Karlsruhe Institute of Technology

Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Higgs boson pair production in HEFT and SMEFT:

Truncation and other uncertainties

Gudrun Heinrich

Institute for Theoretical Physics
Karlsruhe Institute of Technology

EFT WG Open Meeting

September 23, 2024

Outline

- Status SM uncertainties in Higgs boson pair production
  - scale uncertainties
  - EW corrections
  - scheme uncertainties
  - PDF + alpha_s uncertainties
- EFT-related:
  - HEFT versus SMEFT
  - truncation uncertainties
  - subleading operators
  - scheme uncertainties
  - running Wilson coefficients

based on work in collaboration with

https://arxiv.org/abs/2204.13045 GH, Jannis Lang, Ludovic Scyboz ggHH_SMEFT code

https://arxiv.org/abs/2311.15004 GH, Jannis Lang subleading operators

https://arxiv.org/abs/2310.18221 Stefano Di Noi, Ramona Gröber, GH, Jannis Lang, Marco Vitti

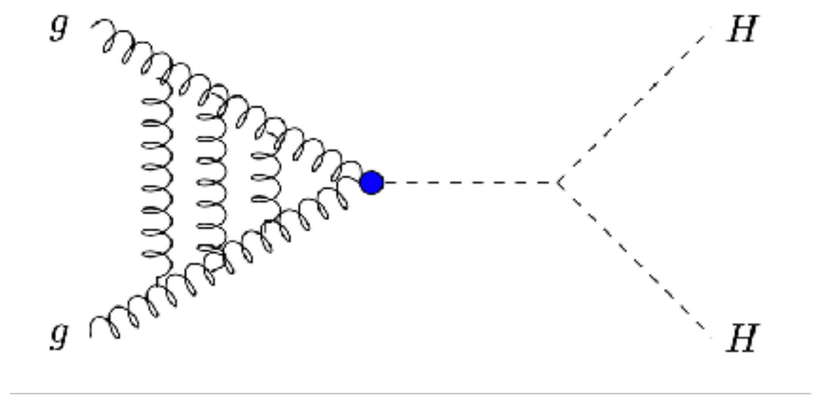
gamma5 scheme (in)dependence

https://arxiv.org/abs/2407.04653 Stephen Jones, Matthias Kerner, GH, Tom Stone, Augustin Vestner

EW corrections in gaugeless limit

also: CERN WG4 note about EFT descriptions of HH production https://arxiv.org/abs/2304.01968

ggHH: higher order QCD corrections in the SM



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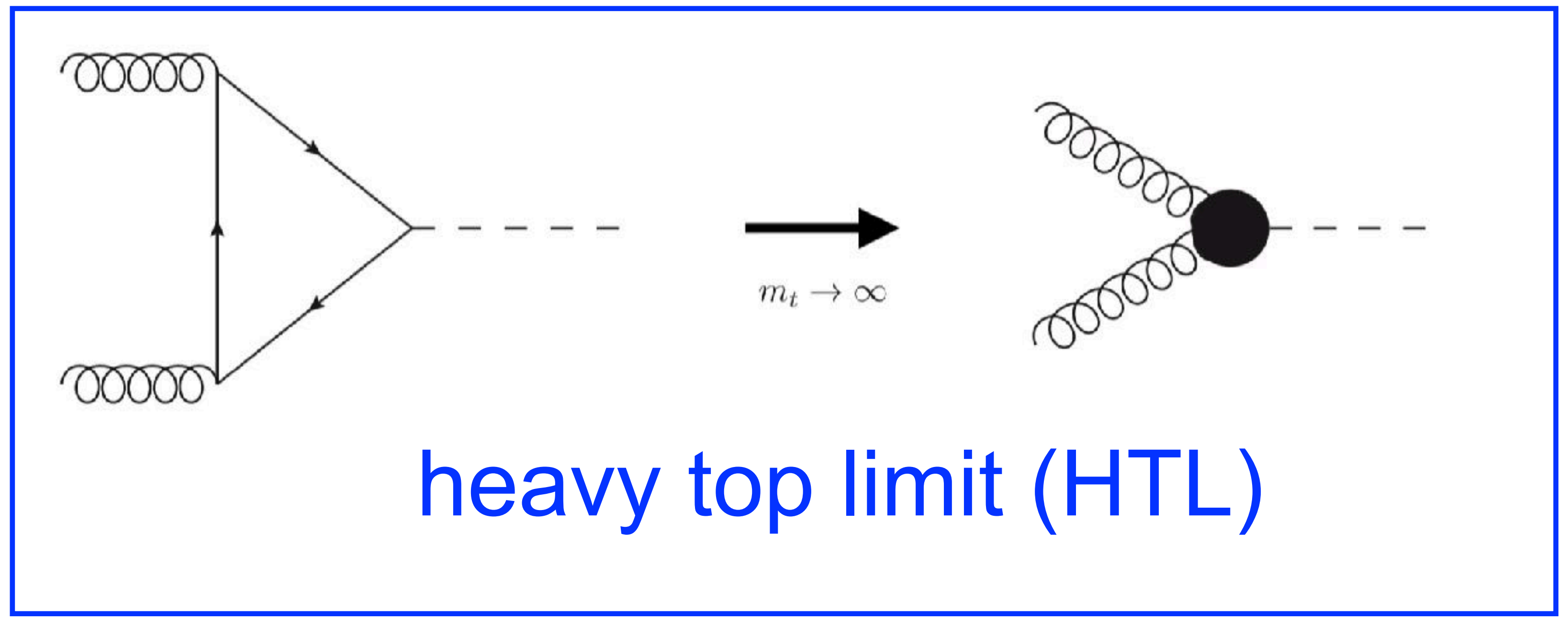

heavy top limit (HTL)

NLO full m_t

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$\mathrm{NNLO}_{\mathrm{FTapprox}}$ Grazzini, Kallweit, GH, Jones, Kerner, Lindert, Mazzitelli ‘18

inclusion of top quark mass dependence except in virtual $\mathcal{O}(\alpha_s^3)$

top quark mass scheme uncertainties: pole mass versus $\overline{\mathrm{MS}}$ mass

Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira ’18, ‘20

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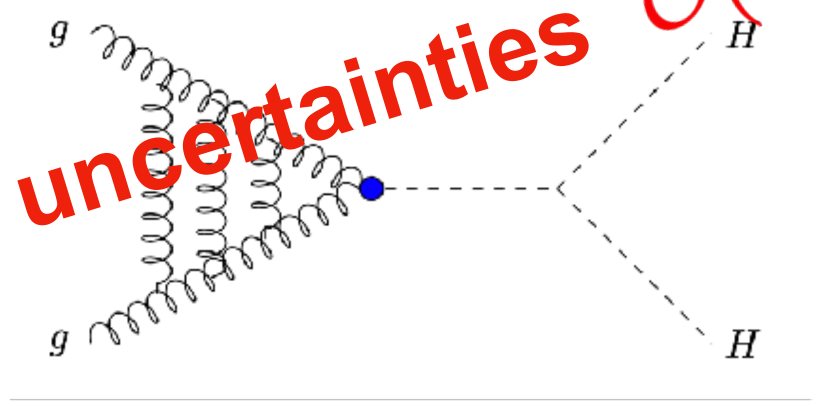
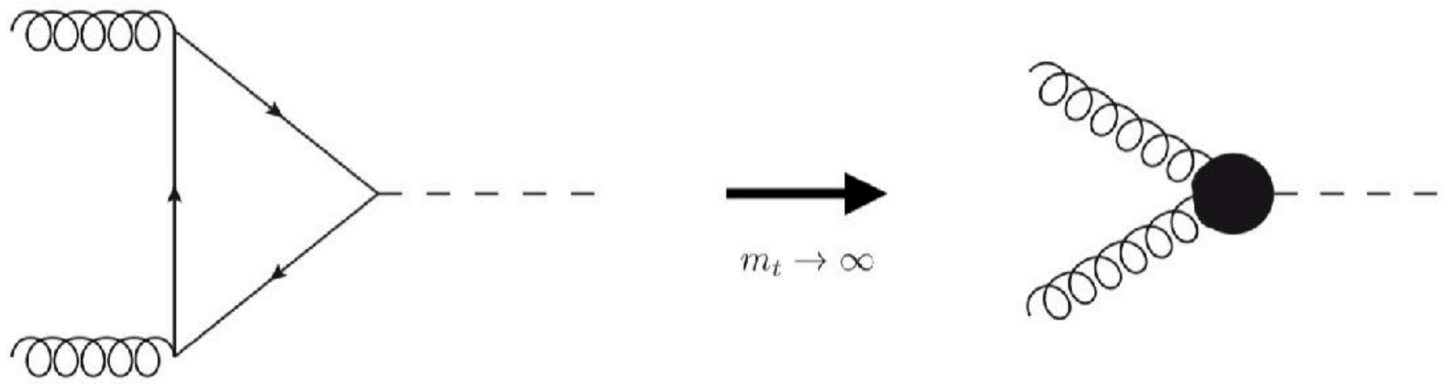


scale uncertainties $\mathcal{O}(3\%)$



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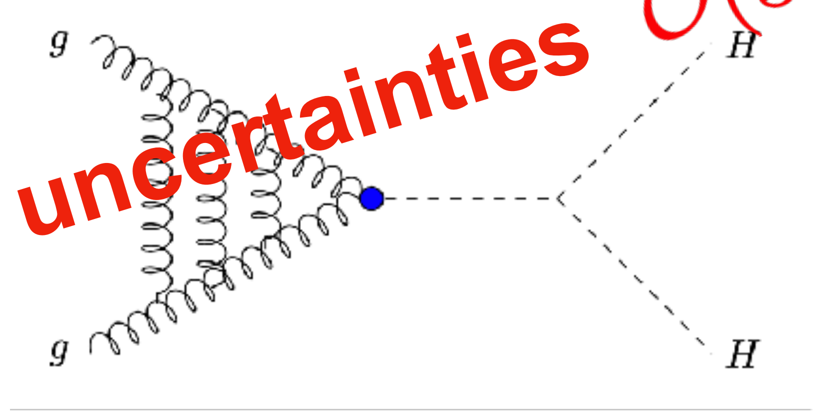

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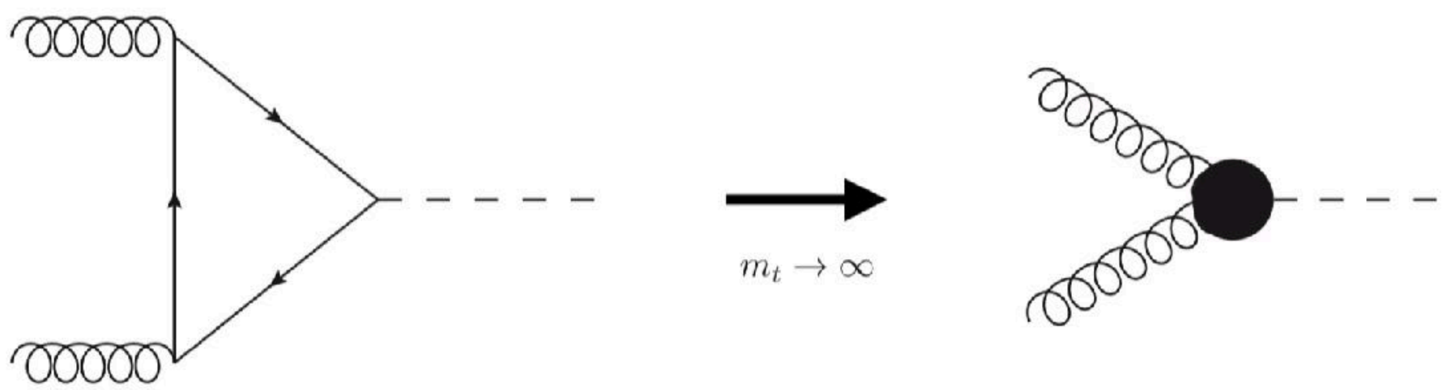

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residual missing top mass effects, estimated to $\mathcal{O}(5\%)$

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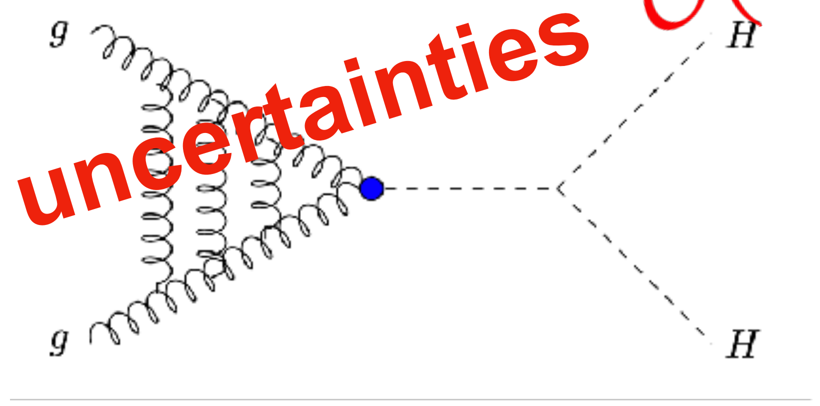

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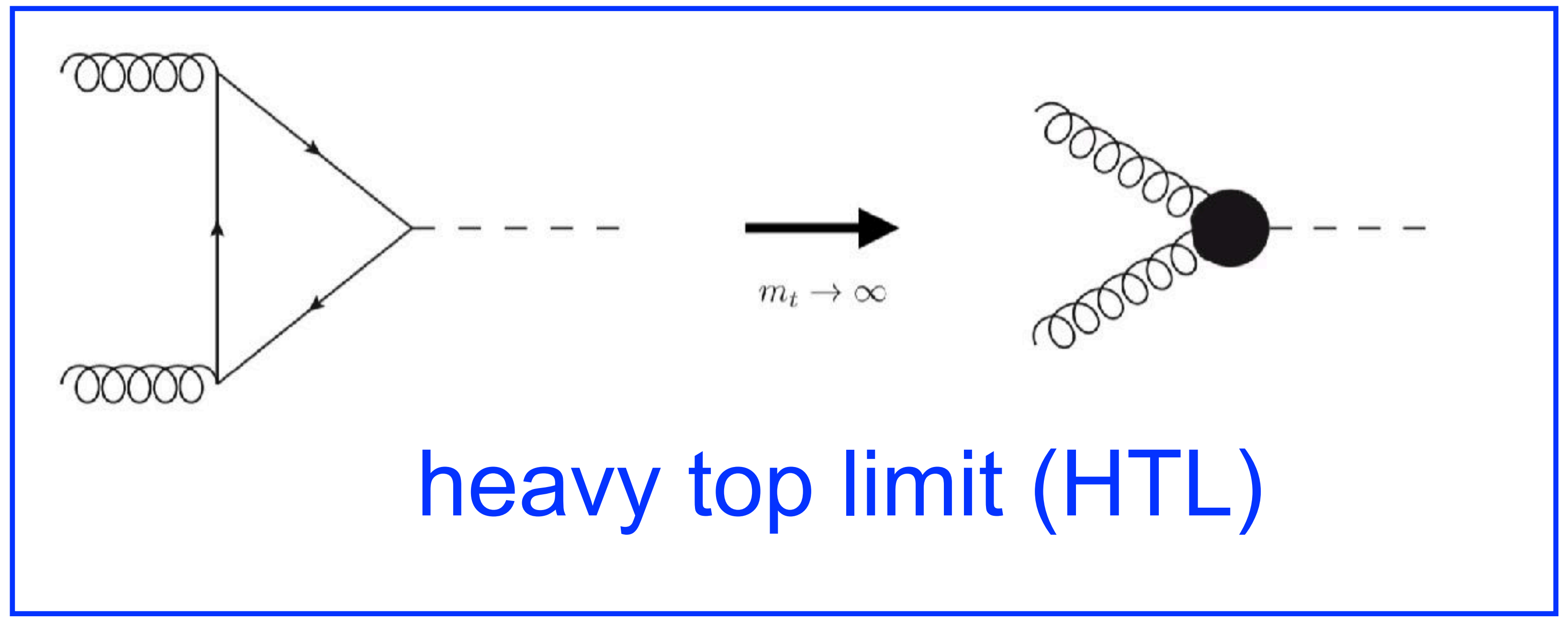

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uncertainty due to top mass scheme $\mathcal{O}(20\%)$

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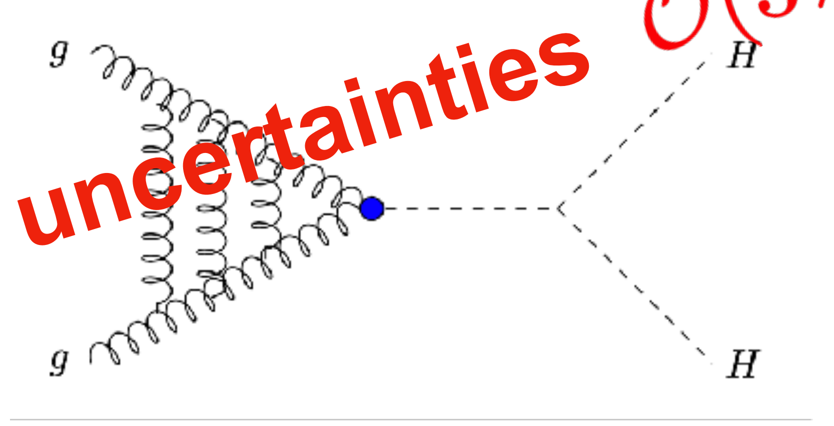

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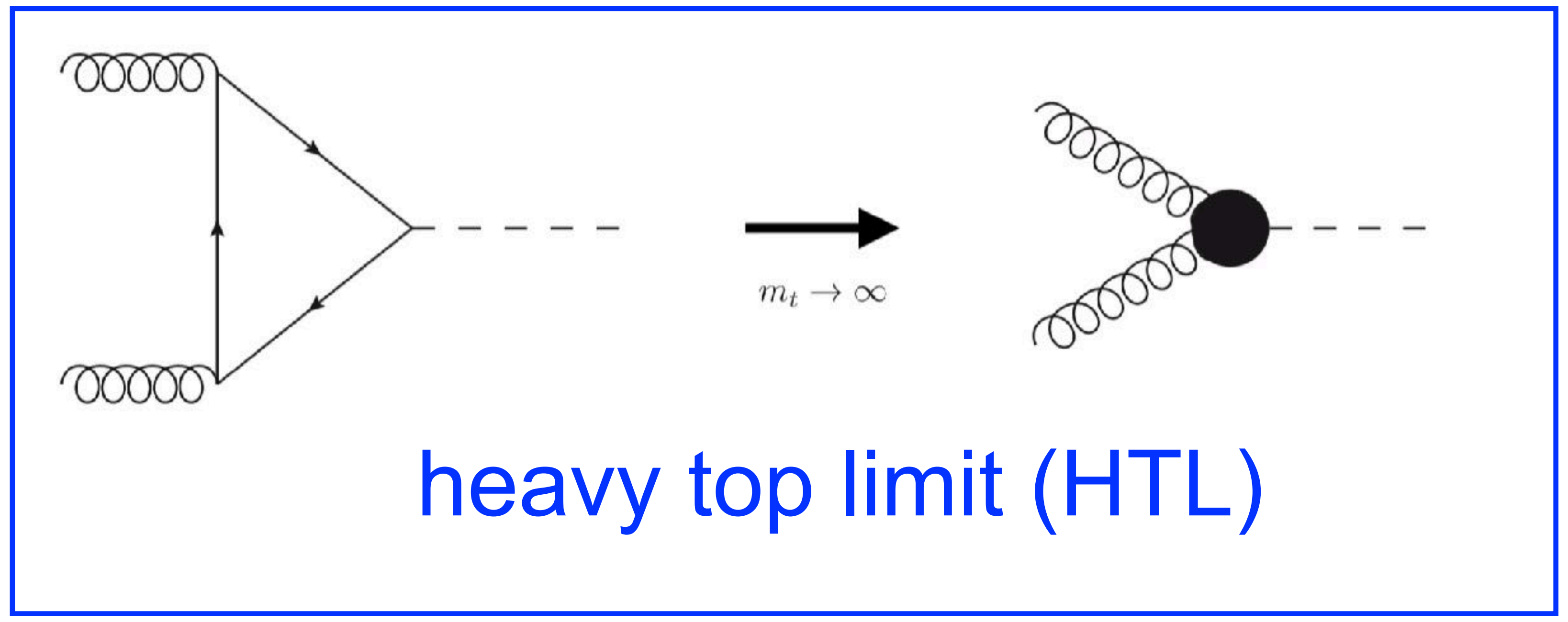

