J/Ψ-plus-jet at large rapidity difference

Michael Fucilla

Laboratoire de Physique des 2 Infinis Irène Joliot-Curie

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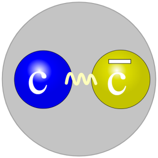
Motivations

- Heavy-flavored productions are recognized as excellent probe channels of Quantum Chromodynamics
- This resulted in remarkable interest over the last decades on both their formal and phenomenological aspects
- At modern colliders heavy-flavor production enters the two-scale regime: $S \gg M^2 \gg \Lambda^2_{QCD} \rightarrow$ large $\ln(S/M^2)$

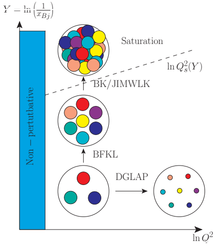
$$\downarrow$$

BFKL/saturation domain

- Quarkonium and small-x have an intriguing overlap

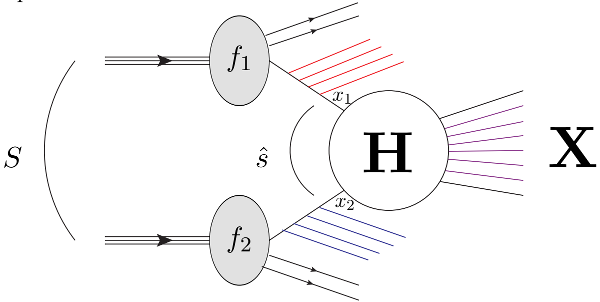


$\longleftrightarrow$



High-energy inclusive reactions

- Proton-proton collision



- High-energy logaritms

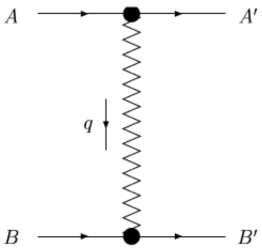
$$\ln\left(\frac{S}{Q^2}\right) = \ln\left(\frac{1}{x_1}\right) + \ln\left(\frac{1}{x_2}\right) + \ln\left(\frac{\hat{s}}{Q^2}\right)$$

- $\ln(1/x_i) \rightarrow$ **small-x evolution** of the parton distribution $f_i(x_i)$
- $\ln\left(\frac{\hat{s}}{Q^2}\right) \rightarrow$ logarithmically enanched contributions in coefficients functions

The Reggeized gluon

Scattering process $A + B \longrightarrow A' + B'$

- Gluon quantum numbers in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t fixed (i.e not growing with s)
- All-order resummation:
  leading logarithmic approximation (LLA): $(\alpha_s \ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s(\alpha_s \ln s)^n$



$$(\mathcal{A})_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t), \qquad j(0) = 1$$

$j(t)$-**Reggeized gluon trajectory**

$$\Gamma^c_{A'A} = g\,\langle A'|T^c|A\rangle\,\Gamma_{A'A}$$

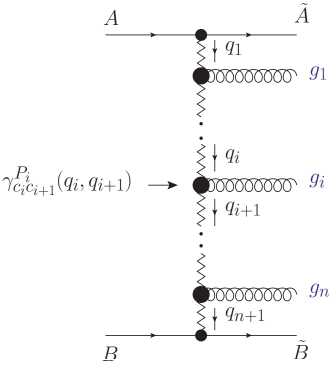
T^c- fundamental(quarks) or adjoint(gluons)

- LLA **[Balitsky, Fadin, Kuraev, Lipatov (1979)]**

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A}, \quad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}}\frac{N}{2}\int\frac{d^{D-2}k_\perp}{k_\perp^2(q-k)_\perp^2} = -g^2\frac{N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon$$

BFKL in LLA

- Inelastic scattering process $A + B \longrightarrow \tilde{A} + \tilde{B} + n$ in the LLA



i. ***Leading-logarithm resummation***

$\downarrow$

Multi-Regge kinematics (MRK)

ii. Exchange of fermions suppressed in LLA

iii. Vertical gluons become Reggeized due to loop radiative corrections

iv. $\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \rightarrow$ ***Lipatov vertex***

- ***Multi-Regge form of inelastic amplitudes***

$$\Re \mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\Gamma^{c_1}_{\tilde{A}A} \left(\prod_{i=1}^{n} \gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \left(\frac{s_i}{s_0}\right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{\tilde{B}B}$$

