Impact of the uncertainty in the ISR predictions on the determination of centre-of-mass energy spread using dimuon events.

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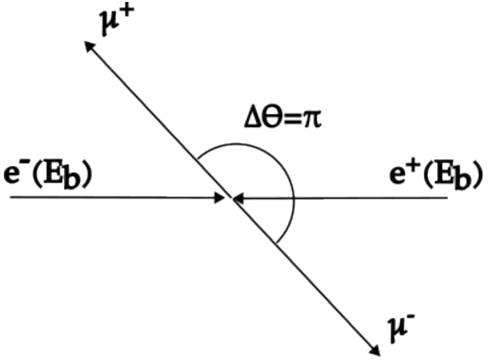
Motivation

- The measurements most affected by the energy calibration uncertainties are those associated with a phenomenon having a strong variation with $\sqrt{s}$ (m_Z, Γ_Z, Z peak cross section σ^0_{had}, $A^{\mu\mu}_{FB}$, $\sin^2\theta^{eff}_W$, $\alpha_{QED}(m^2_Z)$)
- absolute energy scale and its reproducibility can be controlled with the frequent resonant depolarization measurements of the three-point scan (at, below and above the Z peak), point-to-point uncertainty can be controlled with the dimuon invariant mass distribution (See Emmanuel's presentation today)
- If unknown, or poorly measured, the energy spread will dominate the Z width uncertainty - need to find independent measurements to extract the energy spread to reduce uncertainties (e.g. from muon pairs)

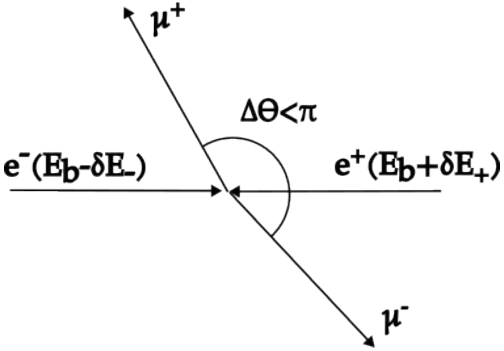
Results from previous studies can be found in: https://arxiv.org/abs/2106.13885

Effect of beam energy spread (BES) and ISR on dimuon final state for head-on collisions

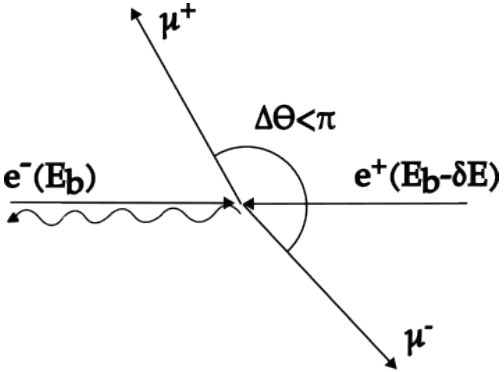
Perfect situation



with BES



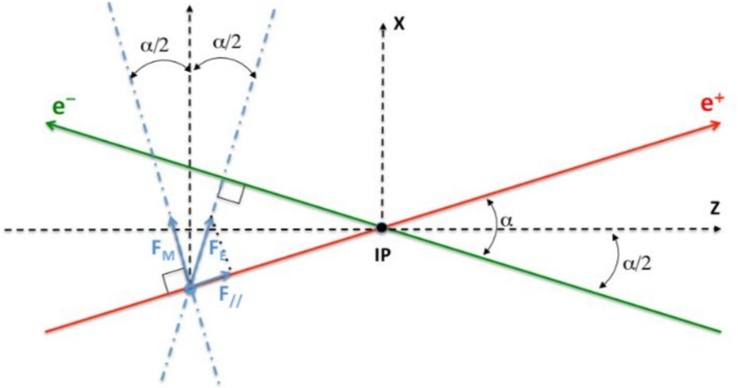
with ISR



The situation at FCC-ee is more complicated due to the large crossing angle and an additional longitudinal boost from RF uneven distribution along the ring but we can still use energy-momentum conservation equations

