UC DAVIS
UNIVERSITY OF CALIFORNIA

QMAP

EFT Truncation Uncertainties at the LHC

Markus Luty
UC Davis/QMAP

Work in progress with F. Montagno, S. Chang, T. Ma, A. Wulzer

The Problem

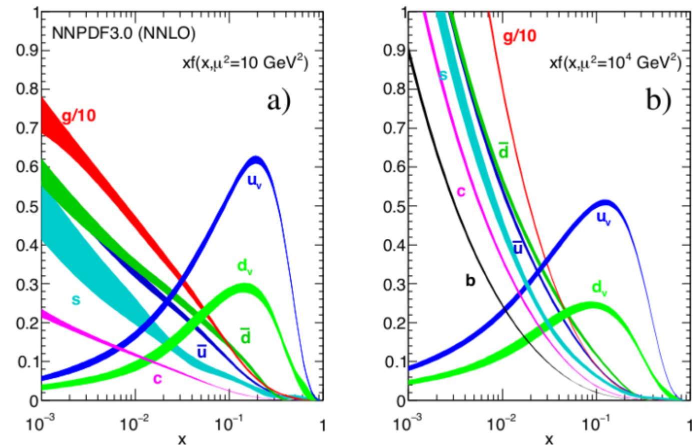
Need a *quantitative* method to treat EFT truncation uncertainties at the LHC.

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{A=1}^{N} G_A \mathcal{O}_A$$

$$\mathcal{M}(\hat{s}, \hat{t}, \hat{u}) = \mathcal{M}_{\text{SM}} + \sum_{A} G_A \mathcal{M}_A + \underbrace{O(G_A^2) + \cdots}_{= \, ?}$$

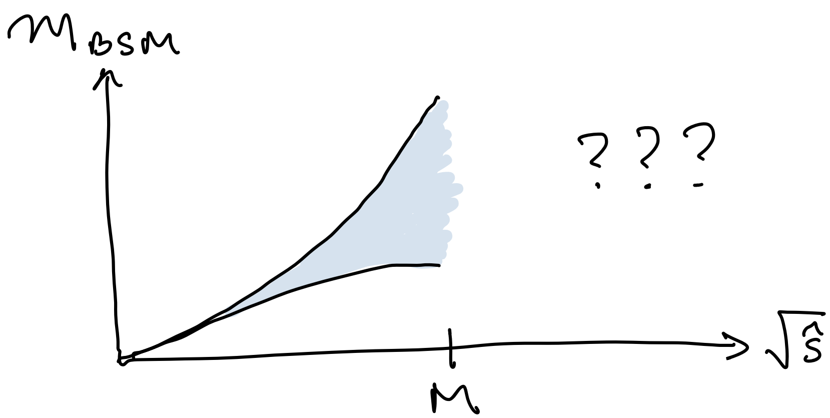
Our Proposal

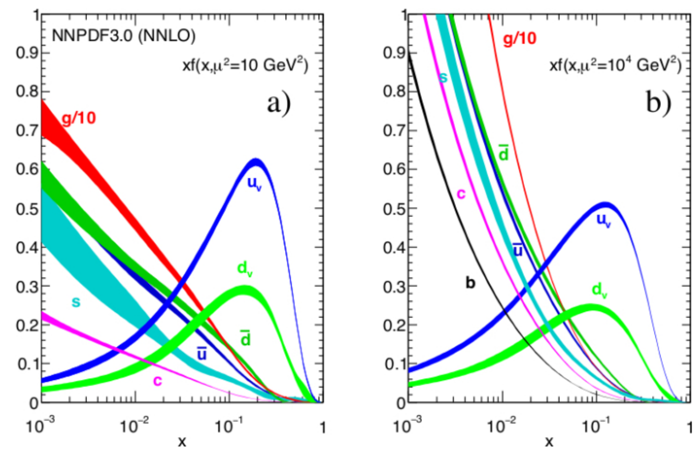
Treat as a theory uncertainty:



Our Proposal

Treat as a theory uncertainty:

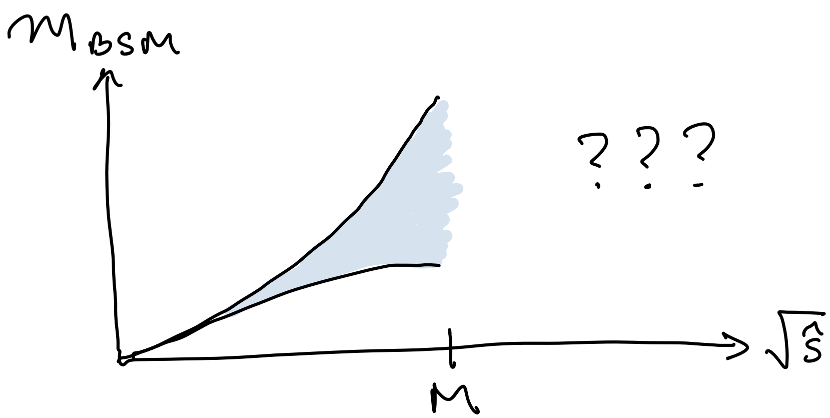


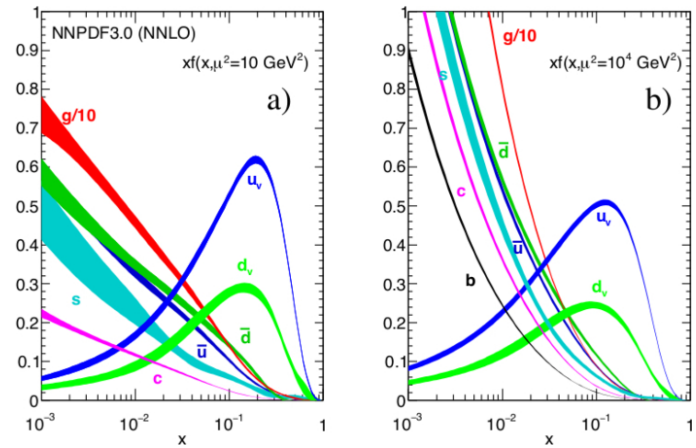


M = energy scale where EFT breaks down

Our Proposal

Treat as a theory uncertainty:

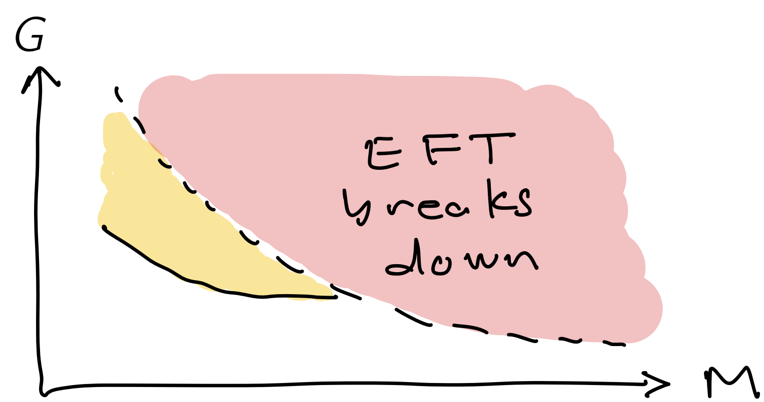




M = energy scale where EFT breaks down

Perform search in (M, G) plane

$$\mathcal{L}_{\text{BSM}} = G\mathcal{O}$$



Our Proposal

Parameterize EFT truncation uncertainty using nuisance parameters

Likelihood: $L = P(\boldsymbol{\nu})P(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\nu})$

$x_a =$ event

$\boldsymbol{\theta} =$ parameters of interest $(G,\ M,\ \ldots)$

$\boldsymbol{\nu} =$ nuisance parameters

Constrain $\boldsymbol{\theta}$, marginalizing over $\boldsymbol{\nu}$

Our Proposal

Parameterize EFT truncation uncertainty using nuisance parameters

Likelihood: $L = P(\boldsymbol{\nu}) P(\boldsymbol{x} | \boldsymbol{\theta}, \boldsymbol{\nu})$

