



# T2K+NOvA



## Joint Analysis Lessons Learned

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NuFact Multi-Experiment Satellite Workshop

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# Overview

- Where were T2K and NOvA when we started joint analysis and what were we trying to do
  - For how we ended up where we did see Ryan's talk
- Some key choices we made and why
- What might we do differently if circumstances were different
- Caveat: These are my personal opinions having been through the process and not official views

# Background

# Long-baseline oscillation experiments

Previous Generation

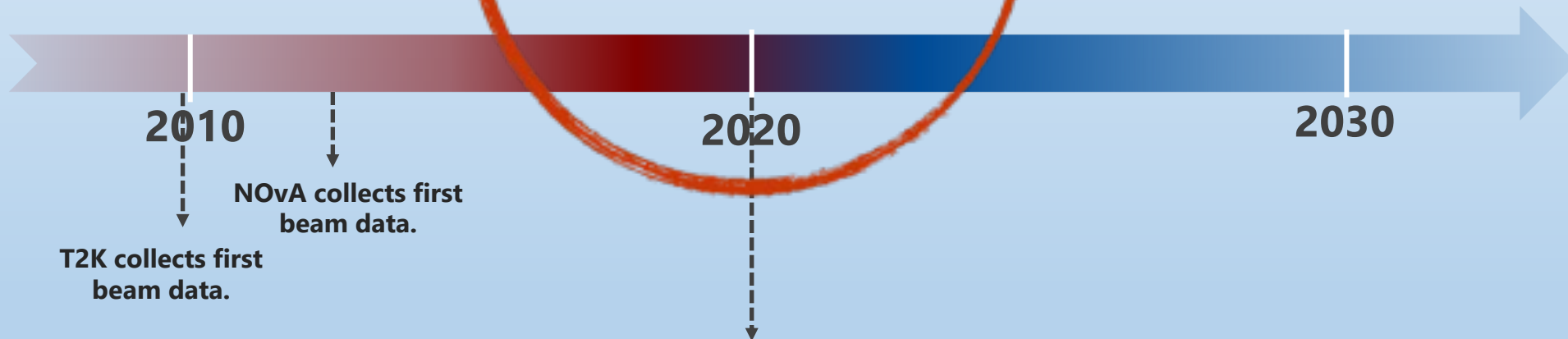


OPERA

Current Generation



Next Generation



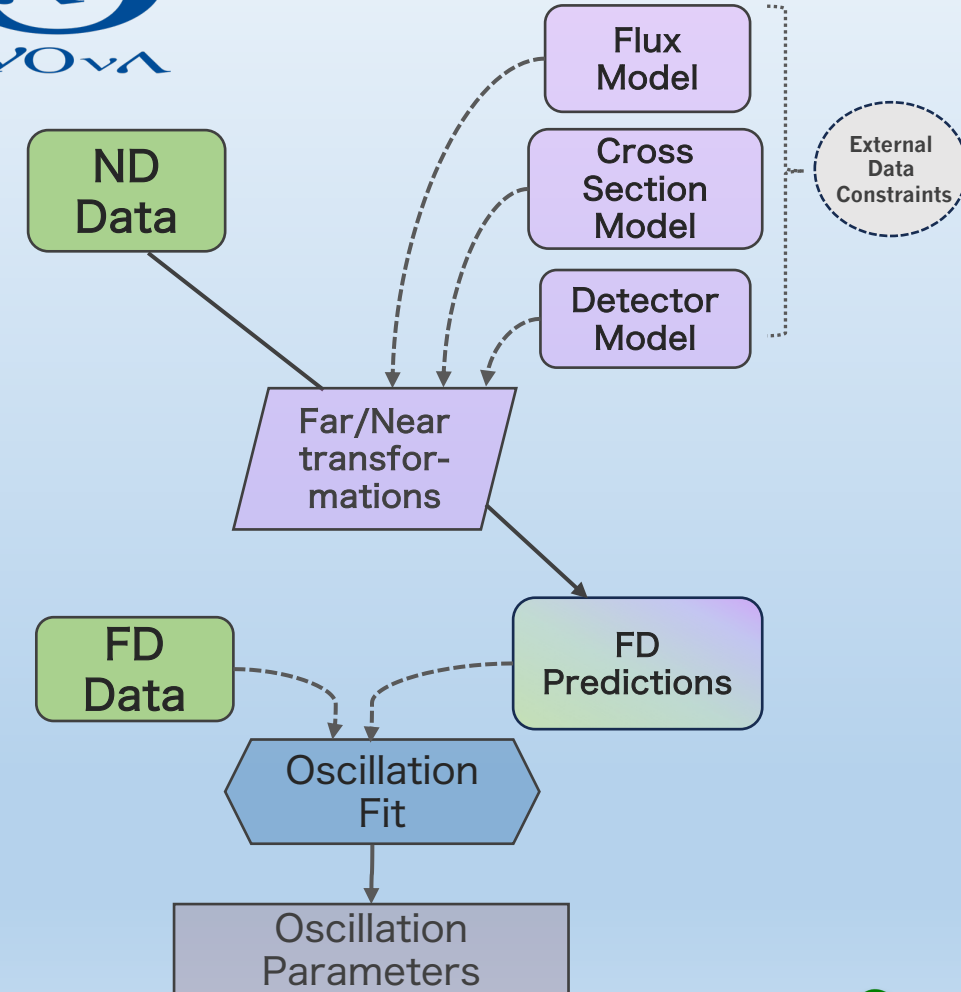
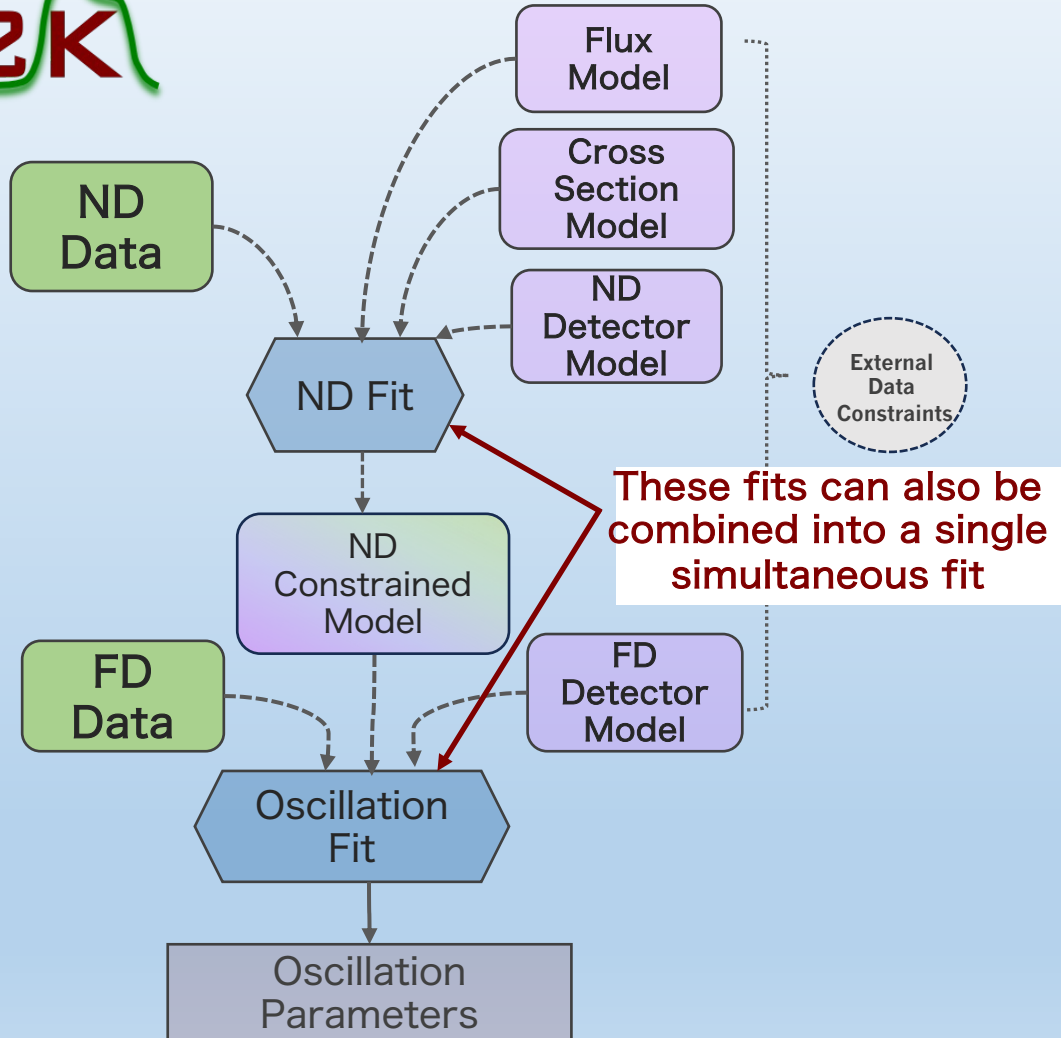
**Published dataset<sup>[1,2]</sup> until 2020 by both experiments. Our results use this dataset!**

- [1] T2K: [Eur. Phys. J. C \(2023\) 83:782 \(2023\)](#)
- [2] NOvA: [Phys. Rev D 106, 032004 \(2022\)](#) (Frequentist) and [arXiv:2311.07835](#) (Bayesian)



# Analysis Strategy

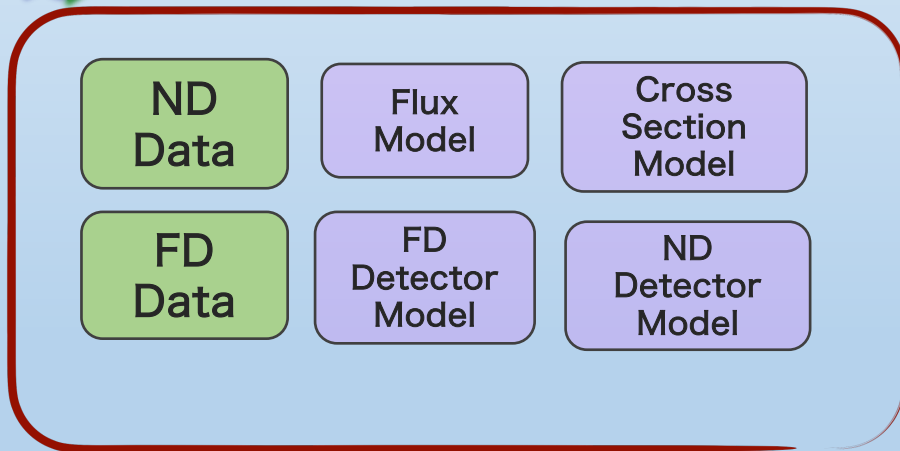
- The experiments have **different analysis approaches** driven by **contrasting detector designs**.



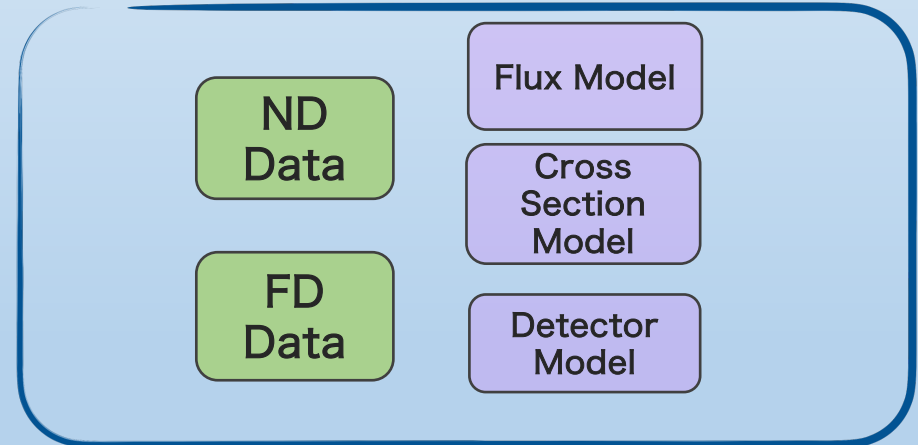
# Construction of the individual analyses

- Central value models and systematic uncertainties around them are not the same
- Different MC generators are used by both experiments (T2K: NEUT, NOvA: Genie)
- Analysis codes used are not the same and do not simultaneously compile against each other

