The Standard Model
of the Electroweak
and Strong Interactions

The Basics of its Construction

ASP2012: KNUST, Kumasi, 15 July – 4 August 2012

Part II: Jan GOVAERTS

Reference Material

- **Many Textbooks**
  From the basics to the advanced

- **Available from** arXiv.org: many Schools and Conferences
  1. **Antonio Pich**, *Aspects of Quantum Chromodynamics*, hep-ph/0001118
  2. **Antonio Pich**, *The Standard Model of Electroweak Interactions*, arXiv:1201.0537 [hep-ph]
  3. **Peter Z. Skands**, *Introduction to QCD*, arXiv:1207.2389 [hep-ph].

Outline

- **The gauge symmetry principle**:
  Abelian and non-abelian (Yang–Mills) gauge theories

- **Symmetry breaking**:
  The Brout-Englert-Higgs mass generation mechanism for gauge bosons

- **The Periodic Table of elementary particles**:
  Quarks, Leptons, and (gauge, Higgs) Bosons

- **The Basics of the SM: Electroweak Interactions**:
  EW SU(2)$_L\times$U(1)$_Y$

- **The Basics of the SM: Strong Interactions**:
  QCD SU(3)$_c$

The Gauge Symmetry Principle

Symmetry + Locality $\longrightarrow$ Interactions

U(1) Abelian gauge invariance (and QED)

$$\mathcal{L}_{\text{Dirac}} = \overline{\psi}\, i\gamma^\mu \partial_\mu \psi \;-\; m\overline{\psi}\,\psi$$

Global symmetry:

$$\psi'(x) = e^{i\alpha}\,\psi(x),\; \overline{\psi}'(x) = \overline{\psi}(x)\,e^{-i\alpha}$$

Conserved current:

$$J_\mu(x) = \overline{\psi}(x)\,\gamma_\mu\,\psi(x),\; \partial^\mu J_\mu(x) = 0$$

Local gauge transformation: $\alpha = \alpha(x)$

Infinitesimal transformation:

$$\delta\psi(x) = i\alpha(x)\,\psi(x),\ \delta\overline{\psi}(x) = -i\alpha(x)\,\overline{\psi}(x)$$

$$\delta\mathcal{L}_{\text{Dirac}} = -\partial_\mu\alpha\, J^\mu,\ \delta J_\mu = 0$$

$$\Longrightarrow \mathcal{L}_{\text{int}} = -eA_\mu J^\mu,\ \delta A_\mu = -\tfrac{1}{e}\partial_\mu\alpha$$

A_μ: vector (gauge) field; field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,
$\delta F_{\mu\nu} = 0$

Gauge invariant Lagrangian density (QED)

$$\boxed{\mathcal{L}_{\text{QED}} = \overline{\psi}\, i\gamma^\mu\partial_\mu\,\psi - m\overline{\psi}\,\psi - eA_\mu\,\overline{\psi}\,\gamma^\mu\,\psi - \tfrac{1}{4}F_{\mu\nu}\,F^{\mu\nu}}$$

Massless gauge boson: mass protected by gauge invariance
$\frac{1}{2}M^2 A_\mu A^\mu$: forbidden by gauge invariance

The secret of gauge invariance: The covariant derivative

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi}\, i\gamma^\mu (\partial_\mu + ie\, A_\mu)\, \psi - m\, \overline{\psi}\, \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Covariant derivative: $\partial_\mu \longrightarrow D_\mu = \partial_\mu + ieA_\mu$

Finite gauge transformations:

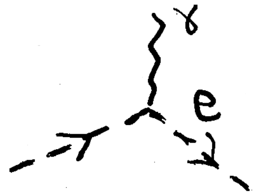
$$\psi'(x) = e^{i\alpha(x)}\, \psi(x),\ \overline{\psi}'(x) = \overline{\psi}(x)\, e^{-i\alpha(x)},$$
$$A'_\mu(x) = A_\mu(x) - \tfrac{1}{e}\partial_\mu \alpha(x),\ F'_{\mu\nu} = F_{\mu\nu}$$