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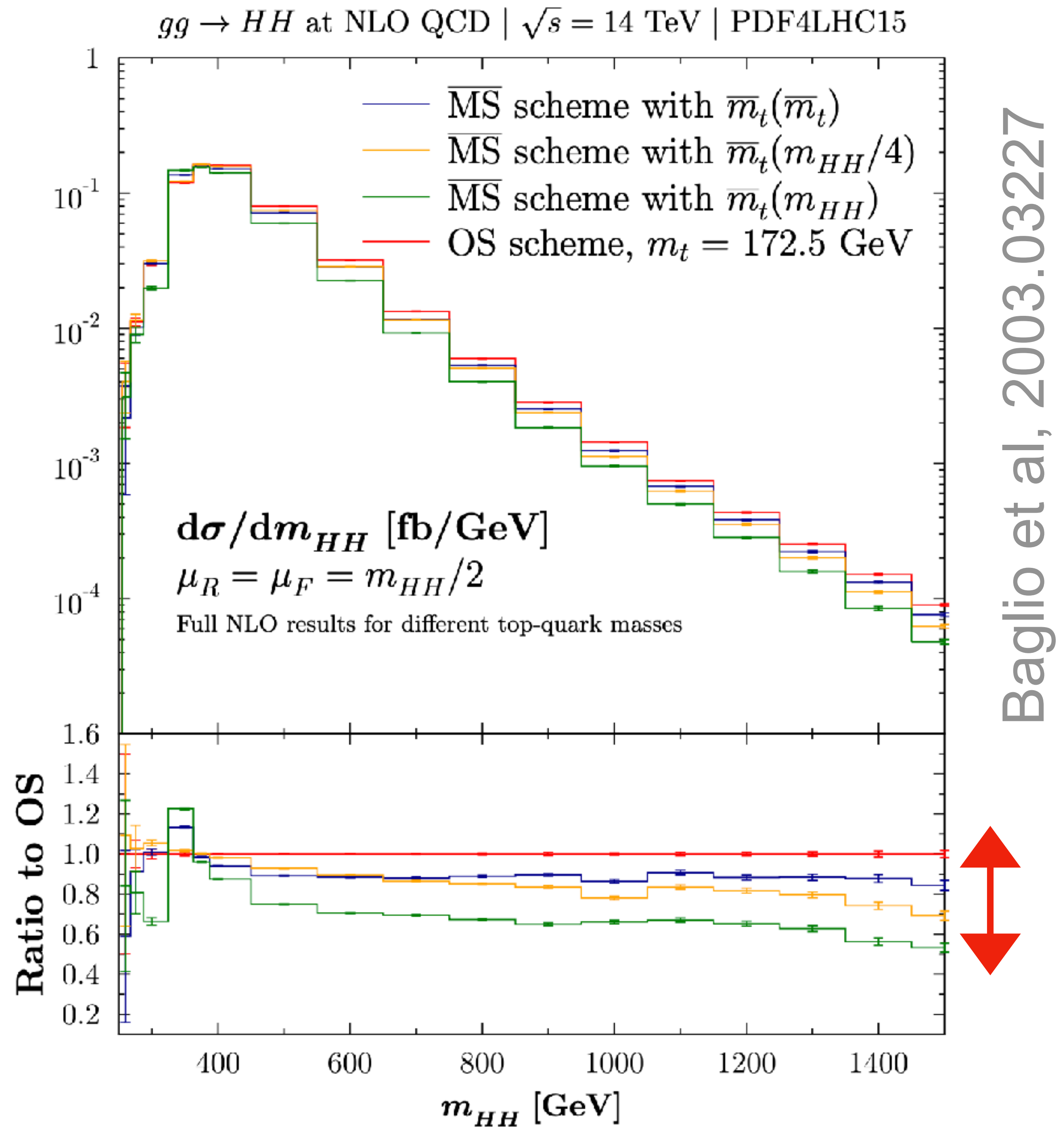
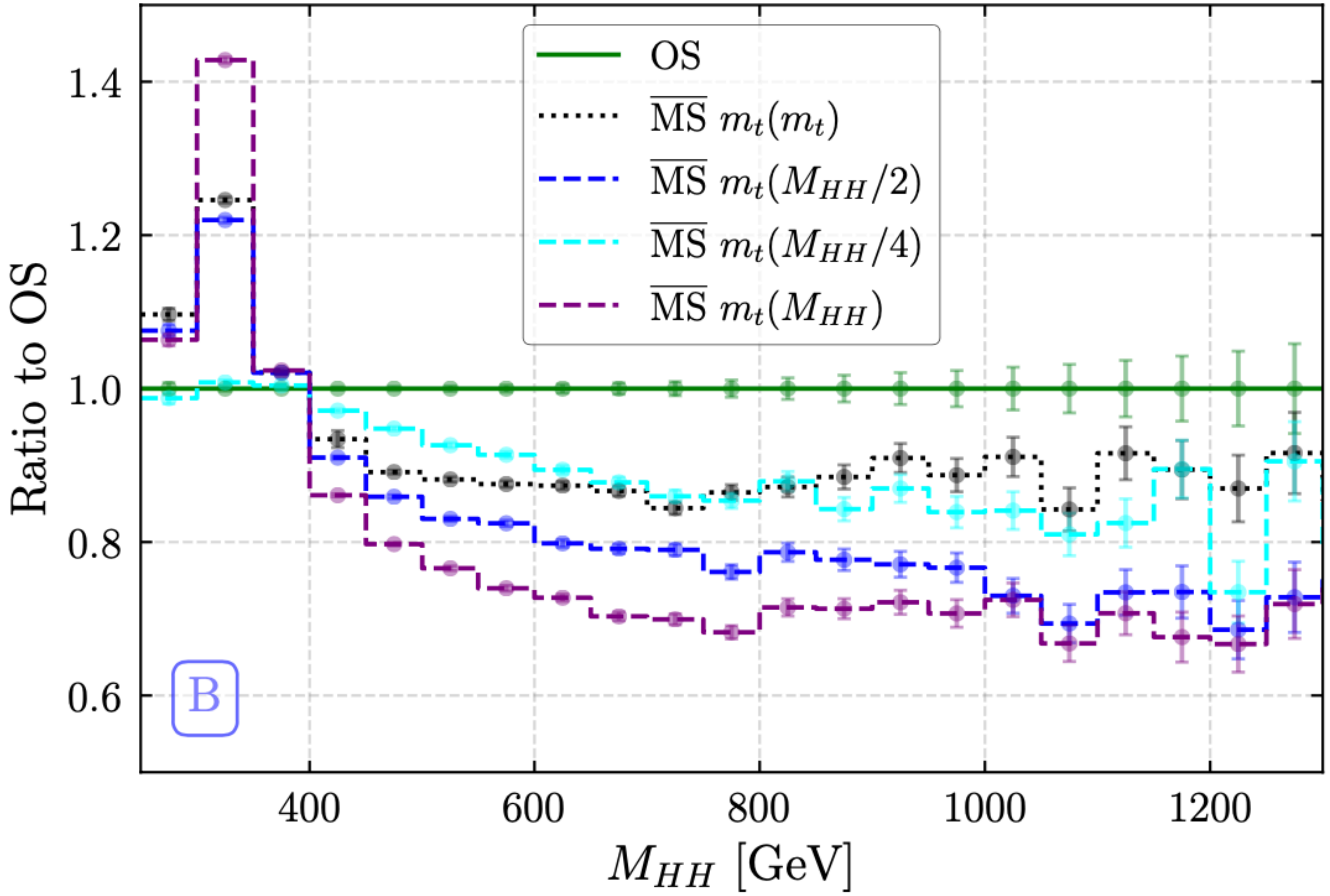


Baglio et al, 2003.03227

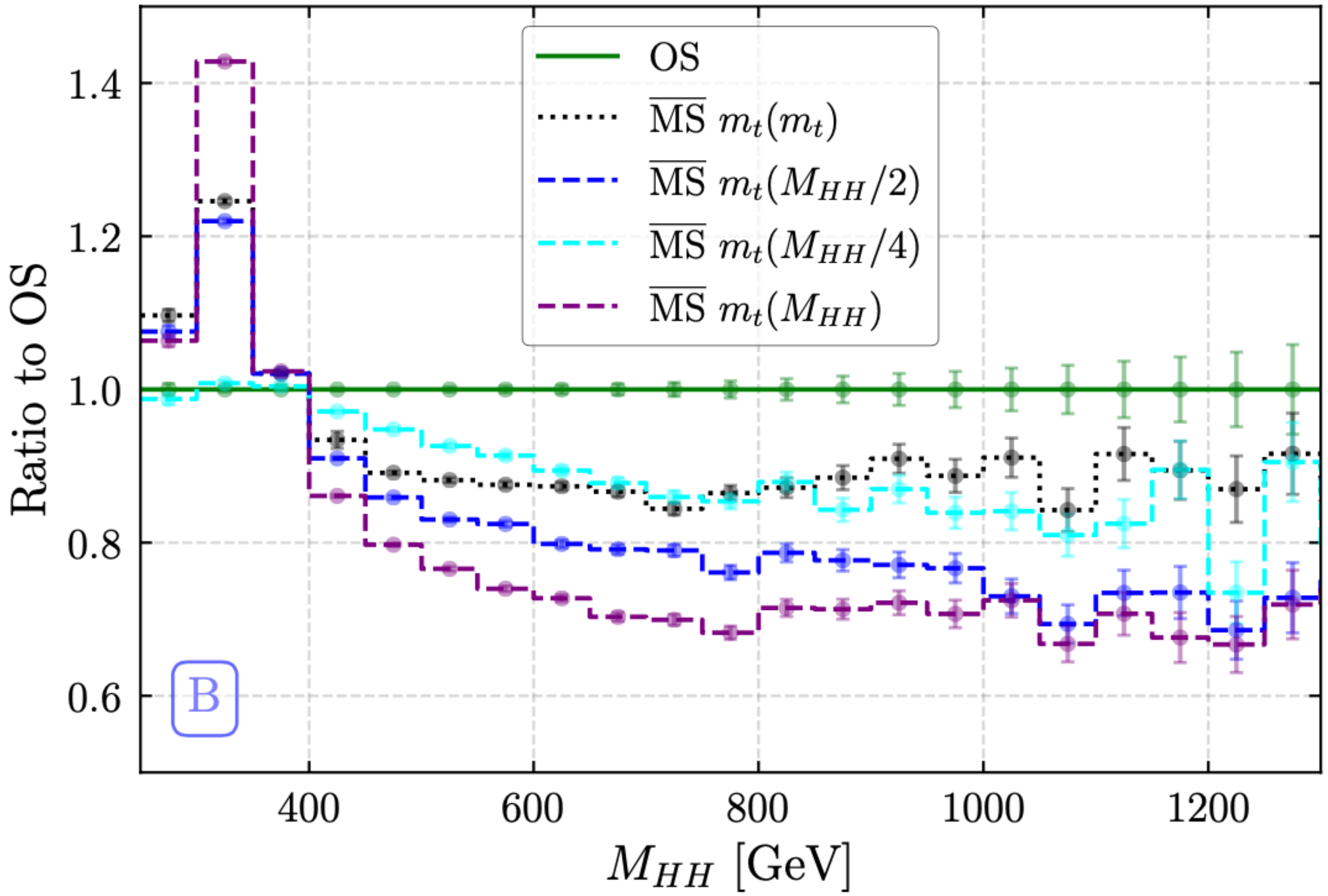
scheme uncertainties (top mass)



Bagnaschi, Degrassi, Gröber 2309.10525

PDF + α_s uncertainties ~ 2.3%

scheme uncertainties (top mass)



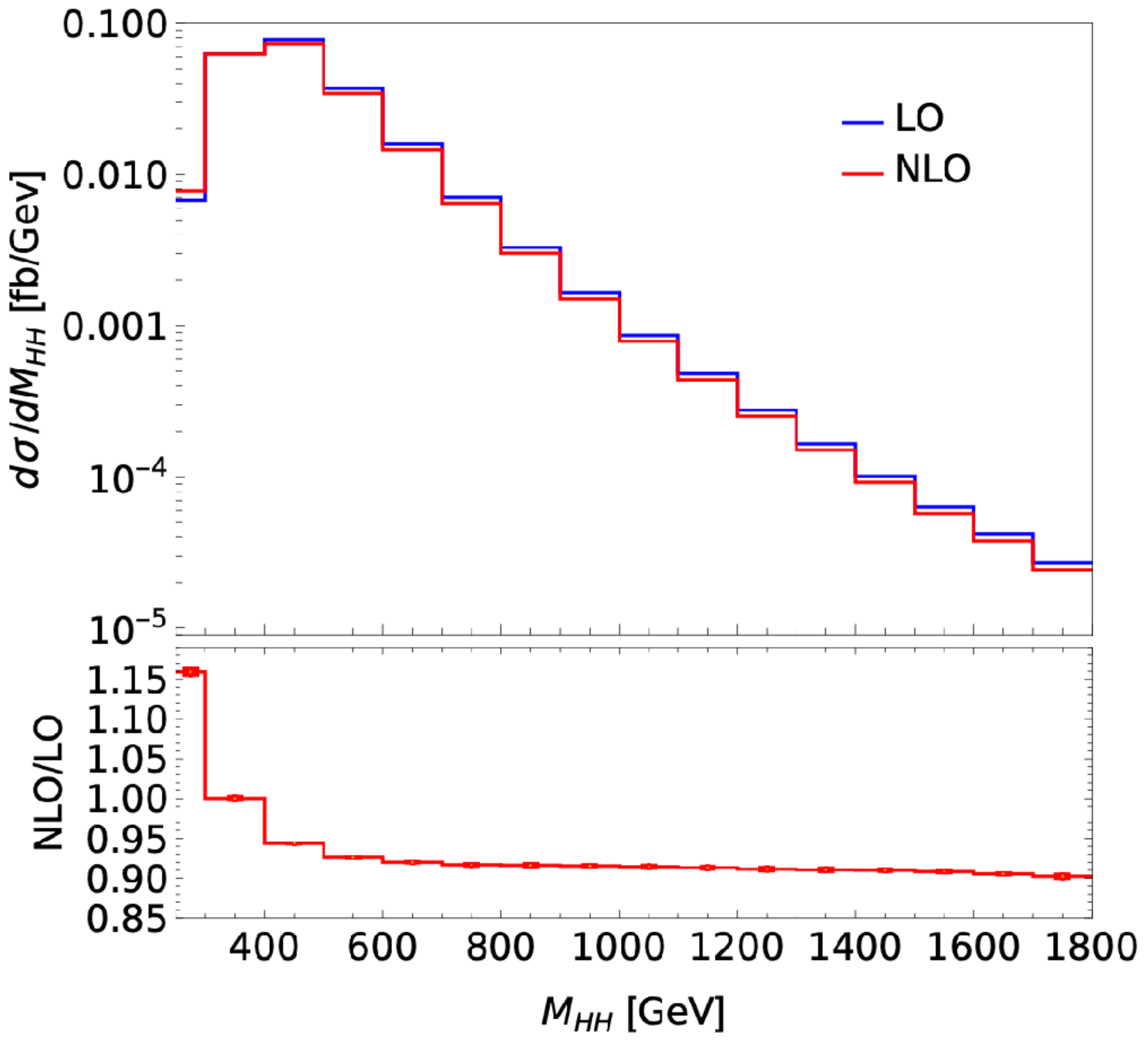
Bagnaschi, Degrassi, Gröber 2309.10525

PDF + α_s uncertainties ~ 2.3%

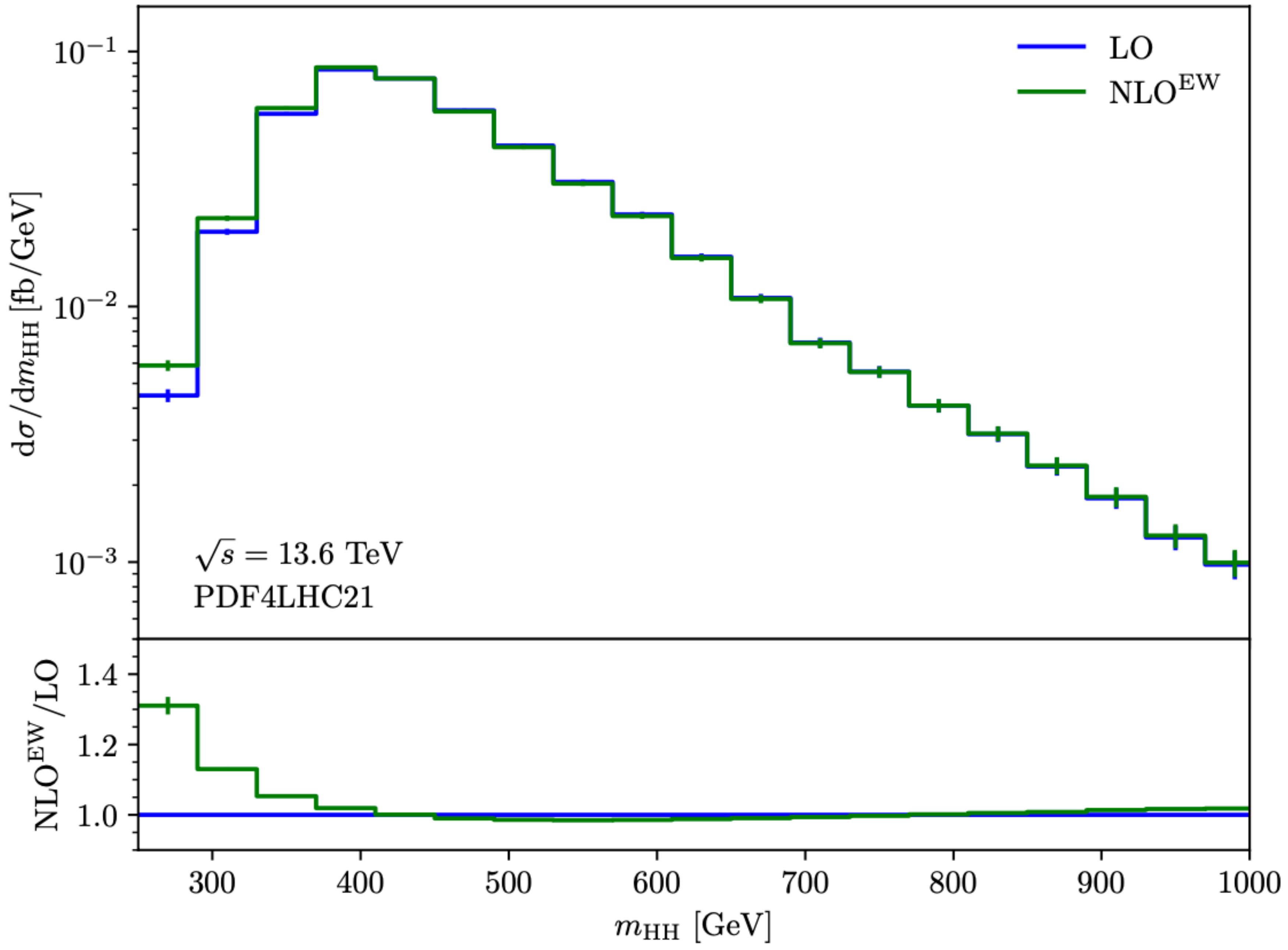
top mass scheme uncertainty currently
largest uncertainty in
Higgs boson pair production

SM electroweak corrections

full EW: Bi, Huang, Huang, Ma, Yu ’23



Yukawa- and Higgs self-coupling type corrections:



GH, Jones, Kerner, Stone, Vestner ‘24

see also

heavy top limit, high energy expansion

Davies, Mishima, Schönwald, Steinhauser, Zhang ‘22

Yukawa coupling corrections in (partial) HTL

Mühlleitner, Schlenk, Spira ’22

full EW in large-mt expansion ‘23
+factorisable contributions ‘24

Davies, Schönwald, Steinhauser, Zhang

cancellations between gauge-boson and Yukawa-type corrections

partial EW corrections, with coupling modifiers:

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao ’18; Bizon, Haisch, Rottoli ’18, ’24 therefore not very conclusive

EFT descriptions of ggHH

SMEFT (Standard Model Effective Field Theory):

Buchmüller, Wyler ’85; Gradzkowsi et al ’10; Brivio, Trott ’17

$$\boxed{\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{\rm dim6} + \mathcal{O}(\frac{1}{\Lambda^3})}$$

- canonical dimension (mass) counting

HEFT (Higgs Effective Field Theory):

Feruglio ’93; Grinstein, Trott ’07; Contino et al. ’10, Alonso et al. ’13, Brivio et al. ‘13, Buchalla et al. ’13

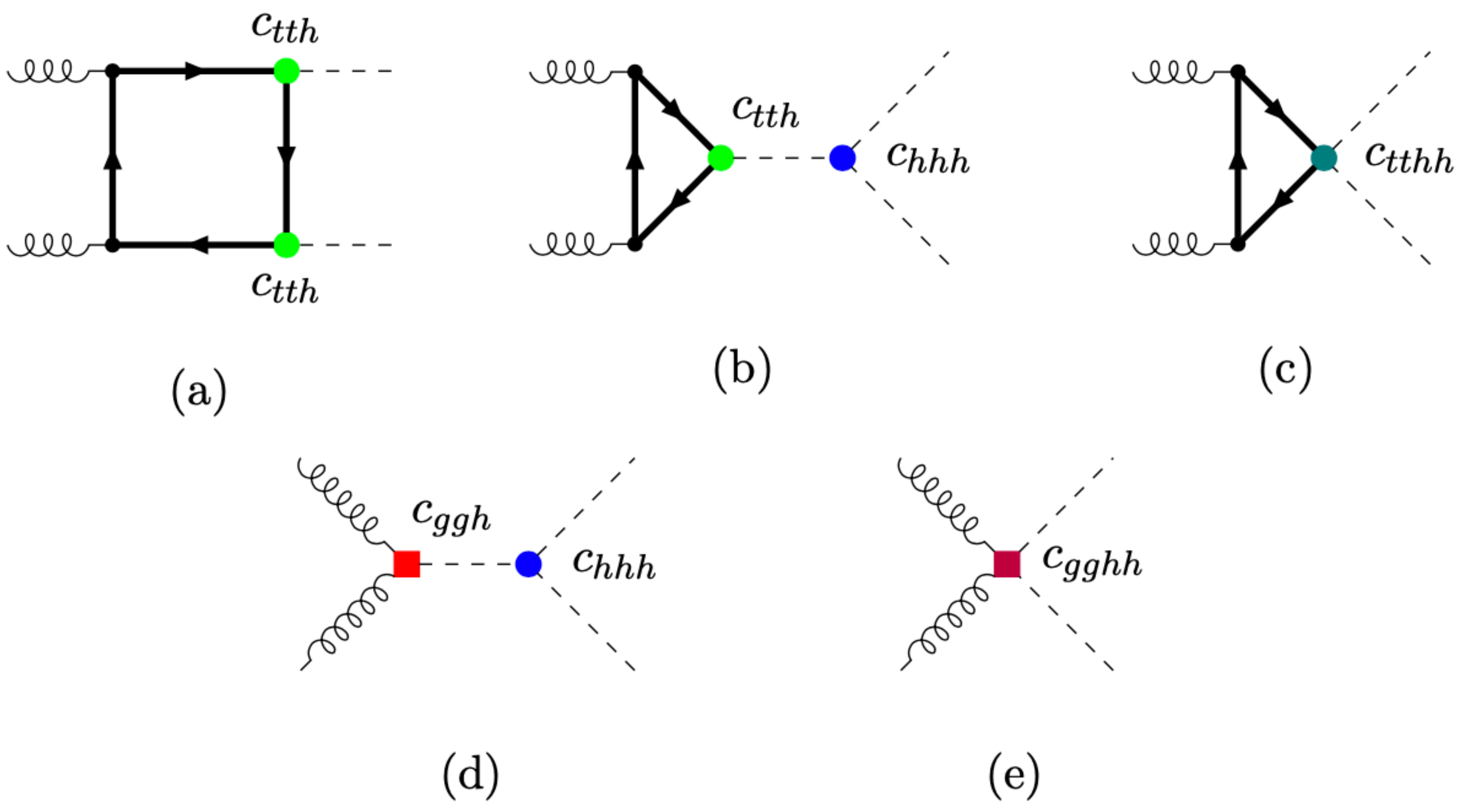
$$\boxed{\mathcal{L}_{d_\chi} = \mathcal{L}_{(d_\chi=2)} + \sum_{L=1}^{\infty}\sum_i \left(\frac{1}{16\pi^2}\right)^L c_i^{(L)} O_i^{(L)}}$$

