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Exotic hadrons

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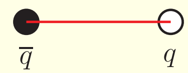
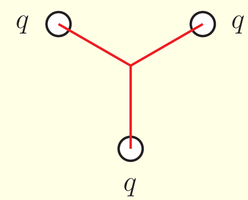
Multiquark states in QCD

Hadrons are color-singlet bound states of quarks and gluons.

Mesons are essentially made of $\overline{q}q$:

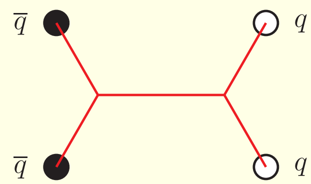


Baryons are essentially made of qqq:

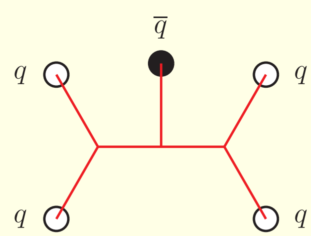


Other types of structure also exist for bound states (exotics)

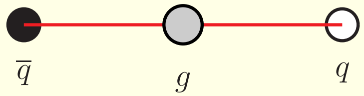
Tetraquarks, made of $\overline{q}\overline{q}qq$:



Pentaquarks, made of $\overline{q}qqqq$:



Hybrids, made of $\overline{q}gq$:



How to distinguish on experimental grounds these new states from ordinary hadrons?

With their masses (heavier), exotic quantum numbers (spin, parity, flavor) not reproduced by the quark model of ordinary hadrons, also corroborated by theoretical predictions from bound state calculations, study of the decay channels and rates, study of their production rates.

Difficulties when some of these states have hidden flavors (of the type $(\overline{c}c)$ or $(\overline{s}s)$ in a color-singlet state), in which case they can be confounded with radial or orbital excitations of ordinary hadrons.

For instance, a state of the type $\overline{u}c\overline{s}s$ could be confounded with an excitation of the D meson; or the state $\overline{c}c\overline{u}u$ with an excitation of the charmonium state.

Observed multiquark states

In the last twenty years, many tetraquark and a few pentaquark candidate states have been observed by several experiments: Belle, BaBar, BESIII, LHCb, CDF, D0, CMS. Ordinary meson and baryon structures could not fit their properties.

Naming: $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{Z}$, $\boldsymbol{T}$ for tetraquarks, $\boldsymbol{P}$ for pentaquarks.

$\boldsymbol{X}$: electrically neutral, $\boldsymbol{I^G(J^{PC}) = 0^+(J^{++})}$, with hidden charm content, same quantum numbers as the $\boldsymbol{\chi_{cJ}}$s.

$\boldsymbol{Y}$: electrically neutral, $\boldsymbol{I^G(J^{PC}) = 0^-(1^{--})}$, similar to vector mesons, with hidden charm content, same quantum numbers as the $\boldsymbol{\Psi}$s.

$\boldsymbol{Z}$: neutral and charged, $\boldsymbol{I^G(J^{PC}) = 1^+(1^{+-})}$, with hidden charm/bottom.

$\boldsymbol{T}$: generally with open charm content.

Attention. Naming in PDG different: $\chi_{cJ}(\mathrm{mass})$ for the $\boldsymbol{X}$s, $\psi(\mathrm{mass})$ for the $\boldsymbol{Y}$s, $\boldsymbol{T}$ for all others.

$\boldsymbol{X(3872)}$, $\mathbf{0^+(1^{++})}$.

$\boldsymbol{M = 3871.64 \pm 0.06}$ MeV, $\boldsymbol{\Gamma = 1.19 \pm 0.21}$ MeV.

$\boldsymbol{M_X - (M_{\bar{D}^0} + M_{D^{*0}}) = 0.00 \pm 0.18}$ MeV.

Main decays: $\boldsymbol{D^0\bar{D}^0\pi^0}$, $\boldsymbol{D^{*0}\bar{D}^0}$, (both together, 80%), $\boldsymbol{\omega J/\psi}$, $\boldsymbol{\rho J/\psi}$, (important isospin violation).

$\boldsymbol{X(4140)}$, $\mathbf{0^+(1^{++})}$.

$\boldsymbol{M = 4146.5 \pm 3.0}$ MeV, $\boldsymbol{\Gamma = 19 \pm 6}$ MeV.

$\boldsymbol{X(4274)}$, $\mathbf{0^+(1^{++})}$.

$\boldsymbol{M = 4286 \pm 9}$ MeV, $\boldsymbol{\Gamma = 51 \pm 7}$ MeV.

$\boldsymbol{X(4500)}$, $\mathbf{0^+(0^{++})}$.

$\boldsymbol{M = 4474 \pm 4}$ MeV, $\boldsymbol{\Gamma = 77 \pm 11}$ MeV.

