Quarkonium Fragmentation Functions

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Quarkonia as Tools
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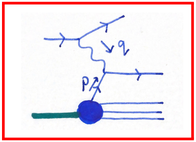
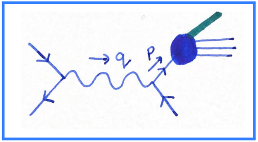
Introduction

Fragmentation functions: final-state counterpart to PDFs

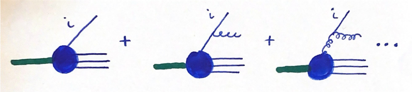


$e^- H \to e^- X$

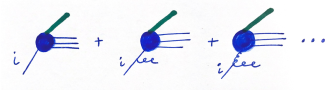


$e^+ e^- \to HX$

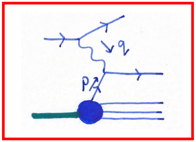
- **P**arton **D**istribution **F**unction: $f_{i/H}(x, \mu^2)$
  parton i is emitted from hadron H carrying longitudial momentum fraction x of H
  - DGLAP evolution amounts to resumming initial-state collinear divergences:



- **F**ragmentation **F**unction: $D_i^H(z, \mu^2)$
  hadron H is emitted from parton i carrying longitudial momentum fraction z of i
  - DGLAP evolution amounts to resumming final-state collinear divergences:



Fragmentation functions: final-state counterpart to PDFs



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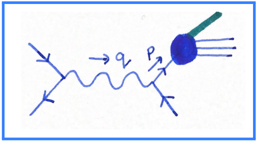


$e^+ e^- \to HX$

- **Parton Distribution Function:** $f_{i/H}(x, \mu^2)$
  parton i is emitted from hadron H carrying longitudial momentum fraction x of H
  - Scale: $\mu^2 = -q^2$ [space-like]
  - DGLAP evolution with space-like (S) splitting kernels:

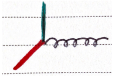
$$\frac{\partial}{\partial \ln \mu^2} f_{i/H}(x, \mu^2) = \sum_j \int_x^1 \frac{dx'}{x'} P_{ij}^S \left(\frac{x}{x'}, \alpha_s(\mu^2) \right) f_{j/H} \left(x', \mu^2 \right)$$

- **Fragmentation Function:** $D_i^H(z, \mu^2)$
  hadron H is emitted from parton i carrying longitudial momentum fraction z of i
  - Scale: $\mu^2 = q^2$ [time-like]
  - DGLAP evolution with time-like (T) splitting kernels:

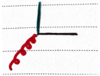
$$\frac{\partial}{\partial \ln \mu^2} D_i^H(z, \mu^2) = \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}^T \left(z', \alpha_s(\mu^2) \right) D_j^H \left(\frac{z}{z'}, \mu^2 \right)$$

Splitting kernals

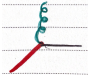
- The kernals $P_{ij}(x)$ describes the splitting of parton j into parton i carrying momentum fraction x of j
- At LO accuracy in α_s $P^S_{ij} = P^T_{ij} = P_{ij}$:



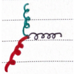
$$P_{qq}(x) = 2C_F \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right)$$



$$P_{qg}(x) = 2T_R \left(x^2 + (1-x)^2 \right)$$



$$P_{gq}(x) = 2C_F \left(\frac{1+(1-x)^2}{x} \right)$$



$$P_{gg}(x) = 4C_A \left(\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right) + \delta(1-x)\frac{11C_A + 4N_f T_R}{3}$$

where $C_F = \frac{4}{3}$, $T_R = \frac{1}{2}$, and $C_A = 3$

Heavy hadron (H_Q) production: $p_T \not\gg m_Q$

Nucl.Phys.B 421 (1994) 530-544; slides from Ingo Schienbein

H_Q production via **F**ixed **F**lavour **N**umber **S**cheme (FFNS):

$$\frac{d\sigma}{dp_{T,H_Q}} = \sum_{i,j,Q} f_{i/A}(\mu_F) \otimes f_{j/B}(\mu_F) \otimes \frac{d\hat{\sigma}_{ij\to QX}}{dp_{T,Q}}(\mu_F, \mu_R, m_Q) \otimes D_Q^{H_Q}$$

- $\otimes$ denotes a Mellin Convolution: $f \otimes g(x) = \int_0^1 dy \int_0^1 dz\, f(y)g(z)\delta(x - yz)$
- PDF:
  - Only light flavours in initial state: $i, j \in \{q, \bar{q}, g\}$, where $q = u, d, s$
  - **perturbative** μ_F evolution which absorbs initial-state collinear singularities
  - **non-perturbative** boundary condition: $f_{i/H}(x, \mu_0)$ at $\mu_0 = \mathcal{O}(1\text{ GeV})$
- Owing to m_Q, no final-state collinear singularities in $\hat{\sigma}$ or $D_Q^{H_Q}$!
- However, logs of the kind $\alpha_s \ln(p_T/m_Q)$ appear in $\hat{\sigma}$
- For $p_T \gg m_Q$, these logs are large and should be resummed

Heavy hadron (H_Q) production: $p_T \gg m_Q$

H_Q production via **Z**ero **M**ass **V**ariable **F**lavour **N**umber **S**cheme (ZM-VFNS):

