Pol B Gossiaux, SUBATECH (NANTES)

Semi classical approximation of quantum master equations

A buffet of Quarkonium dynamics

Quarkonia as Tools 2025 (Aussois)

and Pays de la Loire

IMT Atlantique
Bretagne-Pays de la Loire
École Mines-Télécom

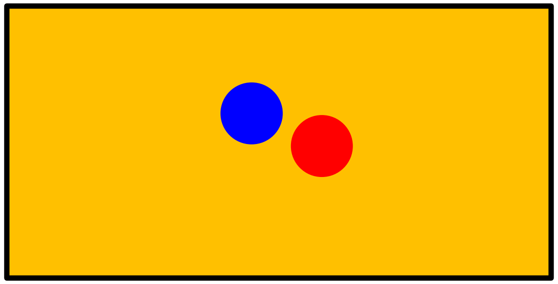
cnrs IN2P3

Nantes Université

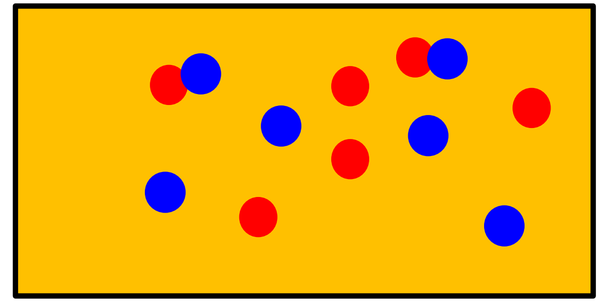
STRONG 2020

Regeneration: Dilute vs Dense

Bottomia (single pair)



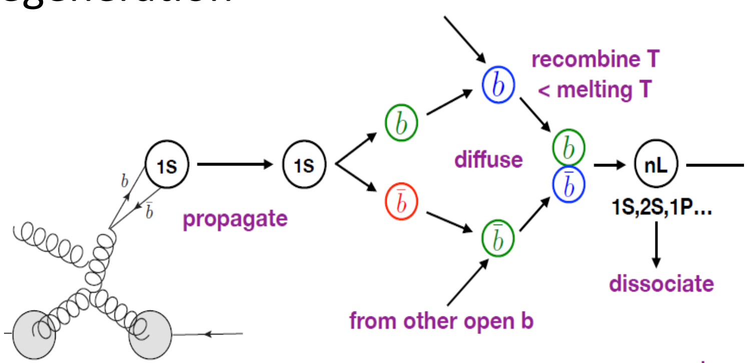
Charmonia (many pairs)



No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states.

Full quantum treatment affordable

N.B.: In some SC formalisms : intermediate regeneration



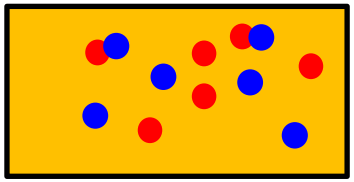
Yao, Mehen, Müller (2019)

Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

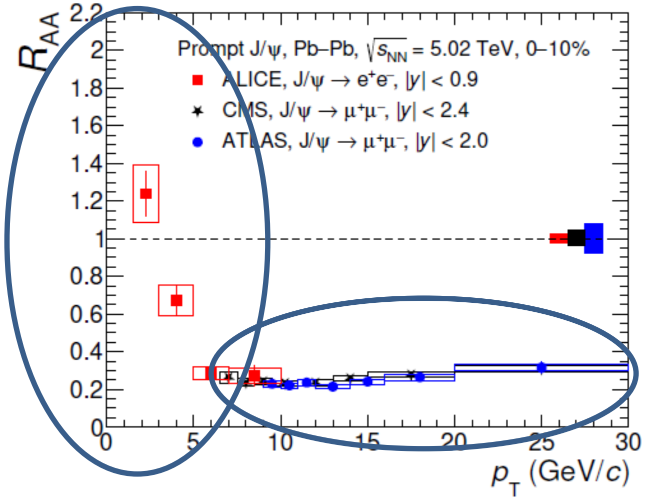
No full quantum treatment possible … but some inspiration from simpler situations…

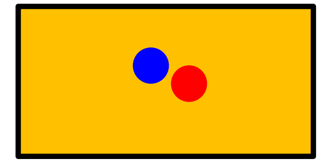
What experiment tells us

ALICE Collab. JHEP02 (2024)



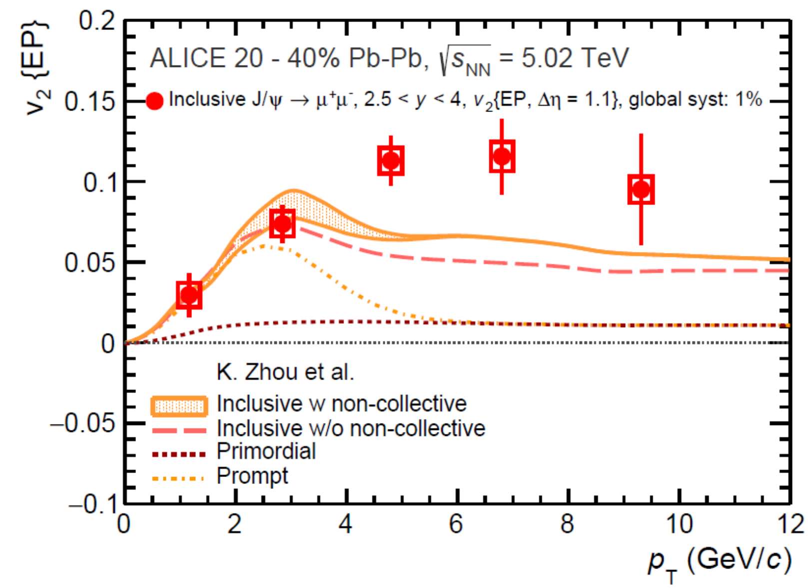
Dense (in phase space)
=> recombination





Dilute (in phase space)

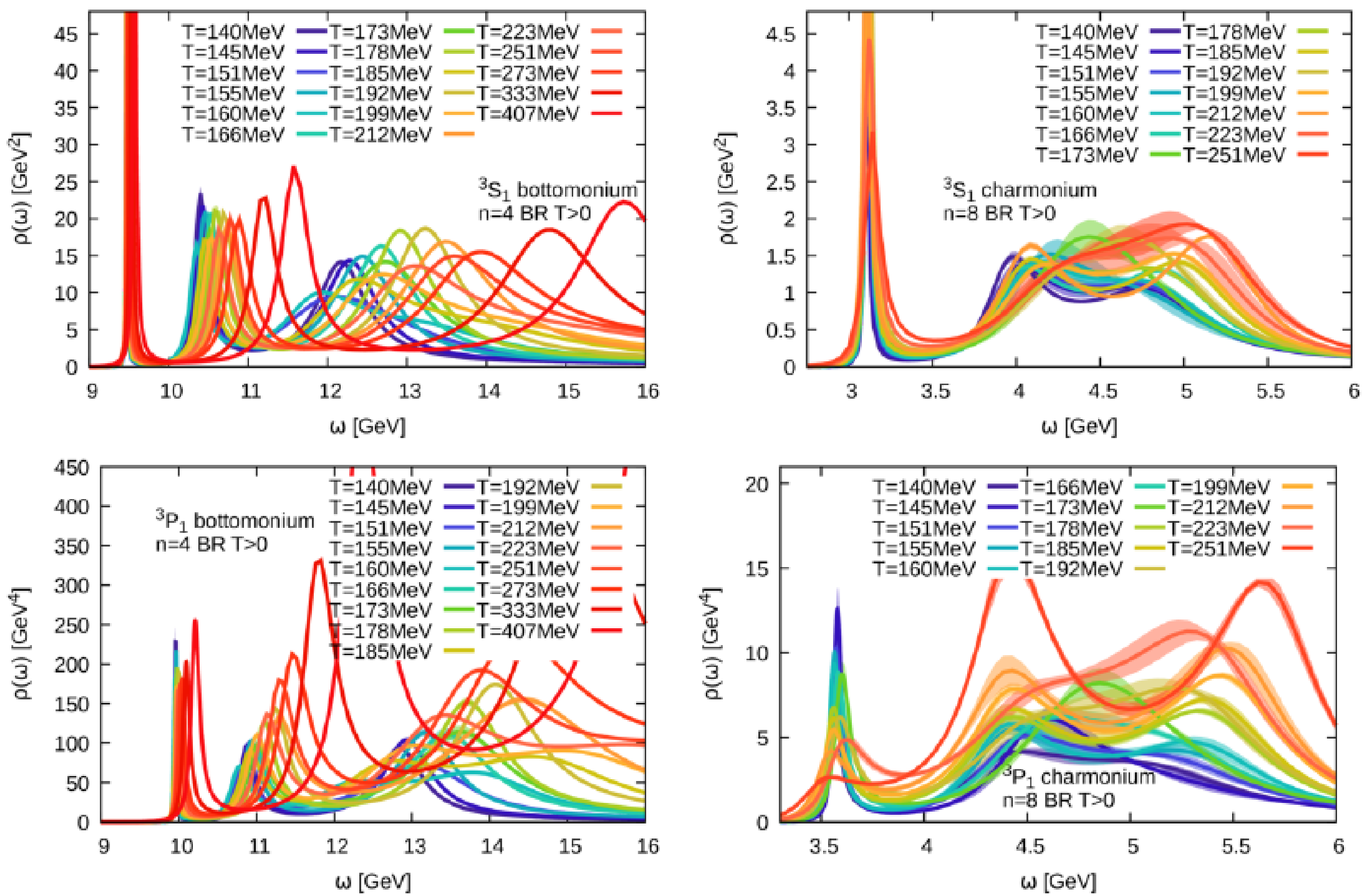
Alternate possible explanation : p_T-dependent absorption cross section : not excluded, but not favored by the finite v_2 observed for J/ψ by ALICE



What theory tells us

In-medium “quarkonia” is a complicated concept…

Kim et al, JHEP11(2018)088



Rich structure : broadening and mass shift. What are the underlying “ingredients’” ?

a) Screened real potential and b) inelastic interactions with the QGP

Once upon a time …

Stochastic Langevin Equation in *evolving QGP*

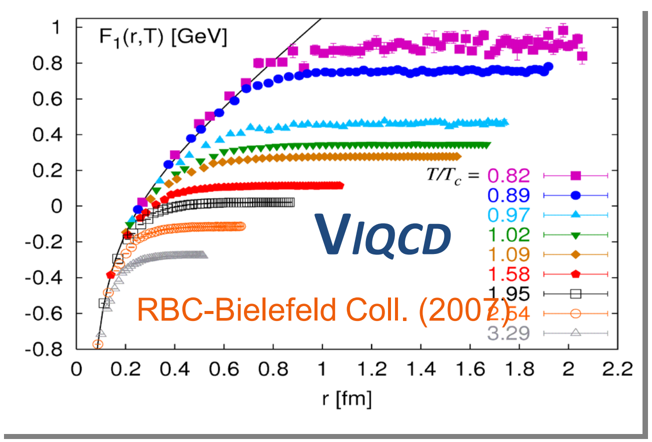
Inner dynamics: Schrödinger-Langevin (SL) equation

Derived from the Heisenberg-Langevin equation, in Bohmian mechanics …

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}_{\mathrm{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

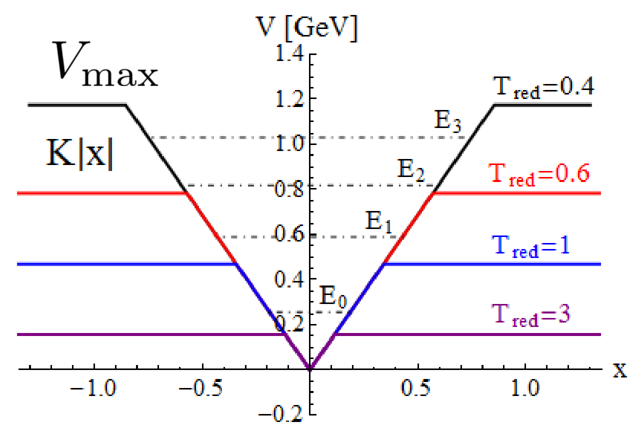
Hamiltonian: Mean Field: T-dependent color screened potential
Generally taken from lattice-QCD. Only singlet for now.

3D not easy to implement => 1D simplified model
not aim to reproduce the data but rather gives insights on the dynamics.



1D

simplification



Linear approx

Screening(T) as VlQCD

Parameters (K, Vmax) chosen to reproduce quarkonium spectrum + $B\bar{B}$ or $D\bar{D}$ threshold

Inner dynamics: SL equation

Derived from the Heisenberg-Langevin equation, in Bohmian mechanics …

$$i\hbar\frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}_{\mathrm{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Fluctuations: thermal excitation

Taken as a « classical » white stochastic force/noise scaled such as to obtain $T_{Q\bar{Q}} = T_{QGP}$ at equilibrium

The noise operator is assumed here to be a commutating c-number whereas it is a non-commutating q-number within the Heisenberg-Langevin framework.

I. R. Senitzky, Phys. Rev. 119, 670 (1960); 124}, 642 (1961).
G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. 6, 504 (1965).
R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

Inner dynamics: SL equation

Derived from the Heisenberg-Langevin equation, in Bohmian mechanics …

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}_{\mathrm{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Dissipation: thermal de-excitation

$$S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$$

✓non-linearly dependent on $\Psi_{Q\bar{Q}}$

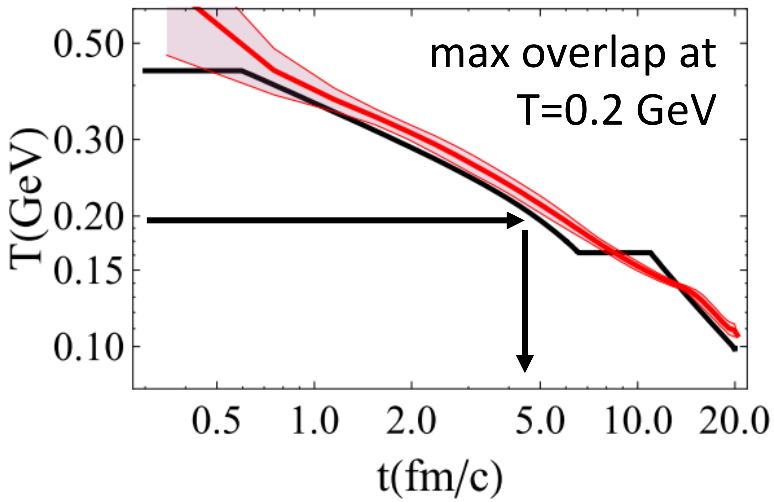
✓ real and ohmic

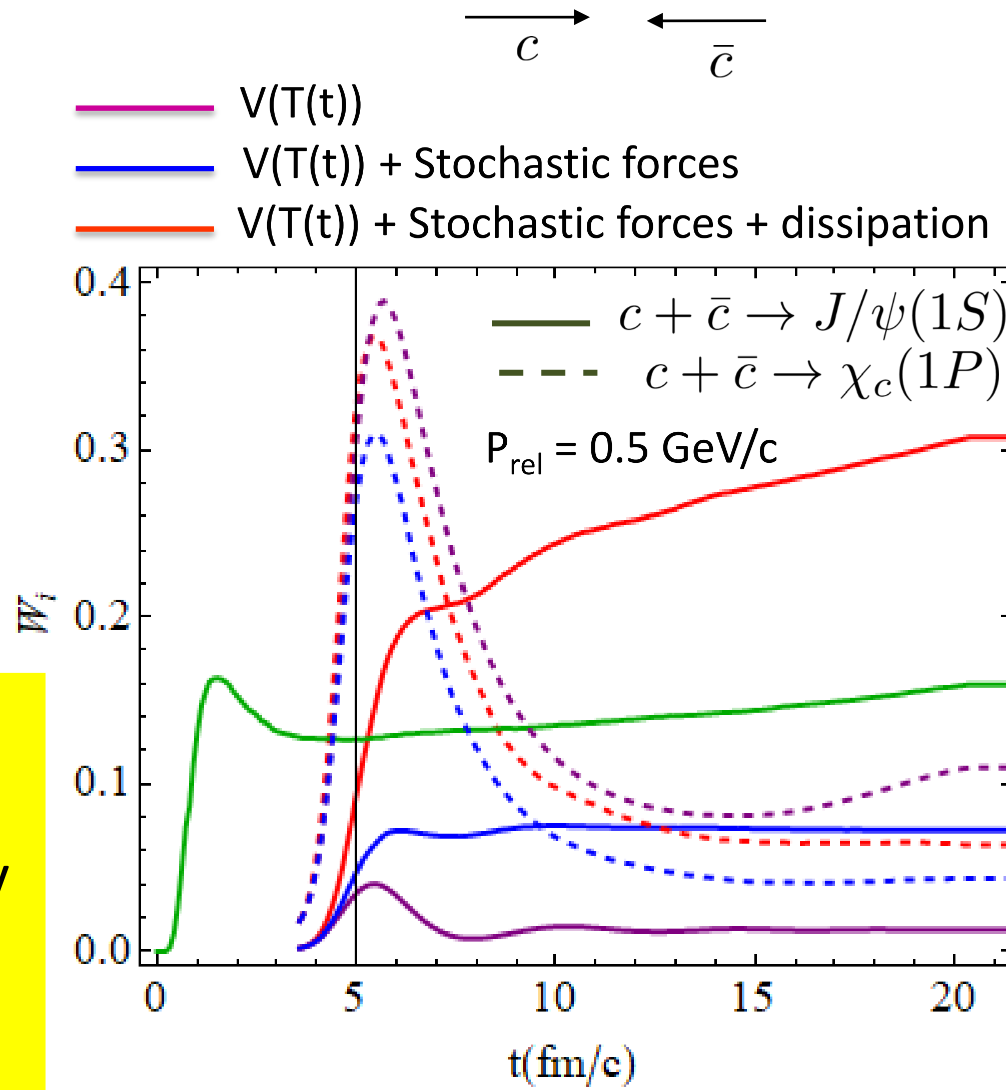
✓ brings the system to the lowest state

✓with A(T) α T^2 the Drag coefficient from a microscopic model (pQCD - HTL) by Gossiaux and Aichelin

P.B. Gossiaux and J. Aichelin, Phys.Rev. C78 (2008) 014904
R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

Stochastic Langevin Equation in *evolving QGP*





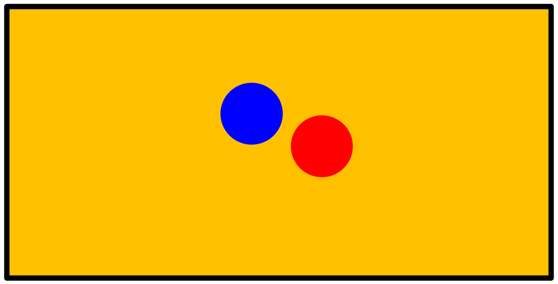
✓ Need the full combination (reconfining V(t) + equilibration with environment) to substantially produce lowest state…

✓ Possible cross talk with fragment production at lower energies

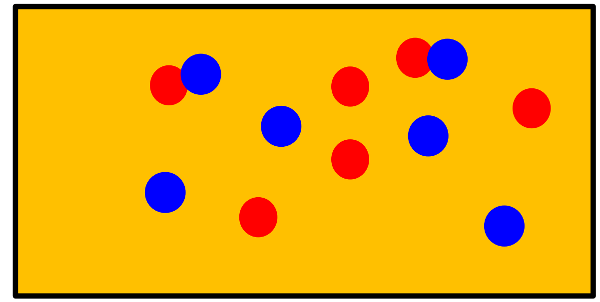
Semi-classical treatment through HQ "trajectories" ?