$\boldsymbol{X(4700)}$, $\mathbf{0^+(0^{++})}$.

$\boldsymbol{M = 4694 \pm 11}$ MeV, $\boldsymbol{\Gamma = 87 \pm 14}$ MeV.

Main decays of the last four $\boldsymbol{X}$s: $\boldsymbol{\phi J/\psi}$.

$Y(4230)$, $0^-(1^{--})$.
$M = 4222 \pm 2.3$ MeV, $\Gamma = 49 \pm 7$ MeV.

$Y(4360)$, $0^-(1^{--})$.
$M = 4374 \pm 7$ MeV, $\Gamma = 118 \pm 12$ MeV.

$Y(4660)$, $0^-(1^{--})$.
$M = 4641 \pm 10$ MeV, $\Gamma = 73 \pm 12$ MeV.

Main decays: ψs, h_c, plus pseudoscalars.

$Z_c(3900)$, $1^+(1^{+-})$.
$M = 3887.1 \pm 2.6$ MeV, $\Gamma = 28.4 \pm 2.6$ MeV.

$Z_{cs}(4000)$, $\frac{1}{2}(1^+)$, $c\bar{c}q\bar{s}$.
$M = 3980 - 4010$ MeV, $\Gamma = 5 - 150$ MeV.

$Z_c(4020)$, $1^+(?^-)$.
$M = 4024.1 \pm 1.9$ MeV, $\Gamma = 13 \pm 5$ MeV.

$Z_c(4240)$, $1^+(0^{--})$.
$M = 4239 \pm 35$ MeV, $\Gamma = 220 \pm 110$ MeV.

$Z_c(4430)$, $1^+(1^{+-})$.
$M = 4478 \pm 17$ MeV, $\Gamma = 181 \pm 31$ MeV.

Main decays: ψs, h_c, plus pseudoscalars.

$Z_b(10610)$, $1^+(1^{+-})$.

$M_{Z_b^0} = 10607.2 \pm 2.0$ MeV, $M_{Z_b^+} = 10609 \pm 6$ MeV, $\Gamma = 18.4 \pm 2.4$ MeV.

Decays: $B^+\bar{B}^{*0} + B^{*+}\bar{B}^0$ (85%), $h_b\pi^+$, $\Upsilon\pi^+$.

$Z_b(10650)$, $1^+(1^{+-})$.

$M = 10652 \pm 1.5$ MeV, $\Gamma = 11.5 \pm 2.2$ MeV.

Decays: $B^{*+}\bar{B}^{*0}$ (74%), $h_b\pi^+$, $\Upsilon\pi^+$.

$T_{cs0}(2870)$, $?(0^+)$, $\bar{c}d\bar{s}u$ (4 different quark flavors).
$M = 2866 \pm 7$ MeV, $\Gamma = 57 \pm 13$ MeV.
Decays: $D^- K^+$.

$T_{c\bar{s}0}(2900)$, $1(0^+)$, $c\bar{s}d\bar{u}$ and $c\bar{s}u\bar{d}$.
$M(T^0_{c\bar{s}0} = 2892 \pm 21$ MeV, $\Gamma = 119 \pm 29$ MeV.
Decays: $D_s^+ \pi^-$.
$M(T^{++}_{c\bar{s}0} = 2921 \pm 26$ MeV, $\Gamma = 140 \pm 40$ MeV.
Decays: $D_s^+ \pi^+$.

$T^*_{cs1}(2900)$, $?(1^+)$.
$M = 2904 \pm 5$ MeV, $\Gamma = 110 \pm 12$ MeV.
Decays: $D^- K^+$.

$T_{cc}(3875)^+$, $?(?)$.
$M = 3874.83 \pm 0.11$ MeV, $\Gamma = 0.41 \pm 0.17$ MeV.
$M_{T_{cc}} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06$ MeV.
Decays: $D^0 D^0 \pi^+$.

$T_{b\bar{s}}(5568)^+$, $1(?)$, $b\bar{s}u\bar{d}$ (4 different quark flavors).

$M = 5566.9 \pm 3.3$ MeV, $\Gamma = 19 \pm 4$ MeV.

Decays: $\bar{B}_s^0 \pi^\pm$.

$T_{cc\bar{c}\bar{c}}(6900)$, $0^+(?^+)$.

$M = 6899 \pm 12$ MeV, $\Gamma = 153 \pm 29$ MeV.

Decays: $J/\psi J/\psi$.

$P_{c\bar{c}}(4312)$, $\frac{1}{2}(?)$.
$M = 4312 \pm 4$ MeV, $\Gamma = 10 \pm 5$ MeV.

$P_{c\bar{c}s}(4338)^0$, $0(\frac{1}{2}^-)$.
$M = 4338.2 \pm 0.8$ MeV, $\Gamma = 7.0 \pm 1.8$ MeV.

