the neutron life time such as the fraction at T = 0.08 MeV is still significant? Shorter life-time will result in no matter

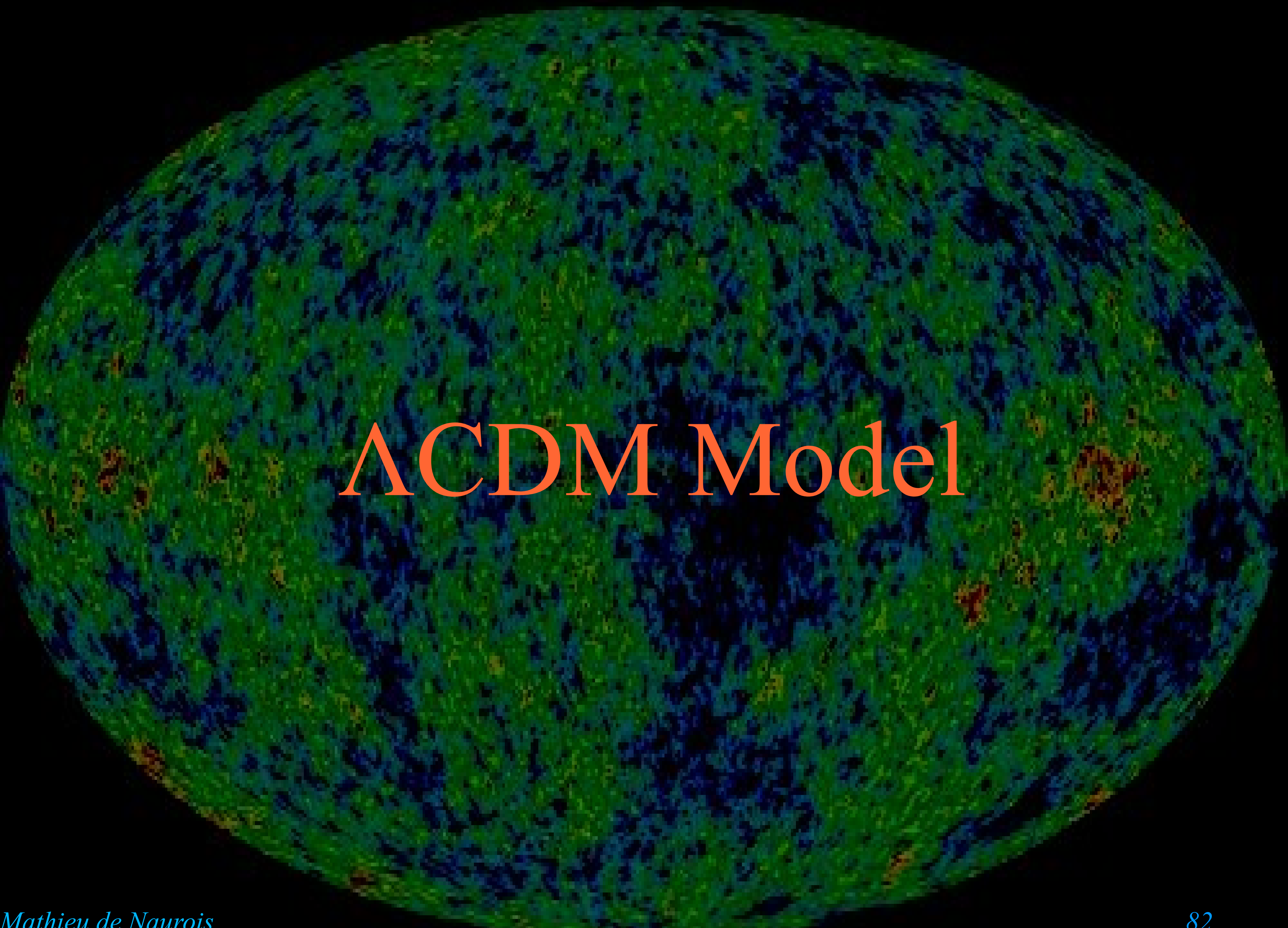
Relative Abundances

- Evolution of abundances as function of baryonic content of the Universe
- When taking everything into account, observed abundances match well the predictions
- They are self-consistent and give $\Omega_B \sim 0.04\text{-}0.05$, consistent with other measurements
- Only free parameter: photon to baryon ration