$x_a = \text{event}$

$\boldsymbol{\theta} = \text{parameters of interest } (G, M, \ldots)$

$\boldsymbol{\nu} = \text{nuisance parameters}$

Constrain $\boldsymbol{\theta}$, marginalizing over $\boldsymbol{\nu}$

$$\mathcal{M}_{\text{BSM}} = \mathcal{M} - \mathcal{M}_{\text{SM}}$$

$$\rightarrow \mathcal{M}_{\text{BSM}} \cdot \underbrace{\mathcal{F}(x, y | \boldsymbol{\nu})}_{\text{form factor}}$$

$$x = \frac{\hat{s}}{M^2} \quad y = \frac{\hat{t}}{M^2}$$

Our Proposal

Parameterize EFT truncation uncertainty using nuisance parameters

Likelihood: $L = P(\boldsymbol{\nu})P(\boldsymbol{x}|\boldsymbol{\theta},\boldsymbol{\nu})$

$x_a =$ event

$\boldsymbol{\theta} =$ parameters of interest $(G,\ M,\ \ldots)$

$\boldsymbol{\nu} =$ nuisance parameters

Constrain $\boldsymbol{\theta}$, marginalizing over $\boldsymbol{\nu}$

$$\begin{aligned} \mathcal{M}_{\text{BSM}} &= \mathcal{M} - \mathcal{M}_{\text{SM}} \\ &\to \mathcal{M}_{\text{BSM}} \cdot \underbrace{\mathcal{F}(x, y|\boldsymbol{\nu})}_{\text{form factor}} \end{aligned}$$

$$x = \frac{\hat{s}}{M^2} \quad y = \frac{\hat{t}}{M^2}$$

Note: can use reweighting to generate events for different choices of $\mathcal{F}$

Our Proposal

Parameterize EFT truncation uncertainty using nuisance parameters

Likelihood: $L = P(\boldsymbol{\nu}) P(\boldsymbol{x}|\boldsymbol{\theta}, \boldsymbol{\nu})$

x_a = event

$\boldsymbol{\theta}$ = parameters of interest $(G, M, \ldots)$

$\boldsymbol{\nu}$ = nuisance parameters

Constrain $\boldsymbol{\theta}$, marginalizing over $\boldsymbol{\nu}$

$$\mathcal{M}_{\text{BSM}} = \mathcal{M} - \mathcal{M}_{\text{SM}}$$

$$\to \mathcal{M}_{\text{BSM}} \cdot \underbrace{\mathcal{F}(x, y|\boldsymbol{\nu})}_{\text{form factor}}$$

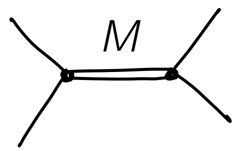
$$x = \frac{\hat{s}}{M^2} \quad y = \frac{\hat{t}}{M^2}$$

Note: can use reweighting to generate events for different choices of $\mathcal{F}$

$$\mathcal{F}(x, y|\boldsymbol{\nu}) = ?$$

EFT Corrections

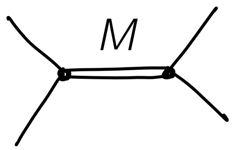
UV complete models will generate EFT operators + 'descendants'



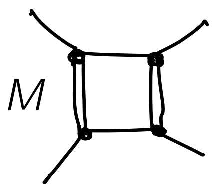
$$= -\frac{g^2}{M^2}\left[1 + \frac{s}{M^2} + \frac{s^2}{M^4} + \cdots\right]$$

EFT Corrections

UV complete models will generate EFT operators + ‘descendants’



$$= -\frac{g^2}{M^2}\left[1 + \frac{s}{M^2} + \frac{s^2}{M^4} + \cdots\right]$$



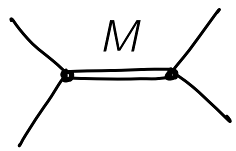
$$= \frac{g^2}{16\pi^2 M^2}\left[1 + c_1\frac{s}{M^2} + c_2\frac{t}{M^2} + c_3\frac{st}{M^2} + \cdots\right]$$

M = mass of heavy particle

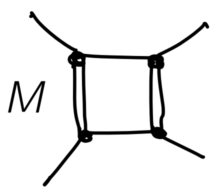
$c_1, c_2, c_3, \ldots \sim 1$

EFT Corrections

UV complete models will generate EFT operators + ‘descendants’



$$= \underbrace{-\frac{g^2}{M^2}}_{= \mathcal{M}_{\mathcal{O}}}\left[1 + \frac{s}{M^2} + \frac{s^2}{M^4} + \cdots\right]$$



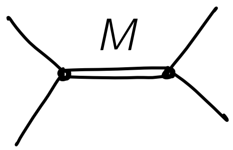
$$= \underbrace{\frac{g^2}{16\pi^2 M^2}}_{= \mathcal{M}_{\mathcal{O}}}\left[1 + c_1\frac{s}{M^2} + c_2\frac{t}{M^2} + c_3\frac{st}{M^2} + \cdots\right]$$

$M =$ mass of heavy particle

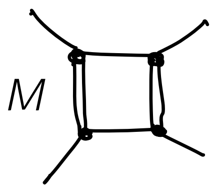
$c_1, c_2, c_3, \ldots \sim 1$

EFT Corrections

UV complete models will generate EFT operators + ‘descendants’



$$= \underbrace{-\frac{g^2}{M^2}}_{= \mathcal{M}_{\mathcal{O}}}\left[1 + \underbrace{\frac{s}{M^2} + \frac{s^2}{M^4} + \cdots}_{\text{descendants}}\right]$$



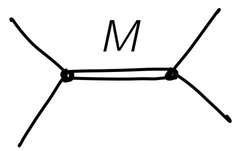
$$= \underbrace{\frac{g^2}{16\pi^2 M^2}}_{= \mathcal{M}_{\mathcal{O}}}\left[1 + \underbrace{c_1\frac{s}{M^2} + c_2\frac{t}{M^2} + c_3\frac{st}{M^2} + \cdots}_{\text{descendants}}\right]$$

M = mass of heavy particle

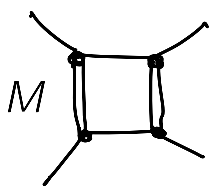
$c_1, c_2, c_3, \ldots \sim 1$

EFT Corrections

UV complete models will generate EFT operators + ‘descendants’



$$= \underbrace{-\frac{g^2}{M^2}}_{= \mathcal{M}_{\mathcal{O}}} \left[1 + \underbrace{\frac{s}{M^2} + \frac{s^2}{M^4} + \cdots}_{\text{descendants}} \right]$$



$$= \underbrace{\frac{g^2}{16\pi^2 M^2}}_{= \mathcal{M}_{\mathcal{O}}} \left[1 + \underbrace{c_1 \frac{s}{M^2} + c_2 \frac{t}{M^2} + c_3 \frac{st}{M^2} + \cdots}_{\text{descendants}} \right]$$

