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# Determination of the Z width at FCC-ee

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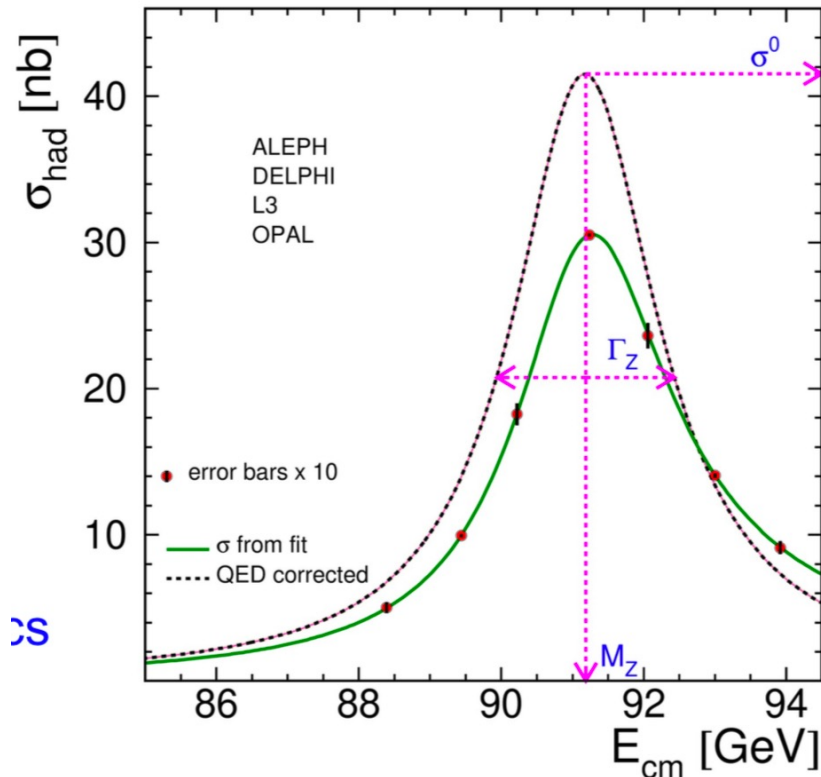
Physics Performance meeting, September 16, 2024

Work in progress...

# Determination of the Z width from the line-shape

Expected statistical uncertainty: 5 keV ( cf e.g. mid-term report )

- Absolute calibration of  $\sqrt{s}$ : key for the determination of the Z mass
- But for the Z width: what matters is the relative, point-to-point uncertainty on  $\sqrt{s}$ , between the off-peak points used in the line-shape scan
  - Other important systematic: BES
  - With BES known to 0.5 per-mill: uncertainty on  $\Gamma_Z$  is 10 keV



With  $\delta(\sqrt{s})_{\text{ptp}} \sim 10$  keV, syst. uncertainty on  $\Gamma_Z$  would be 5 keV, at the level of the stat. !

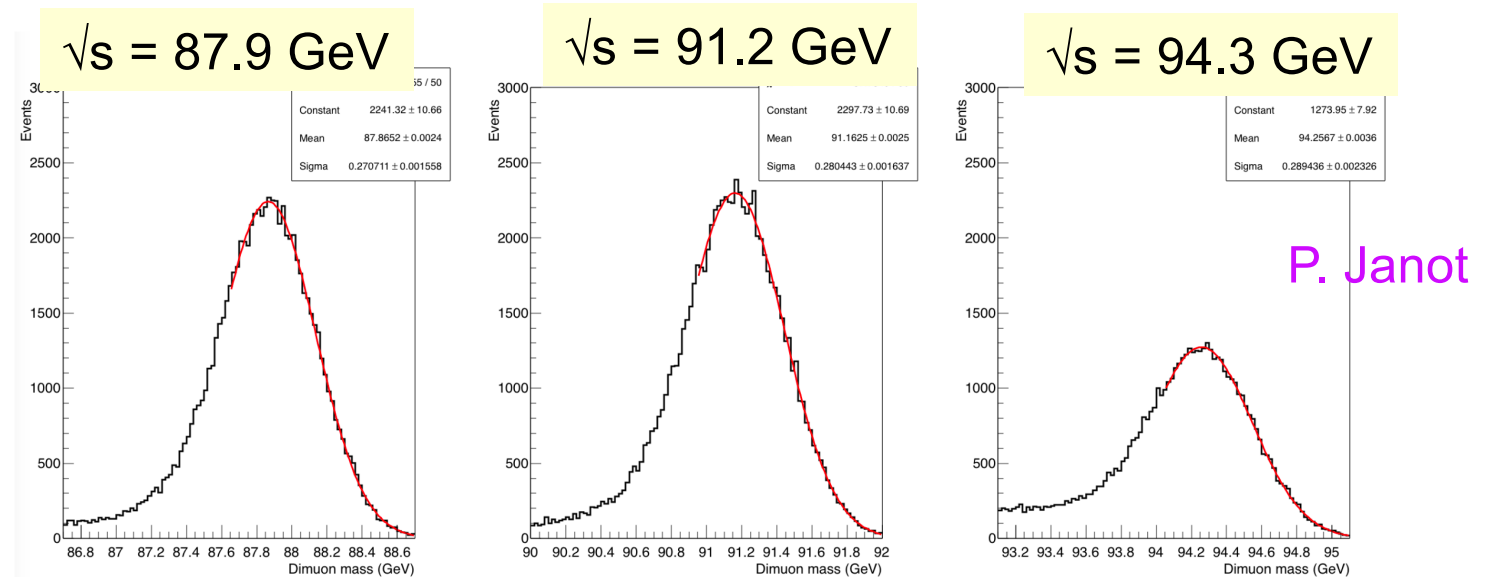
NB:  $\delta(\sqrt{s})_{\text{ptp}}$  also important systematic for  $\sin^2\theta_W$  from  $\text{AFB}(\mu\mu)$ . Need a few 10's of keV to reach the stat. uncertainty of  $2e-6$

# Point-to-point uncertainty on $\sqrt{s}$ from dimuon events

Use e.g. the "peak position" of the  $M_{\mu\mu}$  distribution in dimuon events, at  $\sqrt{s} = M_Z$  and at the off-peak points

arXiv:1909.12245

Key = exquisite momentum resolution



**Figure 58.** Invariant mass distribution of  $10^5$  muon pairs in the CLD detector, at centre-of-mass energies of (left-to-right) 87.9, 91.2 and 94.3 GeV respectively; the width of the distribution is dominated by the muon momentum measurement uncertainty. The data correspond to  $521 \text{ pb}^{-1}$ ,  $69 \text{ pb}^{-1}$ , and  $257 \text{ pb}^{-1}$ , which can be acquired in 4 minutes, 35 seconds and 2 minutes respectively

