Factorization for J/ψ leptoproduction at small transverse momentum

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Quarkonia as Tools 2025
Aussois

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Particle Physics Group

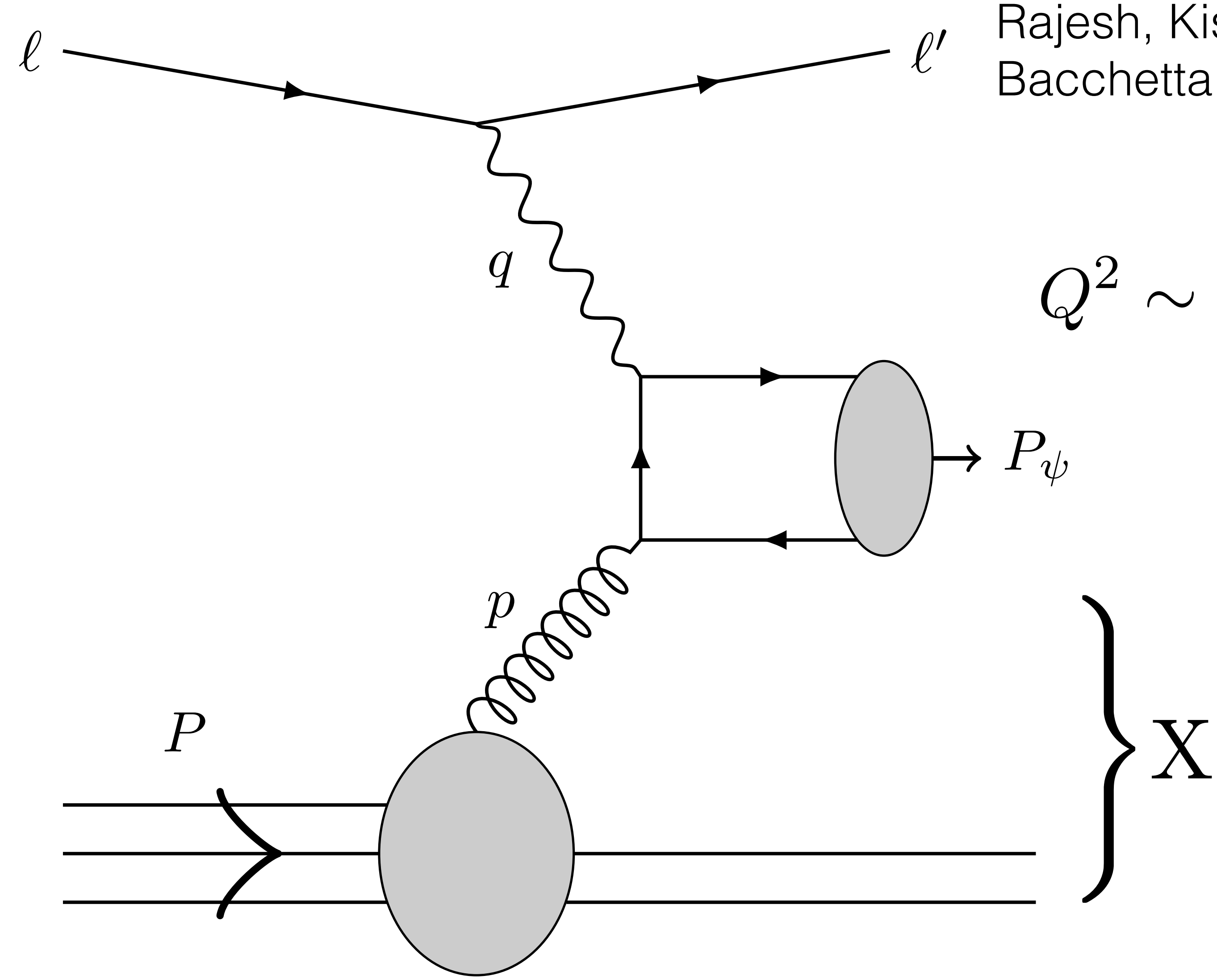
$\ell + p \to \ell + J/\psi + X$ at leading order

Rajesh, Kishore, Mukherjee (2018)
Bacchetta, Boer, Pisano, PT (2018)