**T2K**

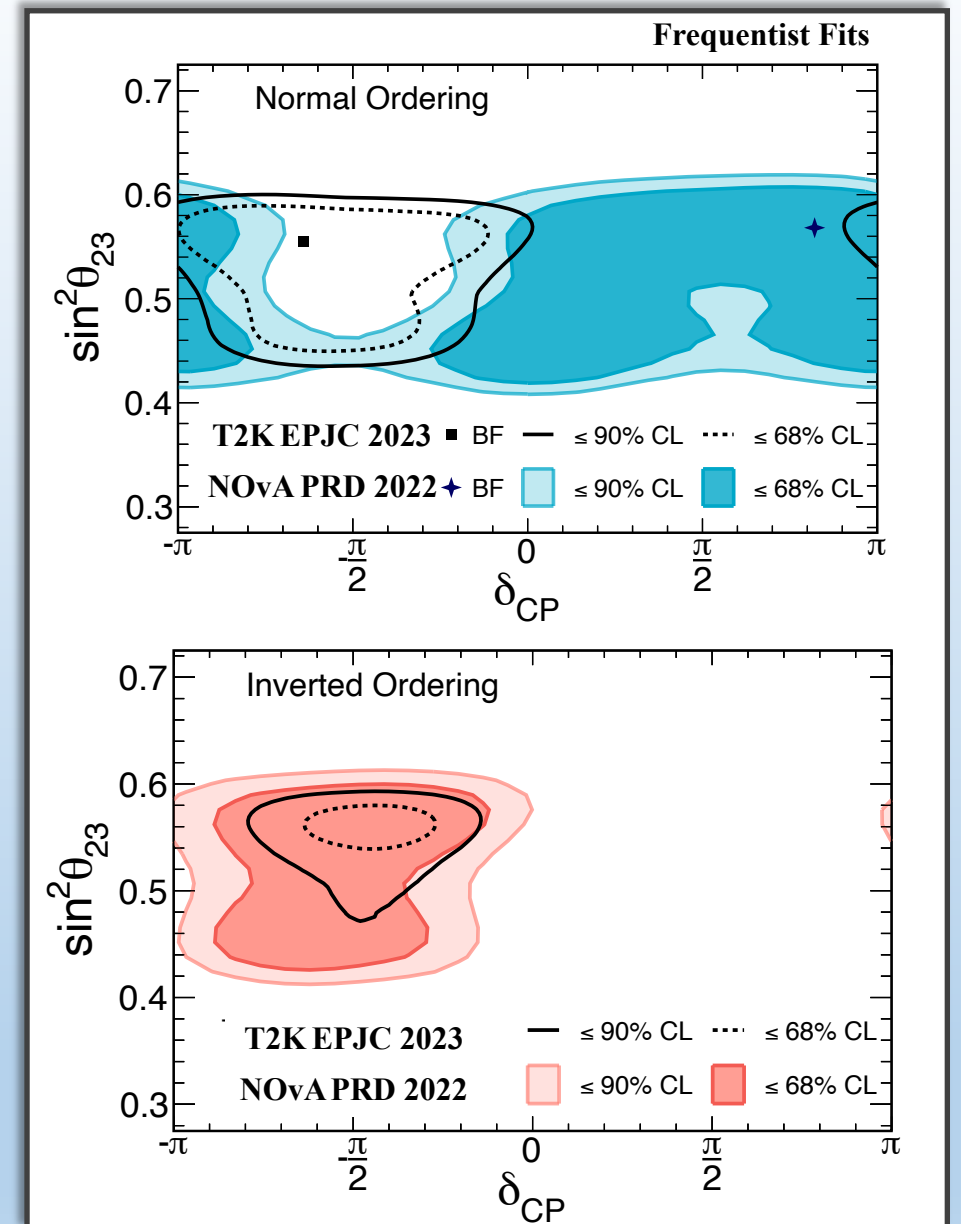


  
NOvA



# Why NOvA-T2K joint fit?

- The complementarity between the experiments provides the power to **break degeneracies and improve oscillation sensitivity**
- Full implementation of:
  - ❑ **Energy reconstruction and detector response**
  - ❑ **Detailed likelihood** from each experiment
  - ❑ **Consistent statistical inference across the full dimensionality**
- In-depth review of:
  - ❑ **Models, systematic uncertainties and possible correlations**
  - ❑ **Different analysis approaches** driven by contrasting detector designs



Results from NOvA and T2K from 2020 datasets

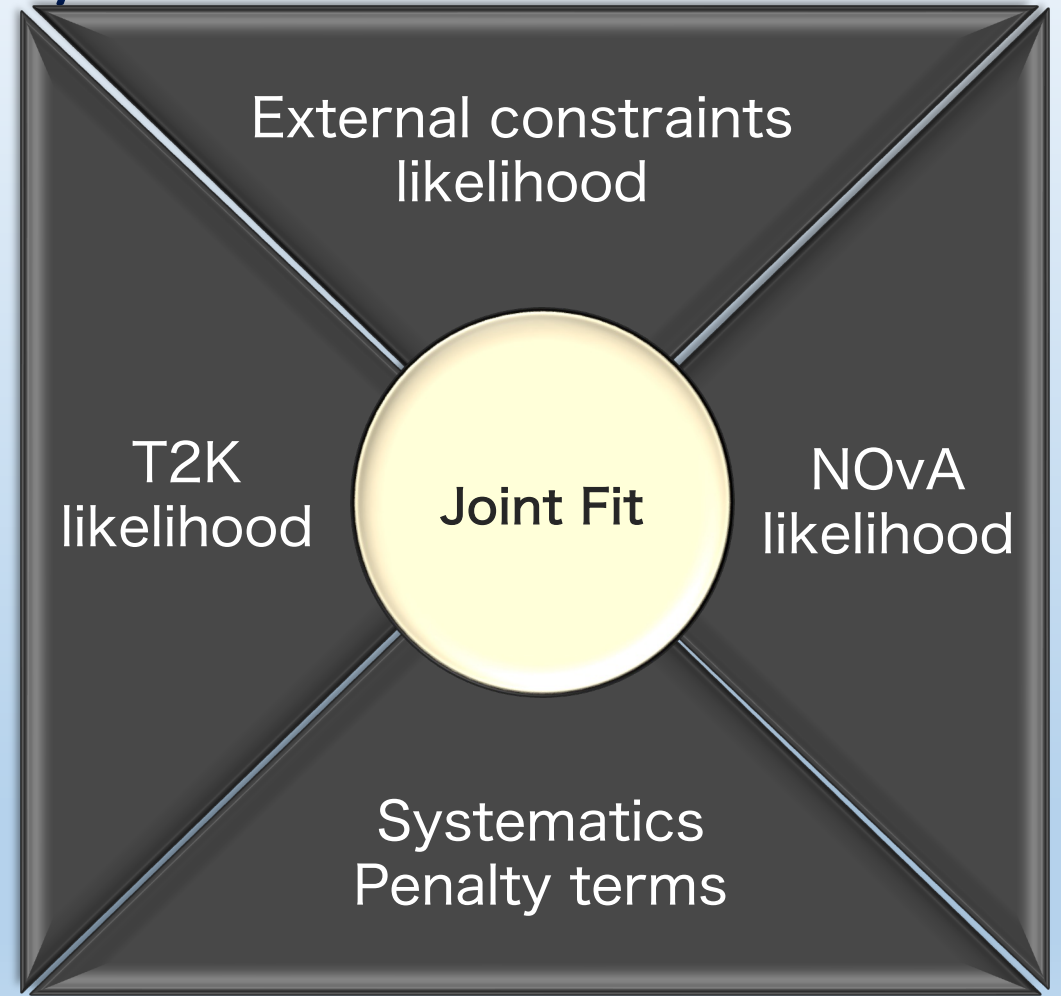
# Constructing the NOvA-T2K joint analysis

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# Constructing the joint-analysis

- A joint fit needs a joint likelihood to interrogate
- Components of this are:
  1. Poisson likelihood comparing data to predictions as a function of model parameters
  2. Penalty terms from the priors on those parameters
  3. External constraints on  $\theta_{13}$ ,  $\theta_{12}$ ,  $\Delta m_{21}^2$  from solar and reactor neutrino experiments



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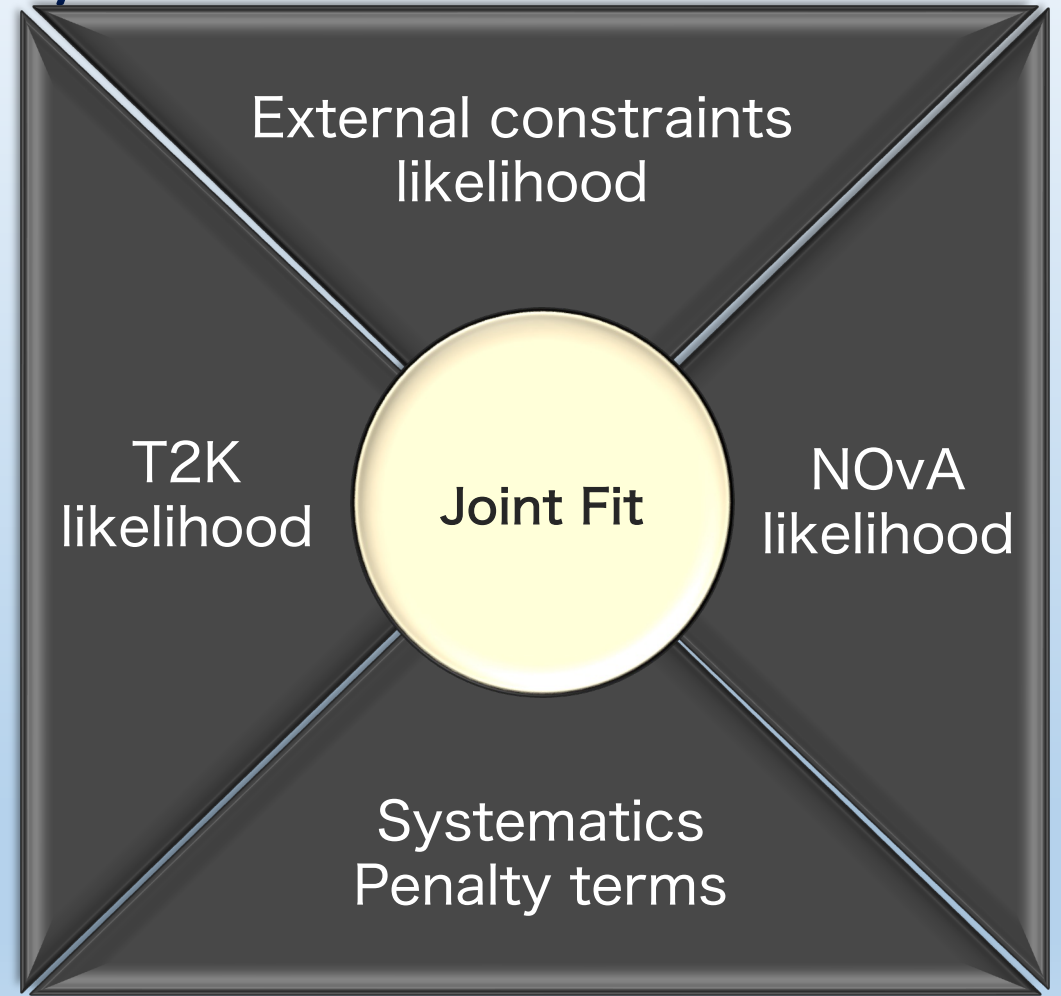
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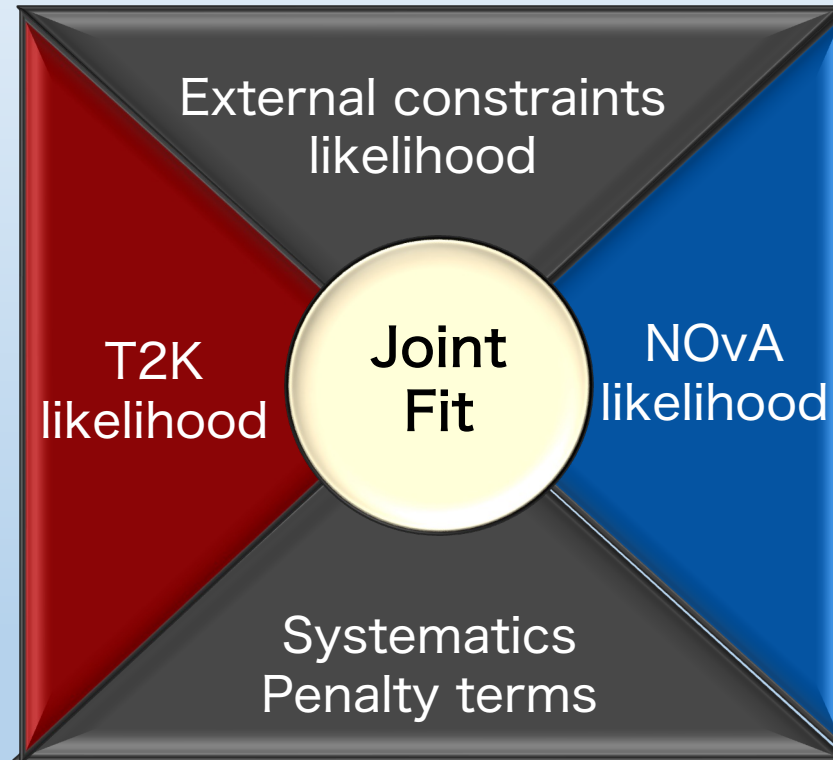
3. External constraints on  $\theta_{13}$ ,  $\theta_{12}$ ,  $\Delta m_{21}^2$  from solar and reactor neutrino experiments

- 2 and 3 are easy, 1 is hard



# Sharing the sample likelihood

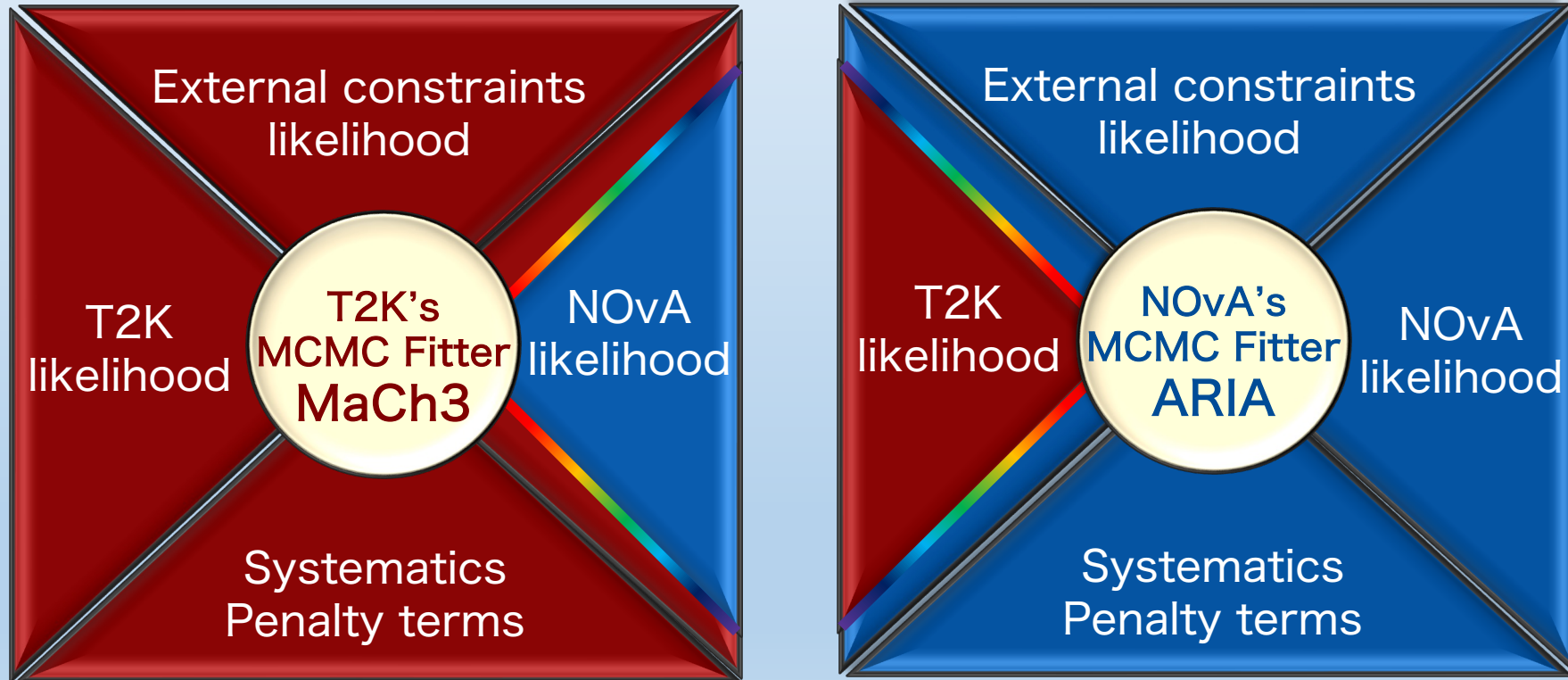
- Each experiment has spent a lot of time and personpower making their 'fitter' able to predict their FD event rates
- This is not implemented in a way that makes it easily portable to another codebase
- These codebases do not compile against each other simultaneously due to dependency versioning etc.