Definitions and assumptions

- The horizontal plane is defined as the plane subtended the two beams
- Beams cross at an angle α in the horizontal plane, z axis is the bisector between the two beams
- The y axis is perpendicular to the (x,z) plane
- Polar angle θ defined wrt the z axis
- Azimuthal angle φ defined in the (x,y) plane
- ε - relative average difference between the two beam energies (relative longitudinal boost)



$$s = (p_e^+ + p_e^-)^2$$

$$e^+(E_e^+ \sin\frac{\alpha}{2}, 0, E_e^+ \cos\frac{\alpha}{2}, E_e^+)$$

$$e^-(E_e^- \sin\frac{\alpha}{2}, 0, E_e^- \cos\frac{\alpha}{2}, E_e^-)$$

$$E_e^\pm = E(1 \pm \epsilon)$$

$$\sqrt{s} = 2E\sqrt{1-\epsilon^2}\cos\frac{\alpha}{2}$$

Total energy and momentum conservation

$$E^+ \cos\theta^+ + E^- \cos\theta^- = (E_e^+ - E_e^-)\cos\alpha/2 = \epsilon\sqrt{s/1-\epsilon^2}$$
$$E^+ \sin\theta^+ \sin\varphi^+ + E^- \sin\theta^- \sin\varphi^- = 0$$
$$E^+ \sin\theta^+ \cos\varphi^+ + E^- \sin\theta^- \cos\varphi^- = (E_e^+ + E_e^-)\sin\alpha/2 = \epsilon\sqrt{s/1-\epsilon^2}\tan\alpha/2$$
$$E^+ + E^- = E_e^+ + E_e^- = \epsilon\sqrt{s/1-\epsilon^2}/\cos\alpha/2$$

where θ^+, θ^-, φ^+, φ^- are $\mu^\pm$ angles and E^+, E^- are $\mu^\pm$ energies.

From the above system of equations one can determine:

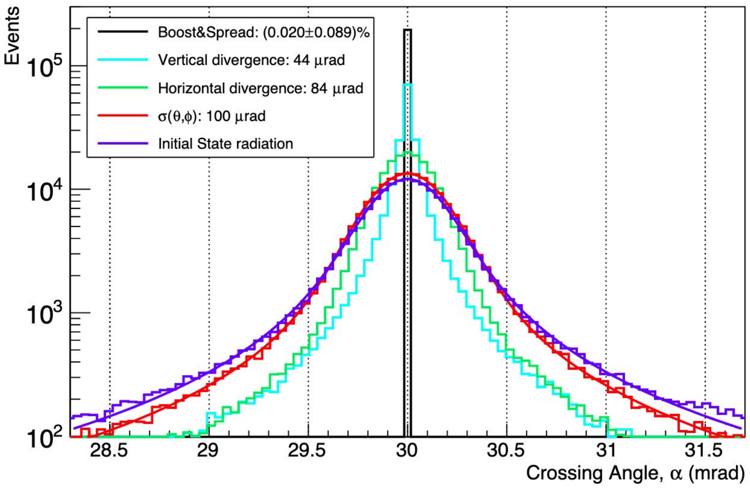
$$\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

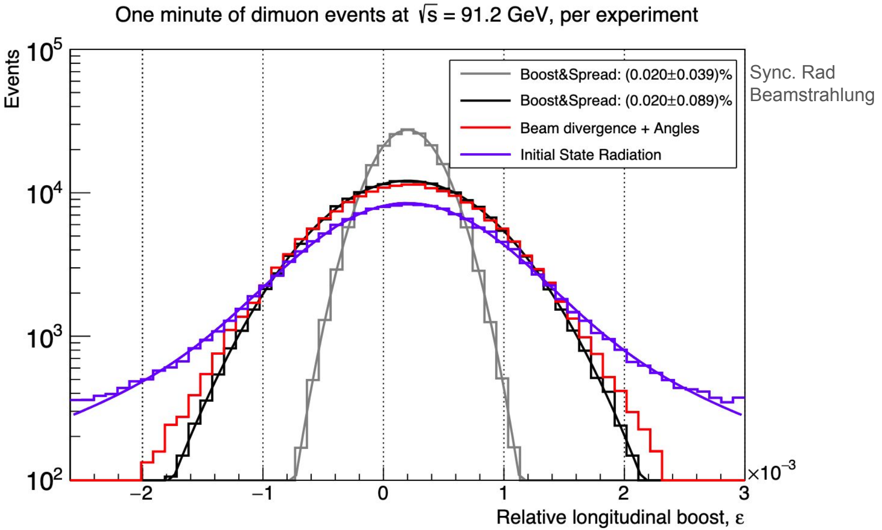
$$\boxed{\varepsilon = \frac{x_+\cos\theta^+ + x_-\cos\theta^-}{\cos\alpha/2},}$$