- For large scale ($p_T \gg m_Q$) we can treat the quarks as massless in $\hat{\sigma}$ up to corrections $\mathcal{O}((m_Q/p_T)^2)$:

$$\frac{d\sigma}{dp_{T,H_Q}} \simeq \sum_{i,j,k} f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i}) \otimes \frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i}, \mu_{F_f}, \mu_R) \otimes D_k^{H_Q}(\mu_{F_f})$$

- In $\hat{\sigma}$ take $i, j, k \in \{q, \bar{q}, g, Q, \bar{Q}\}$ but consider them to be **massless**
- We introduce an additional scale, μ_{F_f}, and the large logs from the prevoious partonic cross section are effectively split into 2 terms $\ln(p_T/m_Q) = \ln(p_T/\mu_{F_f}) + \ln(\mu_{F_f}/m_Q)$:
  - $\ln(p_T/\mu_{F_f})$: contained within $\hat{\sigma}$, this is small provided $\mu_F \sim p_T$
  - $\ln(\mu_{F_f}/m_Q)$: resummed to all orders by evolution equations in $D_k^{H_Q}(\mu_{F_f})$
- The mass dependance is absorbed into the FF
- This results in a better control of the theortical uncertainty at large p_T

Quarkonia in NRQCD in FFNS and ZM-VFNS picture

- **FFNS** approach valid for $p_T \not\gg m_Q$

$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} = \sum_{i,j,n} f_{i/A}(\mu_{F_i}) \otimes f_{j/B}(\mu_{F_i}) \otimes \frac{d\hat{\sigma}_{ij\to Q\bar{Q}[n]X}}{dp_{T,Q\bar{Q}[n]}}(\mu_{F_i}, \mu_R, m_Q)\langle \mathcal{O}^{\mathcal{Q}}_{Q\bar{Q}[n]} \rangle$$

  - $\hat{\sigma}_{ij\to Q\bar{Q}[n]X}$ computed within NRQCD
  - Non-pertubative physics contained in LDME **no convolution, just a number**

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  - In practice, fragmentation functions are computed in NRQCD up to the LDME

$$D^{\mathcal{Q}}_i(z, \mu_0) = \sum_n D^{Q\bar{Q}[n]}_i(z, \mu_0)\langle \mathcal{O}^{\mathcal{Q}}_{Q\bar{Q}[n]}\rangle$$

Quarkonia in NRQCD in **FFNS** and **ZM-VFNS** picture

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  - In practice, fragmentation functions are computed in NRQCD up to the LDME

$$D_i^{\mathcal{Q}}(z,\mu_0) = \sum_n D_i^{Q\bar{Q}[n]}(z,\mu_0)\langle \mathcal{O}^{\mathcal{Q}}_{Q\bar{Q}[n]}\rangle$$

- To describe the full p_T spectrum should combine **FFNS** & **ZM-VFNS** using a matching scheme
- However, in what follows we focus on the large p_T region/**ZM-VFNS**

Fragmentation functions

Inclusive quarkonium production cross section at large p_T

Fragmentation function enters cross section as a convolution with $d\tilde{\sigma}_k$

$$\frac{d\sigma}{dp_{T,\mathcal{Q}}} \simeq \sum_k \underbrace{\sum_{i,j} f_i(\mu_{F_i}) \otimes f_j(\mu_{F_i}) \otimes \underbrace{\frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_{F_i}, \mu_{F_f}, \mu_R)}_{\propto p_{T,k}^{-4} \text{ at LO}}}_{d\tilde{\sigma}_k \propto p_{T,k}^{-n}} \otimes D_k^{\mathcal{Q}}(\mu_{F_f})$$

where $p_{T,k} = \frac{p_{T,\mathcal{Q}}}{z}$

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where $p_{T,k} = \frac{p_{T,\mathcal{Q}}}{z}$

$$\begin{aligned} \frac{d\sigma}{dp_{T,\mathcal{Q}}} &\simeq \sum_k d\tilde{\sigma}_k \otimes D_k^{\mathcal{Q}} \\ &\propto \sum_k \left(p_{T,k} = \frac{p_{T,\mathcal{Q}}}{z}\right)^{-n} \otimes D_k^{\mathcal{Q}} \\ &\propto \sum_k \underbrace{\int dz\, z^{n-1} D_k^{\mathcal{Q}}(z)}_{n^{\text{th}} \text{ Mellin Moment!}} \end{aligned}$$

- Cross section sensitive to the n^{th} Mellin Moment of the fragmentation function: $n = 5.5, 4.5$ according to fits to HERA, LHC data

$\mathcal{Q}$ fragmentation function shapes

- Different channels have different shapes at μ_0

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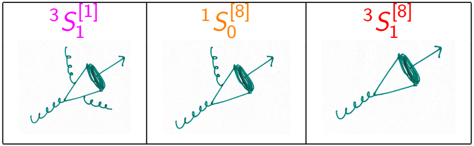

