Probing the odderon through χ_c production at the EIC

Abhiram Kaushik
University of Jyväskylä

Benić, Dumitru, Kaushik, Motyka, Stebel, Phys.Rev.D 110 (2024), 014025

Benić, Horvatić, Kaushik, Vivoda, Phys.Rev.D 108 (2023), 074005

Centre Paul Langevin, Aussois, 6-11 January 2025

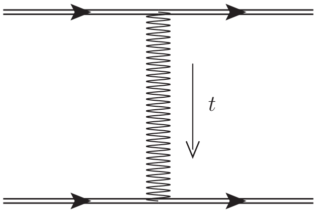
What is the Odderon?

The **Odderon** and (it's more well known cousin) the **Pomeron** originate from pre-QCD approaches to collider phenomenology. Let's consider some history...

- By the 50s experimentalists observed that total hadronic cross-sections had a power law dependence on energy:

$$\sigma_{\text{tot}} \propto s^{\alpha_P - 1} \qquad (\alpha_P - 1) \approx 0.08$$

- To explain this behaviour a t-channel exchange with vaccuum quantum numbers (no charge, no flavour, no spin) was proposed: the **Pomeron**



(Above diagram represents hadron-hadron forward scattering amplitude in the Regge limit (increasing s, t fixed). Can be related to total cross-section through Optical theorem.)

Background

- In terms of QCD: the (bare) Pomeron interpreted as a t-channel exchange of two gluons in a colour singlet state.

What about hadron-antihadron total cross-sections? Do they also have a power law dependence on s?

- **Pomeranchuk theorem (1958):**

$$\lim_{s\to\infty} \frac{\sigma_{p\bar{p}}}{\sigma_{pp}} = 1$$

Background

- In terms of QCD: the (bare) Pomeron interpreted as a t-channel exchange of two gluons in a colour singlet state.

What about hadron-antihadron total cross-sections? Do they also have a power law dependence on s?

- **Pomeranchuk theorem (1958):**

$$\lim_{s\to\infty} \frac{\sigma_{p\bar{p}}}{\sigma_{pp}} = 1$$

- It appears that they do.

Background

- In terms of QCD: the (bare) Pomeron interpreted as a t-channel exchange of two gluons in a colour singlet state.

What about hadron-antihadron total cross-sections? Do they also have a power law dependence on s?

- **Pomeranchuk theorem (1958):**

$$\lim_{s \to \infty} \frac{\sigma_{p\bar{p}}}{\sigma_{pp}} = 1$$

- It appears that they do.
- One might (naively) conclude that proton-proton and proton-antiproton total cross-sections become the same in the high energy limit!

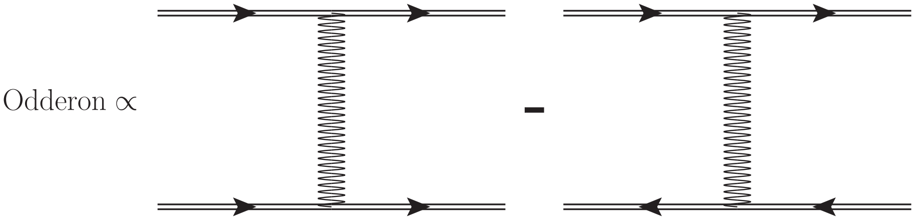
Odderon

- Not quite: $\lim \frac{a}{b} = 1 \;\not\Rightarrow\; a = b$
- For example:

$$\lim_{x\to\infty} \frac{x + 0.2}{x} = 1$$

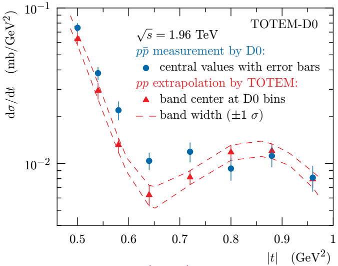
Can we explain the difference in hadron-hadron and hadron-antihadron total cross-sections?