$M =$ mass of heavy particle

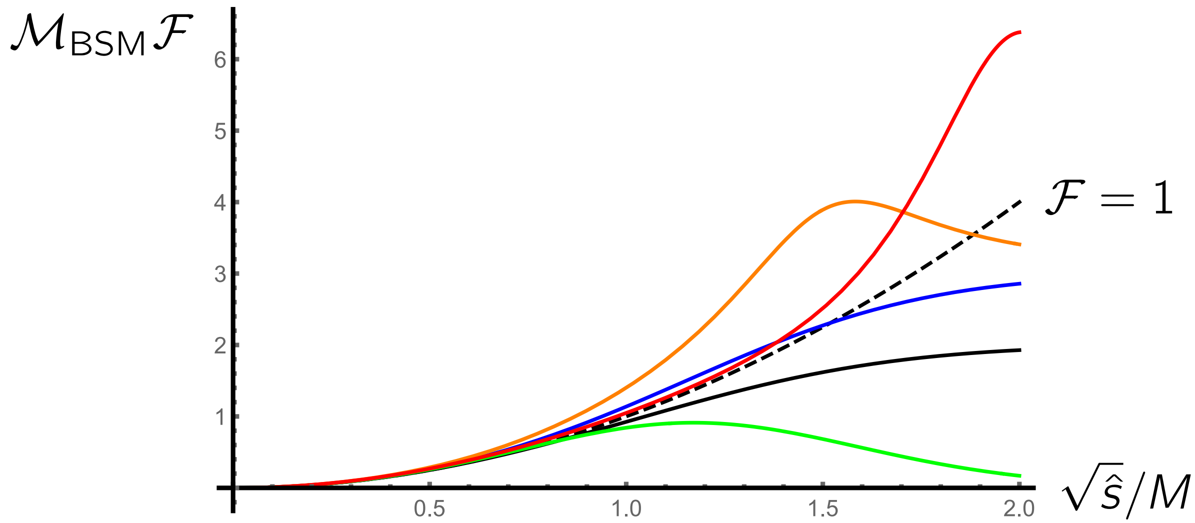
$c_1, c_2, c_3, \ldots \sim 1$

Use descendants to parameterize higher derivative effects in EFT

Form Factors

Form factors parameterize possible high energy behavior

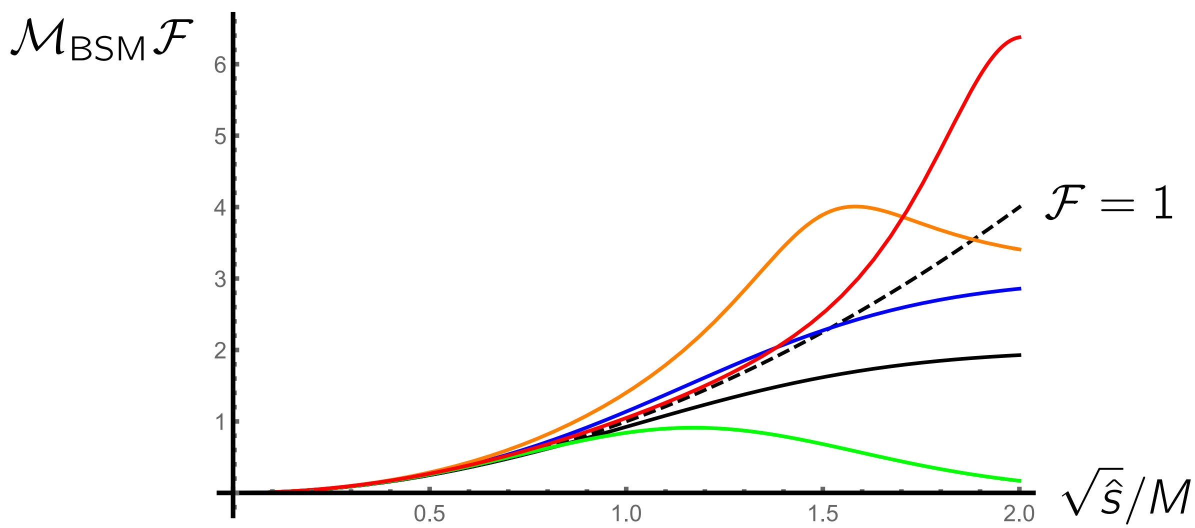
$$\mathcal{M}_{\text{BSM}}(\hat{s},\hat{t}) \to \mathcal{M}_{\text{BSM}}(\hat{s},\hat{t})\mathcal{F}(x,y) \qquad x=\frac{\hat{s}}{M^2} \quad y=\frac{\hat{t}}{M^2} \quad z=\frac{\hat{u}}{M^2}$$



Form Factors

Form factors parameterize possible high energy behavior

$$\mathcal{M}_{\mathrm{BSM}}(\hat{s},\hat{t}) \to \mathcal{M}_{\mathrm{BSM}}(\hat{s},\hat{t})\mathcal{F}(x,y) \qquad x = \frac{\hat{s}}{M^2} \quad y = \frac{\hat{t}}{M^2} \quad z = \frac{\hat{u}}{M^2}$$



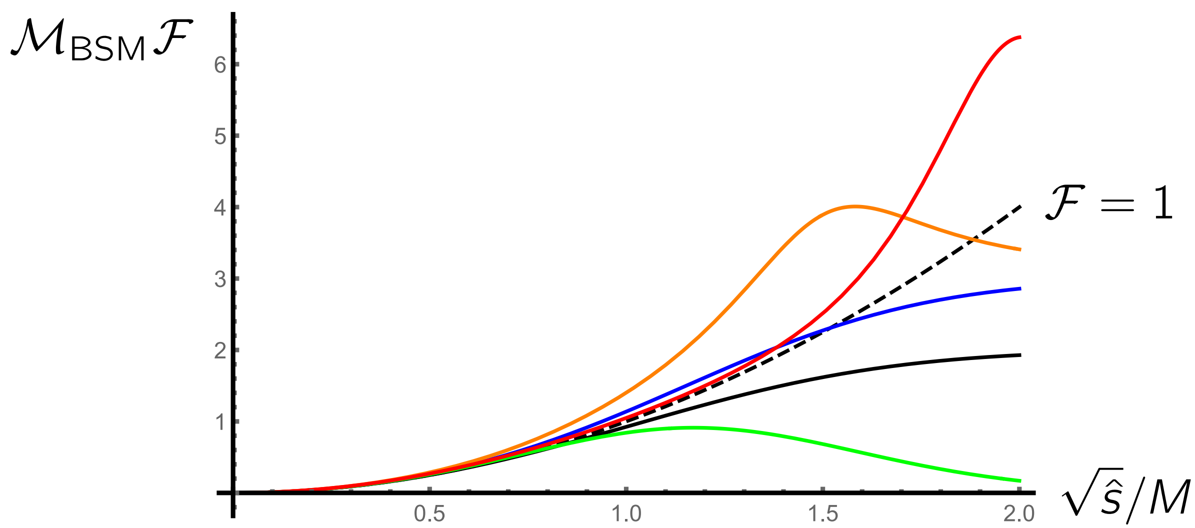
- $x, y, z \ll 1$: Parameterize descendants

$$\Rightarrow \mathcal{F}(x,y) = 1 + \nu_1 x + \nu_2 y + \nu_3 xy + \cdots \qquad \nu_a \sim 1$$

Form Factors

Form factors parameterize possible high energy behavior

$$\mathcal{M}_{\text{BSM}}(\hat{s},\hat{t}) \to \mathcal{M}_{\text{BSM}}(\hat{s},\hat{t})\mathcal{F}(x,y) \qquad x=\frac{\hat{s}}{M^2} \quad y=\frac{\hat{t}}{M^2} \quad z=\frac{\hat{u}}{M^2}$$



- $x, y, z \ll 1$: Parameterize descendants

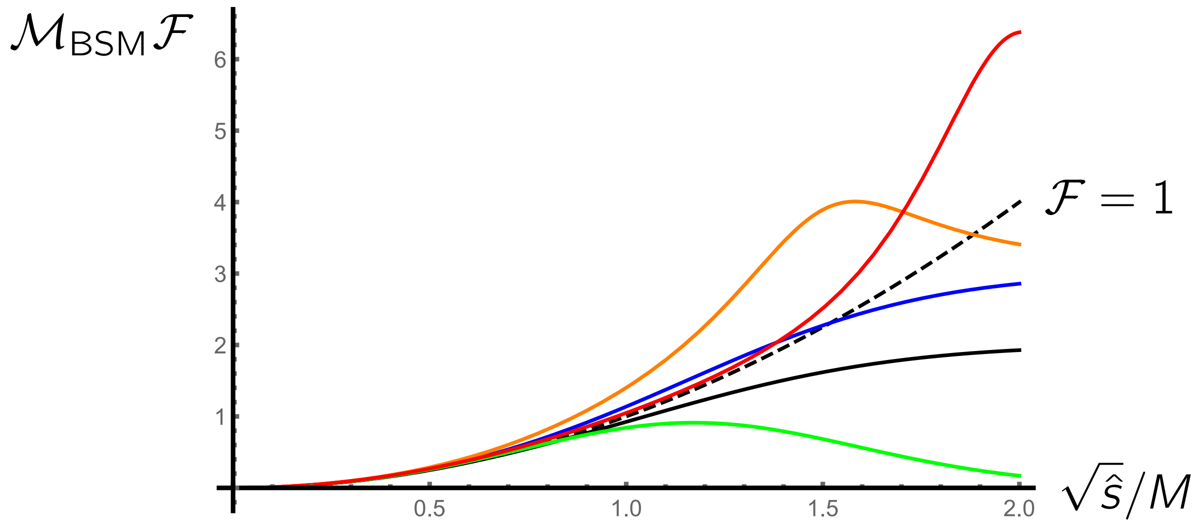
  $\Rightarrow \mathcal{F}(x,y) = 1 + \nu_1 x + \nu_2 y + \nu_3 xy + \cdots \qquad \nu_a \sim 1$

- $x, y, z \gtrsim 1$: $\mathcal{M}_{\text{BSM}}\mathcal{F}$ has 100% uncertainty

Form Factors

Form factors parameterize possible high energy behavior

$$\mathcal{M}_{\text{BSM}}(\hat{s},\hat{t}) \to \mathcal{M}_{\text{BSM}}(\hat{s},\hat{t})\mathcal{F}(x,y) \qquad x=\frac{\hat{s}}{M^2} \quad y=\frac{\hat{t}}{M^2} \quad z=\frac{\hat{u}}{M^2}$$