Multi-Regge kinematics

- ***Sudakov decomposition***

$$k_i = z_i p_A + \lambda_i p_B + k_{i\perp} \qquad p_A^2 = p_B^2 = 0$$

- ***Multi-Regge kinematics (MRK)***

$$z_0 \gg z_1 \gg \dots \gg z_n \gg z_{n+1}$$

$$\lambda_{n+1} \gg \lambda_n \gg \dots \gg \lambda_1 \gg \lambda_0$$

$$k_{0\perp} \sim k_{1\perp} \sim \dots \sim k_{n\perp} \sim k_{n+1\perp}$$

- Cutkosky rules

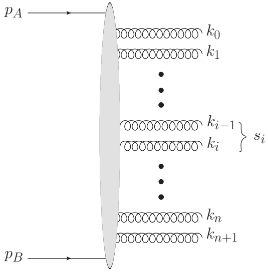
$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{1}{2} \sum_n \int d\Phi_{\tilde{A}\tilde{B}+n} \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} \left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n} \right)^*$$

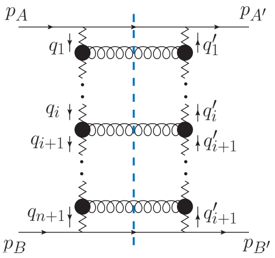
- Integration over phase space

Each integration over s_i (or z_i)

$$\downarrow$$

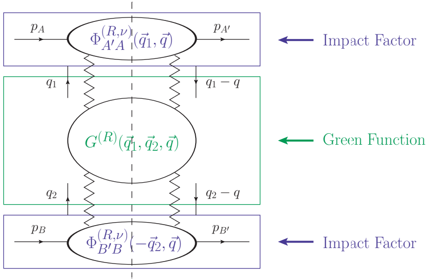
One energy logarithm





BFKL resummation

- Diffusion $A + B \longrightarrow A' + B'$ in the ***Regge kinematical region***
- BFKL factorization for $\Im\mathcal{A}_{AB}^{A'B'}$ $\rightarrow$ convolution of a ***Green function*** (process independent) with the ***Impact factors*** of the colliding particles (process dependent)



$$\Im\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2} \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q}\,)^2}$$

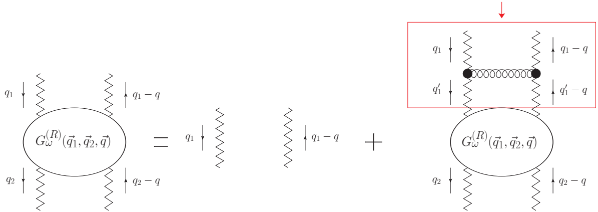
$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

- $\mathcal{R} = 1^{+}(\text{singlet}), 8^{-}(\text{octect}), \ldots$

BFKL resummation

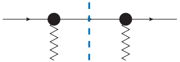
- $G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$-Mellin transform of the Green function for the Reggeon-Reggeon scattering

$$\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q}\,)^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_1'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime} - \vec{q}\,)^2} \mathcal{K}^{(R)}(\vec{q}_1, \vec{q}_1^{\,\prime}; \vec{q}\,) G_\omega^{(R)}(\vec{q}_1^{\,\prime}, \vec{q}_2; \vec{q}\,)$$



- $\Phi_{P'P}^{(R,\nu)}$- LO impact factor in the t-channel color state (R, ν)

$$\Phi_{PP'}^{(R,\nu)} = \langle cc' | \hat{\mathcal{P}} | \nu \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma_{\{f\}P}^{c} (\Gamma_{\{f\}P'}^{c'})^*$$



Rise in s and DIS

- BFKL equation: $\vec{q}^{\,2} = 0$ and singlet color state representation
  [Balitsky, Fadin, Kuraev, Lipatov (1976-1978)]

$$\Im \mathcal{A}_{AB}^{AB(0)} \sim s \int \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(0)}(\vec{q}_1, \vec{q}_2; \vec{0}\,) \right] \sim s^{1+\omega_0}$$

- $\omega_0 = \dfrac{g^2 N_c \ln 2}{\pi^2} \simeq 0.4$ (for $\alpha_s = 0.15$) is the rightmost singularity of G_ω
- **Optical theorem**