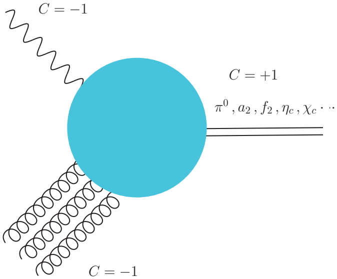
- 50 years ago Lukaszuk and Nicolescu proposed t-channel exchange with vacuum quantum numbers and negative charge parity $C = -1$:



Experimental searches

Recent discovery of the odderon at 5-sigma!





D0 and TOTEM, Phys. Rev. Lett. 127, 062003 (2021) arXiv: 2012.03981

- Hard to interpret in terms of pQCD...

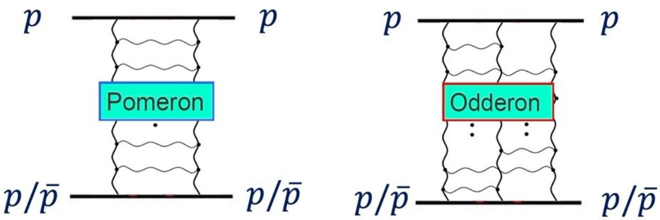
Exclusive DIS offers an alternative

- Possible to select charge parity of interaction
- Interpretation in pQCD terms since hard scale possible

Odderon in pQCD

In QCD terms: (bare) odderon can be understood as **three gluons in a colourless state** $d^{abc} = 2\mathbf{tr}(t^a, \{t^b, t^c\})$

- Need SU(3) or higher



https://blog.hip.fi/the-discovery-of-the-odderon/

- Energy evolution due to s-channel emissions of gluons between the (reggeized) t-channel gluons $\rightsquigarrow$ Balitky-Fadin-Kuraev-Lipatov (BFKL) equation for in the pomeron case (talk by Michael Fucilla yesterday).
- **BFKL** resums the gluon ladder.

Odderon in pQCD

Energy evolution of the odderon given by the Bartels-Jaroszewicz-Kwiecinski-Praszalowicz (BJKP) equation.

- BJKP resums pairwise BFKL ladders amongst the three reggeized gluons.
- Odderon can be considered a solution of the BJKP equation.

Two major solutions to BJKP:

- Janik and Wosiek (1998): Intercept $\alpha_{\text{odd}} - 1 = -0.2472\frac{\alpha_s}{N_c} \implies$ Odderon decreasing with energy.
- Bartels, Lipatov and Vacca (BLV, 1998): Intercept $\alpha_{\text{odd}} - 1 = 0 \implies$ Energy independent Odderon.
- Saturation corrections lead to BLV solution also decreasing with energy.
- BLV solution is relevant for DIS.
- For more information on the odderon and the recent discovery, see lectures by Kovchegov and Royon on CTEQ Youtube page.

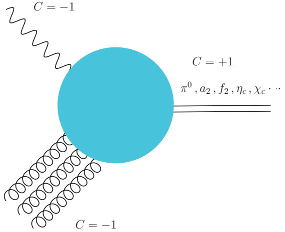
Exclusive η_c production: $ep \to e + p + \eta_c$

Production of C-even mesons in exclusive ep collisions offers a clean environment to probe the odderon: Meson has $C = +1$, virtual photon has $C = -1$, therefore strong exchange should have $C = -1$ selecting the Odderon.

- In particular, charmonium η_c (1S, $J^{PC} = 0^{-+}$) has been suggested as a golden probe. Charm quark production ensures sensitivity to gluon content of proton.
- So far no exclusive measurements of η_c production. Could be measured at the Electron-Ion Collider.