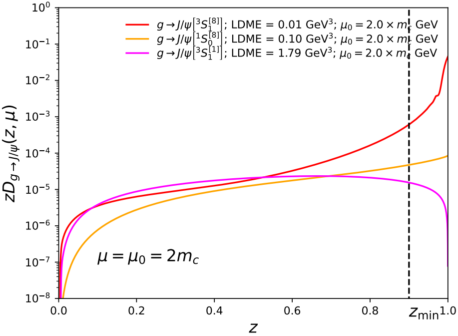

$\mathcal{Q}$ fragmentation function shapes

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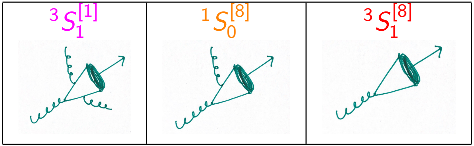
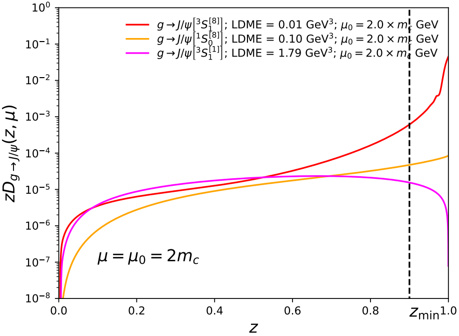




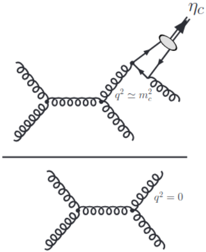
- Observable discriminant in the different production channels?
  - Isolation-like observable: amounts to a $z_{\min}$ cut

Computation of fragmentation functions

- From the decay of a virtual particle:
  - computed as the ratio of the cross sections
    - Example $g \to \eta_c$

Int.J.Mod.Phys.A 21 (2006) 3857-3916

$$\int_0^1 D(z, m_c)dz \simeq$$



- Using the Collins-Soper definition **Nucl. Phys. B 194 (1982) 445**
  - Gauge-invariant definition that includes an eikonal coupling in Feynman rules

Fragmentation functions at lowest order in α_s

⊗ $g \to c\bar{c}(^3S_1^8)$: **Phys. Rev. Lett. 74 (1995) 3327**

$$D_g^{J/\psi[^3S_1^{[8]}]}(z,\mu_0) = \delta(1-z)\frac{\pi\alpha_s(\mu_0)}{24m_Q^3}\langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle \quad (1)$$

⊗ $g \to c\bar{c}(^1S_0^8)$: **Phys. Rev. D 89 (2014) 094029, Phys. Rev. D 55 (1997) 2693, JHEP 11 (2012) 020**

$$D_g^{J/\psi[^1S_0^{[8]}]}(z,\mu_0) = = \frac{(N_c^2-4)\alpha_s^2(\mu_0)}{4N_c m_Q^3}[2(1-z)\log(1-z)+3z-2z^2]\langle\mathcal{O}_8^{J/\psi}(^1S_0)\rangle \quad (2)$$

⊗ $g \to c\bar{c}(^3S_1^1)$: **Phys. Rev. Lett. 71 (1993) 1673, Phys. Rev. D 96, 094016 (2017)**

$$D_g^{J/\psi[^3S_1^{[1]}]}(z,\mu_0) = \frac{128(N_c^2-4)\pi^3\alpha_s^3(\mu_0)}{3N_c^2(2m_Q)^3}\left(Cl_{13}+\sum_{i=0}^{11}C_i\,L_i\right)\langle\mathcal{O}_1^{J/\psi}(^3S_1)\rangle$$

$$L_0=1\,,\; L_1=\ln z\,,\; L_2=\ln(1-z)\,,\; L_3=\ln(2-z)\,,\; L_4=\ln^2 z\,,\; L_5=\ln^2(1-z)\,,\; L_6=\ln^2(2-z)\,, \quad (3)$$

$$L_7=\ln z\,\ln(1-z)\,,\; L_8=\ln z\,\ln(2-z)\,,\; L_9=Li_2(1-z)\,,\; L_{10}=Li_2\left(\frac{z-1}{z-2}\right)\,,\; L_{11}=Li_2\left(\frac{2(z-1)}{z-2}\right)\,\ldots$$

- All LO expressions for g, q, c, Q to $^3S_1^{[1]}$, $^3S_1^{[8]}$, $^3P_J^{[8]}$, and $^1S_0^{[8]}$ collected in **Phys. Rev. D 89, 094029 (2014)**

Evolution of fragmentation function I

- The fragmentation function is computed at $\mu_0 \sim m_Q$ and is convoluted with the hard partonic cross section at $\mu_F \sim p_T$ where $p_T \gg m_{\mathcal{Q}}$

$$\frac{d\hat{\sigma}_{ij\to kX}}{dp_{T,k}}(\mu_F) \otimes D_k^{\mathcal{Q}}(\mu_F)$$