Red represents must be T2K codebase & blue shows must be NOvA codebase.

# Constructing the joint-analysis

- Solution is to compartmentalise analysis using containers
- Both T2K and NOvA's Bayesian analysis frameworks were packaged into containers
- 'Bridge' into and out of the container was constructed that returned LLH when given parameter values
  - Link to bifrost: <https://github.com/nova-t2k/bifrost/tree/main>
- Existing analyses then able to interrogate other experiment's container



Red represents T2K codebase & blue shows NOvA codebase.

# Was this a good idea?

## Pros:

- Existing analyses can be used as is
  - No arguing over who overhauls code to be compatible
- Internals are hidden from the other experiment easing political issues around data sharing

## Cons:

- Internals are hidden from the other experiment making validation harder
  - Eg checking all data spectra being fit for a given study cannot be done by one person

## Conclusion:

- Right thing for T2K+NOvA:
  - We started after experiments had been running for years, had existing analyses, no common/compatible fitting frameworks
  - Political will was not there for full openness at start of effort and personpower was not present for full code overhaul
  - This approach let us surmount real obstacles we had no other solution for

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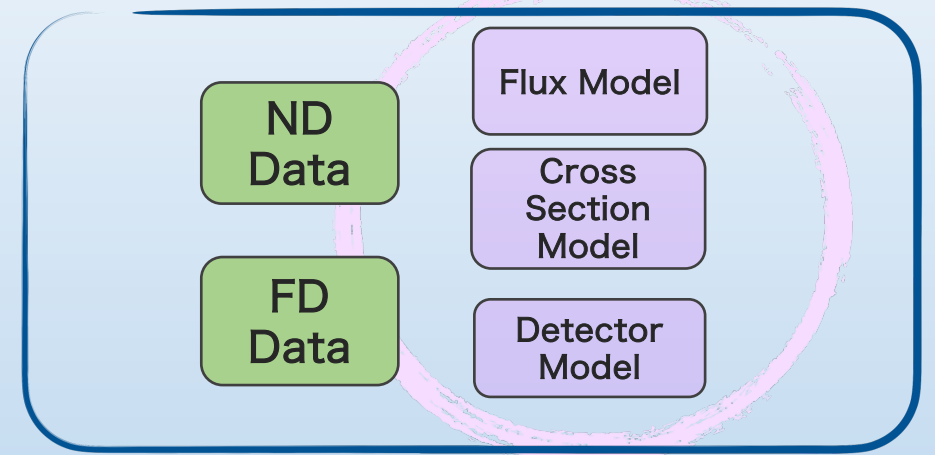
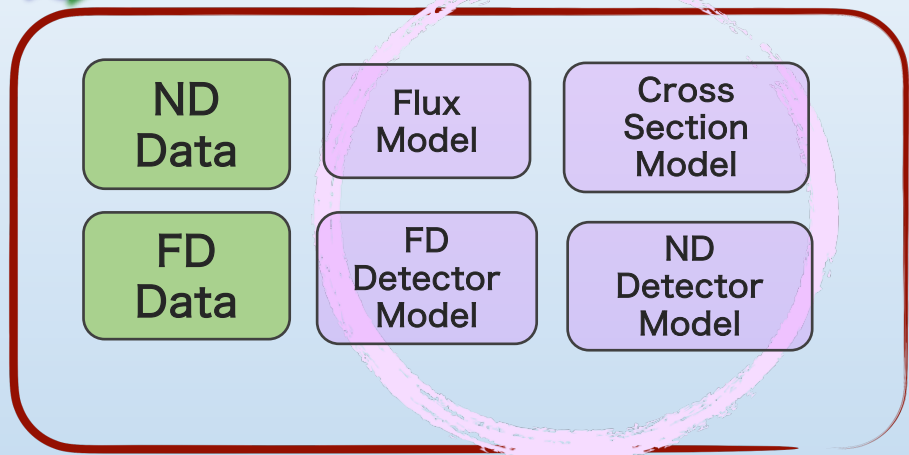
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  - Eg checking all data spectra being fit for a given study cannot be done by one person

## Conclusion:

- Maybe not the right thing if you start earlier
  - Feedback and validation loops being long caused real issues taking up analyser time and making studies take longer to complete
  - We never had problems with one collaboration trying to reverse engineer the other's result and data sharing worries relaxed fairly quickly once we were working together

# What are you actually combining?

T2K



Challenge: When? What? How? to correlate common physics parameters between the two experiments.



T2K

# Cross-section: Impact of correlations

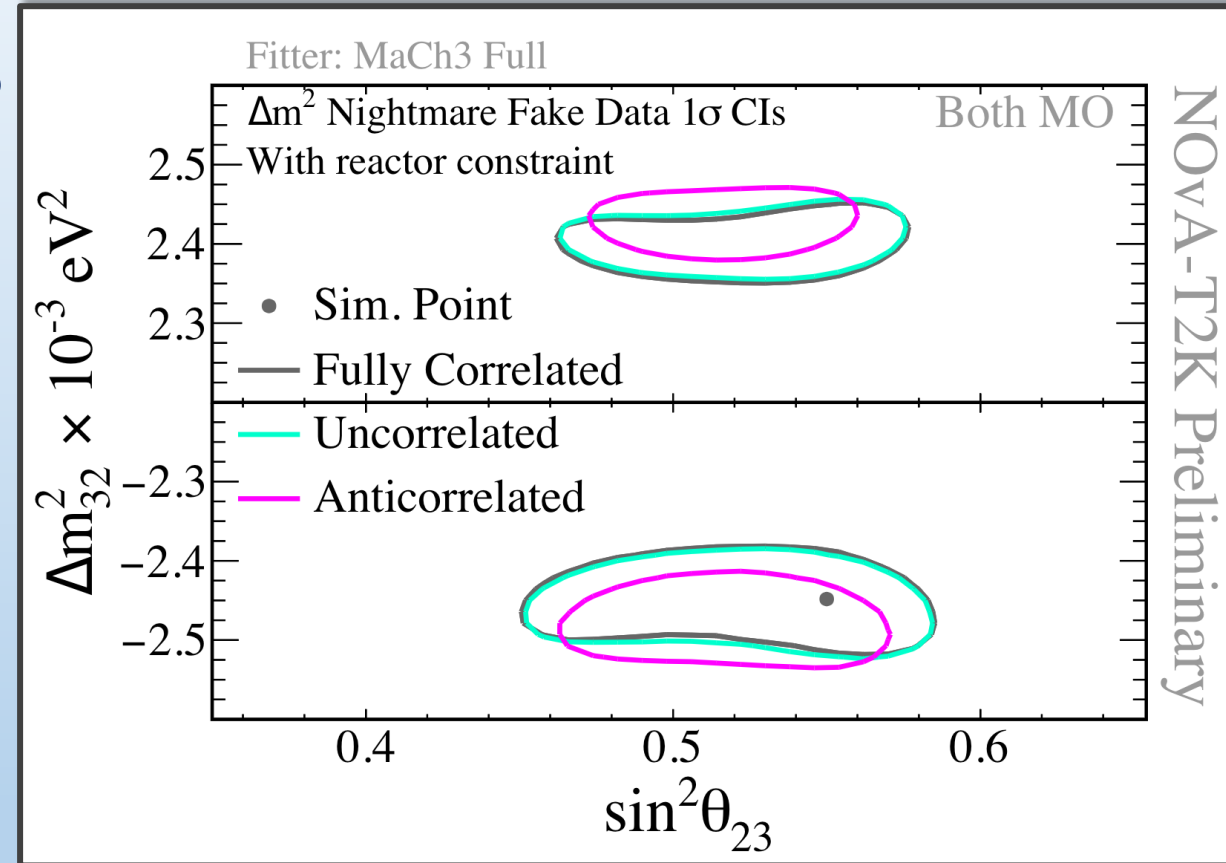
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\*Phys. Rev. D **86**, 053003



# Cross-section: Impact of correlations

- **Challenge:** No direct mapping between the cross-section systematics parameters
- **Strategy:** Explore a range of artificially crafted scenarios to bracket the impact of possible correlations
  - Example: **Fabricated systematics equal in size to total statistical uncertainty**, causing a correlated bias in the oscillation dip across both experiments.
- **Uncorrelated and correctly correlated (full correlation) credible intervals agree with negligible differences**, while **incorrectly correlating systematics shows a bias**.



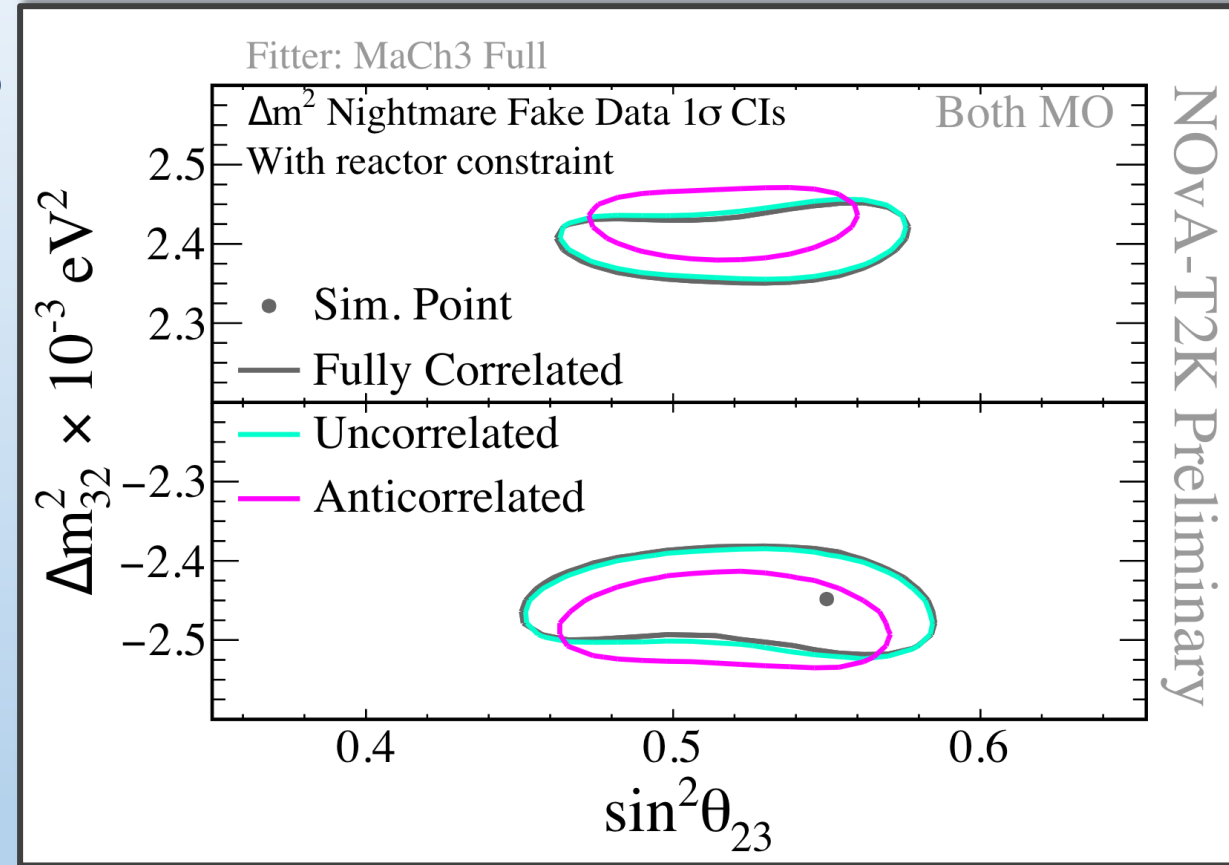
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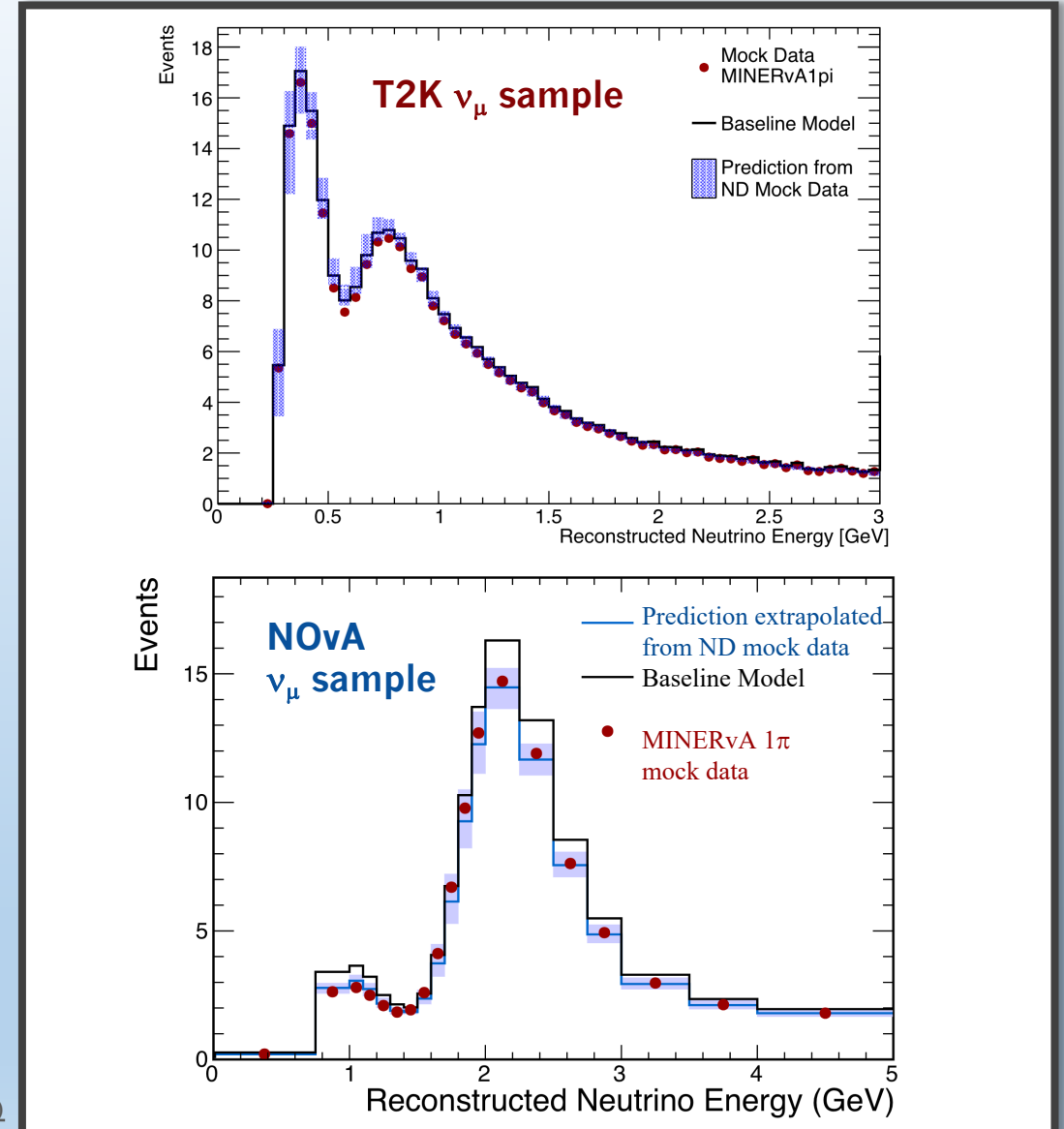
**Lessons:** Shared models with ability to describe all targets and energy ranges of T2K and NOvA not currently mature so these studies were key to joint fit feasibility