  Null result from HERA for π^0

  H1, PLB 544 (2002) 35-43

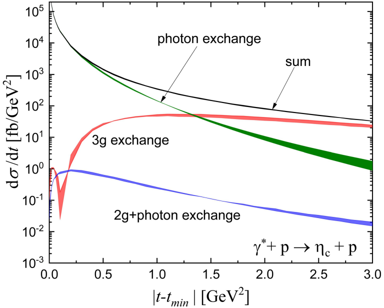


Exclusive η_c production: $ep \to e + p + \eta_c$

Lots of work done on this probe:

Czyzewski, Kwiecinski, Motyka, Sadzikowski, PLB 398 (1997) 400
Bartels, Braun, Colferai, Vacca, EPJC 20 (2001) 323
Dumitru, Stebel, PRD 99 (2019) 094038
Benić, Horvatić, AK, Vivoda (2003)

- Newer calculations sugggest far smaller differential cross-sections than older calculations: $d\sigma/d|t| \sim$ 10-100 fb/GeV^2 vs $\mathcal{O}(\text{pb/GeV}^2)$
- Bounds on π^0 production by HERA: $d\sigma/d|t| \lesssim O(\text{nb/GeV}^2)$



Dumitru, Stebel, Phys. Rev. D 99, 094038 (2019)

Exclusive χ_c production: $ep \to e + p + \chi_c$

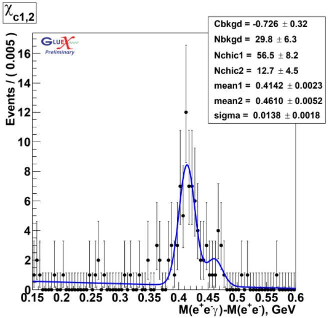
Unfortunately η_c is hard to measure:

$\text{BR}(\eta_c \to \gamma\gamma) \sim 10^{-4}$, $\text{BR}(\eta_c \to \rho\rho) \approx 1.5\%$, feed-down from larger exclusive J/ψ production ($J/\psi \to \eta_c + \gamma$).

χ_{cJ} offer an alternative!

- P-wave quarkonia with good branching channels: $\text{BR}(\chi_{c1} \to J/\psi + \gamma) \sim 34\%$.
- Recently been detected in exclusive *ep* (near threshold) by GlueX in JLab.

Pentchev, PoS SPIN2023 (2024) 152

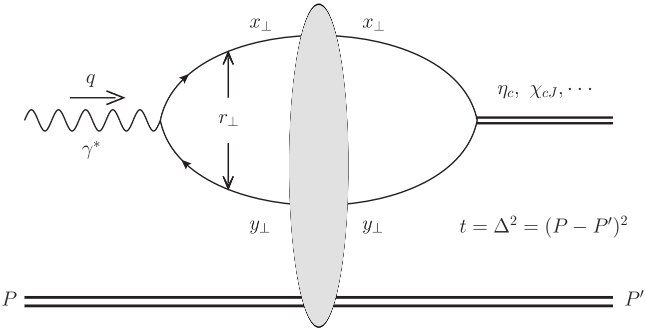


Calculation: Dipole framework

Consider the $\gamma^* p$ scattering in the rest frame of the proton.

- Can orient the coordinate system such that $q^\mu = (q^+, Q^2/q^+, 0_\perp)$. q^+ is typically very large for high energy collisions.
- Coherence length of photon larger than size of proton $x^+ \approx 2/|q^-| = 2q^+/Q^2$.
- Fluctuation of virtual photon into $q\bar{q}$ pair will be long lived.
- In this frame DIS can be seen as virtual photon splitting into a long lived $q\bar{q}$ dipole which interacts with the gluon field of the target (high energy scattering so gluons more relevant). Gribov 1970, Bjorken and Kogut 1973, Frankfurt and Strikman 1988

Calculation: Dipole framework



- Scattering of dipole is eikonal, i.e, tranverse positions of quark and antiquark don't change when passing through the colour field.
- Net effect is a colour rotation of the quark and antiquark $\Longrightarrow$ Wilson lines

$$V(\mathbf{x}_\perp) = \mathcal{P} \exp\left\{ -ig \int dz^- A^{a,+}(x^-, \mathbf{x}_\perp) t^a \right\}$$

Calculation: Dipole framework

Interaction can be characterized by the dipole S-matrix

- $\mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) = \frac{1}{N_c}\mathrm{tr}[V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)]$
- In momentum space:

$$\mathcal{D}(\mathbf{k}_\perp, \boldsymbol{b}_\perp) = \int_{\mathbf{k}_\perp \boldsymbol{b}_\perp} \mathrm{e}^{-\mathrm{i}\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\mathrm{e}^{\mathrm{i}\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\langle P|\mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)|P\rangle$$

In this framework the Odderon is the imaginary part of the dipole distribution

$$\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) \equiv -\frac{1}{2iN_c}\mathrm{tr}\langle V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - V(\mathbf{y}_\perp)V^\dagger(\mathbf{x}_\perp)\rangle$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)
Hatta, Itakura, McLerran, Nucl.Phys.A 760 (2005) 172-207

- Under charge conjugation $\mathbf{x}_\perp \leftrightarrow \mathbf{y}_\perp$ Odderon flips sign!