- Must evolve from μ_0 to μ_F

Evolution of fragmentation function I

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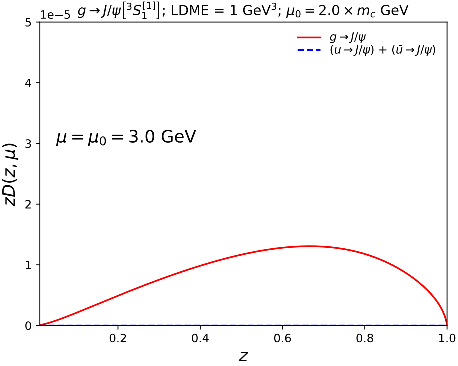
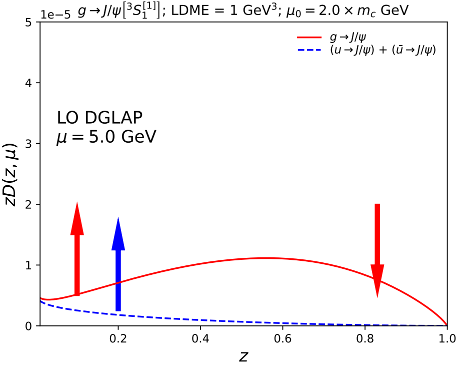


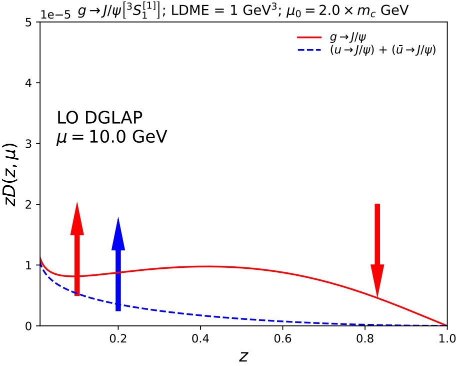
- Initial condition for $D_g^{\mathcal{Q}}(\mu_0)$ via $^3S_1^{[1]}$ chanel **Phys. Rev. Lett. 71 (1993) 1673, Phys. Rev. D 89 (2014) 094029**
- $D_k^{\mathcal{Q}}(\mu_0) = 0$ for $k \in \{q, \bar{q}, Q, \bar{Q}\}$

Evolution of fragmentation function II



- Effect of evolution:
  - Large-z gluon shrinks
  - Low-z gluon grows
  - Low-z quark grows

Evolution of fragmentation function III



- Effect of evolution:
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  - Low-z gluon grows
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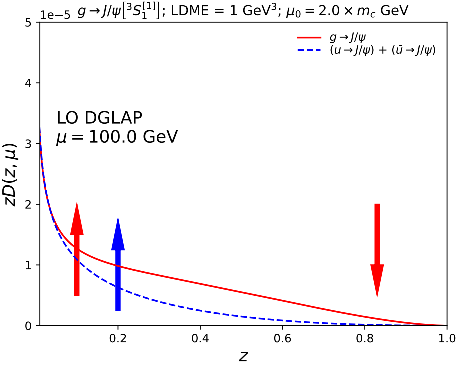
Evolution of fragmentation function IV



- Effect of evolution:
  - Large-z gluon shrinks
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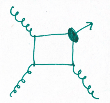
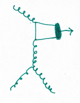
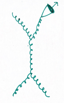
FFNS vs. ZM-VFNS: p_T hierarchy

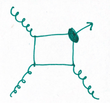
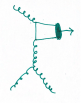
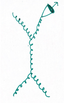
Fixed **F**lavour **N**umber **S**cheme:

$^3S_1^{[1]}$	$^1S_0^{[8]}$	$^3S_1^{[8]}$
		
$\alpha_s^3 p_T^{-8}$	$\alpha_s^3 p_T^{-6}$	$\alpha_s^3 p_T^{-4}$

- v^2-supressed terms ($^1S_0^{[8]}$, $^3S_1^{[8]}$) are leading and subleading in p_T

FFNS vs. ZM-VFNS: p_T hierarchy

Fixed Flavour Number Scheme:

${}^3S_1^{[1]}$	${}^1S_0^{[8]}$	${}^3S_1^{[8]}$
		
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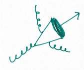
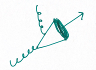
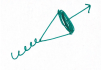
- v^2-supressed terms (${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$) are leading and subleading in p_T

Zero Mass Variable Flavour Number Scheme:

...

- All contributions enter with same scaling in p_T
- Number of couplings modifies FF at μ_0

${}^3S_1^{[1]}$	${}^1S_0^{[8]}$	${}^3S_1^{[8]}$
		
$\alpha_s^3(\mu_0)$	$\alpha_s^2(\mu_0)$	$\alpha_s(\mu_0)$

What about higher order terms?

- The first term is valid up to corrections $\mathcal{O}(m_{\mathcal{Q}}^2/p_T^2)$

$$d\sigma_{AB\to\mathcal{Q}X} = \sum_i d\tilde{\sigma}_{AB\to iX} \otimes \underline{D_{i\to\mathcal{Q}}} + \sum_\kappa d\tilde{\sigma}_{AB\to Q\bar{Q}[\kappa]X} \otimes \underline{D_{Q\bar{Q}[\kappa]\to\mathcal{Q}}} + \mathcal{O}(m_{\mathcal{Q}}^4/p_T^4)$$

- **Leading power** (single parton fragmentation): a single parton i decays into the observed $\mathcal{Q}$
- **Next-to-leading power** (double parton fragmentation): two partons in a spin and colour state κ decay into the observed $\mathcal{Q}$
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- Expect that double parton fragmentation of a $Q\bar{Q}$ is more likely than single parton fragmentation
  - ${}^3S_1^{[1]}$: LP first appears at $\mathcal{O}(\alpha_s^3)$ vs. NLP first appears at $\mathcal{O}(\alpha_s^1)$