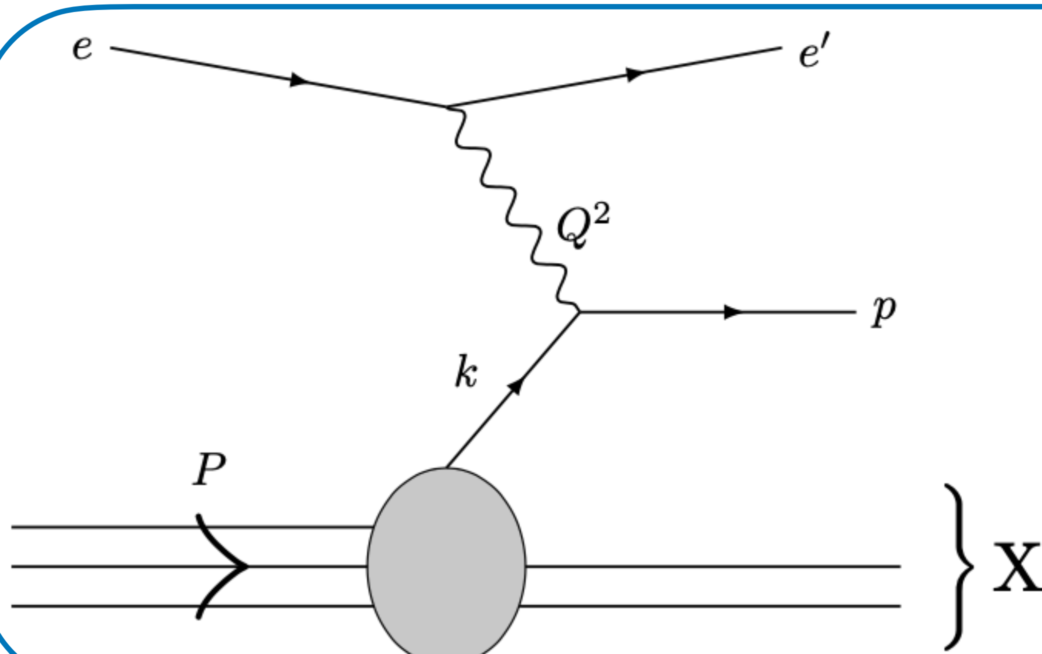
$$y = \frac{P \cdot q}{P \cdot l}$$

$$Q^2 = x_B y s$$

$$z = \frac{P \cdot P_\psi}{P \cdot q}$$

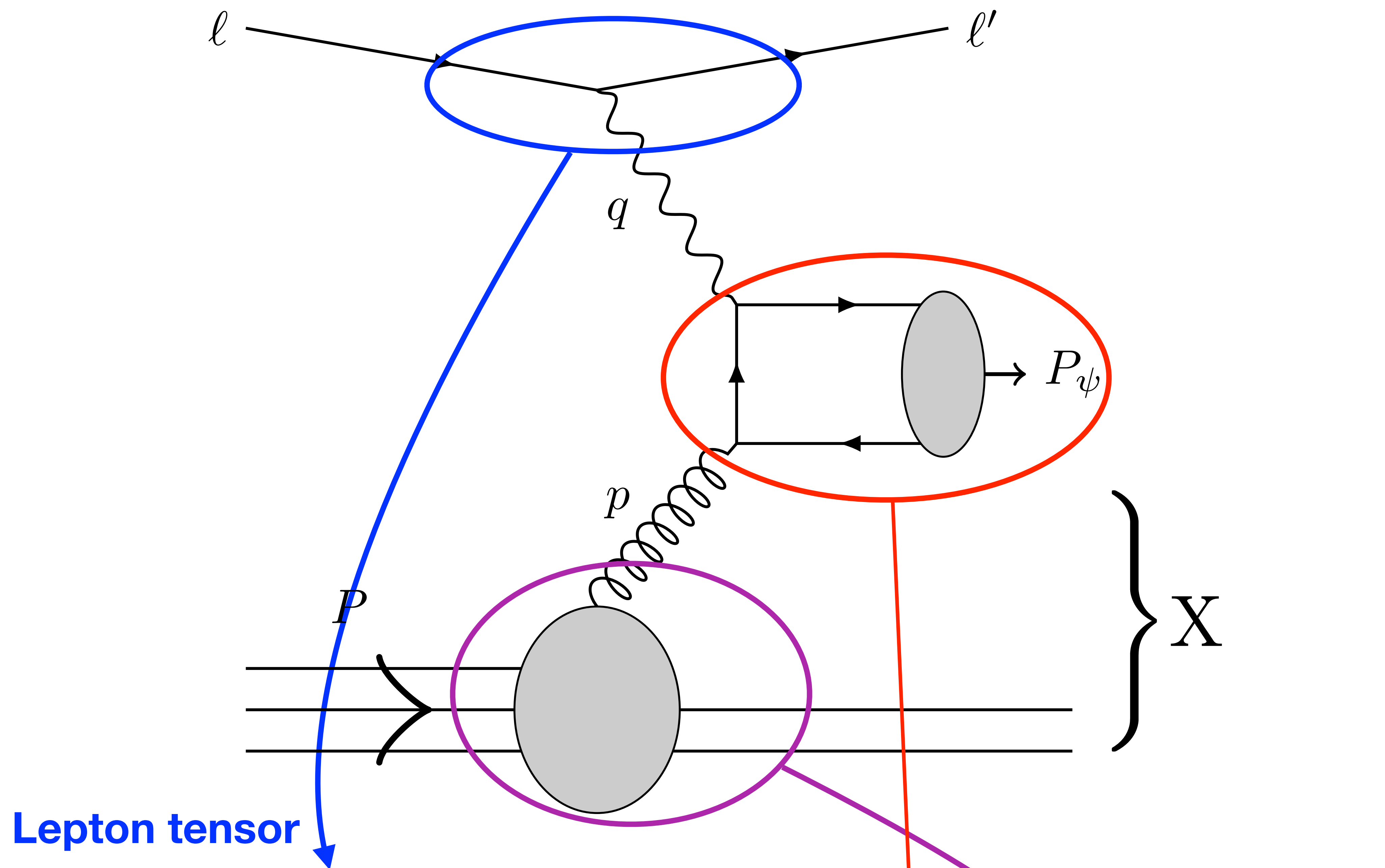


$$Q^2 \sim M_\psi^2 \gg P_{\psi\perp}^2$$



Idea: apply Transverse-Momentum-Dependent (TMD) factorization for Semi-Inclusive DIS (SIDIS) to production of quarkonium, in order to probe gluon TMD PDFs.