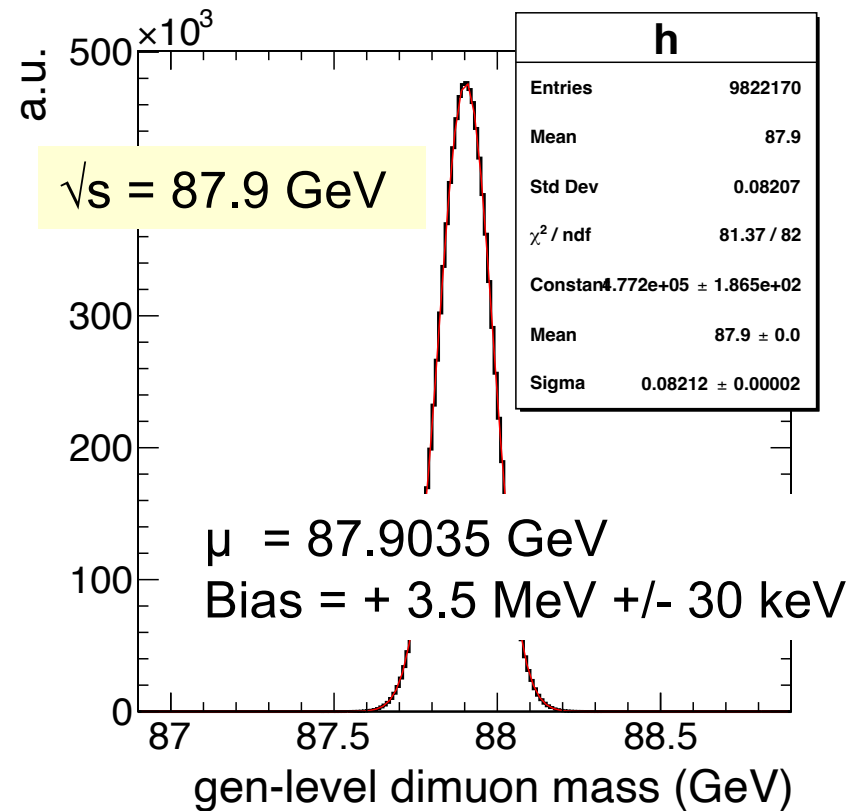
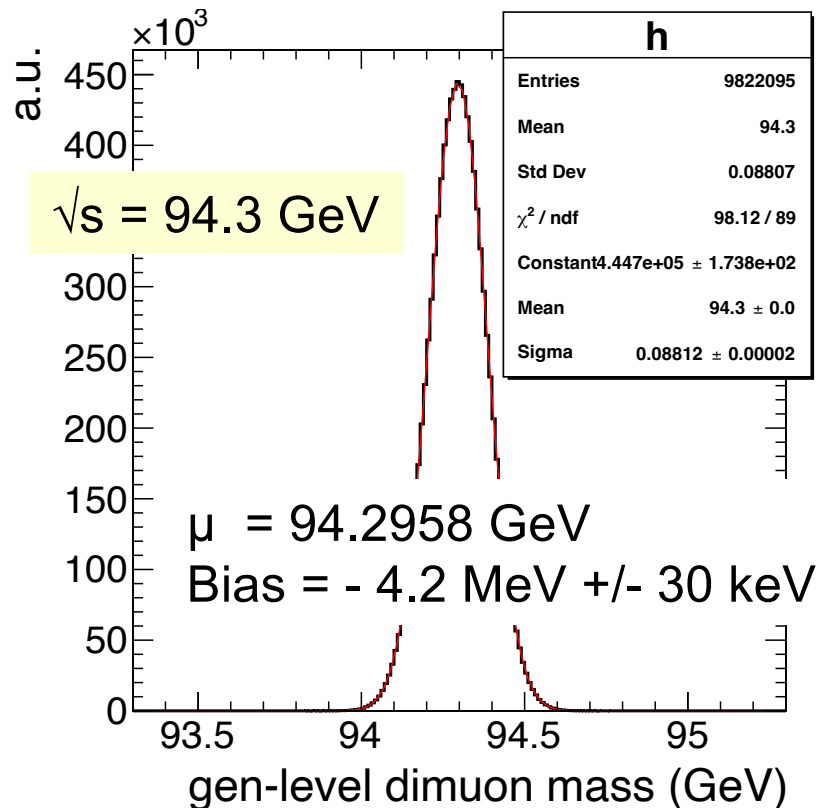
May not be good enough for an absolute calibration of  $\sqrt{s}$ , but could provide  $\delta(\sqrt{s})_{\text{ptp}}$  to better than  $\sqrt{2}$  x RDP uncertainty.

First follow-up: E. Leogrande, E.P, Dec. 2020...

## Bias of the estimator of $\sqrt{s}$

- Any proxy to  $\sqrt{s}$  (e.g. the “peak position” of the  $M_{\mu\mu}$  distribution, or some parameter extracted from a fit) will show a bias
  - in particular due to ISR/FSR
- And this bias depends on  $\sqrt{s}$  !

Example: no ISR, no FSR, gen-level dimuon mass. Simple gaussian fit:



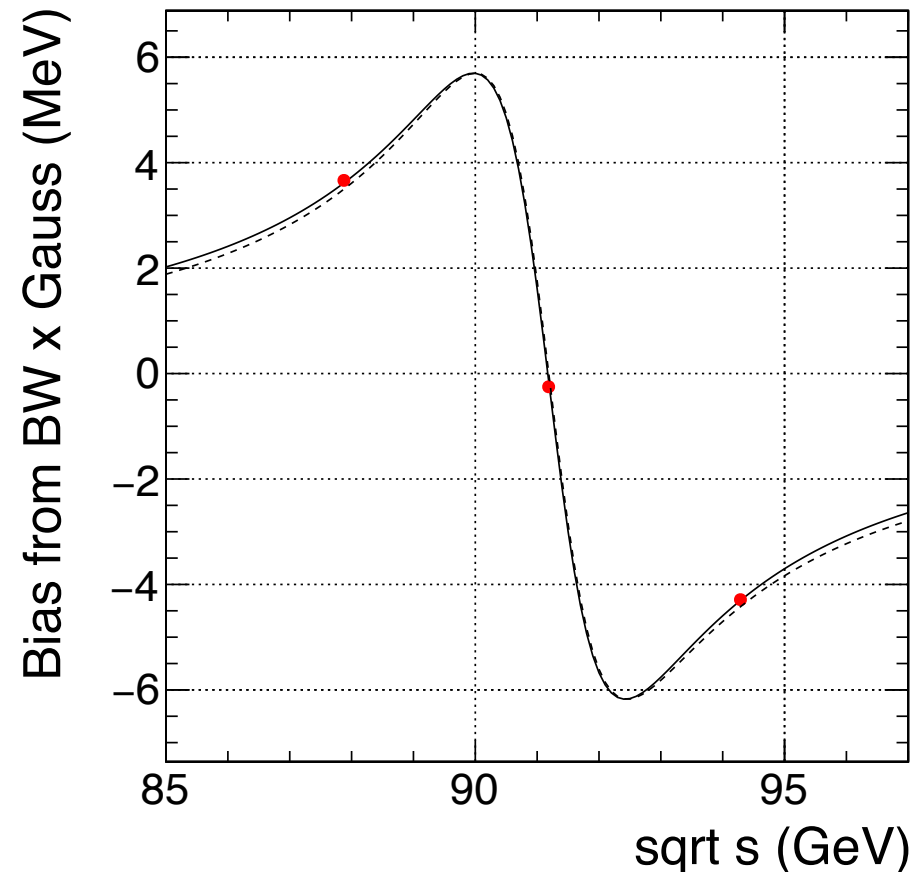
## Bias of the estimator of sqrts: simplest case

The bias in the previous plots comes from the product of the Breit-Wigner with the Gaussian that represents the beam-energy spread (BES).

- Below MZ : the BW pulls the distribution towards MZ, positive bias
- Above MZ : negative bias

The value of the bias can be determined analytically by maximizing  $BW \times \text{Gauss}(\text{BES})$ .

The bias varies quadratically with the BES.