- chiral dimension (loop) counting

Lagrangians relevant for HH production

HEFT:

$$\mathcal{L}_{d_{\chi\leq 4}} \supset -m_t\left(c_t\frac{h}{v}+c_{tt}\frac{h^2}{v^2}\right)\bar{t}\,t - c_{hhh}\frac{m_h^2}{2v}h^3+\frac{\alpha_s}{8\pi}\left(c_{ggh}\frac{h}{v}+c_{gghh}\frac{h^2}{v^2}\right)G^a_{\mu\nu}G^{a,\mu\nu}$$



Lagrangians relevant for HH production

SMEFT: Warsaw basis Grzadkowski et al. 1008.4884

$$\Delta\mathcal{L}_{\text{Warsaw}} = \frac{C_{H,\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2}(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2}(\phi^\dagger\phi)^3 + \left(\frac{C_{uH}}{\Lambda^2}\phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c.\right) + \frac{C_{HG}}{\Lambda^2}\phi^\dagger\phi G^a_{\mu\nu}G^{\mu\nu,a}$$

canonical normalisation

$$C_{H,\text{kin}} := C_{H,\Box} - \frac{1}{4}C_{HD}$$

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$$+\frac{C_{uG}}{\Lambda^2}\left(\bar{q}_L\sigma^{\mu\nu}T^a G^a_{\mu\nu}\phi^c\, t_R + \text{h.c.}\right)$$

(chromomagnetic operator)

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(chromomagnetic operator) **+ 4-fermion operators**

chromo is loop-generated if UV completion a weakly coupled, renormalisable gauge theory

in the HH case, it is inserted already into a SM loop → should be subleading compared to $C_{uH}, C_{HG}, \ldots$

Leading Wilson coefficients relevant for HH production

naive translation HEFT -> SMEFT at dim6 (comparing coefficients at Lagrangian level):

HEFT	Warsaw
c_{hhh}	$1 - 2\frac{v^2}{\Lambda^2}\frac{v^2}{m_h^2}C_H + 3\frac{v^2}{\Lambda^2}C_{H,\mathrm{kin}}$
c_t	$1 + \frac{v^2}{\Lambda^2}C_{H,\mathrm{kin}} - \frac{v^2}{\Lambda^2}\frac{v}{\sqrt{2}m_t}C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2}\frac{3v}{2\sqrt{2}m_t}C_{uH} + \frac{v^2}{\Lambda^2}C_{H,\mathrm{kin}}$
c_{ggh}	$\frac{v^2}{\Lambda^2}\frac{8\pi}{\alpha_s}C_{HG}$
c_{gghh}	$\frac{v^2}{\Lambda^2}\frac{4\pi}{\alpha_s}C_{HG}$

problems:

- two field theories with different assumptions
- valid HEFT point can be invalid after translation to SMEFT
- translation depends on Λ
- treatment of strong coupling

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ggHH and **ggHH_SMEFT** codes

- both codes: NLO QCD with full top quark mass dependence Borowka et al. 2016

implemented in

http://powhegbox.mib.infn.it/User-Process-V2

HEFT: **ggHH** code GH, Jones, Kerner, Scyboz, 2006.16877

5 anomalous couplings

SMEFT: **ggHH_SMEFT** GH, J. Lang, L. Scyboz, 2204.13045

4 leading operators, different truncation options

+ 6 subleading operators (chromo, 4-top) GH, J. Lang, 2311.15004

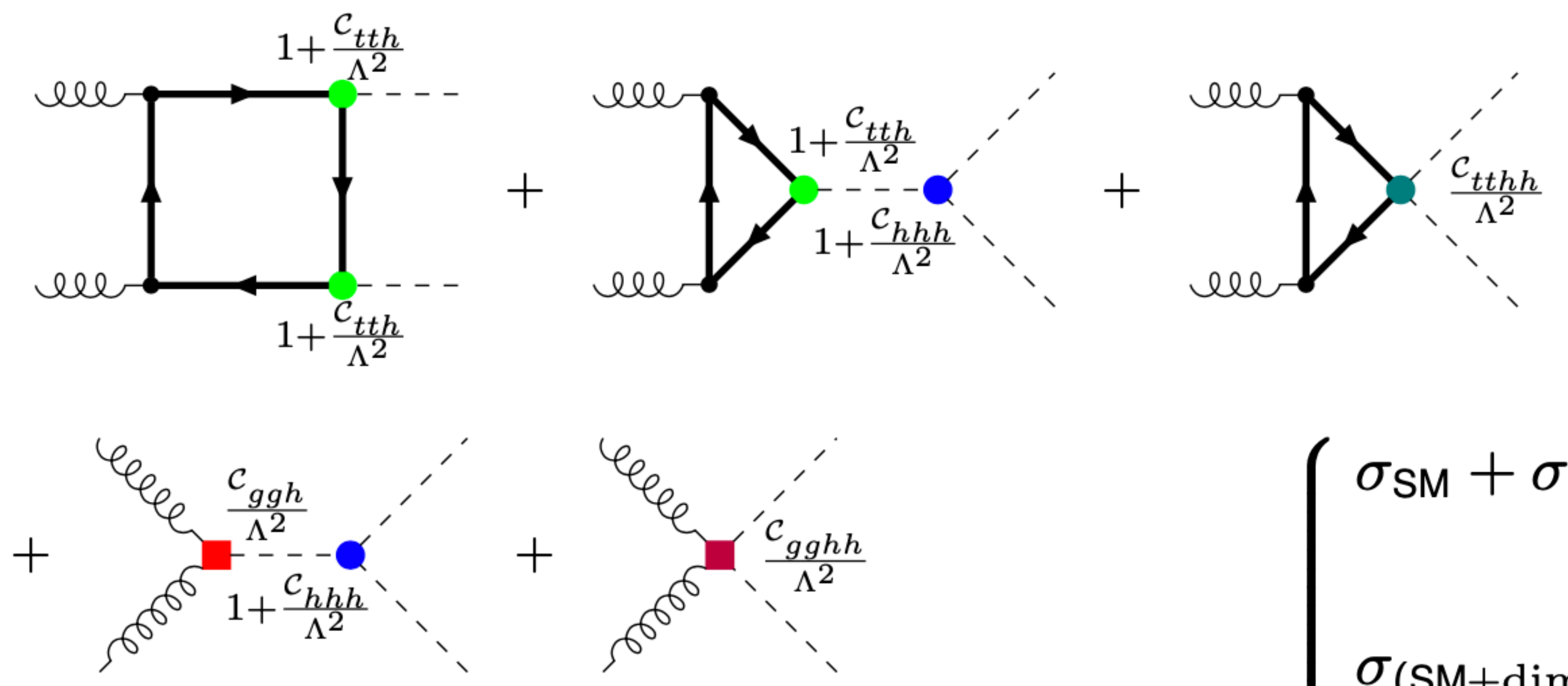
+ running Wilson coefficients coming soon GH, J. Lang

note: bug in 2-loop triangle contribution (in both codes) corrected September 2023

(thanks to Ramona Gröber, Emanuele Bagnaschi, Guiseppe Degrassi, 2309.10525)

SMEFT truncation

$\mathcal{M}^{\rm LO}_{\rm SMEFT} =$



$$= \mathcal{M}_{\rm SM} + \mathcal{M}_{\rm dim6} + \mathcal{M}_{\rm (dim6)^2}$$

$$\sigma \simeq \begin{cases} \sigma_{\rm SM} + \sigma_{\rm SM\times dim6} & \text{(a)} \\ \sigma_{\rm (SM+dim6)\times(SM+dim6)} & \text{(b)} \\ \sigma_{\rm (SM+dim6)\times(SM+dim6)} + \sigma_{\rm SM\times dim6^2} & \text{(c)} \\ \sigma_{\left(\rm SM+dim6+dim6^2\right)\times\left(\rm SM+dim6+dim6^2\right)} & \text{(d)} \end{cases}$$

(a) **“linear”**

(b) **“quadratic”**

(c) double insertions

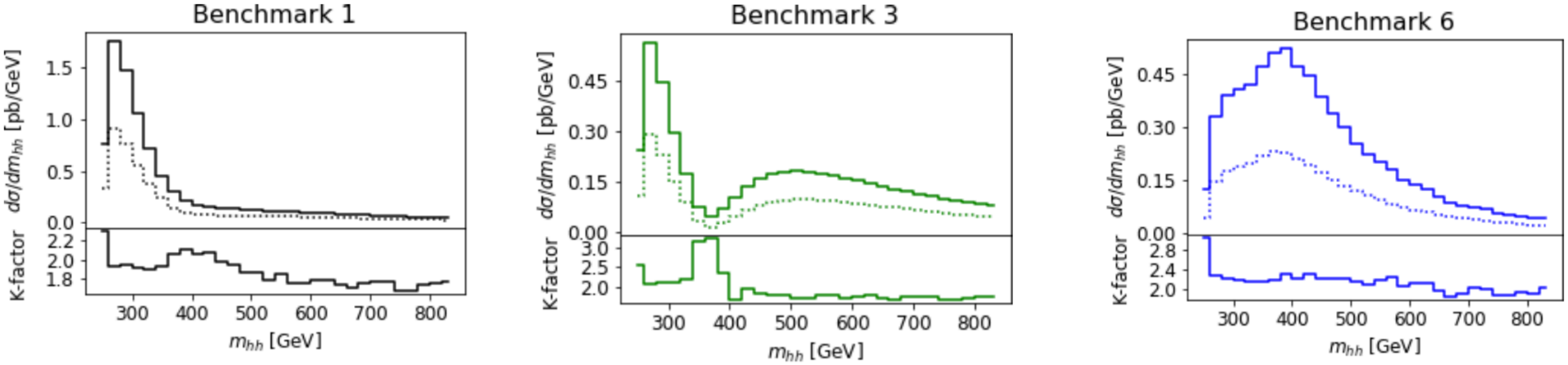
(d) HEFT situation (up to treatment of α_s)

all options available in **ggHH_SMEFT** code

(c) and (d) inconsistent in SMEFT (canonical orders messed up)

HEFT mhh-shape benchmark points

consider benchmark points characteristic for a certain mhh **shape**



Capozi, GH, 1908.08923

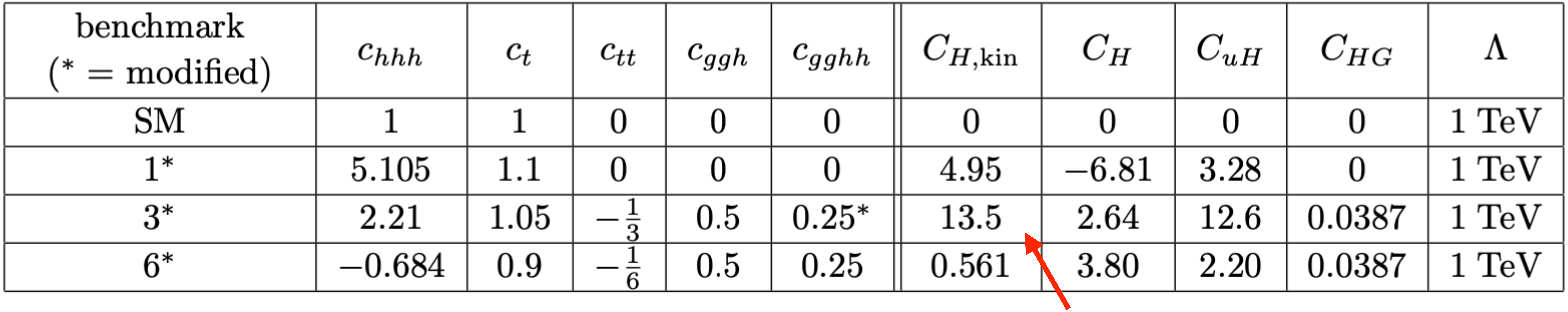
benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}
SM	1	1	0	0	0
1*	5.105	1.1	0	0	0
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25

- benchmark **1**: enhanced low mHH
- benchmark **3**: dip
- benchmark **6**: SM-like except for shoulder left of peak

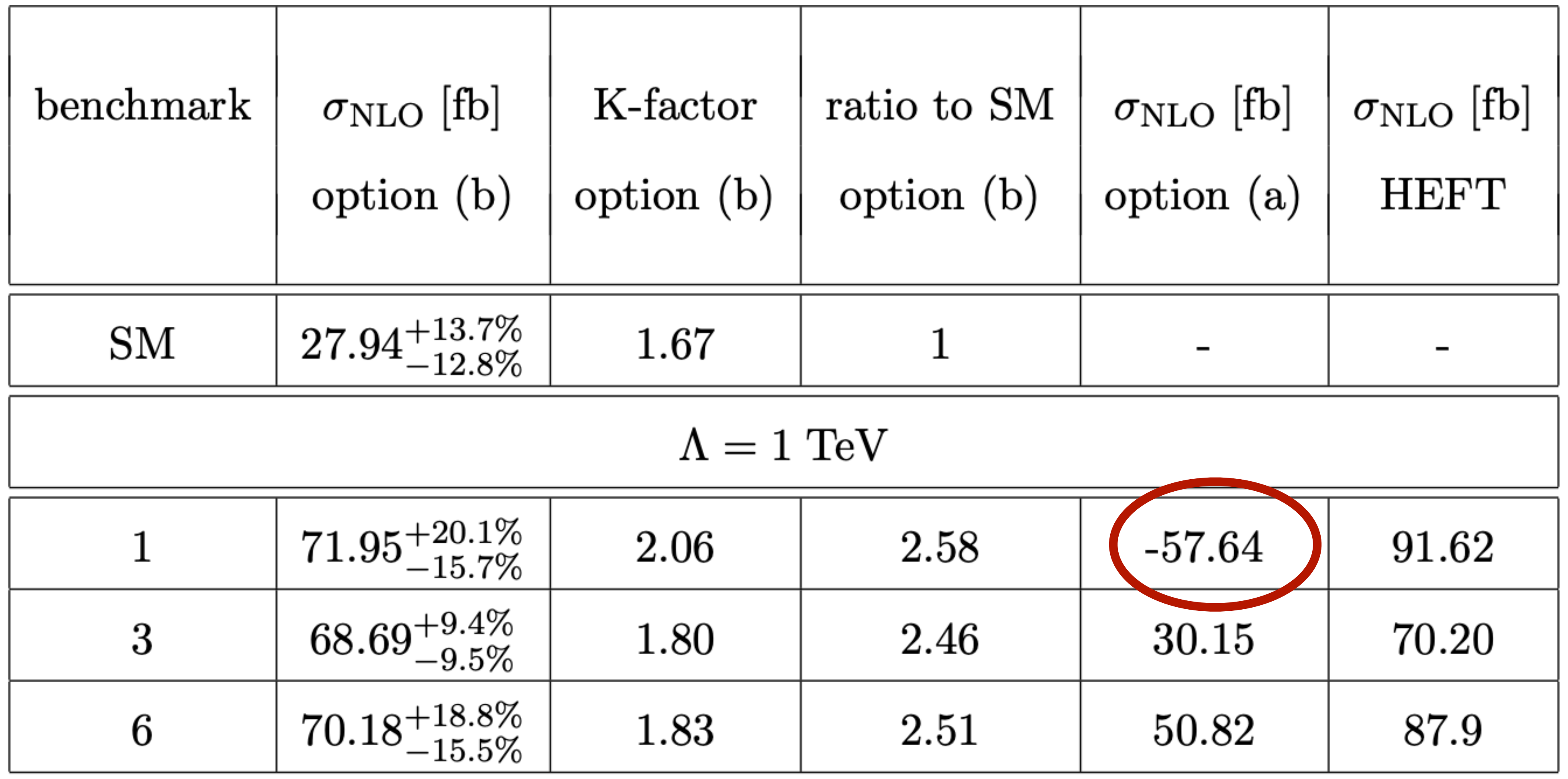
modified: to fulfil SMEFT relation $c_{ggh} = 2c_{gghh}$ and constraints after 2019

see also LHC Higgs WG4 note, 2304.01968

Naive translation HEFT to SMEFT



benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}	$C_{H,\text{kin}}$	C_H	C_{uH}	C_{HG}	Λ
SM	1	1	0	0	0	0	0	0	0	1 TeV
1*	5.105	1.1	0	0	0	4.95	-6.81	3.28	0	1 TeV
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25*	13.5	2.64	12.6	0.0387	1 TeV
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV



benchmark	σ_{NLO} [fb] option (b)	K-factor option (b)	ratio to SM option (b)	σ_{NLO} [fb] option (a)	σ_{NLO} [fb] HEFT
SM	$27.94^{+13.7\%}_{-12.8\%}$	1.67	1	-	-
$\Lambda = 1$ TeV					
1	$71.95^{+20.1\%}_{-15.7\%}$	2.06	2.58	-57.64	91.62
3	$68.69^{+9.4\%}_{-9.5\%}$	1.80	2.46	30.15	70.20
6	$70.18^{+18.8\%}_{-15.5\%}$	1.83	2.51	50.82	87.9

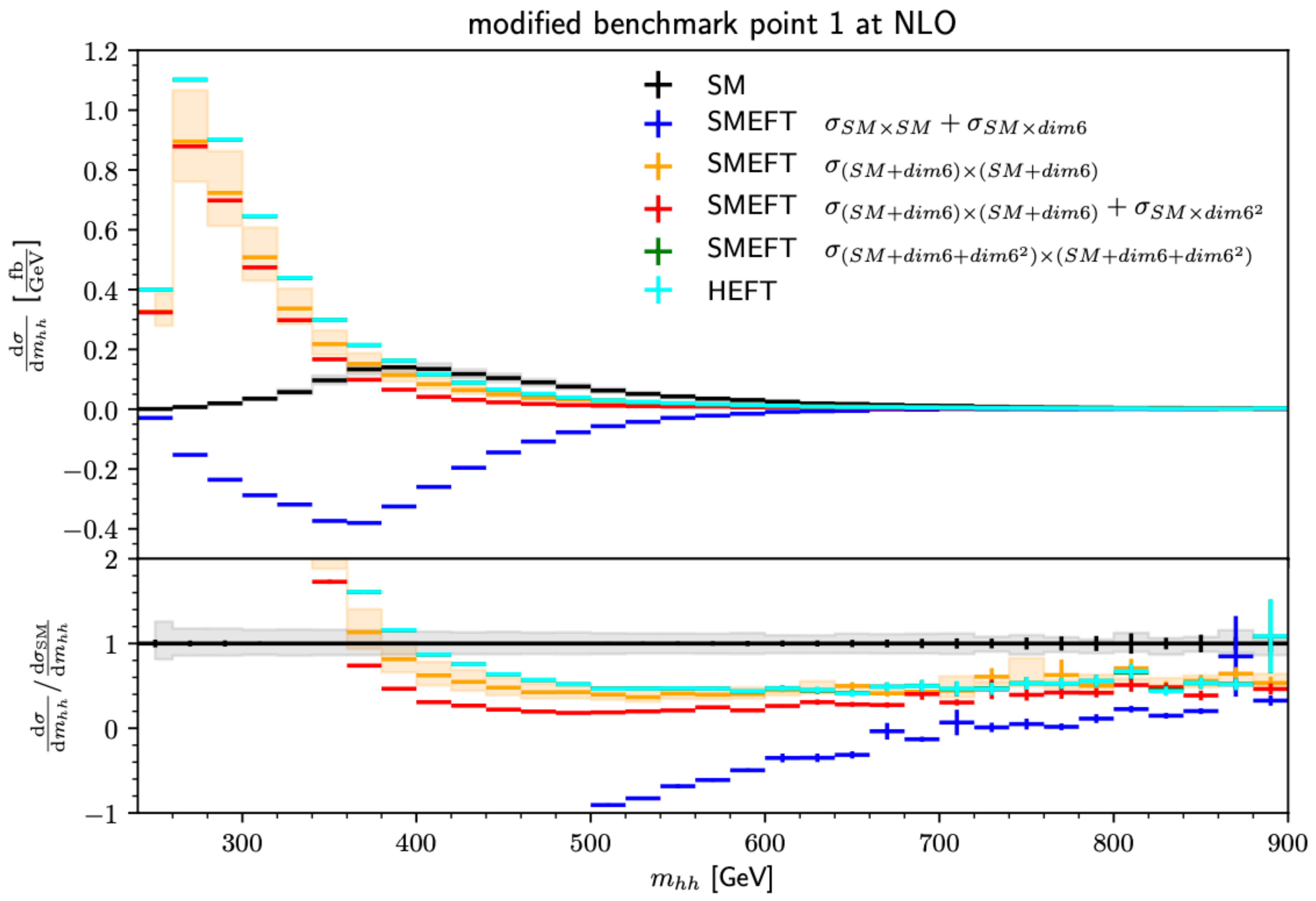
$$E^2\frac{|C_i|}{\Lambda^2} \ll 1$$ not fulfilled for $\Lambda \simeq 1\,\text{TeV}$