$\ell + p \to \ell + J/\psi + X$ at leading order



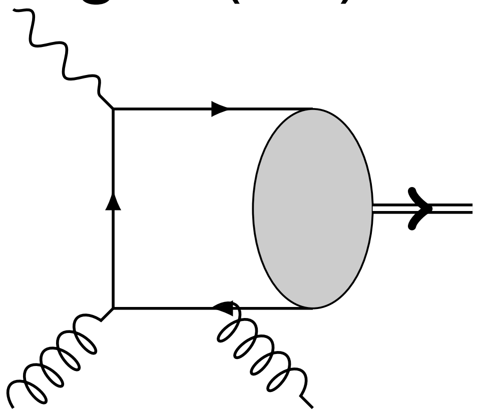
$$d\sigma = L \otimes |M|^2 \otimes \Gamma$$

$$= L \otimes \sum_{[i]} \langle \mathcal{O}_{[i]} \rangle \cdot |\mathcal{H}_{[i]}|^2 \otimes \Gamma$$

LDME Hard part

Hard part: non-relativistic QCD (NRQCD)

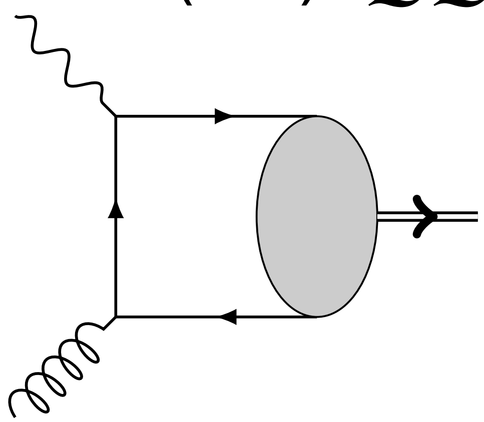
Color Singlet (CS): colorless bound state in quantum numbers of J/ψ



$$\langle \mathcal{O}_1^{J/\psi}(^3S_1) \rangle \quad \sim v^0$$

$$v_c^2 \approx 0.3$$
$$v_b^2 \approx 0.1$$

Color Octet (CO): $Q\bar{Q}$ pair in some colored excited state, hadronizes later



$$\langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle \quad \sim v^3$$

$$\langle \mathcal{O}_8^{J/\psi}(^3P_J) \rangle \quad \sim v^4$$

CS mechanism requires emission of additional hard gluon (Landau-Yang theorem) -> is suppressed by power P_ψ^2/Q^2. Typical factorization established at leading power -> provisionally only CO mechanism.

Bodwin, Braaten and Lepage (1995)
Fleming, Mehen (1998)

$\ell + p \to \ell + J/\psi + X$ at leading order

Final cross section can be written in terms of structure functions, similar to SIDIS:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}^2\mathbf{p}_T} = \frac{\alpha^2}{yQ^2}\left\{[1+(1-y)^2]\,\mathcal{F}_{UU,T} + 4(1-y)\,\mathcal{F}_{UU,L} + (1-y)\,\cos 2\phi_\psi\,\mathcal{F}_{UU}^{\cos 2\phi_\psi}\right\}$$

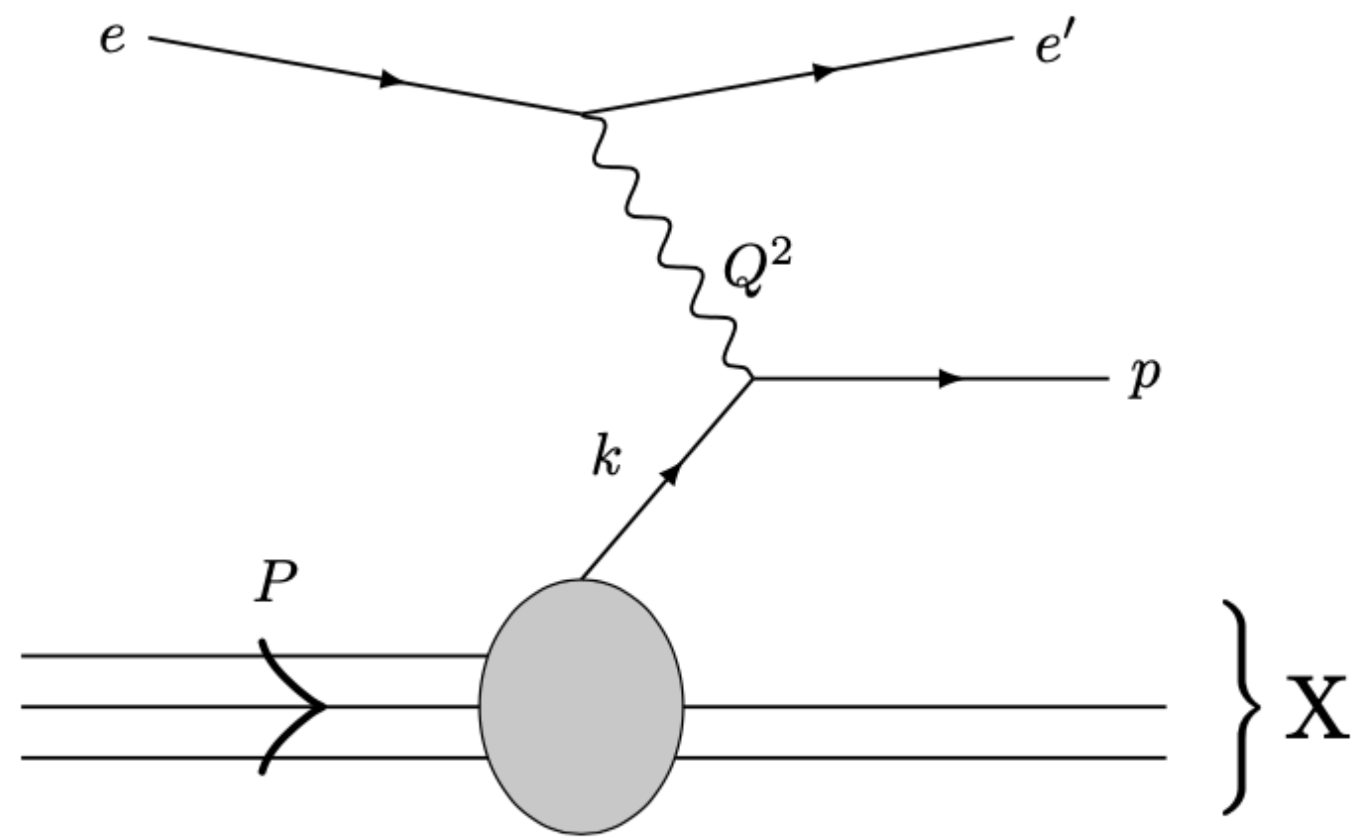
$$\mathcal{F}_{UU,T} = \frac{2\pi^2\alpha_s e_c^2}{M_\psi(M_\psi^2+Q^2)}\left[\langle 0|\mathcal{O}(^1S_0^{[8]})|0\rangle + 4\,\frac{(7M_\psi^4+2M_\psi^2Q^2+3Q^4)}{M_\psi^2(M_\psi^2+Q^2)^2}\langle 0|\mathcal{O}(^3P_0^{[8]})|0\rangle\right] f_1^g(x,p_T^2)$$