Regeneration: Dilute vs Dense

Bottomia (single pair)



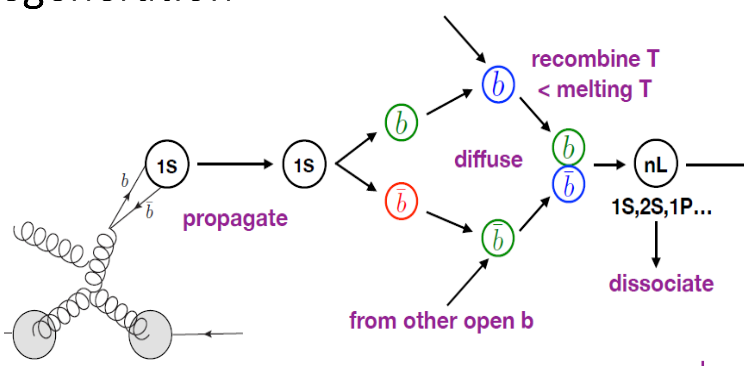
Charmonia (many pairs)



No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states.

Full quantum treatment affordable

N.B.: In some SC formalisms : intermediate regeneration



Yao, Mehen, Müller (2019)

Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

No full quantum treatment possible => semi-classical approximation (to be specified later)

Level of accuracy ?

Structure of the talk

1. Solve the quantum problem in a “simple” situation (single pair)
2. Use this solution to benchmark the semi-classical approximation

QCD time scales

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ_S: system intrinsic time scale

$$\tau_S \approx \underbrace{\frac{1}{\Delta E}}_{\text{Difference btwn energy levels}} \approx \frac{1}{m_Q v^2}$$ with $v \approx \alpha_S$ … at the beginning of the evolution

τ_R: system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi\left(\frac{CT}{m_Q \alpha_s}\right)$$

At "small" T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

$$\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1$$ And $$\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

Fine with the Markovian assumption

Two types of dynamical modelling

(and a 3rd class of its own: statistical hadronization)

$m_D \gg E_{\rm bind}$ $m_D \sim E_{\rm bind}$ $m_D \ll E_{\rm bind}$

Quantum Brownian Motion

Quantum Optical Regime

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations in position-momentum space

?

- Well identified resonances
- Time long enough wrt quantum decoherence time

Good description with transport models (TAMU, Tsinghua, Duke)

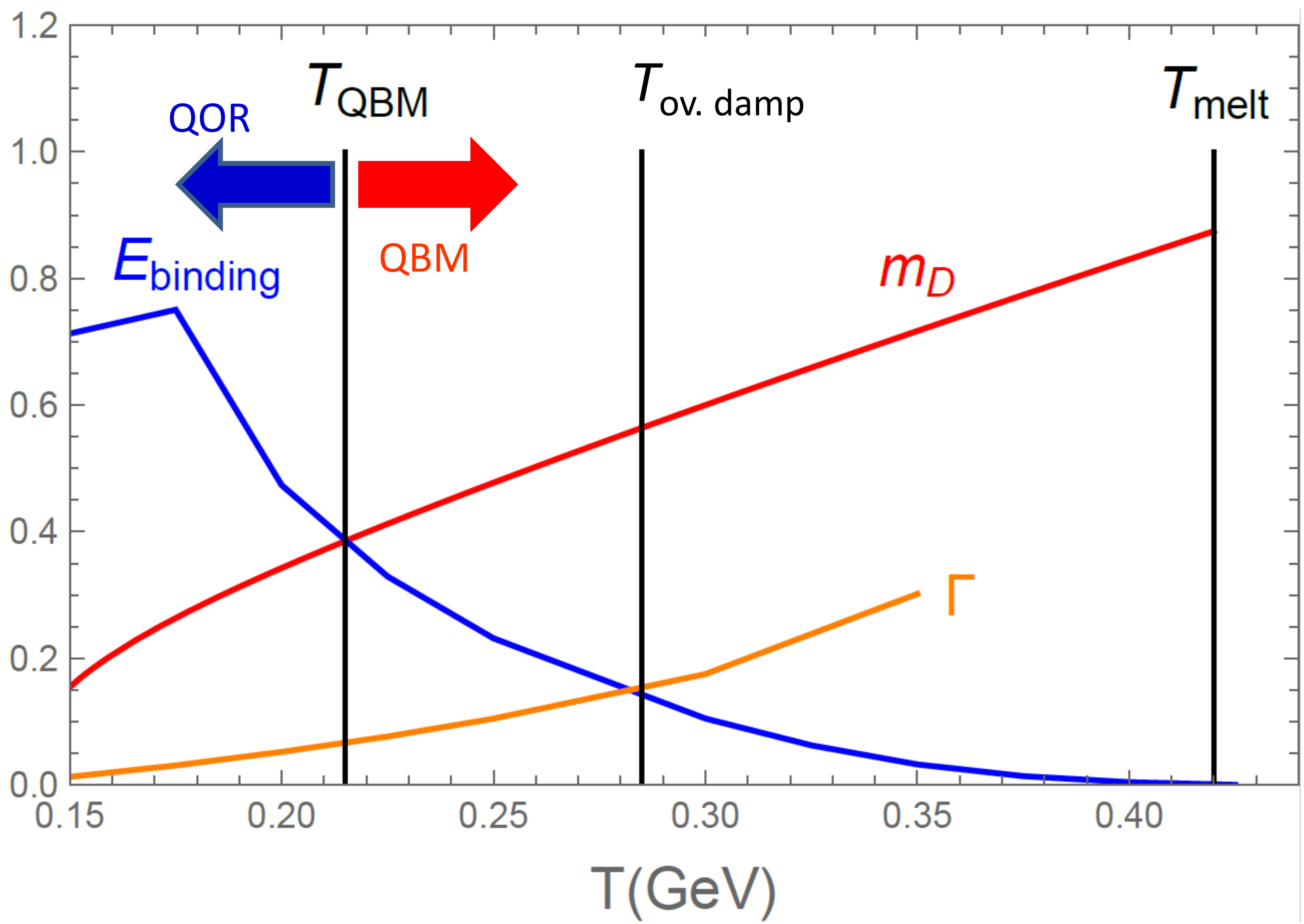
Central quantities :
2->2 and 2->3 Cross sections, decay rates

Equilibrium : $\exp(-E_n/T)$ (theorem)

SC Approx: rate equations

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these model is an important prerequisite !!!

Two types of dynamical modelling



Numbers extracted from potential described in Phys. Rev. D 101, 056010 (2020)

Non abelian Quantum Master Equation for a $Q\bar{Q}$ pair (Nantes Saclay)

$$\hat{\rho}_S = \mathcal{D}_\mathrm{s}|1\rangle\langle 1| + \mathcal{D}_\mathrm{o}\sum_a |o_a\rangle\langle o_a|$$

2 coupled color representations (singlet octet)

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

The Linblad Operator contains various terms representing several aspects of HQ physics

Unitary dynamics:

- $\mathcal{L}_0$: kinetic term
- $\mathcal{L}_1$: (screened) real potential term

Imaginary potential W → Non-Unitary dynamics:

- $\mathcal{L}_2$: fluctuations => heating and decoherence
- $\mathcal{L}_3$: dissipation
- $\mathcal{L}_4$: mandatory to preserve positivity (but sub-dominant)

J-P.Blaizot & M.A. Escobedo, JHEP 2018.6 (2018)

Non abelian Quantum Master Equation for a $Q\bar{Q}$ pair (Nantes Saclay)

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

Dissipation

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price of extra subleading terms :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_4$

Sketch of the appearance of an imaginary part to V

$$\frac{d}{dt}\hat{\rho}_S^{\rm red}(t) = -i[\hat{H}_S^{(0)}, \hat{\rho}_S^{\rm red}] + \sum_i \gamma_i \left[\hat{L}_i \hat{\rho}_S^{\rm red} \hat{L}_i^\dagger - \frac{1}{2}\left(\hat{L}_i^\dagger \hat{L}_i \hat{\rho}_S^{\rm red} + \hat{\rho}_S^{\rm red} \hat{L}_i^\dagger \hat{L}_i\right)\right]$$

$$\hat{H} = \hat{H}_S + \hat{H}_{\rm sto}$$

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] \Rightarrow \hat{\rho}(t+dt) = \hat{\rho}(t) + dt\,\frac{d\hat{\rho}}{dt} + \frac{dt^2}{2}\frac{d^2\hat{\rho}}{dt^2}$$

0 stochastic average

At 2nd order : $$\frac{d^2\hat{\rho}}{dt^2} = \frac{1}{\hbar^2}\left(2\hat{H}_{\rm sto}\hat{\rho}\hat{H}_{\rm sto} - \hat{H}_{\rm sto}^2\hat{\rho} - \hat{\rho}\hat{H}_{\rm sto}^2\right)$$

$$\frac{d^2\langle x|\hat{\rho}|x'\rangle}{dt^2} = \frac{1}{\hbar^2}\Big(2\underbrace{\hat{H}_{\rm sto}(x)\langle x|\hat{\rho}|x'\rangle\hat{H}_{\rm sto}(x')}_{W(x-x')/dt} - \underbrace{\hat{H}_{\rm sto}^2}_{W(x-x)/dt}(x)\langle x|\hat{\rho}|x'\rangle - \langle x|\hat{\rho}|x'\rangle\underbrace{\hat{H}_{\rm sto}^2}_{W(x'-x')/dt}(x')\Big)$$

$$\frac{d\rho(x,x')}{dt} = \cdots - 2\underbrace{\left(W(x-x') - W(0)\right)}_{\Gamma(x-x')}\rho(x,x')$$

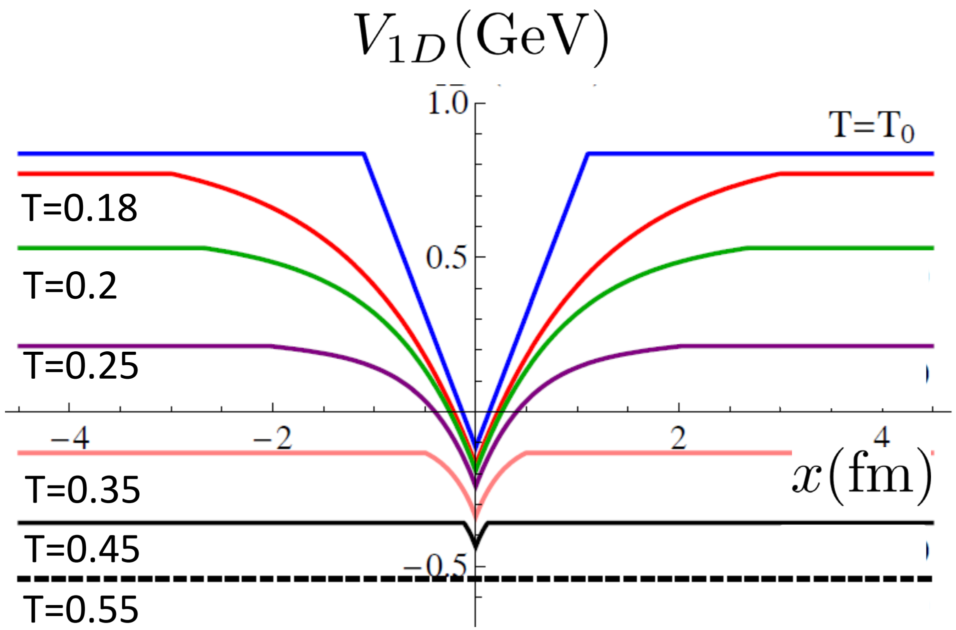
Further implementation features

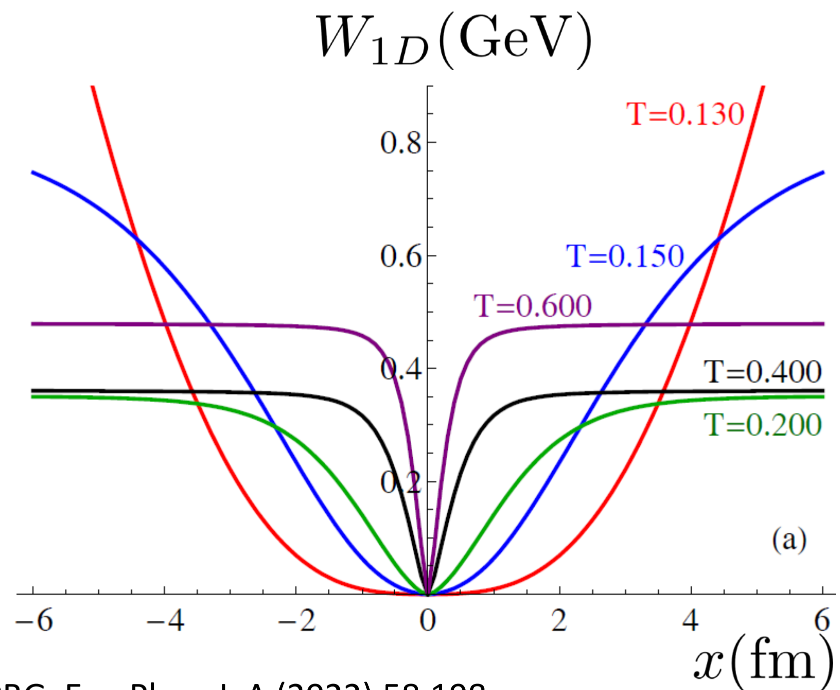
- 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

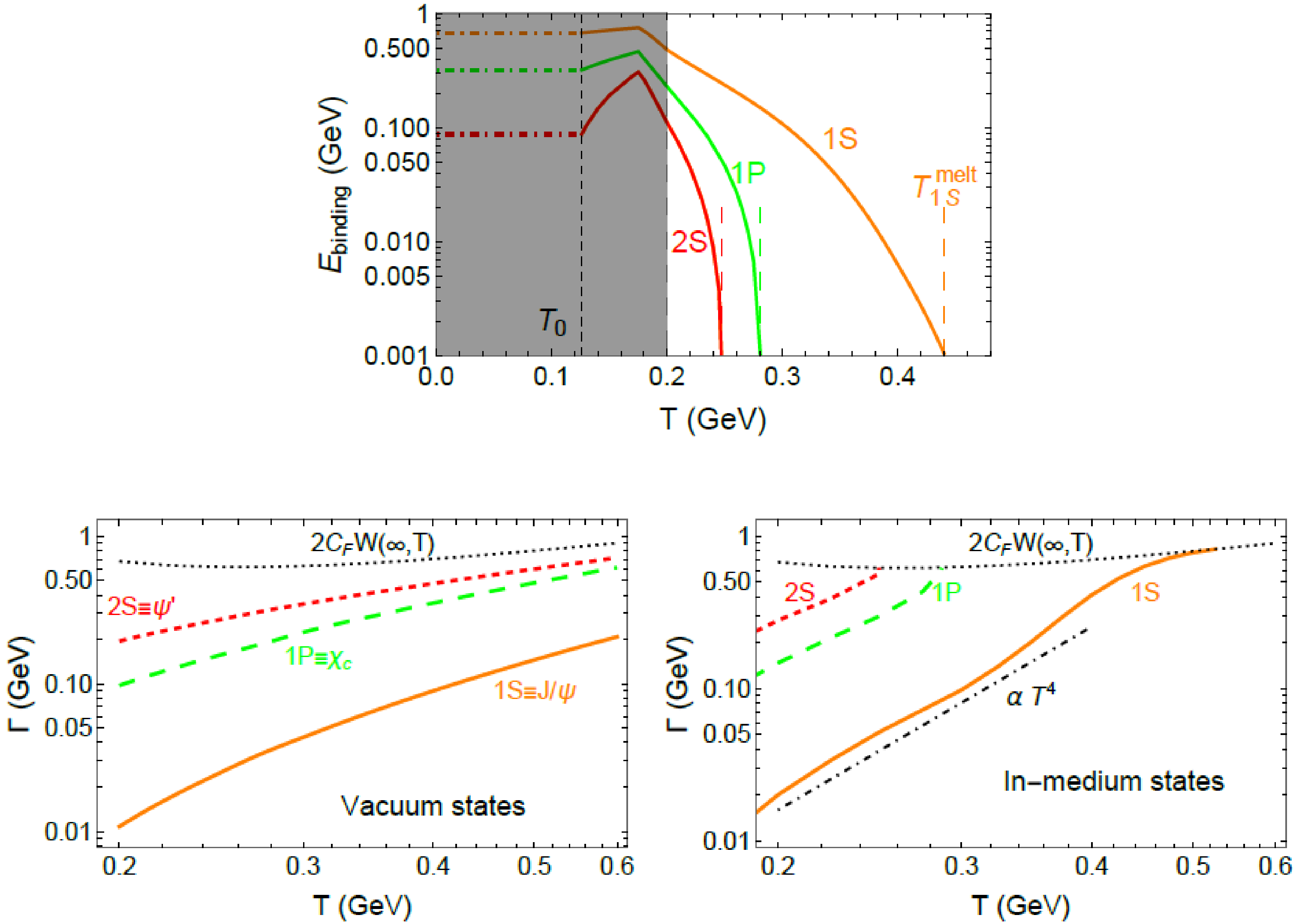
Need to design a realistic 1D bona fide potential V + i W (based on 3D lQCD results, tuned to reproduce 3D mass spectra and decay widths)





1D potential from R. Katz, S. Delorme & PBG, Eur. Phys. J. A (2022) 58:198

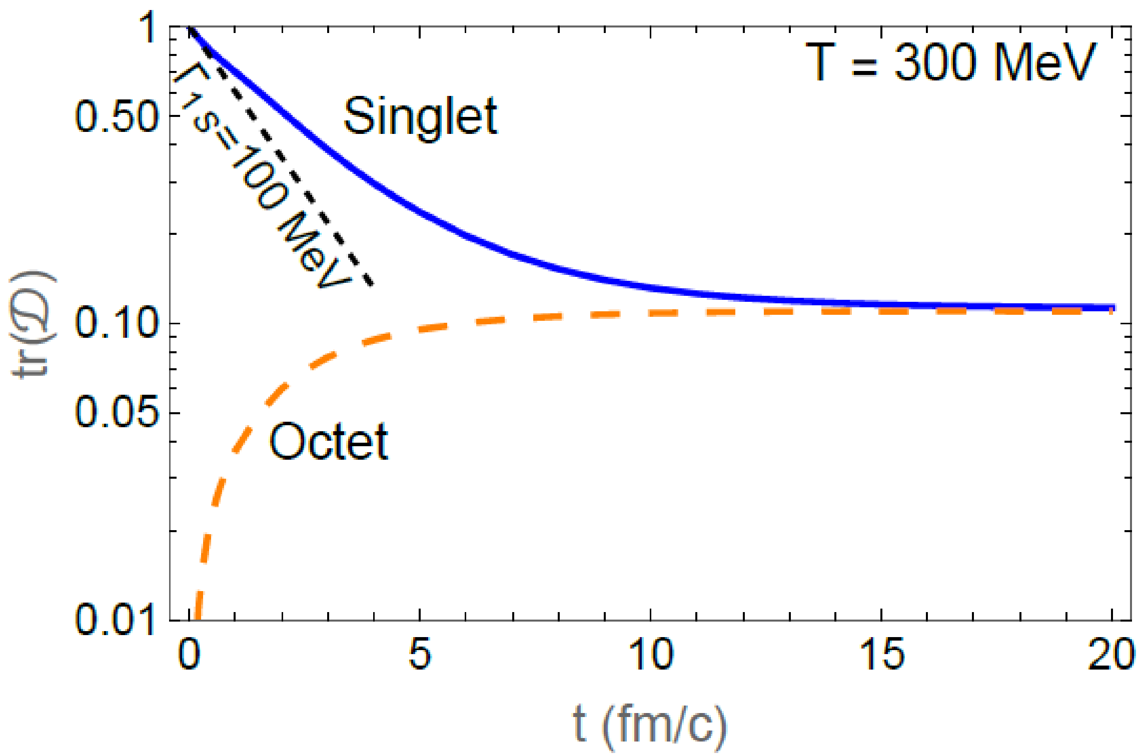
Decay rates from QME



Some selected results for 1 $c\bar{c}$ pair

<u>Color Dynamics : Singlet – octet probabilities:</u>

- Starting from a singlet 1S-like, one expects some equilibration / thermalisation -> asymptotic values : $D_{\rm s}^{\rm eq} = D_{\rm o}^{\rm eq} = \frac{1}{9}$ $\quad (1+8) \times \frac{1}{9}$