Measured abundances

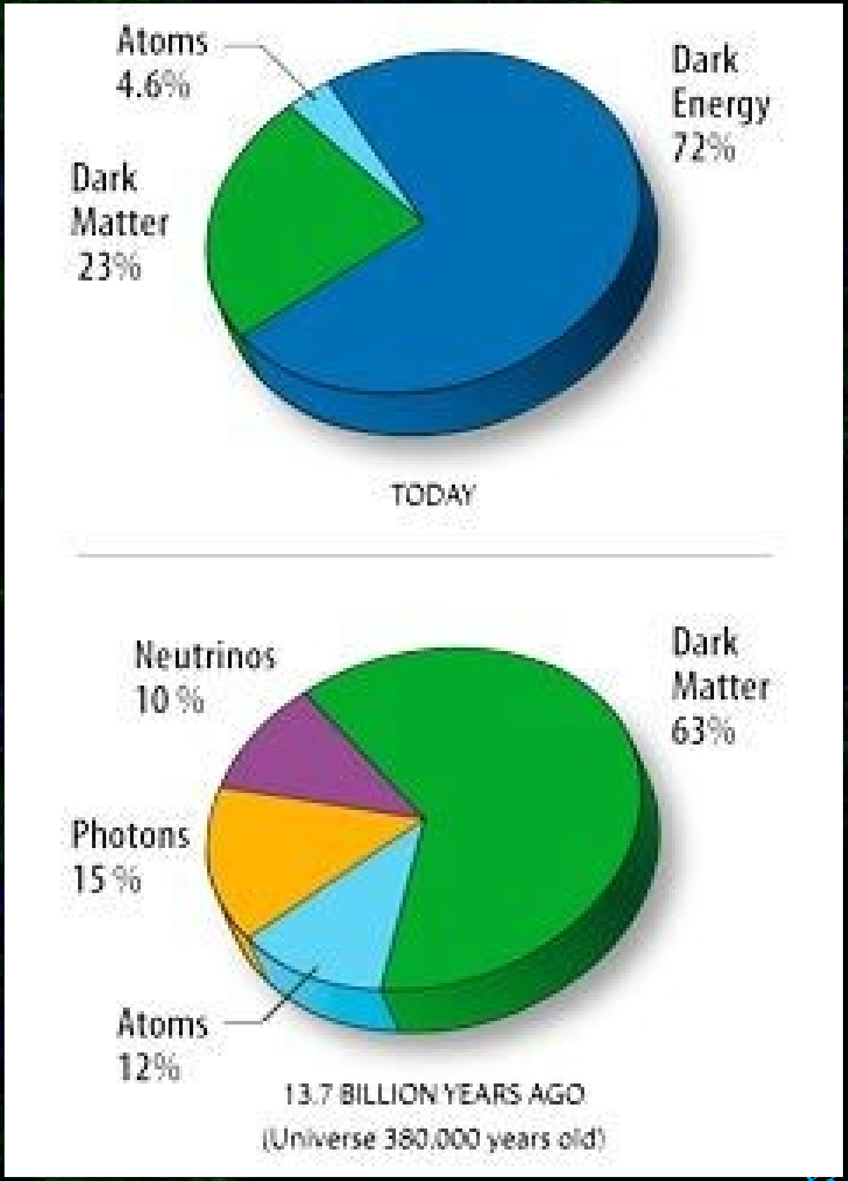


ΛCDM Model



Composition of the Universe

- Robust model based on several pillars:
  - Expansion measurement (Supernova, ..)
  - Astronomical observation of dark matter (rotation curves, ...)
  - CMB
  - Formation of large structures
  - Big bang nucleosynthesis



ΛCDM model

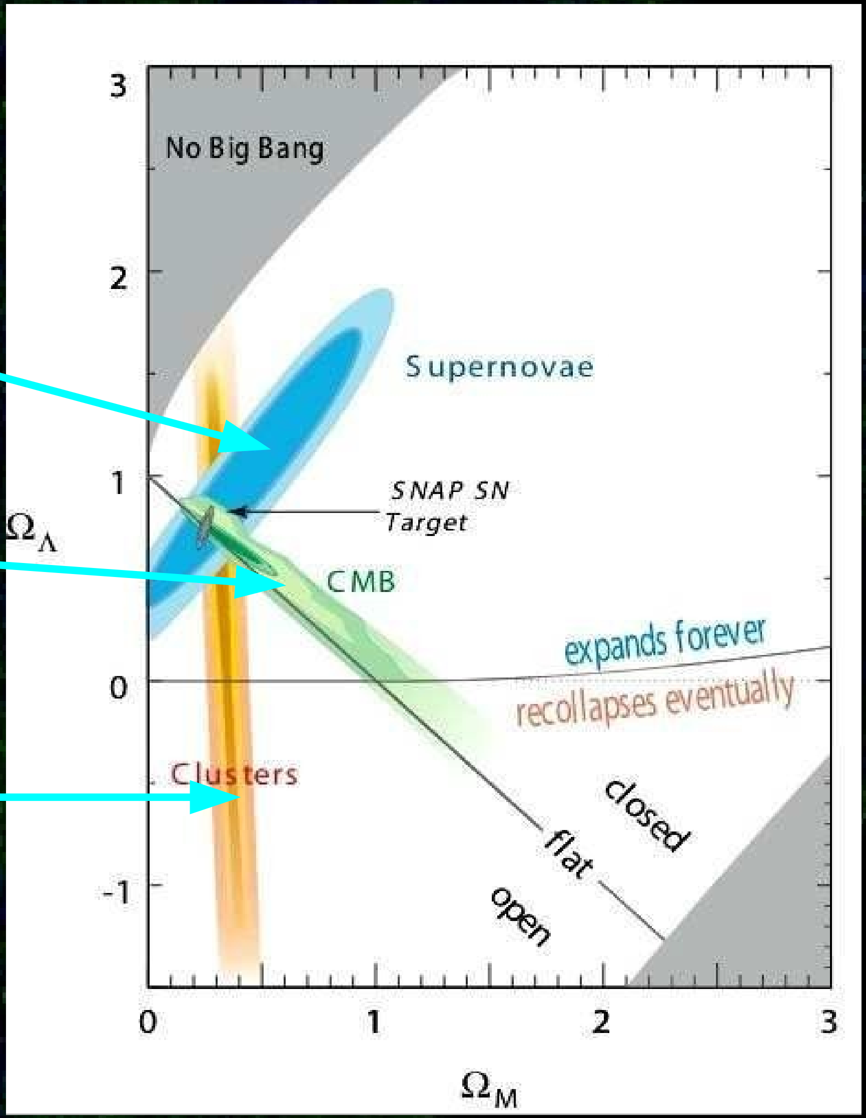
- Baryons,electrons,photons,neutrinos
  - $\Omega_{baryon} = 0.0456 \pm 0.0015$
- Cold Dark Matter
  - $\Omega_{CDM} = 0.228 \pm 0.013$
- Dark Energy (expansion is accelerating !)
  - $\Omega_{\Lambda} = 0.726 \pm 0.015$
- Critical density (spatially flat universe)
  - $\Omega_{T} = 1.01 \pm 0.01$
- Inhomogeneities : gravitational potential flucturations