LDME **LDME**

Unpolarized gluon TMD carrying all the transverse momentum dependence

Why do we need to go further?

Fixed-order calculations are not sufficient. Gluon radiation is enhanced by large logarithms and needs to be treated at all perturbative orders (resummation).



Example of SIDIS

If $p_\perp \ll Q$

↪ need to resum large logs $\ln \frac{Q^2}{p_\perp^2}$

↪ $p_\perp$ can be intrinsic

Transverse momentum dependent (TMD) factorization

$$\sigma^{\gamma^* p \to h+X} = \hat{\sigma}(Q^2) \otimes f(x, k_\perp; Q^2) \otimes D(x, p_\perp; Q^2)$$

partonic cross section — TMD PDFs — TMD FFs

Collins, Soper, Sterman (`85-`89); Ji, Ma, Yuan (2005); Collins (2011)

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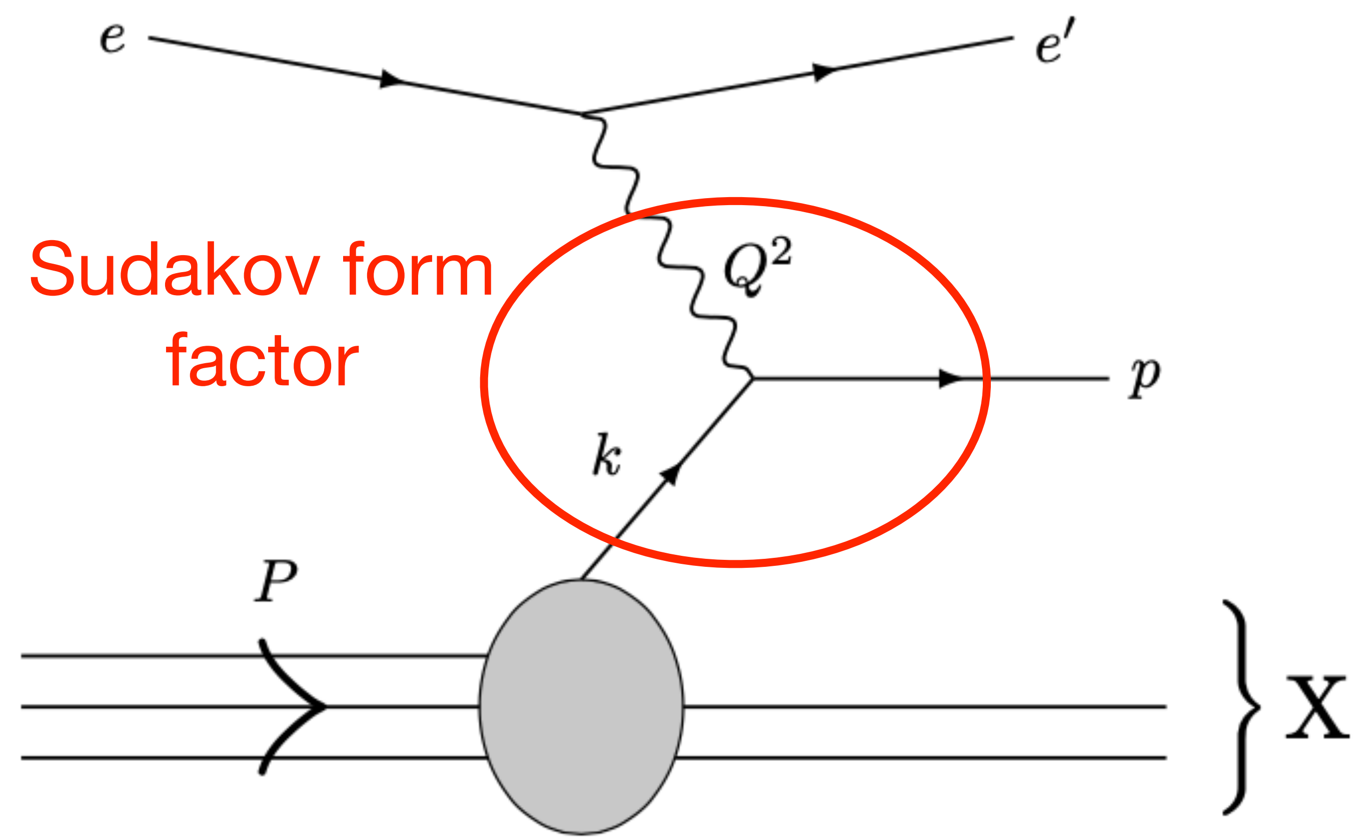