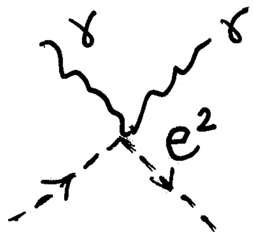
$$\boxed{D'_\mu \psi' = \left(\partial_\mu + ie\left(A_\mu - \tfrac{1}{e}\partial_\mu \alpha\right)\right) e^{i\alpha}\, \psi = e^{i\alpha}\, D_\mu \psi}$$

Scalar quantum electrodynamics

$$\mathcal{L} = |(\partial_\mu + ieA_\mu)\phi|^2 - m^2\,|\phi|^2 - V\,(|\phi|) - \frac{1}{4}F_{\mu\nu}\,F^{\mu\nu}$$

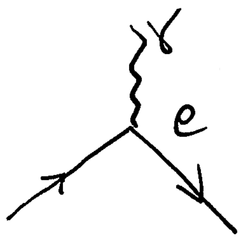
Gauge vertices





$-i\,e\,A_\mu\,\phi^*\,\partial_\mu\,\phi + h.c. = -e\,A_\mu\,J^\mu$ $\qquad e^2\,A_\mu A^\mu \phi^* \phi$

QED:



$-e\,A_\mu\,\overline{\psi}\,\gamma^\mu\,\psi$

Non Abelian gauge invariance

Global internal symmetry: **compact simple Lie group** G

Representation: **generators** T^a $(a = 1, 2, \cdots, \text{dim } G)$

$$(T^a)^\dagger = T^a, \qquad \left[T^a, T^b\right] = i f^{abc} T^c$$

Matter field content: $\phi(x)$

$$\phi'(x) = U(\theta)\,\phi(x), \qquad U(\theta) = e^{i\theta^a T^a}$$

Symmetric dynamics: $\mathcal{L}(\phi, \partial_\mu \phi)$

a) Covariant derivative

$$U\left(\theta^a(x)\right) = e^{i\theta^a(x)\,T^a}$$

$$\phi' = U\phi, \qquad D'_\mu\,\phi' = U\,\left(D_\mu\,\phi\right)$$

$$D_\mu = \partial_\mu + ig\,A^a_\mu\,T^a$$

$$A'^a_\mu\,T^a = U\,\left(A^a_\mu T^a\right)\,U^{-1} + \frac{i}{g}\partial_\mu U\,U^{-1}$$

Infinitesimal transformations:

$$A'^a_\mu = A^a_\mu - \frac{1}{g}\,\partial_\mu\theta^a - f^{abc}\,\theta^b\,A^c_\mu$$

Adjoint representation: $(T^a)^{bc} = -i\,f^{abc}$

b) Gauge field strength

$$[\,D_\mu\,,\,D_\nu\,] = ig\,F_{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\,[A_\mu, A_\nu] = F^a_{\mu\nu}\,T^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\,f^{abc}\,A^b_\mu\,A^c_\nu$$

Gauge transformations:

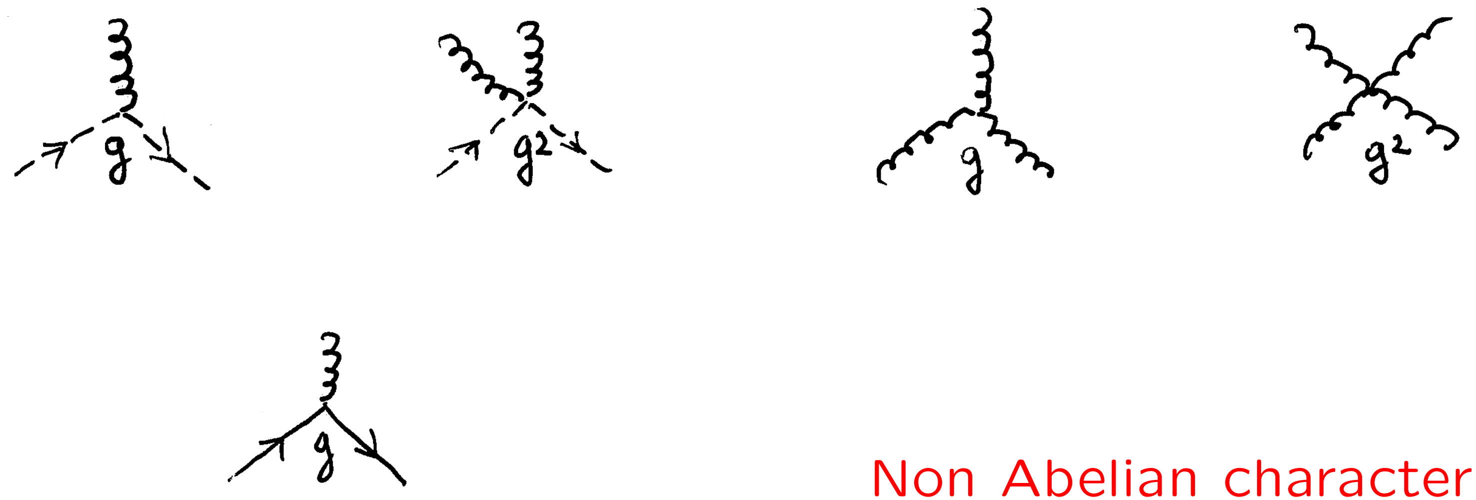
$$F'_{\mu\nu} = U\,F_{\mu\nu}\,U^{-1}$$

(Adjoint representation)

c) Gauge invariant dynamics: Yang-Mills theory

$$\mathcal{L}_{\mathrm{YM}} = \mathcal{L}\left(\phi, D_\mu \phi\right) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$D_\mu \phi = (\partial_\mu + i g A^a_\mu T^a)\phi \quad ; \quad -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} = -\tfrac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu\right)^2$$



Non Abelian character

Unification of interactions: A single coupling constant g

Massless gauge bosons:

$\frac{1}{2} A^a_\mu M^2_{ab} A^{b\mu}$: forbidden by gauge invariance

Symmetry Breaking

Spontaneous symmetry breaking: Goldstone's theorem

Global symmetry: $[T^a, H] = 0$

a) Wigner mode: manifest realisation of the symmetry

Invariant vacuum: $T^a|0\rangle = 0$, $U(\theta)|0\rangle = |0\rangle$

Spectrum: **Multiplets of the symmetry:** $H|E\rangle = E|E\rangle$

$$H\, T^a\, |E\rangle = E\, T^a\, |E\rangle, \qquad H\, U(\theta)\, |E\rangle = E\, U(\theta)\, |E\rangle$$

b) Goldstone mode: spontaneous symmetry breaking

(perturbative, non perturbative mechanisms)

Non invariant vacuum: infinite degenerate

Massless Goldstone modes:

quantum numbers of the broken symmetry

Vacuum: $H\,|0\rangle = E_0\,|0\rangle$

$$T^a|0\rangle = |0,a\rangle \neq |0\rangle, \quad |\theta^a\rangle = U(\theta)\,|0\rangle, \quad H|\theta^a\rangle = E_0|\theta^a\rangle$$

Goldstone modes:

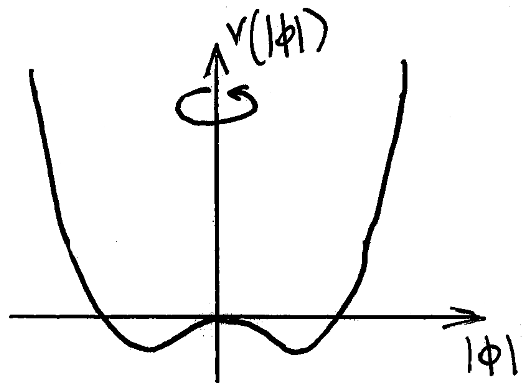
$$\delta^{ab} = \langle 0,a|0,b\rangle = \langle 0|T^a|0,b\rangle$$

$$P^\mu\,|0,a\rangle = k^\mu\,|0,a\rangle, \qquad k^\mu = (E_0,\vec{0})\ (E_0 = 0,\ \text{massless})$$

c) U(1) spontaneous symmetry breaking

$$\mathcal{L} = |\partial_\mu \phi|^2 \underbrace{-\mu^2|\phi|^2 - \lambda|\phi|^4}_{-V(|\phi|)}$$

$$\boxed{\mu^2 < 0}, \ \lambda > 0$$



Classical vacuum configuration: $|\phi| = v/\sqrt{2}$, $v = \sqrt{-\mu^2/\lambda}$