→ can lead to negative cross sections

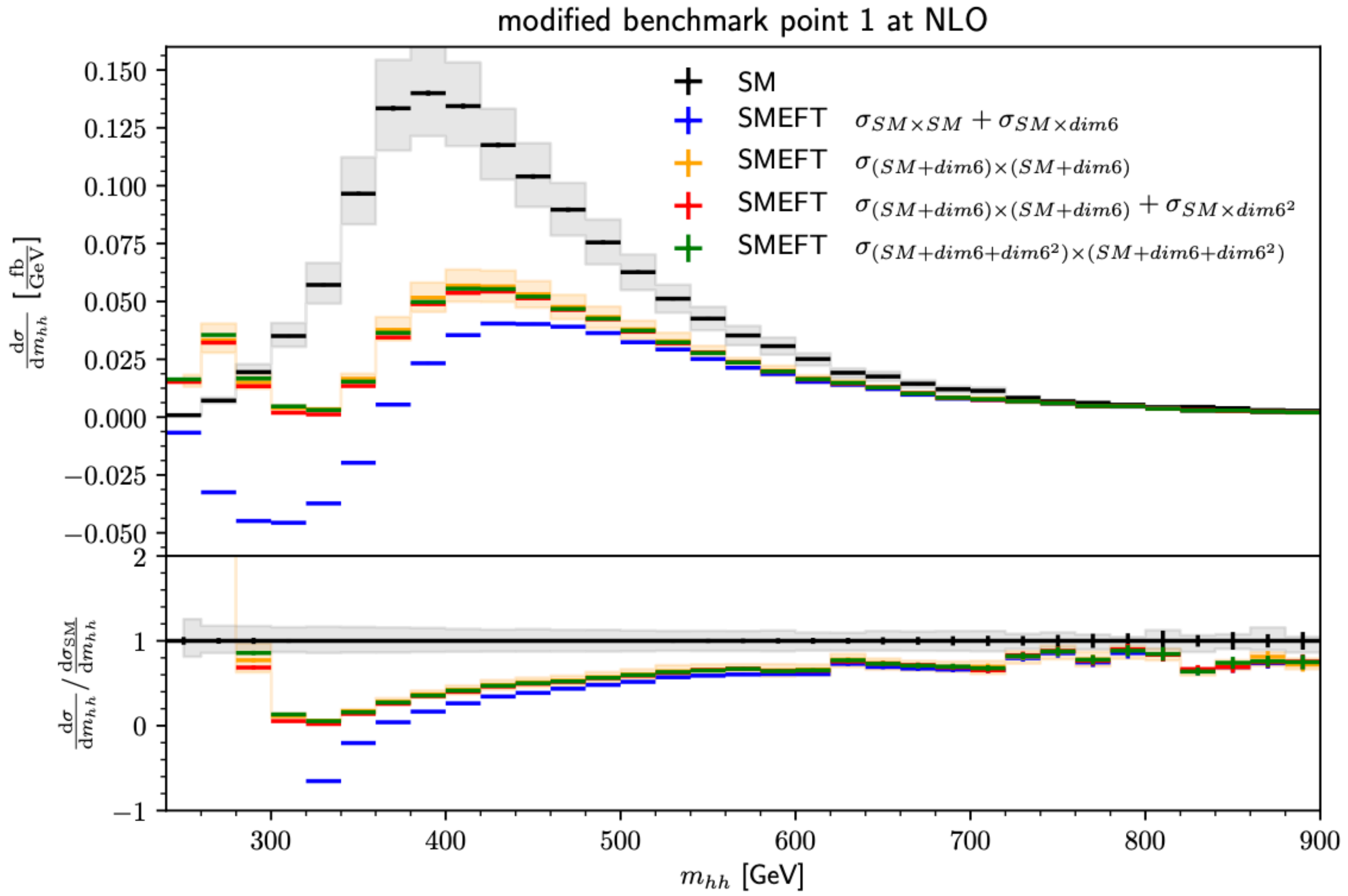
Higgs boson pair invariant mass spectrum

benchmark point 1

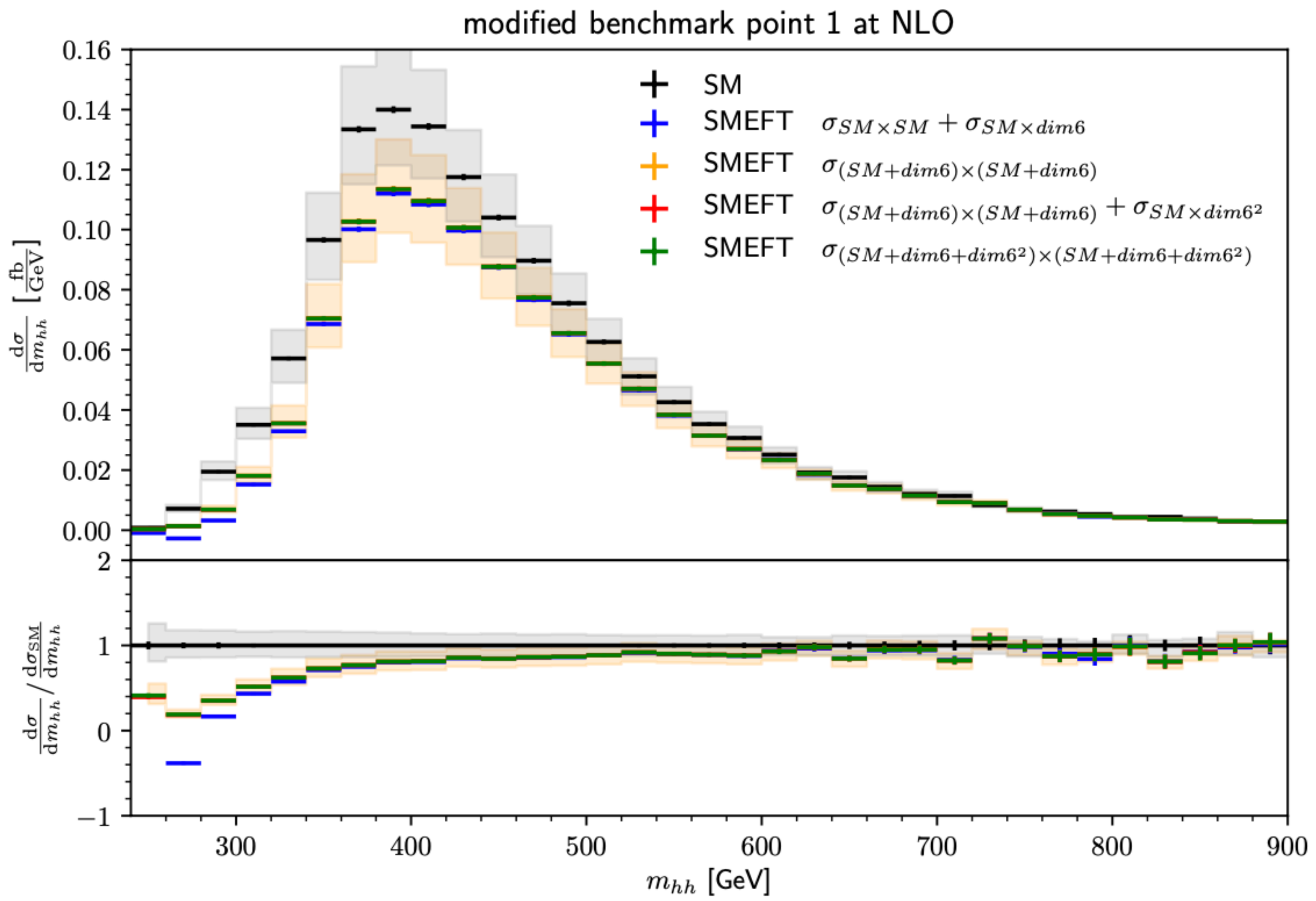
figures: Jannis Lang



$\Lambda = 1\,\text{TeV}$



$\Lambda = 2\,\text{TeV}$



$\Lambda = 4\,\text{TeV}$

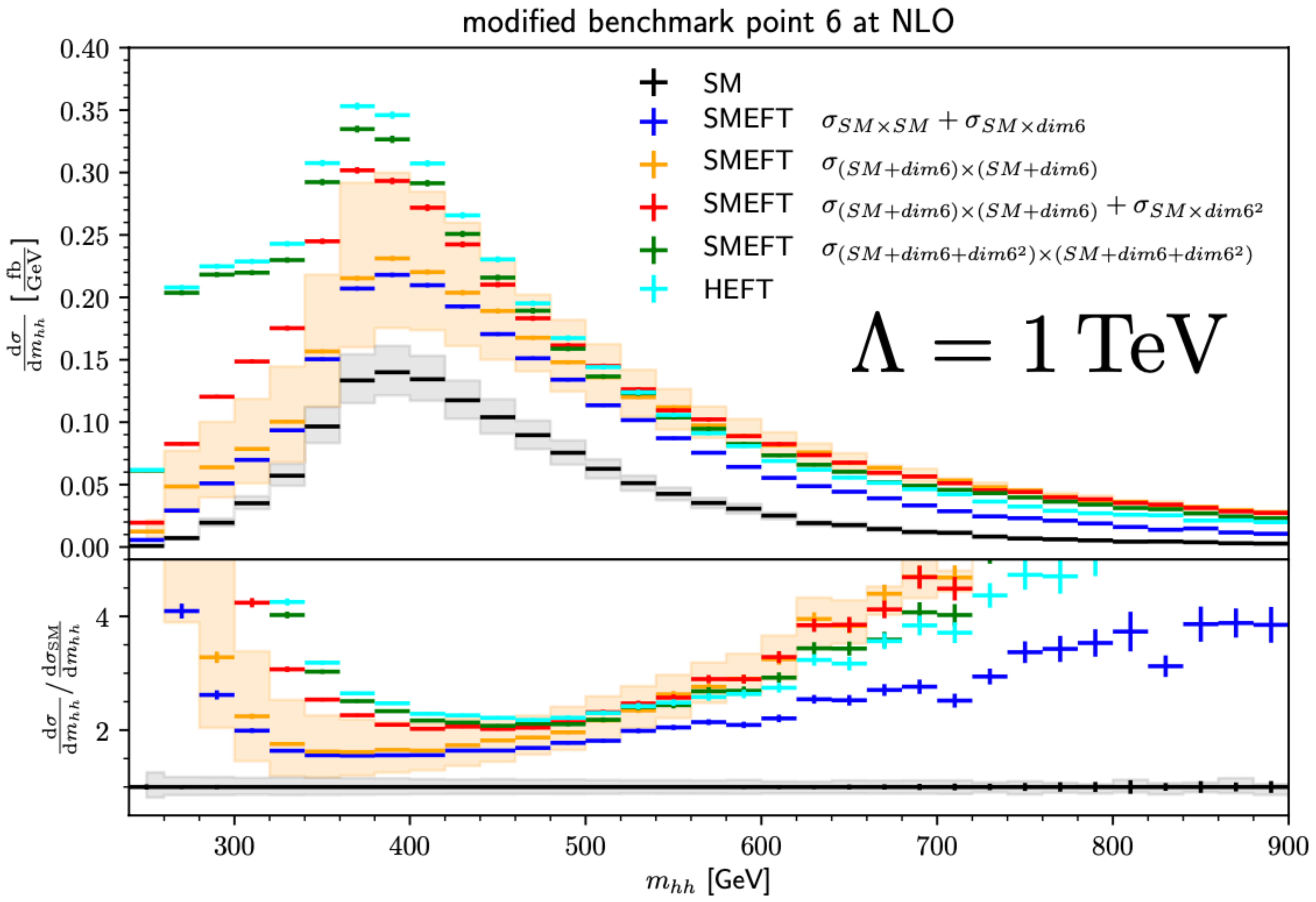
linear dim6 (blue): negative cross sections; quadratic dim6 (orange) looks reasonable even though not a valid SMEFT point

shape changes as Λ is increased (obviously, approaching SM shape)

→ for low values of Λ: parameter point valid in HEFT can be invalid in SMEFT

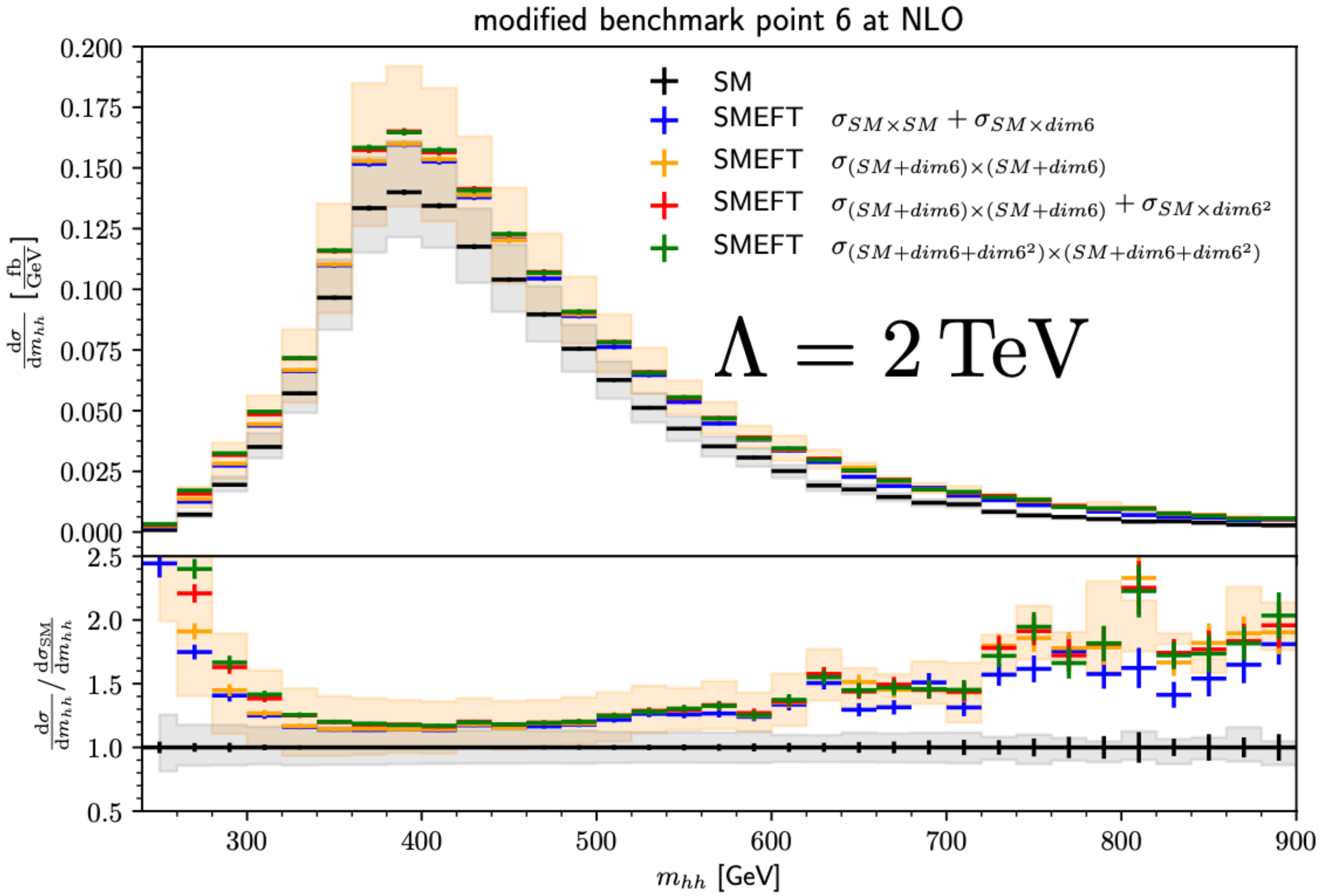
Truncation effects on Higgs boson pair invariant mass

benchmark point 6 $c_{hhh} = -0.684, c_t = 0.9, c_{tt} = -1/6, c_{ggh} = 0.5, c_{gghh} = 0.25$



characteristic shape not present in SMEFT,

large difference between linear and quadratic truncation



differences between truncation options smaller, but

shape very SM-like, difference to SM
in peak region within NLO scale uncertainties

figures: Jannis Lang

EFT expansion + higher orders in QCD

(SM)EFT expansion parameters:

$$\boxed{\Lambda^{-d_c}(g_s^2\,L)^{l_{\mathrm{QCD}}}\,\mathbf{L}^{l_{\mathrm{not\_QCD}}}}$$

d_c : canonical dimension

This is an expansion in several parameters

g_s : strong coupling

$L = (16\pi)^{-1}$: loop factor (QCD)

$\mathbf{L} = (16\pi)^{-1}$: loop factor (new physics)

l_{QCD} : number of QCD loops

$l_{\mathrm{not\_QCD}}$: number of loops involving new particles or new interactions (or EW corrections)

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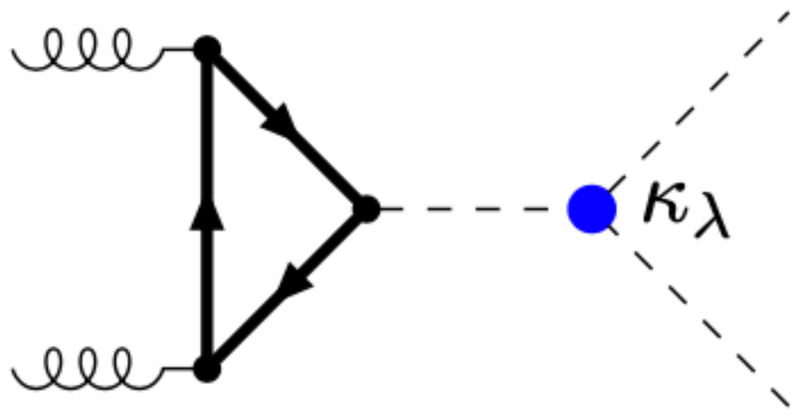
$l_{\mathrm{not\_QCD}}$: number of loops involving new particles or new interactions (or EW corrections)

In renormalisable, weakly coupled UV completions:

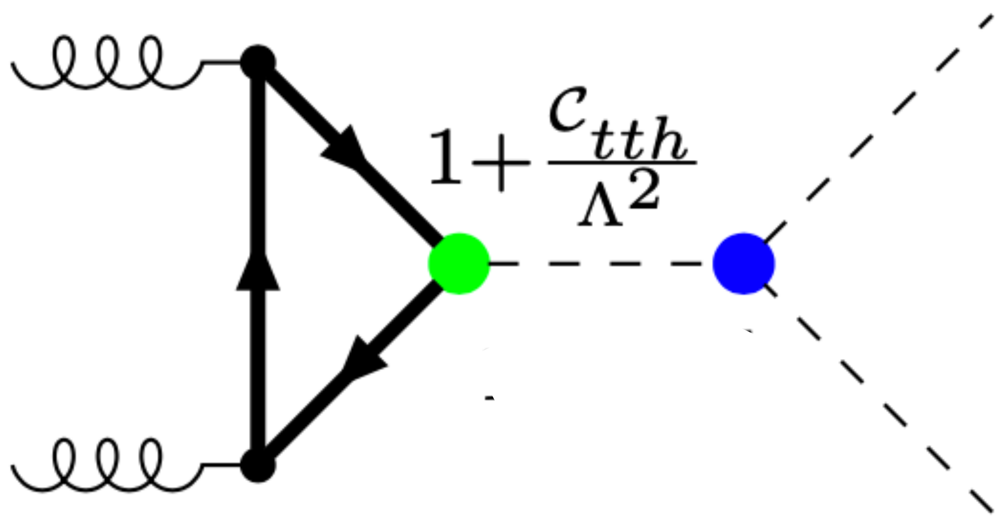
Operators containing field strength tensors are loop-generated $\Rightarrow$ get a loop suppression factor