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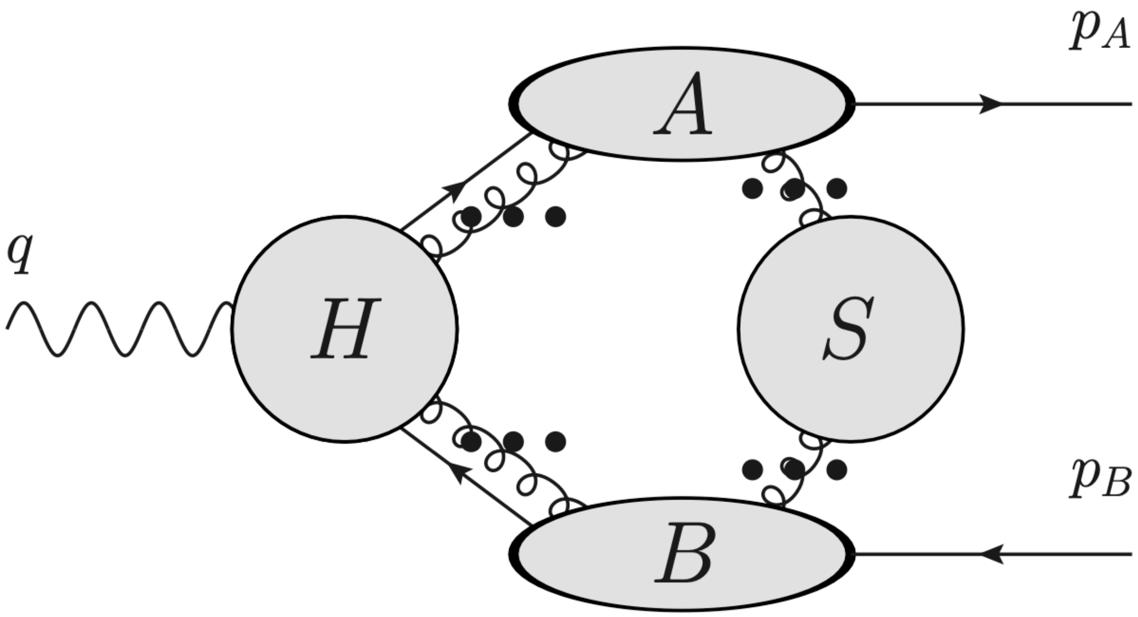
Sudakov form factor



Essential ingredient of factorization

$$-q^2 = Q^2 \gg p_A^2 \sim p_B^2$$

DIS, SIDIS, DY…

At all orders in perturbation theory, the dominant contribution factorizes in a hard part, a soft factor, and two collinear parts.

Dominant = up to *power corrections* $\mathcal{O}(\lambda)$ with $\lambda^2 \sim p_A^2/Q^2 \sim p_B^2/Q^2$

$$F(Q^2, p_A^2, p_B^2) = H(Q^2, \mu^2) \otimes A(p_A^2, \mu^2) \otimes B(p_B^2, \mu^2) \otimes S(p_A^2 p_B^2/Q^2, \mu^2)$$

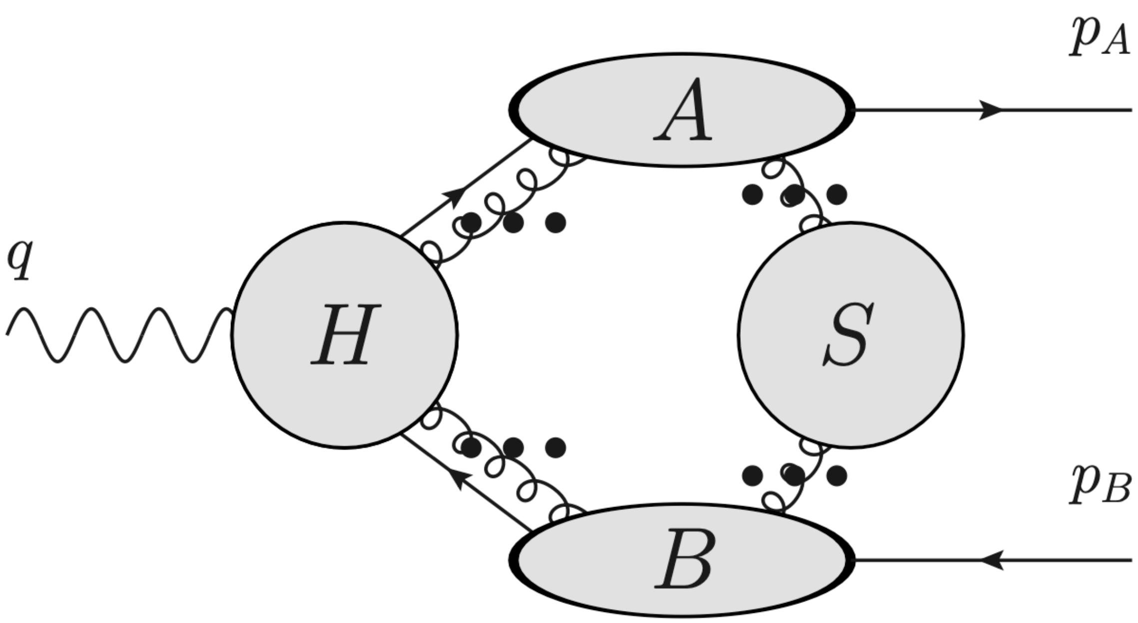
quarks/gluons collinear to p_A^μ, p_B^μ

virtual corrections w/o soft or collinear enhancement

soft gluon radiation

See, e.g.: Bertone, *PhD Course,* 2019; Collins, *Sudakov form factors*, 2003; Sterman, *TASI lectures*, 1995

Soft-Collinear Effective Theory (SCET)



Work with operators that directly reflect the factorized structure

Determine effective couplings (=Wilson coefficients) by comparing with full QCD à la operator product expansion

$$\lim_{x\to 0} \hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0) = \sum_i C_i(x)\hat{\mathcal{A}}_i(0) + \text{power corrections}$$