$(\Omega_M , \Omega_\Lambda)$ constraints (2003)

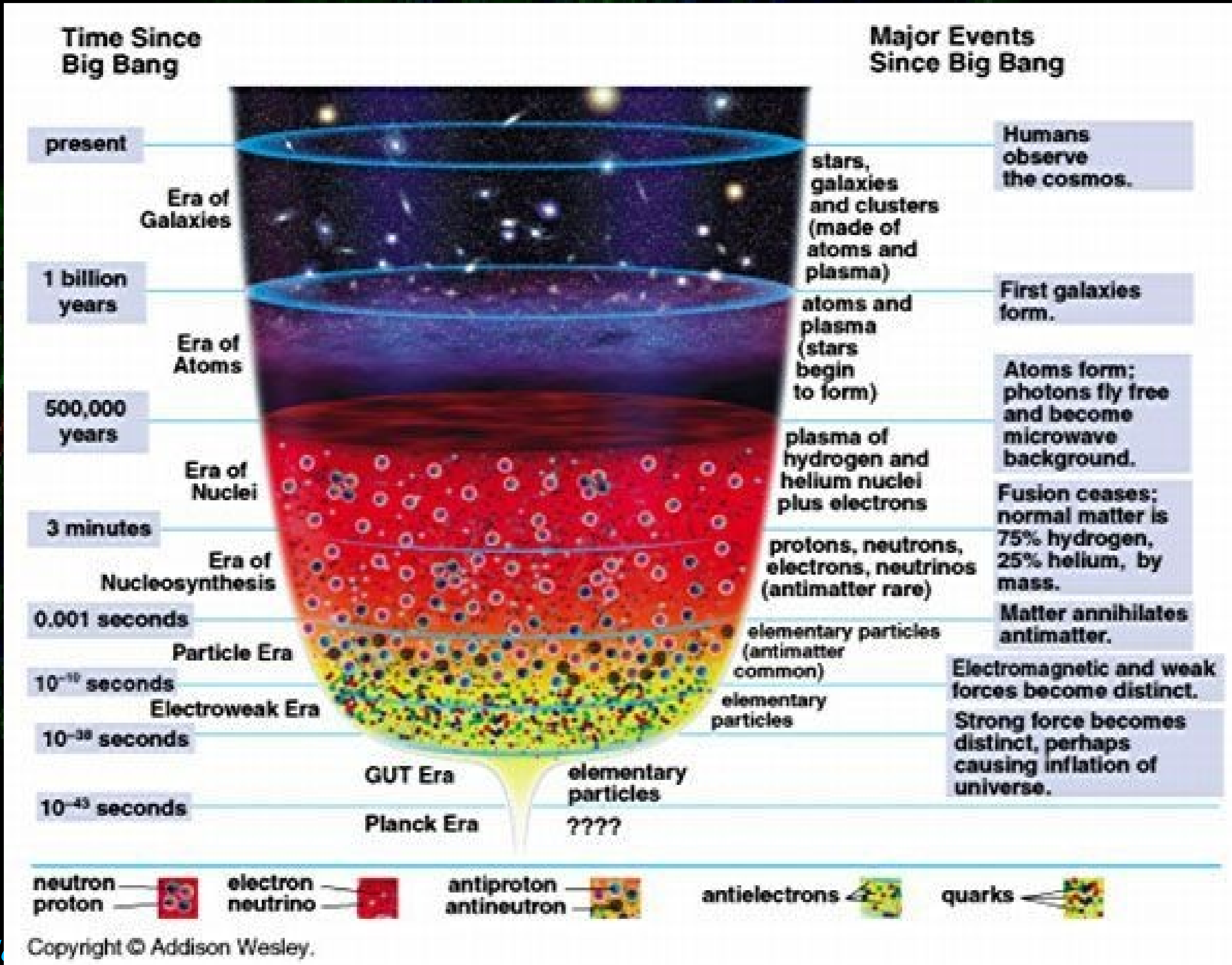
Supernova Cosmology Project

CMB (WMAP)