Phys. Rev. Lett. 113, 142002 (2014)

Relevance of higher order terms

- Therefore at intermediate p_T, NLP contributions are important
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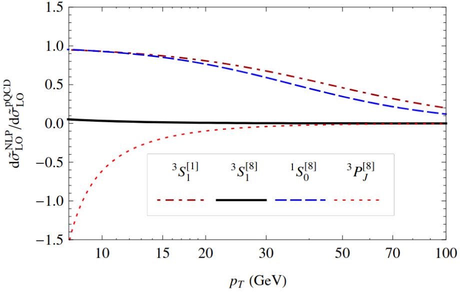
Ma et. al., Phys.Rev.Lett. 113 (2014) 14, 142002



p_T[GeV]	NLP/(LP + NLP) $^3S_1^{[1]}$	$^1S_0^{[8]}$	$^3S_1^{[8]}$	$^3P_J^{[8]}$
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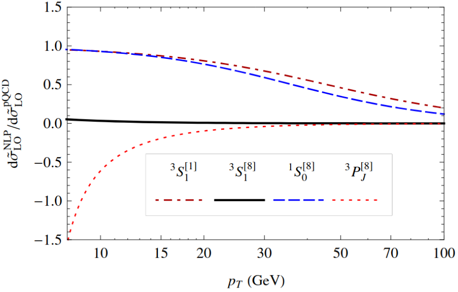

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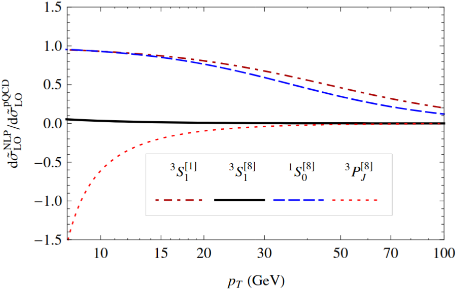

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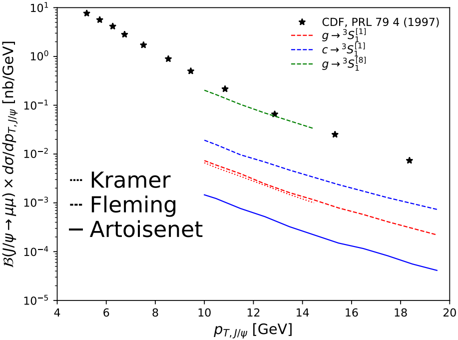
No **phenomenology** at next-to-leading power accuracy with **evolution**!

Phenomenological applications of fragmentation functions

$p\bar{p}$ data comparisons



- Kramer: **Prog. Part. Nucl. Phys., 47:141–201, 2001**
  - CTEQ5l
  - LO evolution
  - $\langle \mathcal{O}_1^{J/\psi}({}^3S_1) \rangle = 1.16\ \text{GeV}^3$
  - $d\hat{\sigma}$ at α_s^2
- Fleming: **PhD thesis, 1995**
  - MRS-D0
  - LO evolution* [only diagonal splitting functions P_{ii}]
  - $\langle \mathcal{O}_1^{J/\psi}({}^3S_1) \rangle = 1.00\ \text{GeV}^3$
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  - $d\hat{\sigma}$ at α_s^{2*} [$d\hat{\sigma}_c$ at α_s^3]
- Artoisenet, Lansberg, Maltoni: **Phys. Lett. B, 653:60–66, 2007**
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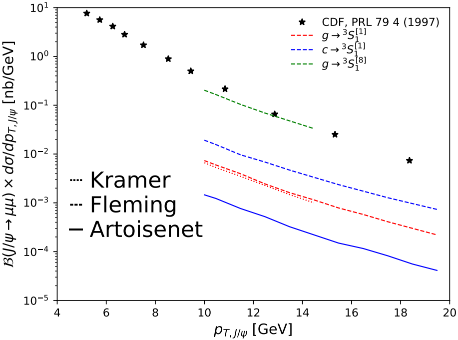

- $g \to J/\psi$: Kramer and Fleming are comparable
- $c \to J/\psi$: Fleming and Artoisenet are ~ 10 apart
- $g \to J/\psi$: LDME is fit to data

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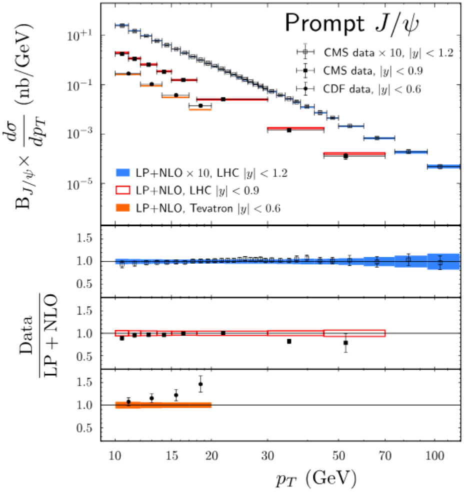
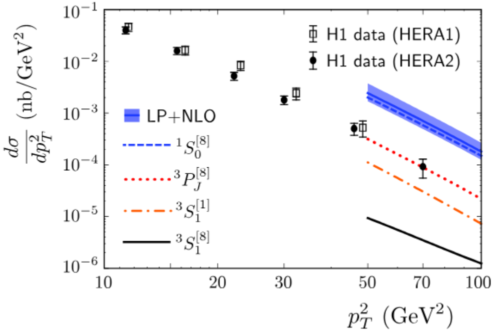
More recent data comparisons