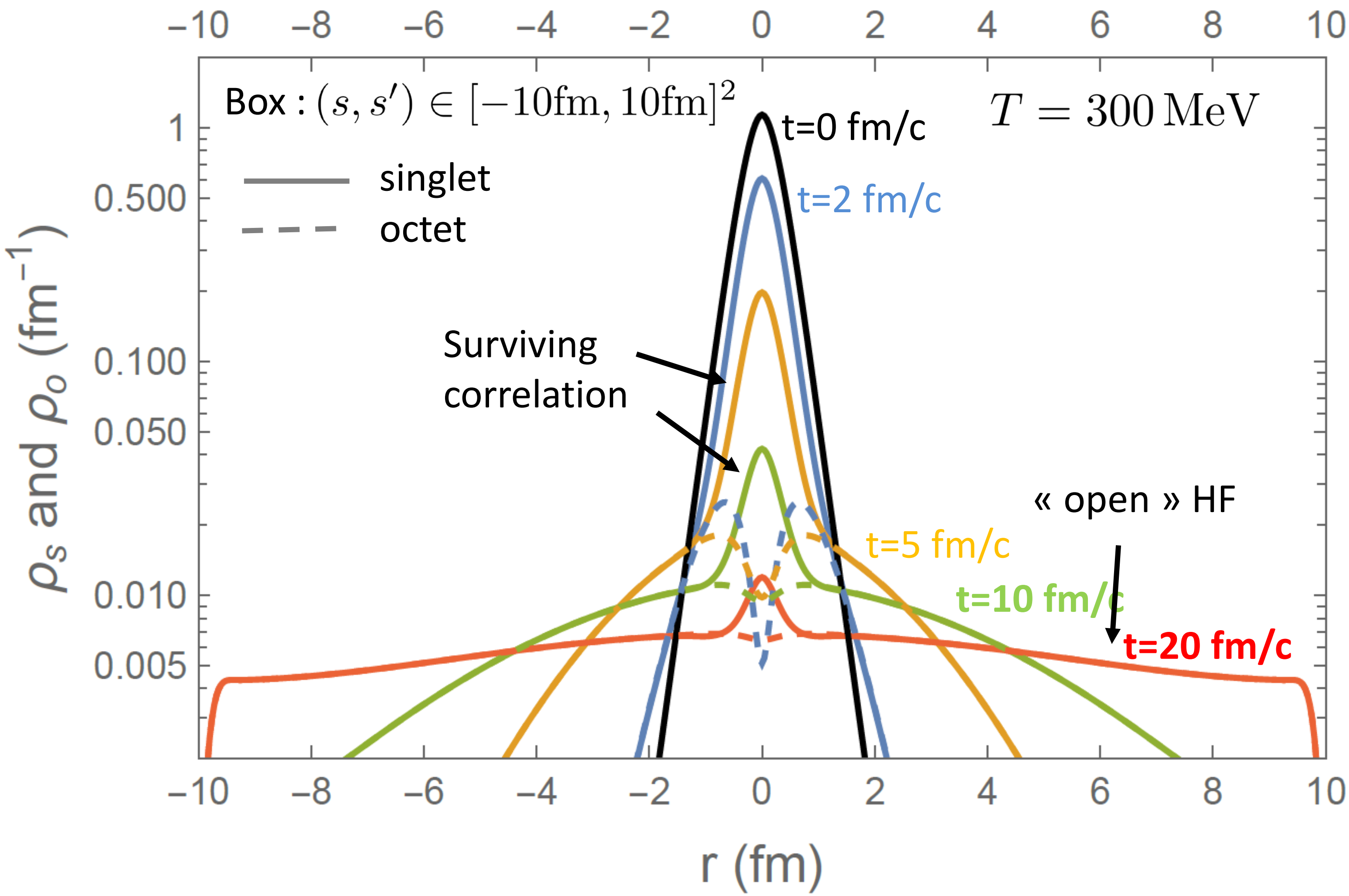


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.
- C-cbar can interact with the surrounding QGP as an octet => energy loss

S.Delorme et al. JHEP 06 (2024) 060

Evolution of the spatial density

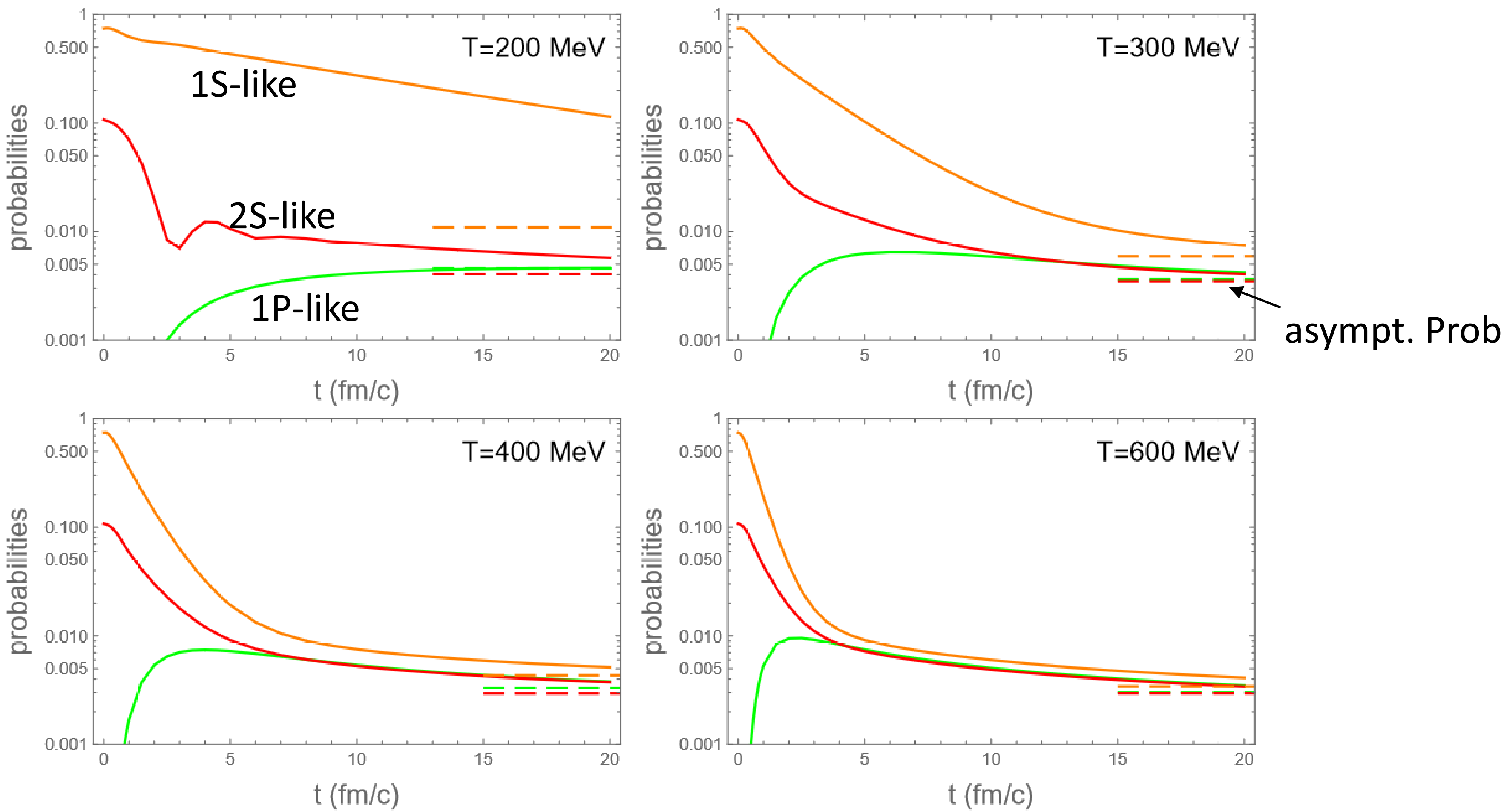
1S singlet initial state:



Some c-cbar stay at intermediate distance (“recombination”) … remaining peak in the asymptotic distribution

Results for projection on vacuum states

Starting from a compact S-like state : $\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}$ $\sigma = 0.165\,\mathrm{fm}$ $p_\Phi = \mathrm{tr}(\mathcal{D}_s D_\Phi)$

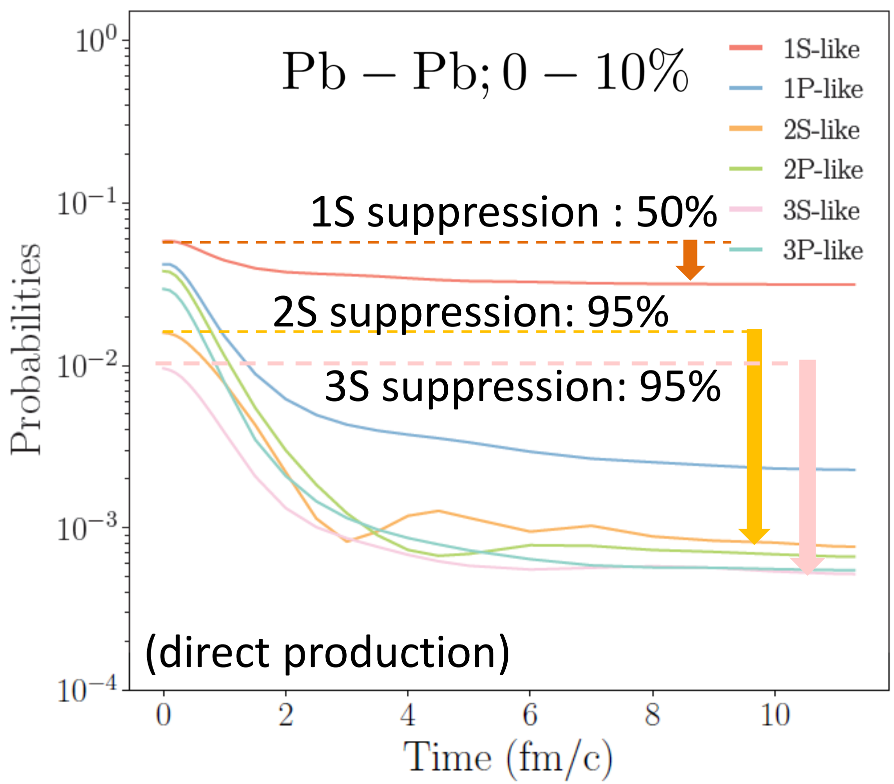


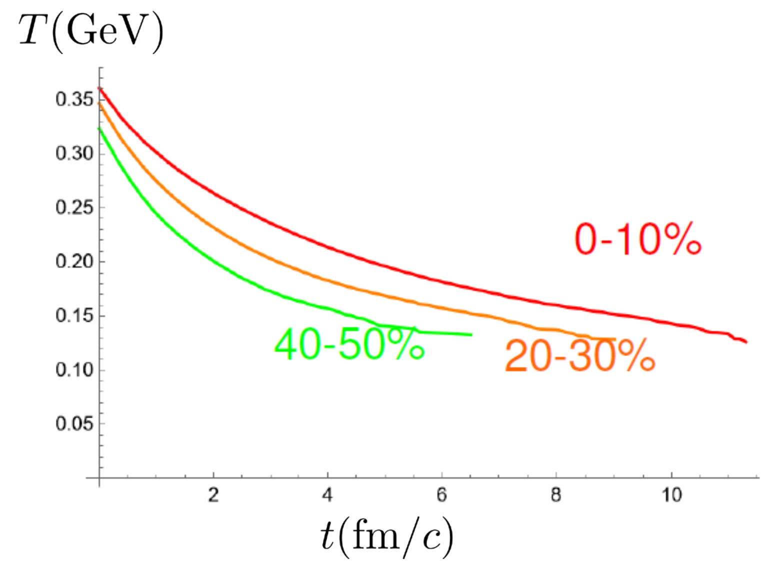
- Natural evolution for 1S-like suppression, from low to high T
- 2S state do not decay $\alpha e^{-\Gamma_{2S} t}$ at early time… partly driven by the ground state at later time.

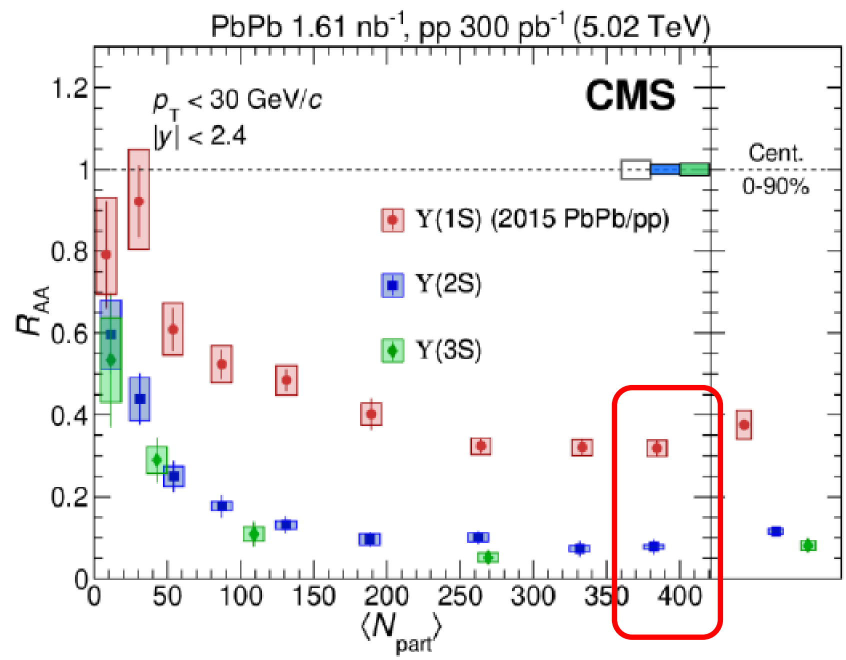
Contact with experiment ($b\bar{b}$)

- Bottomonia yield using the QME with EPOS4 (T,v) profiles and starting from a compact $b\bar{b}$ state.

$$\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}} \left(1 + a_{\rm odd} \frac{s}{\sigma}\right) \quad \sigma = 0.045\,\mathrm{fm}\; a_{\rm odd} = 3.5$$

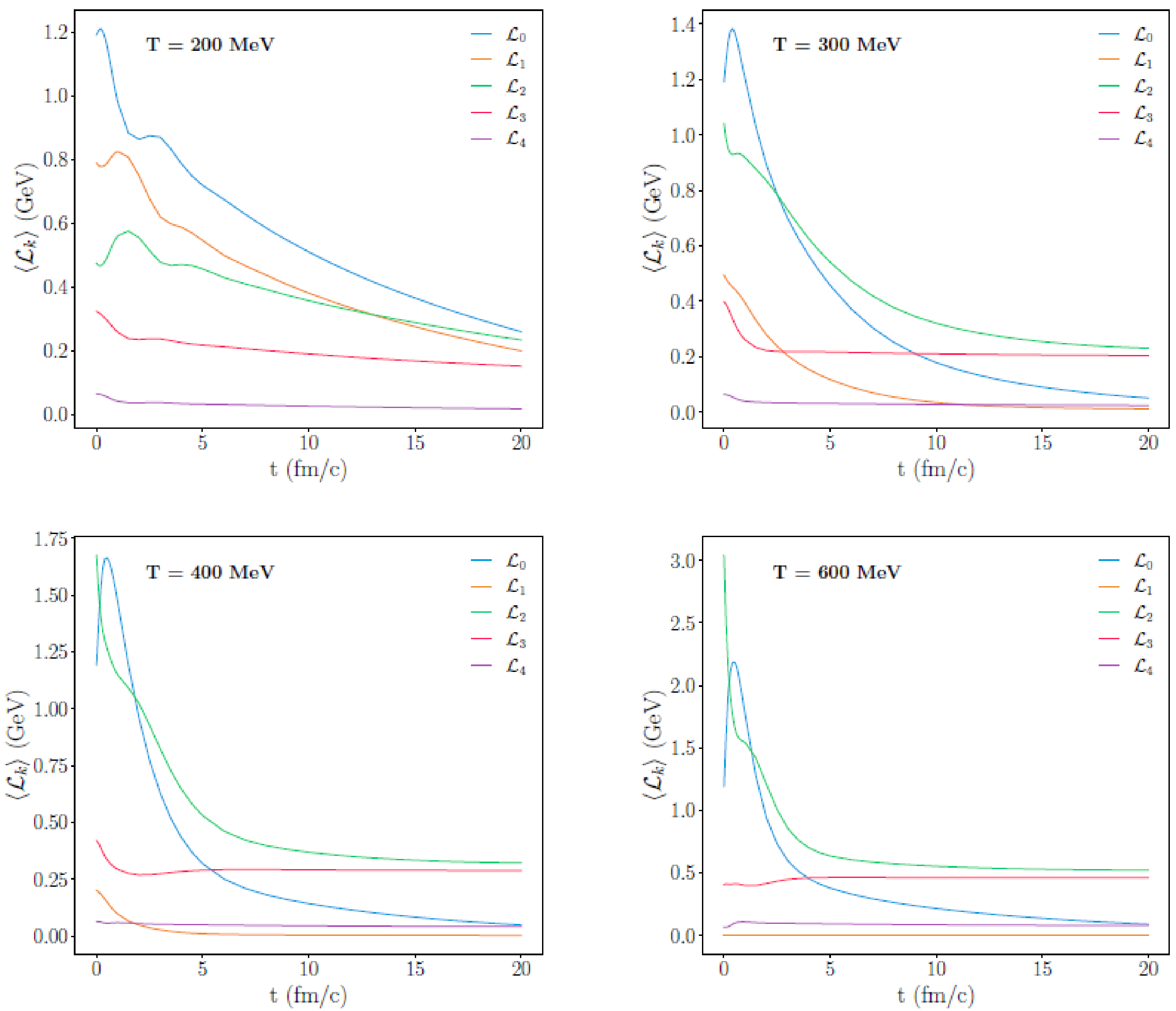






- See Stephane Delorme's talk at SQM24 for more details.

Contributions from the various terms



At large T : kinetic + diffusion dominate at all time

A buffet of Quarkonium dynamics

MENU Ô SAVOYARD

Entrée + Plat ou Plat + Dessert .... 24,00 €
Entrée + Plat + Dessert .... 32,00 €
Hors boisson
Non cumulable avec d'autres offres ou gratuités.