**Caveat:** All of these studies are specific to T2K and NOvA at current exposure. Assumption will break down at some point



# Cross-section: Impact of alternate models

- Evaluate the robustness of the fit against various alternate models
- Generated simulated fake data using reweighting to alternate models for both the near and far detector, then analyze the credible intervals of the full joint-fit
- Compare results against nominal fit and make sure certain criteria are met

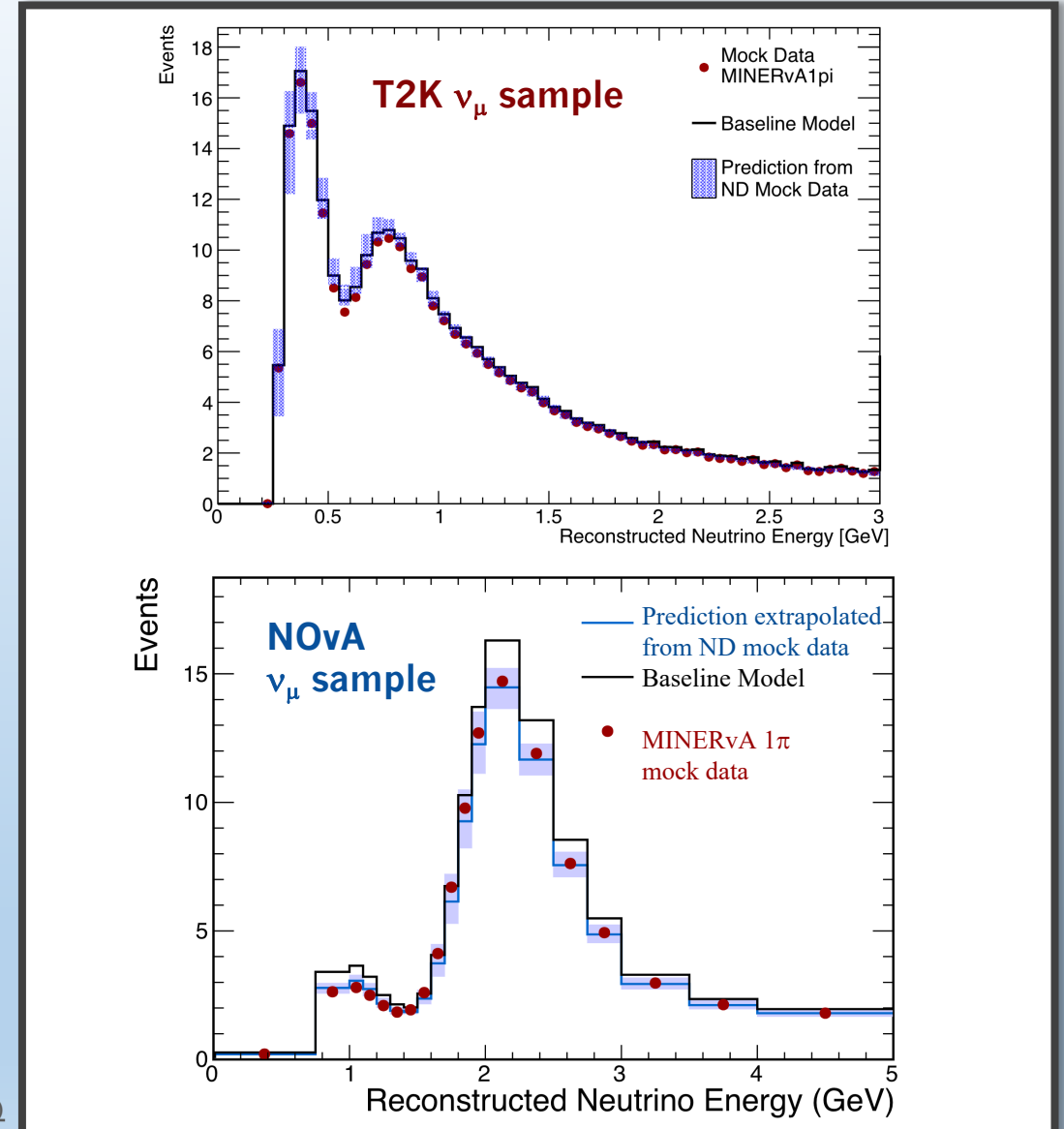


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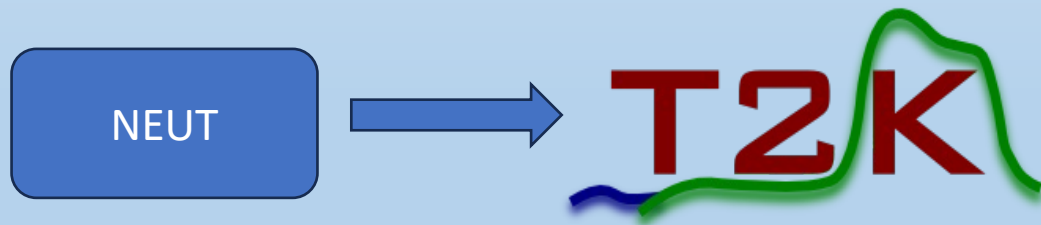
- Evaluate the robustness of the fit against various alternate models
  - **Generated simulated fake data using reweighting to alternate models for both the near and far detector, then analyze the credible intervals of the full joint-fit**
  - Compare results against nominal fit and make sure certain criteria are met
- Today's focus is on ability to do these studies in a joint fit, many people present who can discuss pros/cons of these studies in oscillation analyses generally
  - Key problem is ability to actually look at the same model in both experiments at the same time

*\*Phys. Rev. D 100, 072005 (2019)*



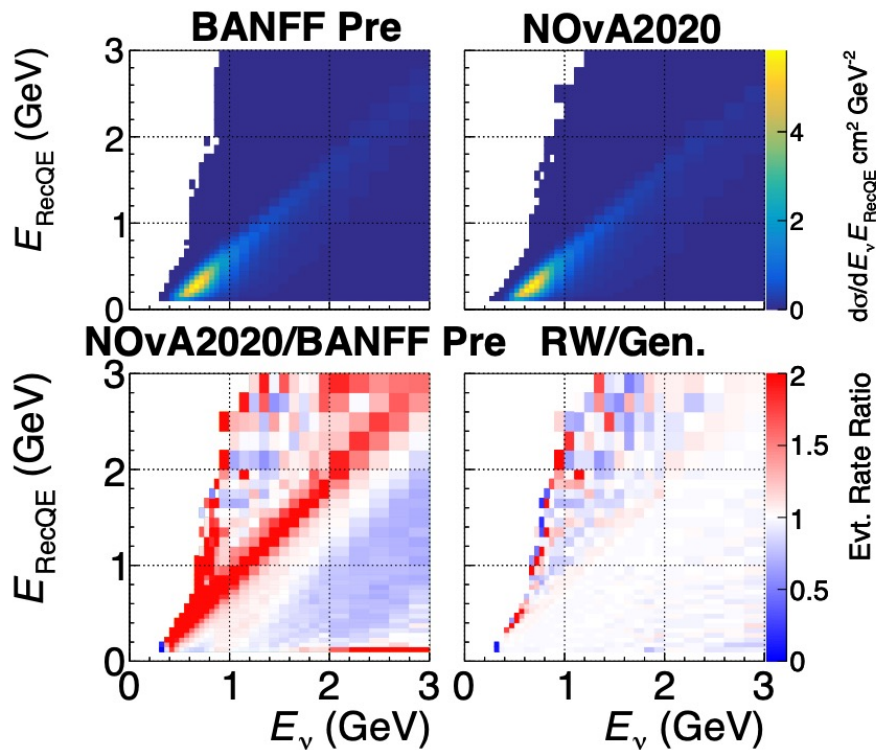
# Alternate model tests

- T2K uses Neut, NOvA uses Genie. Neither experiment has run a production with the other generator for oscillation analysis for a long time
- Each experiment's analysis and simulation toolchain cannot easily take input from the other's generator
- How then do you actually fit data generated with the same model in both experiments at the same time?



# Alternate model tests

- Answer is reweighting with a careful choice of variables and binning to ensure fit variables are properly described
- Long process requiring much care and hard validations as answer isn't obvious

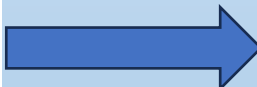


Analysis	Analysis Variable	Truth Proxy
T2K ND + FD	$E_\nu$	$E_\nu$
	$p_\ell^{\text{reco}}$	$p_\ell$
	$\theta_\ell^{\text{reco}}$	$\theta_\ell$
NOvA ND + FD	$p_{T,\ell}^{\text{reco}}$	$p_{T,\ell}$
	$E_\nu^{\text{reco}}$	$E_\nu$
	$y^{\text{Vis.}} = E_{\text{Had}}^{\text{reco}}/E_\nu^{\text{reco}}$	$E_{\text{Av.}} = \sum_{p,\pi^\pm} T + \sum_{e,\gamma,\pi^0,K,\text{Other}} E + \sum_{B_{\text{strange}}} (E - m_p) + \sum_{\bar{p}} (E + m_p)$

# A better way! - NuHEPMC

- If you were starting from scratch why not agree on the format for generator output/analysis simulation input to directly generate data with same model
- Collider physics world has had this for a long time starting with the Les Houches accords ~2001 and now leading to LHE files/HEPMC3
- NuHEPMC effort uses HEPMC3 format from collider physics with agreed extra neutrino information to make a common generator output format

```
common /HEPRUP/ IDBMUP(2), EBMUP(2), PDFGUP(2), PDFSUP(2),  
+ IDWTUP, NPRUP, XSECUP(MAXPUP), XERRUP(MAXPUP),  
+ XMAXUP(MAXPUP), LPRUP(MAXPUP)  
common /HEPEUP/ NUP, IDPRUP, XWGTUP, SCALUP, AQEDUP, AQCDUP,  
+ IDUP(MAXNUP), ISTUP(MAXNUP), MOTHUP(2,MAXNUP),  
+ ICOLUP(2,MAXNUP), PUP(5,MAXNUP), VTIMUP(MAXNUP),  
+ SPINUP(MAXNUP)
```