Fluctuations around the vacuum configuration:

$$\phi(x) = \frac{1}{\sqrt{2}} e^{i\xi(x)/v} \left(\rho(x) + v\right)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{1}{2}\left(1+\frac{\rho}{v}\right)^2 \partial_\mu\xi\partial^\mu\xi - \frac{1}{2}\mu^2\left(\rho+v\right)^2 - \frac{1}{4}\lambda\left(\rho+v\right)^4$$

Goldstone mode:

$\xi(x)$, $m_\xi^2 = 0$ [U(1) symmetry]

Massive mode:

$\rho(x)$, $m^2 = -2\mu^2 > 0$

Global symmetry:

$\xi'(x) = \xi(x) + \alpha v$, $\rho'(x) = \rho(x)$ [derivative couplings of ξ]

The Brout-Englert-Higgs mechanism and the Higgs boson

Example: U(1) gauge symmetry
(Landau-Ginzburg model, superconductivity)

$$\mathcal{L} = |(\partial_\mu + ieA_\mu)\phi|^2 - \mu^2|\phi|^2 - \lambda|\phi|^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad \mu^2 < 0, \quad \lambda > 0$$

$$\phi(x) = \frac{1}{\sqrt{2}} e^{i\xi(x)/v}(\rho(x) + v)$$

Gauge transformations:

$$\phi'(x) = e^{i\alpha(x)}\,\phi(x), \qquad A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

Choose: $\alpha(x) = -\frac{1}{v}\,\xi(x)$ [Unitary gauge]

$$\overline{\phi}(x) = \frac{1}{\sqrt{2}}(\rho(x) + v)\,, \qquad \overline{A}_\mu(x) = A_\mu(x) + \frac{1}{ev}\partial_\mu\xi(x)$$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\rho - ie\overline{A}_\mu(\rho+v)\right)\left(\partial^\mu\rho + ie\overline{A}^\mu(\rho+v)\right) - \frac{1}{2}\mu^2(\rho+v)^2 - \frac{1}{4}\lambda(\rho+v)^4 - \frac{1}{4}\overline{F}_{\mu\nu}\overline{F}^{\mu\nu}$$

Physical spectrum: [Unitary gauge]

Massive scalar mode:
$\rho(x)$, $m^2 = -2\mu^2$: **Higgs boson:** $h(x) = \rho(x)$

Massive gauge boson mode:
$\overline{A}_\mu(x)$, $M_A^2 = e^2 v^2$: **Without explicit gauge symmetry breaking**

No Goldstone mode:
Longitudinal component of $\overline{A}_\mu$

Nonlinear realisation of the gauge symmetry: $\rho' = \rho$, $\overline{A}'_\mu = \overline{A}_\mu$

$$\xi'(x) = \xi(x) + v\alpha(x), \quad A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

The Periodic Table of elementary particles

	Generations			Q
Leptons	ν_e	ν_μ	ν_τ	0
	e^-	μ^-	τ^-	-1
Quarks	u	c	t	$+2/3$
	d	s	b	$-1/3$

Fundamental interactions: carriers

Bosons: γ, $W^{\pm}$, Z_0, 8 gluons, **Higgs** (?), (graviton ?)

The Basics of the Standard Model

Choice of gauge group, of matter field content, of Higgs sector, of the most general renormalisable Lagrangian

The gauge group: $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\boxed{Q = T_3 + \frac{1}{2} Y}$$

$U(1)_Y$: g', B_μ; $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$SU(2)_L$: g, W^i_μ, $i = 1, 2, 3$; $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\epsilon^{ijk} W^j_\mu W^k_\nu$

Fundamental representation: $T^i = \frac{1}{2}\sigma^i$, $[T^i, T^j] = i\epsilon^{ijk} T^k$

$SU(3)_c$: g_s, G^a_μ, $a = 1, 2, \cdots, 8$; $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s\, f^{abc} G^b_\mu G^c_\nu$

Fundamental representation: $T^a = \frac{1}{2}\lambda^a$, $[T^a, T^b] = i f^{abc} T^c$

The matter fields: leptons and quarks [non Minimal SM]