$P_{c\bar{c}}(4380)^+$, $\frac{1}{2}(?)$.
$M = 4380 \pm 30$ MeV, $\Gamma = 210 \pm 90$ MeV.

$P_{c\bar{c}}(4440)^+$, $\frac{1}{2}(?)$.
$M = 4440 \pm 5$ MeV, $\Gamma = 21 \pm 11$ MeV.

$P_{c\bar{c}}(4457)^+$, $\frac{1}{2}(?)$.
$M = 4457.3 \pm 4.0$ MeV, $\Gamma = 6.4 \pm 6.0$ MeV.

$P_{c\bar{c}s}(4459)^0$, $0(?)$.
$M = 4458.8 \pm 6.0$ MeV, $\Gamma = 17 \pm 10$ MeV.
Decays: $p\, J/\psi$ or $\Lambda J/\psi$.

Theoretical calculations

- Constituent quark model.
- Born-Oppenheimer effective theories of mesons and baryons.
- Heavy quark effective theory with spin symmetry.
- QCD sum rules.
- Lattice theory calculations.
- Large N_c.

One of the main difficulties comes from the cluster reducibility of multiquark states.

Contrary to ordinary hadrons, multiquark states are not color-irreducible, in the sense that they can be decomposed along a finite number of combinations of ordinary mesonic or baryonic clusters.

$$(\bar{q}\bar{q}qq) = \sum(\bar{q}q)(\bar{q}q),$$

$$(\bar{q}qqqq) = \sum(\bar{q}q)(qqq),$$

Hadronic clusters, being color-singlets, mutually interact by means of short-range forces, like meson-exchanges or contacts. They would form loosely bound states, in similarity with atomic molecules. These are called hadronic molecules or molecular states. (Törnqvist, 1994.)

In contrast, multiquark states formed directly from confining interactions acting on all quarks (diquark mechanism), would form compact bound states, called compact multiquark states. (Maiani *et al.*, 2005.)

Multiquark states can thus be formed by two different mechanisms, each leading to a different structure. The issue is to find, by theoretical justification, the most faithful description.

Compact tetraquarks are expected to have small sizes: $\mathbf{0.3 - 0.5}$ fm.

Molecular tetraquarks are expected to have large sizes: $\mathbf{2 - 20}$ fm.

Phenomenological differences between compact and molecular states

Compact multiquark states are expected to have small sizes. For tetraquarks: $R \sim 0.3 - 0.5$ fm.

Molecular multiquark states are expected to have large sizes. For tetraquarks: $R \sim 2 - 20$ fm.

Compact multiquark states would have larger flavor multiplicities (confining forces do not depend on flavor). For molecular states, the binding energies are more sensitive to the quark masses, reflected by the constituent and exchanged hadron masses.

Also, there is a difference in the production rates in hadron-hadron collisions. At high energies, compact states behave like elementary particles and should be produced promptly with high transverse momenta. Molecular states need time to be formed from other elementary particles during the production process and should be produced with low transverse momenta.

Energy balance

Compact states and molecular states are formed with different mechanisms. Below the case of tetraquarks.

In the compact scheme, two quarks, which together are in the color-gauge group antisymmetric representation $\bar{\mathbf{3}}$, are submitted to a mutual attractive confining force which allows them to form an intermediate compact diquark bound state. Similarly for two antiquarks, which are in the color-gauge group representation $\mathbf{3}$, forming an antidiquark bound state. Then, the diquark and the antidiquark form together a final color-singlet compact bound state.

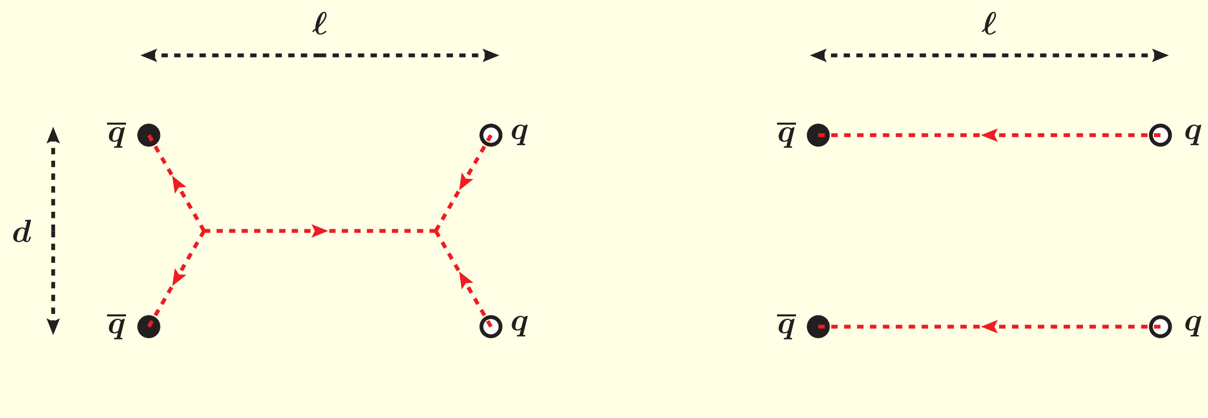
In themolecular scheme, the quarks are separately linked to antiquarks, forming mesonic clusters, which now mutually interact with short-range forces (meson-exchange, or contact).