QCD **SCET**

Effective theory needs to reproduce long-distance structure of QCD order-by-order

Bauer, Fleming, Luke, Pirjol, Stewart (2000-2002)
See, e.g.: Becher, Broggio, Ferroglia, 2014

Factorization theorem for $\ell + p \to \ell + J/\psi + X$

Differential cross section:

$$\frac{d^5\sigma}{dx_B\, dy\, dz\, d^2\boldsymbol{P}_{\psi\perp}} = \frac{\pi\alpha_{em}^2 e_c^2}{2Q^4}\frac{y}{z}\frac{1}{2\gamma_\psi - 1}L_{\mu\nu}W^{\mu\nu}$$

Hadron operator:

$$W^{\mu\nu} = \sum\!\!\!\!\!\!\!\int_X \delta^{(4)}\big(P_N + q - P_\psi - P_X\big)\langle N|J^{\mu\dagger}(0)|J/\psi, X\rangle\langle J/\psi, X|J^\nu(0)|N\rangle$$

$$= \sum\!\!\!\!\!\!\!\int_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4} e^{iq\cdot b}\langle N|J^{\mu\dagger}(b)|J/\psi, X\rangle\langle J/\psi, X|J^\nu(0)|N\rangle$$

Instead of standard current operator $J^\mu = \bar{\psi}\gamma^\mu\psi$, introduce a current build from SCET and NRQCD operators

Echevarria, Romera, PT, 2024

Factorization theorem for $\ell + p \to \ell + J/\psi + X$

$$W^{\mu\nu} = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4} e^{iq\cdot b} \langle N | J^{\mu\dagger}(b) | J/\psi, X\rangle \langle J/\psi, X | J^\nu(0) | N\rangle$$

Soft Wilson lines in direction of onium (v^μ) and proton (n^μ)

NRQCD heavy-quark fields

$$J^\mu_{[i]}(0) = C_{[i]}(Q, M; \mu^2) \left[S_v S_n \psi^\dagger_{\vec{p}} (\Gamma \cdot \mathcal{K})_{[i]} T B^\dagger_{n\perp} \chi_{\vec{p}} \right](0) \rangle$$

Wilson coefficient

NRQCD projection operators and matching tensors

Gluon field collinear to proton, dressed with Wilson lines

$$[i] = {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

! Wilson coefficient = coupling of effective field theory, obtained from matching with full QCD.

! Wilson line = path-ordered exponential of soft, collinear,… gluons

$$S_v(x) = P \exp\left[ig \int_{-\infty}^0 d\tau\, v \cdot A_s(x + v\tau) \right]$$

Factorization theorem for $\ell + p \to \ell + J/\psi + X$

$$W^{\mu\nu} = \sum\!\!\!\!\!\!\int_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4} e^{iq\cdot b} \langle N | J^{\mu\dagger}(b) | J/\psi, X\rangle \langle J/\psi, X | J^\nu(0) | N\rangle$$

At leading power, undetected particles are soft or collinear to the incoming proton: $X = X_n + X_s$. We get $|J/\psi, X\rangle = |X_n\rangle \times |J/\psi, X_s\rangle$

$$\langle N | J^{\mu\dagger}_{[i]}(b) | J/\psi, X\rangle$$
$$= C_{[i]}(Q, M; \mu^2) \times \langle N | B^\dagger_{n\perp}(b) | X_n\rangle \times \langle 0 | \big[S_v S_n \psi^\dagger_{\vec{p}} \big(\Gamma \cdot \mathcal{K}\big)_{[i]} T \chi_{\vec{p}}\big](b) | J/\psi, X_s\rangle$$

Expand in leading momentum components:

$$\int \frac{\mathrm{d}^4 b}{(2\pi)^4} e^{iq\cdot b} \langle N | J^{\mu\dagger}_{[i]}(b) | J/\psi, X\rangle$$
$$= C_{[i]} \times \int \frac{\mathrm{d}^4 b}{(2\pi)^4} e^{iq\cdot b} \langle N | B^\dagger_{n\perp}(b^-, b_\perp) | X_n\rangle \times \langle 0 | \big[S_v S_n \psi^\dagger_{\vec{p}} \big(\Gamma \cdot \mathcal{K}\big)_{[i]} T \chi_{\vec{p}}\big](b_\perp) | J/\psi, X_s\rangle + \mathcal{O}(\lambda)$$

Factorization theorem for $\ell + p \to \ell + J/\psi + X$

We recognize the unsubtracted gluon TMD:

$$\mathcal{J}^{(0)}_{n,\alpha\beta}(x,b_\perp) = \frac{1}{2}\int \frac{\mathrm{d}b^-}{2\pi} e^{\frac{1}{2}i(q^+ - P^+_\psi)b^-} \langle N | B^\dagger_{n\perp\alpha}(b^-, b_\perp) B_{n\perp\beta}(0) | N \rangle$$

and we define, for every CO state [i], an unsubtracted TMD shape function:

$$S^{(0)}_{[i]\to J/\psi}(b_\perp) \propto \mathrm{Tr}\langle 0 | \left[S^\dagger_n S^\dagger_v \chi^\dagger_{\vec{p}} \Lambda_{[i]} T \psi_{\vec{p}} \right](b_\perp) \mathcal{N}_\psi \left[S_v S_n \psi^\dagger_{\vec{p}} \Lambda_{[i]} T \chi_{\vec{p}} \right](0) | 0 \rangle$$