Arzt, Einhorn Wudka ’94; Buchalla, GH, Müller-Salditt, Pandler 2204.11808

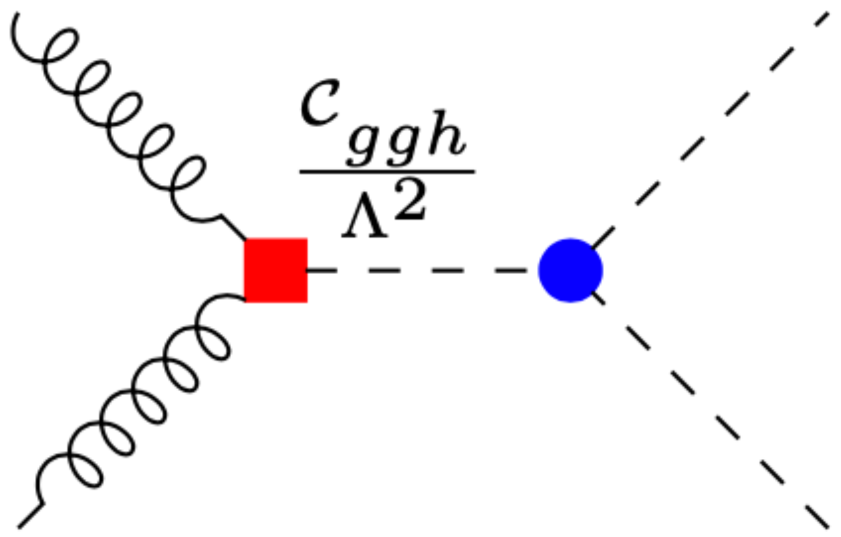
Loop counting in SMEFT



$$\frac{1}{16\pi^2} \quad l_{\mathrm{QCD}} = 1$$

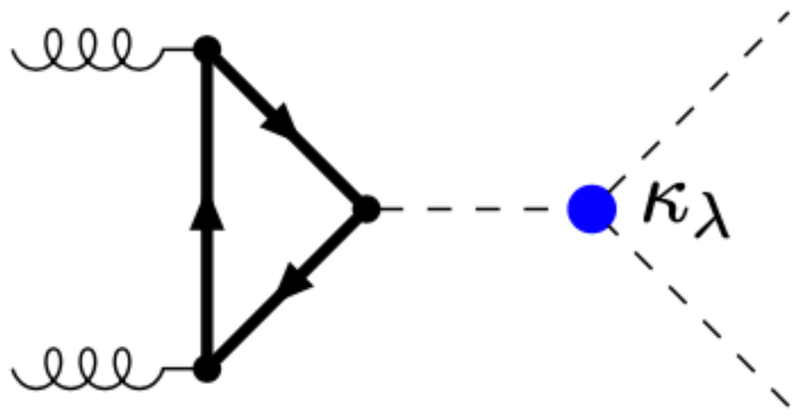


$$\frac{1}{\Lambda^2}\frac{1}{16\pi^2} \quad l_{\mathrm{QCD}} = 1$$

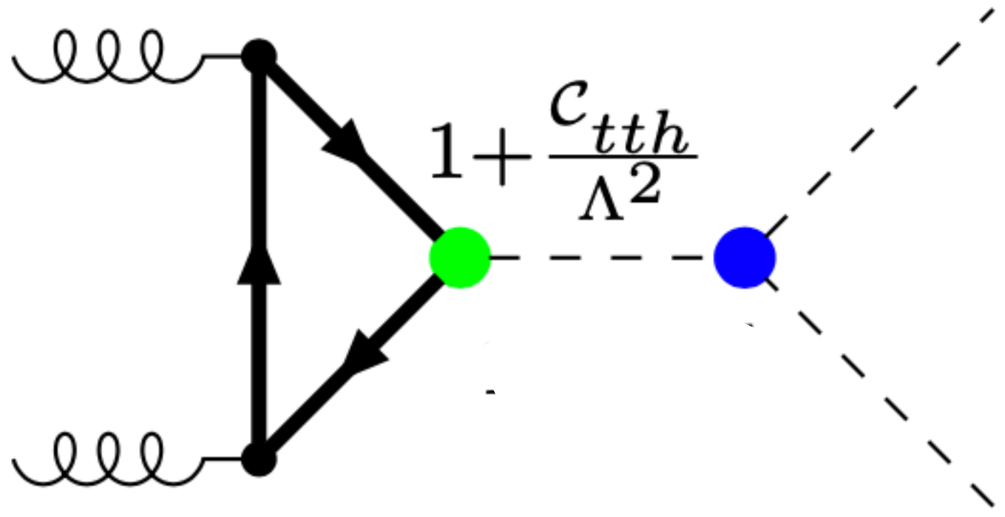


$$\frac{1}{\Lambda^2}\frac{1}{16\pi^2} \quad l_{\mathrm{not\_QCD}} = 1$$

Loop counting in SMEFT



$$\frac{1}{16\pi^2} \quad l_{\rm QCD} = 1$$



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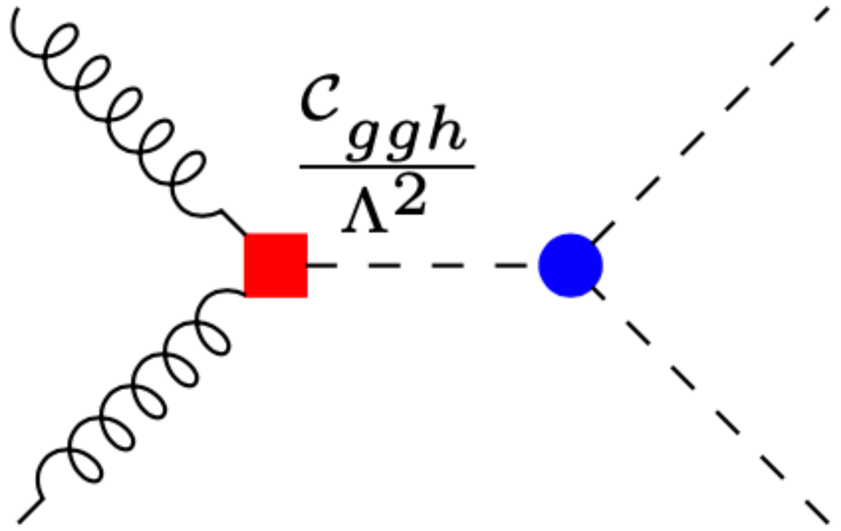
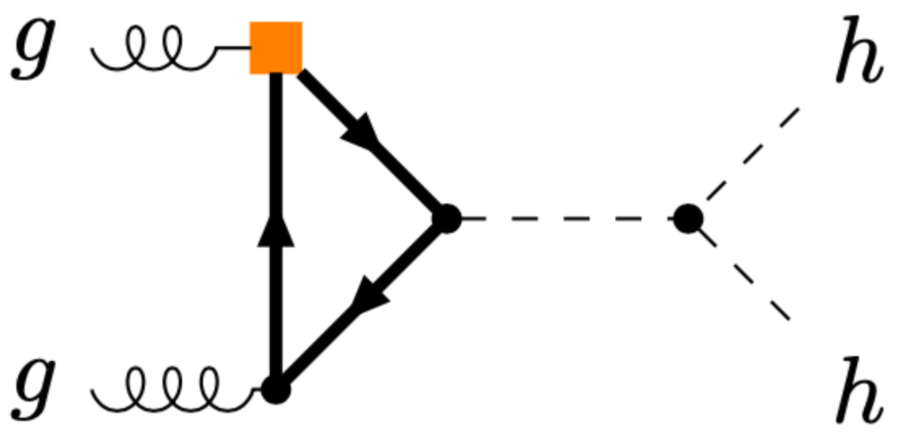


$$\frac{1}{\Lambda^2}\frac{1}{16\pi^2} \quad l_{\rm not\_QCD} = 1$$

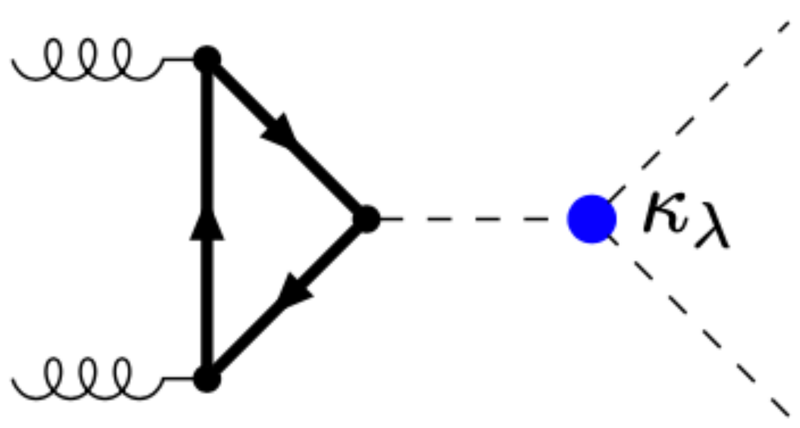


chromomagnetic
operator

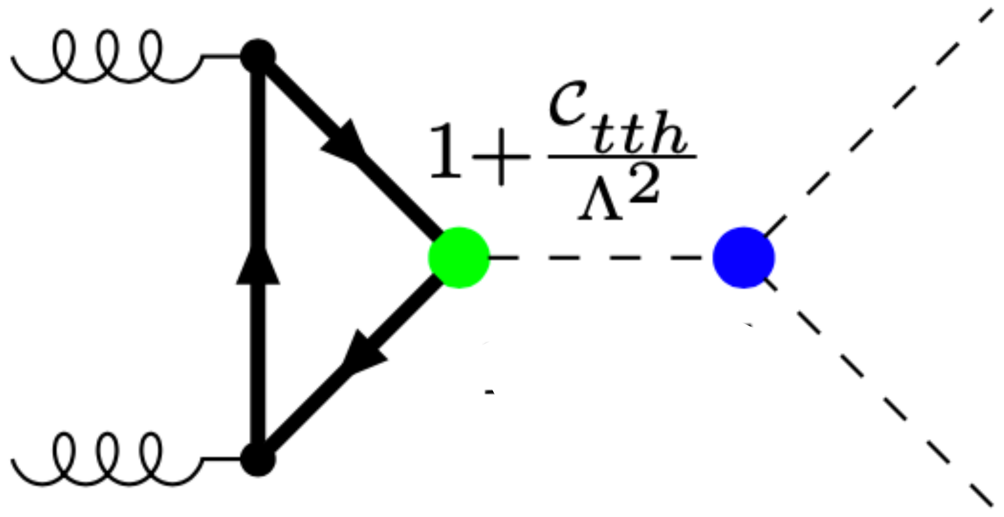
$$\frac{1}{\Lambda^2}\frac{1}{(16\pi^2)^2} \quad l_{\rm QCD} = 1, l_{\rm not\_QCD} = 1$$

explicit implicit

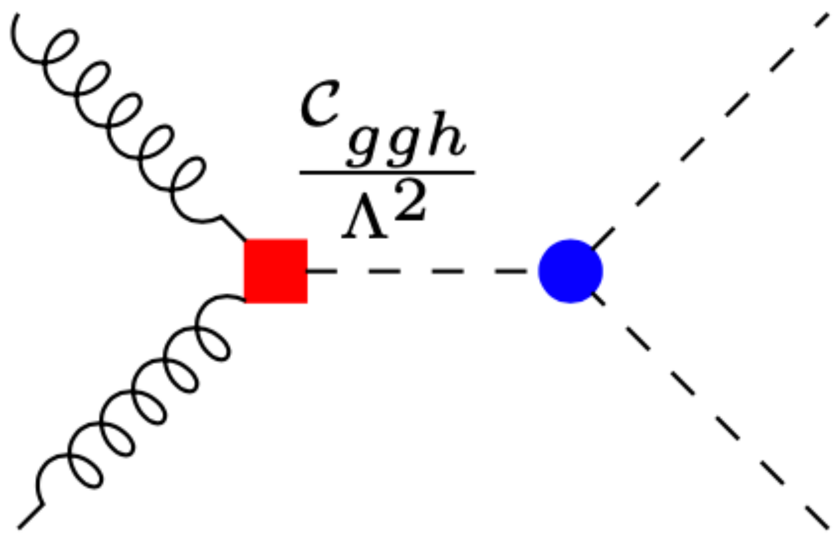
Loop counting in SMEFT



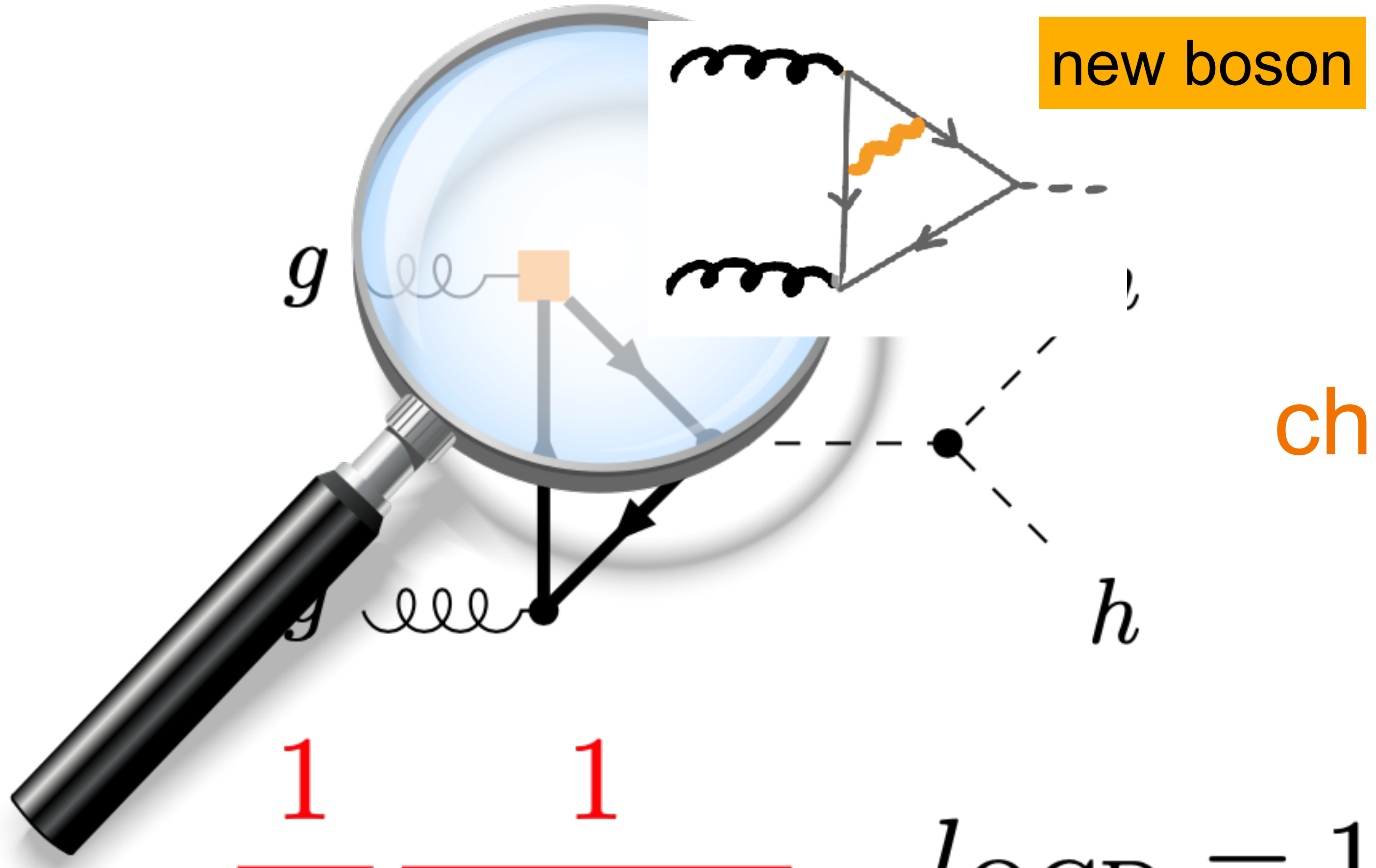
$$\frac{1}{16\pi^2} \qquad l_{\rm QCD} = 1$$



$$\frac{1}{\Lambda^2}\,\frac{1}{16\pi^2} \qquad l_{\rm QCD} = 1$$



$$\frac{1}{\Lambda^2}\,\frac{1}{16\pi^2} \qquad l_{\rm not\_QCD} = 1$$



chromomagnetic
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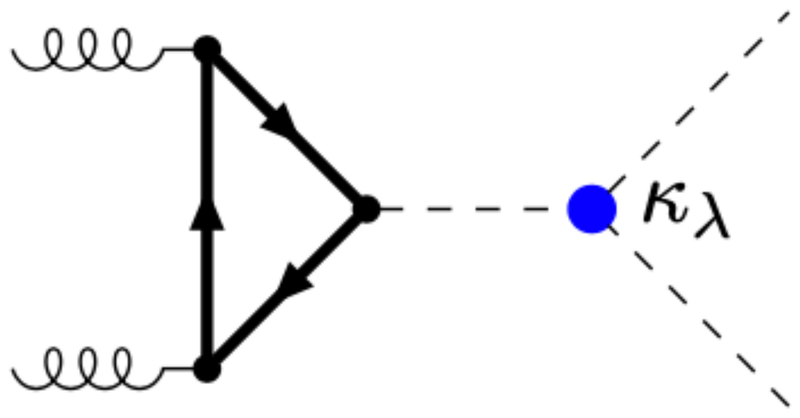
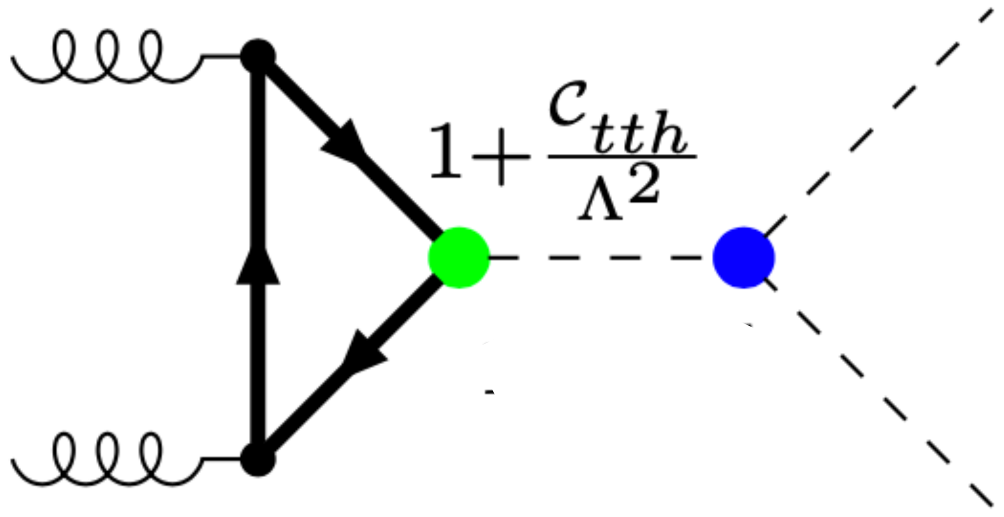
explicit implicit

Loop counting in SMEFT



$\frac{1}{16\pi^2}$ $l_{\rm QCD}=1$



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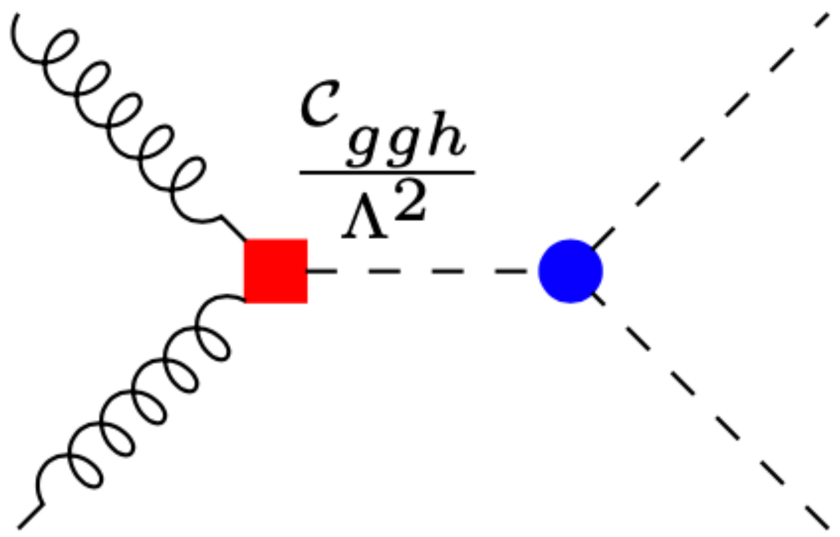
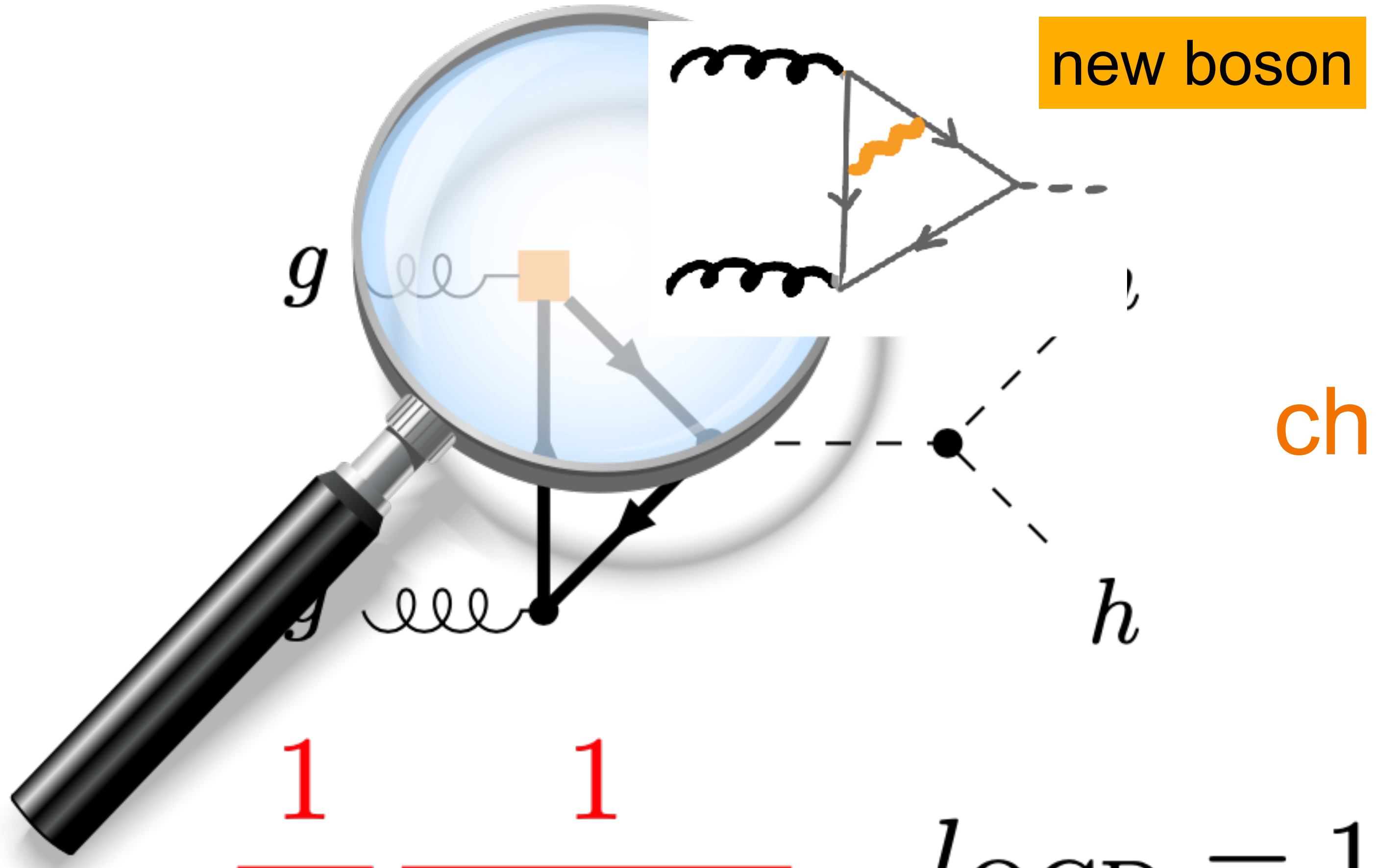


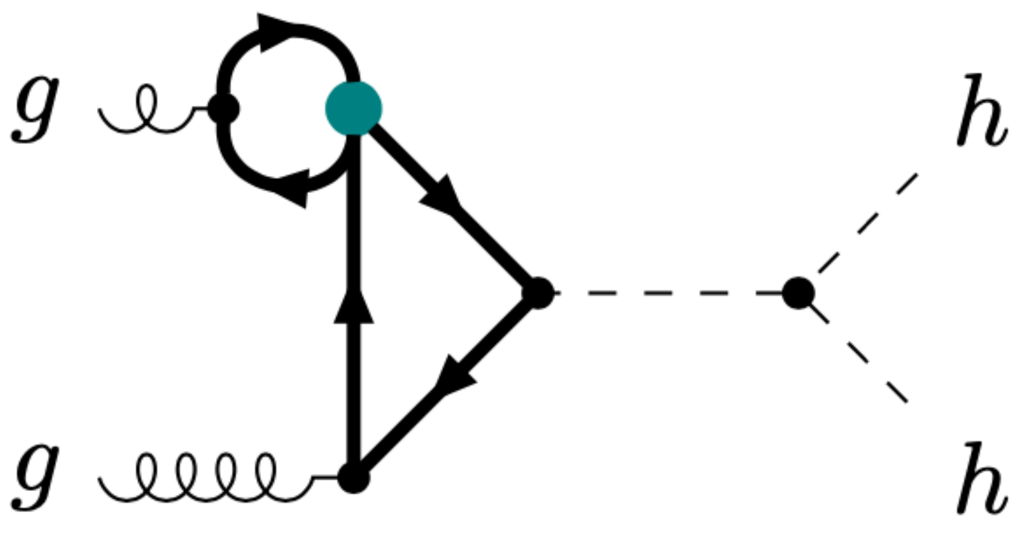
$\frac{1}{\Lambda^2}\frac{1}{16\pi^2}$ $l_{\rm not\_QCD}=1$



chromomagnetic operator

$\frac{1}{\Lambda^2}\frac{1}{(16\pi^2)^2}$ $l_{\rm QCD}=1, l_{\rm not\_QCD}=1$

explicit implicit



4-top operators enter at the same order!