				$SU(3)_c \times SU(2)_L \times U(1)_Y$
$\begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}$	$\begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix}$	$\begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}$	L'_L	$(\mathbf{1}, \mathbf{2}, -1)$
e'_R	μ'_R	τ'_R	E'_R	$(\mathbf{1}, \mathbf{1}, -2)$
$\begin{pmatrix} \nu'_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c'_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t'_L \\ b'_L \end{pmatrix}$	Q'_L	$(\mathbf{3}, \mathbf{2}, \frac{1}{3})$
u'_R	c'_R	t'_R	U'_R	$(\mathbf{3}, \mathbf{1}, \frac{4}{3})$
d'_R	s'_R	b'_R	D'_R	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$	N'_R	$(\mathbf{1}, \mathbf{1}, 0)$

All **matter** fields: **massless** because of **gauge invariance**

All **gauge** fields: **massless** because of **gauge invariance**

The Higgs sector: Minimal content: a single SU(2)$_L$ doublet

Yukawa coupling: $\lambda\overline{\psi_L}\,\phi\,\psi_R \to \lambda\overline{\psi_L}\,\langle\phi\rangle\,\psi_R$

$$Q = T_3 + \tfrac{1}{2}Y$$

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}, \qquad (\mathbf{1},\mathbf{2},1)$$

$$\tilde{\phi} = i\sigma_2\,\phi^* = \begin{pmatrix} \phi_0^* \\ \phi_- \end{pmatrix} = \begin{pmatrix} \phi_0^* \\ -\phi_+^* \end{pmatrix}, \qquad (\mathbf{1},\mathbf{2},-1)$$

The Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$\mathcal{L}_{\text{Higgs}} = |(\partial_\mu + igW^i_\mu \frac{\sigma^i}{2} + ig' B_\mu \frac{1}{2})\phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4, \quad \mu^2 < 0, \quad \lambda > 0$$

$$\begin{aligned}
\mathcal{L}_{\text{fermions}} \;=\; & \overline{L'_L}\, i\gamma^\mu (\partial_\mu + igW^i_\mu \frac{\sigma^i}{2} + ig' B_\mu \frac{-1}{2}) L'_L \;+ \\
+\; & \overline{E'_R}\, i\gamma^\mu (\partial_\mu + ig' B_\mu \frac{-2}{2}) E'_R \;+ \\
+\; & \overline{Q'_L}\, i\gamma^\mu (\partial_\mu + ig_s G^a_\mu \frac{\lambda^a}{2} + igW^i_\mu \frac{\sigma^i}{2} + ig' B_\mu \frac{1}{6}) Q'_L \;+ \\
+\; & \overline{U'_R}\, i\gamma^\mu (\partial_\mu + ig_s G^a_\mu \frac{\lambda^a}{2} + ig' B_\mu \frac{2}{3}) U'_R \;+ \\
+\; & \overline{D'_R}\, i\gamma^\mu (\partial_\mu + ig_s G^a_\mu \frac{\lambda^a}{2} + ig' B_\mu \frac{-1}{3}) D'_R \;+ \\
+\; & \overline{N'_R}\, i\gamma^\mu (\partial_\mu) N'_R
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} \;=\; & -\overline{L'_L}\, \lambda_\ell\, \phi\, E'_R - \overline{Q'_L}\, \lambda_{\text{d}}\, \phi\, D'_R - \overline{Q'_L}\, \lambda_{\text{u}}\, \tilde{\phi}\, U'_R - \overline{L'_L}\, \lambda_\nu\, \tilde{\phi}\, N'_R - \\
& -\overline{E'_R}\, \lambda_\ell^\dagger\, \phi^\dagger\, L'_L - \overline{D'_R}\, \lambda_{\text{d}}^\dagger\, \phi^\dagger\, Q'_L - \overline{U'_R}\, \lambda_{\text{u}}^\dagger\, \tilde{\phi}^\dagger\, Q'_L - \overline{N'_R}\, \lambda_\nu^\dagger\, \tilde{\phi}^\dagger\, L'_L
\end{aligned}$$

λ_ℓ, λ_{d}, λ_{u}, λ_ν: inter-generational (Yukawa) coupling matrices

The Higgs and gauge sectors

Spontaneous gauge symmetry breaking

$$V(|\phi|) = -\mu^2|\phi|^2 - \lambda|\phi|^4, \qquad \mu^2 < 0, \quad \lambda > 0$$