with $x_\pm = \dfrac{\mp\sin\theta^\mp\sin\varphi^\mp}{\sin\theta^+\sin\varphi^+ - \sin\theta^-\sin\varphi^-}$

See Patrick’s talk from EPOL workshop:
https://indico.cern.ch/event/1181966/contributions/5049894/attachments/2512991/4319811/WG4_EnergySpread.pdf

Nuisances for α and ε





For α, the nuisances are:

- spread of the crossing angle itself (in the vertical and the horizontal plane)
- angular resolution
- ISR

For ε, the nuisances are:

- synchrotron radiation
- beamstrahlung

} Responsible for the beam energy spread (BES)

- angular resolution of the tracker for muons
- ISR

Strategy

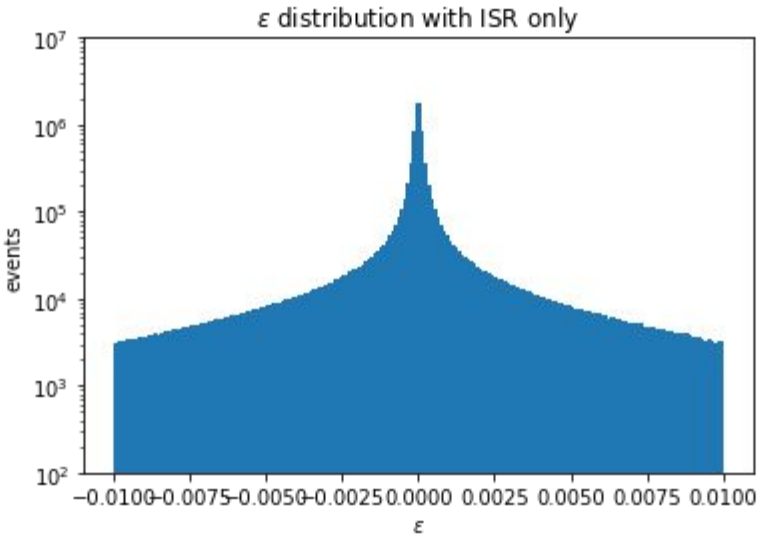
In the absence of ISR and perfect muon angular resolutions, ε distribution is the distribution of centre-of-mass energy spread (the centre-of-mass energy spread equals BES/√2). In the study muon angular resolution of 0.1 mrad is assumed to be known perfectly, BES distribution is assumed to be Gaussian, but any distribution can be assumed a priori. To assess impact of ISR uncertainty on this distribution following steps were made:

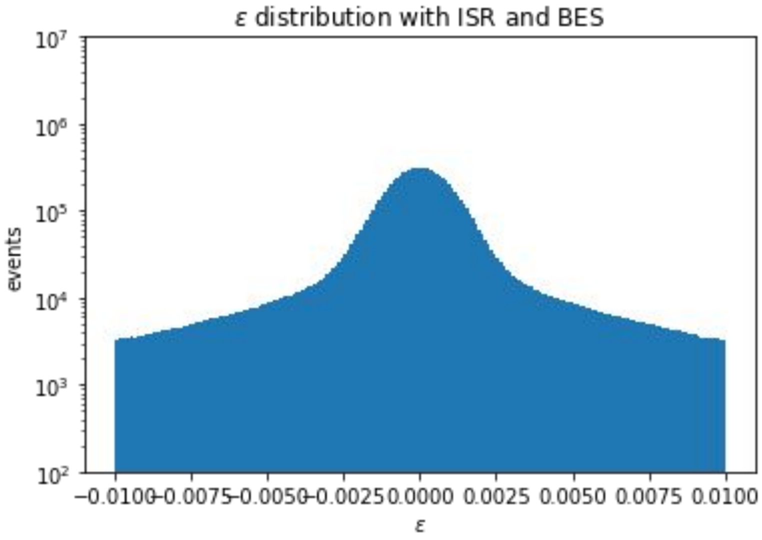
- Find ε distribution for two samples (with ISR, with ISR and BES)
- Unfold the ISR and the angular resolution effects from the ε distribution, to obtain the beam energy spread distribution
- Fit Gaussian to unfolded distribution
- See the effect of a different ISR spectrum on Gaussian fit parameters by repeating unfolding procedure for reweighted ε distributions with assumption of the default ISR spectrum. The choice of the different ISR spectra is guided by the current theoretical uncertainties, as implemented in KKMC.

Simulation

- two samples of 10^7 $e^+e^- \rightarrow \mu^+\mu^-$ events were generated at the Z pole using KKMCee:
  - with ISR,
  - with ISR and gaussian beam energy spread of 0.132% (BES/√2 = 0.132%/√2 = 0.093338%)
- to account for different ISR spectra events were weighted:
  - CEEX2 - $O(\alpha^2)$ QED matrix element (default)
  - EEX2 - $O(\alpha^2)$ QED matrix element (alternative)
  - CEEX1 - $O(\alpha^1)$ QED matrix element
- after generating:
  - the system was boosted to account for 30 mrad crossing angle

Epsilon distributions (with CEEX2 matrix element)





Unfolding

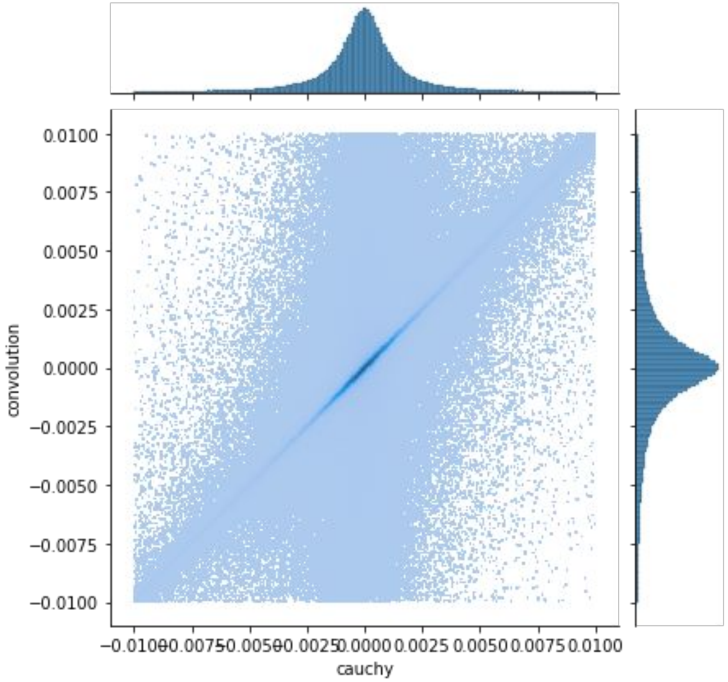
For unfolding TUnfold class was used (https://root.cern.ch/doc/master/classTUnfold.html)

Matrix of migrations (matrix of probabilities describing the migrations from bin j of original distribution to any of the n bins of the affected distribution) used for unfolding is constructed as 2D histogram:

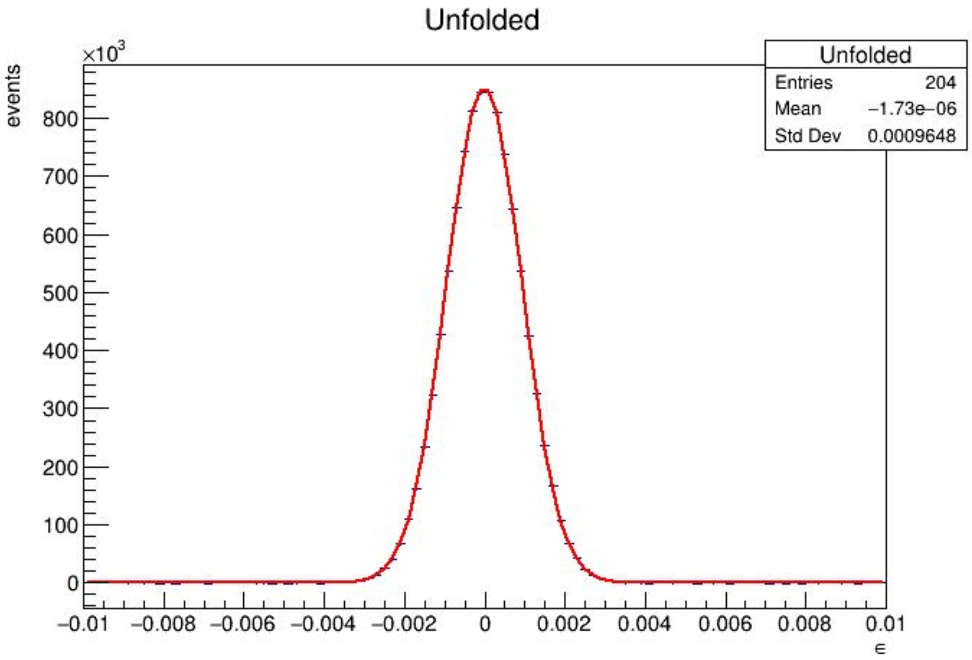
- Cauchy distribution (arbitrary choice) on x axis - sampling distribution
- convolution of the sampling (Cauchy) distribution with ISR only (CEEX2 matrix element) on y axis.

The matrix was applied to ε distribution with all effects (with CEEX2, EEX2 and CEEX1 weights respectively) to get the unfolded energy spread distribution.

Matrix of migrations



Unfolding (with CEEX2 matrix element in the simulated events and in the unfolding procedure)



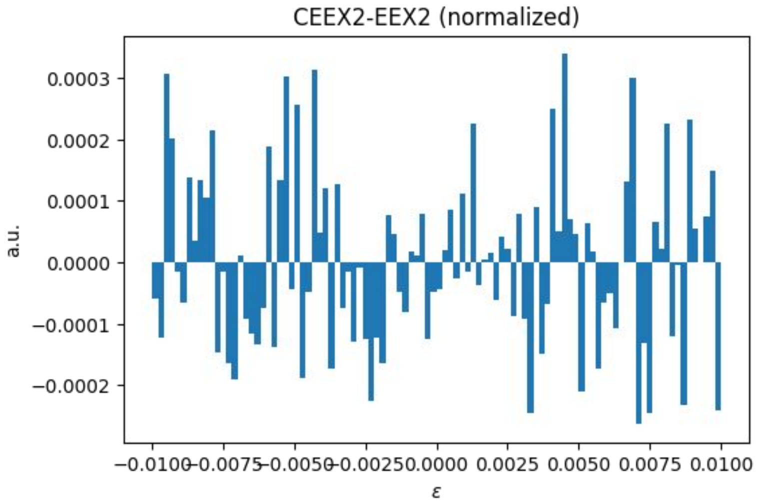
Fit results:

NO.	NAME	VALUE	ERROR
1	Constant	8.53097e+05	5.77793e+02
2	Mean	-3.50388e-07	6.60867e-07
3	Sigma	9.34855e-04	6.09540e-07

Sigma differs from the expected value by 10^{-6}, i.e., 1.5 standard deviation (stat. of the dimuon sample)