Entrée au choix
Friture d'ablettes sauce tartare
Moflet savoyard
Pâté croûte artisanal poulet citron bio
Petite salade Ô savoyard

The (semi) classical

Plat au choix
Tartiflette spéciale Ô Savoyard
Filet frais de féra grillé à la plancha, légumes frais (+ 2 €)
Fricassée de caïon
Tartare de bœuf charolais

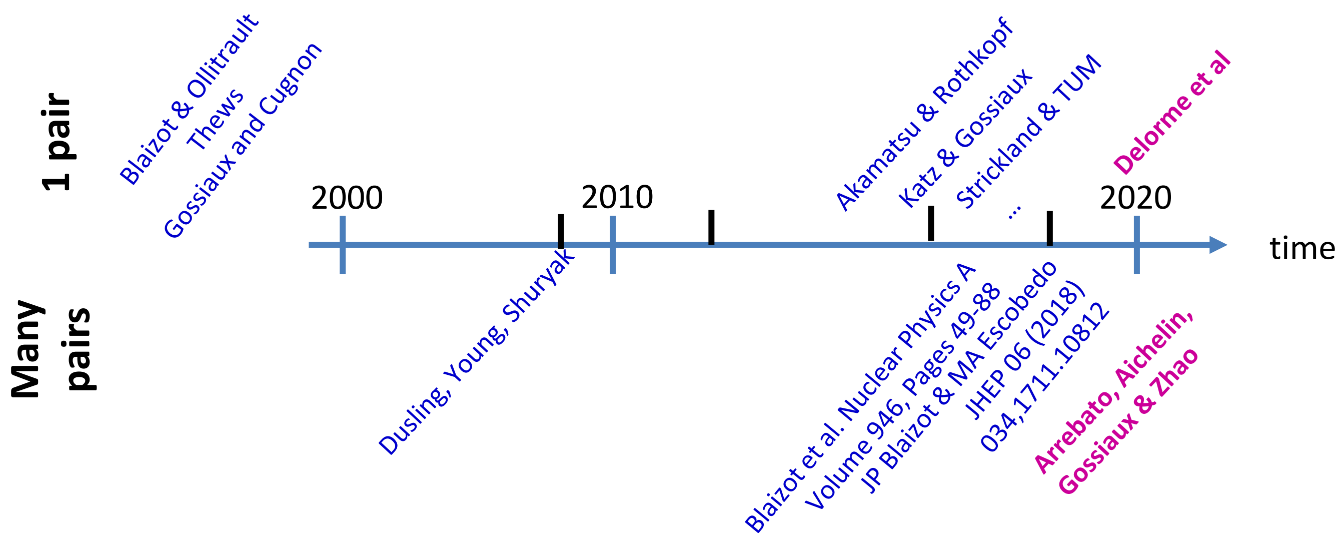
Dessert au choix
Glace Alpage au Génépi
Soupe de fraises et glace vanille-framboise meringuée
Tarte aux myrtilles fraîches
Mojito tendresse citron

Semiclassical approximation (meant for charmonia)

… and now, we will consider the semi-classical evolution of the lowest $Q\bar{Q}$ bound state

Several motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**… In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions**)
- Better suited for **« from small to large »**
- Extra complication: For RHIC and LHC : many c-cbar pairs !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs (NRQCD) => mixed Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

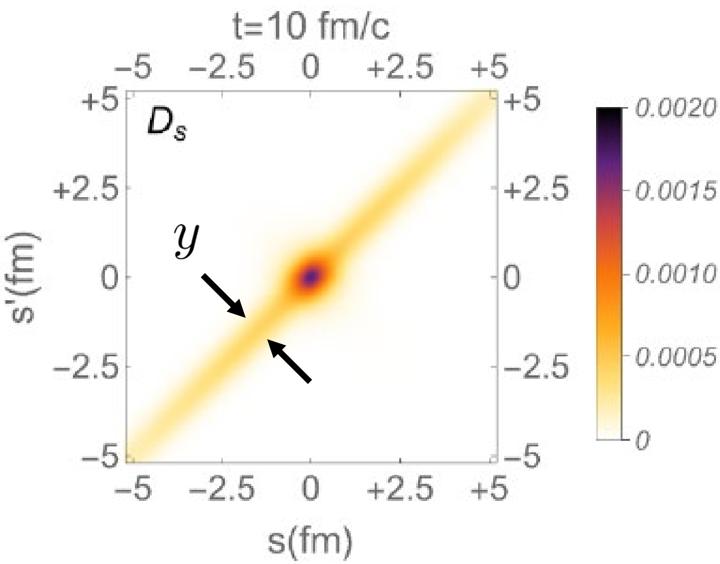
Semiclassical approximation (meant for charmonia)

… and now, we will consider the semi-classical evolution of the lowest $Q\bar{Q}$ bound state

SC = limit of small $\hbar$ ⇔ large action of the system… ok for ground state ?

Semiclassical approximation

- For the relative motion (2 body): $\left.\begin{array}{l}\vec{s} = \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' = \vec{x}_1' - \vec{x}_2'\end{array}\right\} \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2}$ and $\vec{y} = \vec{s} - \vec{s}\,'$



$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** of the Linblad equation: power series in *y* up to 2nd order)

- Wigner transform : $\mathcal{D}(\vec{r},\vec{y}) \rightarrow W(\vec{r},\vec{p})$ and $\{\vec{y},\nabla_y\} \rightarrow \{\nabla_p,\vec{p}\}$
- => Usual Fokker Planck equation :

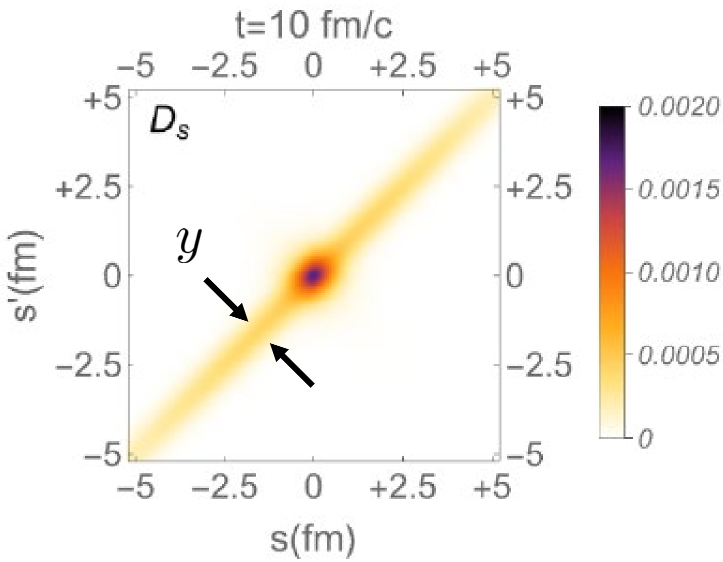
fluctuations dissipation

$$\frac{\partial W}{\partial t} = \left[-\frac{2\vec{p}\cdot\nabla_r}{M} - \nabla_r V \cdot \nabla_p + \frac{\eta(r)}{2}\nabla_p^2 + \frac{\gamma(r)}{M}\nabla_p \cdot \vec{p}\right] W$$

- Easy MC implementation + generalization for N body system (c-cbar @ LHC)

Semiclassical approximation

- For the relative motion (2 body): $\left.\begin{array}{l}\vec{s} = \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' = \vec{x}_1' - \vec{x}_2'\end{array}\right\} \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2}$ and $\vec{y} = \vec{s} - \vec{s}\,'$



$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

Near thermal equilibrium, Density operator is nearly diagonal => **semi-classical expansion** of the Linblad equation: power series in *y* up to 2nd order)

- When / why does it work ?
  - The unitary term : $\mathcal{L}_1[\rho] = [V,\rho] = \rho(s,s')(V(s) - V(s')) = V'(r)y + \mathcal{O}(y^3)$

    Wigner-Moyal expansion, valid when y << variation scale of the real potential

  - The interaction with the environment : L_2

$$\Gamma(y)\rho(s,s') \approx \Gamma''(0)y^2 \times \left(1 + \mathcal{O}(y^2 m_D^2)\right)\rho(s,s')$$

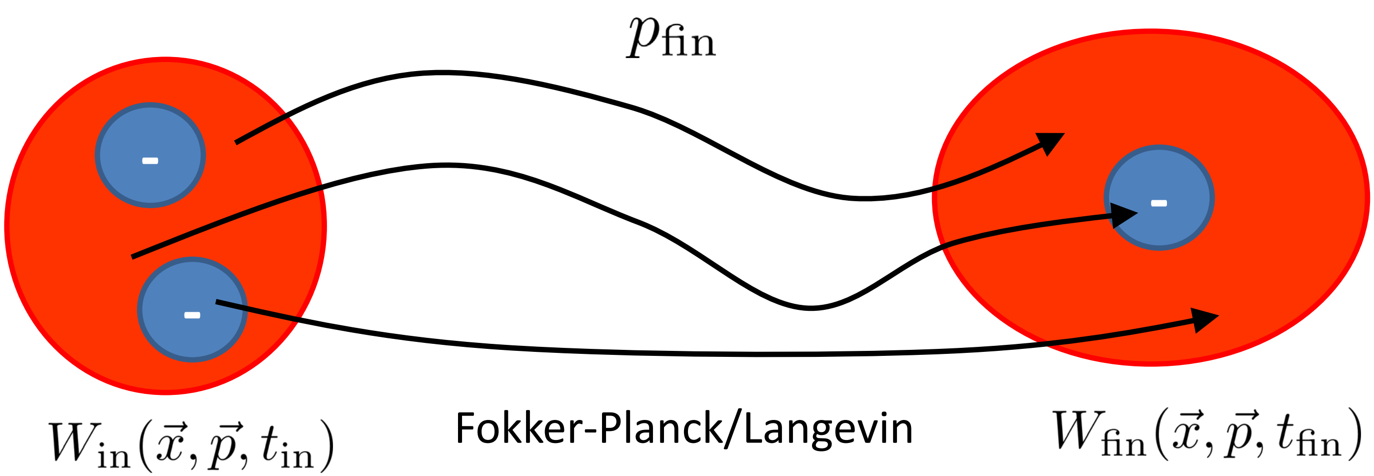
$$\Gamma''(0)\partial_p^2 W(r,p)$$

$$\approx \frac{T}{m_Q} \ll 1$$

Classical noise

Quantum vs SC dynamics

- SCA : linear mapping



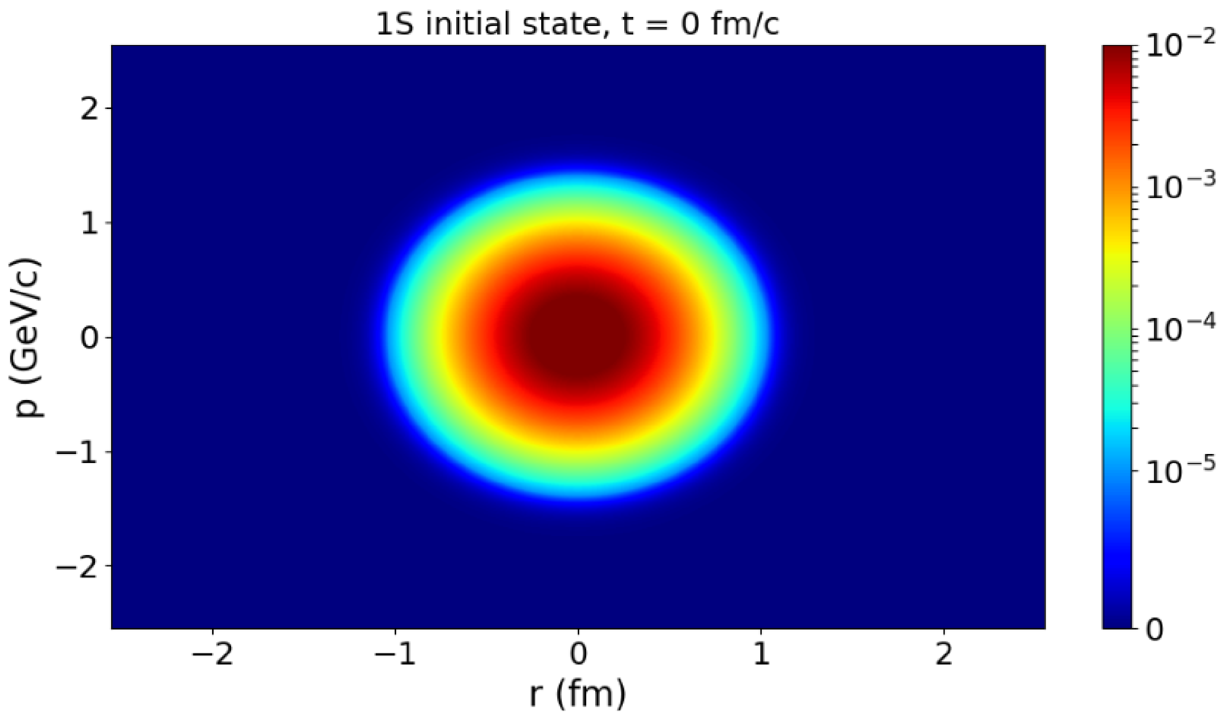
Always positive defined even if W_{in} and W_{fin} are not positive defined

- Several aspects :
  - Temperature
  - Initial state
  - Property considered

In the following : only a limited set of results; manuscript to come soon

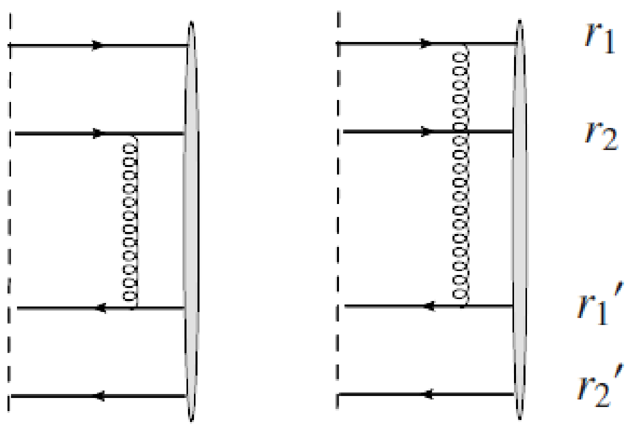
Concrete implementation

- 1D (same as for the QME), **1 $c\bar{c}$ pair**
- Same real potential, W in the QME ⇔ η and γ in the FP
- **Abelian case** (for the time, not clear how to deal with the singlet <-> octet transition in a semiclassical approach)
- Yet, not trivial…
- Initial state vacuum 1S state : $\psi(s) \propto e^{-\frac{s^2}{2\sigma^2}}$ $\sigma = 0.38\,\mathrm{fm}$



QED-like (abelian) vs genuine QCD case

Genuine QCD

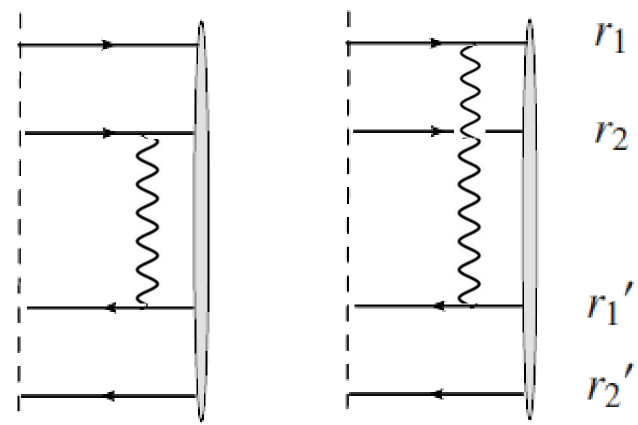


- Scattering from gluons change the color respresentation : o <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \\ \mathcal{D}_o \end{pmatrix}$$

- No binding potential in the octet chanel => « large » energy gap

QED-like



- Scattering from photons do not change the Casimir : s <-> s

$$\mathcal{D}_Q = \begin{pmatrix} \mathcal{D}_s \end{pmatrix}$$

- Usual 1S <-> 1P transitions between bound states.

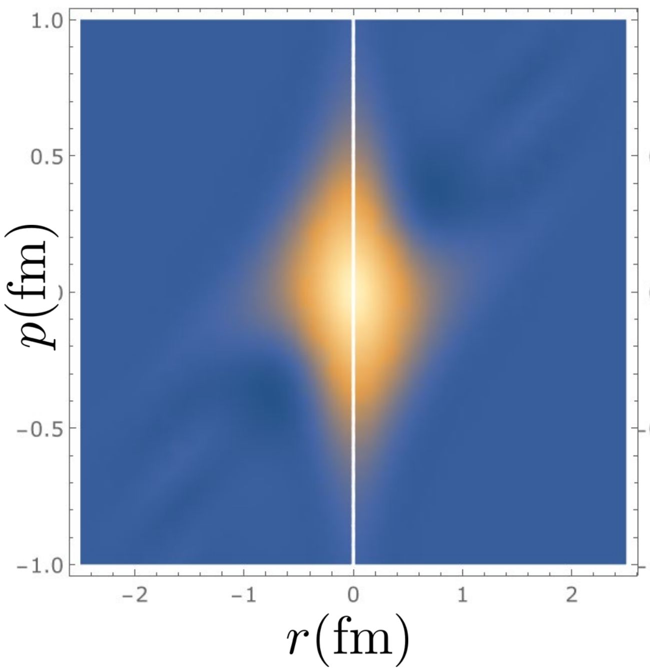
Unitary evolution

Unitary ⇔ no fluctuation and dissipation by coupling with the QGP

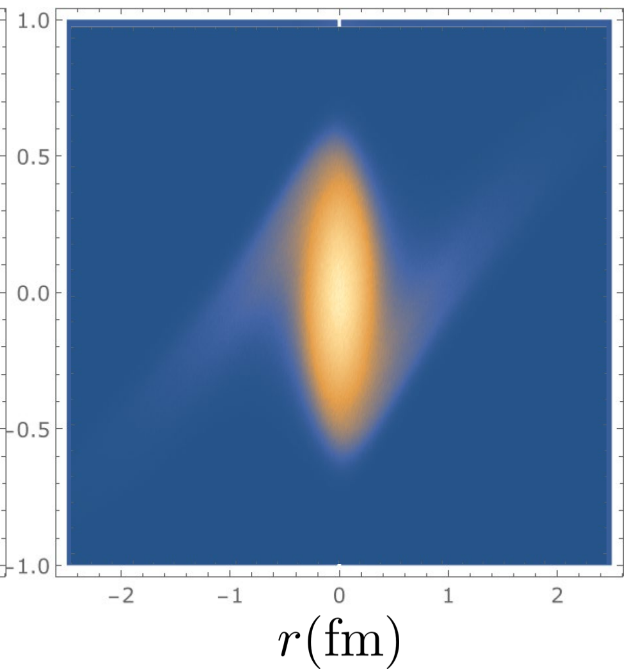
evolution of a vacuum 1S state in a screened V (T=200MeV) … still some evolution

$$H(T)|\psi\rangle_0 \neq E|\psi_0\rangle$$

W_{QME}(t=2fm/c)

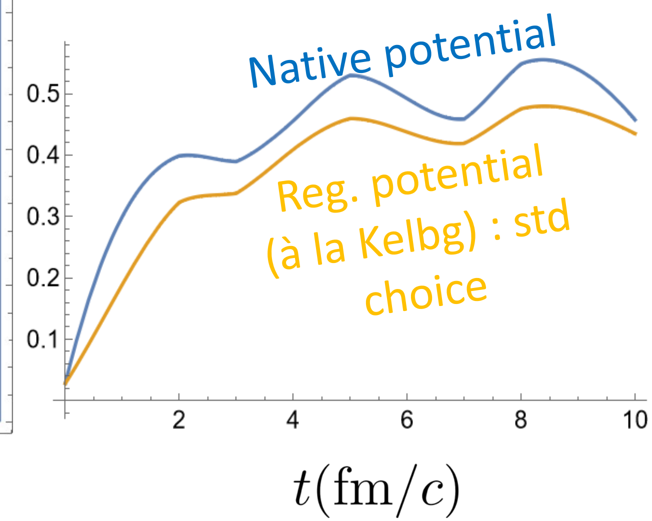


W_{SC}(t=2fm/c)