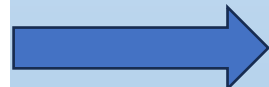


Computer Physics Communications  
Volume 260, March 2021, 107310

**The HepMC3 event record library for Monte Carlo event generators**

☆☆☆

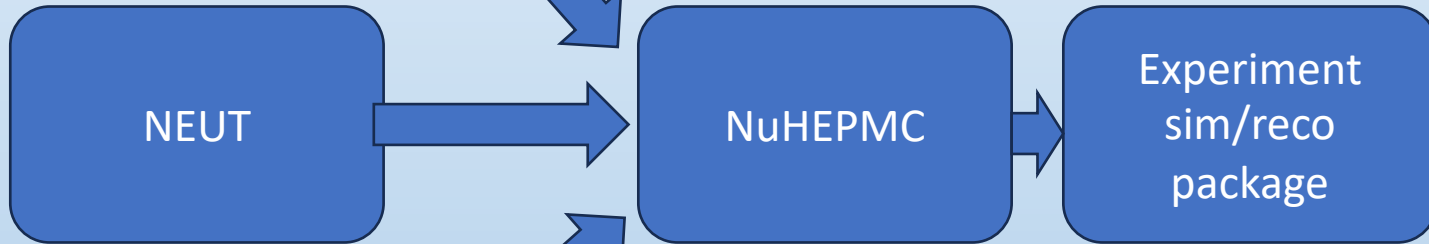
Andy Buckley<sup>a</sup>, Philip Ilten<sup>b</sup>, Dmitri Konstantinov<sup>c</sup>, Leif Lönnblad<sup>d</sup>,  
James Monk<sup>e,f</sup>, Witold Pokorski<sup>g</sup>, Tomasz Przedzinski<sup>h</sup>,  
Andrii Verbytskyi<sup>i</sup>  



NuHEPMC



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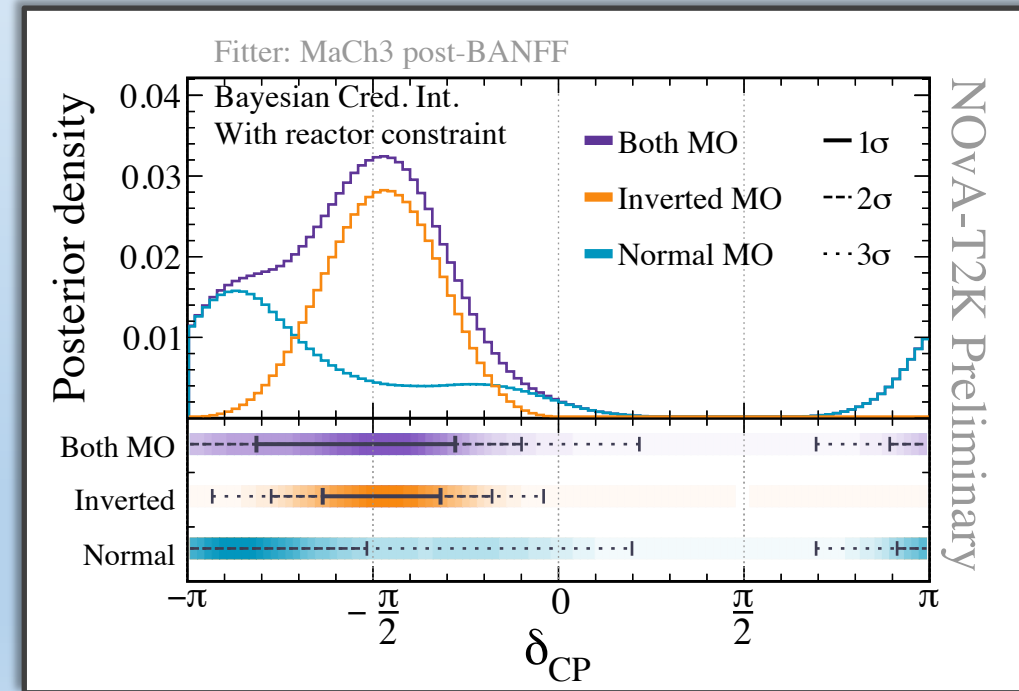
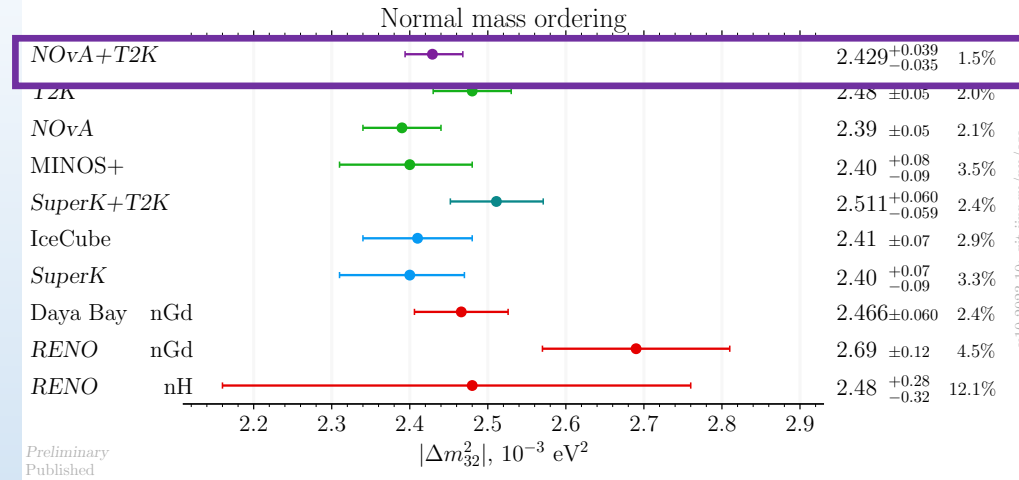


- If all experiments going forward agree to be able to take in this format, alternate model studies can be done much faster and much more robustly
  - With another hat on the DUNE Phase II ND reco can already use NuHEPMC
- Not only useful for alternate model studies in joint fits but also in standalone analyses
- Also opens up the possibility for shared central value models and uncertainties



# Summary

- T2K+NOvA analysis used several innovative methods that allowed us to combine already designed analyses
- Choices driven by context when analysis started that one might revisit if starting from scratch with new experiments:

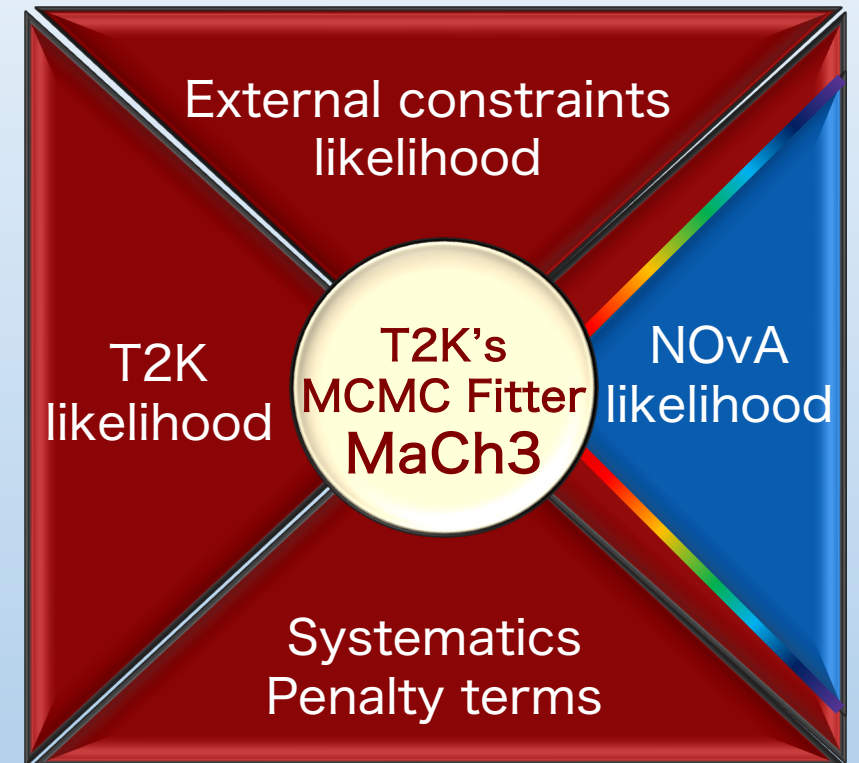


# Summary

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## 1. Containerised LLHs

- Let you simultaneously call code that isn't compatible
- Meaningfully increase validation cycles and make some studies hard to run

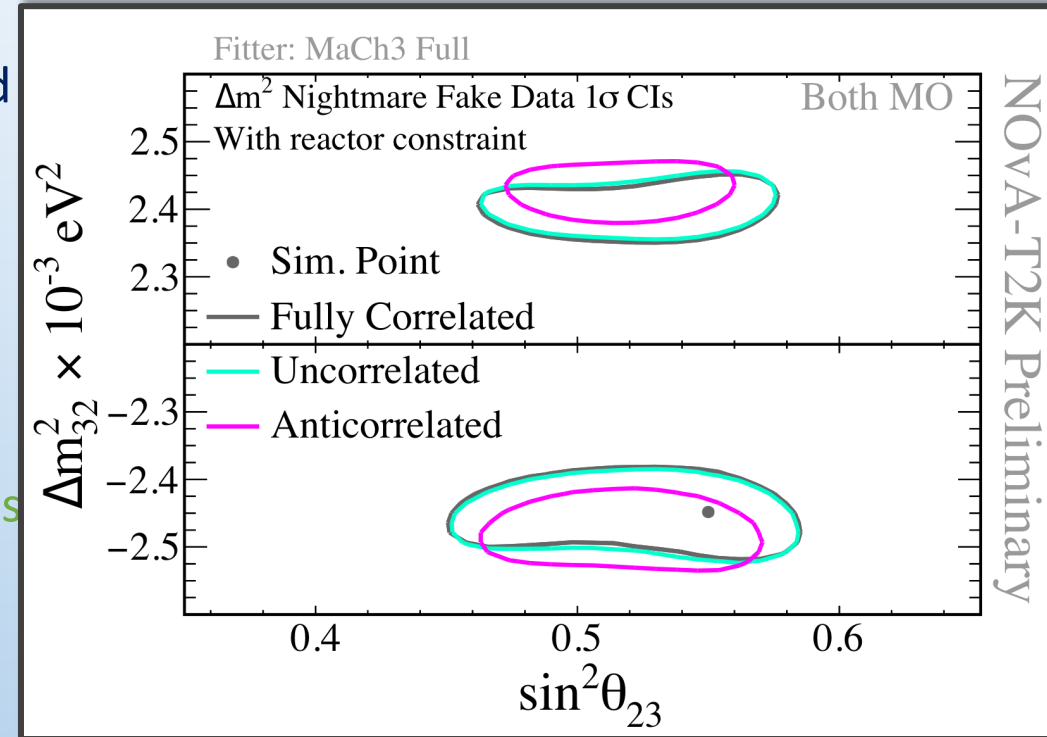


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## 2. Correlation impact checks

- Systematics are sub-dominant for our input analyses so it was plausible that correlations could be negligible
- This made it worth testing if they were to avoid needing unified xsec model
- This assumption will break at some point plus the next generation of experiments might need the more sophisticated unified models anyway