Baryon fraction in galaxy clusters

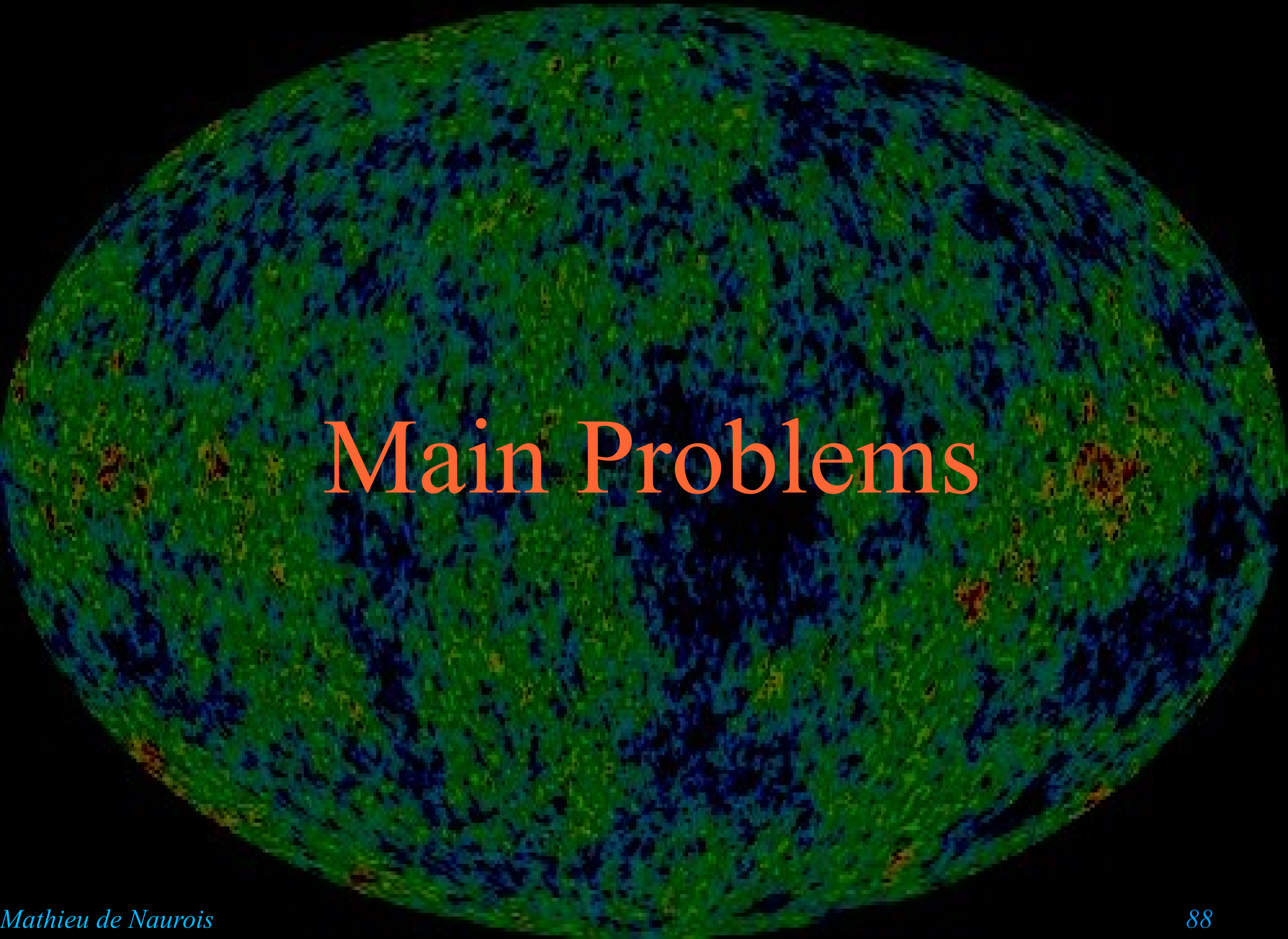


Thermal History



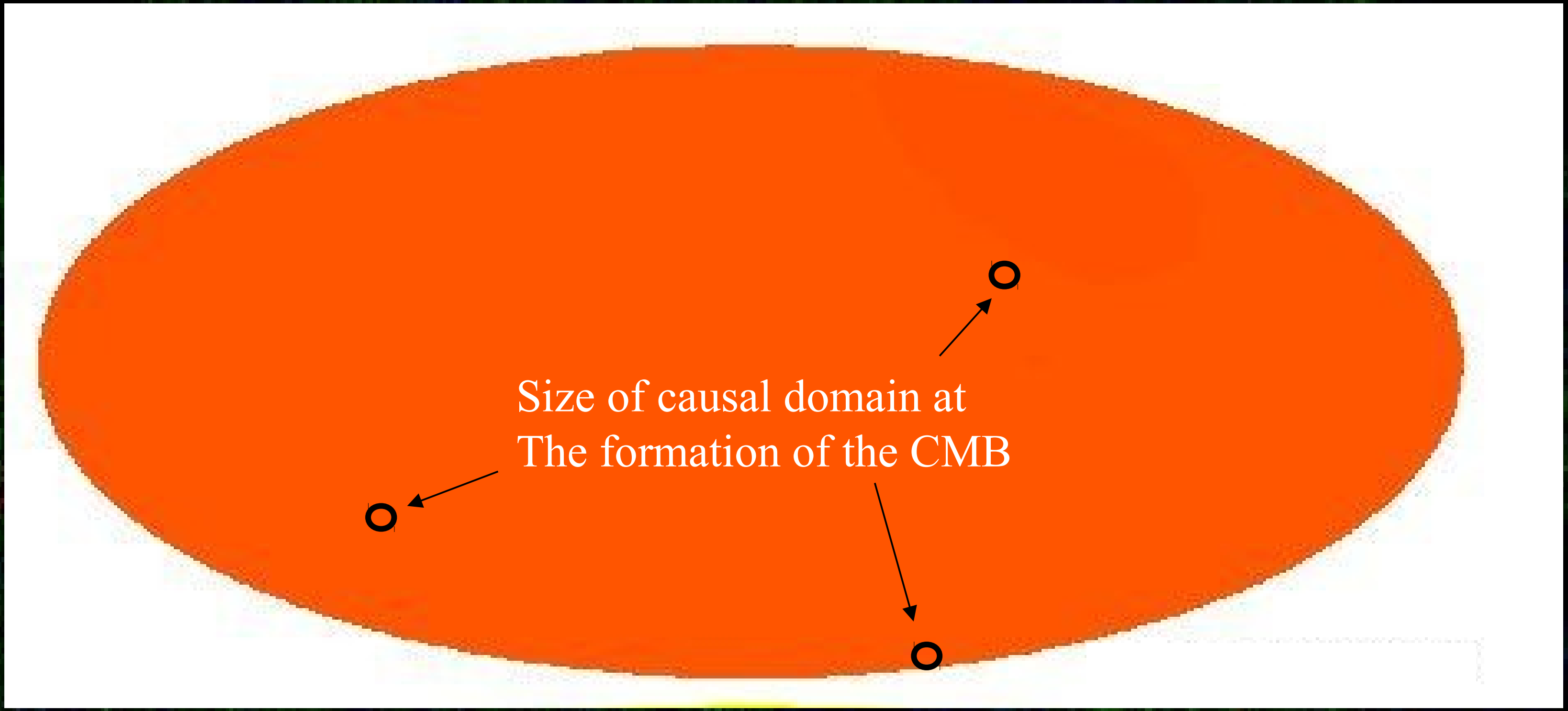
Brief thermal history of the Universe

- 10^{19} GeV – The Planck energy. Quantum gravity required.
- 10^{16} GeV – The GUT scale; inflation
- 100 GeV – Electroweak symmetry breaking
- 100 MeV – Quark-gluon plasma
- 1 MeV – Big Bang Nucleosynthesis
- 1 eV – Formation of the CMB
- 10^{-3} eV – Cosmic acceleration, dark energy