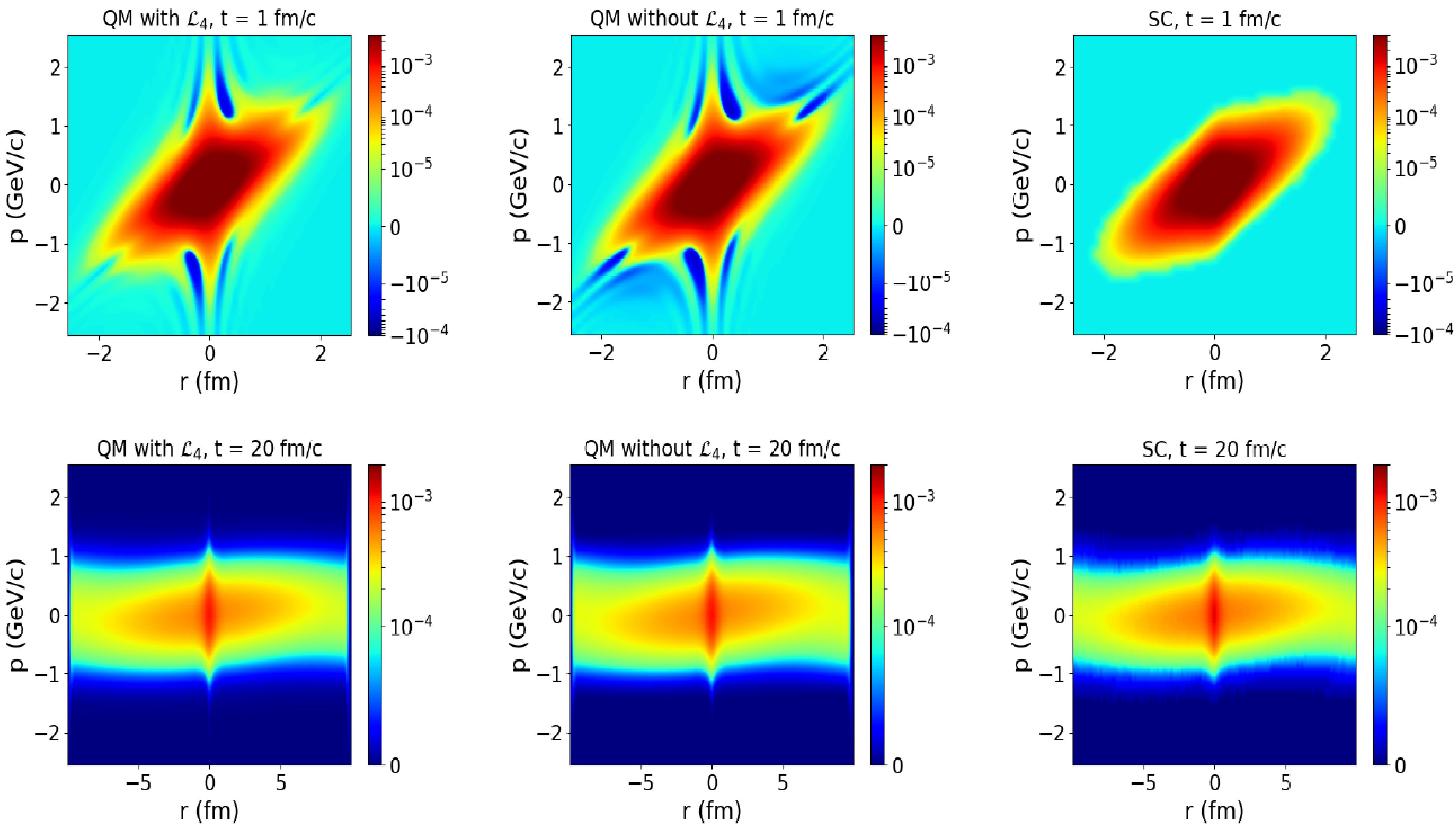
Normalized to 1

$$d_1 = \int dr dp |W_{\rm QME} - W_{\rm SC}|$$



Non-unitary evolution

Full coupling with the QGP

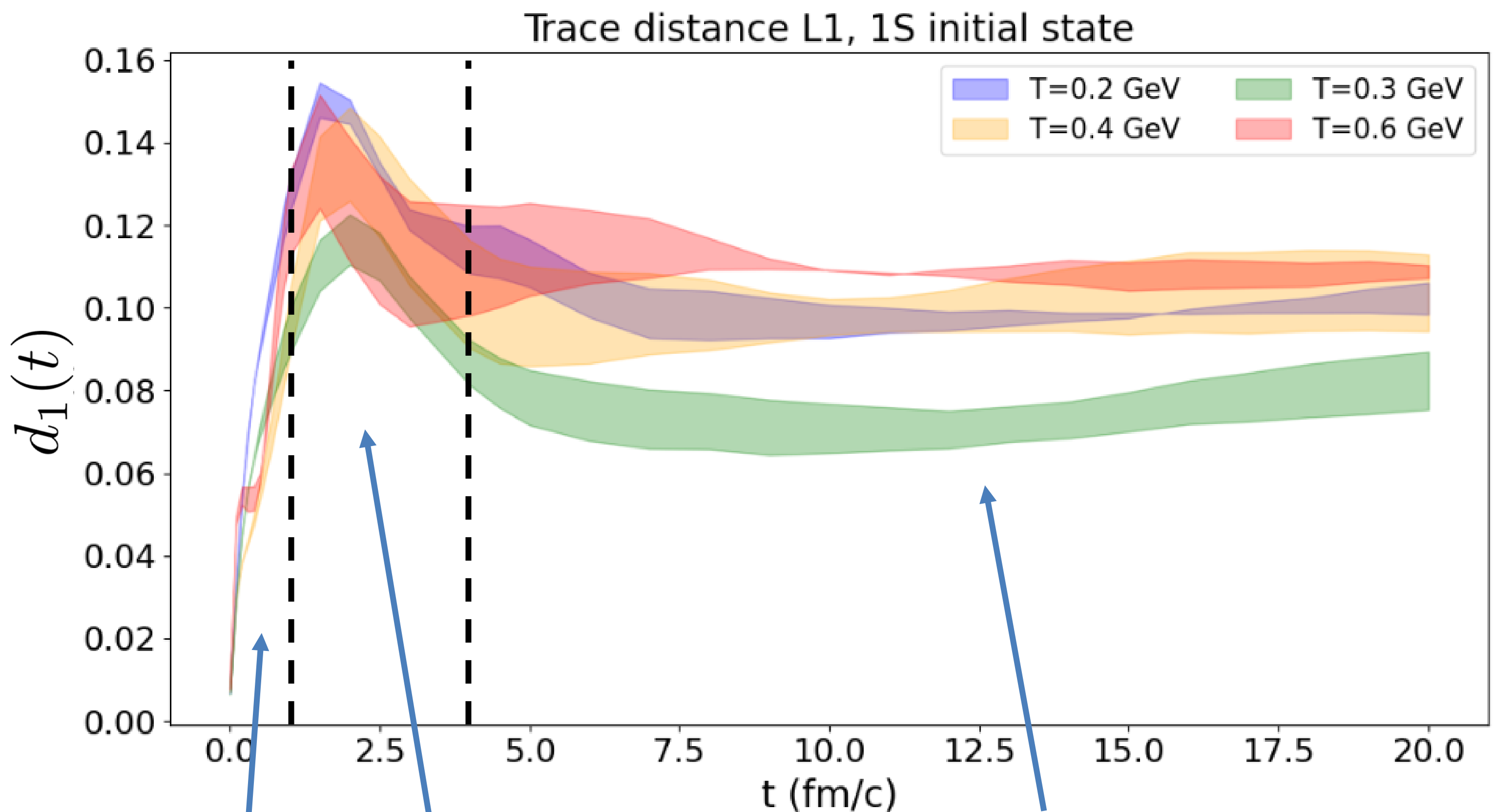


- Some specific signs of genuine QM evolution at small time
- Better agreement between the two descriptions at late times.

Non-unitary evolution

Full coupling with the QGP

$d_1 = \int dr dp |W_{\rm QME} - W_{\rm SC}|$



I. Growth of the ≠ between QME and SC evolution : genuine QM features

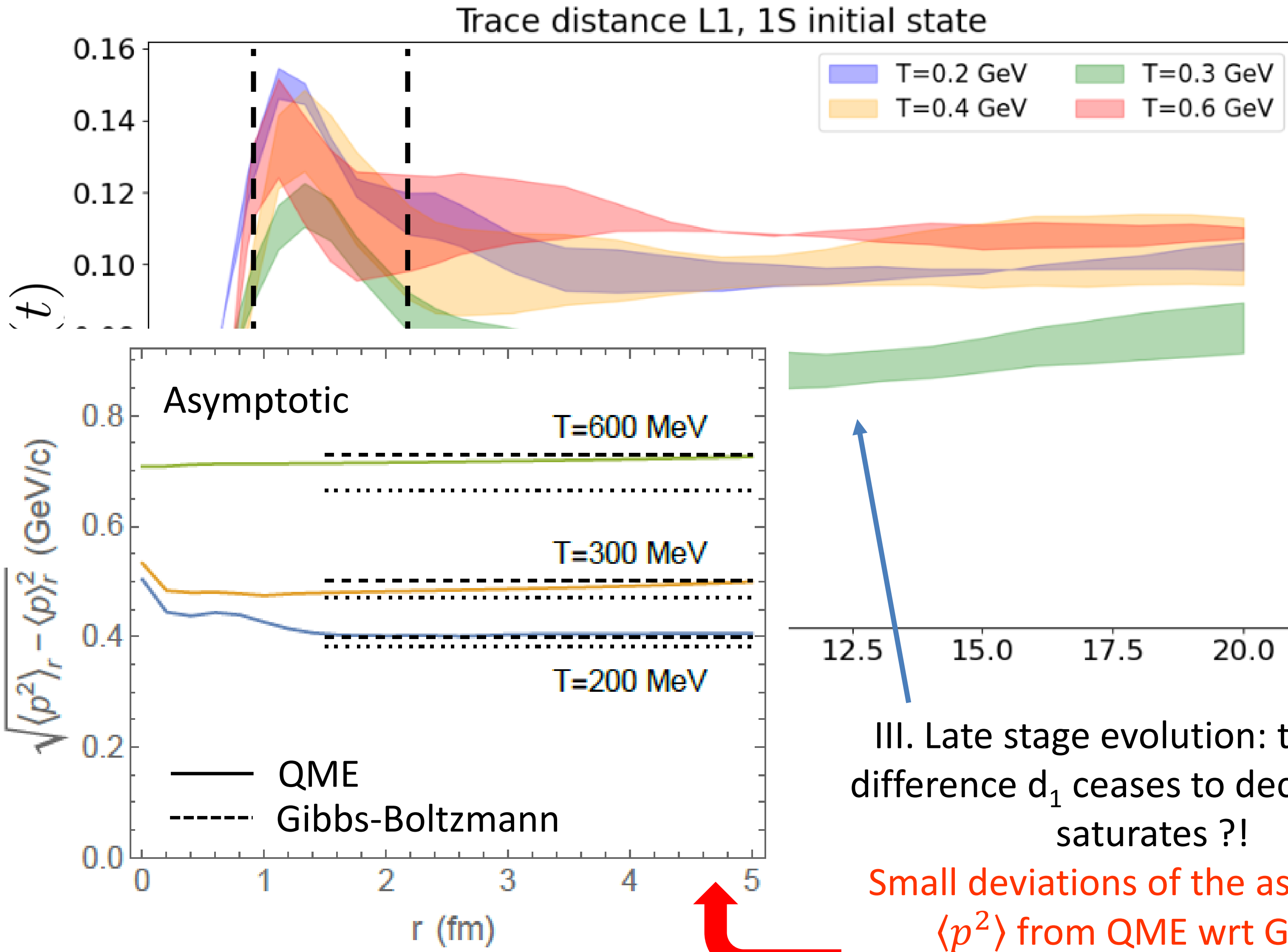
II. Saturation and decrease of the ≠ between QME and SC evolution : **classicalization**

III. Late stage evolution: the norm difference d_1 ceases to decrease and saturates ?!

Non-unitary evolution

Full coupling with the QGP

$$d_1 = \int drdp|W_{\mathrm{QME}} - W_{\mathrm{SC}}|$$

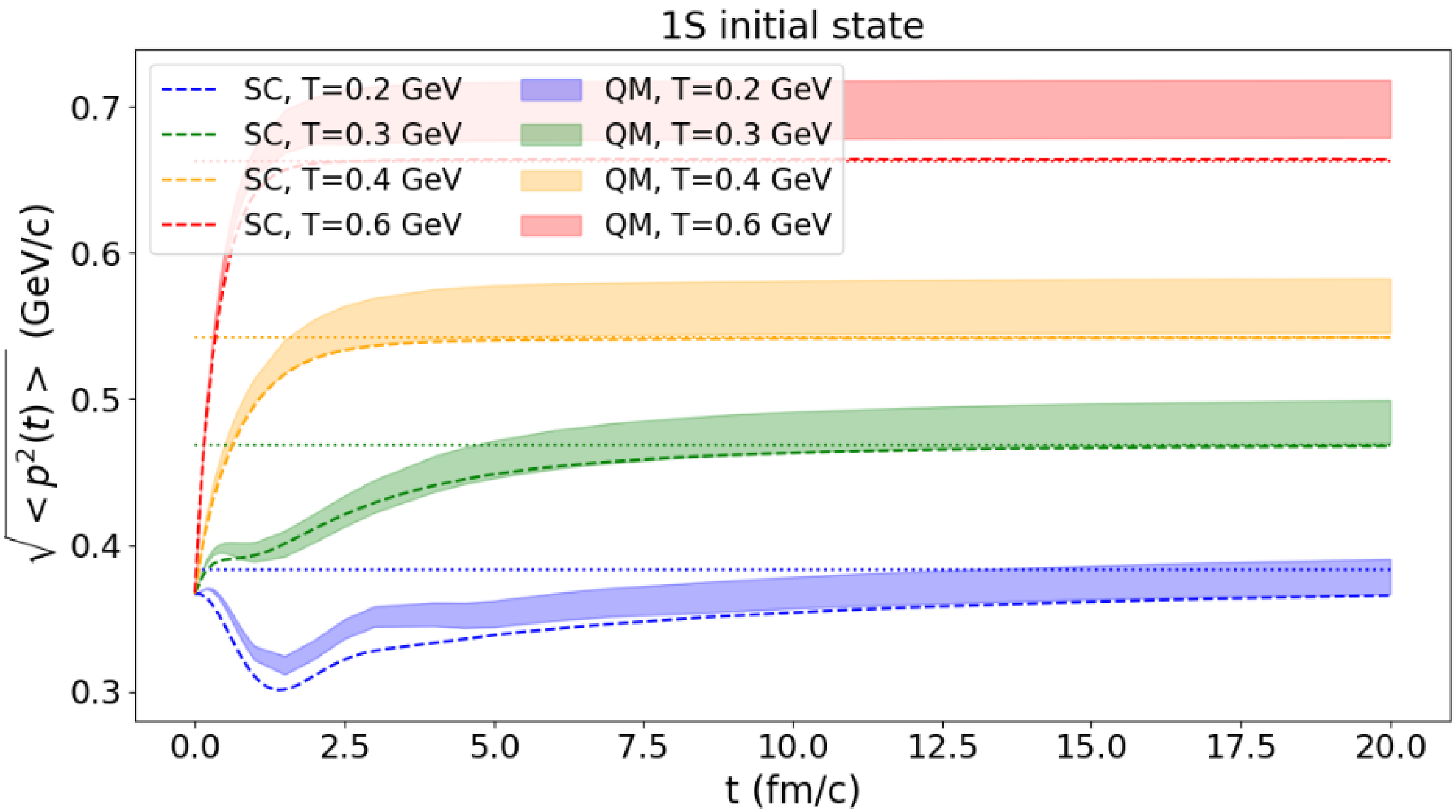


I. (
betv
evo

III. Late stage evolution: the norm difference d_1 ceases to decrease and saturates ?!

Small deviations of the asymptotic $\langle p^2 \rangle$ from QME wrt Gibbs-Boltzmann

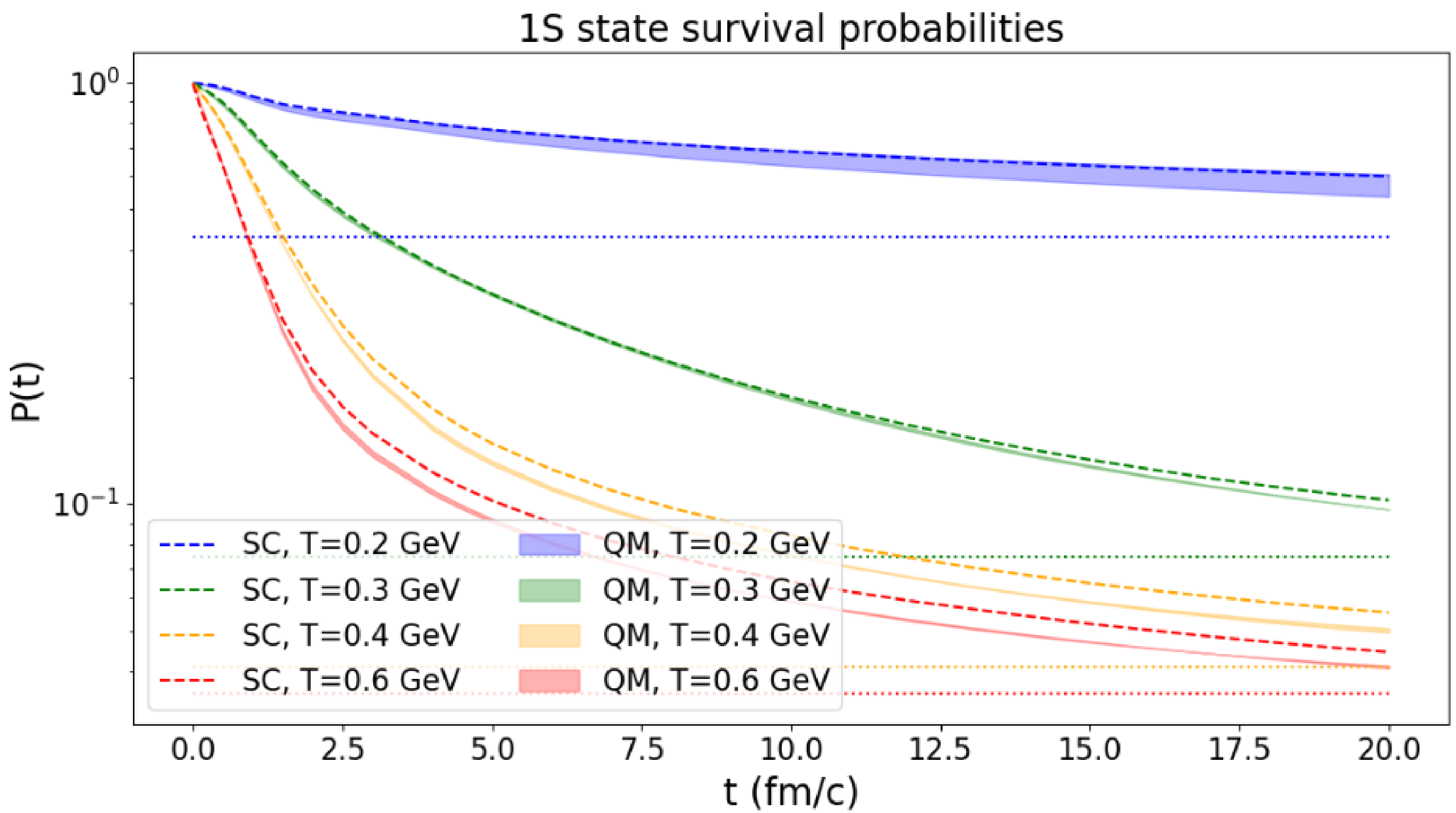
Quantum vs SC dynamics (QED-like)



Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **Survival probability of the 1S initial state**:

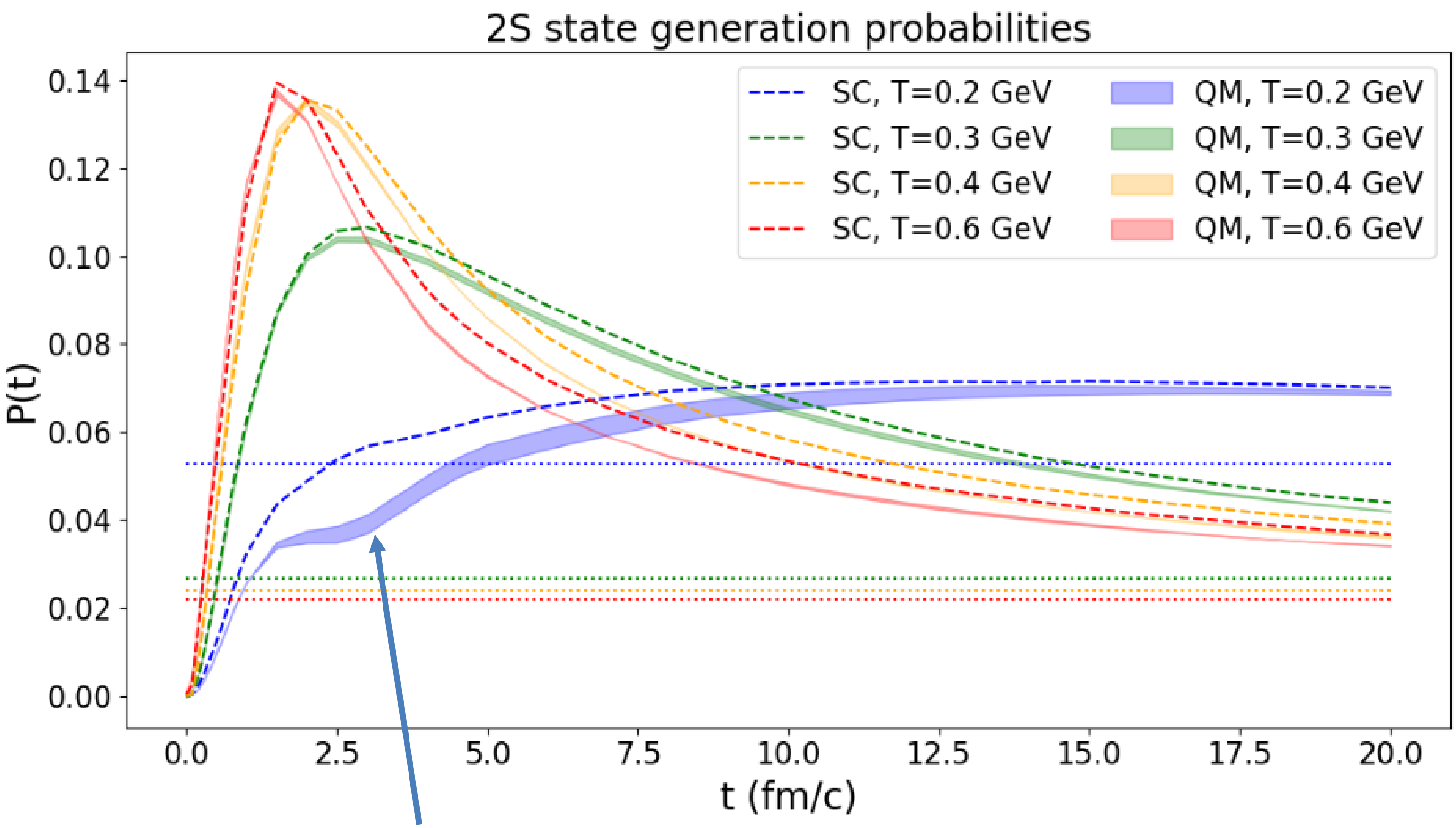


- Good agreement of the SC calculation with the QME benchmark
- Slight over suppression for the QME (overheating)

Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **(Re)generate 2S state**
- Good agreement of the SC calculation with the QME benchmark, especially at large T

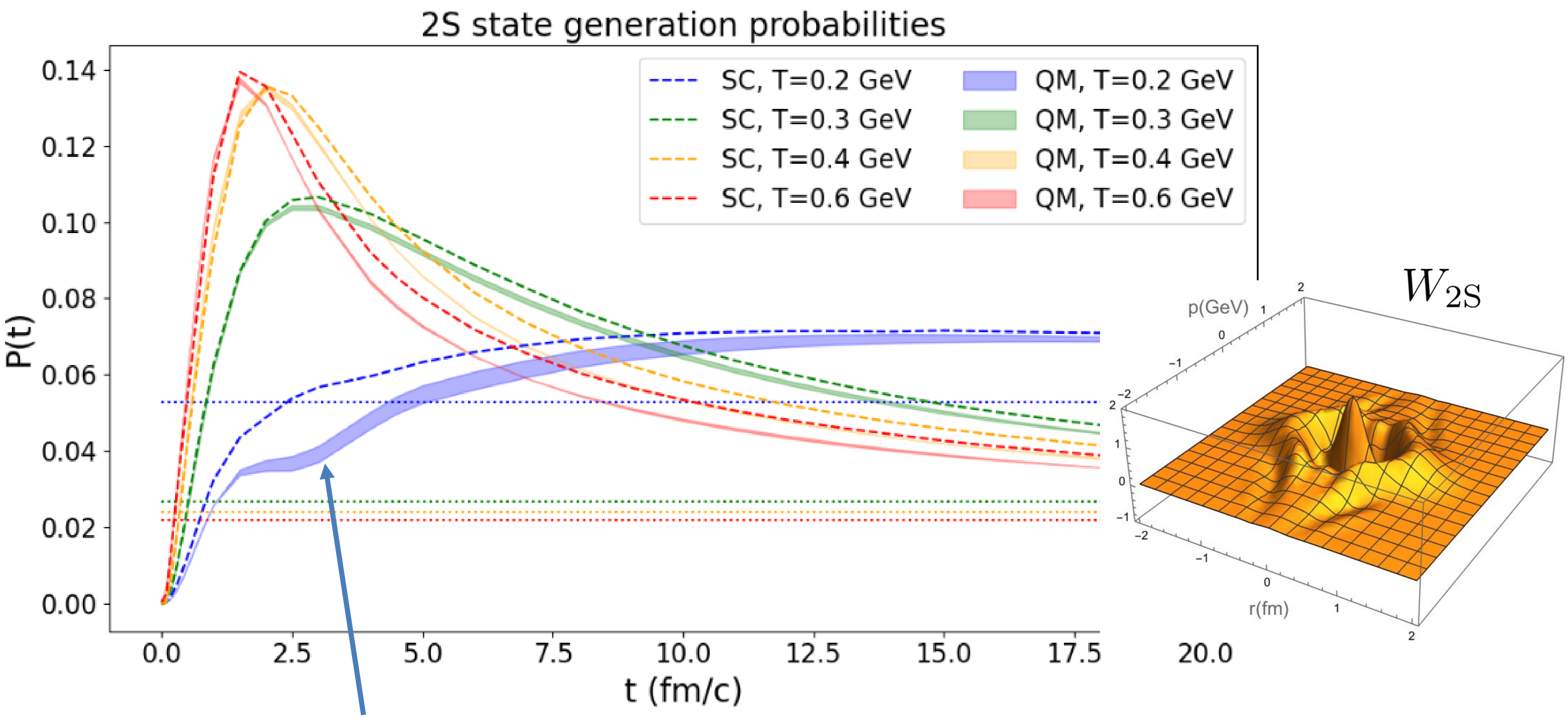


- Most significant disagreement for low T, around t = 2.5 fm/c (beginning of the classicalization)

Non-unitary evolution

Full coupling with the QGP

- More concrete observable : **(Re)generate 2S state**
- Good agreement of the SC calculation with the QME benchmark, especially at large T

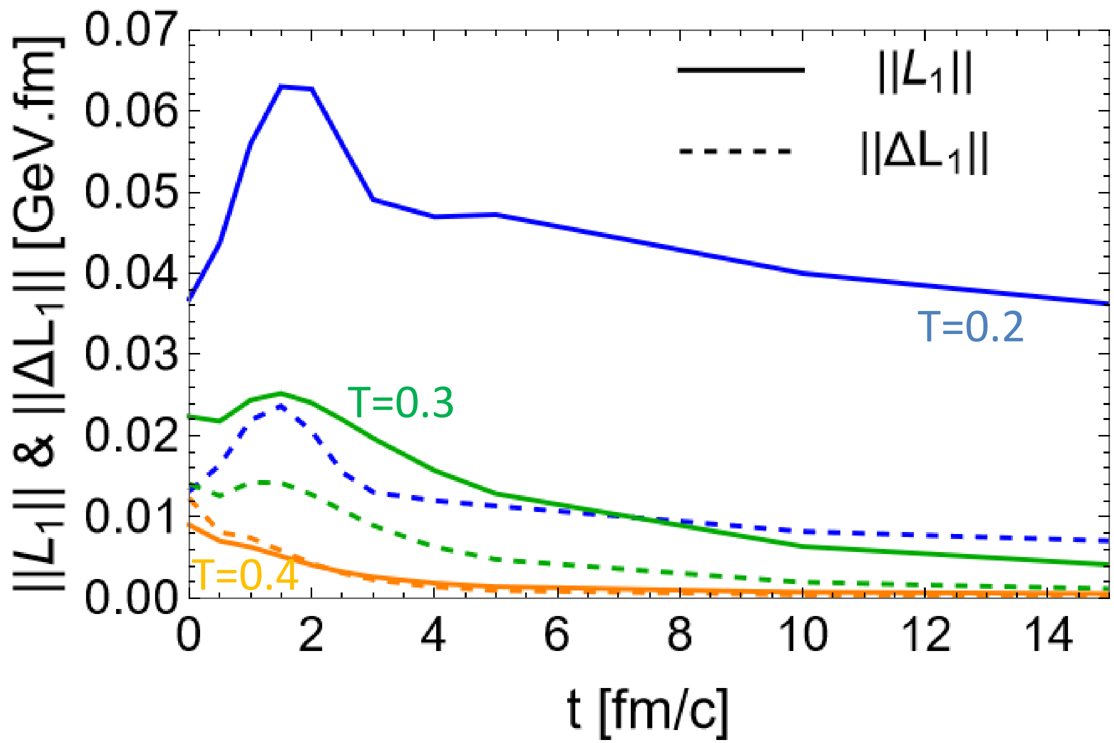


- Higher effect of the genuine interference quantum effects due to the mixture of positive and negative regions in W_{2S}.

Why does it work ?

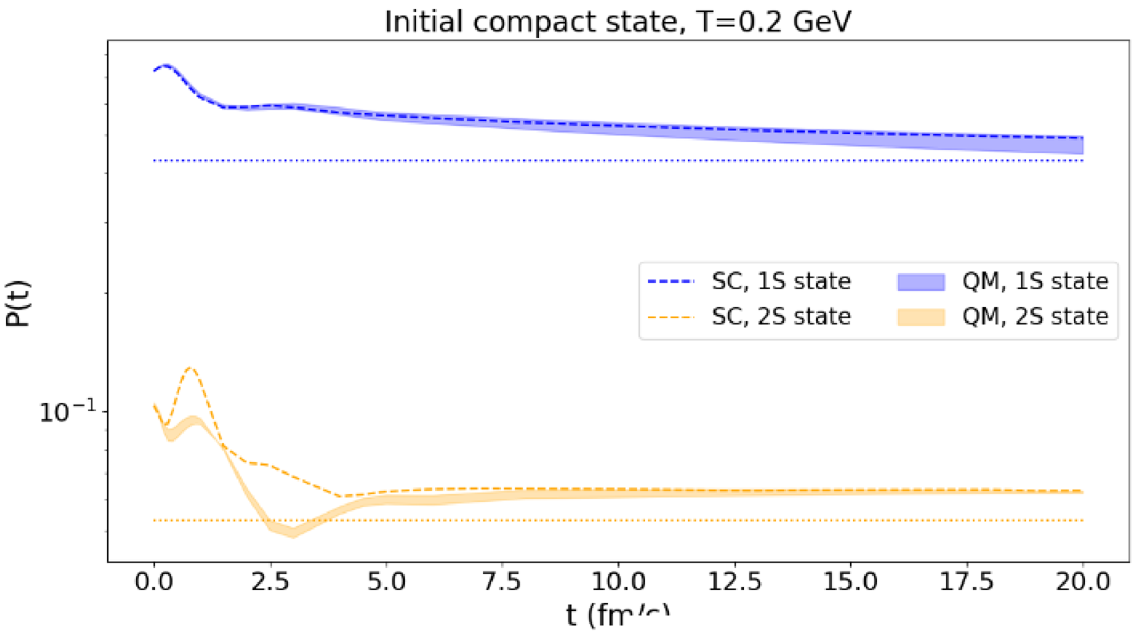
- When / why does it work ?
  - The unitary term : $\mathcal{L}_1[\rho] = [V,\rho] = \rho(s,s')(V(s) - V(s')) = V'(r)y + \mathcal{O}(y^3)$

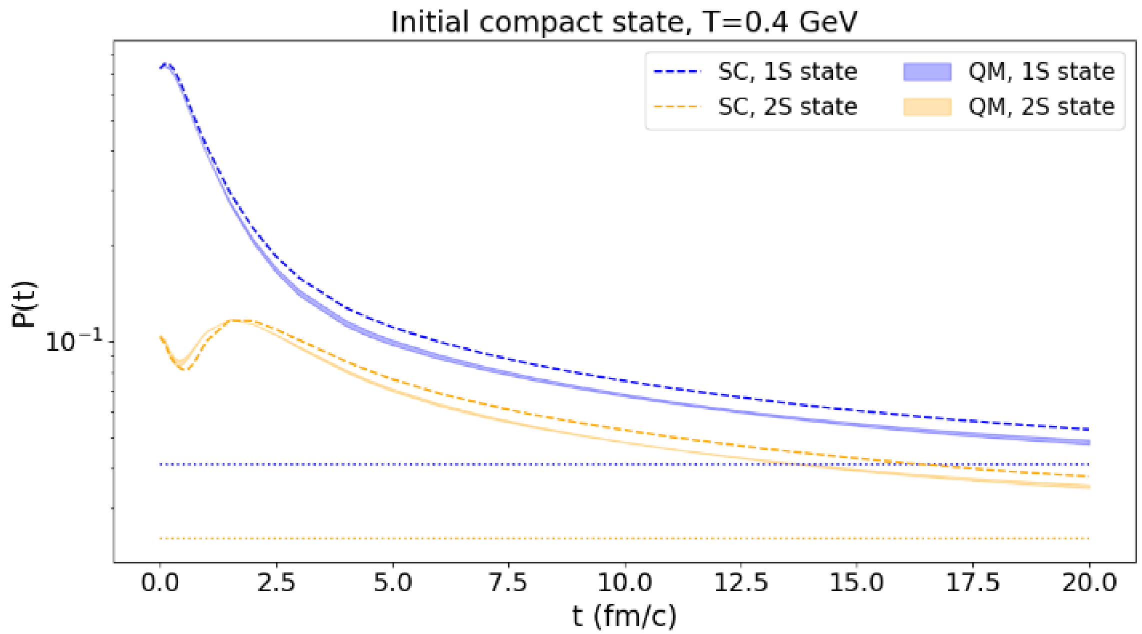
    Wigner-Moyal expansion, valid when y << variation scale of the real potential



➢ With increasing time, <y^2> decreases -> the de Broglie thermal length $\propto \sqrt{\frac{1}{Tm_Q}}$ and the Wigner-Moyal expansion works better and better.

Initial compact state





A buffet of Quarkonium dynamics

MENU Ô SAVOYARD

Entrée + Plat ou **Plat + Dessert** .... 24,00 €
Entrée + Plat + Dessert .... 32,00 €
Hors boisson
Non cumulable avec d'autres offres ou gratuités.

Entrée au choix

Friture d'ablettes sauce tartare
Moflet savoyard
Pâté croûte artisanal poulet citron bio
Petite salade Ô savoyard

The (semi) classical

Plat au choix

Tartiflette spéciale Ô Savoyard
Filet frais de féra grillé à la plancha, légumes frais (+ 2 €)

Fricassée de caïon
Tartare de bœuf charolais

Dessert au choix

Glace Alpage au Génépi
Soupe de fraises et glace vanille-framboise meringuée
Tarte aux myrtilles fraîches
Mojito tendresse citron

The sweet illusion of touching the truth

The spirit of the method…

$$P^{\Psi}(t) = \mathrm{Tr}\left[\hat{\rho}^{\Psi}_{Q\bar{Q}}\hat{\rho}_N(t))\right]$$

Simply taken at the end of the evolution (ideal world)

$$\hat{\rho}^{\Psi_i}_{Q\bar{Q}} = \sum_i |\Psi^i_{Q\bar{Q}}\rangle\langle\Psi^i_{Q\bar{Q}}|$$

$$\frac{d\hat{\rho}_N(t)}{dt} = -i\left[\hat{H}_N, \hat{\rho}_N(t)\right]$$

Various Quarkonia bound states (in vacuum)

Unfortunately… all N-body practionners know that **modelling the full system up to the last stage is quite challenging !** Issues of stability, energy conservation,…

Clear lesson from the « old » cascade and QMD codes for fragment formation

Replace « final » => « initial » + Sum of time steps and chop off at the appropriate time scale

$$P^{\Psi}(t) = P^{\Psi}(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$$

Caution : Not the usual decay rate

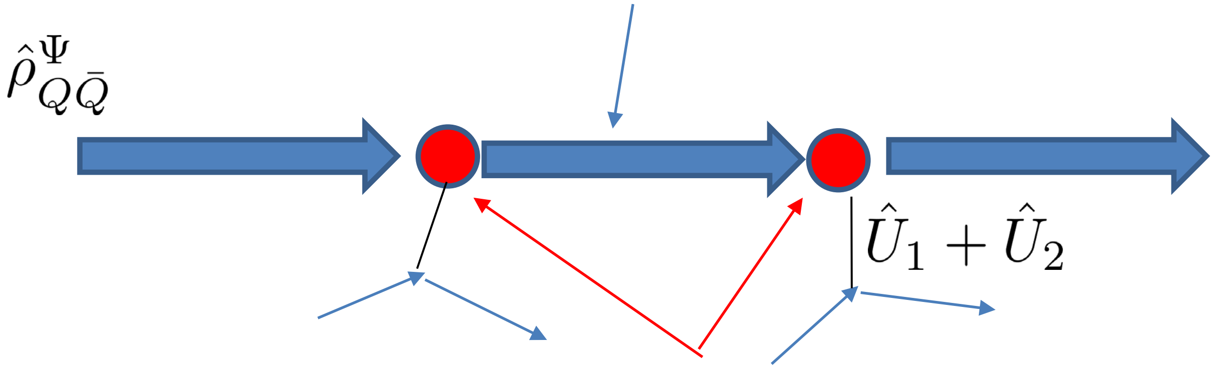
Convergence towards statistical equilibrium in a fixed temperature QGP recently demonstrated in arxiv 2302.14001

The spirit of the method…

Dealing with the dynamics ? **Von Neumann equation**

If eigenstates of the « internal » 2-body (QQbar) interaction

Free evolution => Γ=0



No « late time » evolution in this eigenbasis

Interaction with a 3rd body => modification of the $\hat{\rho}^{\Psi}_{Q\bar{Q}}$

$$\Rightarrow \ \ldots \ \Rightarrow \quad \Gamma^{\Psi}(t) = -iTr[\hat{\rho}^{\Psi}[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Total interaction of Q and Qbar with all light partons

Source of « destruction » ⇔ imaginary potential

Remler Formalism at work

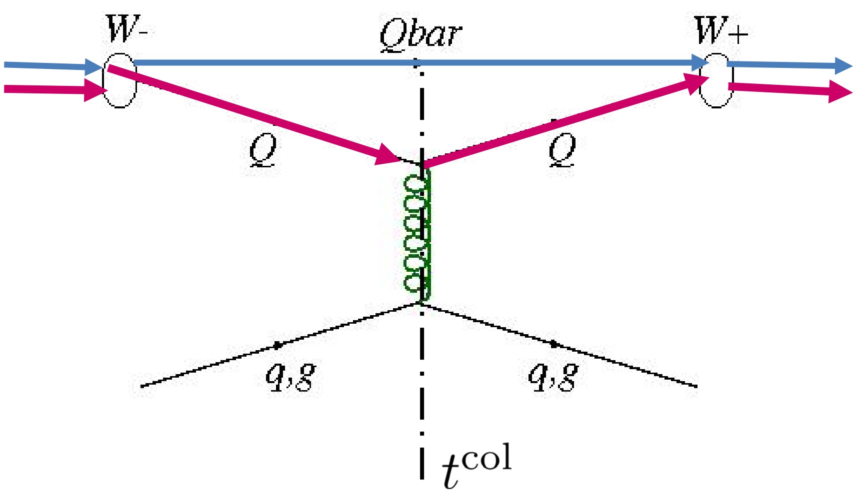
E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

Level of the modelling : **semi classical** for the Q-Qbar evolution => Wigner distributions instead of density operators