Bodwin et. al.; Phys.Rev.D 93 (2016) 3, 034041, Phys.Rev.D 92 (2015) 7, 074042





- LP+NLO: FFNS $^1S_1^{[1]}$ + matched FFNS and LP $^3P_J^{[8]}$, $^3S_1^{[8]}$, $^1S_0^{[8]}$
  - **LP/ZM-VFNS**:
    - $d\hat{\sigma}$ at α_s^3
    - $D_i^{\mathcal{Q}}$ at α_s^2
    - LO evolution/LL resummation
  - **FFNS**:
    - $d\hat{\sigma}$ at α_s^4

Matching Scheme Bodwin et. al.; Phys.Rev.D 93 (2016) 3, 034041, Phys.Rev.D 92 (2015) 7, 074042

- In order to describe the whole p_T region one should combine the FFNS and ZM-VFNS contributions
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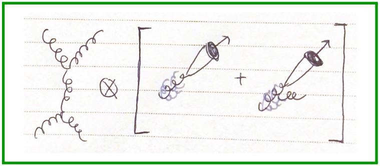
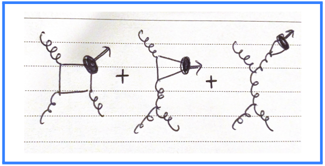
$$d\sigma^{\text{LP+NLO}} = \underbrace{d\sigma^{\text{ZM-VFNS}}}_{\alpha_s^2 \otimes \alpha_s^2} + \underbrace{d\sigma^{\text{FFNS}}}_{\alpha_s^3} - d\sigma_{\text{matching}}$$





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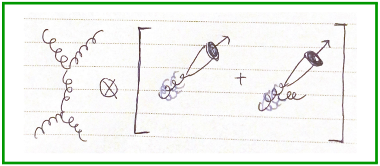
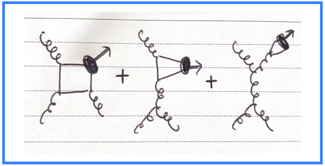
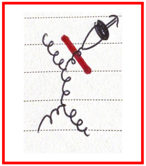




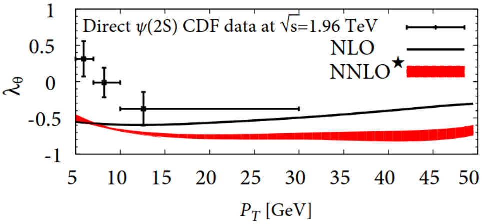


- Double counting is $\mathcal{O}(\alpha_s^3)$
- Matching term is the $\mathcal{O}(\alpha_s^3)$ component of the ZM-VFNS contribution without evolution

$\mathcal{Q}$ polarisation at large P_T

Phys.Rept. 889 (2020) 1-106, Phys. Rev. D 96, 094016 (2017)

- $\frac{dN}{d\cos\theta} \propto 1 + \lambda_\theta \cos^2\theta$ where $\lambda_\theta = \frac{1/2\sigma_T - \sigma_L}{1/2\sigma_T + \sigma_L}$
- $\lambda_\theta = +1$ transverse; $\lambda_\theta = -1$ longitudinal; $\lambda_\theta = 0$ unpolarised
- **F**ixed **F**lavour **N**umber **S**cheme results:
  - transversely polarised at LO
  - longitudinaly polarised at NLO, NNLO*



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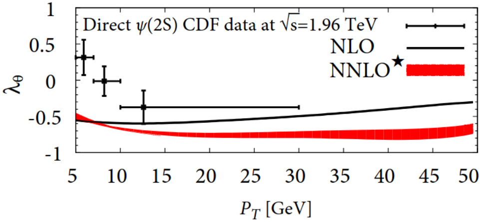

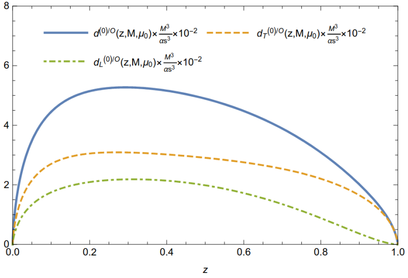

- What about FF?