Main Problems

Cosmic Problem 1 : Isotropy & Horizon



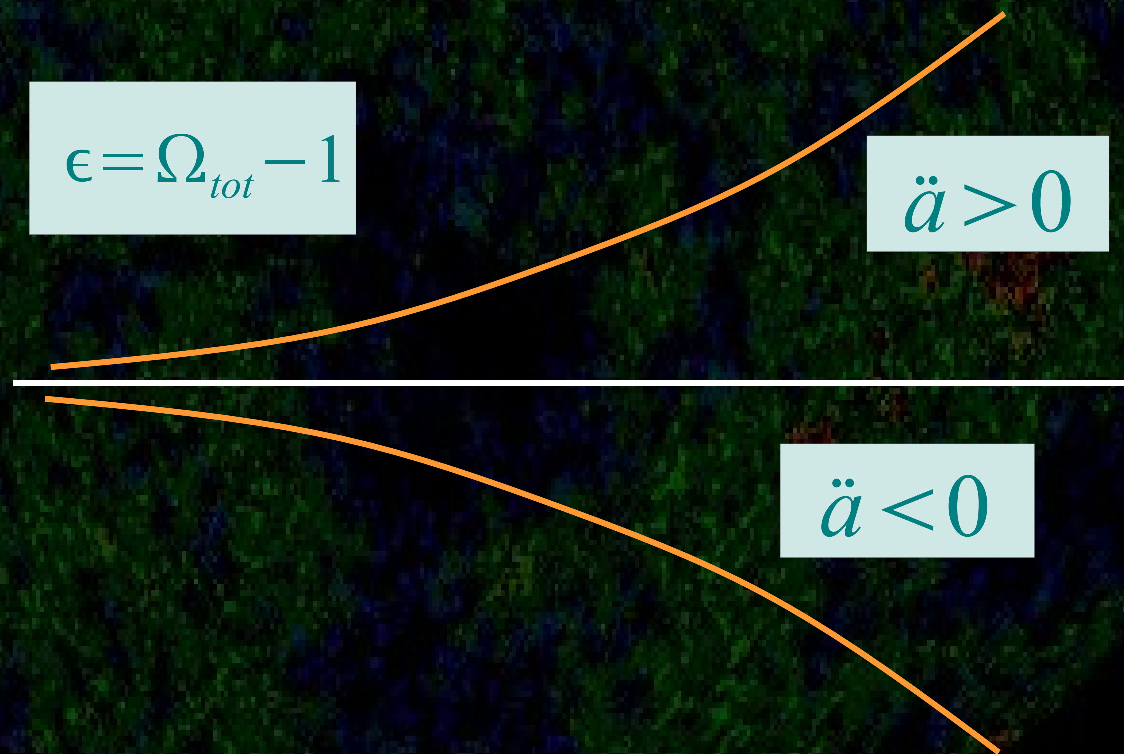
- The Universe is surprisingly homogeneous at large scale, though the horizon at decoupling time was about 1 degree
- How is it possible?

Cosmic Problem 2: flatness

- We know that our universe is flat to within a few percent…
- But gravity makes space curve…So the flatness of the cosmos is is a mystery

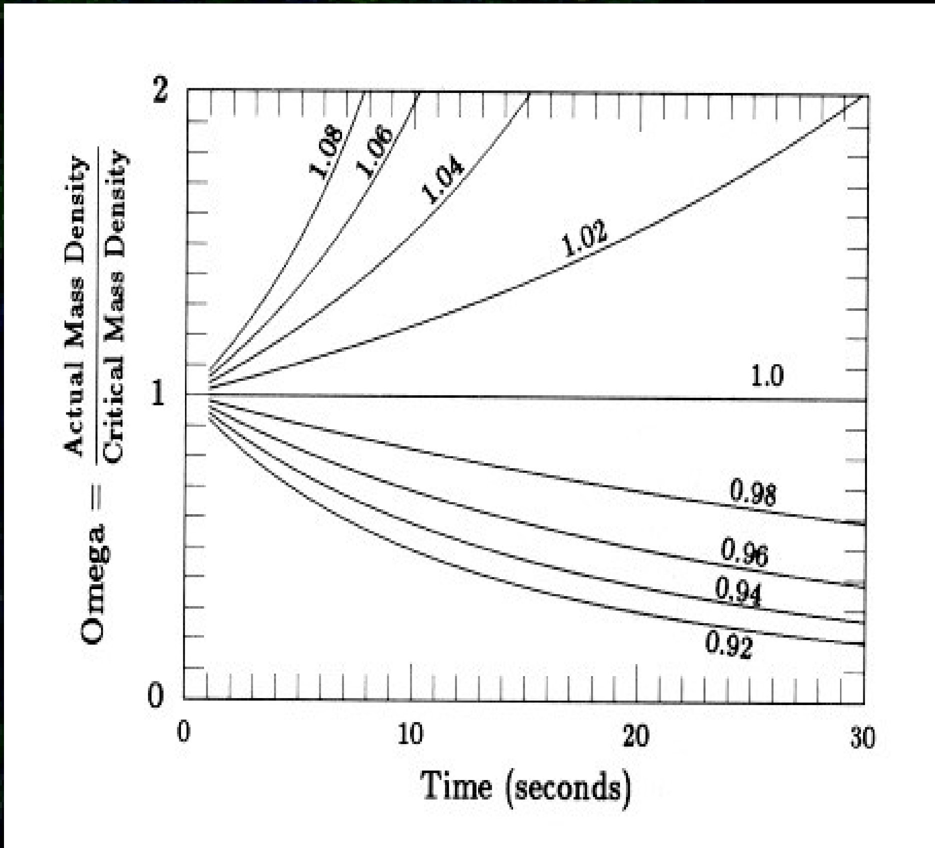
$$\dot{\epsilon} = -2\,\epsilon\left(\frac{\ddot{a}}{\dot{a}}\right)$$