## Samples, fit procedure

Delphes samples of  $ee \rightarrow \mu\mu$  from Whizard and KKMC

- BES, ISR and FSR

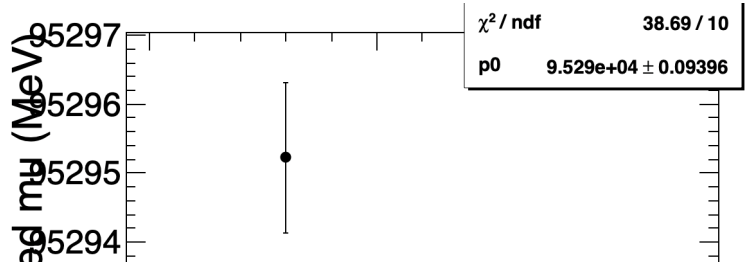
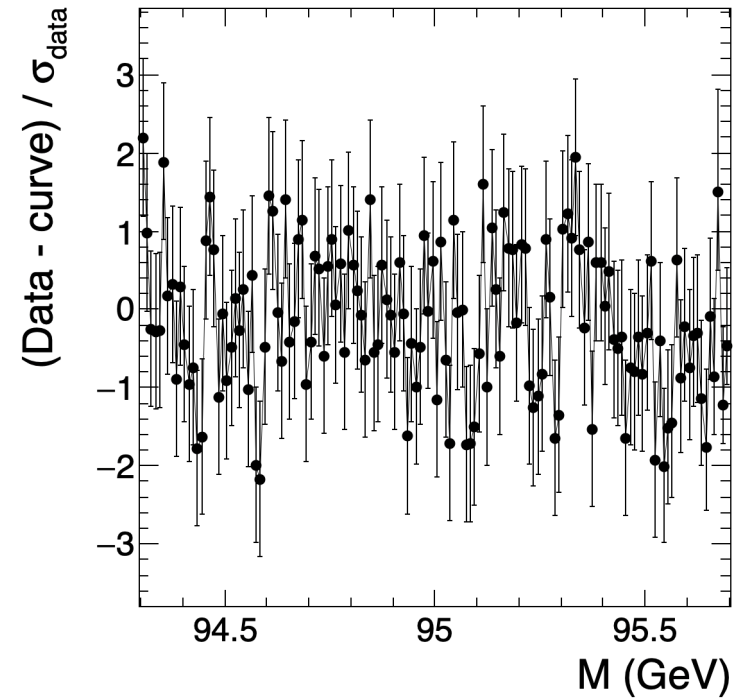
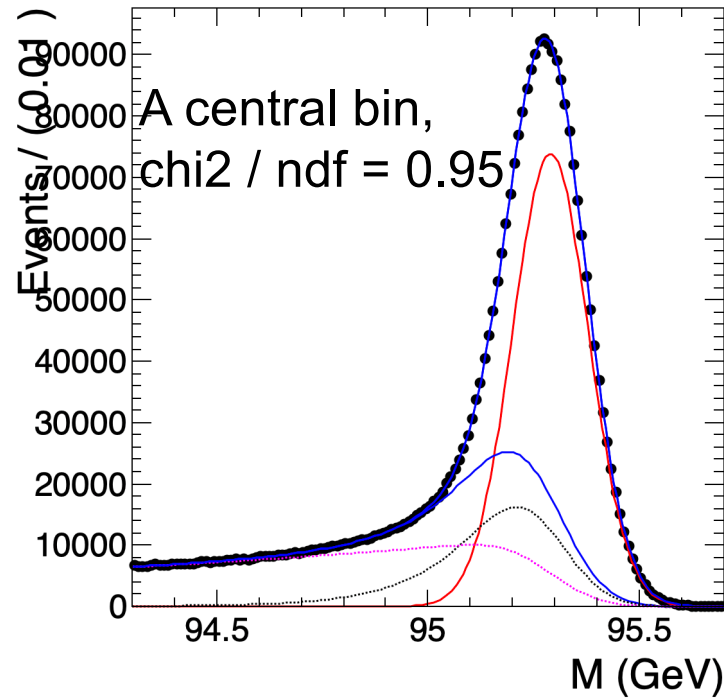
Energies: 91.188 GeV, 87.9 GeV and 94.3 GeV ; +/- 300 keV around these values;  
and a few other off-peak points for checks

About 100 M events for each sample

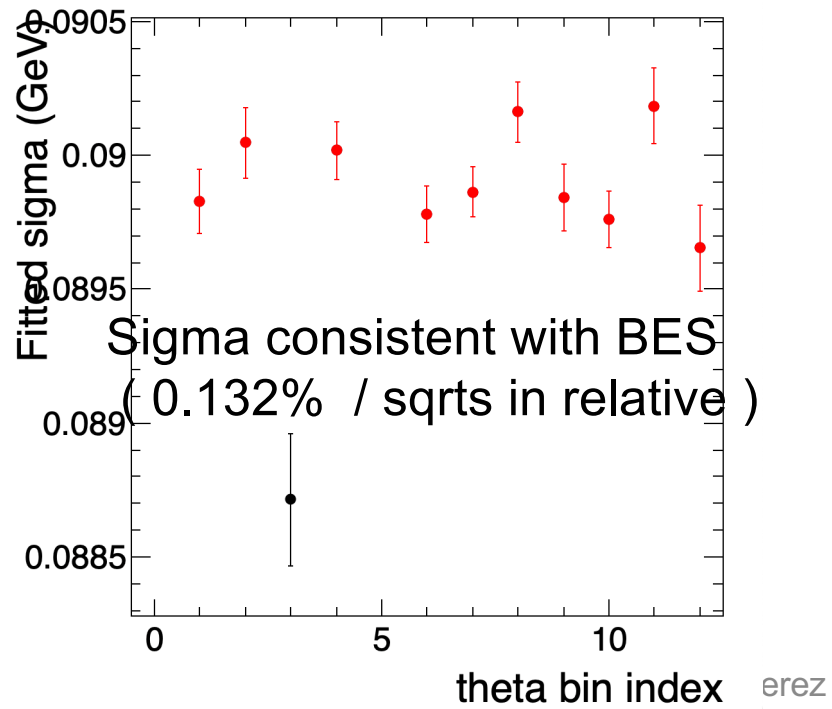
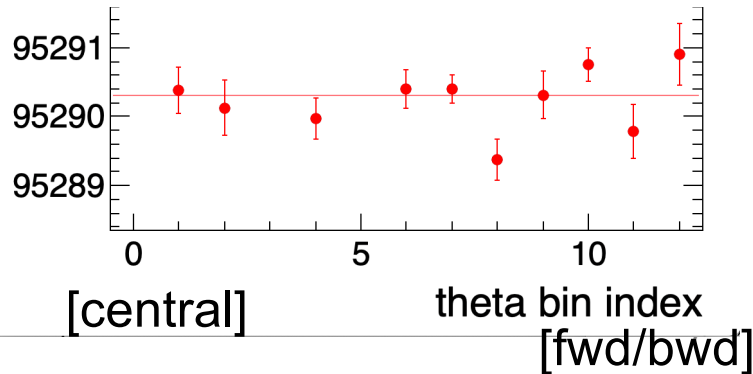
- Fit the dimuon mass distribution
  - so far, only the “raw” dimuon mass (no “correction” for colinear ISR photon, no  $S_p$  yet)
- Fit model: Gauss  $\otimes$  ( delta + two exponentials )
  - cf 2022 paper from G. Wilson & B. Madison, arXiv:2209.03281
  - Provides good fits – for this MC statistics
- Fits done in theta bins (angular dependence of the momentum resolution)
- To have 1D bins only: demand that the  $\mu^+$  and the  $\mu^-$  be in the “same” theta bin (accop cut :  $|\theta^+ + \theta^- - \pi| < 0.1$  rad )
- Keep only good fits
  - Equivalent :  $\chi^2 < N_{df} + 3 \times \sqrt{2 \times N_{df}}$
- Proxy for  $\sqrt{s}$ : weighted average of the means of the Gaussian in the various theta bins