Hadron tensor now cast in factorized form:

$$W^{\mu\nu} = \frac{x_B P^+_N}{Q^2} \delta(z - 1 + \mathcal{O}(\lambda^2)) \times \sum_{[i]} \underbrace{\left| C_{[i]} \right|^2 \Gamma^{\dagger\mu\alpha}_{[i]} \Gamma^{\nu\beta}_{[i]}} \int \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2} e^{i b_\perp \cdot q_\perp} J^{(0)}_{n\alpha\beta}(x, b_\perp) S^{(0)}_{[i]\to J/\psi}(b_\perp)$$

from matching with on QCD

Factorization theorem for $\ell + p \to \ell + J/\psi + X$

Now it gets technical: the matrix elements defined above contain *rapidity* divergences, which can be cancelled with the help of the *soft function*

$$\mathcal{S}(b_\perp) = \frac{1}{N_c^2 - 1} \mathrm{Tr}\langle 0|[S_n^\dagger S_{\bar{n}}](b_\perp)[S_{\bar{n}}^\dagger S_n](0)|0\rangle$$

Hence, we write:

$$J^{(0)}_{n\alpha\beta}(x, b_\perp) \times S^{(0)}_{[i]\to J/\psi}(b_\perp) = J^{(0)}_{n\alpha\beta}(x, b_\perp)\sqrt{\mathcal{S}(b_\perp)} \times \frac{S^{(0)}_{[i]\to J/\psi}(b_\perp)}{\sqrt{\mathcal{S}(b_\perp)}}$$

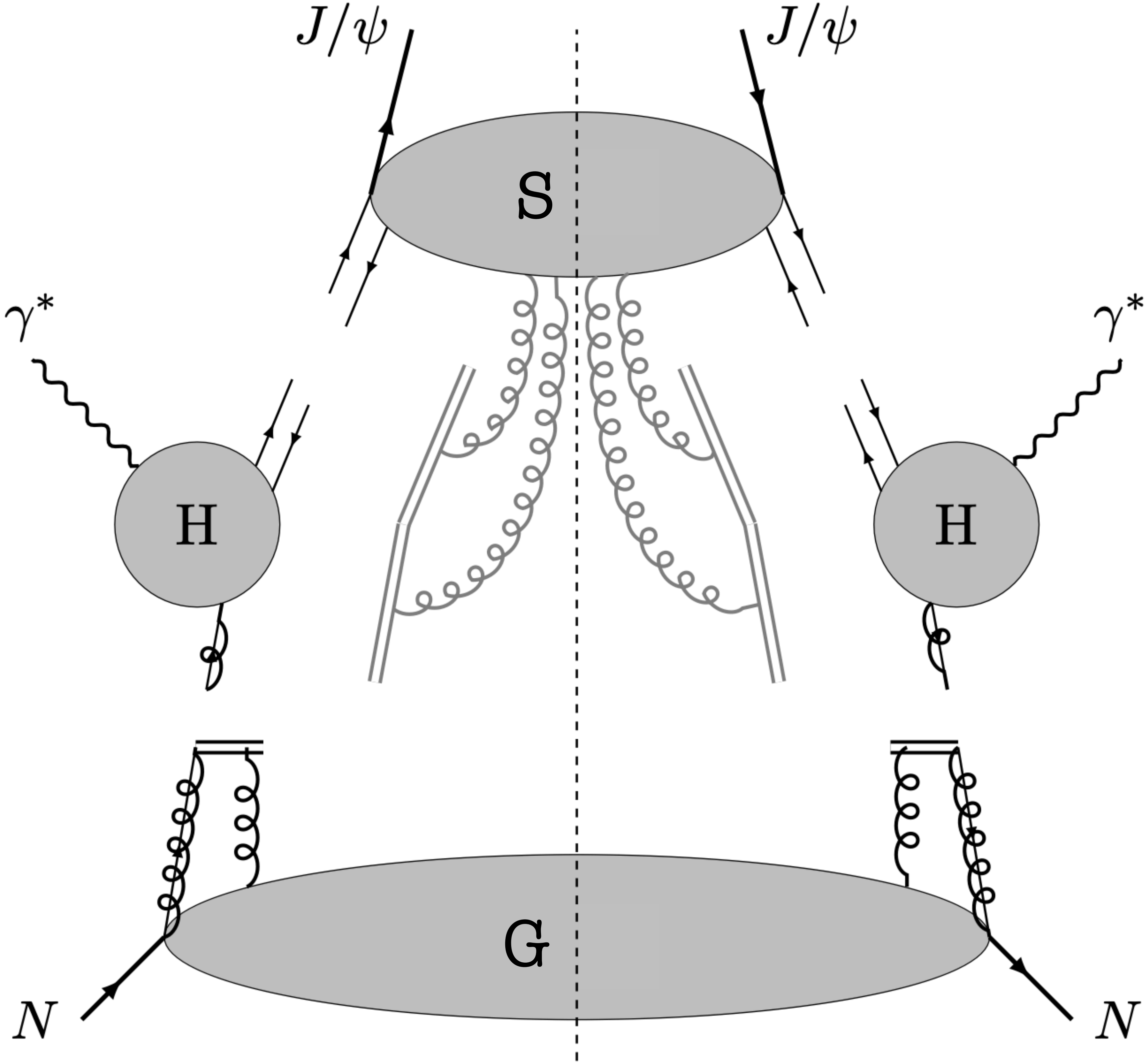
$$= G_{\alpha\beta}(x, b_\perp) \times S_{[i]\to J/\psi}(b_\perp)$$