Energy evolution (small-x): BK equation

The Balitsky-Kovchegov equation describes the small-x evolution of the dipole distribution:

$$\frac{\partial \mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \left[\mathcal{D}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp})\mathcal{D}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}) - \mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)\right]$$

$$\boldsymbol{r}_{2\perp} = \mathbf{r}_\perp - \boldsymbol{r}_{1\perp}$$

$$\mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) \equiv \frac{1}{N_c} \mathrm{tr} \left\langle V\left(\boldsymbol{b}_\perp + \frac{\mathbf{r}_\perp}{2}\right) V^\dagger \left(\boldsymbol{b}_\perp - \frac{\mathbf{r}_\perp}{2}\right)\right\rangle = 1 - \mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) + i\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)$$

BK nonlocal in $\boldsymbol{b}_\perp$: $\boldsymbol{b}_{1\perp} = \boldsymbol{b}_\perp + (\mathbf{r}_\perp - \boldsymbol{r}_{1\perp})/2\,,\ \boldsymbol{b}_{2\perp} = \boldsymbol{b}_\perp - \boldsymbol{r}_{1\perp}/2$

and Odderon explicitly depends on $\boldsymbol{b}_\perp$

- In principle, we need to solve the fully impact parameter dependent BK
- In practice, we treat impact parameter $\boldsymbol{b}_\perp$ as an external parameter
  Lappi, Mäntysaari, PRD 88 (2013) 114020

$$\boldsymbol{r}_{1\perp},\, \boldsymbol{r}_{2\perp} << \boldsymbol{b}_\perp$$

BK equation

$$\frac{\partial \mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2}\int_{r_{1\perp}} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2}\left[\mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) + \mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)\right.$$
$$\left. - \mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp)\right],$$
$$\frac{\partial \mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2}\int_{r_{1\perp}} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2}\left[\mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) + \mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)\right.$$
$$\left. - \mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp)\right].$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)
Hatta, Itakura, McLerran, NPA 760 (2005) 172-207
Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)
Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366

- Odderon and pomeron evolution coupled by nonlinear terms

Small $r_\perp$ limit: system decouples, odderon exponentially suppressed

$$\mathcal{O} \sim \exp(-cY)$$

Large $r_\perp$ limit: $\mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) \to 1$, nonlinear terms result in exponential suppression

$$\mathcal{O} \sim \exp(-cY)$$

(In numerical computations we replace $\frac{\alpha_S N_c}{2\pi^2}\frac{\mathbf{r}_\perp^2}{r_{1\perp}^2 r_{2\perp}^2}$ by Balitsky's prescription for the running coupling kernel.)

Amplitude

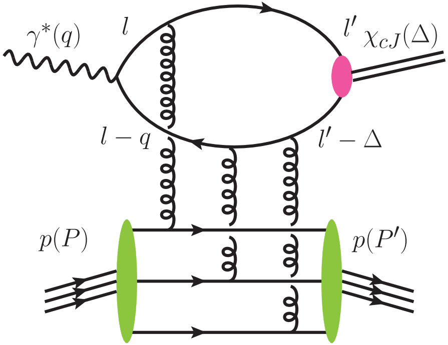
$$\langle \mathcal{M}_{\lambda\bar{\lambda}}(\gamma^* p \to \mathcal{H} p)\rangle = 2q^- N_c \int_{\mathbf{r}_\perp \boldsymbol{b}_\perp} \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp\cdot \boldsymbol{b}_\perp} \mathrm{i}\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)\mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \boldsymbol{\Delta}_\perp)\,,$$