Example fits

Gen-level mass  
 $\sqrt{s} = 95.3$  GeV



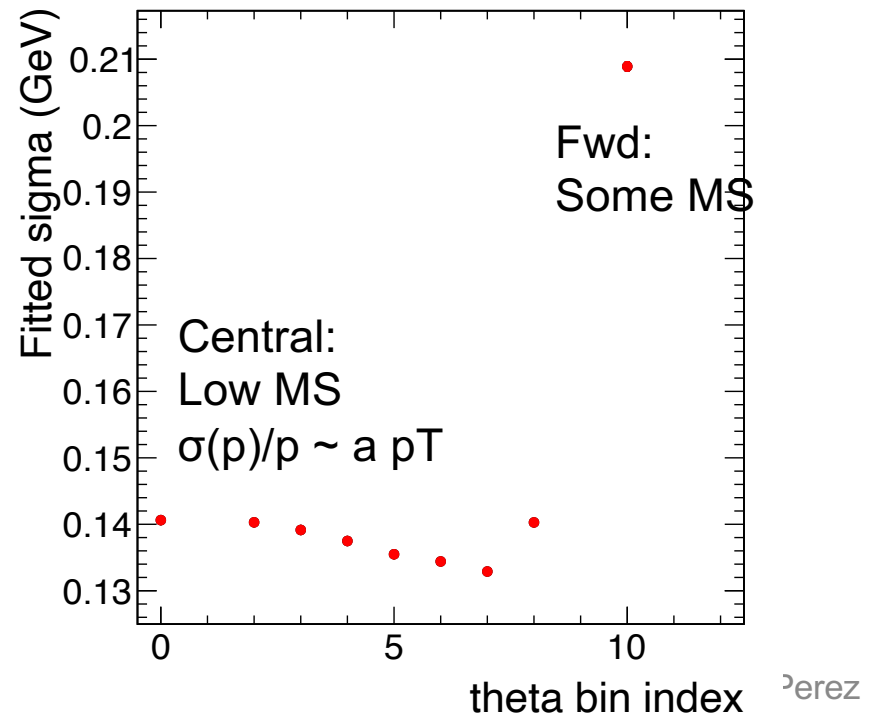
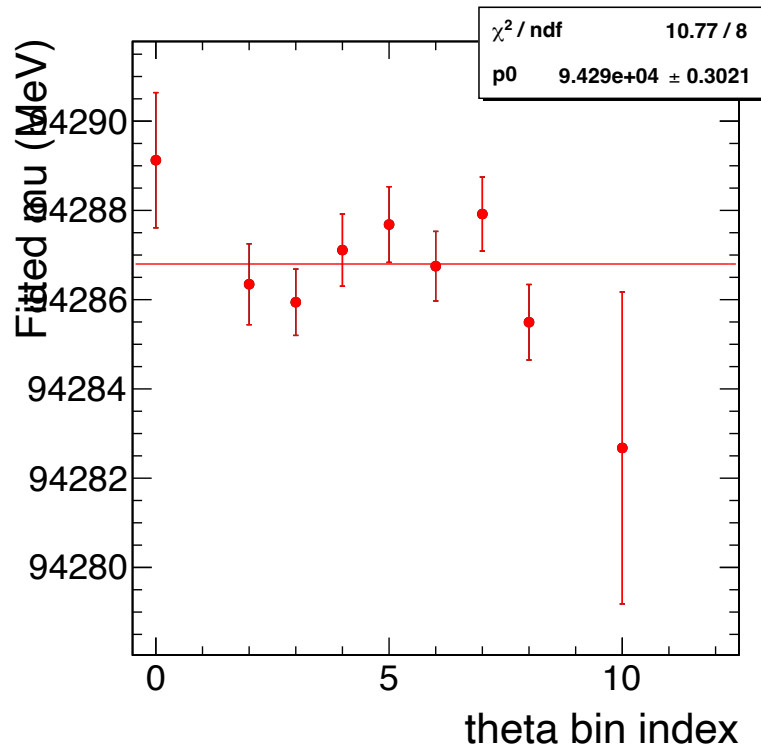
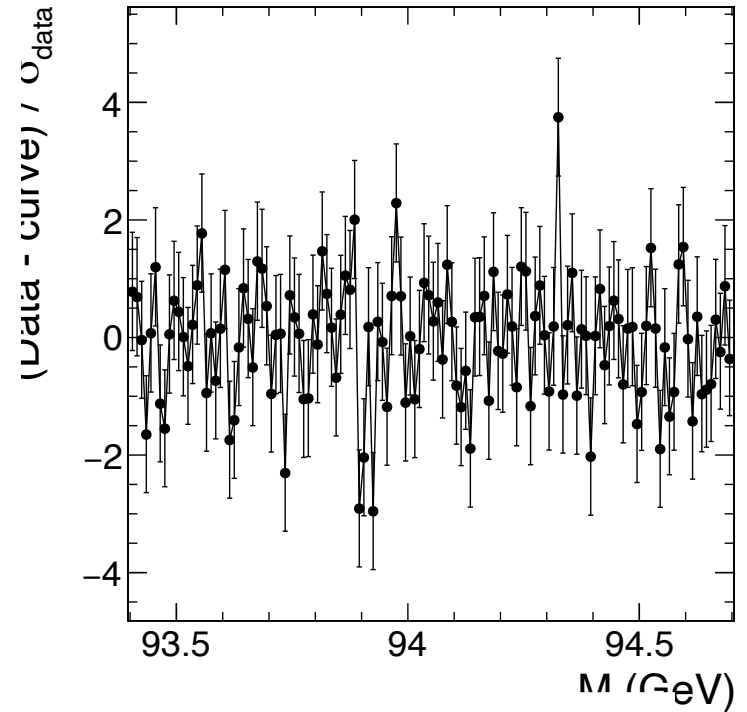
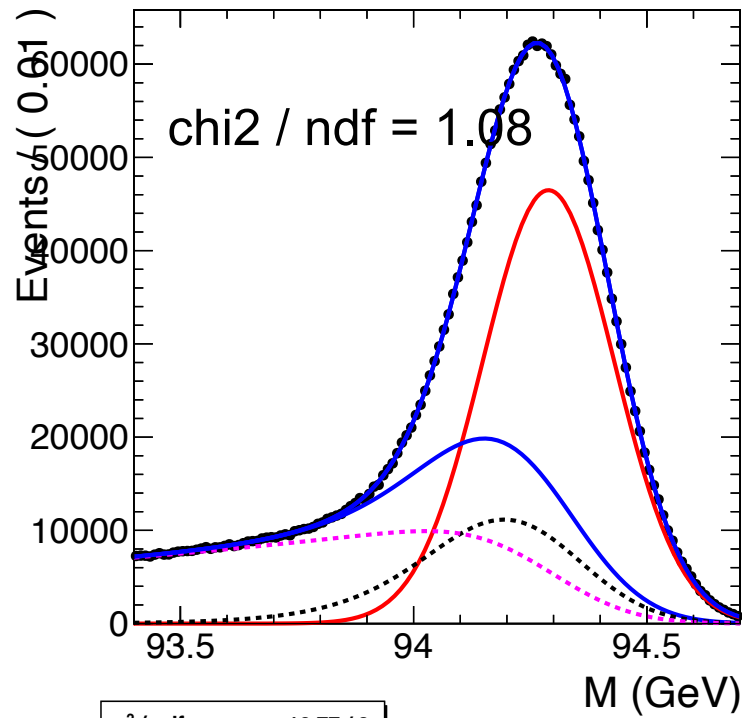
Fitted mu is the same in all bins  
 ( outliers are removed )



Sigma consistent with BES  
 ( 0.132% / sqrts in relative )

Example fits (2)