The mechanism that is favored is the one that corresponds to the lowest energy of the system.

The energy balance of the system is most easily studied in the static limit of the theory, with very heavy quarks, fixed at spatial positions. The system would choose configurations with minimal energy.

The problem is analytically solved in the strong coupling limit of lattice theory (Dosch, 1983) and confirmed by direct lattice numerical calculations (Alexandrou *et al.*, 2005; Suganuma *et al.*, 2005; Bicudo *et al.*, 2011).

In the strong coupling limit, the potential energy is concentrated on the Wilson lines (path-ordered gluon field phase factors) with constant linear energy density (linear confinement).



(a) (b)

(a): Compact tetraquark, formed by confining interactions, through diquark–antidiquark global interaction.

(b): Two meson clusters.

The compact tetraquark formation is energetically favored if the diquark interdistance d is much smaller than the quark-antiquark interdistance ℓ:

$$d \ll \ell.$$

However, quarks have finite masses and move in space. Even if a compact tetraquark has been formed, there is a probability that the quarks, during their motion, reach the two-meson-cluster configuration, whose presence could not be ignored. On the quantum level, this is realized by means of fluctuations represented by the couplings of the compact object with its internal mesonic clusters.

This will have the tendancy to deform the initial compact system into a more loosely bound system, having similarities with a molecular-type state.

This is a dynamical mechanism, whose precise outcome necessitates the resolution of the four-body bound state problem, in the presence of the confining forces. For the time being, this problem has not yet been solved in its generality. One at least deduces that it seems difficult to realize, without additional constraints, a pure compact multiquark state.

Compositeness

The comparison of the molecular and compact schemes is reminiscent of a general problem, already raised in the past in the case of the deuteron state, denoted under the term of compositeness (Weinberg, 1965).

Weinberg has shown that this question can receive, in the nonrelativistic limit, a precise and model-independent answer, by relating the probability of a state as being elementary (or compact) to observable quantities, represented by the scattering length and the effective range of the two constituents of the molecular scheme in the S-wave state of their scattering amplitude. Designating by Z this probability, one has the following relations for the scattering length a and the effective range r_e, adapted to the tetraquark problem (with small binding energy):

$$a = \frac{2(1-Z)}{(2-Z)}R, \qquad r_e = -\frac{Z}{(1-Z)}R, \qquad R = (-2m_r E_t)^{-1/2},$$

where R is the radius of the bound state, m_r the reduced mass of the two-meson system, E_t the tetraquark nonrelativistic energy.

One notices that the effective range is the most sensitive quantity to $\boldsymbol{Z}$, which, in case $\boldsymbol{Z \neq 0}$, is manifested by a sizeable negative value. One can also express the compositeness factor in a combined form with respect to $\boldsymbol{a}$ and $\boldsymbol{r_e}$:

$$\mathbf{1 - Z = \frac{1}{\sqrt{1 - 2r_e/a}}}.$$

$\boldsymbol{Z = 1} \iff$ compact tetraquark.
$\boldsymbol{Z = 0} \iff$ molecular state.

In the case of the deuteron, the experimental data about a and r_e rule out a nonzero value of Z and confirm its composite nature Weinberg, 1965.

The above formulas can also be continued to the resonance region.

Therefore, the measurement of the scattering length and of the effective range of the scattering of two hadrons gives information about the internal structure of the bound state or the resonance observed in the vicinity of the two-hadron threshold.

Extraction of a and r_e from experimental data demands much work (many uncertainties) and sometimes coupled-channel analyses. For the $X(3872)$, estimates for Z vary between 0 and 1. For many other states, Z is determined with more precision and according to the states, it may take definite values lying between 0 and 0.8. (Oller *et al.*, 2016-2022.) Presence of compact multiquark states not excluded. Interpretation must take into account the fact that pure compact multiquark states cannot survive: they are deformed under the influence of clustering.

In hadron-hadron high-energy collisions, many multiquark states, which have the same quantum numbers as ordinary quarkonia, may be confounded with the latter and their contribution may be included in the quarkonia production rates. Separation of the two categories of states on experimental grounds seems necessary.

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Reviews and general.

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