- $x, y, z \ll 1$: Parameterize descendants

$$\Rightarrow \mathcal{F}(x,y) = 1 + \nu_1 x + \nu_2 y + \nu_3 xy + \cdots \qquad \nu_a \sim 1$$

- $x, y, z \gtrsim 1$: $\mathcal{M}_{\text{BSM}}\mathcal{F}$ has 100% uncertainty

- $x, y, z \gg 1$: $\mathcal{M}_{\text{BSM}}\mathcal{F} \to$ constant (scale invariant)

Form Factors

$$F(x) = \text{sigmoid} = \begin{cases} x + O(x^2) & x \to 0, \\ 1 + O(x^{-1}) & x \to \infty, \end{cases}$$

Form Factors

$$F(x) = \text{sigmoid} = \begin{cases} x + O(x^2) & x \to 0, \\ 1 + O(x^{-1}) & x \to \infty, \end{cases}$$

$$\mathcal{F}(x,y) = \frac{F(x)^\alpha F(y)^\beta F(z)^\gamma}{x^\alpha y^\beta z^\gamma} \Bigg\{ \sum_{a,b,c=0}^{\infty} \nu_{abc} F(x)^a F(y)^b F(z)^c$$

$$+ \sum_a \mathcal{S}_a \left[\frac{1}{x - x_a + i\gamma_a} - (x = 0) \right]$$

$$+ \text{crossed} \Bigg\}$$

Similar to L. Allwicher, D. Faroughy, F. Jaffredo, O. Sumensari, F. Wilsch (2023)

Form Factors

$$F(x) = \text{sigmoid} = \begin{cases} x + O(x^2) & x \to 0, \\ 1 + O(x^{-1}) & x \to \infty, \end{cases}$$

$$\mathcal{F}(x,y) = \underbrace{\frac{F(x)^\alpha F(y)^\beta F(z)^\gamma}{x^\alpha y^\beta z^\gamma}}_{} \Bigg\{ \sum_{a,b,c=0}^{\infty} \nu_{abc} F(x)^a F(y)^b F(z)^c$$

$\mathcal{M}_{\mathcal{O}}(\hat{s},\hat{t}) \sim \hat{s}^\alpha \hat{t}^\beta \hat{u}^\gamma$

$\Rightarrow$ cancels high energy growth

$$+ \sum_a \mathcal{S}_a \left[\frac{1}{x - x_a + i\gamma_a} - (x=0) \right]$$

$$+ \text{crossed} \Bigg\}$$

Similar to L. Allwicher, D. Faroughy, F. Jaffredo, O. Sumensari, F. Wilsch (2023)

Form Factors

$$F(x) = \text{sigmoid} = \begin{cases} x + O(x^2) & x \to 0, \\ 1 + O(x^{-1}) & x \to \infty, \end{cases}$$

$$\mathcal{F}(x,y) = \underbrace{\frac{F(x)^\alpha F(y)^\beta F(z)^\gamma}{x^\alpha y^\beta z^\gamma}}_{\substack{\mathcal{M}_{\mathcal{O}}(\hat{s},\hat{t}) \sim \hat{s}^\alpha \hat{t}^\beta \hat{u}^\gamma \\ \Rightarrow \text{ cancels high} \\ \text{energy growth}}} \Bigg\{ \underbrace{\sum_{a,b,c=0}^{\infty} \nu_{abc} F(x)^a F(y)^b F(z)^c}_{= \begin{cases} \text{descendants} & x,y,z \ll 1 \\ \text{constant} & x,y,z \gg 1 \end{cases}}$$

$$+ \sum_a \mathcal{S}_a \left[\frac{1}{x - x_a + i\gamma_a} - (x = 0) \right]$$

$$+ \text{ crossed} \Bigg\}$$

Similar to L. Allwicher, D. Faroughy, F. Jaffredo, O. Sumensari, F. Wilsch (2023)

Form Factors

$$F(x) = \text{sigmoid} = \begin{cases} x + O(x^2) & x \to 0, \\ 1 + O(x^{-1}) & x \to \infty, \end{cases}$$

$$\mathcal{F}(x,y) = \underbrace{\frac{F(x)^\alpha F(y)^\beta F(z)^\gamma}{x^\alpha y^\beta z^\gamma}}_{\substack{\mathcal{M}_\mathcal{O}(\hat{s},\hat{t}) \sim \hat{s}^\alpha \hat{t}^\beta \hat{u}^\gamma \\ \Rightarrow \text{ cancels high} \\ \text{energy growth}}} \Bigg\{ \underbrace{\sum_{a,b,c=0}^{\infty} \nu_{abc} F(x)^a F(y)^b F(z)^c}_{= \begin{cases} \text{descendants} & x,y,z \ll 1 \\ \text{constant} & x,y,z \gg 1 \end{cases}}$$

$$+ \sum_a \mathcal{S}_a \underbrace{\left[\frac{1}{x - x_a + i\gamma_a} - (x = 0) \right]}_{\text{resonances}}$$

$$+ \text{crossed} \Bigg\}$$

$$x_a = \frac{m_a^2}{M^2} \qquad \gamma_a = \frac{m_a \Gamma_a}{M^2}$$

Similar to L. Allwicher, D. Faroughy, F. Jaffredo, O. Sumensari, F. Wilsch (2023)

Toy Analysis

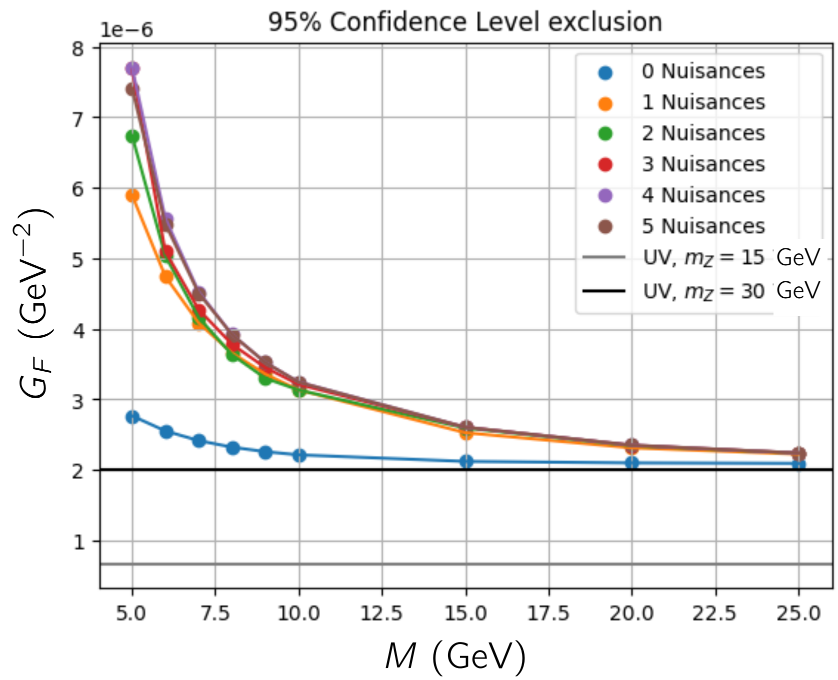
SM $\rightarrow$ QED   LHC $\rightarrow$ 30 GeV pp collider   BSM $\rightarrow$ Z boson