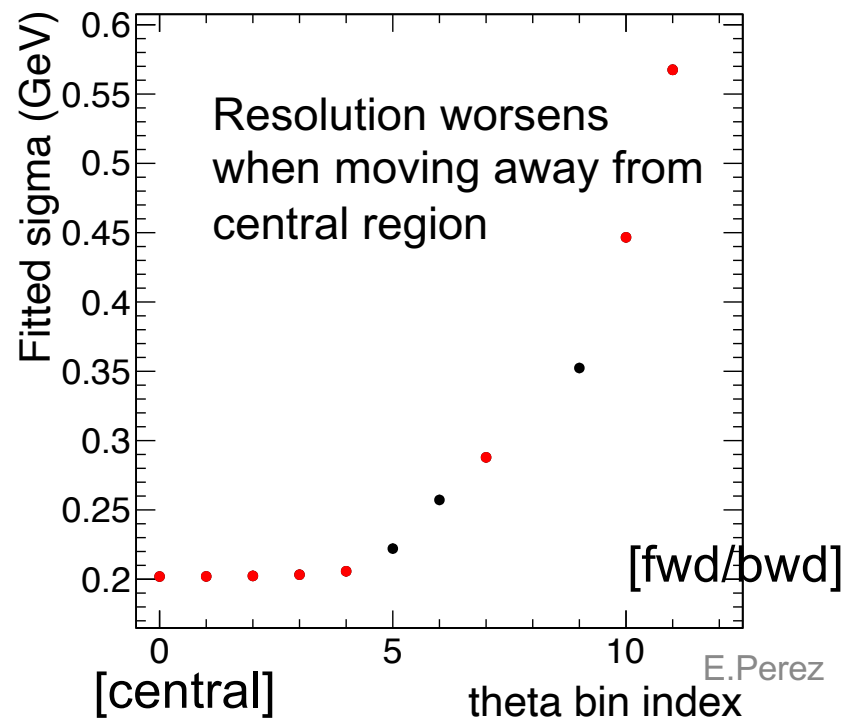
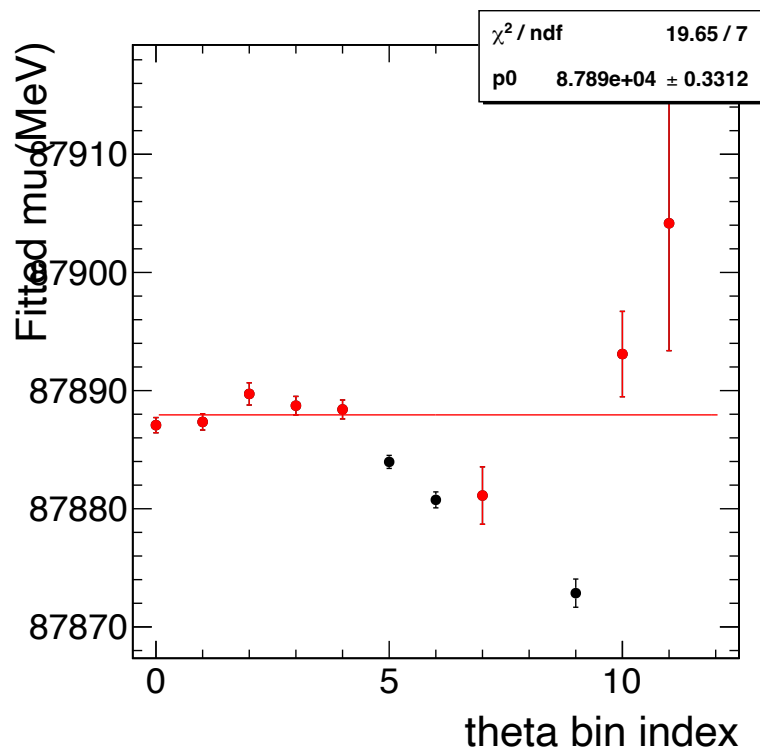
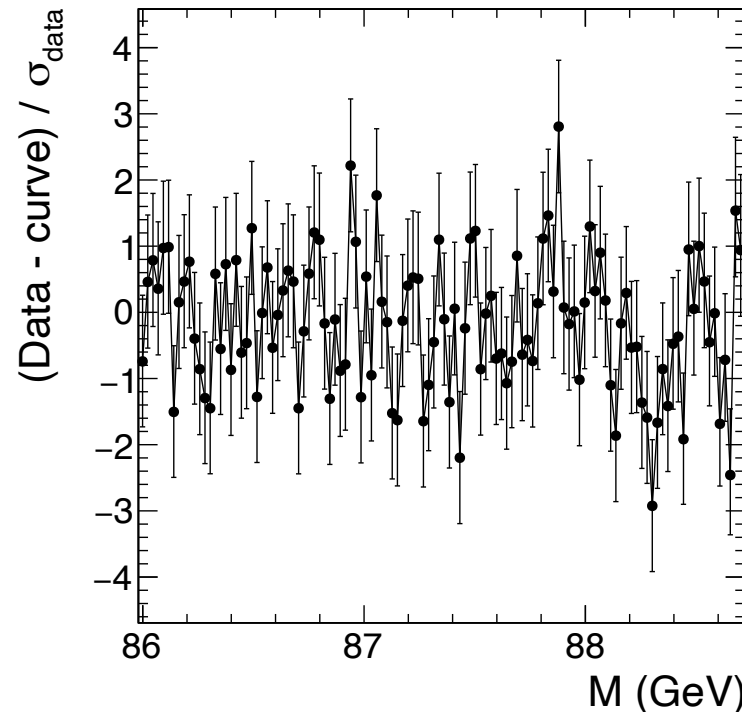
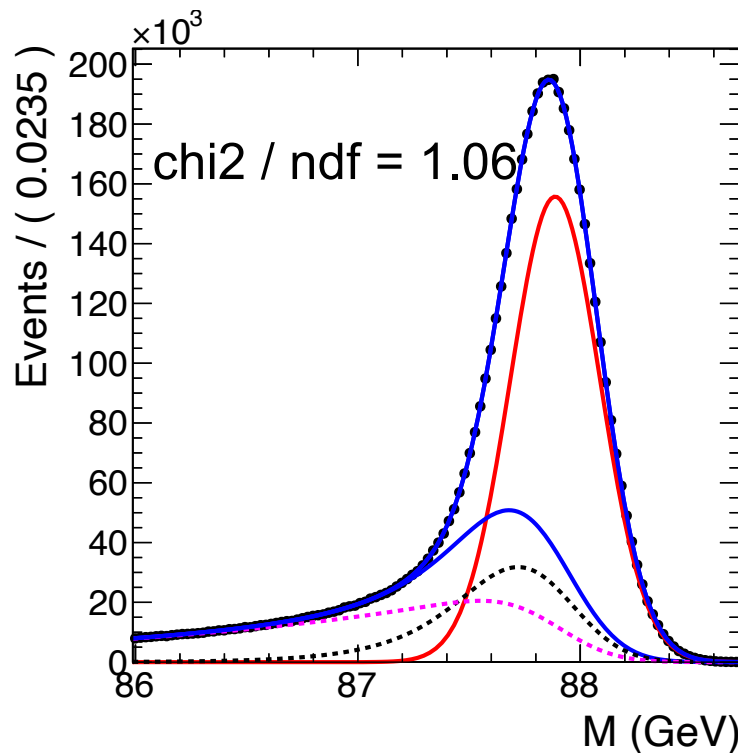
Reco'd mass,  
IDEA  
 $\sqrt{s} = 94.3 \text{ GeV}$