Vacuum configuration:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad v = \sqrt{\frac{-\mu^2}{\lambda}}$$

Physical content:

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_{+1}(x) + i\phi_{+2}(x) \\ h(x) + v + i\varphi_0(x) \end{pmatrix}$$

i) $\phi_{+1}(x)$, $\phi_{+2}(x)$, $\varphi_0(x)$: longitudinal components of the massive gauge bosons (would-be Goldstone bosons)

ii) $h(x)$: **Higgs boson, massive neutral scalar particle**

$$\boxed{m_h^2 = -2\mu^2 = 2\lambda\, v^2}$$

$v \simeq 247$ GeV, $\boxed{m_h \simeq 125 \text{ GeV}}$ [4 July 2012]

The gauge boson sector: Mass terms follow from $|D_\mu\langle\phi\rangle|^2$

EW gauge symmetry breaking: $SU(2)_L \times U(1)_Y \to U(1)_Q$

$$D_\mu\langle\phi\rangle = \frac{i}{2\sqrt{2}}\begin{pmatrix} \sqrt{2}gv\ W_\mu^+ \\ -(gW_\mu^3 - g'B_\mu)v \end{pmatrix}, \qquad W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

a) Charged gauge bosons: $W^\pm$ $\boxed{M_W = \frac{1}{2}gv}$

b) Neutral gauge bosons: γ, Z_0

$$A_\mu = \cos\theta_W\, B_\mu + \sin\theta_W\, W_\mu^3, \quad Z_\mu = -\sin\theta_W\, B_\mu + \cos\theta_W\, W_\mu^3$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}, \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \qquad \tan\theta_W = \frac{g'}{g}$$

$$\boxed{M_A = 0, \quad M_Z = \frac{1}{2}\frac{gv}{\cos\theta_W} = \frac{M_W}{\cos\theta_W}}$$

The fermionic sector

Fermion mass eigenstates: Mass terms follow from $\overline{\psi}\,\lambda\langle\phi\rangle\,\psi$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad \langle\tilde{\phi}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}} = -\frac{v}{\sqrt{2}}\overline{E'_L}\lambda_\ell E'_R - \frac{v}{\sqrt{2}}\overline{D'_L}\lambda_{\text{d}} D'_R - \frac{v}{\sqrt{2}}\overline{U'_L}\lambda_{\text{u}} U'_R + h.c.$$

a) Bi-unitary transformations: Diagonalisation

$$D = U\,M\,V^\dagger, \qquad U^\dagger = U^{-1}, \qquad V^\dagger = V^{-1}$$

$$D_1 = U\,(M M^\dagger)\,U^\dagger, \qquad D_2 = V\,(M^\dagger M)\,V^\dagger$$

$$D_1 = D D^\dagger, \qquad D_2 = D^\dagger D$$

b) Diagonalisation of the Yukawa coupling matrices

$$\lambda_\ell^{(d)} = U_L^{(\ell)\dagger} \lambda_\ell V_R^{(\ell)}, \quad \lambda_{\mathrm{d}}^{(d)} = U_L^{(\mathrm{d})\dagger} \lambda_{\mathrm{d}} V_R^{(\mathrm{d})}, \quad \lambda_{\mathrm{u}}^{(d)} = U_L^{(\mathrm{u})\dagger} \lambda_{\mathrm{u}} V_R^{(\mathrm{u})}$$

c) Fermion mass eigenstates

$$L_L = \begin{pmatrix} U_X^\dagger & 0 \\ 0 & U_L^{\ell\,\dagger} \end{pmatrix} L'_L, \qquad E_R = V_R^{(\ell)} E'_R$$

$$Q_L = \begin{pmatrix} U_L^{(\mathrm{u})\dagger} & 0 \\ 0 & U_L^{(\mathrm{d})\dagger} \end{pmatrix} Q'_L, \qquad U_R = V_R^{(\mathrm{u})} U'_R, \qquad D_R = V_R^{(\mathrm{d})} D'_R$$

$$\mathcal{L}_{\mathrm{mass}} = -\frac{v}{\sqrt{2}} \overline{E} \lambda_\ell^{(d)} E - \frac{v}{\sqrt{2}} \overline{U} \lambda_{\mathrm{u}}^{(d)} U - \frac{v}{\sqrt{2}} \overline{D} \lambda_{\mathrm{d}}^{(d)} D$$

$$\boxed{m_{\mathrm{fermion}} = \frac{v}{\sqrt{2}} (\text{Yukawa coupling})}$$

[Mass hierarchy problem]