$$\epsilon = \Omega_{tot} - 1$$



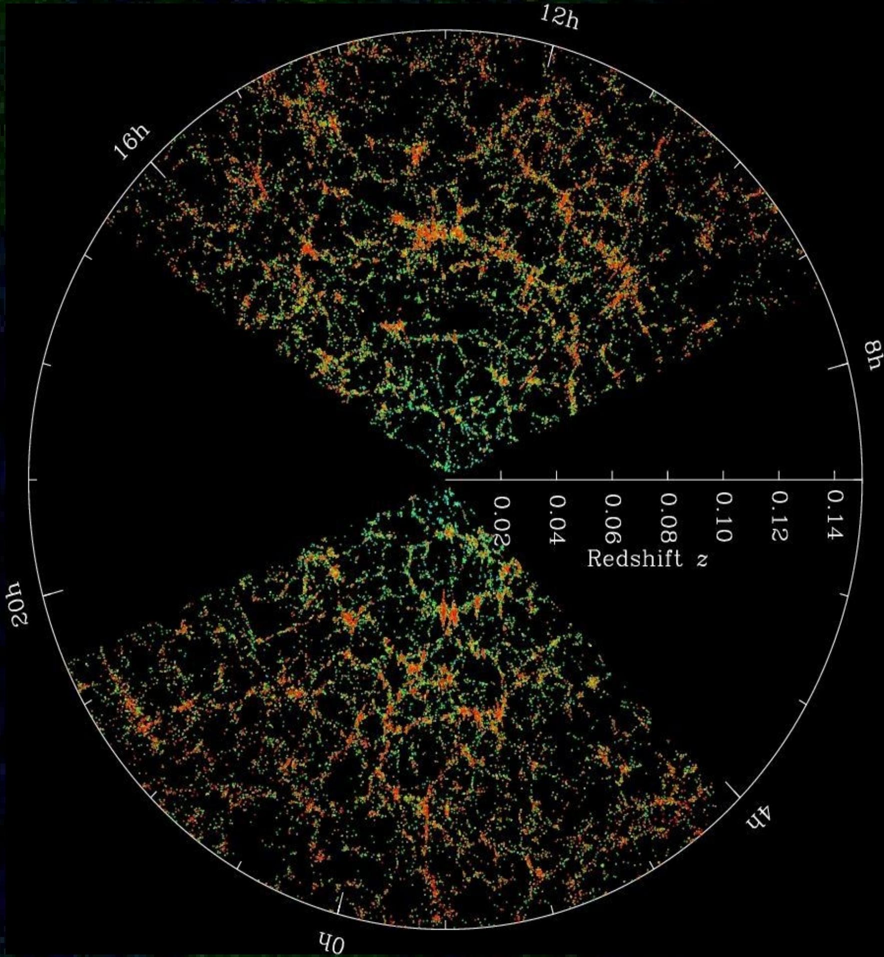
Flatness problem

- Today $\epsilon = 0.01 \pm 0.02$
- @ t = 10^{-43} s, this requires $\epsilon < 10^{-60}$
- Such a precise tuning seems completely unlikely