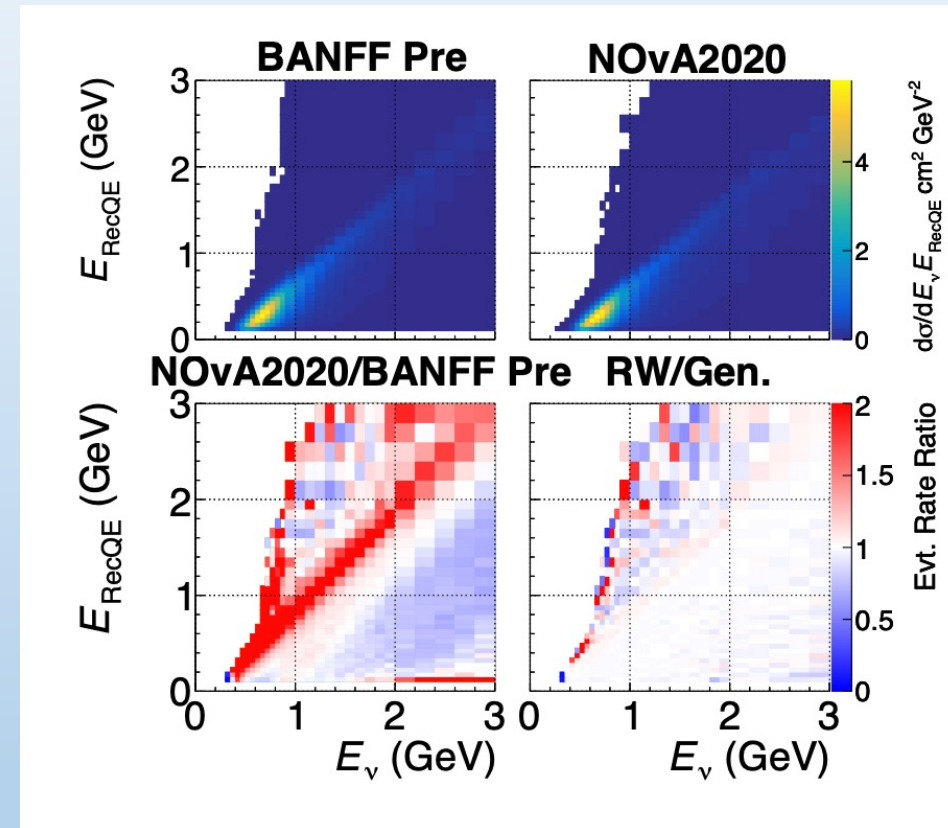


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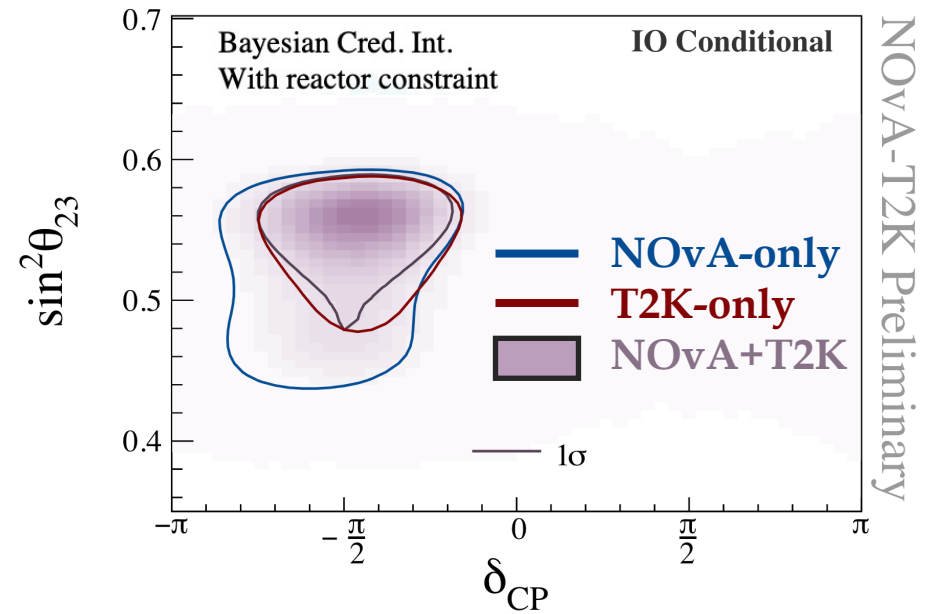
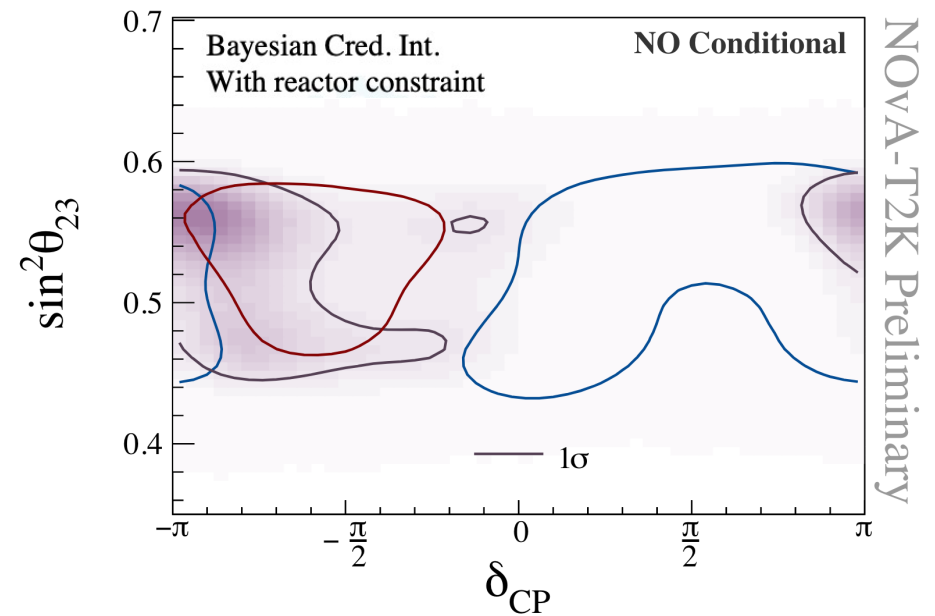
## 3. Parameterised reconstruction mapping

- Allowed models not simulated by other experiment to be tested
- Not exact and needed careful thought for each study on whether variables used were right
- Newer techniques like NuHEPMC would allow other generator models to go through exact reco



# Summary

- Resulting analysis lifted degeneracies and meaningfully improved oscillation parameter constraints
- Process itself was valuable: I certainly learned a lot about both experiments and the working group continues to be a great source of new ideas for oscillation analyses



# Backup

# Models & Systematics

Flux Model

Detector Model

Cross Section  
Model

- **Challenge: When? What? How? to correlate common physics parameters between the two experiments.**

- Strategy:

- Is the overall impact negligible on the result?
- Do we expect any correlations between the experiments?
- Is the impact of the correlations negligible on the result?

# Models & Systematics

Flux Model

- Different energies
- Different tuning to external data
  - thin target vs thick target data
- Enters the analysis differently

❑ No significant correlations between the experiments



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- Different detector design and targets
- Different selections
  - inclusive vs exclusive outgoing pions
- Different energy reconstruction
  - calorimetric vs lepton kinematics

❑ Explored possible correlations between leptonic energy scales; pion and neutron secondary interactions

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## Cross Section Model

- As the underlying physics is fundamentally the same, we expect correlations
- Different neutrino interaction models
  - optimized for different energy ranges
- Systematics are designed for individual models and analysis strategies

❑ Investigate the impact of models and correlations on the joint analysis

# Cross-section: Impact of correlations

- **Challenge:** No direct mapping between the cross-section systematics parameters
  - Exception: **Uncertainties in  $\nu_e/\nu_\mu$  and  $\bar{\nu}_e/\bar{\nu}_\mu$**  cross-section have identical origin\* and similar treatment
    - **Fully correlated in the joint fit.**

\*Phys. Rev. D **86**, 053003

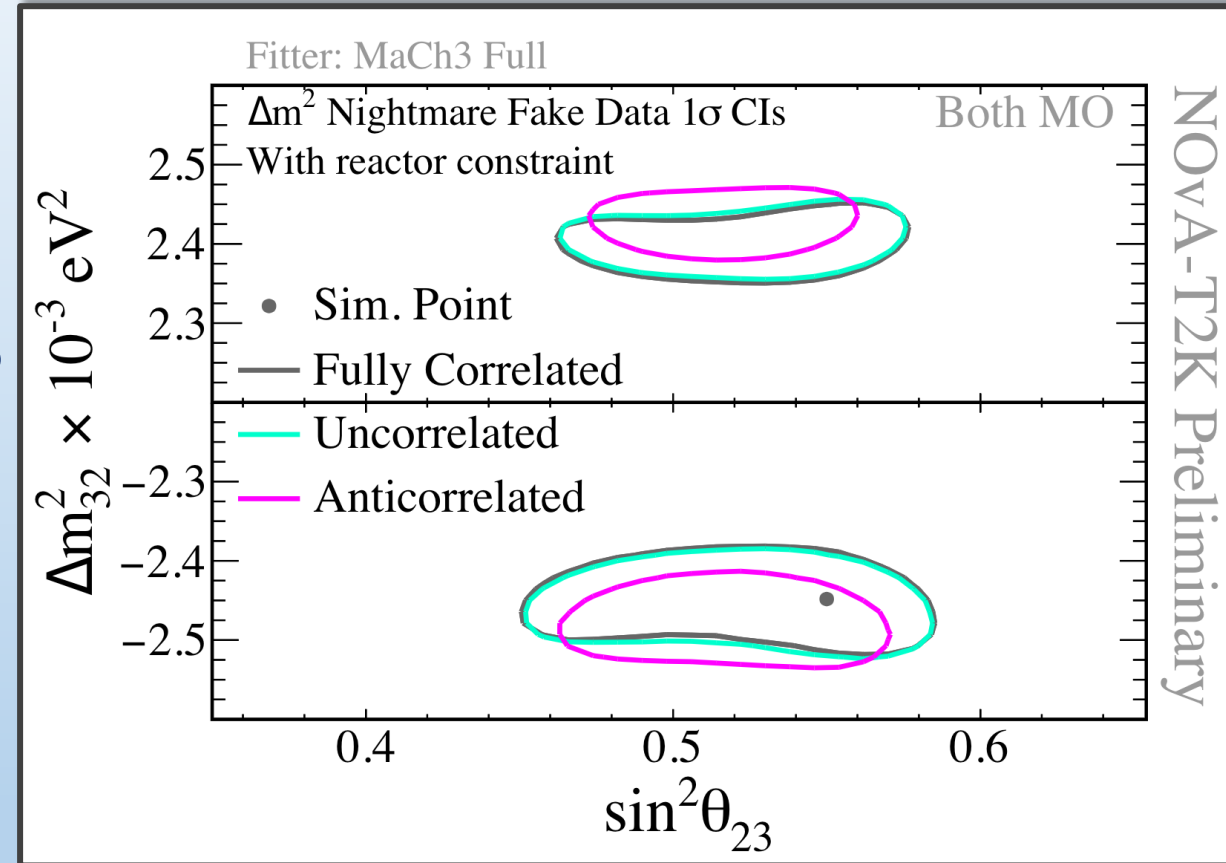
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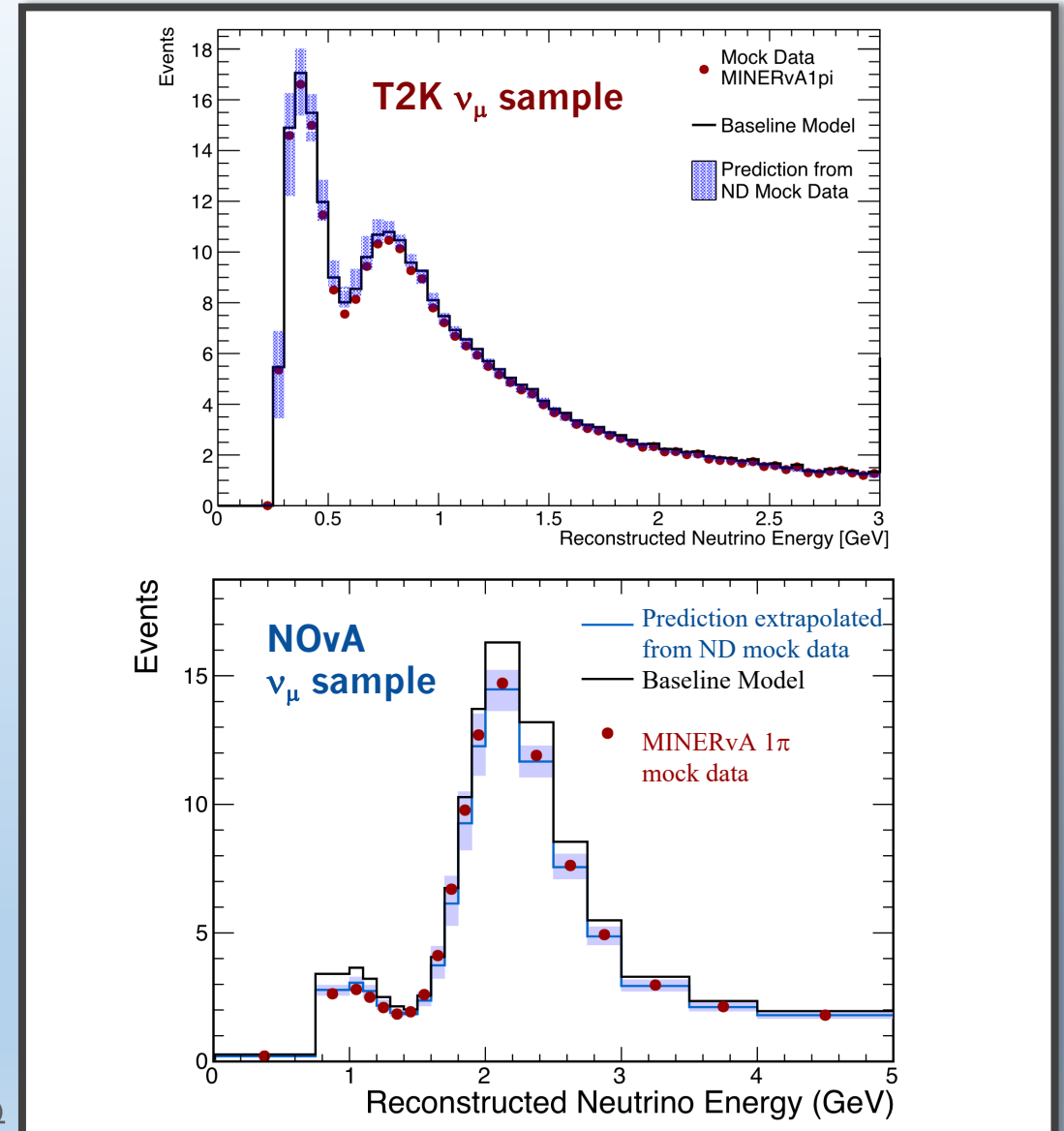


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# Cross-section: Impact of alternate models

- Evaluate the robustness of the fit against various alternate models
- Generated simulated fake data using reweighting to alternate models for both the near and far detector, then analyze the credible intervals of the full joint-fit
- Pre-decided thresholds for bias:
  - Change in the width of the 1D intervals <10%
  - Change in central value < 50% of systematic uncertainty
- **Example: Suppression in single pion channel based on tune to the MINERvA data\***

\*Phys. Rev. D 100, 072005 (2019)