Final, leading-power factorization theorem in terms of gluon TMD PDF and TMD Shape Functions (TMD ShF)

$$W^{\mu\nu} = \frac{x_B P_N^+}{Q^2}\delta(z - 1 + \mathcal{O}(\lambda^2))$$

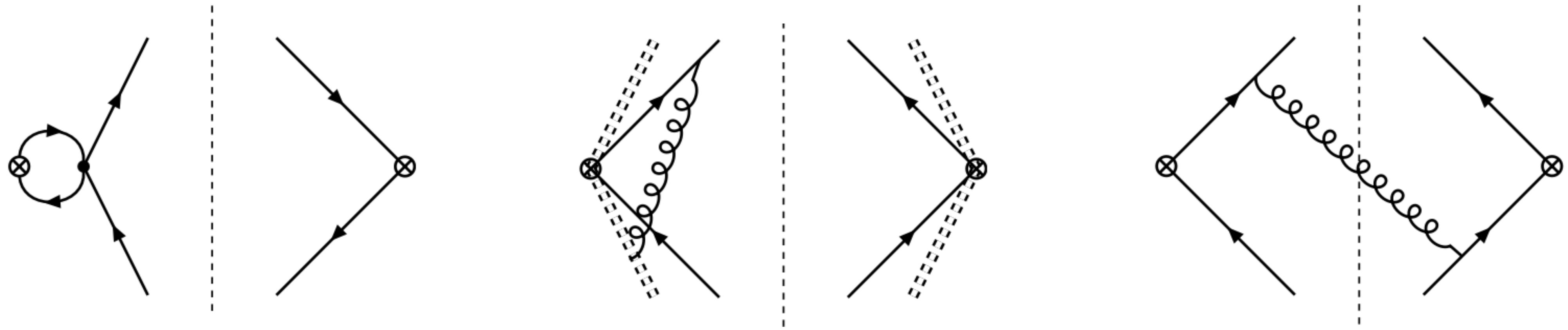
$$\times \sum_{[i]} \left|C_{[i]}\right|^2 \Gamma^{\dagger\mu\alpha}_{[i]} \Gamma^{\nu\beta}_{[i]} \int \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2} e^{ib_\perp \cdot q_\perp} G_{\alpha\beta}(x, b_\perp, \mu, \zeta_A) S_{[i]\to J/\psi}(b_\perp, \mu, \zeta_B)$$

Factorization theorem for $\ell + p \to \ell + J/\psi + X$



NLO calculation

Typical diagrams:



Result features ultraviolet (UV), infrared (IR), and Coulomb divergences.

Dependence on dimensional regularization scale μ, and on rapidity scale ζ_B

$$S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} + \frac{\alpha_s}{2\pi}\Bigg[\frac{C_A}{\varepsilon_{\mathrm{UV}}}(1-\ln\zeta_B)\left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}}$$

$$+ C_A L_T (1-\ln\zeta_B)\left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} - \frac{8}{3m_c^2} L_T \left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\mathrm{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\mathrm{LO}} \right)$$

$$+ \frac{\pi^2}{\mathrm{v}}(C_F - C_A/2)\left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} - \frac{8}{3m_c^2}\frac{1}{\varepsilon_{\mathrm{IR}}}\left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\mathrm{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\mathrm{LO}} \right)\Bigg]$$

$L_T = \ln(\mu^2 b_T^2 e^{2\gamma_E}/4)$ Sudakov logarithm

Evolution equations

UV divergence can be renormalized with counterterm:

$$S_{[i]}(b_T, \zeta_B) \equiv Z_{[i]}(b_T, \zeta_B, \mu) S_{[i]}(b_T, \zeta_B, \mu)$$

Renormalization group equation (RGE): $\frac{\mathrm{d}}{\mathrm{dln}\mu} S_{[i]}(b_T, \zeta_B) = 0$

leads to:

$$\frac{\mathrm{d}}{\mathrm{dln}\mu} S_{[i]}(b_T, \zeta_B, \mu) = -\underbrace{\frac{\mathrm{d} \ln Z_{[i]}(b_T, \zeta_B, \mu)}{\mathrm{dln}\mu}} S_{[i]}(b_T, \zeta_B, \mu)$$

Anomalous dimension

$$\gamma_{\mathrm{Sh}} = \frac{\alpha_s C_A}{\pi} (1 - \ln \zeta_B)$$

turns out to be the same for all CO states

Evolution equations

Similarly, the shape functions run with the rapidity scale:

$$\frac{\mathrm{d}}{\mathrm{dln}\zeta_B} S_{[i]}(b_T, \zeta_B, \mu) = -\mathcal{D}_g(b_T, \mu) S_{[i]}(b_T, \zeta_B, \mu)$$

Known as the Collins-Soper kernel, turns out to be the same as the one for gluon TMD PDFs.

Complete evolution for shape functions, the same independently of CO state

$$S_{^1S_0^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = \exp\left[\int\left(\gamma_{\mathrm{Sh}}(\mu,\zeta_B)\frac{d\mu}{\mu} - \mathcal{D}_g(b_T;\mu)\frac{d\zeta_B}{\zeta_B}\right)\right] S_{^1S_0^{[8]}\to J/\psi}(b_T;\mu_f,\zeta_f)$$

$$S_{^3P_J^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = \exp\left[\int\left(\gamma_{\mathrm{Sh}}(\mu,\zeta_B)\frac{d\mu}{\mu} - \mathcal{D}_g(b_T;\mu)\frac{d\zeta_B}{\zeta_B}\right)\right] S_{^3P_J^{[8]}\to J/\psi}(b_T;\mu_f,\zeta_f)$$

! At large transverse momentum, there is still mixing between the LDMEs.

Conclusions

Factorization theorem at leading power: all radiation accounted for!

Not a rigorous 'proof' since derived in effective theory, but can be validated order by order in perturbation theory.

In paper (2024), everything NLO except for finite corrections to hard part. Those are recently obtained by the Bilbao group.

Need next-to-leading power factorization theorem to include CS mechanism, highly-non trivial!

In memoriam

Tom Mehen (1970-2024)

https://trinity.duke.edu/news/physics-professor-tom-mehen-passes-away