Combining the expression of the Wigner's distribution and substituting in the **effective rate equation** :

$$\Gamma^{\Psi}(t) \approx \sum_{i=1,2} \sum_{j\geq 3} \delta(t - t_{ij}^{\rm col}) \int \frac{d^3p_i d^3x_i}{h^3} \left[W_{Q\bar{Q}}^{\Psi}(p_1,x_1;p_2,x_2)\Big|_{t+\epsilon} - W_{Q\bar{Q}}^{\Psi}(p_1,x_1;p_2,x_2)\Big|_{t-\epsilon} \right]$$

- The quarkonia production in this model is a three body process; the HQs interact only by collisions with the QGP !!!
- The "details" of H_{int} between HQ and bulk partons are incorporated into the evolution of W_N after each collision / time step (nice feature for the MC simulations)
- Dissociation and recombination treated in the same scheme



Then: $P^{\Psi}(t) = P^{\Psi}(t_0) + \int_{t_0}^{t} \Gamma(t')dt'$

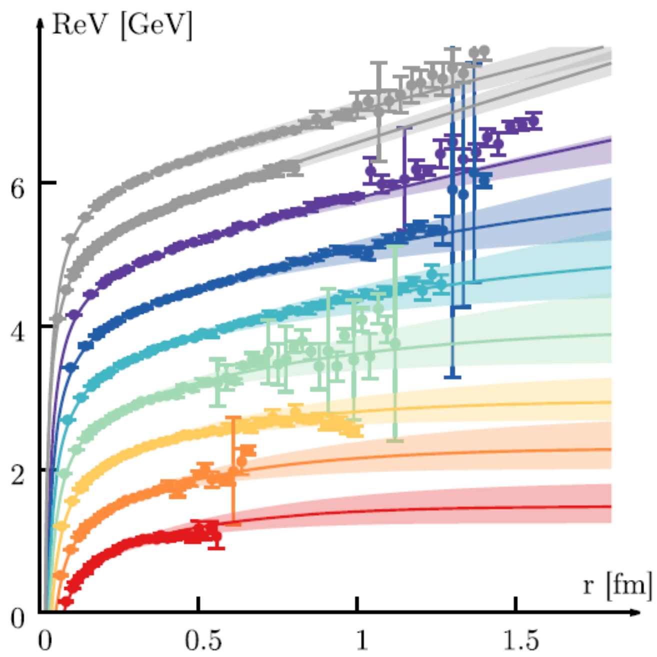
NB: Also possible to generate similar relations for differential rates

Interaction of HQ with the QGP are carried out by EPOS2+MC@HQ (good results for D and B mesons production) Eur. Phys. J. C (2016) 76:107

Subatech

Extension of the Remler formalism

- **Confining $Q\bar{Q}$ forces inside the MC evolution ; large impact on the # of close pairs… and correlated trajectories.**



(No internal potential in early applications dedicated to deuteron production in low energy AA collisions; advocated to be negligible… as only the « hot zone » was contributing

But for quarkonia, it turns out not to be the case => need for in-medium potential

D. Lafferty and A. Rothkopf,
PHYS. REV. D 101, 056010 (2020)

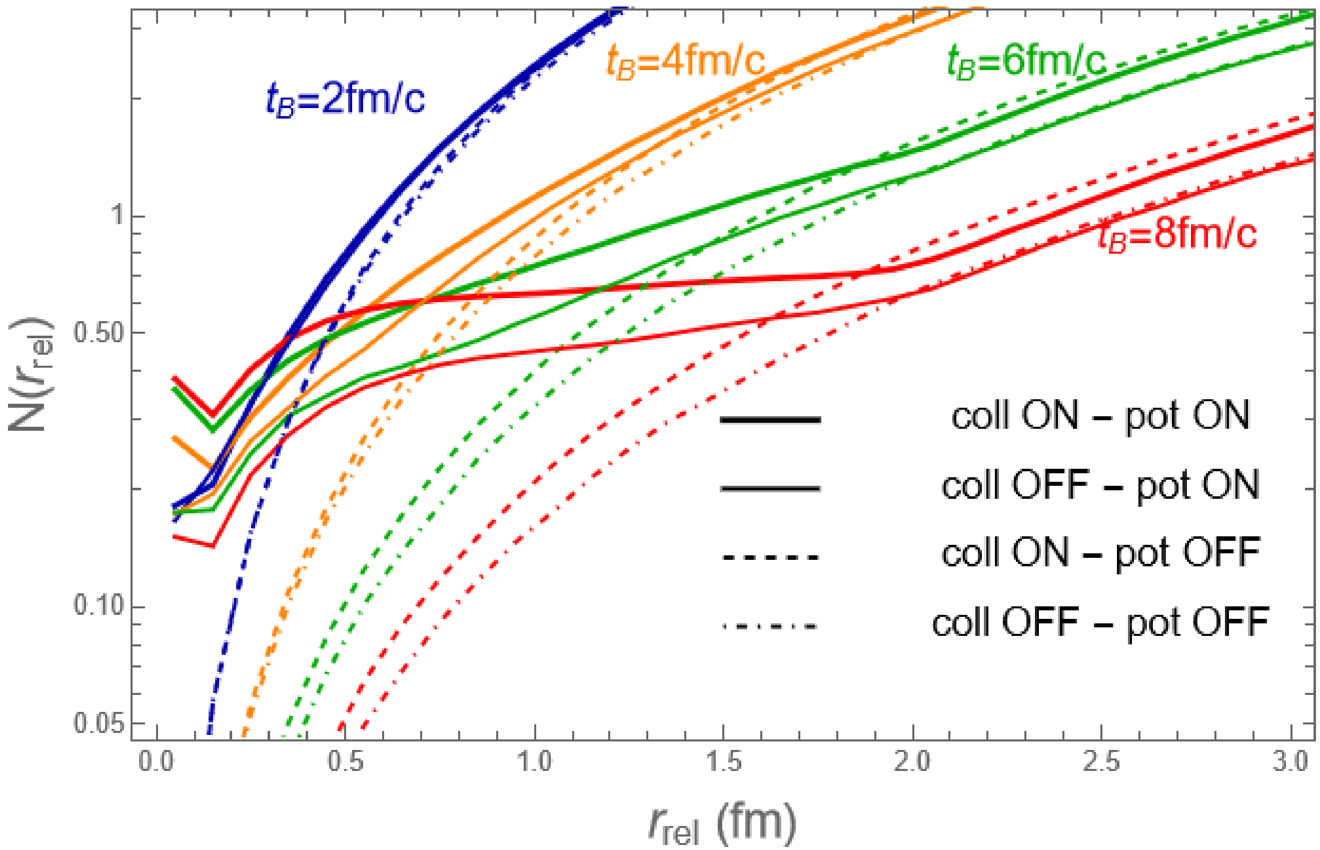
Correlated SC trajectories

Complicated relativistic N-body problem… Only stable at « not too high » p_T

The 3 layers of the numerical modelling

	Initial state		evolution
Background	EPOS initial condition +transverse positions of the NN collisions	→	Fluid dynamics
	↓		↓ {T(x),v(x)}
HQ level	Initialisation of c-cbar pairs according to FONLL[2] ⊗ EPOS positions	→	Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**
			↓
			Correlated SC trajectories

The dynamics of c-cbar correlation



- The c-cbar potential (« pot ON ») leads to a **huge increase of the c-cbar probability at close distance** at large times (not a random Poisson distribution !)…
- … Especially when the collisions with the QGP (« coll ») are switched ON as well

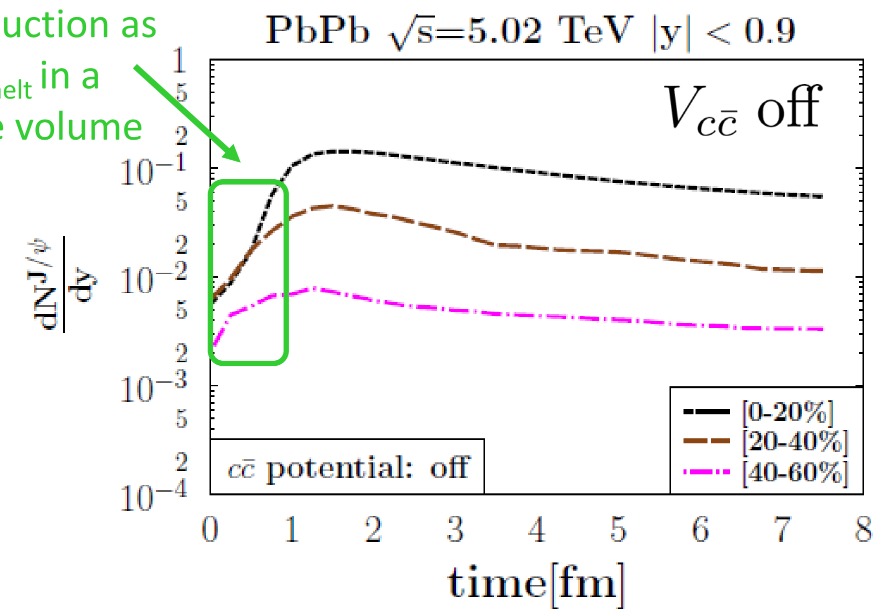
The 3 layers of the numerical modelling

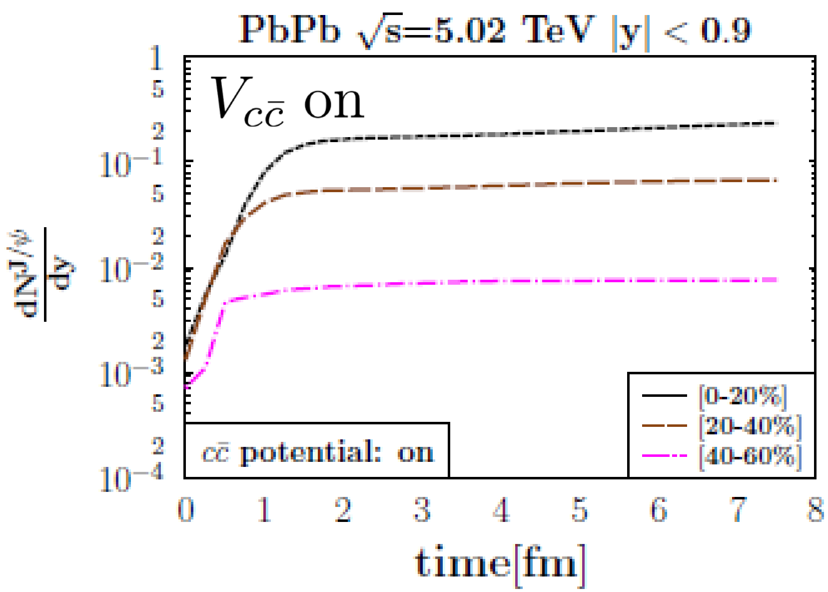
	Initial state		evolution
Background	EPOS initial condition +transverse positions of the NN collisions	→	Fluid dynamics
	↓		↓ {T(x),v(x)}
HQ level	Initialisation of c-cbar pairs according to FONLL[2] ⊗ EPOS positions	→	Evolution of HQ according to MC@HQ **+ c-cbar in-medium potential**
	↓		↓ Correlated SC trajectories
Quarkonia level	Initial J/ψ(T) P (t_0) when T(x) < T_{diss}	→	Evaluation of Γ according to improved Remler + Γ_{loc}.

We do not have J/ψ quasi particles in our approach, just correlated c-cbar trajectories

Results : J/ψ production vs time

Delayed initial production as $T>T_{melt}$ in a large volume

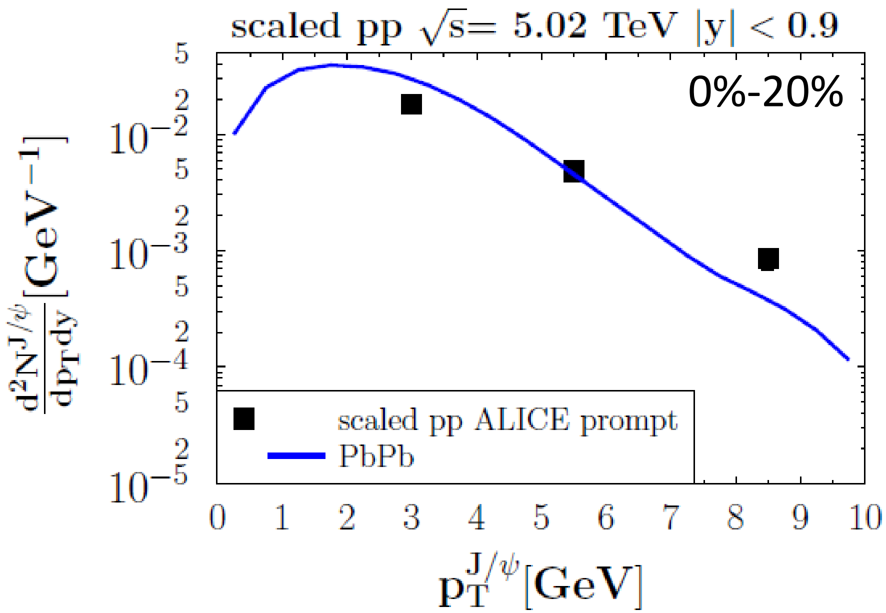


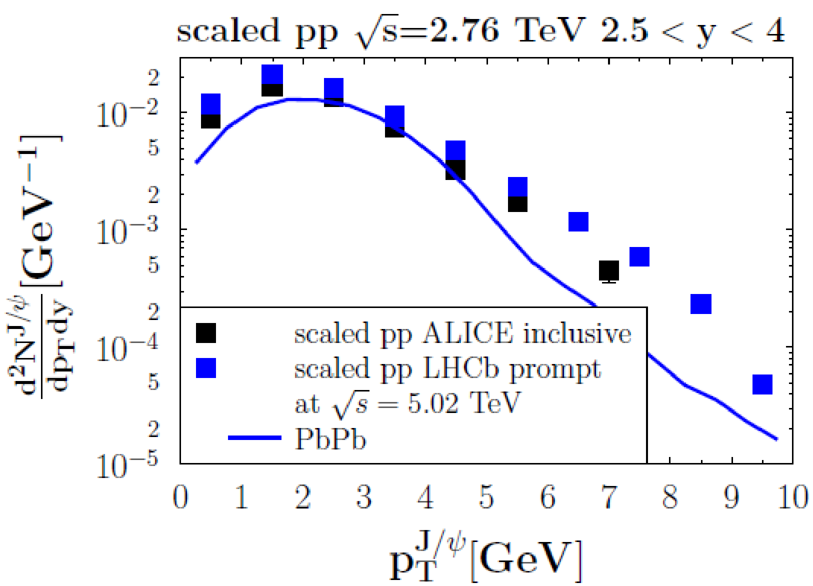


- Without interaction potential between c and cbar, the collisions with the medium manage to destroy the native J/ψ (left)
- With the interaction potential between c and cbar « on », one observes a steady rate of J/ψ creation (increase of Γ^{col}, increase of Γ^{local})… No adiabaticity, but **no instantaneous formation** either.

Subatech

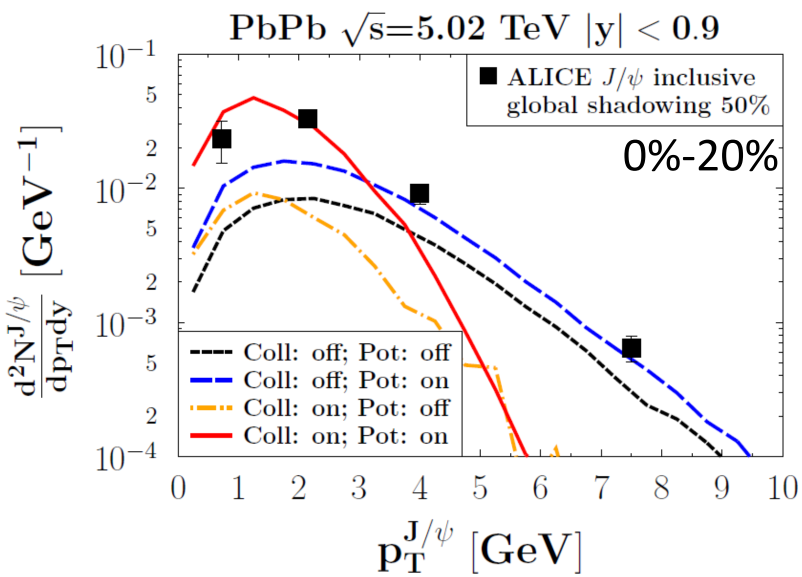
Results : J/ψ production vs p_T

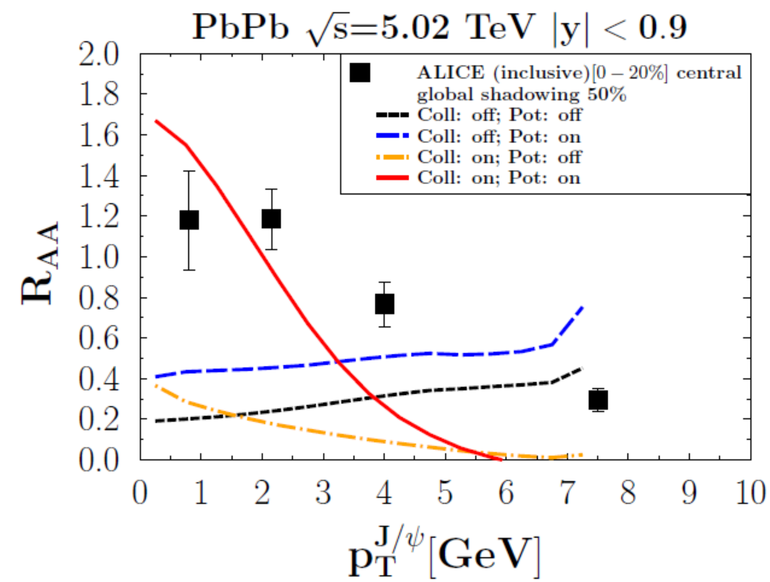


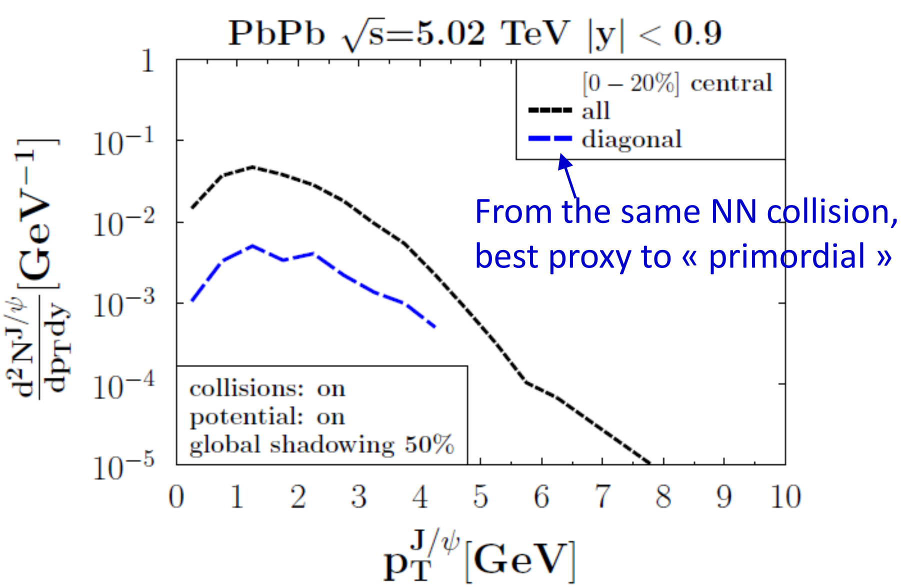


- **Equivalent pp production** (the denominator of the R_{AA}) : c-cbar according to FONLL[2] without any correlation, then coalescence with the Wigner distribution.
- No feed-down from higher states (to be implemented)
- Acceptable for p_T < 5 GeV/c, but deviations for higher p_T.
- To investigate : more appropriate scheme for c-cbar production, including c-cbar correlation : EPOS4