Cosmic Problem 3: Birth of fluctuations

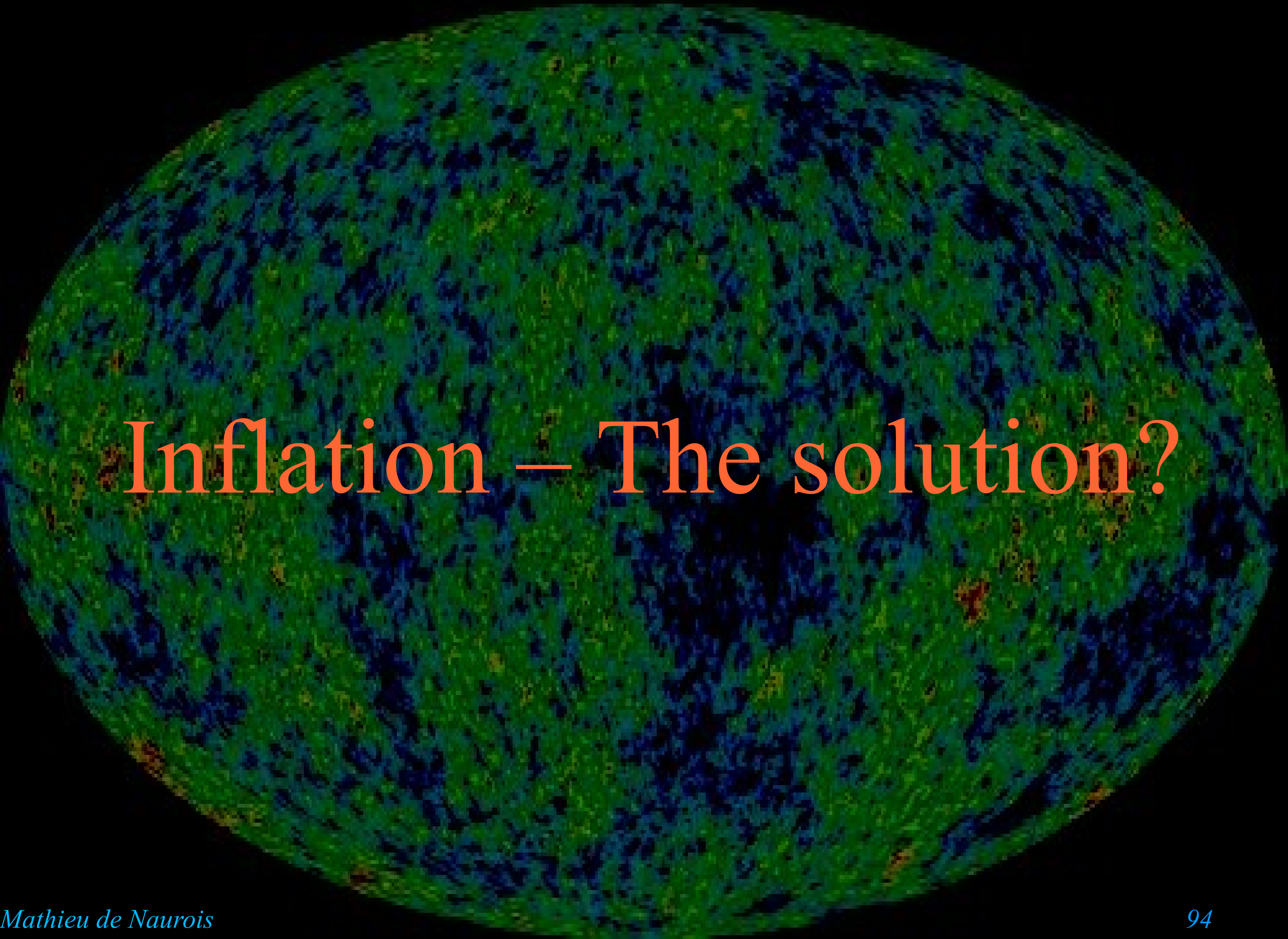
- The simple big-bang Model does not provide enough seeds for the formation of structures
- One need to assume seed fluctuation much larger than simple quantum fluctuation at decoupling time



Cosmic Problem 4: the baryonic universe

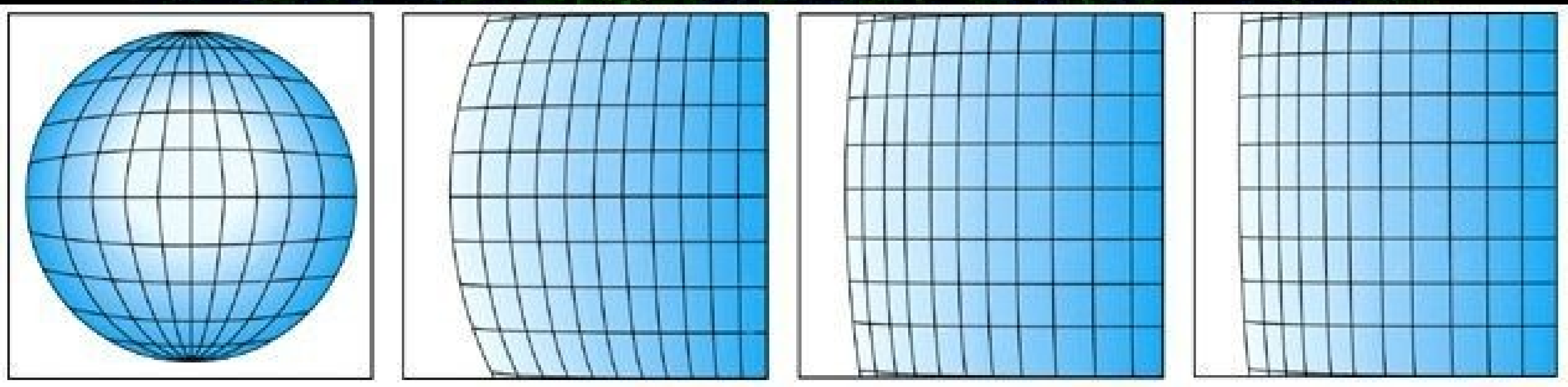
- There must have been a tiny matter—anti-matter asymmetry in the early universe which yielded 1 proton per ~ 10^{10} photons today – why and how did this happen?
- We should expect no baryons at all…since they should have annihilated with an equal number of anti-baryons…
- To get an asymmetry requires non-equilibrium physics and violation of CP and B conservation

Inflation – The solution?



Inflation – the solution?