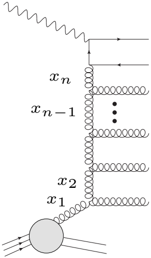
$$\sigma_{AB \to X} = \frac{\Im \mathcal{A}_{AB}^{AB(0)}}{s} = s^{\omega_0}$$

- DIS cross-section

$$\sigma_{\gamma^* P}(x) = \Phi_{\gamma^* \gamma^*}(\vec{k}\,) \otimes_{\vec{k}} \mathcal{F}(x, \vec{k})$$

$$\downarrow$$

$$\sigma_{\gamma^* P}(x) \sim \left(\frac{s}{Q^2} \right)^{\omega_0} = \left(\frac{1}{x} \right)^{\omega_0}$$



$$1 \gg x_1 \gg x_2 \gg \ldots \gg x_{n-1} \gg x_n \equiv x_{Bj} = Q^2/s$$

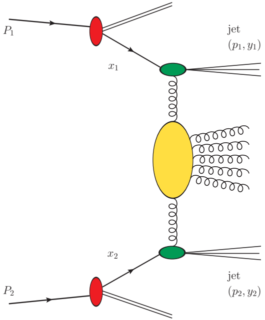
Forward/backward: Mueller-Navelet jets

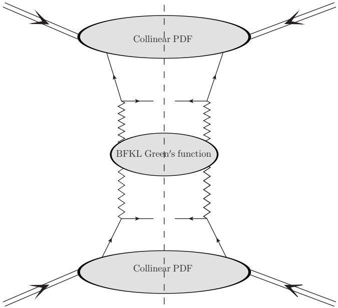
- Inclusive production of **rapidity-separated jets** in proton-proton collision
- Large energy logarithms → *BFKL resummed partonic cross section*
- Moderate values of parton x → *collinear PDFs*

[Mueller, Navelet (1987)]

- **Full next-to-leading analysis**

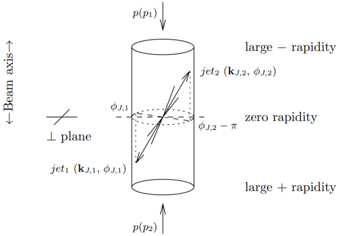
[Ducloué, Szymanowski, Wallon (2013,2014)]
[Caporale, Ivanov, Murdaca, Papa (2014,2015)]

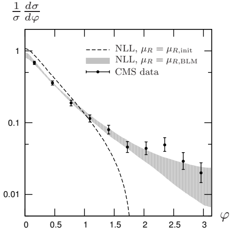


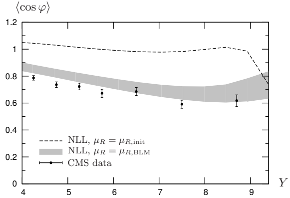


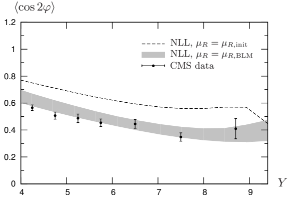
Muller-Navelet: Theory vs Experiment

- Relative angle in the azimuthal plane $\varphi = \phi_{J_1} - \phi_{J_2} - \pi$







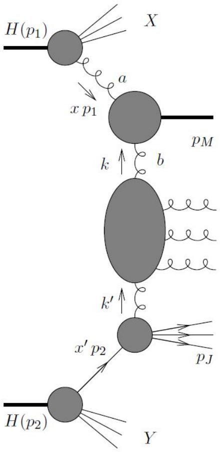


In this slide: **[Ducloué, Szymanowski, Wallon (2013)]**
[Caporale, Ivanov, Murdaca, Papa (2014)]

J/ψ plus jet production

- Process: proton(p_1) + proton(p_2) $\to J/\psi + X +$ jet

- **Hybrid collinear/BFKL approach**
- High-energy hadroproduction of a J/Ψ meson and a jet, with a remnant X
- Both the J/Ψ and the jet emitted with large transverse momenta and well separated in rapidity
- NLA BFKL + NLO jet + LO J/Ψ
  - LO J/Ψ IF calculated in **NRQCD** (Color-singlet and Color-octect)
  - LO J/Ψ IF calculated in **color evaporation model (CEM)**
- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states