$\frac{1}{\Lambda^2}\frac{1}{(16\pi^2)^2}$ $l_{\rm QCD}=1, l_{\rm not\_QCD}=1$

explicit explicit

Four-top operators

$$\mathcal{L}_{4t} = \frac{C_{Qt}^{(1)}}{\Lambda^2} \underbrace{\bar{t}_L \gamma^\mu t_L \, \bar{t}_R \gamma_\mu t_R}_{\bar{t}\mathbb{P}_R \gamma^\mu \mathbb{P}_L t \, \bar{t}\mathbb{P}_L \gamma_\mu \mathbb{P}_R t} + \frac{C_{Qt}^{(8)}}{\Lambda^2} \bar{t}_L \gamma^\mu T^a t_L \, \bar{t}_R \gamma_\mu T^a t_R + \ldots$$

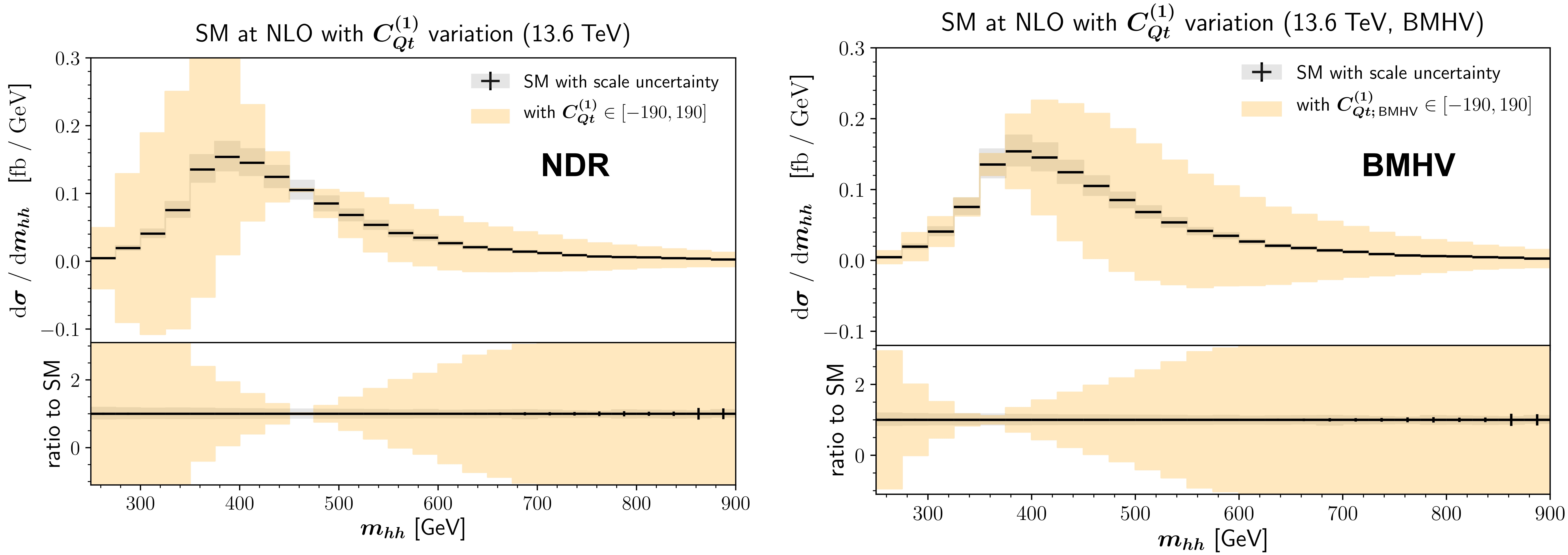
$$; \quad \mathbb{P}_{L/R} = (\mathbb{I} \mp \gamma_5)/2$$

- 4-top operators occur in 2-loop diagrams

- treatment of γ_5 matters! Di Noi, Gröber, GH, Lang, Vitti ‘23

- translation between schemes also affects other operators (e.g. chromomagnetic) and parameters

Example: $C_{Qt}^{(1)}$ in different gamma5 schemes



GH, J. Lang, 2311.15004

large effect and very different behaviour in the two schemes → specify scheme for global fits

Running Wilson coefficients

coming soon in the ggHH_SMEFT code (Powheg-Box-V2): [GH, Jannis Lang]

$$\mu \frac{\partial \mathcal{C}_i}{\partial \mu} = \frac{\gamma_{\mathcal{C}_i}^{\mathcal{C}_j}}{16\pi^2} \mathcal{C}_j \quad , \quad \gamma_{\mathcal{C}_i}^{\mathcal{C}_j}$$: anomalous dimension

new options for users:

WCscaledependence:

0: no running, $\mu_{\mathrm{EFT}} = \mu_R$

1: $\mu_{\mathrm{EFT}} = \mu_0 \cdot \mathtt{EFTscfact}$, μ_0 fixed by the user

2: $\mu_{\mathrm{EFT}} = m_{hh}/2 \cdot \mathtt{EFTscfact}$

EFTscfact: variation factor

inputscaleEFT: scale where the running starts

see also

Maltoni, Ventura, Vryonidou 2406.06670

Gröber, Di Noi 2312.11327

Aoude et al. 2212.05067

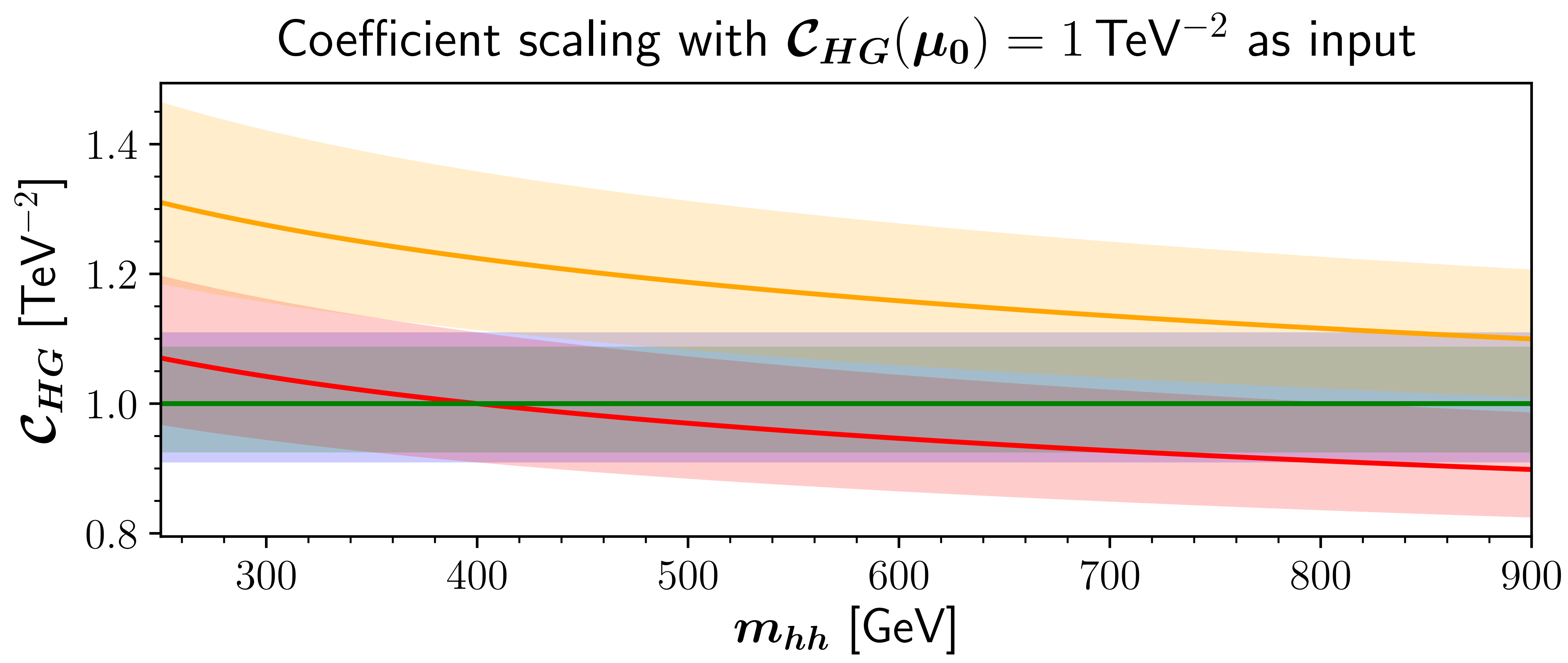
Battaglia, Grazzini, Spira, Wiesemann 2109.02987

Deutschmann, Duhr, Maltoni, Vryonidou 1708.00460

Maltoni, Vryonidou, Zhang 1607.05330

Running Wilson coefficients

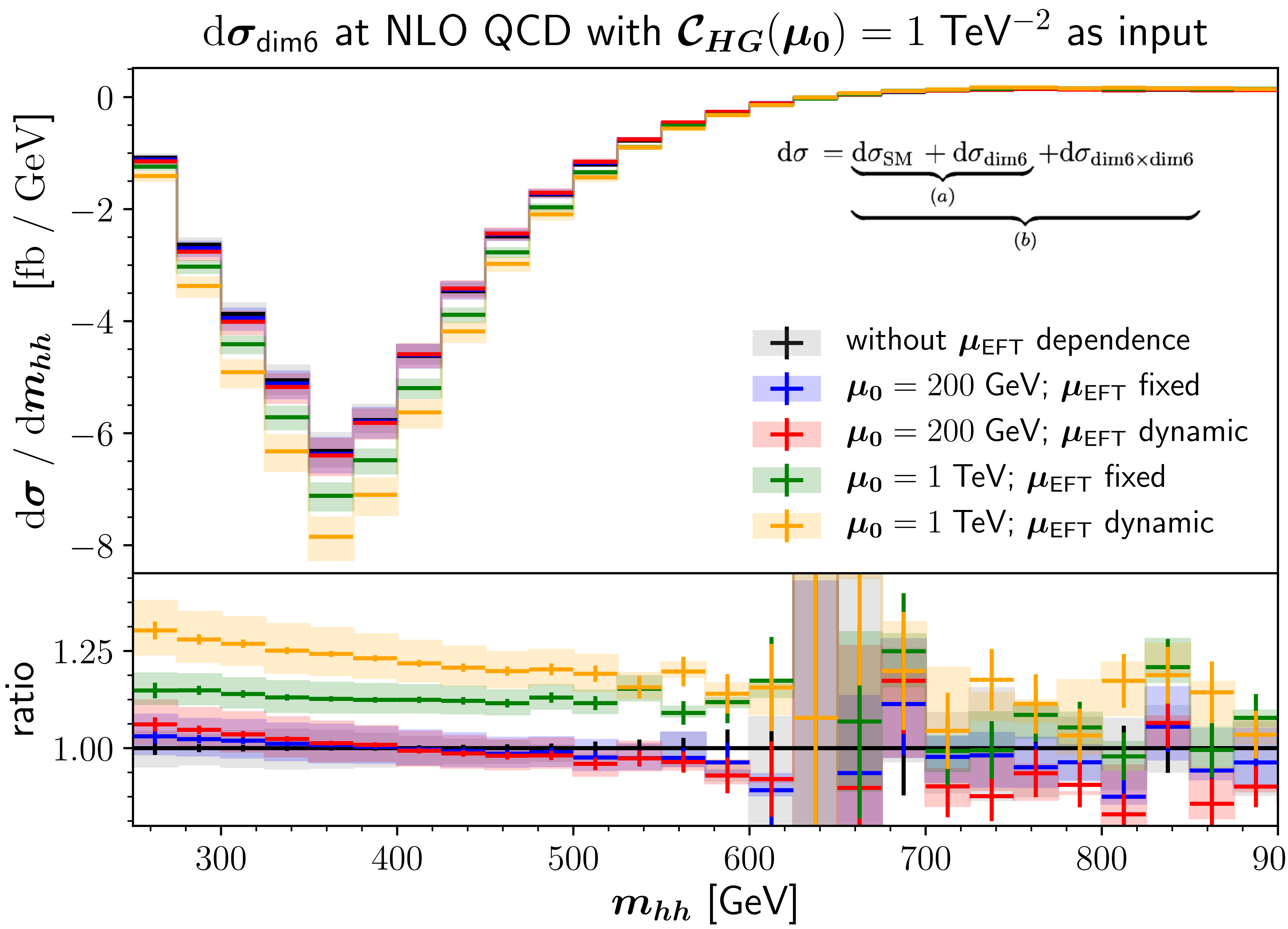
effect of running on CHG only:



grey band: usual μ_r, μ_f variations

coloured bands: variations around μ_{EFT}

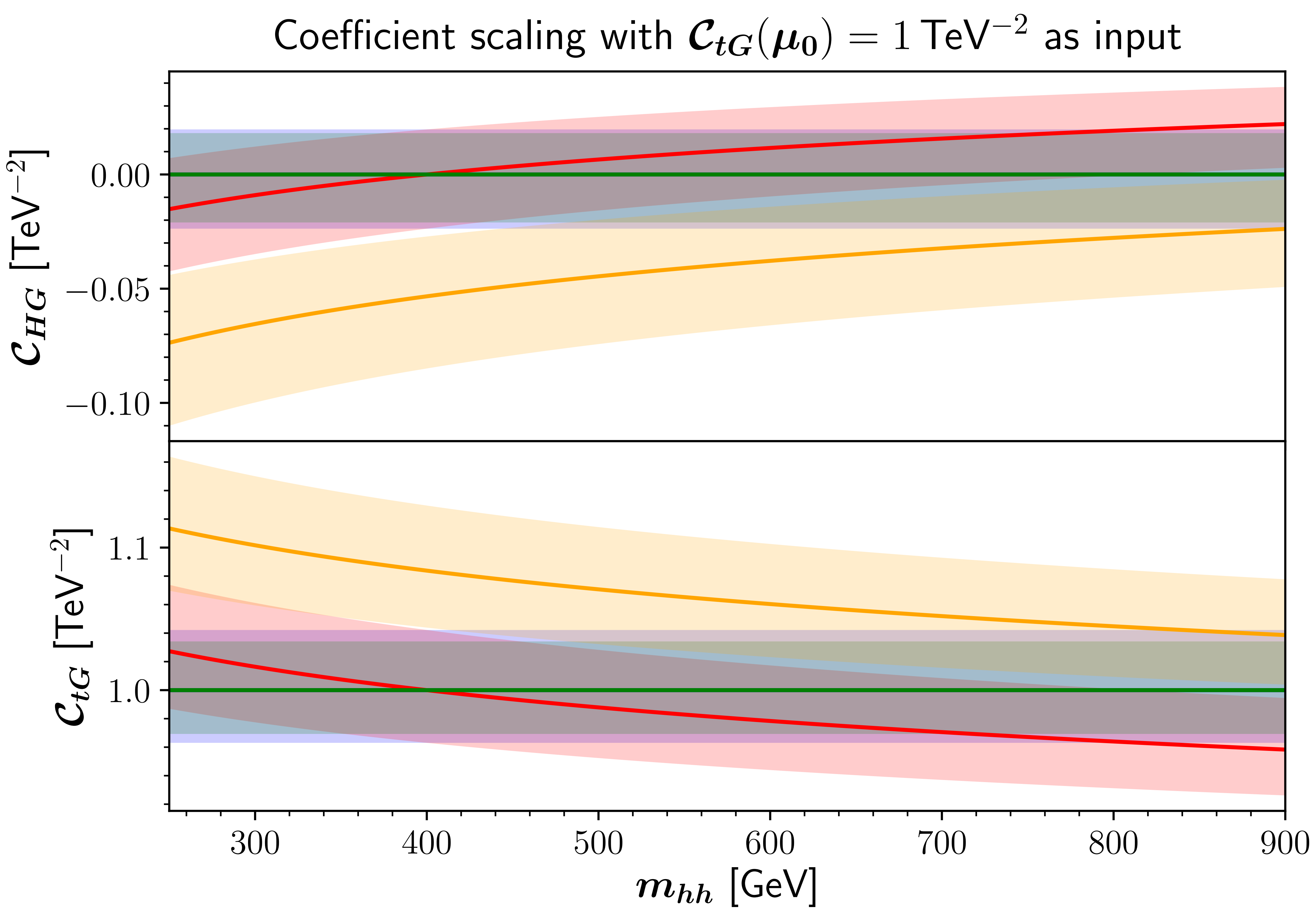
dynamic: $\mu_{\mathrm{EFT}} = \dfrac{m_{hh}}{2}$



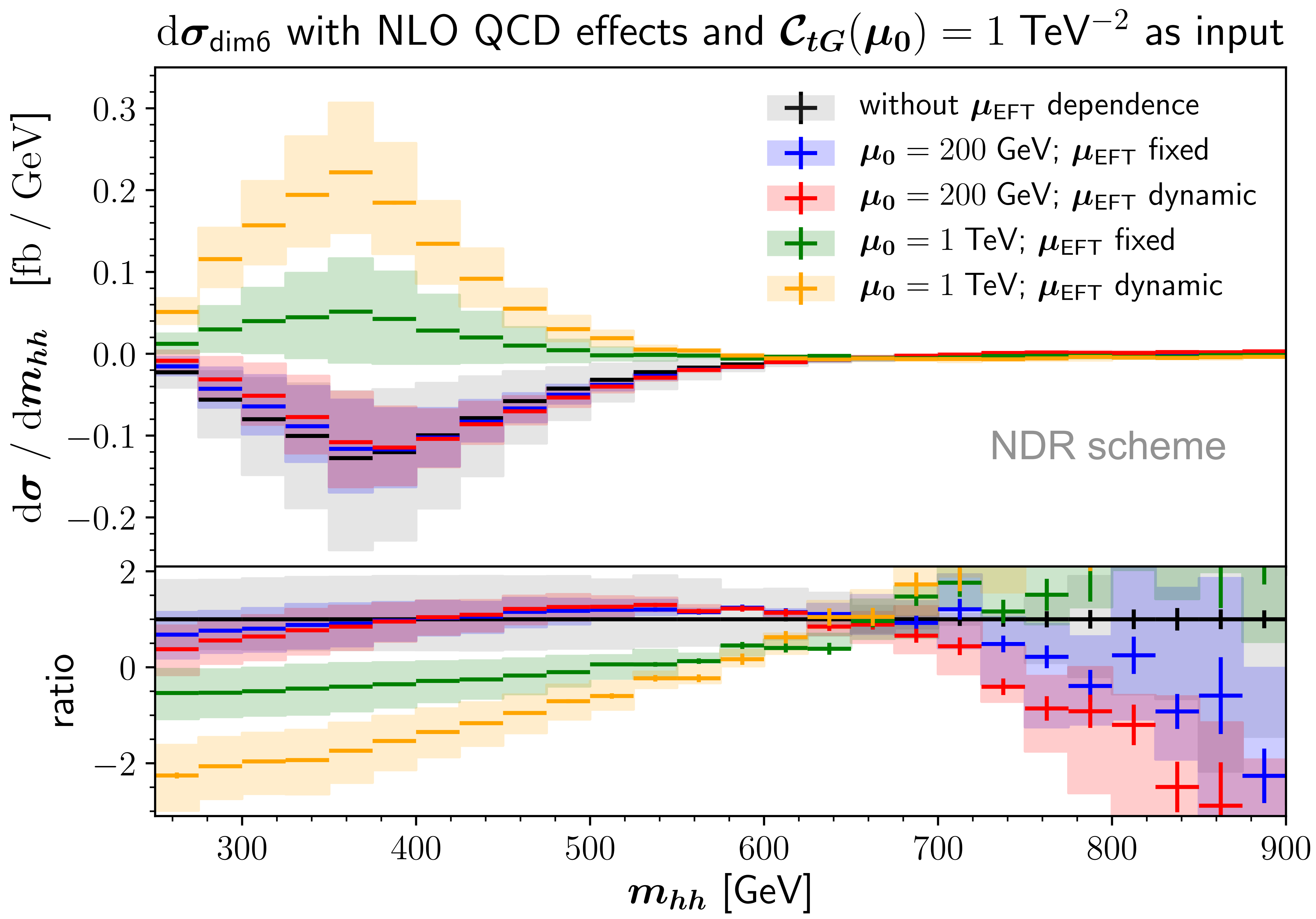
figures: Jannis Lang

large effects only for starting scale 1TeV

Running Wilson coefficients


Coefficient scaling with $\mathcal{C}_{tG}(\mu_0) = 1\,\mathrm{TeV}^{-2}$ as input

running of $\mathcal{C}_{tG}$ induces non-zero $\mathcal{C}_{HG}$


$\mathrm{d}\sigma_{\mathrm{dim6}}$ with NLO QCD effects and $\mathcal{C}_{tG}(\mu_0) = 1\,\mathrm{TeV}^{-2}$ as input

sign change for large μ_0

figures: Jannis Lang

Summary & outlook

- mHH shape benchmarks in HEFT cannot be translated to SMEFT
- truncation uncertainties can exceed all other uncertainties if parameter point is close to the border of the SMEFT validity range
- chromomagnetic operator and 4-top operators are linked through renormalisation -> gamma5-scheme dependence at loop level
- coming soon in `ggHH_SMEFT`: running of Wilson coefficients