- If the universe was accelerating it would become flat…

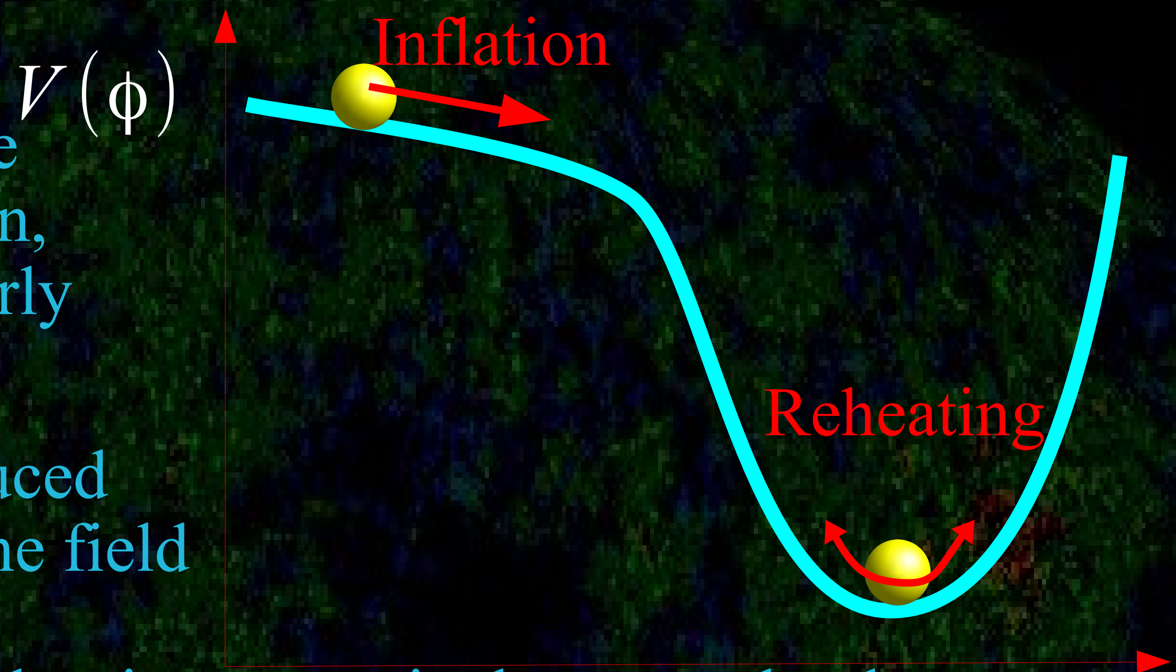


- Acceleration: Raychaudhuri equation (c=1)

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4}{3}\pi G \sum_i \left(\rho_i + 3p_i\right)$$

- Inflation requires negative pressure: $\rho_i + 3p_i < 0$
- But cosmological constant in negligible in early Universe

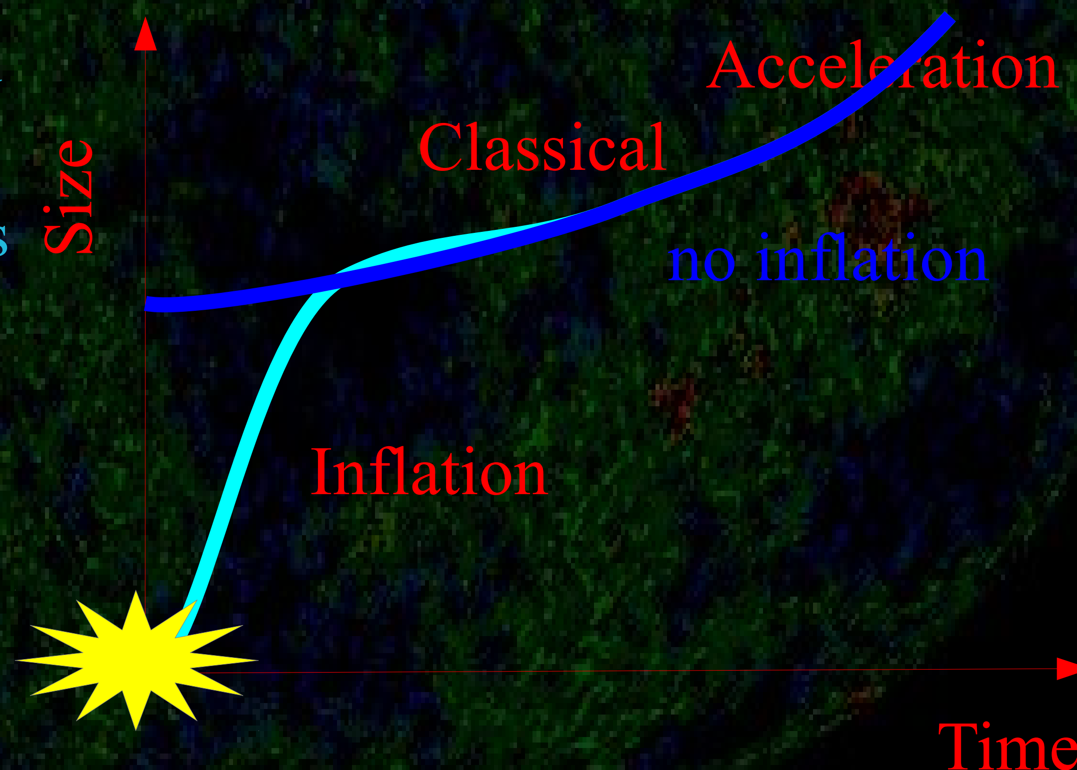
Inflation field



- A scalar field, the so-called inflaton, dominates the early Universe
- Inflation is produced by slow-roll of the field
  Flat potential =>
  Uniform energy density => equivalent to a slowly decreasing cosmological constant
- Inflation stops around minimum of potential, released energy by inflaton decay reheats the medium and gives rise to particle production
- Then Universe follows a classical Universe

Inflation

- Inflation begins around 10^{-33} seconds after the big bang and expands the Universe by a factor 10^{30} to solve the cosmological problems
- The quantum seeds for structures are expanded by the same factor
- Inflation naturally leads to a flat Universe
- Inflation increases naturally the size of homogeneous regions
- It could be related to the Higgs field



Conclusion

- The ΛCDM hot big-band model is well established by a large number of observations, relying on several consistent pillars
- The Universe has entered an accelerated expansion phase
- BUT the very early days of the Universe remains mysterious. Several problems point toward an inflation.
  - What is the inflation field? What is its potential form? Where does it come from? Do we actually need inflation?
  - What is the Dark Matter?
  - What is the Dark Energy?