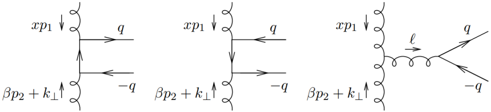


[Boussarie, Ducloué, Szymanowski, Wallon (2018)]

Impact factors in NRQCD

- Impact factor in **Color Evaporation Model**

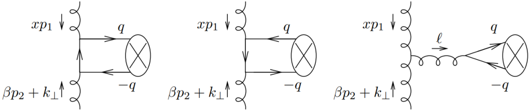
$$\mathcal{V}_{J/\psi} = F_{J/\psi} \int_{4m_c^2}^{4m_D^2} dM^2 \, \frac{d\mathcal{V}_{c\bar{c}}}{dM^2}$$



- **NRQCD expansion**

$$|J/\psi\rangle = O(1) \left| Q\bar{Q} \left[{}^3S_1^{(1)} \right] \right\rangle + O(v) \left| Q\bar{Q} \left[{}^3P_1^{(8)} \right] g \right\rangle + O\left(v^2\right)$$

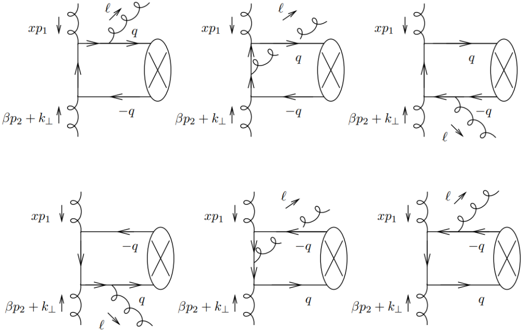
- Impact factor in the *Color Octect* case



$$\left[v\left(q\right) \bar{u}\left(q\right) \right]_{\alpha\beta}^{ji \to a} \to t_{ji}^a d_8 \left(\frac{\langle \mathcal{O}_8 \rangle_{J/\psi}}{m} \right)^{\frac{1}{2}} \left[\hat{\varepsilon}_{J/\psi}^* \left(2\hat{q} + 2m\right) \right]_{\alpha\beta}$$

Impact factors in NRQCD

- Impact factor in the *Color Singlet* case

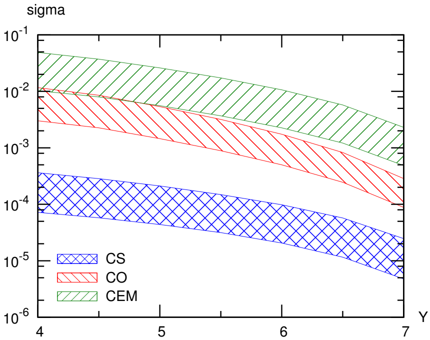


$$\left[v\left(q\right)\bar{u}\left(q\right)\right]^{ij}_{\alpha\beta} \to \frac{\delta^{ij}}{4N_c}\left(\frac{\langle\mathcal{O}_1\rangle_{J/\psi}}{m}\right)^{\frac{1}{2}}\left[\hat{\varepsilon}^*_{J/\psi}\left(2\hat{q}+2m\right)\right]_{\alpha\beta}$$

J/ψ plus jet production

- Realistic CMS and CASTOR rapidity ranges, fixed p_T final states

$$\frac{d\sigma}{d|k_{J/\psi}|d|k_{\text{jet}}|dY}[\text{nb GeV}^{-2}] \qquad |\mathrm{k}_{J/\psi}| = |\mathrm{k}_{\text{jet}}| = 10 \text{ GeV}$$



[Boussarie, Ducloué, Szymanowski, Wallon (2018)]

J/ψ production from single parton fragmentation

- J/Ψ production from single parton fragmentation
  [Celiberto, M.F. (2022)] [Celiberto (2023)]

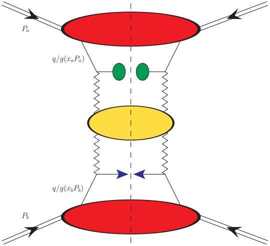
$$p(P_a) + p(P_b) \to \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J)$$

- **Hybrid cross section**

$$\frac{d\sigma}{dy_{\mathcal{Q}}\, dy_J\, d^2\vec{p}_{\mathcal{Q}}\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