Toy Analysis

SM $\rightarrow$ QED

LHC $\rightarrow$ 30 GeV pp collider

BSM $\rightarrow$ Z boson

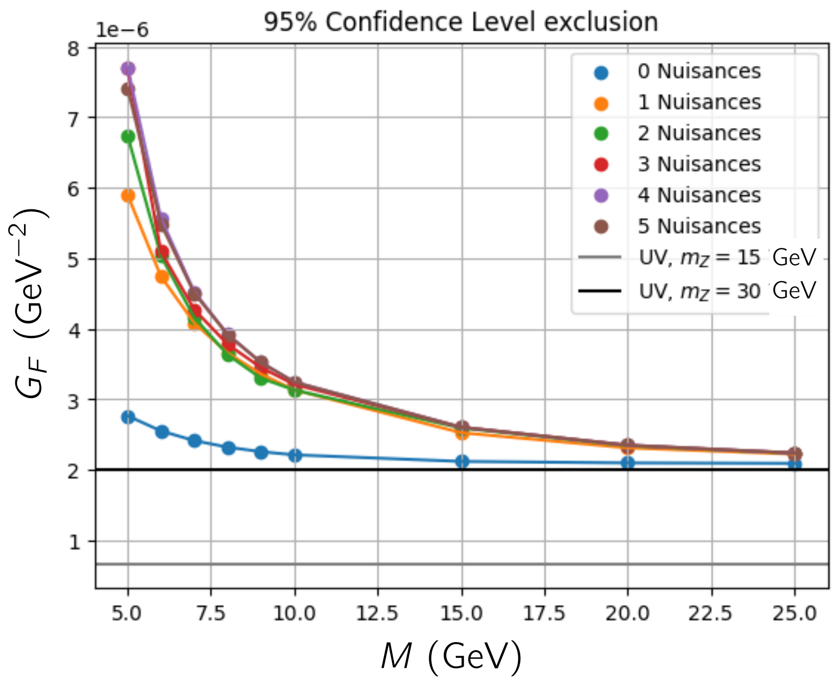


$u\bar{u} \rightarrow e^+e^-$

Binned in $m_{e^+e^-}$

Toy Analysis

SM $\rightarrow$ QED   LHC $\rightarrow$ 30 GeV pp collider   BSM $\rightarrow$ Z boson



$u\bar{u} \rightarrow e^+e^-$

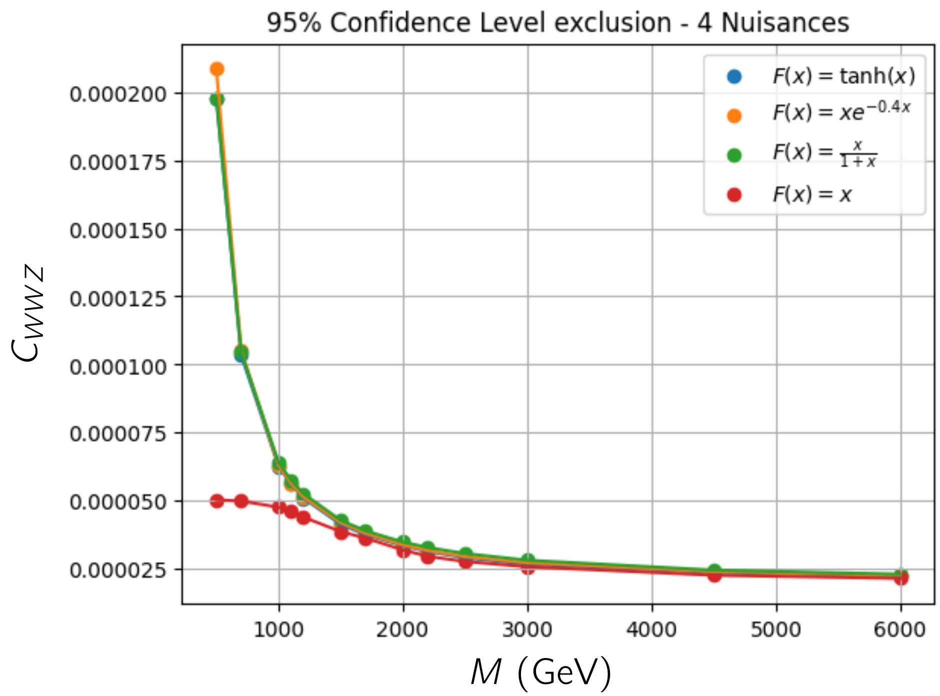
Binned in $m_{e^+e^-}$

- Nuisance parameters weaken the bound

  They reduce the significance of events with large $\sqrt{\hat{s}}$

- Constraints are weaker at small M
- Results converge as more niusance parameters are added
- EFT constraints are weaker than constraints from search for UV model

Form Factor Dependence

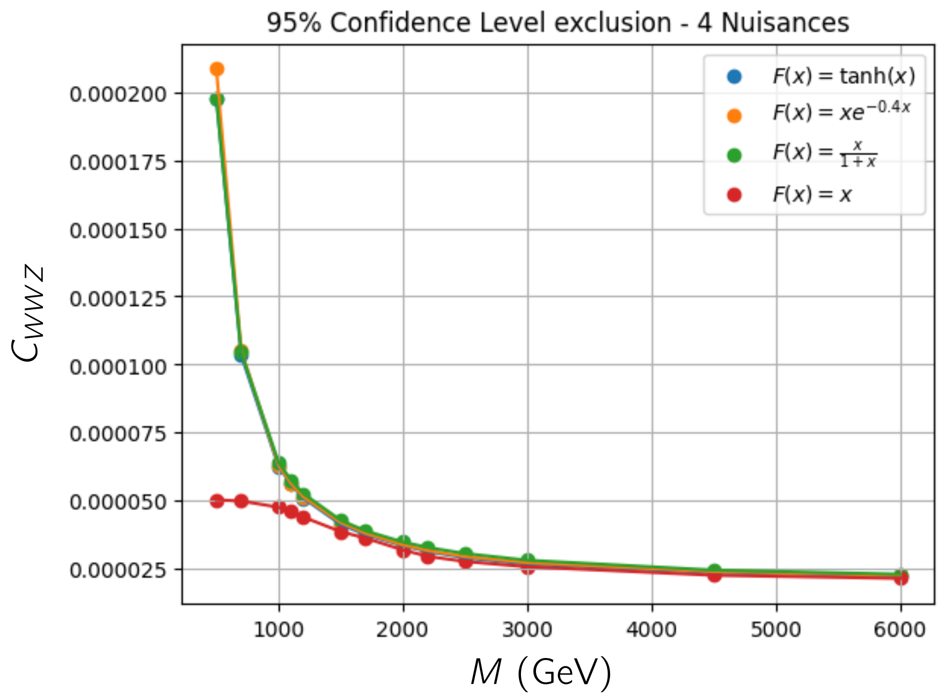


$pp \to WZ$ toy analysis

HL-LHC, 3 ab^{-1}

$$\mathcal{L}_{\text{BSM}} = C_{WWZ}\left[i(W^{+})^{\mu\nu}W^{-}_{\mu}Z_{\nu} + \text{h.c.}\right]$$

Form Factor Dependence



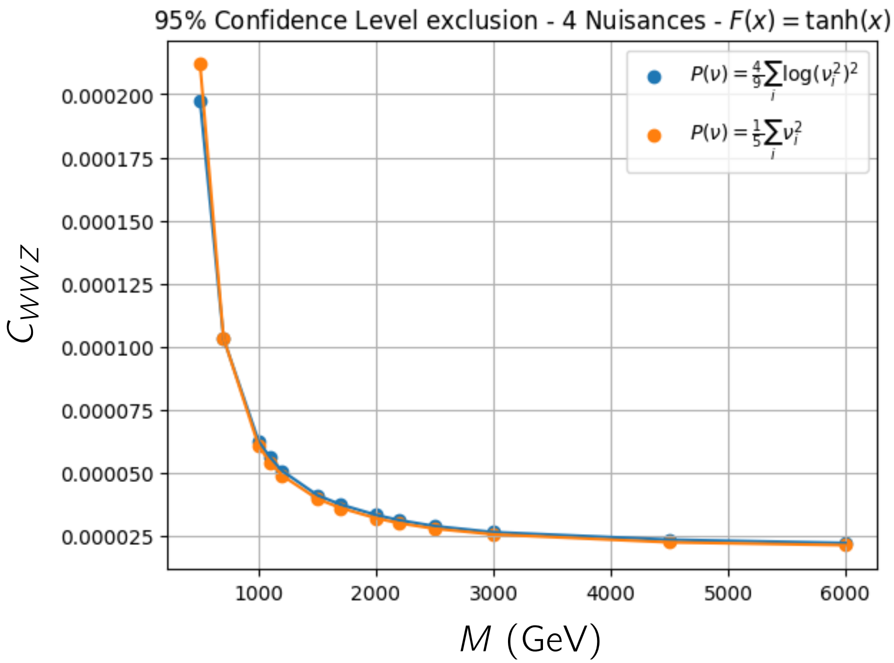
$pp \to WZ$ toy analysis

HL-LHC, 3 ab^{-1}

$$\mathcal{L}_{\mathrm{BSM}} = C_{WWZ}\left[i(W^+)^{\mu\nu}W^-_\mu Z_\nu + \mathrm{h.c.}\right]$$