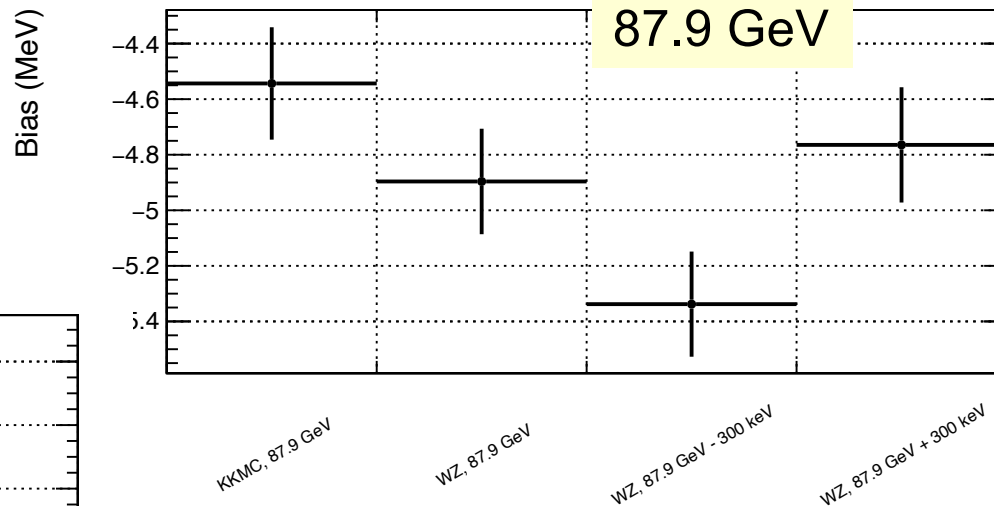
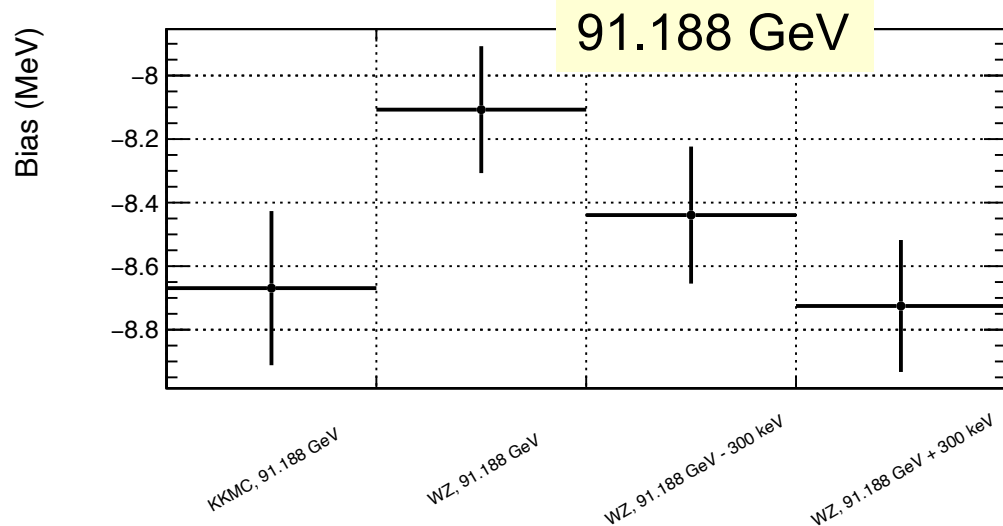
Example fits (3)

Reco'ed mass,  
CLD  
 $\sqrt{s} = 87.9 \text{ GeV}$



## Bias, numerical values (IDEA)

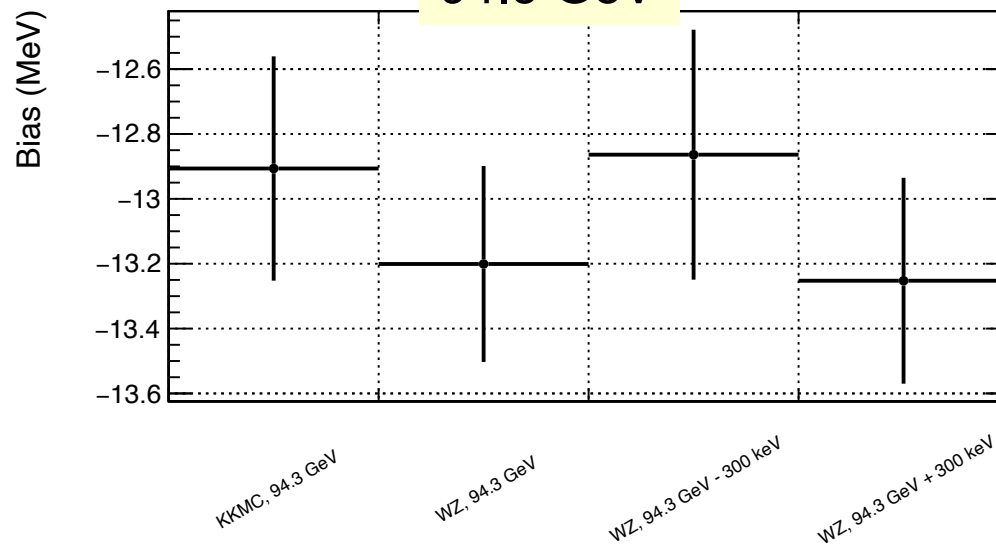
Reco'd mass, IDEA



$$\text{Bias} = \text{proxy (average mu)} - \sqrt{s}$$

Within uncertainties ( $\sim 200$  keV with current MC statistics) :

- KKMC  $\sim$  Whizard
- Bias is “locally constant” when  $\sqrt{s}$  varies by  $\pm 300$  keV ( $\sqrt{s}$  will be known from RDP to within 100 keV or better)



## Statistical uncertainties on the $\sqrt{s}$ proxy

- With  $1e8$  events, uncertainties of 200 – 300 keV
- Rescaling to the number of events expected with 40 / 125 / 40  $ab^{-1}$  at 87.9 / 91.2 / 94.3 GeV :  
     $\langle \mu \rangle$  would be known to  $\sim 4$  keV at 91.2 GeV,  $\sim 20$  keV off-peak

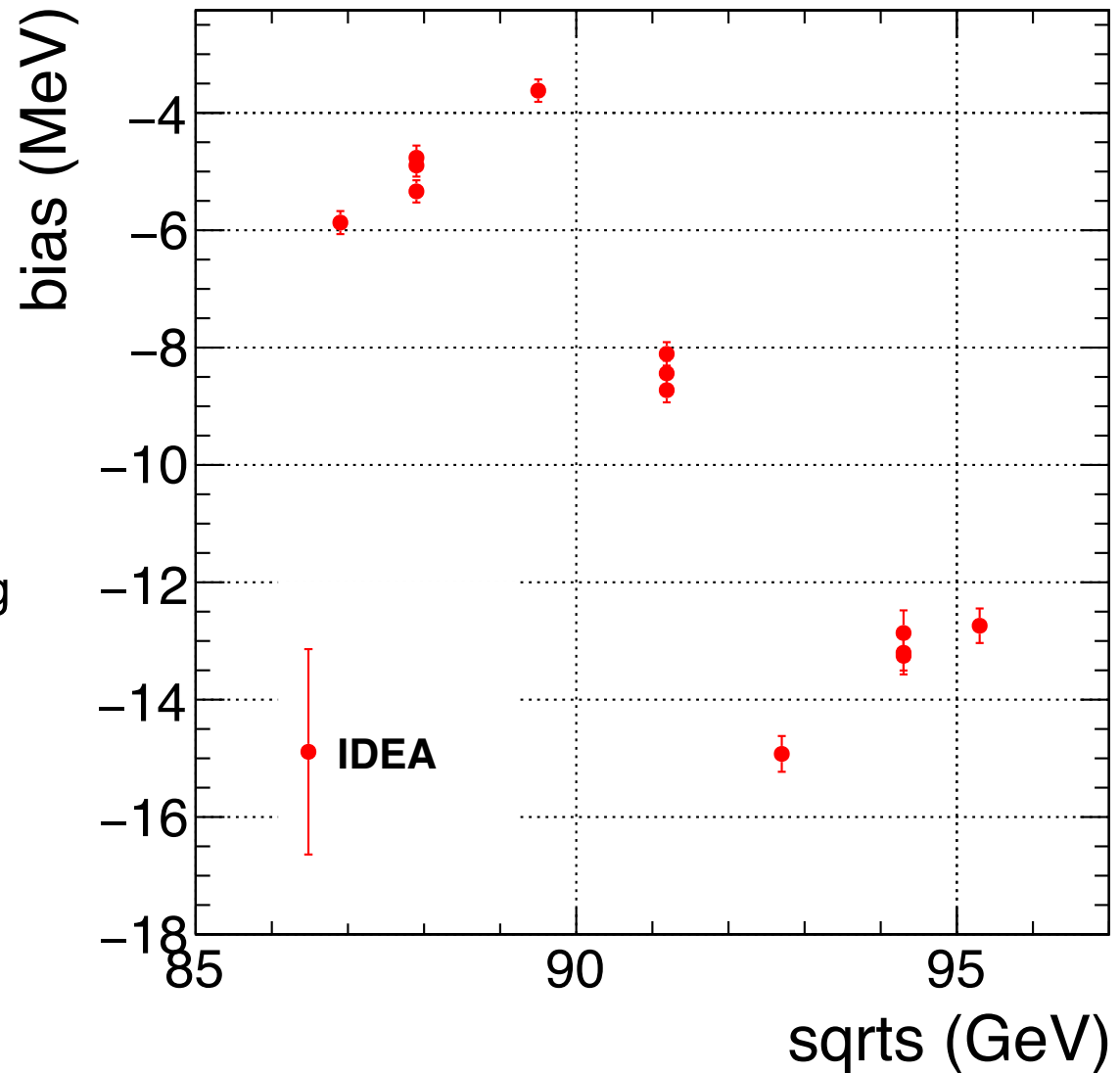
If the bias is known (e.g. from MC-based calibration) to better than that, one would know  $\sqrt{s^+} - \sqrt{s^-}$  to  $\sim 28$  keV from the difference  $\langle \mu (s^+) \rangle - \langle \mu (s^-) \rangle$

The bias determination relies on MC modeling, esp. ISR. Need a good theoretical control, see later.