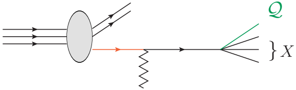
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} V_{\mathcal{Q}}(\vec{q}_1, x_g, \vec{p}_{\mathcal{Q}}) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} V_J(\vec{q}_2, x_J, \vec{p}_J)$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_{\mathcal{Q}} x_J s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

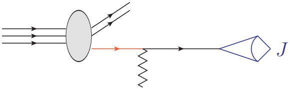


- Impact factors **[Ivanov, Papa (2012)]**

$$V_{\mathcal{Q}}(\vec{q}_1, x_{\mathcal{Q}}, \vec{p}_{\mathcal{Q}}) = f_{q/g} \otimes H \otimes D^{\mathcal{Q}}_{q/g}$$



$$V_J(\vec{q}_2, x_J, \vec{p}_J) = f_{q/g} \otimes H \cdot \mathcal{J}$$



LO Impact factors

- Parton-Parton-Reggeon (PPR) vertices

- LO jet impact factor

$$c_J(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x) + \sum_{\beta=q,\bar{q}} f_\beta(x)\right) \;,$$

- LO light hadron impact factor

$$c_{\mathcal{Q}}(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\int_x^1 \frac{d\zeta}{\zeta}\left(\frac{\zeta}{x}\right)^{2i\nu-1}$$

$$\times\left[\frac{C_A}{C_F}f_g(\zeta)D_g^{\mathcal{Q}}\left(\frac{x}{\zeta}\right) + \sum_{\alpha=q,\bar{q}} f_\alpha(\zeta)D_\alpha^{\mathcal{Q}}\left(\frac{x}{\zeta}\right)\right]$$

- Both known at NLO

[Bartels, Colferai, Vacca (2003)]
[Caporale, Ivanov, Murdaca, Papa (2014)]
[Ivanov, Papa (2012)]

J/Ψ *fragmentation: heavy-quark channel*

- NRQCD FFs

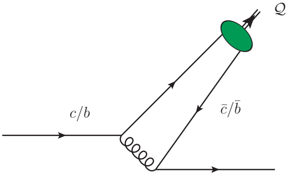
[talk by Kate Lynch on Wednesday]

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n]) \langle \mathcal{O}^{\mathcal{Q}}([n]) \rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, ${}^3S_1^{(1)}$
- Heavy-quark fragmentation function computed at $\mu_0 = 3m_Q$ in NRQCD

$$D_Q^{\mathcal{Q}}(z,\mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z)$$

$$+\frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \, \Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$



- LO fragmentation function

[Braaten, Cheung, Yuan (1993)]

$$D_Q^{\mathcal{Q},\mathrm{LO}}(z) = \frac{\alpha_s^2(\mu_R)}{m_Q^3} \frac{8z(1-z)^2}{27\pi(2-z)^6} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \left(5z^4 - 32z^3 + 72z^2 - 32z + 16\right)$$

- The NLO correction is given by a polynomial function

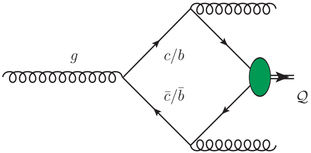
[X.C. Zheng, C.H. Chang, and X.G. Wu (2019)]

J/Ψ fragmentation: gluon and light-quark channels

- Gluon fragmentation function computed at $\mu_0 = 2m_Q$ in NRQCD
  [Braaten, Yuan (1993)]

$$D_g^{\mathcal{Q}}(z, 2m_Q) = \frac{5}{36(2\pi)^2}\alpha_s^3(2m_Q)\frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_{\mathcal{Q}}^3}\int_0^z d\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} d\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$

$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau)\frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}}\right)\right]$$



- FFs evolved from the initial scale through the DGLAP evolution equations
- Light-quarks FFs (LQFFs) are zero at the initial scale: $D_q^{\mathcal{Q}}(z, 2m_Q) = 0$
- At higher scales LQFFs dynamically generated by evolution