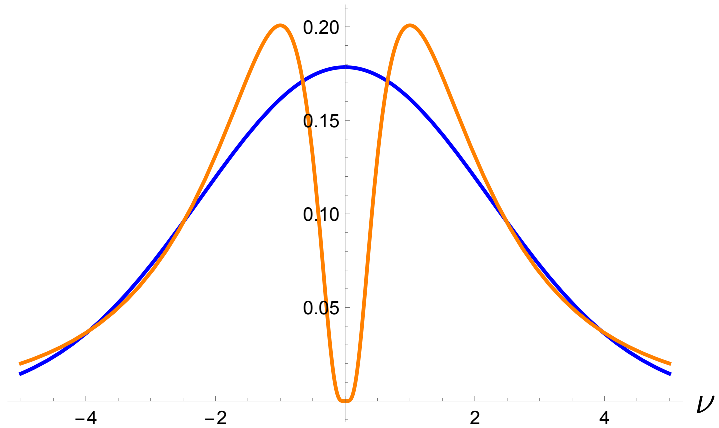
Behavior of $F(x)$ for $x \gtrsim 1$ does not affect final results

Nuisance Probability

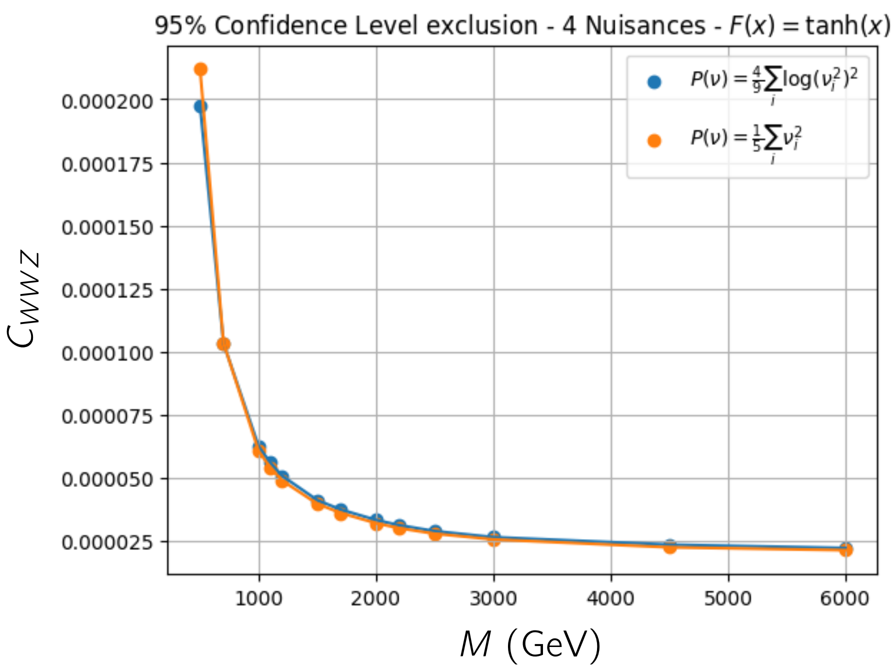


$P(\nu) \propto e^{-2\log(\nu^2)^2/9}$

$P(\nu) \propto e^{-\nu^2/10}$

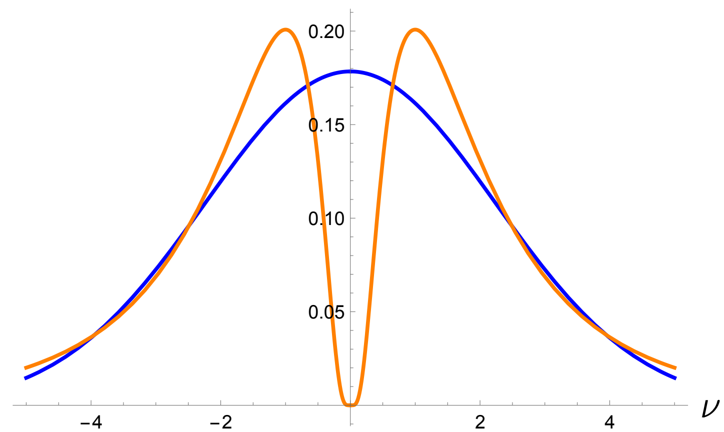


Nuisance Probability



$P(\nu) \propto e^{-2\log(\nu^2)^2/9}$

$P(\nu) \propto e^{-\nu^2/10}$

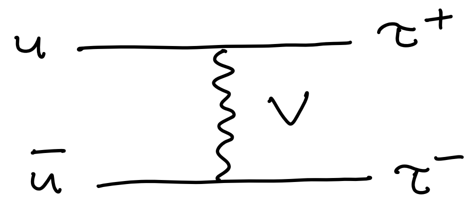


Results are insensitive to form of $P(\nu)$ if we fix tail of distribution

e.g. $\text{Prob}(\nu > \#) = \#$

Realistic Case Study

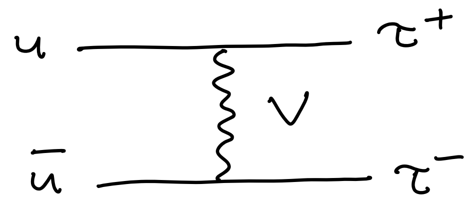
Fourth generation vector leptoquark model



$$\mathcal{L}_{\text{UV}} = \frac{g}{\sqrt{2}}\bar{b}\gamma^{\mu}\tau V_{\mu} + \text{h.c.} + \cdots \quad \rightarrow \quad \mathcal{L}_{\text{EFT}} = \underbrace{\frac{g^2}{2m_V^2}}_{= C_{\bar{b}b\bar{\tau}\tau}}|\bar{b}\gamma^{\mu}\tau|^2 + \cdots$$

Realistic Case Study

Fourth generation vector leptoquark model



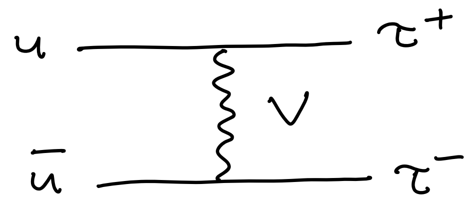
$$\mathcal{L}_{\text{UV}} = \frac{g}{\sqrt{2}}\bar{b}\gamma^{\mu}\tau V_{\mu} + \text{h.c.} + \cdots \quad \rightarrow \quad \mathcal{L}_{\text{EFT}} = \underbrace{\frac{g^2}{2m_V^2}}_{= C_{\bar{b}b\bar{\tau}\tau}} |\bar{b}\gamma^{\mu}\tau|^2 + \cdots$$

- Motivated by *b* anomalies

  Can be UV-completed into '4321' model

Realistic Case Study

Fourth generation vector leptoquark model



$$\mathcal{L}_{\text{UV}} = \frac{g}{\sqrt{2}} \bar{b}\gamma^{\mu}\tau V_{\mu} + \text{h.c.} + \cdots \quad \rightarrow \quad \mathcal{L}_{\text{EFT}} = \underbrace{\frac{g^2}{2m_V^2}}_{= C_{\bar{b}b\bar{\tau}\tau}} |\bar{b}\gamma^{\mu}\tau|^2 + \cdots$$