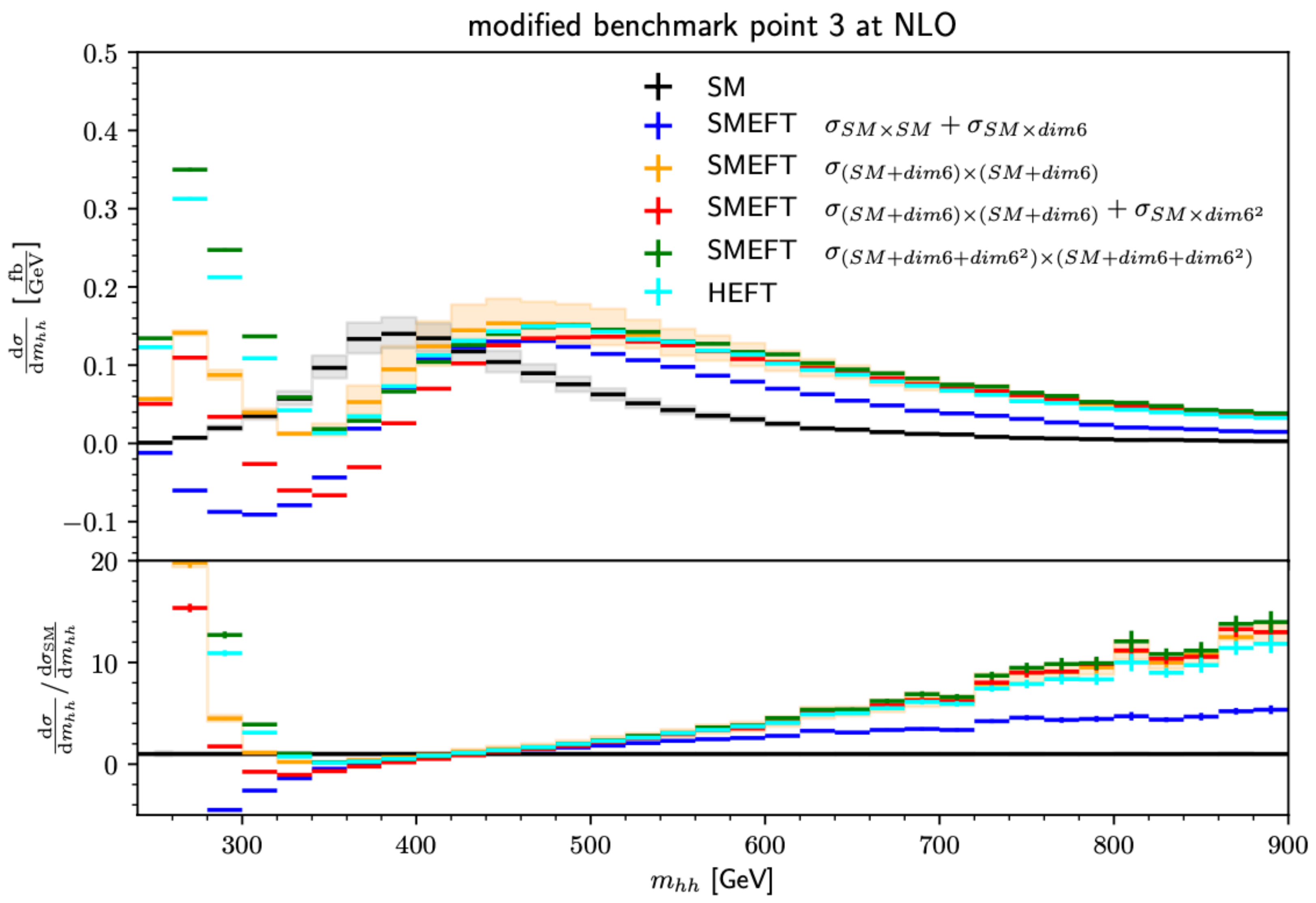
Thank you for your attention !

image: Laura Vogiatzis

Higgs boson pair invariant mass spectrum

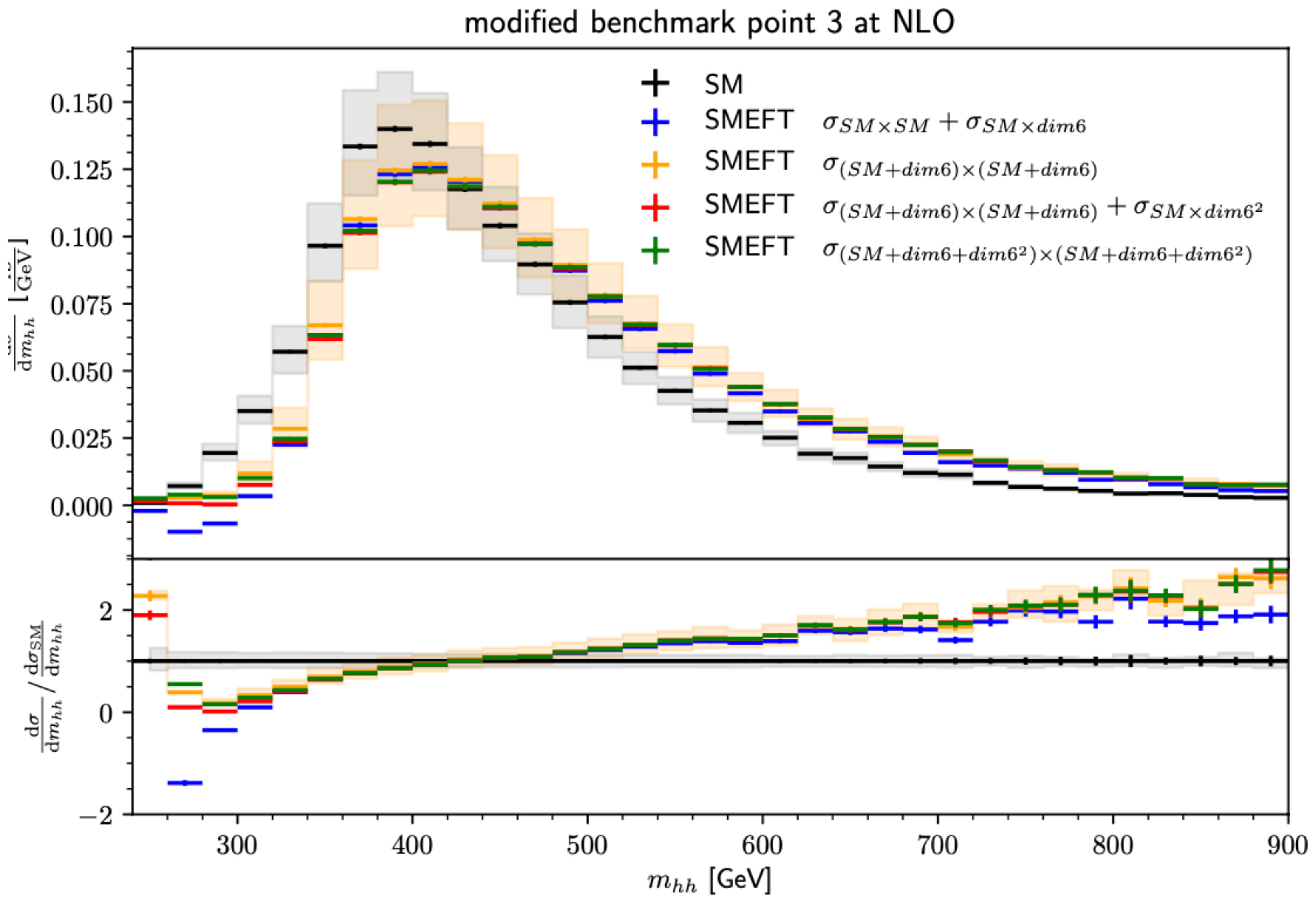
figures: Jannis Lang

benchmark point 3



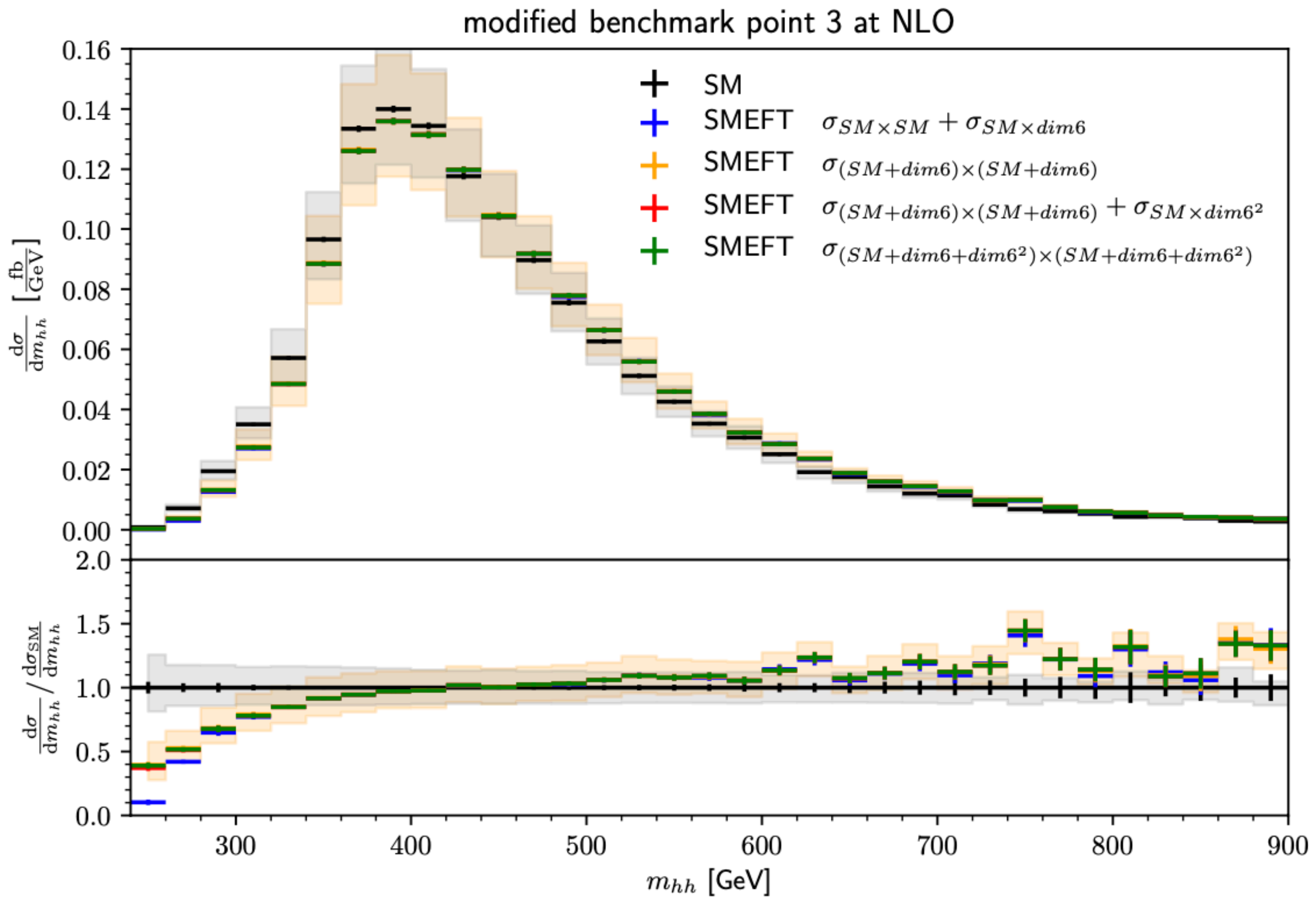
$$\Lambda = 1\,\mathrm{TeV}$$

double operator insertions have large effect



$$\Lambda = 2\,\mathrm{TeV}$$

distinguishable from SM within NLO uncertainties



$$\Lambda = 4\,\mathrm{TeV}$$

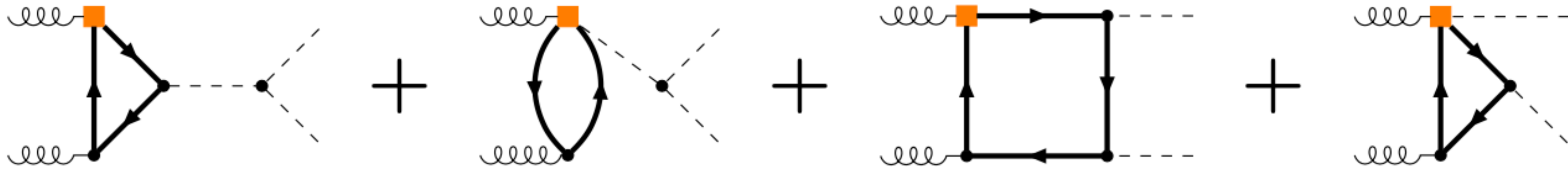
can only be distinguished from SM in low mHH region

Subleading operators in SMEFT

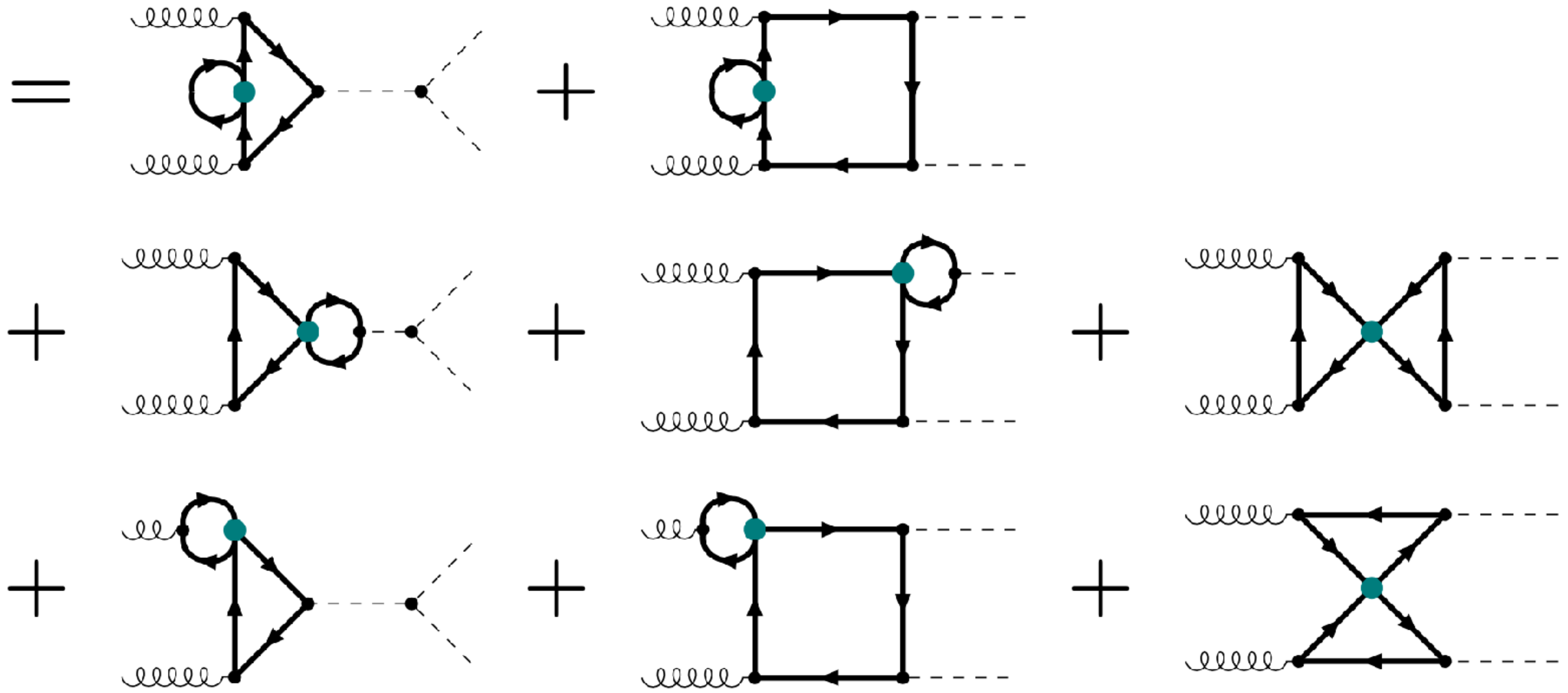
in a renormalisable, weakly coupling UV completion

$$\mathcal{L}_{tG} = \frac{C_{tG}}{\Lambda^2}\left(\bar{Q}_L\sigma^{\mu\nu}T^a G^a_{\mu\nu}\tilde{\phi}t_R + \text{h.c.}\right)$$

$\mathcal{M}_{\text{tG}} =$



$\mathcal{M}_{\text{4t}} =$



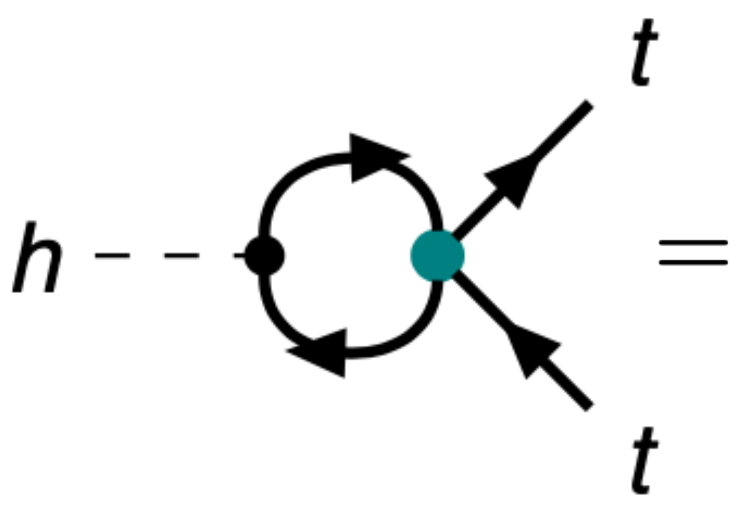
$$\begin{aligned}\mathcal{L}_{4t} = {} & \frac{C^{(1)}_{Qt}}{\Lambda^2}\bar{Q}_L\gamma^\mu Q_L\bar{t}_R\gamma_\mu t_R + \frac{C^{(8)}_{Qt}}{\Lambda^2}\bar{Q}_L\gamma^\mu T^a Q_L\bar{t}_R\gamma_\mu T^a t_R \\ & + \frac{C^{(1)}_{QQ}}{\Lambda^2}\bar{Q}_L\gamma^\mu Q_L\bar{Q}_L\gamma_\mu Q_L + \frac{C^{(8)}_{QQ}}{\Lambda^2}\bar{Q}_L\gamma^\mu T^a Q_L\bar{Q}_L\gamma_\mu T^a Q_L \\ & + \frac{C_{tt}}{\Lambda^2}\bar{t}_R\gamma^\mu t_R\bar{t}_R\gamma_\mu t_R\end{aligned}$$

Scheme dependence induced by 4t operators

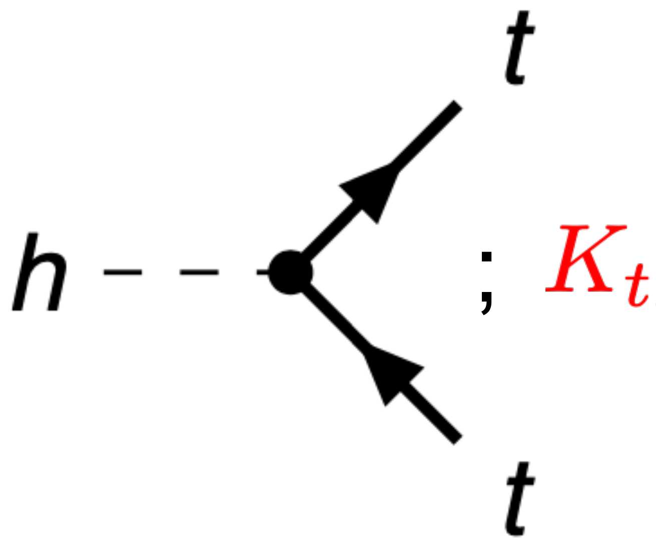
scheme dependent part

$$= \frac{C^{(1)}_{Qt} + c_F C^{(8)}_{Qt}}{\Lambda^2}\left(B_{m_t} + K_{m_t}\right) \times$$