- Odderon: Eikonal interaction of dipole with nuclear shockwave
- Reduced amplitude: Overlap of photon and quarkonium lightcone wavefunctions

$$\mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \boldsymbol{\Delta}_\perp) = \int_z \int_{\boldsymbol{l}_\perp \boldsymbol{l}'_\perp} \sum_{h\bar{h}} \Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{l}_\perp, z)\Psi^{\mathcal{H}*}_{\bar{\lambda},h\bar{h}}(\boldsymbol{l}'_\perp - z\boldsymbol{\Delta}_\perp, z)\mathrm{e}^{\mathrm{i}(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp + \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot \mathbf{r}_\perp}$$

Photon wavefunction:

$$\Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{k}_\perp, z) \equiv \sqrt{z\bar{z}}\,\frac{\bar{u}_h(k) e q_c \not{\epsilon}(\lambda, q) v_{\bar{h}}(q-k)}{\boldsymbol{k}_\perp^2 + \varepsilon^2}$$



Calculating χ_c wavefunctions

$\chi_{c,J}$ wavefunction:

$$\Psi^{\mathcal{H}}_{\bar{\lambda},h\bar{h}}(\boldsymbol{k}_\perp, z) \equiv \frac{1}{\sqrt{z\bar{z}}} \bar{u}_h(k)\Gamma^{\mathcal{H}}_{\bar{\lambda}}(k,k')v_{\bar{h}}(k')\phi_{\mathcal{H}}(\boldsymbol{k}_\perp, z)$$

Spin structure Nonperturbative scalar part

- Spin structure motivated by ensuring C-even wavefunctions:

$$\Gamma^{\mathcal{H}}_{\bar{\lambda}}(k,k') = \begin{cases} 1\ , & \mathcal{H} = \mathcal{S}\ (J=0) \\ \mathrm{i}\gamma_5 \not{E}(\bar{\lambda}, \Delta_0)\ , & \mathcal{H} = \mathcal{A}\ (J=1) \\ \frac{1}{2}\left(\gamma_\mu(k_\nu - k'_\nu) + \gamma_\nu(k_\mu - k'_\mu)\right)E^{\mu\nu}(\bar{\lambda}, \Delta_0)\ , & \mathcal{H} = \mathcal{T}\ (J=2) \end{cases}$$

- $E(\bar{\lambda}, \Delta_0)$: Spin 1 polarization vector
- $E^{\mu\nu}(\bar{\lambda}, \Delta_0)$: Constructed from spin 1 using Clebsch-Gordan coefficients. Contracted with energy-momentum tensor.

Calculating χ_c wavefunctions

- Scalar part: boosted Gaussian ansatz

$$\phi_{\mathcal{H},B}(r_\perp, z) = \mathcal{N}_{\mathcal{H},B} z\bar{z} \exp\left(-\frac{m_c^2 \mathcal{R}_\mathcal{H}^2}{8z\bar{z}} - \frac{2z\bar{z}r_\perp^2}{\mathcal{R}_\mathcal{H}^2} + \frac{1}{2} m_c^2 \mathcal{R}_\mathcal{H}^2\right)$$

Parameters $\mathcal{N}_{\mathcal{H},B}$ and $\mathcal{R}_\mathcal{H}$ fixed by considerations of

- normalization of the wavefunction:

$$1 = N_c \sum_{h\bar{h}} \int_z \int_{\mathbf{r}_\perp} \left| \Psi^{\mathcal{H}}_{\lambda,h\bar{h}}(\mathbf{r}_\perp, z) \right|^2$$

- $\chi_c \to \gamma\gamma$ decay width

$$\Gamma(\mathcal{S} \to \gamma\gamma) = \frac{\pi\alpha^2}{4} M_\mathcal{S}^3 F_\mathcal{S}^2$$

$$F_\mathcal{S} \equiv 4q_c^2 m_c N_c \int_z \int_{\boldsymbol{k}_\perp} \frac{\boldsymbol{k}_\perp^2 + (z-\bar{z})^2 m_c^2}{(\boldsymbol{k}_\perp^2 + m_c^2)^2} \frac{\phi_\mathcal{S}(\boldsymbol{k}_\perp, z)}{z\bar{z}}$$

- $\chi_{c1} \to \gamma\gamma$ forbidden due to Landau-Yang theorem but we assume that decay width is same as χ_{c2}.