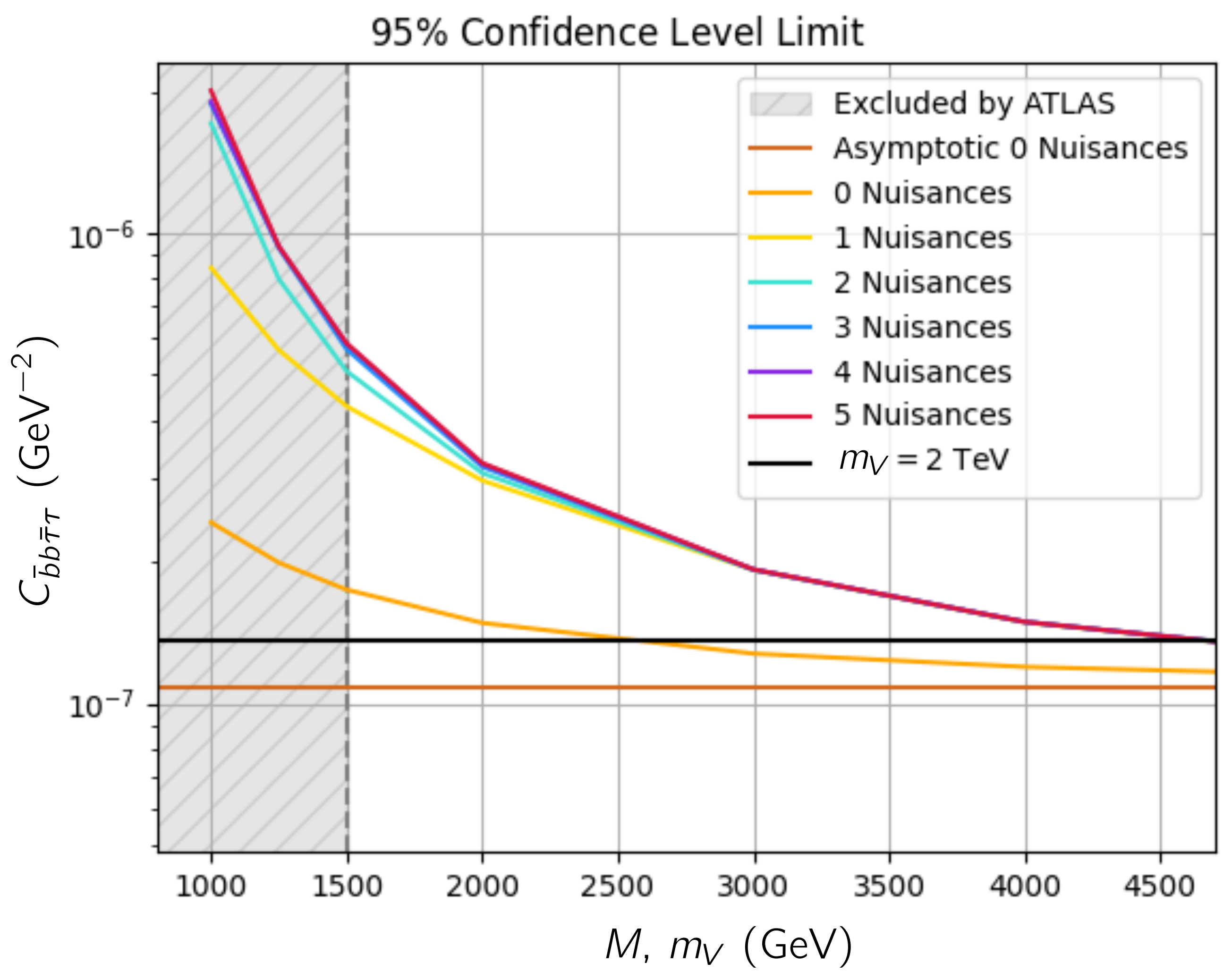
- Motivated by b anomalies

  Can be UV-completed into '4321' model

- Used data from CMS-HIG-21-001

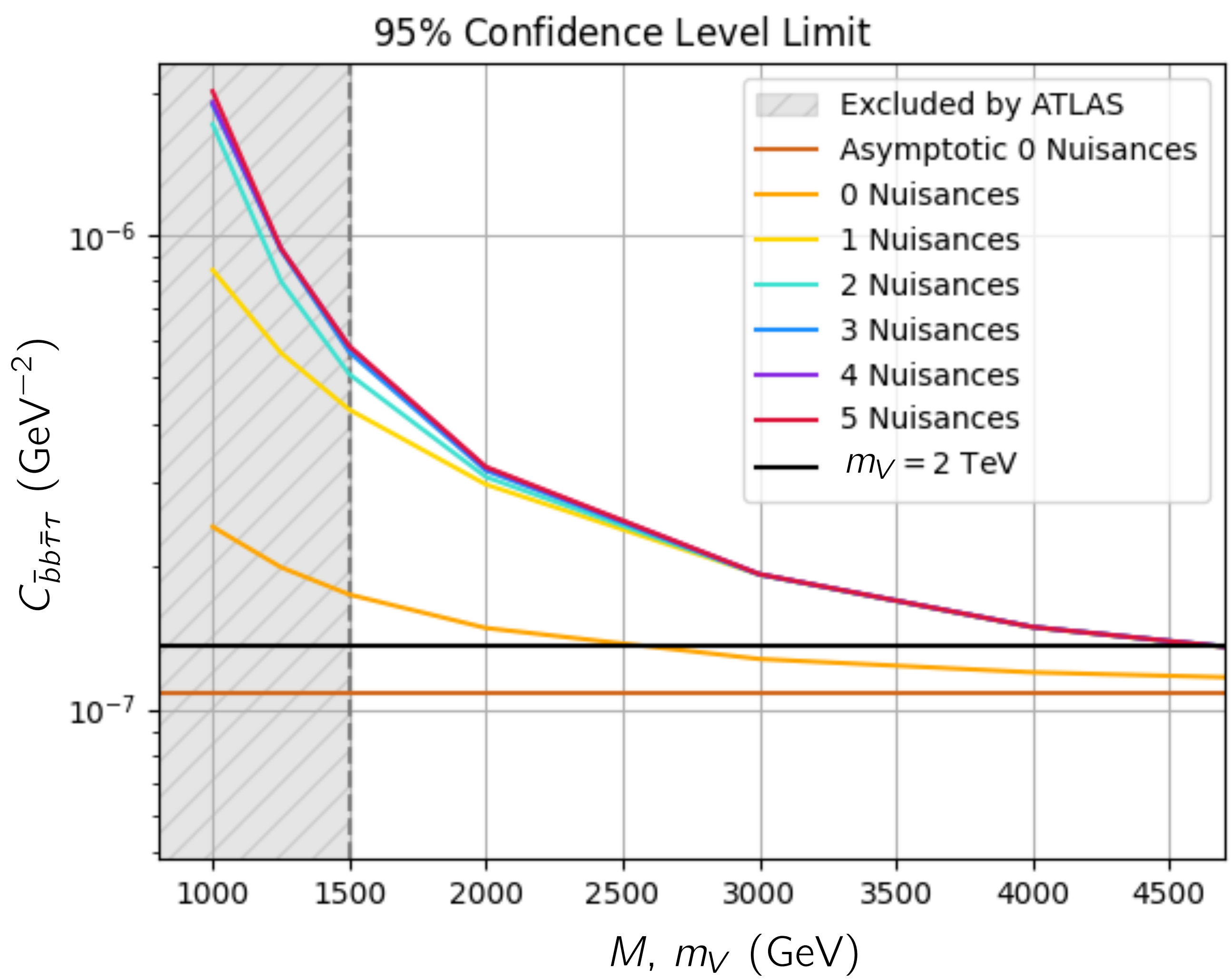
  Verified agreement with generated signal, background