${}^3S_1^{[1]}$ FF at μ_0

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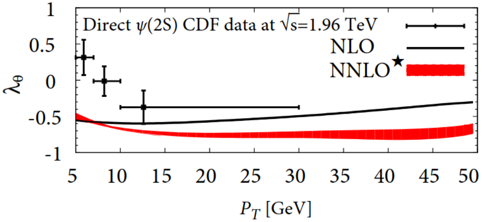
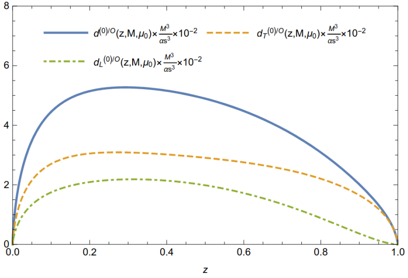




- What about FF?
  - $z = 0.1$: $\lambda_\theta \approx -0.1$ and $z = 0.9$: $\lambda_\theta \approx 0.4$

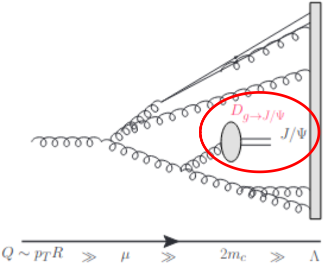
${}^3S_1^{[1]}$ FF at μ_0

$\mathcal{Q}$ in jet and fragmentation functions

See talk of Paul Caucal on Monday

Quarkonia in jets - formalism

- J/Ψ at high p_T is expected to predominantly come from jet fragmentation.
- Formalism based on the jet evolution outlined above + FF at the scale $\sim m_c$.



$$\frac{\mathrm{d}\sigma^{pp\to j_1+j_2(J/\Psi)+X}}{\mathrm{d}p_T \mathrm{d}z_{J/\Psi}} = H_{ab\to ij} \otimes f_a \otimes f_b \otimes J_j \otimes \mathcal{G}_i^{J/\Psi}(p_T, R, z, \mu)$$

$$\mathcal{G}_i^{J/\Psi} \sim C_{ij}(p_T, R, \mu) \otimes K_{\mathrm{DGLAP}} \left[D_{j\to J/\Psi}(2m_c)\right]$$

State κ	${}^3S_1^{[1]}$	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^3P_J^{[8]}$
$g \to c\bar{c}(\kappa)$	α_s^3	α_s	α_s^2	α_s^2
LDME $\langle O_\kappa^{J/\Psi}\rangle$	$(v/c)^3$	$(v/c)^7$	$(v/c)^7$	$(v/c)^7$

$\Rightarrow$ competing orders of magnitudes between $g \to c\bar{c}(\kappa)$ and LDME in NRQCD.

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Available computing tools for the study of fragmentation functions

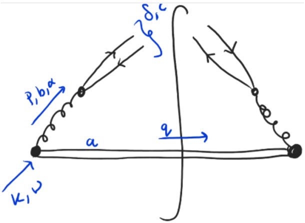
- Fragmentation function evolution (LHAPDF grid format):
  - APFEL++ (https://github.com/vbertone/apfelxx)
    - Input: $zD_i^{\mathcal{Q}}(z, \mu_0)$
    - Must be a continuous function
  - MELA (https://github.com/vbertone/MELA)
    - Input: $\tilde{D}_i^{\mathcal{Q}}(N, \mu_0)$
    - Can be discontinuous (e.g. contain δ functions/plus distributions)
- Tools for phenomenological studies:
  - INCNLO (https://lapth.cnrs.fr/PHOX_FAMILY/readme_inc.html)
  - FMNLO (https://fmnlo.sjtu.edu.cn/)

Conclusion

- Fragmentation functions (FF) appear as a natural description of heavy-hadron production for $p_T \gg m_Q$
- It is believed that $\mathcal{Q}$ FF can be computed within the NRQCD framework **modulo the LDMEs**
- There is no existing phenomenology for NLP (double parton) FF **evolution is complicated**
  - What is the relative size of the evolved LP and NLP contributions?
- LP FF should be sufficient to describe latest ATLAS data, which extends up to 360 GeV **Eur.Phys.J.C 84 (2024) 169, 2024 (in this region we assume $\sigma_{\text{LP}} \gg \sigma_{\text{NLP}}$)**
- Large-p_T observables can be described using $\mathcal{Q}$ FF
  - Isolated $\mathcal{Q}$
  - $\mathcal{Q}$ in jets
  - $\mathcal{Q}$ polarisation
  - ...
- We are currently re-examining existing phenomenology with LP FF

Backup

Example: computation of $g \to J/\psi(^3S_1^{[8]})$ FF using Collins-Soper definition I



1. Compute Amplitude on LHS of cut line: [eikonal coupling]

$$\mathcal{A}_{\nu\alpha} = -i\delta^{ab}\left[g_{\nu\alpha}(n\cdot k) - p_\nu n_\alpha\right]\left(ig\mu^\epsilon\gamma^\alpha T^b\right)$$

Example: computation of $g \to J/\psi({}^3S_1^{[8]})$ FF using Collins-Soper definition II

2. Contract with colour and spin projector:

$$\mathrm{Tr}\left[\mathcal{A}_{\nu\alpha}\Pi_8^c\Pi_1^\delta\right],$$

$$\Pi_8^c = \sqrt{2}T^c, \quad \Pi_1^\delta = \frac{1}{4m_Q^2}\left(\frac{\not{p}_Q}{2} - m_Q\right)\gamma_\delta\frac{(\not{p}_Q + 2m_Q)}{4m_Q}\left(\frac{\not{p}_Q}{2} + m_Q\right)$$