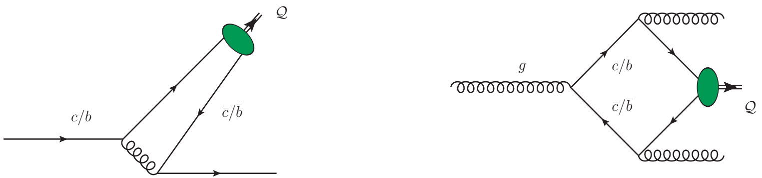
$$D_q^{\mathcal{Q}}(z, \mu > 2m_Q) = \underbrace{D_q^{\mathcal{Q}}(z, 2m_Q)}_{0} + \frac{\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{(2m_Q)^2}\right)\int_z^1 \frac{dx}{x} P_{qg}(x) D_g^{\mathcal{Q}}\left(\frac{z}{x}, 2m_Q\right)$$

J/ψ production from single parton fragmentation

- J/Ψ fragmentation functions in the **NRQCD** framework

$$D_i^{\mathcal{Q}}(z,\mu_F) = \sum_{[n]} \mathcal{D}_i^{Q\bar{Q}}(z,\mu_F,[n])\langle \mathcal{O}^{\mathcal{Q}}([n])\rangle$$

- Spin-triplet (vector) and color-singlet quarkonium state, ${}^3S_1^{(1)}$
- Initial inputs for the DGLAP evolution



[Braaten, Cheung, Yuan (1993)] [Zheng, Chang, Wu (2019)]
[Braaten, Yuan (1993)]

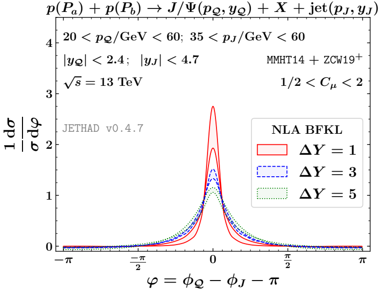
- Evolution performed by **APFEL++** **[Bertone (2017)]**

$$\left\{D_Q^{\mathcal{Q}}(z,3m_Q), D_g^{\mathcal{Q}}(z,2m_Q)\right\}_{\text{NRQCD}} \xrightarrow{\textbf{APFEL++}} \boxed{\textbf{ZCW19}^+ \textbf{ Onium FFs}}$$

J/ψ production from single parton fragmentation

- Azimuthal distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)R_{n0}\right\} .$$



[Celiberto, M.F. (2022)]

- NLO analysis but only in the fragmentation region (high-p_Q^2 region)
- Higher sensitivity to high-energy effects at lower hard scales

$$\alpha_s(Q^2)\ln\left(\frac{\hat{s}}{Q^2}\right) \sim 1$$

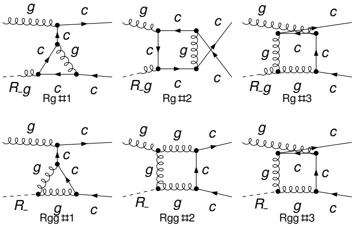
J/Ψ-plus-jet and η_c-plus-jet at full NLL

- NLO impact factor for S-wave Quarkonium hadroproduction

[Nefedov (2023)]

$$gR \to Q\bar{Q}\left[{}^1S_0^{[1]}\right] , \quad gR \to Q\bar{Q}\left[{}^1S_0^{[8]}\right] , \quad gR \to Q\bar{Q}\left[{}^3S_1^{[8]}\right]$$

- Some diagrams of the off-shell coefficient functions



- Gauge invariant **Lipatov effective action** $\to$ eikonal approximation and factorization in rapidity space
- Regularization of rapidity divergences through **tilted Wilson lines** allows automation of computational steps (IBP reductions, ecc...)
- We are currently using these results for a full NLL description of J/Ψ-plus-jet and η_c-plus-jet

[M.F., Lansberg, Nefedov, Szymanowski, Wallon (ongoing)]

Summary and conclusions

- J/Ψ-plus-jet is a good channel to test different Quarkonium production mechanisms and to probe the BFKL dynamics.
- The LLA description of the J/Ψ-plus-jet is established but can provide qualitative predictions only
- A full NLLA treatment can be established easily only in the limit where the *fragmentation* mechanism dominates
- Full-NLA predictions require the inclusion of subleading corrections from the *heavy-quark pair impact factors*

  [Nefedov (2023, ongoing)]
- The implementation of these results will provide **the first full NLL description of J/Ψ-plus-jet and η_c-plus-jet hadroproduction channels**

  [M.F., Lansberg, Nefedov, Szymanowski, Wallon (ongoing)]

Thank you for the attention!

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