Reduced amplitudes: Scalar

$$\mathcal{A}_0(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = eq_c \int_z \mathrm{e}^{-\mathrm{i}\boldsymbol{\delta}_\perp\cdot\mathbf{r}_\perp} \mathcal{A}_L(r_\perp)$$

$$\mathcal{A}_{\lambda=\pm1}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = eq_c \lambda \mathrm{e}^{\mathrm{i}\lambda\phi_r} \int_z \mathrm{e}^{-\mathrm{i}\boldsymbol{\delta}_\perp\cdot\mathbf{r}_\perp} \mathcal{A}_T(r_\perp)$$

Factor out impact parameter $\mathbf{\Delta}_\perp$ dependence into the off-forward phase, $\mathrm{e}^{-\mathrm{i}\boldsymbol{\delta}_\perp\cdot\mathbf{r}_\perp}$ to get $\mathbf{r}_\perp$-dependent amplitudes:

$$\mathcal{A}_L(r_\perp) \equiv -\frac{2}{\pi} m_c Q(z-\bar{z}) K_0(\varepsilon r_\perp)\phi_{\mathcal{S}}(r_\perp, z)$$

$$\mathcal{A}_T(r_\perp) \equiv \frac{\mathrm{i}\sqrt{2}}{2\pi}\frac{m_c}{z\bar{z}}\left[(z-\bar{z})^2 \varepsilon K_1(\varepsilon r_\perp)\phi_{\mathcal{S}}(r_\perp, z) - K_0(\varepsilon r_\perp)\frac{\partial \phi_{\mathcal{S}}}{\partial r_\perp}\right]$$

Full amplitudes in terms of odderon harmonics (using only leading harmonic $k=0$):

$$(\mathcal{O}(r_\perp, \Delta_\perp) = \mathcal{O}_1(r_\perp, \Delta_\perp)\cos(\phi_{rb}) + \mathcal{O}_3(r_\perp, \Delta_\perp)\cos(3\phi_{rb}) + \ldots)$$

$$\widetilde{\mathcal{M}}_L = 8\pi N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp \mathrm{d}r_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp)\mathcal{A}_L(r_\perp)\,\mathrm{sgn}(z-\bar{z}) J_{2k+1}(r_\perp\delta_\perp)$$

$$\widetilde{\mathcal{M}}_T = 4\pi\mathrm{i} N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp \mathrm{d}r_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp)\mathcal{A}_T(r_\perp)\left[J_{2k}(r_\perp\delta_\perp) - J_{2k+2}(r_\perp\delta_\perp)\right]$$

Reduced amplitudes: Axial vector

$$\mathcal{A}_{LL}(r_\perp) \equiv 0$$

$$\mathcal{A}_{LT}(r_\perp) \equiv \frac{\sqrt{2}}{\pi} Q K_0(\varepsilon r_\perp) \frac{\partial \phi_{\mathcal{A},T}}{\partial r_\perp}$$

$$\mathcal{A}_{TL}(r_\perp) \equiv \frac{\sqrt{2}}{\pi} \frac{1}{z\bar{z}} \frac{1}{M_{\mathcal{A}}} \left[-m_c^2 K_0(\varepsilon r_\perp) \frac{\partial \phi_{\mathcal{A},L}}{\partial r_\perp} + \varepsilon K_1(\varepsilon r_\perp) \boldsymbol{\nabla}_\perp^2 \phi_{\mathcal{A},L} \right]$$

$$\mathcal{A}_{TT}(r_\perp) \equiv -\frac{\mathrm{i}}{\pi} \frac{z-\bar{z}}{z\bar{z}} \left[\frac{\partial \phi_{\mathcal{A},T}}{\partial r_\perp} \varepsilon K_1(\varepsilon r_\perp) - m_c^2 K_0(\varepsilon r_\perp) \phi_{\mathcal{A},T} \right]$$