## Dependence of the bias vs $\sqrt{s}$

Most of the dependence seems to come from the interplay of the Breit-Wigner with the Gaussian describing the BES (see slide 5).

Same shape, modulo a constant shift.

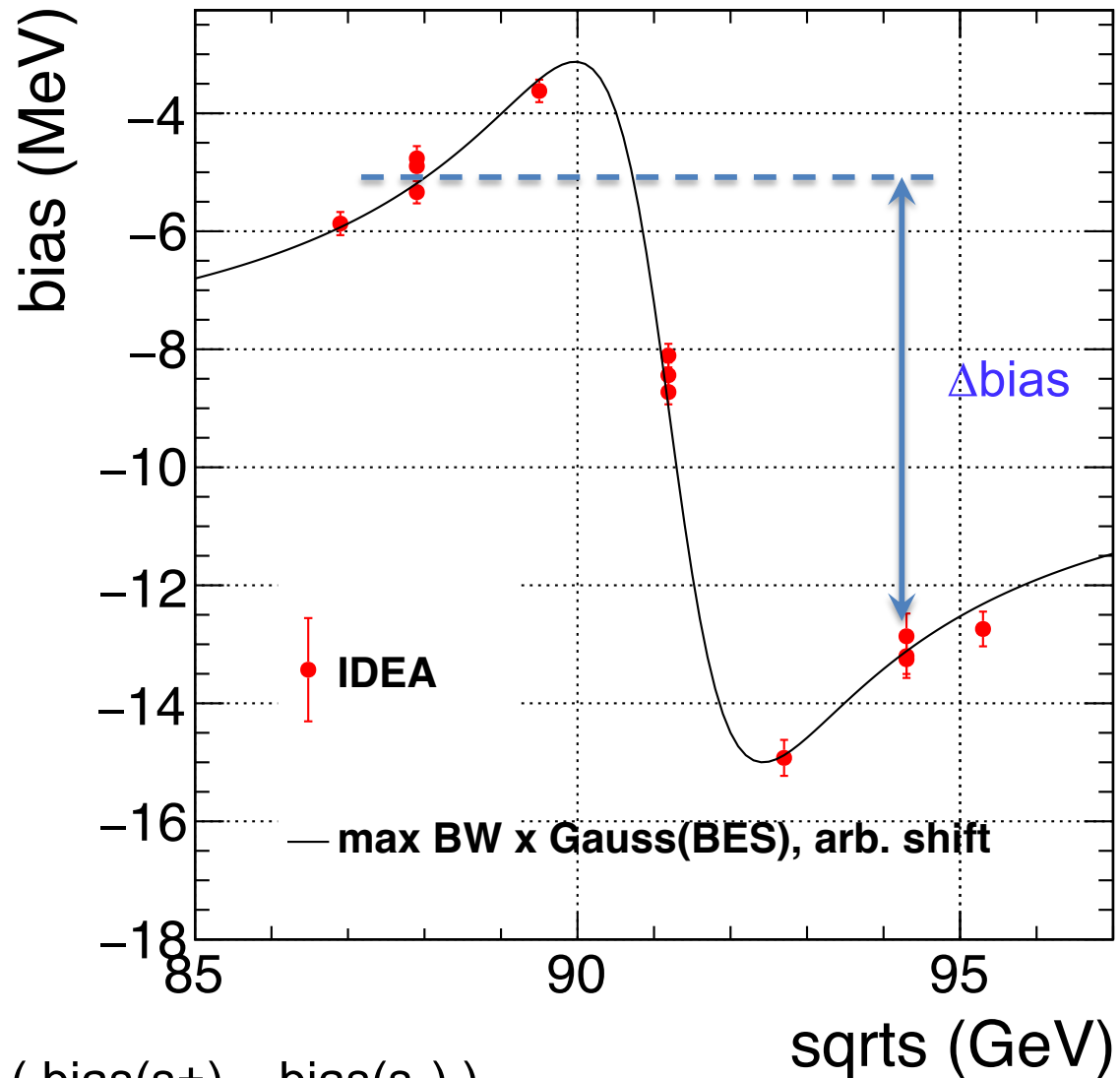


## Dependence of the bias vs $\sqrt{s}$ (IDEA)

The derivative of this dependence can be used to assess the fact that the bias is locally constant (to much better than the  $\sim 200$  keV that we get from the current MC statistics)

Constant to better than 1 keV when  $\sqrt{s}$  varies by  $\pm 100$  keV.

Negligible w.r.t. stat. uncertainty on the proxy ( $\sim 20$  keV)



$$\sqrt{s_+} - \sqrt{s_-} = \mu(s_+) - \mu(s_-) - (\text{bias}(s_+) - \text{bias}(s_-))$$

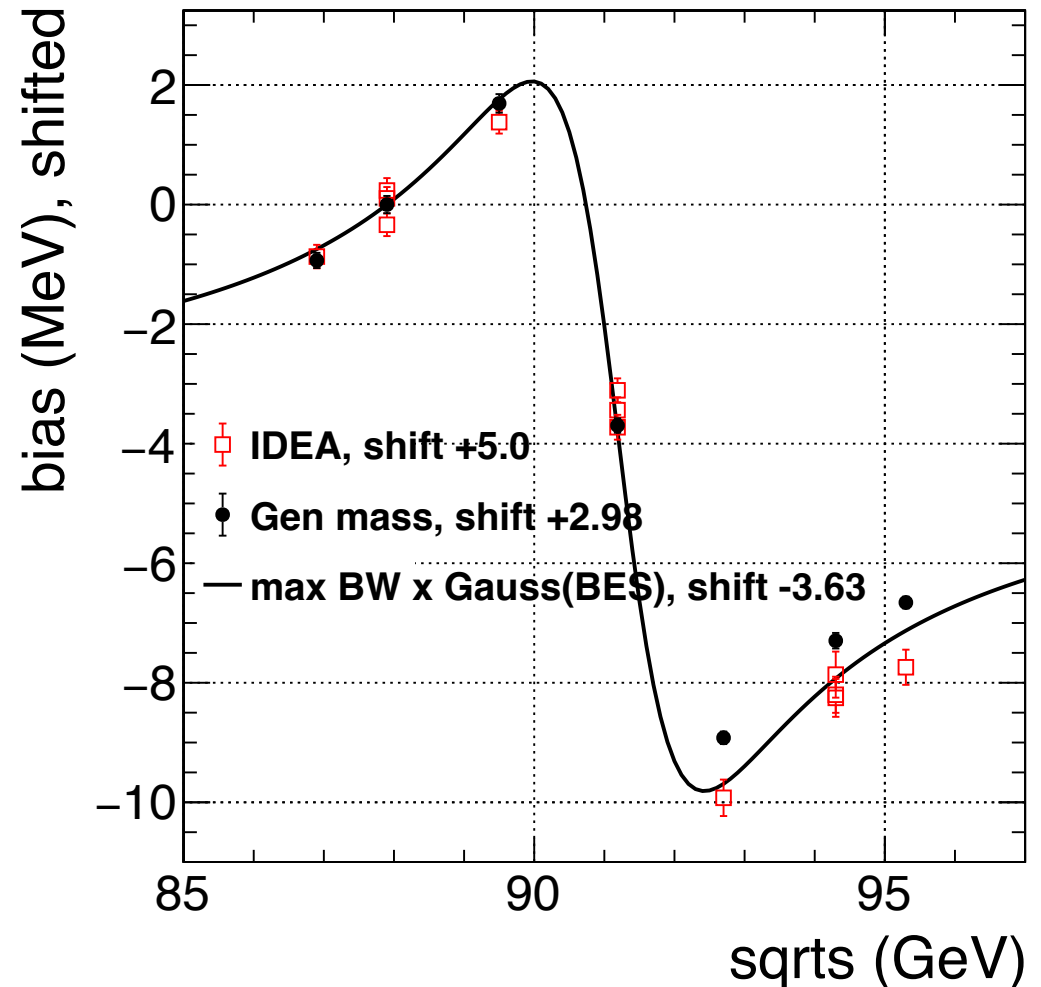
So what matters is  $\Delta\text{bias}$  between the two off-peak points.