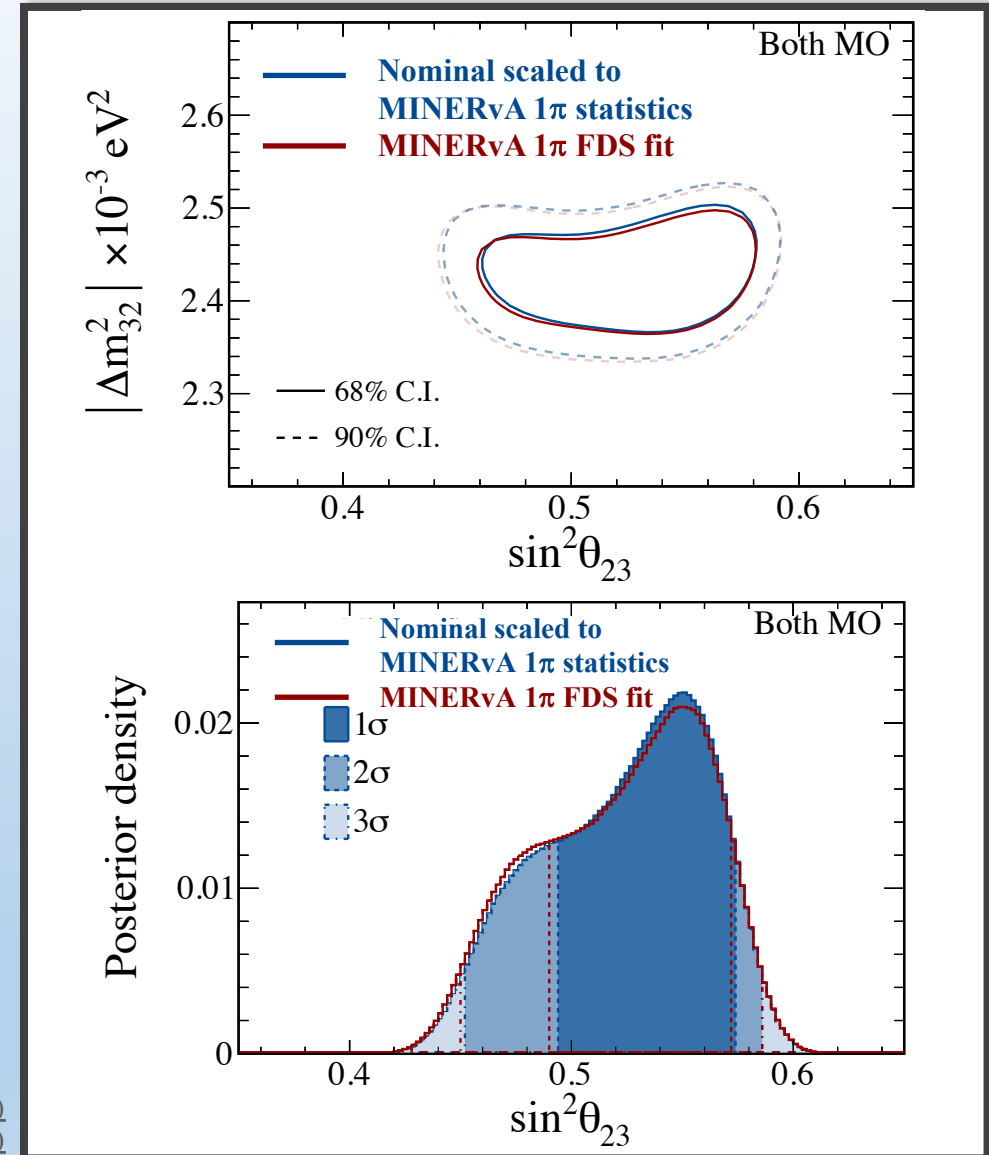


# Cross-section: Impact of alternate models

- **Example: Suppression in single pion channel based on the tune to the MINERvA data\***
- Additional tests:
  - Cross-experiment models after the ND constraint
  - Impact of alternative nuclear response model: HF-CRPA\*\*
  - Full list available in backup
- **No alternate model tests failed the preset threshold bias criteria.**

\*Phys. Rev. D 100, 072005 (2019)

\*\* Phys. Rev. D 106, 073001 (2022)





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## Cross Section Model

- As the underlying physics is fundamentally the same, we expect correlations
- Different neutrino interaction models
  - optimized for different energy ranges
- Systematics are designed for individual models and analysis strategies

- ❑ Impact of correlations is negligible on the results at the current statistical significance.
- ❑ Merits continued investigations for higher data exposures.

# Why NOvA-T2K joint fit?

✓ The complementarity between the experiments provides the power to **break degeneracies**.

• Full implementation of:

✓ **Energy reconstruction and detector response**

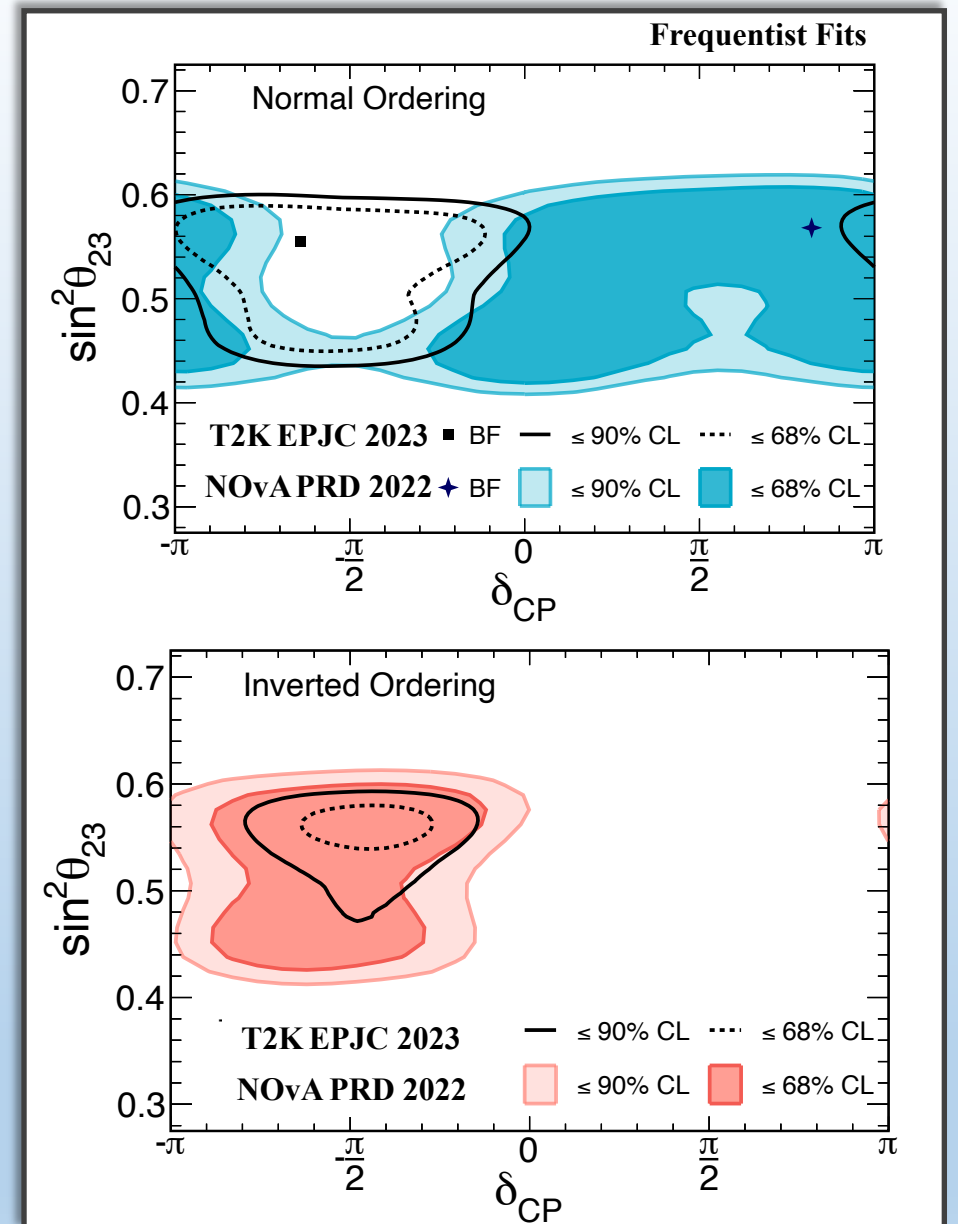
✓ **detailed likelihood** from each experiment

✓ **Consistent statistical inference across the full dimensionality**

• In-depth review of:

✓ **Models, systematic uncertainties and possible correlations**

✓ **different analysis approaches** driven by contrasting detector designs.



Results from NOvA and T2K from 2020 datasets

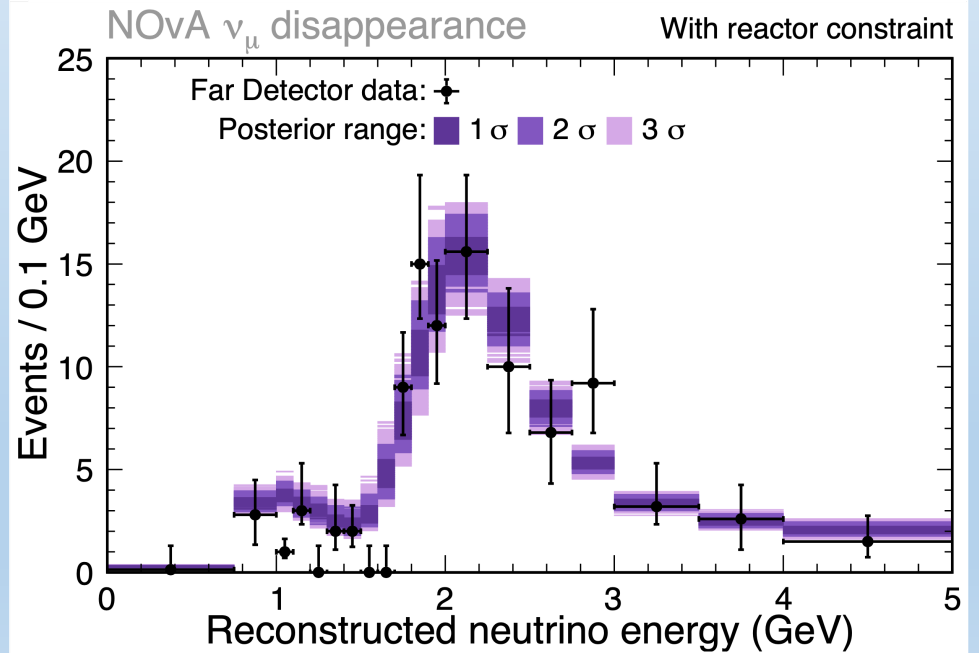
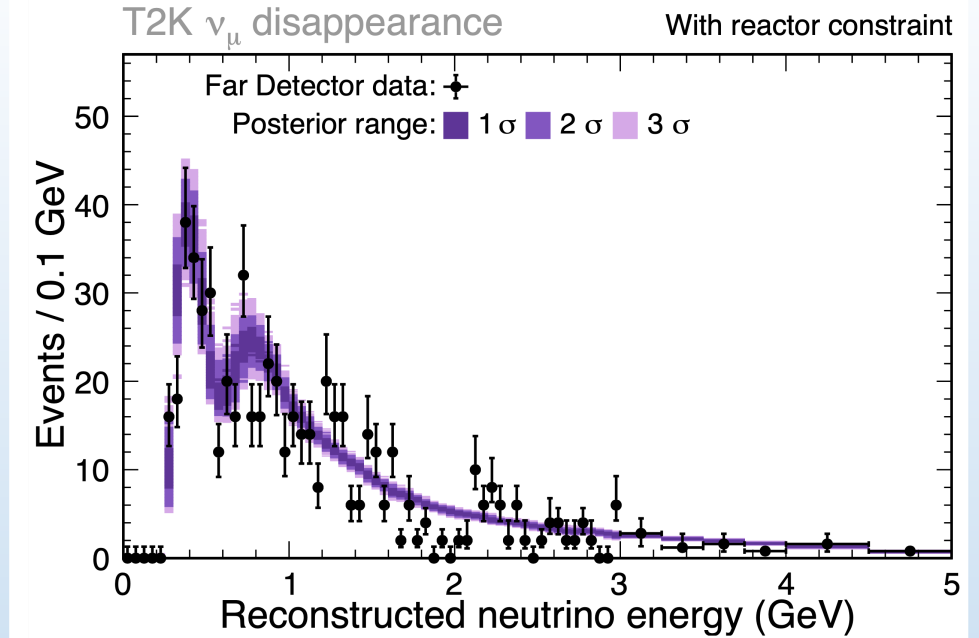
# Data Results

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# FD Data Samples

- The joint-fit uses the data collected by each experiment up until 2020.
- Using both experiments data roughly doubles the total statistics at the far detectors.

Channel	NOvA	T2K
$\nu_e$	82	94 ( $\nu_e 0\pi$ ) 14 ( $\nu_e 1\pi$ )
$\bar{\nu}_e$	33	16
$\nu_\mu$	211	318
$\bar{\nu}_\mu$	105	137



$\bar{\nu}_\mu$  samples in backup

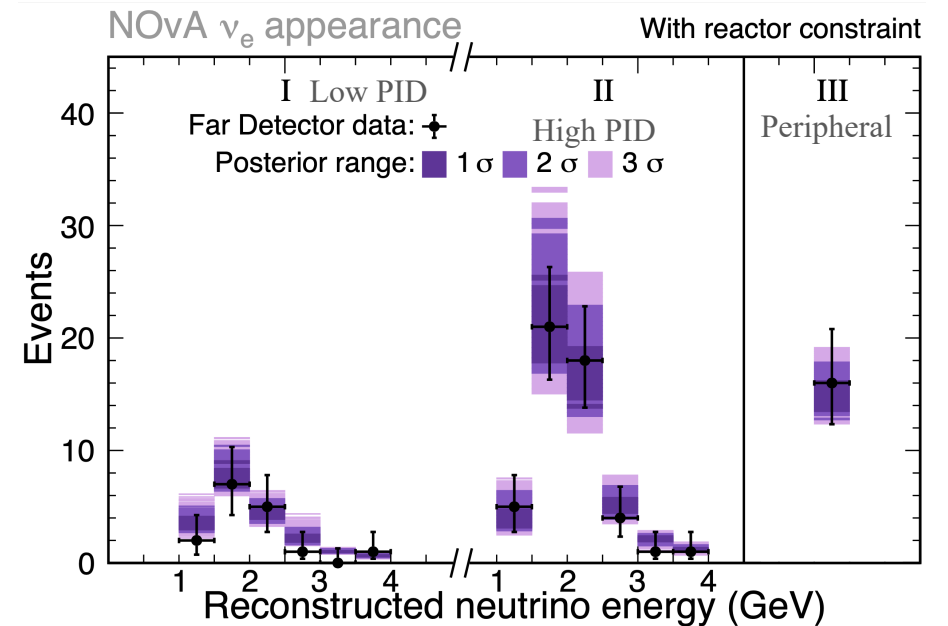
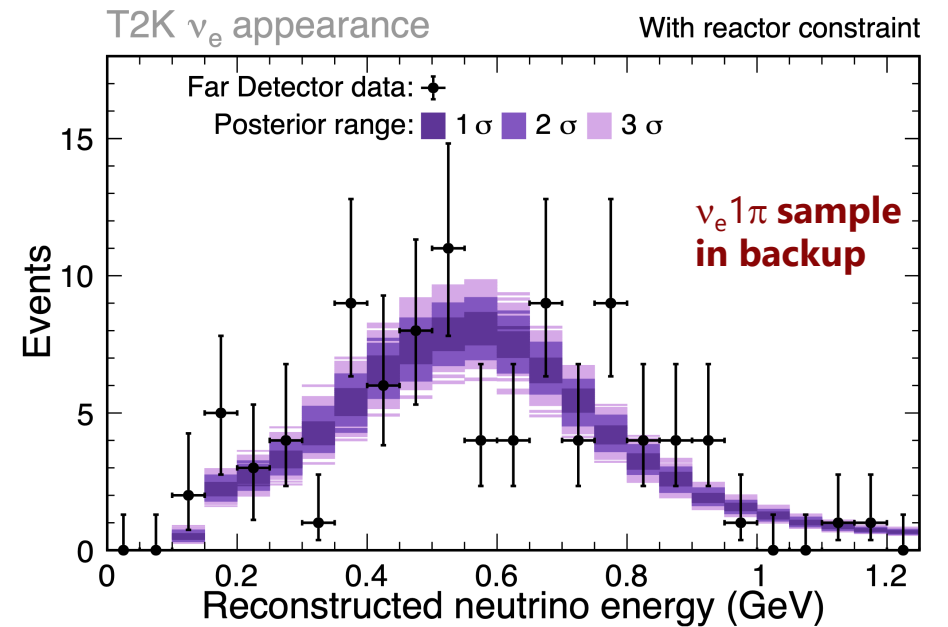
# Compatibility of datasets

- Posterior predictive p-values (PPP)\*
  - Compare likelihood best fit to data and fluctuated predictions
  - A good PPP is around 0.5
- The data from both experiments is described well by the joint fit.