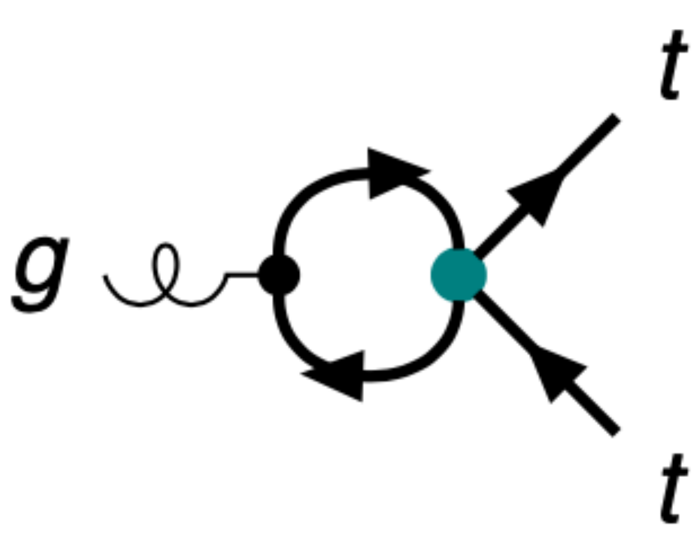
$$; \quad K_{m_t} = \begin{cases} -\frac{m_t^2}{8\pi^2} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}$$



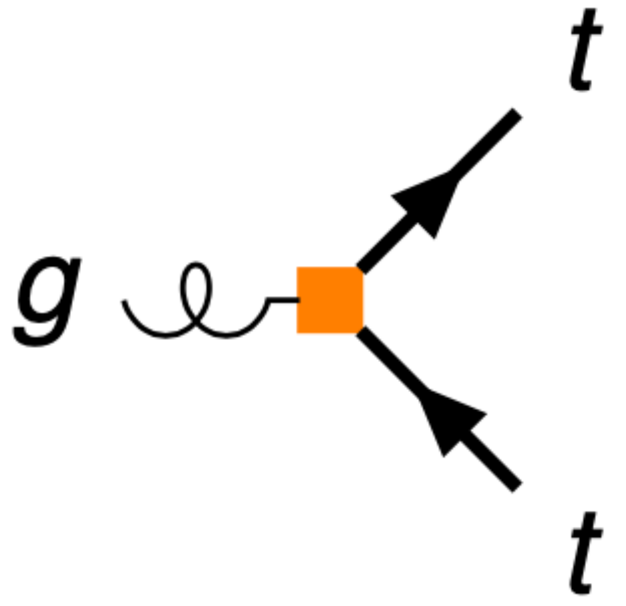
$$= \frac{C^{(1)}_{Qt} + c_F C^{(8)}_{Qt}}{\Lambda^2}\left(B_{ht\bar{t}} + K_{m_t} - \frac{v^3}{\sqrt{2}m_t} K_{tH}\right) \times$$



$$; \quad K_{tH} = \begin{cases} \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}$$



$$= \frac{C^{(1)}_{Qt} + \left(c_F - \frac{c_A}{2}\right) C^{(8)}_{Qt}}{C_{tG}} K_{tG} \times$$



$$; \quad K_{tG} = \begin{cases} -\frac{\sqrt{2}m_t g_s}{16\pi^2 v} & \text{(NDR)} \\ 0 & \text{(BMHV)} \end{cases}$$

Scheme (in)dependence

The renormalised physical amplitude must be scheme-independent

$$
\begin{aligned}
\mathcal{M}^{\mathrm{ren}} = \mathcal{M}^{\text{scheme indep.}} \\
&+ \left(C_{Qt}^{(1)} + c_F\, C_{Qt}^{(8)}\right) \frac{1}{\Lambda^2} \boxed{K_{m_t}} \frac{\partial \mathcal{M}_{\mathrm{SM}}}{\partial m_t} \times m_t \\
&+ \left[1 - \frac{v^3}{\sqrt{2} m_t} \left(\frac{C_{tH}}{\Lambda^2} + \boxed{K_{tH}} \frac{C_{Qt}^{(1)} + c_F\, C_{Qt}^{(8)}}{\Lambda^2} \right)\right] \mathcal{M}_{\mathrm{SM}} \\
&+ \underbrace{\left[C_{tG} + \left(C_{Qt}^{(1)} + (c_F - \frac{c_A}{2}) C_{Qt}^{(8)} \right) \boxed{K_{tG}} \right]}_{\tilde{C}_{tG}} \frac{1}{\Lambda^2} \mathcal{M}_{tG}|_{\mathrm{FIN}}
\end{aligned}
$$

$\Rightarrow$ scheme dependence of K-terms
must be cancelled by
scheme dependence of
Wilson coefficients and parameters

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&+ \left[1 - \frac{v^3}{\sqrt{2} m_t} \left(\frac{C_{tH}}{\Lambda^2} + \boxed{K_{tH}} \frac{C_{Qt}^{(1)} + c_F\, C_{Qt}^{(8)}}{\Lambda^2} \right)\right] \mathcal{M}_{\text{SM}} \\
&+ \underbrace{\left[C_{tG} + \left(C_{Qt}^{(1)} + (c_F - \frac{c_A}{2}) C_{Qt}^{(8)}\right) \boxed{K_{tG}}\right]}_{\tilde{C}_{tG}} \frac{1}{\Lambda^2} \mathcal{M}_{tG}\big|_{\text{FIN}}
\end{aligned}
$$

$\Rightarrow$ scheme dependence of K-terms must be cancelled by scheme dependence of Wilson coefficients and parameters

$\Rightarrow$

define combinations absorbing the scheme dependence or translation table

Di Noi, Gröber, GH, Lang, Vitti
2310.18221

Scheme (in)dependence

possible solution: redefine parameters, absorbing scheme dependent parts

$$\tilde{C}_{tG} = C_{tG} + \left(C_{Qt}^{(1)} + (c_F - \frac{c_A}{2}) C_{Qt}^{(8)} \right) K_{tG}$$

$$\tilde{C}_{tH} = C_{tH} + \left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)} \right) K_{tH}$$

$$\tilde{m}_t = m_t \left(1 + \frac{C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}}{\Lambda^2} K_{m_t} \right)$$

known e.g. in flavour physics
Ciuchini et al. ’93
Herrlich, Nierste ‘94

S. Di Noi et al, 2310.18221

Scheme (in)dependence

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$$\tilde{C}_{tG} = C_{tG} + \left(C_{Qt}^{(1)} + (c_F - \frac{c_A}{2})C_{Qt}^{(8)}\right) K_{tG}$$

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known e.g. in flavour physics
Ciuchini et al. ’93
Herrlich, Nierste ‘94

S. Di Noi et al, 2310.18221

more flexible: derive a **translation dictionary** by requiring $\tilde{X}^{\mathrm{NDR}} \overset{!}{=} \tilde{X}^{\mathrm{BMHV}}$

Translation between BMHV and NDR

4-top operators are linked to other operators through a scheme translation

$$m_t^{\mathrm{BMHV}} = m_t^{\mathrm{NDR}} - \frac{m_t^3}{8\pi^2\Lambda^2}\left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}\right)$$

$$C_{tH}^{\mathrm{BMHV}} = C_{tH}^{\mathrm{NDR}} + \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3}\left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}\right)$$

$$C_{tG}^{\mathrm{BMHV}} = C_{tG}^{\mathrm{NDR}} + \frac{\sqrt{2}m_t g_s}{16\pi^2 v}\left(C_{Qt}^{(1)} + (c_F - \frac{c_A}{2})C_{Qt}^{(8)}\right)$$

S. Di Noi et al, 2310.18221

note: loop suppression factor for C_{tG} not included here (Warsaw basis conventions)

Translation between BMHV and NDR

4-top operators are linked to other operators through a scheme translation

$$m_t^{\mathrm{BMHV}} = m_t^{\mathrm{NDR}} - \frac{m_t^3}{8\pi^2\Lambda^2}\left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}\right)$$

$$C_{tH}^{\mathrm{BMHV}} = C_{tH}^{\mathrm{NDR}} + \frac{\sqrt{2}m_t(4m_t^2 - m_h^2)}{16\pi^2 v^3}\left(C_{Qt}^{(1)} + c_F C_{Qt}^{(8)}\right)$$

$$\frac{C_{tG}^{\mathrm{BMHV}}}{16\pi^2} = \frac{C_{tG}^{\mathrm{NDR}}}{16\pi^2} + \frac{\sqrt{2}m_t g_s}{16\pi^2 v}\left(C_{Qt}^{(1)} + (c_F - \frac{c_A}{2})C_{Qt}^{(8)}\right)$$

S. Di Noi et al, 2310.18221

note: loop suppression factor for C_{tG} not included here (Warsaw basis conventions)

shift can be of same order as Wilson coefficient itself

gamma5 in 4 dimensions

$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ definition in 4 space-time dimensions

in 4 dimensions:

$\{\gamma_5, \gamma^\mu\} = 0$ (1)

$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5] = -4i\epsilon^{\mu\nu\rho\sigma}$ (2)

$\mathrm{Tr}[\Gamma_1\Gamma_2\gamma_5] = \mathrm{Tr}[\gamma_5\Gamma_1\Gamma_2]$ cyclicity of Traces (3)

in $D = 4 - 2\epsilon$ dimensions: (1), (2) and (3) cannot be maintained simultaneously

gamma5 in D dimensions

different schemes to extend γ_5 to D dimensions:

“naive dimensional regularisation” (**NDR**):

keep $\{\gamma_5,\gamma^\mu\}=0$

abandon cyclicity of trace (or fix inconsistencies by hand)

reading point for traces: “Kreimer scheme”

but: ambiguities observed at high loop orders

L. Chen, 2304.13814, J. Davies et al 2110.05496, …

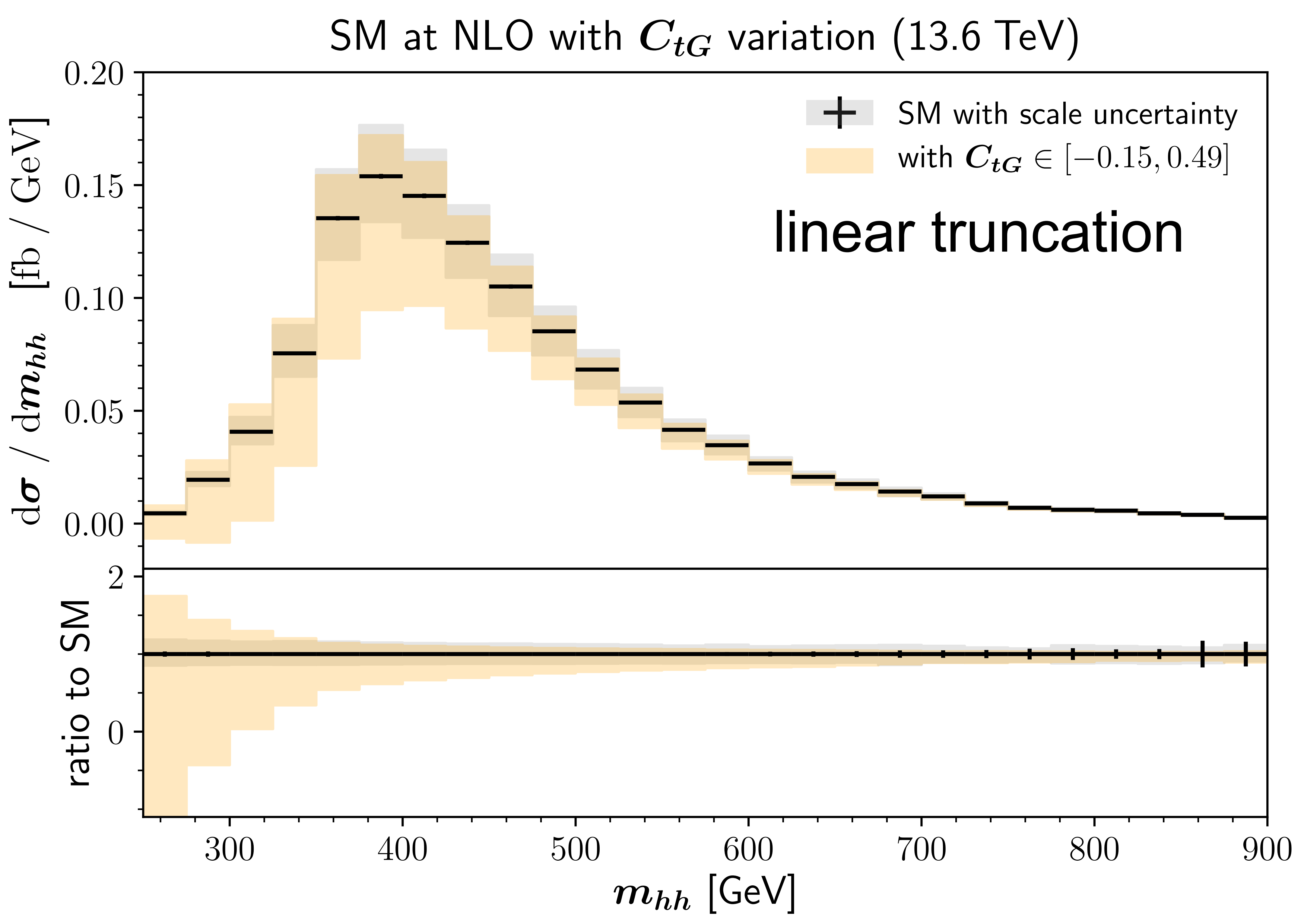
Breitenlohner, Maison; ‘t Hooft, Veltman (**BMHV**):

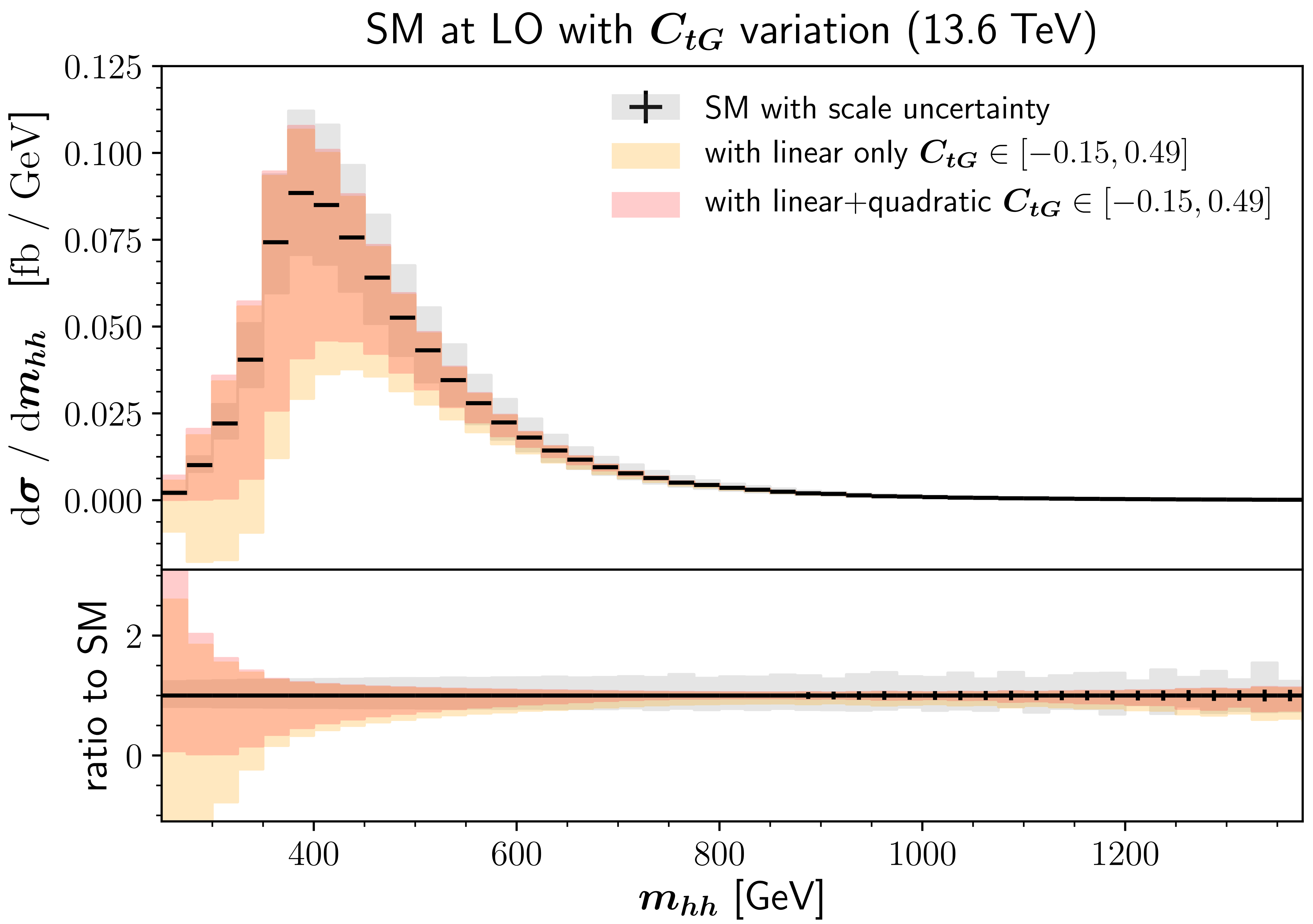
$$\gamma^\mu = \underbrace{\bar\gamma^\mu}_{\text{4-dim.}} + \underbrace{\hat\gamma^\mu}_{\text{(D-4) dim.}} \;;\; \{\gamma_5,\bar\gamma^\mu\}=0 \;;\; [\gamma_5,\hat\gamma^\mu]=0$$

- spurious breaking of gauge invariance
- needs symmetry restoring counterterms
- the latter can be derived algorithmically

Effects of chromomagnetic operator

variation ranges: from global fit (marginalised) , Ethier et al, 2105.00006 [SMEFiT coll.]

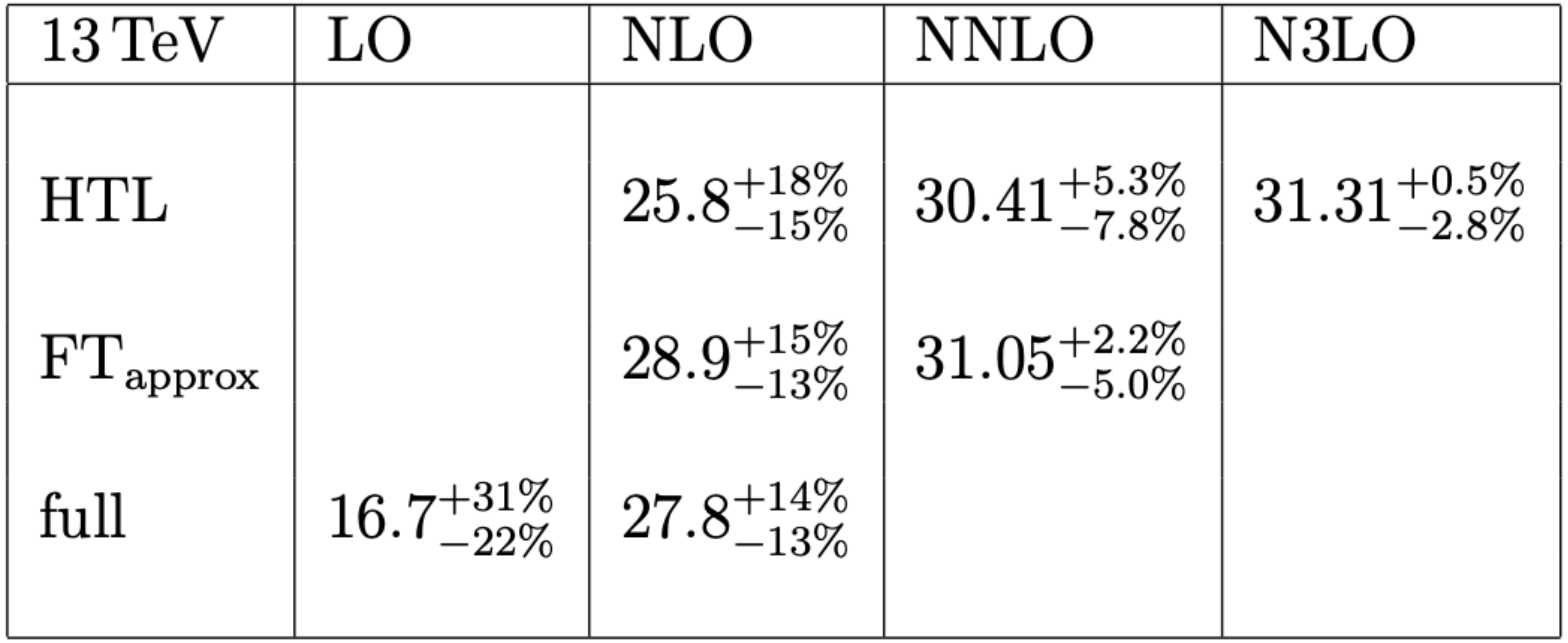




Effect larger than SM scale uncertainties

Effect of linear+quadratic truncation smaller than linear only due to destructive interference

SM status and scale uncertainties



13 TeV	LO	NLO	NNLO	N3LO
HTL		$25.8^{+18\%}_{-15\%}$	$30.41^{+5.3\%}_{-7.8\%}$	$31.31^{+0.5\%}_{-2.8\%}$
$\mathrm{FT}_{\mathrm{approx}}$		$28.9^{+15\%}_{-13\%}$	$31.05^{+2.2\%}_{-5.0\%}$	
full	$16.7^{+31\%}_{-22\%}$	$27.8^{+14\%}_{-13\%}$		

N3LO+N3LL: scale uncertainty <1%

Ajjath, Hua-Sheng Shao 2209.03914

PDF + α_s uncertainties ~ 2.3%