## Dependence of the bias vs $\sqrt{s}$

Shift all curves such that the shifted bias is zero at  $\sqrt{s_-} = 87.9$  GeV.

To which precision do we know the point at  $\sqrt{s_+} = 94.3$  GeV ?

- Black symbols vs curve: difference between radiations and no radiation at all
- Red vs black symbols: difference between detector-level and gen-level



Full difference of  $\sim 500$  keV.

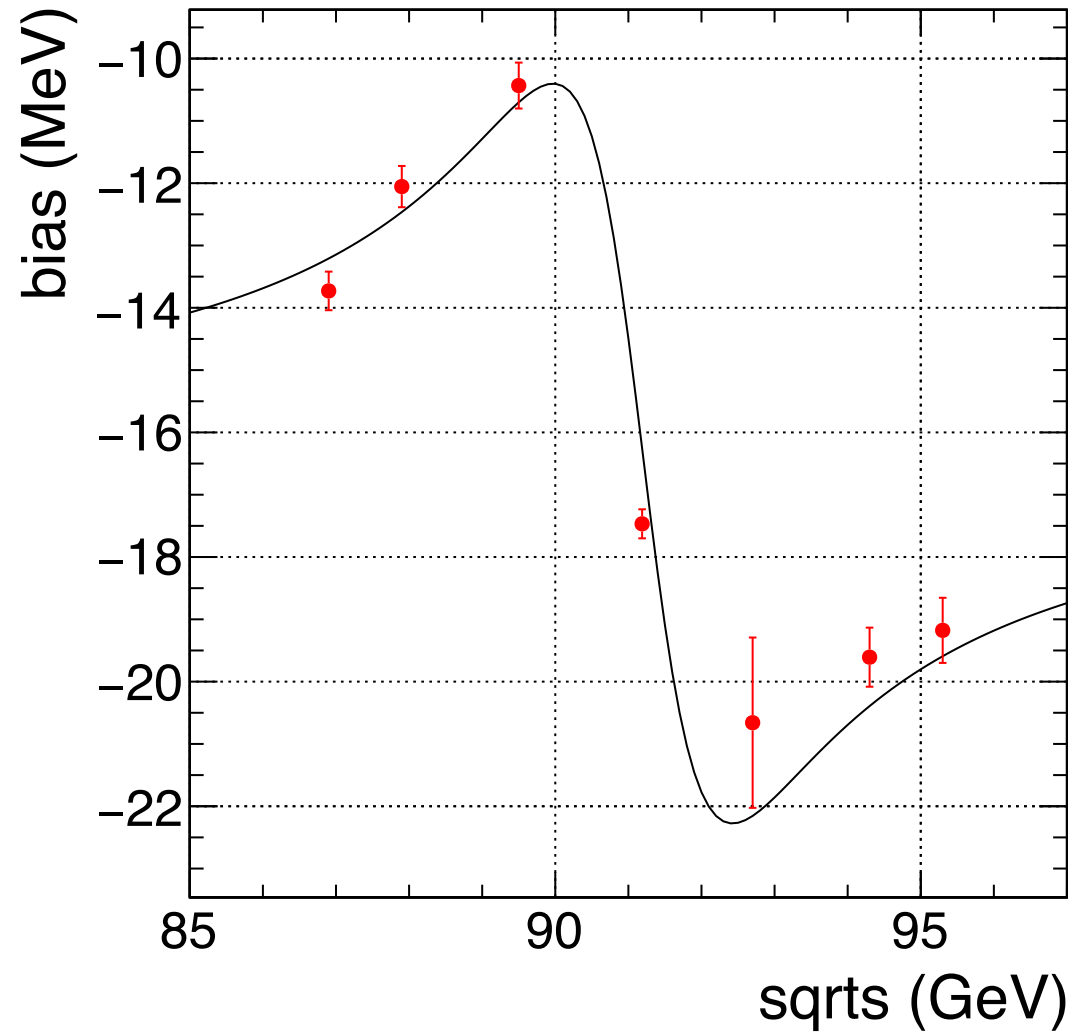
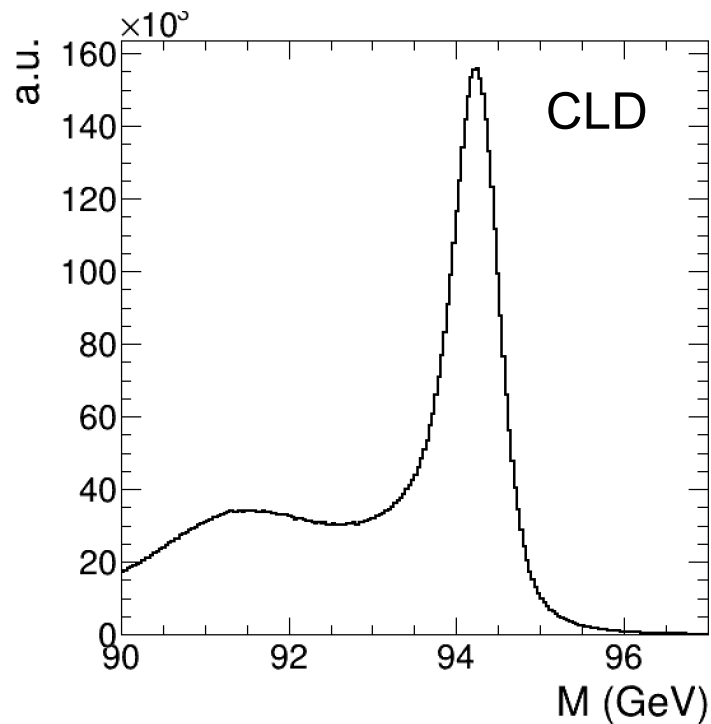
Naively, would need to know the ISR/FSR effects and the detector response to 5% ( 1% ) to ensure a systematic uncertainty on  $\Delta$ bias below 25 keV (5 keV).

Probably within reach.

## CLD samples (Delphes)

Shifted by about -13 MeV compared to the fits to the IDEA samples.

NB: Difficult to fit at  $\sqrt{s}+$  with this function with this exp. resolution. Limited lever-arm to fit the exp. tail w/o seeing the radiative return bump.



## Conclusions and next steps

- Potential to control the point-to-point systematic uncertainty on sqrts to  $\sim 25$  keV with the resolution of the IDEA tracker
  - Currently  $O(3x)$  worse with CLD samples (may be room for improvement from fit model)
  - Some sources of bias are not accounted for in Delphes (e.g muon energy loss in beam pipe). Should not be an issue for the relative bias anyway (t.b.c. e.g. with Bethe-Bloch formula)
- Requires that the momentum scale is stable to  $25 \text{ keV} / 100 \text{ GeV} = \text{a few } 1e-7$  !
  - NMR probes ? ...
  - or in-situ, using low mass resonances
    - demands excellent momentum resolution for soft(er) tracks
    - may also put requirements on PID (e.g. for  $D0 \rightarrow K \pi$  )
    - yet to be quantified