Remark: $\psi' = \gamma_5 \psi$, $\overline{\psi'}\psi' = -\overline{\psi}\psi$

Charged current interactions: [Fermionic matter fields]

$$\begin{aligned}\mathcal{L}_{\text{CC}} \;=\; & -\frac{1}{2\sqrt{2}}\, g\, \overline{E}\, \gamma^\mu (1-\gamma_5)\, W_\mu^- \left(U_L^{(\ell)\dagger} U_X \right) N - \\ & -\frac{1}{2\sqrt{2}}\, g\, \overline{D}\, \gamma^\mu (1-\gamma_5)\, W_\mu^- \left(U_L^{(\text{d})\dagger} U_L^{(\text{u})} \right) U + h.c.\end{aligned}$$

Cabibbo-Kobayashi-Maskawa:

$$U_{\text{CKM}} = U_L^{(\text{d})\dagger} U_L^{(\text{u})}, \qquad U_{\text{CKM}}^\dagger = U_{\text{CKM}}^{-1}$$

$$U_X = U_L^{(\ell)}$$

$$\mathcal{L}_{\text{CC}}^{\text{quarks}} = -\frac{1}{2\sqrt{2}} g W_\mu^- \overline{D} \gamma^\mu (1-\gamma_5) U_{\text{CKM}} U + h.c.$$

$$\mathcal{L}_{\text{CC}}^{\text{leptons}} = -\frac{1}{2\sqrt{2}} g W_\mu^- \overline{E} \gamma^\mu (1-\gamma_5)\, N + h.c.$$

[Accidental global leptonic symmetries: L_e, L_μ, L_τ conserved]

Neutral and electromagnetic current interactions: [Fermionic fields]

Electromagnetic interactions:

$$\mathcal{L}_{\text{em}} = -e\, A_\mu\, \overline{F}\, \gamma^\mu\, Q\, F$$

$$e = g \sin\theta_W = g' \cos\theta_W = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

Neutral current interactions:

$$\mathcal{L}_{\text{nc}} = -\frac{g}{2\cos\theta_W}\, Z_\mu\, \overline{F} \gamma^\mu (g_V - g_A \gamma_5)\, F$$

$$g_V = T_3 - 2\sin^2\theta_W\, Q, \qquad g_A = T_3$$

The Strong interaction sector: Quantum Chromodynamics (QCD)

The QCD Lagrangian:

$$\mathcal{L}_{\text{QCD}} = \sum_f i\overline{q_f}\gamma^\mu \left(\partial_\mu + i g_s G^a_\mu T^a\right) q_f - \sum_f m_f \overline{q_f} q_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

(Approximate global) **Flavour symmetries**:

[Explicit breaking controlled by mass terms, possessing themselves flavour symmetries:

effective chiral Lagrangians]

a) N_f equal mass quarks

SU(N_f)$_V$×U(1)$_B$ vector flavour symmetry:

$$J^\alpha_{V,\mu} = \overline{q}\,\gamma_\mu\, T^\alpha\, q, \qquad J_{B,\mu} = \overline{q}\,\gamma_\mu\, q$$

$N_f = 2$: SU(2)$_I$ of isospin

$N_f = 3$: SU(3)$_f$ of flavour (eightfold way, quark model)

b) N_f massless quarks

Chiral symmetries: $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$

$$q_{L/R} = \frac{1 \mp \gamma_5}{2} q, \qquad J^\alpha_{L/R,\mu} = \overline{q}\,\gamma_\mu \frac{1 \mp \gamma_5}{2}\, T^\alpha\, q$$

$$J^\alpha_{V/A,\mu} = J^\alpha_{L,\mu} \pm J^\alpha_{R,\mu}, \qquad J_{A,\mu} = \overline{q}\,\gamma_\mu \gamma_5\, q$$

Axial symmetry $U(1)_A$:

explicit breaking by nonperturbative QCD dynamics

($SU(3)_c$ instantons)