Results



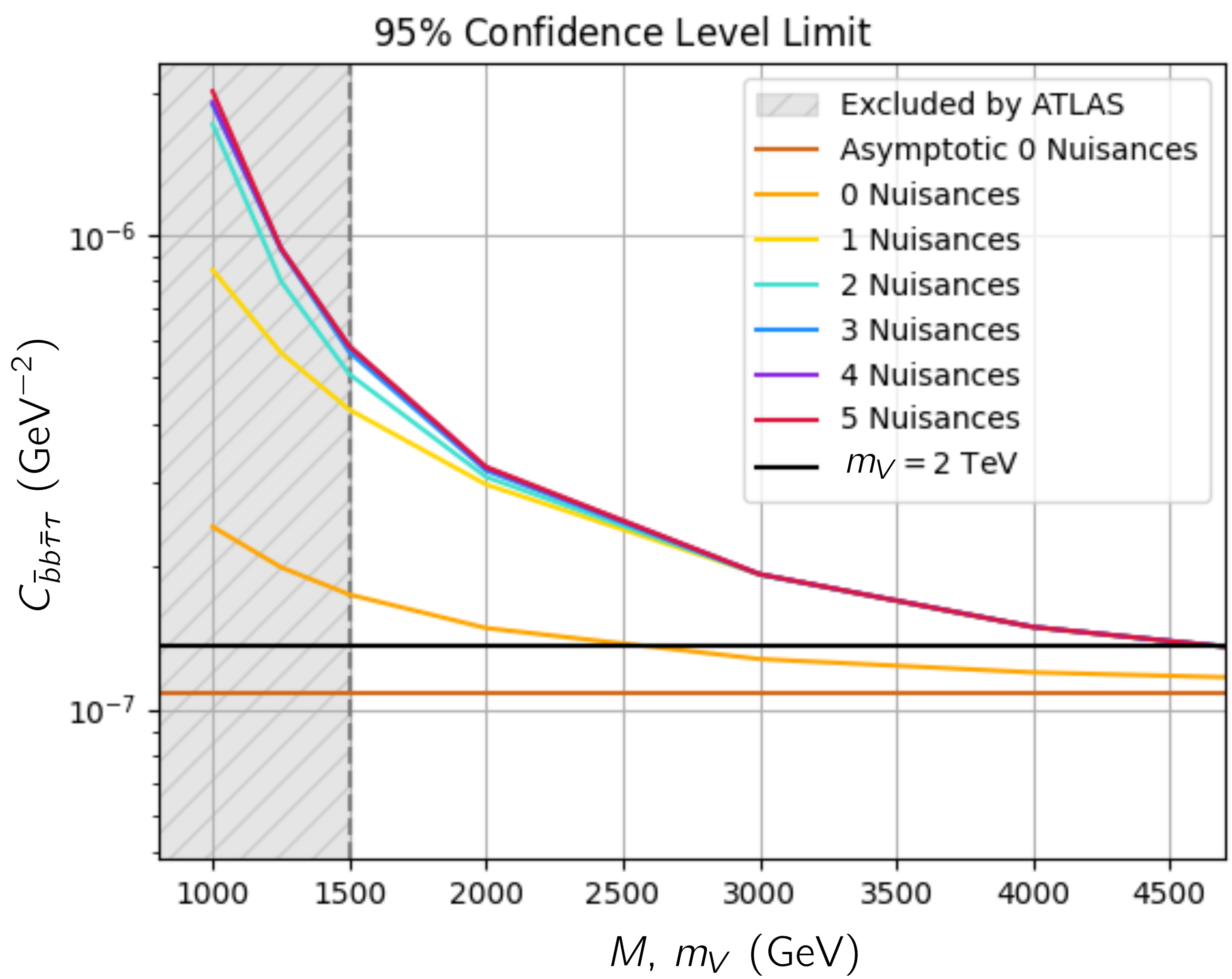
95% Confidence Level Limit
Excluded by ATLAS
Asymptotic 0 Nuisances
0 Nuisances
1 Nuisances
2 Nuisances
3 Nuisances
4 Nuisances
5 Nuisances
$m_V = 2$ TeV
10^{-6}
10^{-7}
$C_{\bar{b}b\bar{\tau}\tau}$ (GeV^{-2})
1000
1500
2000
2500
3000
3500
4000
4500
M, m_V (GeV)

Results



- Standard EFT search gives too-strong bounds

Results



- Standard EFT search gives too-strong bounds
- EFT + nuisance parameter search gives conservative bounds

Conclusions

- Proposed treating EFT truncation as theory uncertainty
- Uses standard statistical methods (nuisance parameters)

$\Rightarrow$ EFT prediction is uncertain at high energies

- Parameterize higher order corrections using 'descendants'
- Method is general and practical
- Results are convergent and robust

To do:

- Discovery examples
- Include nuisance form factors

$\vdots$

?

Backup Slides

Operator Parameterization

Classify EFT operators using amplitude methods:

$$\mathcal{O} \leftrightarrow \mathcal{M}_{\mathcal{O}} = \text{local on-shell amplitude}$$

Write $\mathcal{O}$ in terms of physical fields (massive W, Z, h, ...)

$\Rightarrow \mathcal{M}_{\mathcal{O}}$ is Feynman rule for $\mathcal{O}$ evaluated on shell

Finitely many 'primary' operators with ≤ 4 legs:

$$\underbrace{\sum_{\mathcal{O}} G_A \mathcal{M}_{\mathcal{O}_A}(s,t)}_{\text{infinite sum}} = \underbrace{\sum_{\hat{A}} G_{\hat{A}} \mathcal{M}_{\mathcal{O}_{\hat{A}}}(s,t)}_{\mathcal{O}_{\hat{A}} = \text{primary}} \Big[1 + \underbrace{C_{\hat{A}1} s + C_{\hat{A}2} t + C_{\hat{A}3} st + \cdots}_{\text{'descendants'}} \Big].$$

G. Durieux, T. Kitahara, C. S. Machado, Y. Shadmi, Y. Weiss, arXiv:2008.09652

S. Chang, M. Chen, D. Liu, ML, arXiv:2212.06215

Primary Operators

i	$\mathcal{O}_i^{hZ\bar{f}f}$	CP	$d_{\mathcal{O}_i}$	SMEFT Operator	c Unitarity Bound
1	$hZ^\mu \bar{\psi}_L \gamma_\mu \psi_L$	$+$	5	$i\big(H^\dagger \overleftrightarrow{D}_\mu H\big)\bar{Q}_L\gamma^\mu Q_L$	$\frac{0.6}{E_{\rm TeV}^2}, \frac{5}{E_{\rm TeV}^4}$
2	$hZ^\mu \bar{\psi}_R \gamma_\mu \psi_R$	$+$		$i\big(H^\dagger \overleftrightarrow{D}_\mu H\big)\bar{u}_R\gamma^\mu u_R$	
3	$hZ^{\mu\nu}\bar{\psi}_L\sigma_{\mu\nu}\psi_R + \text{h.c.}$	$+$	6	$\bar{Q}_L\sigma^{\mu\nu}u_R\sigma^a\widetilde{H}W^a_{\mu\nu} + \text{h.c.}$	$\frac{2}{E_{\rm TeV}^2}, \frac{10}{E_{\rm TeV}^4}$
4	$ih\widetilde{Z}_{\mu\nu}\bar{\psi}_L\sigma^{\mu\nu}\psi_R + \text{h.c.}$	$-$		$i\bar{Q}_L\sigma^{\mu\nu}u_R\sigma^a\widetilde{H}\widetilde{W}^a_{\mu\nu} + \text{h.c.}$	
5	$ihZ^\mu\big(\bar{\psi}_L\overleftrightarrow{\partial}_\mu\psi_R\big) + \text{h.c.}$	$+$	6	$\big(H^\dagger \overleftrightarrow{D}_\mu H\big)\big(\bar{Q}_L\overleftrightarrow{D}^\mu u_R\big)\widetilde{H} + \text{h.c.}$	$\frac{0.1}{E_{\rm TeV}^3}, \frac{4}{E_{\rm TeV}^6}$
6	$hZ^\mu\partial_\mu\big(\bar{\psi}_L\psi_R\big) + \text{h.c.}$	$-$		$i\big(H^\dagger \overleftrightarrow{D}_\mu H\big)D^\mu\big(\bar{Q}_L u_R\big)\widetilde{H} + \text{h.c.}$	
7	$ihZ^\mu\partial_\mu\big(\bar{\psi}_L\psi_R\big) + \text{h.c.}$	$+$		$\big(H^\dagger \overleftrightarrow{D}_\mu H\big)D^\mu\big(\bar{Q}_L u_R\big)\widetilde{H} + \text{h.c.}$	
8	$hZ^\mu\big(\bar{\psi}_L\overleftrightarrow{\partial}_\mu\psi_R\big) + \text{h.c.}$	$-$		$i\big(H^\dagger \overleftrightarrow{D}_\mu H\big)\big(\bar{Q}_L\overleftrightarrow{D}^\mu u_R\big)\widetilde{H} + \text{h.c.}$	
9	$ih\widetilde{Z}_{\mu\nu}\big(\bar{\psi}_L\gamma^\mu\overleftrightarrow{\partial}^\nu\psi_L\big)$	$+$	7	$i\lvert H\rvert^2\widetilde{W}^{a\mu\nu}\big(\bar{Q}_L\gamma_\mu\sigma^a\overleftrightarrow{D}_\nu Q_L\big)$	$\frac{0.4}{E_{\rm TeV}^3}, \frac{1}{E_{\rm TeV}^4}$
10	$h\widetilde{Z}_{\mu\nu}\partial^\mu\big(\bar{\psi}_L\gamma^\nu\psi_L\big)$	$-$		$\lvert H\rvert^2\widetilde{W}^{a\mu\nu}D_\mu\big(\bar{Q}_L\gamma_\nu\sigma^a Q_L\big)$	
11	$ih\widetilde{Z}_{\mu\nu}\big(\bar{\psi}_R\gamma^\mu\overleftrightarrow{\partial}^\nu\psi_R\big)$	$+$		$i\lvert H\rvert^2\widetilde{B}^{\mu\nu}\big(\bar{u}_R\gamma_\mu\overleftrightarrow{D}_\nu u_R\big)$	
12	$h\widetilde{Z}_{\mu\nu}\partial^\mu\big(\bar{\psi}_R\gamma^\nu\psi_R\big)$	$-$		$\lvert H\rvert^2\widetilde{B}^{\mu\nu}D_\mu\big(\bar{u}_R\gamma_\nu u_R\big)$	