3. Compute amplitude square:

$$|\mathcal{A}|^2 = \mathrm{Tr}\left[\mathcal{A}_{\nu\alpha}\Pi_8^c\Pi_1^\delta\right]\left(\mathrm{Tr}\left[\mathcal{A}_{\nu'\alpha'}\Pi_8^{c'}\Pi_1^{\delta'}\right]\right)^\dagger \Pi_{\delta\delta'}\delta^{cc'}(-g_{\nu\nu'})\delta^{aa'}$$

- $\Pi_{\delta\delta'}\delta^{cc'}$: colour and spin polarisation of $Q\bar{Q}\left[{}^3S_1^{[8]}\right]$
- $(-g_{\nu\nu'})\delta^{aa'}$: contract eikonal indicies

Example: computation of $g \to J/\psi(^3S_1^{[8]})$ FF using Collins-Soper definition III

4. Integrate over phase space and multiply by normalisation factors:

$$D_g^{J/\psi[^3S_1^{[8]}]}(z,\mu_0) = \frac{N_{\rm CS}}{k^4}|\mathcal{A}|^2 d\phi_0 \frac{\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle}{(D-1)(N_c^2-1)}$$

- $d\phi_0 = \frac{8\pi m_Q}{k\cdot n}\delta(1-z)$: normalisation of 0-body phase space
- $N_{\rm CS} = \frac{z^{D-3}}{(N_c^2-1)(k\cdot n)2\pi(D-2)}$: Collins-Soper normalisation
- $k^4 = (2m_Q)^4$: off-shellness of fragmenting gluon
- $\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle$: LDME
- $(D-1)(N_c^2-1)$: spin and colour averaging

to obtain final expression at $\mu_0 \sim 2m_c$:

$$D_g^{J/\psi[^3S_1^{[8]}]}(z,\mu_0) = \delta(1-z)\frac{\pi\alpha_s(\mu_0)}{24m_Q^3}\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle$$

When is $\sigma_{\text{FFNS}} \lesssim \sigma_{\text{ZM-VFNS}}$?

Phys.Rev.Lett. 71 (1993) 1673-1676

- At what values of p_T does the fragmentation function contribution become important?
  - i.e. for what value of p_T does this hold:

$$\frac{d\sigma_{gg\to g\eta_c}}{dt} \approx \frac{d\sigma_{gg\to gg}}{dt} \times P_{g\to\eta_c} \tag{4}$$

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  - In the limit $m_c \to 0$ and $\hat{s} = 4p_T^2$

$$\frac{d\sigma_{gg\to g\eta_c}}{dt} = \frac{81\pi\alpha_s^3|R(0)|^2}{256M_{\eta_c}p_T^6} \quad \& \quad \frac{d\sigma_{gg\to gg}}{dt} = \frac{243\pi\alpha_s^2}{128p_T^4}$$

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$$\frac{d\sigma_{gg\to g\eta_c}}{dt} \approx \frac{d\sigma_{gg\to gg}}{dt} \times P_{g\to\eta_c} \tag{4}$$

  - In the limit $m_c \to 0$ and $\hat{s} = 4p_T^2$

$$\frac{d\sigma_{gg\to g\eta_c}}{dt} = \frac{81\pi\alpha_s^3|R(0)|^2}{256 M_{\eta_c} p_T^6} \quad \& \quad \frac{d\sigma_{gg\to gg}}{dt} = \frac{243\pi\alpha_s^2}{128 p_T^4}$$

  - The probability for a gluon to decay to η_c is given by:

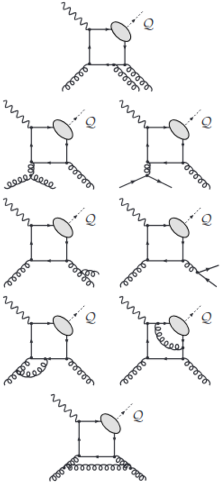
$$P_{g\to\eta_c} = \int_0^1 dz D_g^{\eta_c}(z,\mu_0) = \frac{1}{9\pi}\alpha_s^2(\mu_0)\frac{|R(0)|^2}{M_{\eta_c}^3}$$

$$D_g^{\eta_c}(z,\mu_0) = \frac{1}{3\pi}\alpha_s^2(\mu_0)\frac{|R(0)|^2}{M_{\eta_c}^3}\left(3z - 2z^2 + 2(1-z)\log(1-z)\right)$$

  - Hence we find eq(4) holds for $p_T \approx 3M_{\eta_c}$
  - For J/ψ we find $p_T \approx 2M_{J/\psi}$
- From these arguments FF can be used for $p_T \gtrsim \mathcal{O}(10\text{ GeV})$

General structure of NLO corrections

M. Krämer, Nucl.Phys., B459, 3 (96')



Singularities at NLO [and how they are removed]:

- Real emission
  - Infrared divergences: Soft [cancelled by loop IR contr.]
  - Infrared divergences: Collinear
    - initial state [subtracted via “renormalisation” of collinear PDFs (Altarelli-Parisi counter-terms)]
    - final state [cancelled by loop IR contr.]
- Virtual (loop) contribution
  - Ultraviolet divergences: [removed by renormalisation]
  - Infrared divergences: [cancelled by real Infrared contribution]

[The quark and antiquark attached to the blob are taken as on-shell and their relative velocity v is set to zero.]