Results : J/ψ production vs p_T





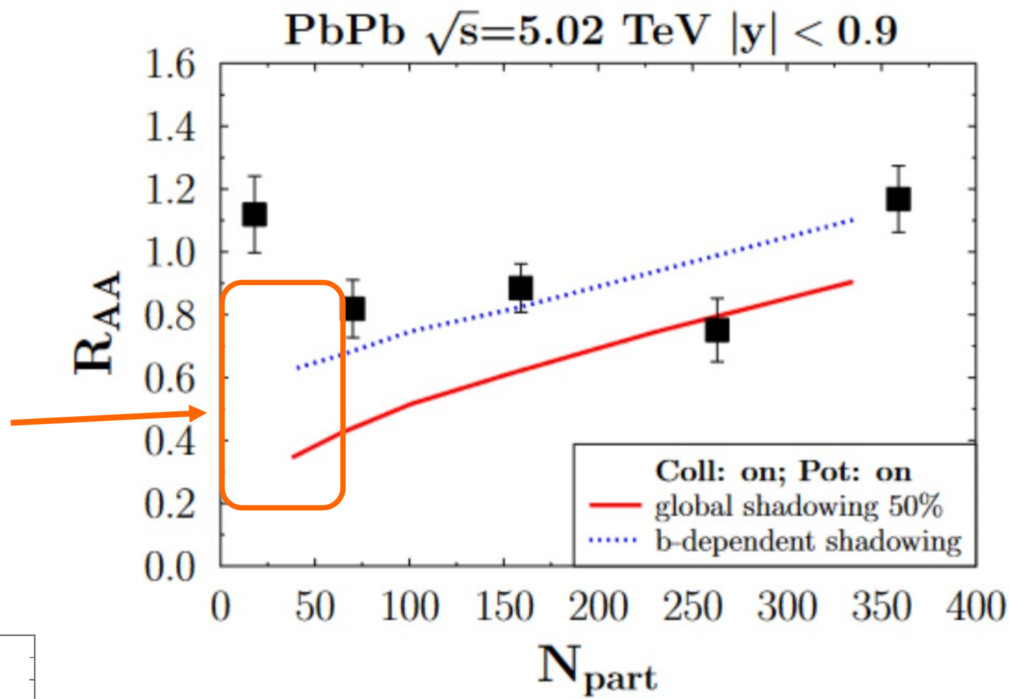


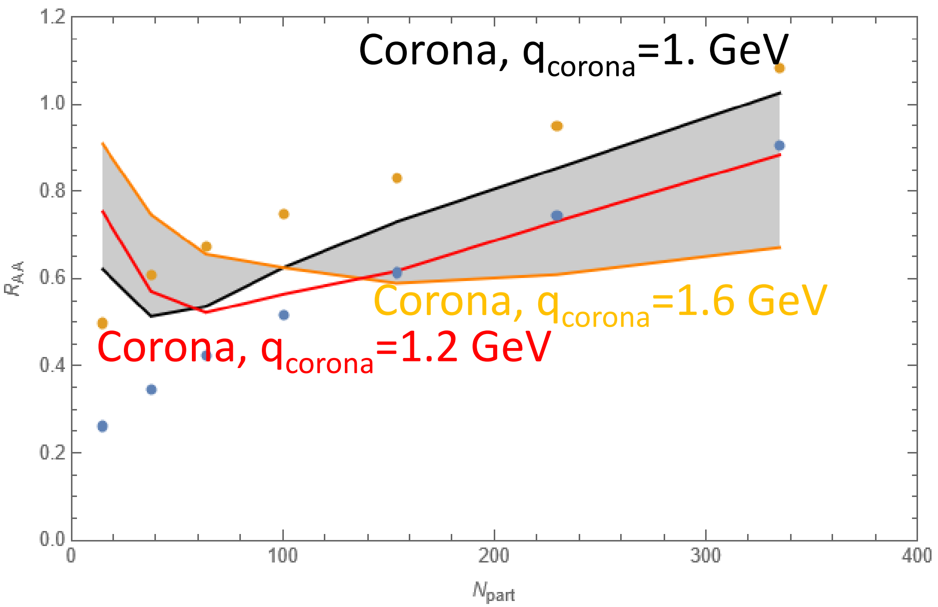
- Dynamical recombination is quite effective at low p_T
- At higher p_T, we are missing J/ψ as compared to the experimental value.
- Several possible reasons, under investigation:
  - in terms of transport model : « primordial too much suppressed »
  - lack of c-cbar correlation in the IS
  - …

R_{AA} vs N_{part}

Caveat : too crude modelling of the thermalization in the bulk… assumed to happen after 0.35 fm/c independent of the centrality

=> c and cbar created at t≈0 have the time to diffuse away => reduction of the production

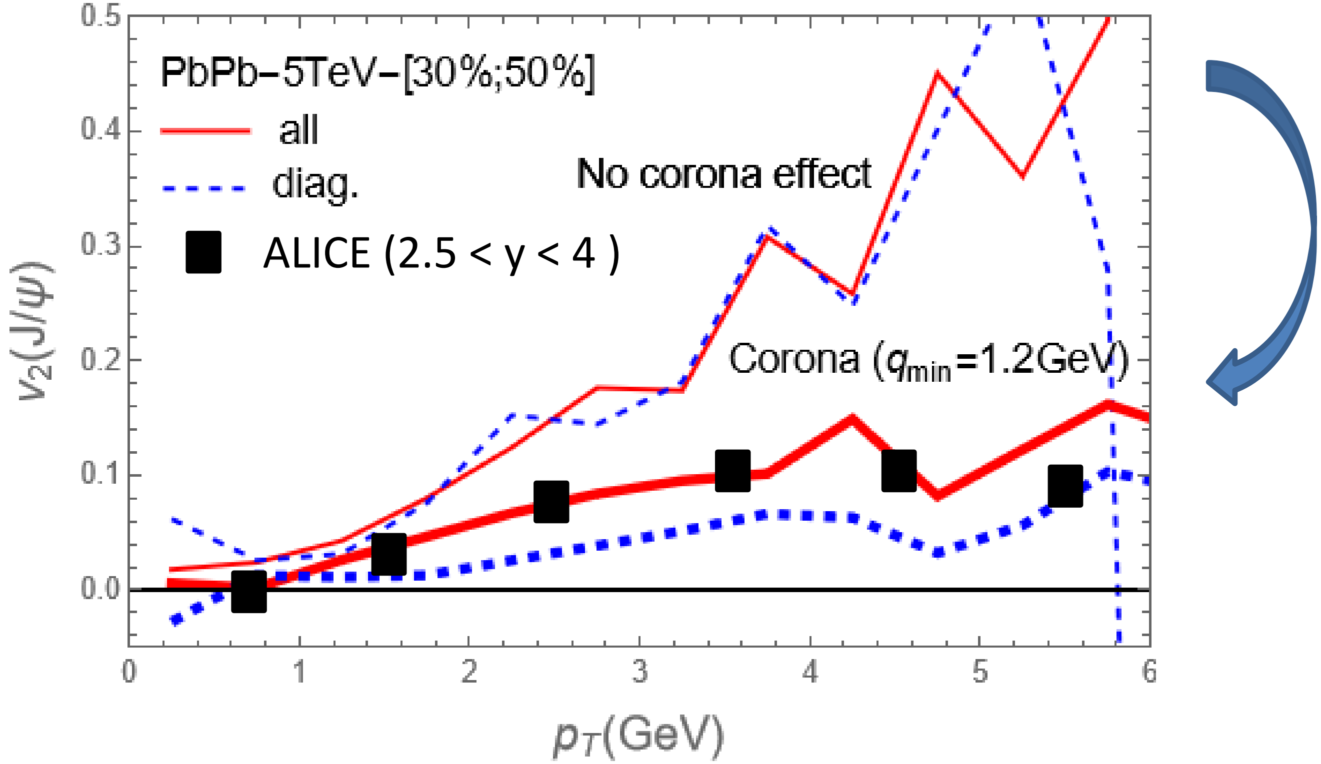




One possible solution : core – corona model for c-quarks : c-quarks with momentum transfer < q_{corona} are considered to combine -> quarkonia as in vacuum…

Optimal value : Corona, q_{corona}=1.2 GeV

Results : J/ψ v_2



- v_2 excess as compared to experimental data (**late formation** of the J/ψ due to binding potential under restoration)
- Without corona, the « diagonal » contribution shows no difference wrt the full production, what is a bit conter-intuitive
- Corona has large effect on v2, even with moderate q_{thresh}. As the corona mostly affects the diagonal part, one recovers $v_2^{\mathrm{diag}} < v_2^{\mathrm{all}}$

Next steps

- The Semi-classical approximation in the genuine QCD case...

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\\ \mathcal{D}_o(\mathbf{r}_{rel},\mathbf{r}'_{rel},t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet density matrix

octet density matrix

singlet-octet transitions

$$P_s = \mathrm{Wi}(\mathcal{D}_f) \qquad P_o = (N_c^2-1)\mathrm{Wi}(\mathcal{D}_o)$$

Wigner Transform

Singlet evolution

$$\mathrm{KK_s} P_s(\vec{p},\vec{r}) = -2C_F\Gamma(r)\left(P_s - \frac{P_o}{N_c^2-1}\right) + \cdots$$

Klein Kramers (⇔ Langevin):

$$\partial_t + \frac{\vec{p}}{m}\cdot\nabla_r - C_F\vec{F}(r)\cdot\nabla_p - \frac{1}{2}\nabla_p\cdot\bar{\bar{\eta}}_s(r)\cdot\left(\nabla_p + \frac{\vec{p}}{mT}\right)P_s(r,p)$$

Easily modelled in MC evolution (Brownian motion)

QCD SC evolution

- The Semi-classical approximation in the genuine QCD case...

$$P_s = \mathrm{Wi}(\mathcal{D}_f) \qquad P_o = (N_c^2 - 1)\mathrm{Wi}(\mathcal{D}_o)$$

Wigner Transform

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\\ \mathcal{D}_o(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

octet density matrix

singlet-octet transitions

Singlet evolution

$$\mathrm{KK_s} P_s(\vec{p}, \vec{r}) = -2C_F\Gamma(r)\left(P_s - \frac{P_o}{N_c^2-1}\right) + \cdots$$

Klein Kramers (⇔ Langevin):

Easily modelled in MC evolution (Brownian motion)

Loss term -> octet and gain term from octet, with rate Γ(r) (aka color rotation)

Easily modelled in MC with conversion probability

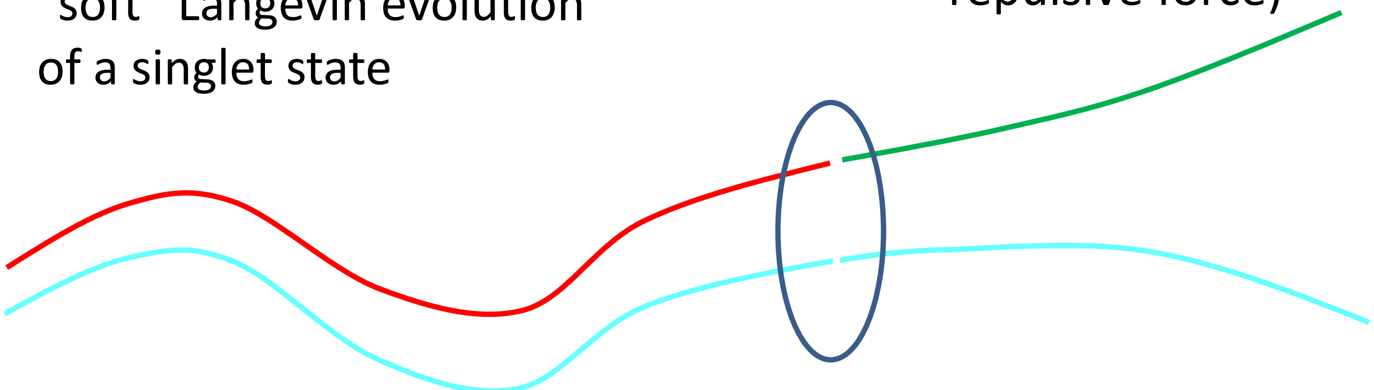
Octet evolution: $$\mathrm{KK_o} P_o(\vec{p}, \vec{r}) = -\frac{1}{N_c}\Gamma(r)\left(P_o - (N_c^2 - 1)P_s\right) + \cdots$$

QCD SC evolution (MC)

Typical evolution :

"soft" Langevin evolution of an octet state (with repulsive force)

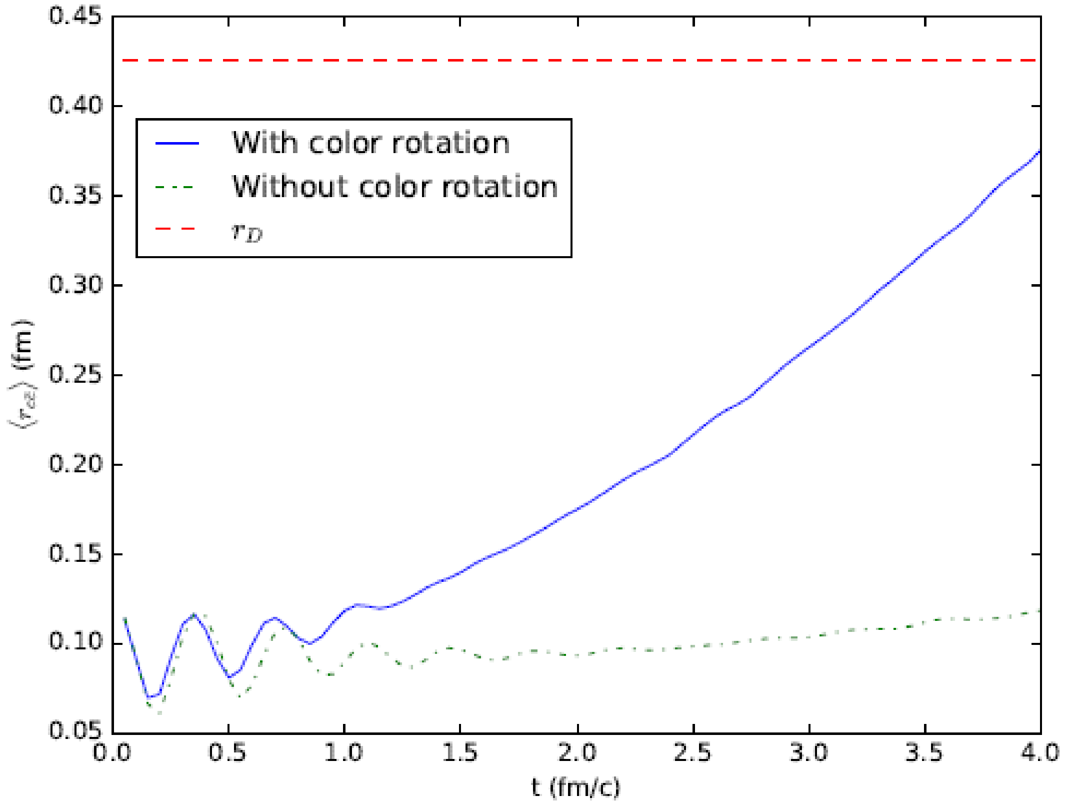
"soft" Langevin evolution of a singlet state



Conversion -> octet

QCD SC evolution

J-P.Blaizot & M.A. Escobedo, JHEP 2018.6 (2018)



Color rotation (neglected in all (to my knowledge) SC calculations drastically increase the delocalization of c-cbar states

However, some terms have been neglected in the calculation…

QCD SC evolution

- The Semi-classical approximation in the genuine QCD case…

$$P_s = \mathrm{Wi}(\mathcal{D}_{\int}) \qquad P_o = (N_c^2 - 1)\mathrm{Wi}(\mathcal{D}_o)$$

Wigner Transform

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s \\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t) \\ \mathcal{D}_o(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\end{pmatrix}$$

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so} \\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet density matrix

octet density matrix

singlet-octet transitions

Singlet evolution

$$\mathrm{KK_s} P_s(\vec{p}, \vec{r}) = -2C_F\Gamma(r)\left(P_s - \frac{P_o}{N_c^2-1}\right) + \frac{1}{2}\nabla_p \cdot \bar{\bar{\eta}}_s(0) \cdot \left(\nabla_p + \frac{\vec{p}}{mT}\right)\frac{P_o}{N_c^2-1}$$

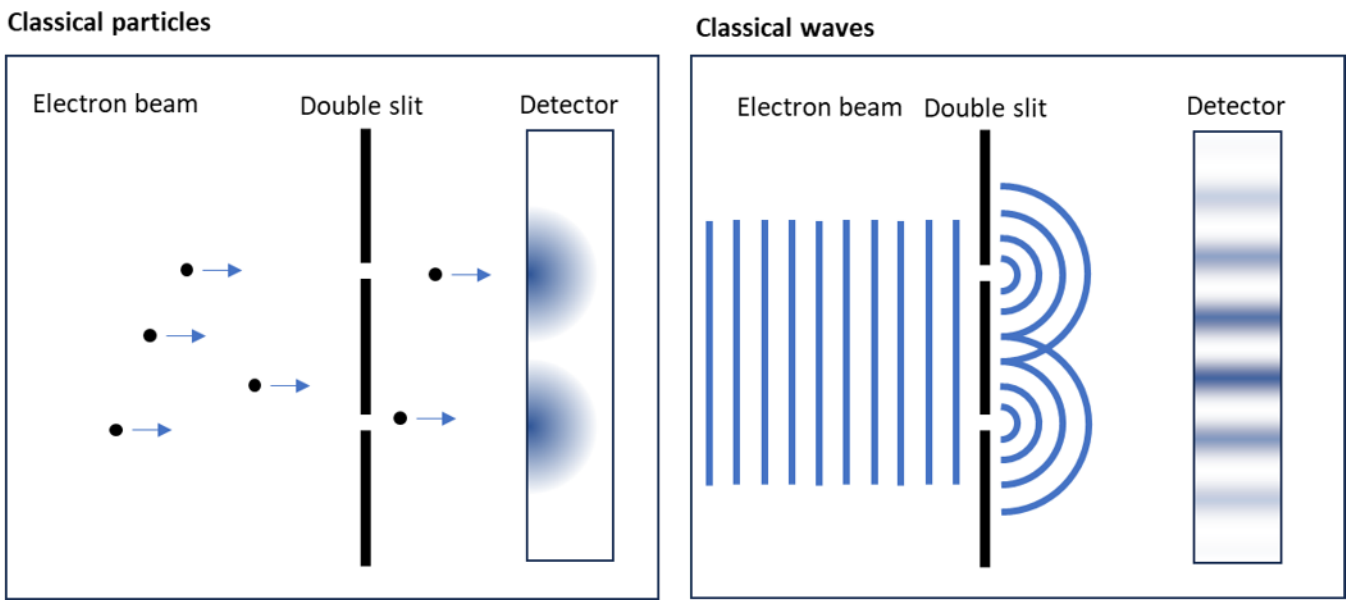
Gain from octet through fluctuation and dissipation !

Klein Kramers (⇔ Langevin):

$$\partial_t + \frac{\vec{p}}{m}\cdot\nabla_r - C_F\vec{F}(r)\cdot\nabla_p - \frac{1}{2}\nabla_p\cdot\bar{\bar{\eta}}_s(r)\cdot\left(\nabla_p + \frac{\vec{p}}{mT}\right)P_s(r,p)$$

Multi-species Fokker-Planck equation…. No analytical solution even for simple expressions of the transport coefficient ; no simple MC modelling to my knowledge

Color is quantum physics !



However, truncation of the Wigner-Moyal equation at the 1rst order in $\hbar$ does not allow to reproduce the interferences in a double slit experiment

One can think of other advocated methods to describe the evolution of the colored quarks like Wong's equations… However, the Casimir of the global color will not be either in a singlet or an octet representation => potential intermediate between binding and unbinding ☹

Summary

- In the (unphysical) abelian representation, the Semi-classical approximation allows to reproduce the results from the exact Quantum Master Equation relevant for phenomenology.

- This is due to the coupling with the heat bath which "classicalizes" the subsystem.

- The genuine QCD case is however one level higher in conceptual difficulty; color is quantum; however preliminary results indicate it needs to be taken into account…

- … and we will benefit from the benchmark of the QME