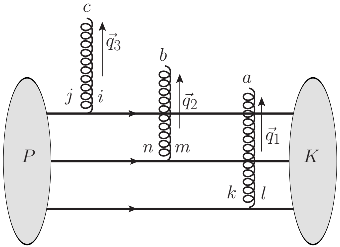
$$\widetilde{\mathcal{M}}_B = 4\pi \mathrm{i} N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp \mathrm{d} r_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \mathcal{A}_B(r_\perp) \left[J_{2k}(r_\perp \delta_\perp) - J_{2k+2}(r_\perp \delta_\perp) \right]$$

where $B = TL, LT$

$$\widetilde{\mathcal{M}}_{TT} = 8\pi N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp \mathrm{d} r_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \mathcal{A}_{TT}(r_\perp) \operatorname{sgn}(z-\bar{z}) J_{2k+1}(r_\perp \delta_\perp)$$

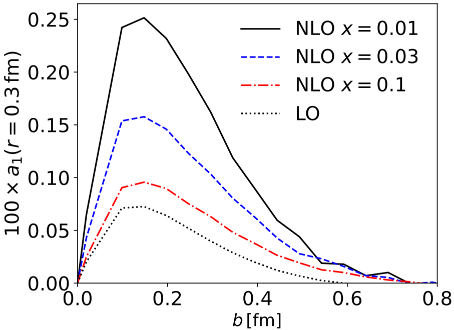
Initial condition for the odderon

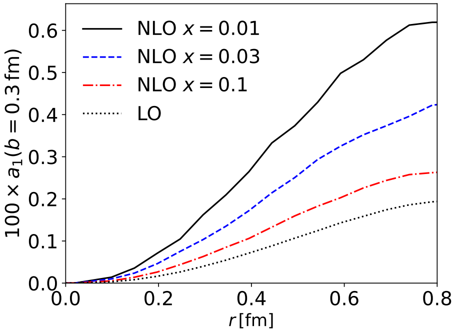
We use a recent quark model calculation of the odderon by Dumitru, Mäntysaari and Paatelainen
Dumitru, Mäntysaari, Paatelainen, PRD 107 (2023) 1, L011501



- Odderon can generate a high-t kick to the proton that doesn't break it up
- Odderon amplitudes will lead to a weak t-dependence

Initial condition for the odderon



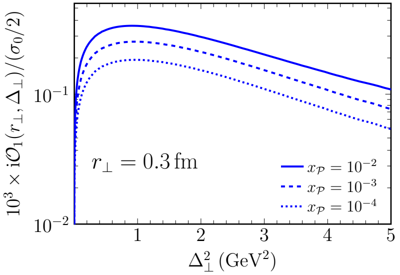


- Initial $x = 0.01$ (black curve)
- Odderon peak lies well within the proton $\sim\ 0.25 \times R_p$

Solutions of BK evolution

- Negligible higher harmonics induced in the odderon by non-linear terms
  Yao, Hagiwara, Hatta PLB 790 (2019) 361   Motyka, PLB 637 (2006) 185

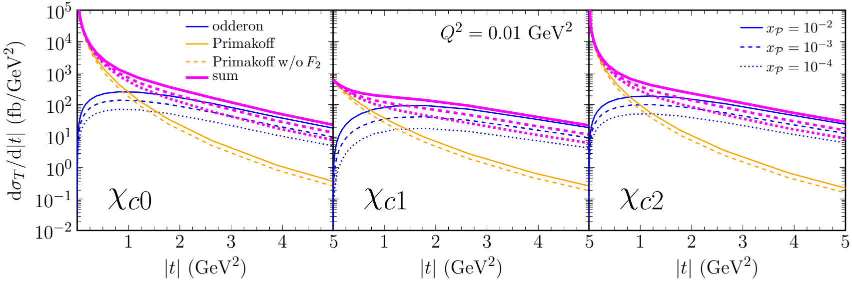
$$\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) = \mathcal{O}_1(r_\perp, b_\perp)\cos(\phi_{rb}) + \mathcal{O}_3(r_\perp, b_\perp)\cos(3\phi_{rb}) + \ldots$$