Chiral symmetries $SU(N_f)_L \times SU(N_f)_R$:

spontaneous breaking by nonperturbative QCD dynamics

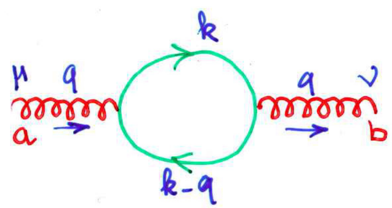
$\langle \overline{u}u \rangle \neq 0$, $\langle \overline{d}d \rangle \neq 0$, $\langle \overline{s}s \rangle \neq 0$

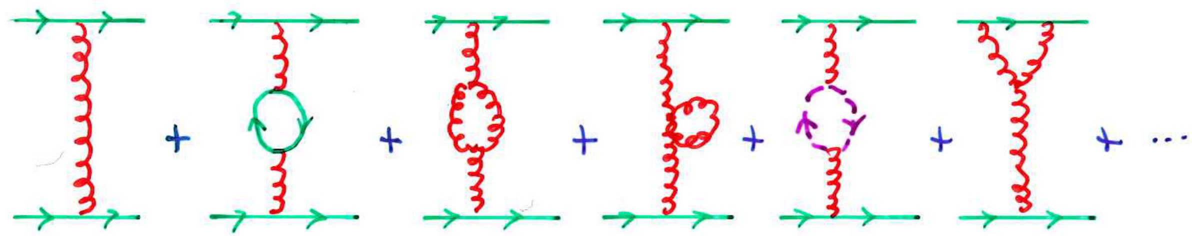
(pseudo)Goldstone bosons: pseudoscalars (π's, K's, η):

$$J^a_{A,\mu} = \overline{q}\,\gamma_\mu \gamma_5 \frac{\lambda^a}{2}\, q$$

Quantum corrections, renormalisation, asymptotic freedom

Application of perturbation theory at the level of quarks and gluons





Renormalisation point, running coupling constant:

$$\alpha_s(\mu^2) = \frac{g_s^2(\mu^2)}{4\pi}, \qquad \alpha_s(Q^2)$$

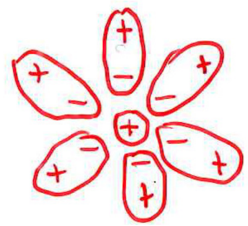
The β-function: scale dependence of $\alpha_s(Q^2)$:

$$\beta(\alpha_s) = \frac{\mu}{\alpha_s(\mu^2)} \frac{d\alpha_s(\mu^2)}{d\mu}, \qquad \beta(\alpha_s) = \beta_1 \left(\frac{\alpha_s}{\pi}\right) + \beta_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \cdots$$

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \frac{1}{2\pi}\beta_1\, \alpha_s(Q_0^2)\ ln\left(\frac{Q^2}{Q_0^2}\right)}$$

Infrared freedom: $\beta_1 > 0$ [QED; Landau pole]

$$\beta_1^{\text{QED}} = \frac{2}{3}$$



Aymptotic freedom: $\beta_1 < 0$ [QCD; quarks]

[dimensional transmutation]

$$\text{SU}(N)_c: \quad \beta_1 = \frac{2N_f - 11N_c}{6} = \frac{2N_f - 33}{6}, \qquad N_f \leq 16$$

$$\alpha_s(Q^2) = \frac{2\pi}{-\beta_1\, ln\left(\frac{Q^2}{\Lambda^2}\right)}, \qquad -\frac{\beta_1}{2\pi}\ ln\left(\frac{Q_0^2}{\Lambda^2}\right) = \frac{1}{\alpha_s(Q_0^2)}$$

The parton model of deep inelastic scattering

Parton distribution or structure functions
of quarks and gluons

Lectures by **Peter Skands**

The SU(3) Lie algebra: The fundamental representation

$$\left[T^a, T^b\right] = i f^{abc} T^c, \qquad T^a = \frac{1}{2}\lambda^a, \qquad a = 1,2,3,4,5,6,7,8$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \; \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\begin{aligned} \frac{1}{2} f^{123} &= f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = \\ &= -f^{367} = \frac{1}{\sqrt{3}} f^{458} = \frac{1}{\sqrt{3}} f^{678} = \frac{1}{2} \end{aligned}$$