BES/$\sqrt{2}$ = 9.3338×10^{-4}

CEEX2 vs EEX2



Difference between normalised ε distributions with CEEX2 and EEX2 weights

CEEX2 fit results

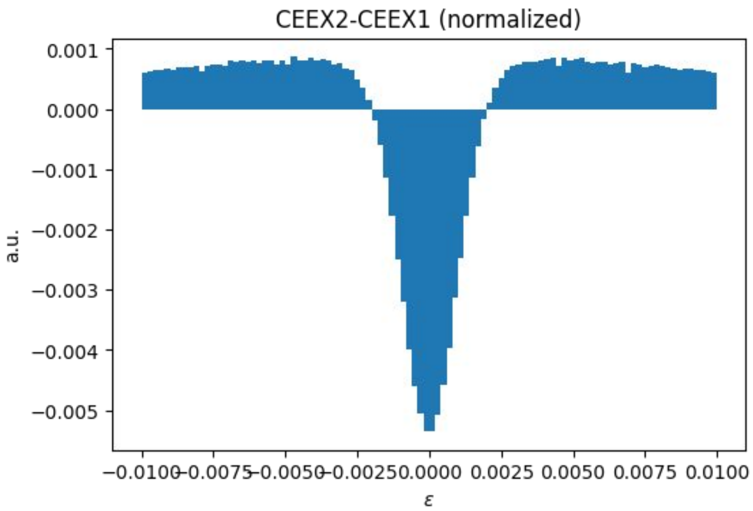
NO.	NAME	VALUE	ERROR
1	Constant	8.53097e+05	5.77793e+02
2	Mean	-3.50388e-07	6.60867e-07
3	Sigma	9.34855e-04	6.09540e-07

EEX2 fit results

NO.	NAME	VALUE	ERROR
1	Constant	8.53047e+05	5.77727e+02
2	Mean	-3.50563e-07	6.60830e-07
3	Sigma	9.34856e-04	6.09506e-07

The difference is at the level of 10^{-9}, which shows the sensitivity of the centre-of-mass energy spectrum determination to the ISR theoretical uncertainty.

CEEX2 vs CEEX1



Difference between normalised ε distributions with CEEX2 and CEEX1 weights

CEEX2 fit results

NO.	NAME	VALUE	ERROR
1	Constant	8.53097e+05	5.77793e+02
2	Mean	-3.50388e-07	6.60867e-07
3	Sigma	9.34855e-04	6.09540e-07

CEEX1 fit results

NO.	NAME	VALUE	ERROR
1	Constant	8.52367e+05	5.77282e+02
2	Mean	-3.50450e-07	6.60845e-07
3	Sigma	9.34851e-04	6.09516e-07

Even with a deliberate and large overestimate of the ISR theoretical uncertainty (assuming that the second order is not known), the effect on the centre of mass energy spread is of the order of $4{\times}10^{-9}$.

Summary

Since the different ISR spectra are applied as weights to generated events and the same matrix of migrations was used in all cases, the difference in Gaussian fit parameters is not affected by statistical uncertainty.

Systematic uncertainty due to ISR spectrum for σ_{gauss} is 5×10^{-9} which is 2 orders of magnitude smaller than statistical error determined with 10^7 dimuon events.
This precision is much better that what is needed for EWPO measurement (the relative centre-of-mass energy spread uncertainty should be smaller than 0.1% to have an effect smaller than 20 keV on Γ_Z, the exercise shows that the relative uncertainty due to ISR knowledge is at most ~ 5×10^{-6}).

	CEEX2 (default)	EEX2	CEEX1
μ_{gauss}	-3.50388e-07	-3.50563e-07	-3.50450e-07
σ_{gauss}	9.34855e-04	9.34856e-04	9.34851e-04

Next steps

Although ISR has dominant impact on centre-of-mass energy spread distribution, we should take into account other nuisances as well. In the study the angular resolution of 0.1 mrad was assumed to be known perfectly. The next steps are:

- to do the same exercise with 0, 0.1, 0.5 and 1 mrad resolutions,
- see the effect of a 10% uncertainty on these resolutions,
- see with which uncertainty these resolutions can be determined from full simulated data

Backup

The code can be found in:

https://gitlab.cern.ch/mkazanec/ffgamma