- Odderon decreases significantly with evolution
- Slope not affected by evolution $\Longrightarrow$ evolution does not alter expected weak t-dependence
- Small-x evolution does not change the sign of the sign of the Odderon

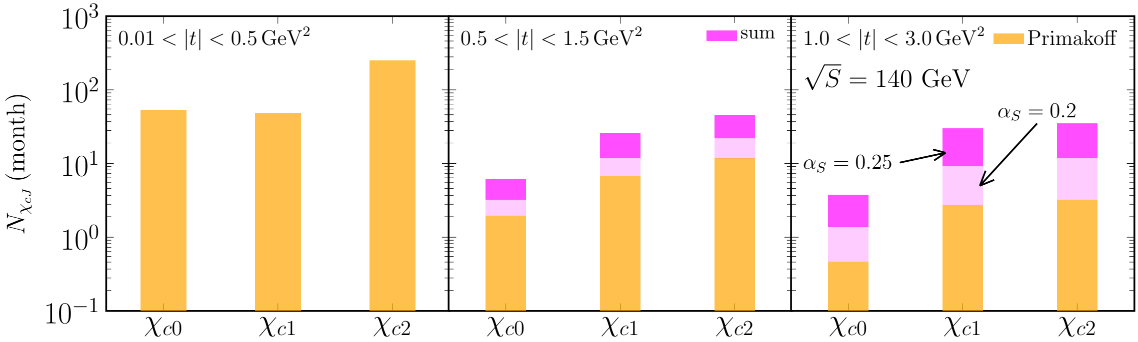
Results: $ep \to e + \chi_{cJ} + p$

Important QED background: Primakoff process. Photon ($C = -1$) from proton can also result in η_c. Can be calculated from well known electromagnetic charge form factor.



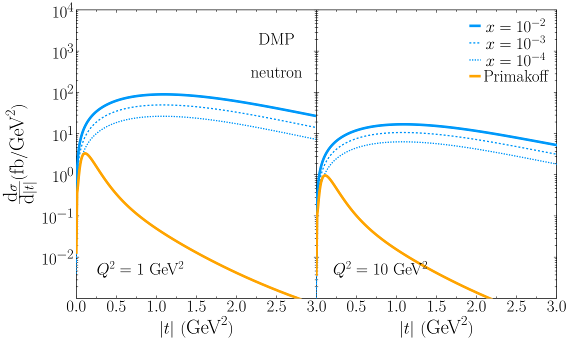
- Odderon contribution has rather small slope in $|t|$ as expected
- Primakoff contribution (photonic background) dominates at small $|t|$. Need $|t| \gtrsim 1$ GeV^2 to access odderon
- Constructive interference between photon and odderon

Predictions for the EIC



- Odderon contribution exceeds the Primakoff background when $|t| > 0.5$ GeV^2
- Around 20 events/month for χ_{c1} at peak EIC energy and luminosity

Suppress Primakoff background with neutron targets



- Primakoff contribution negligible
- Odderon accesible even at low momentum transfers
- In practice, could be done with deuteron or He^3 target with spectator proton tagging in the near forward region

  CLAS, PRL 108, 142001 (2012)
  Friscic et al., PLB 823, 136726 (2021)

Conclusions and Outlook

- Isolating odderon requires large momentum transfer $|t| \gtrsim$ 1-3 GeV^2 for $x \sim 10^{-2} - 10^{-4}$.
- Cross-sections in 10-60 femtobarn range. At most tens of events/month expected per month at EIC with peak luminosity depending (on which χ_{cJ}).
- Detection expected to be challenging due to low rate and feed-down from $\psi(2S) \to \chi_c + \gamma$.
- Cross-sections could be increased by allowing for excitations of the proton while requiring a large rapidity gap.

If using a neutron target:

- Negligible Primakoff component. Can probe odderon at low $|t|$.
- Feasible at EIC for He^3 targets with spectator protons tagged in the near forward direction.

Thank you!