Channel	NOvA	T2K	Combined
$\nu_e$	<b>0.90</b>	<b>0.19</b> ( $\nu_e$ ) <b>0.79</b> ( $\nu_e 1\pi$ )	<b>0.62</b>
$\bar{\nu}_e$	<b>0.21</b>	<b>0.67</b>	<b>0.40</b>
$\nu_\mu$	<b>0.68</b>	<b>0.48</b>	<b>0.62</b>
$\bar{\nu}_\mu$	<b>0.38</b>	<b>0.87</b>	<b>0.72</b>
Total	<b>0.64</b>	<b>0.72</b>	<b>0.75</b>

posterior predictive p-value

\*Statistica Sinica, vol. 6, no. 4, 1996, pp. 733–60. JSTOR



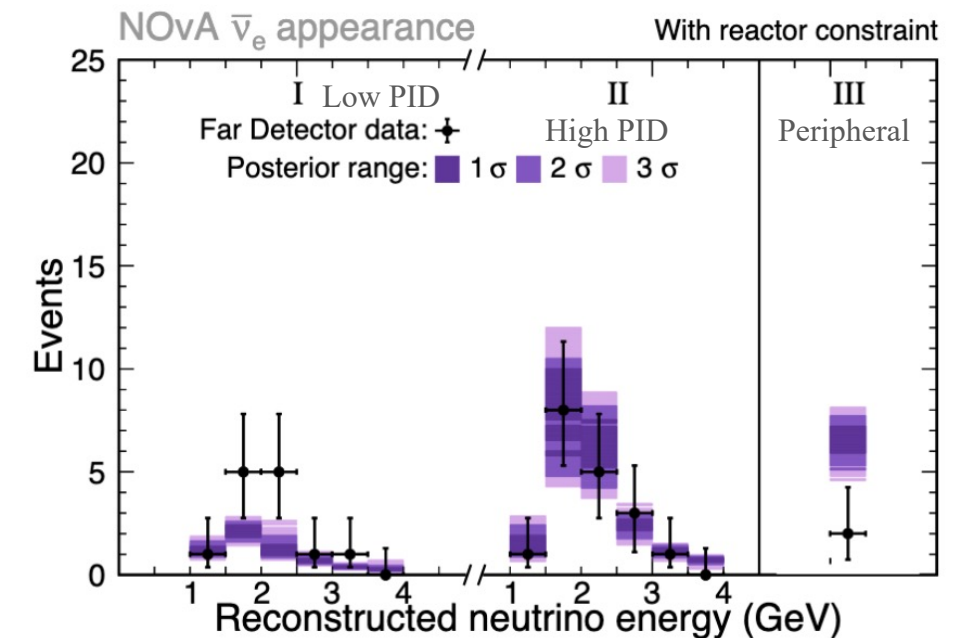
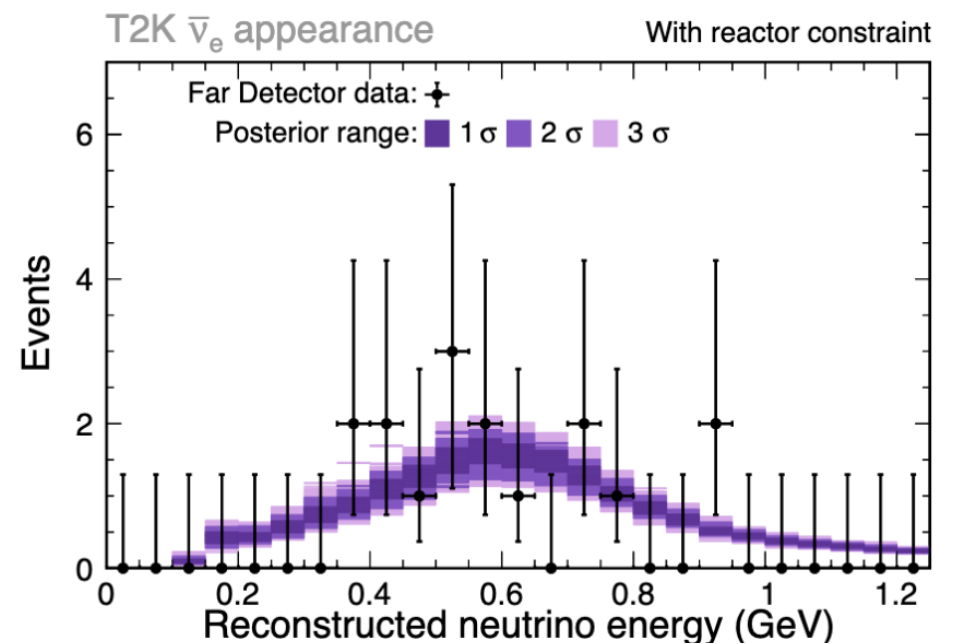
# Compatibility of datasets

- Posterior predictive p-values (PPP)\*
  - Compare likelihood best fit to data and fluctuated predictions
  - A good PPP is around 0.5
- The data from both experiments is described well by the joint fit.

Channel	NOvA	T2K	Combined
$\nu_e$	<b>0.90</b>	<b>0.19</b> ( $\nu_e$ ) <b>0.79</b> ( $\nu_e 1\pi$ )	<b>0.62</b>
$\bar{\nu}_e$	<b>0.21</b>	<b>0.67</b>	<b>0.40</b>
$\nu_\mu$	<b>0.68</b>	<b>0.48</b>	<b>0.62</b>
$\bar{\nu}_\mu$	<b>0.38</b>	<b>0.87</b>	<b>0.72</b>
Total	<b>0.64</b>	<b>0.72</b>	<b>0.75</b>

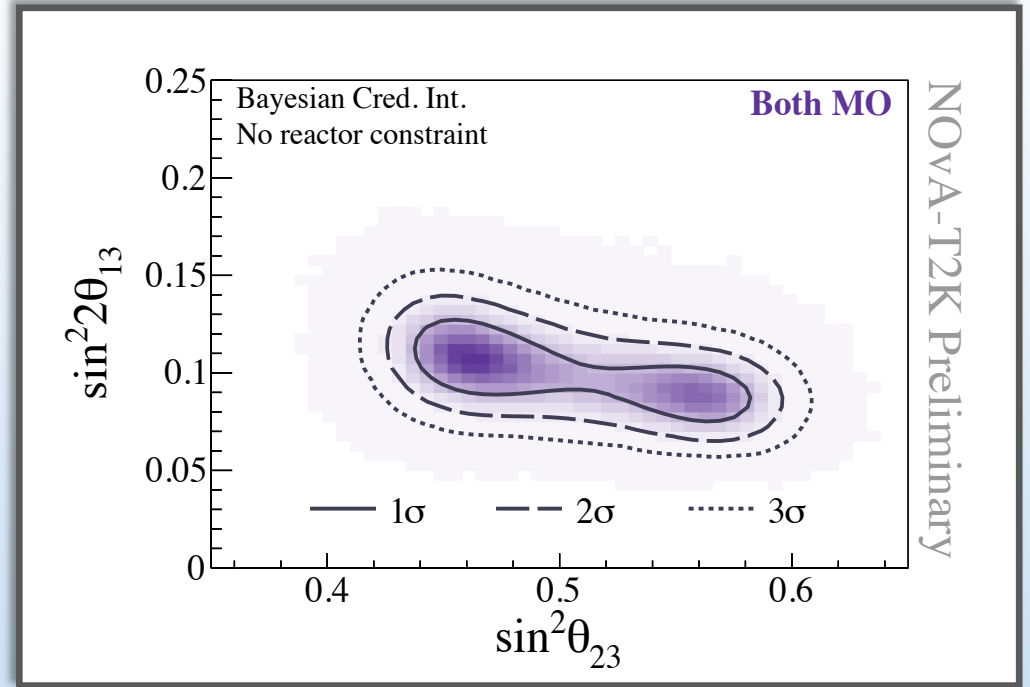
posterior predictive p-value

\*Statistica Sinica, vol. 6, no. 4, 1996, pp. 733–60. JSTOR



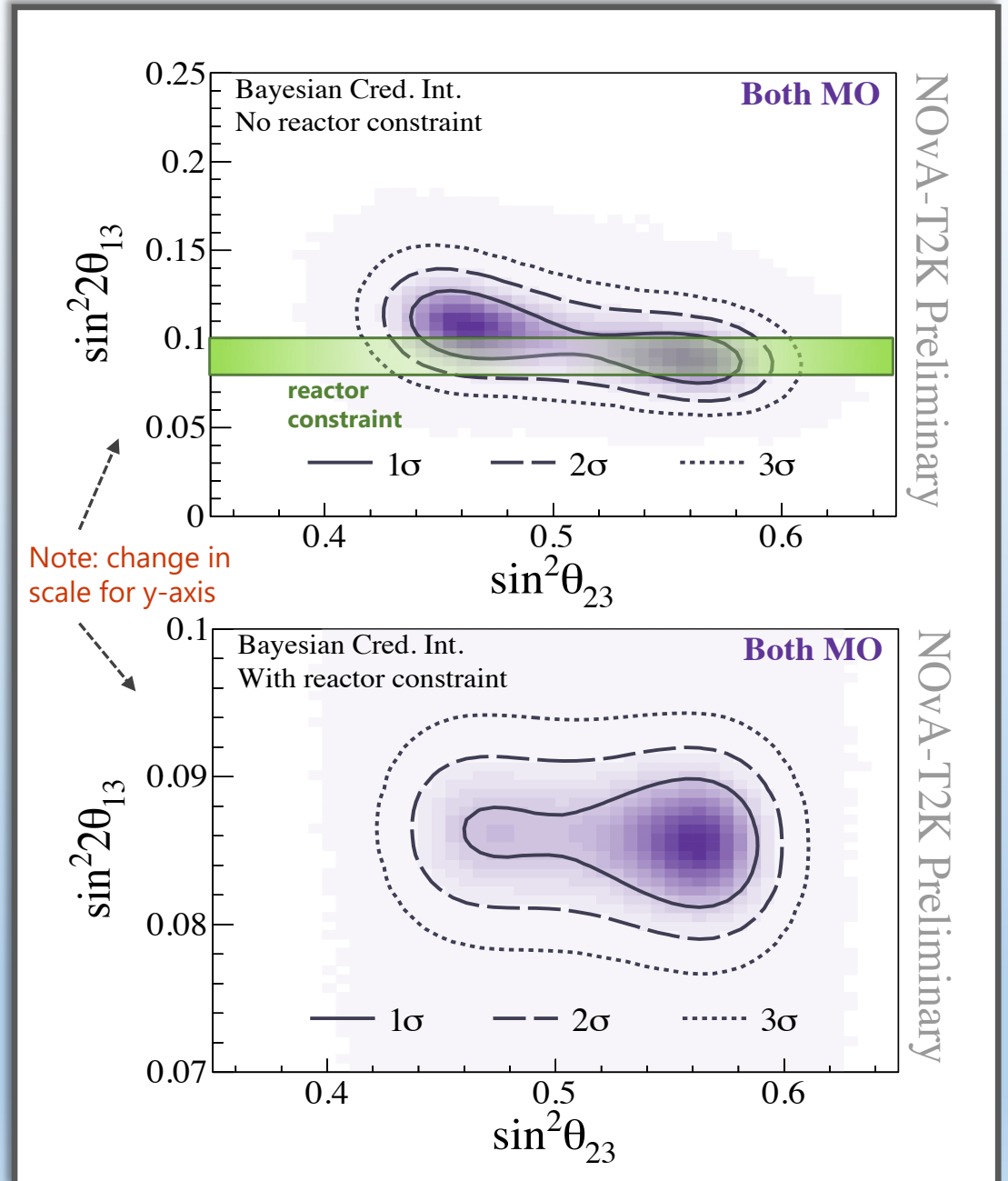
# Mixing angles: $\theta_{23}$ & $\theta_{13}$

- Without any external constraint from reactor experiments, long-baseline measurements have a degeneracy in  $\sin^2 \theta_{23}$  and  $\sin^2 2\theta_{13}$  parameters.



# Mixing angles: $\theta_{23}$ & $\theta_{13}$

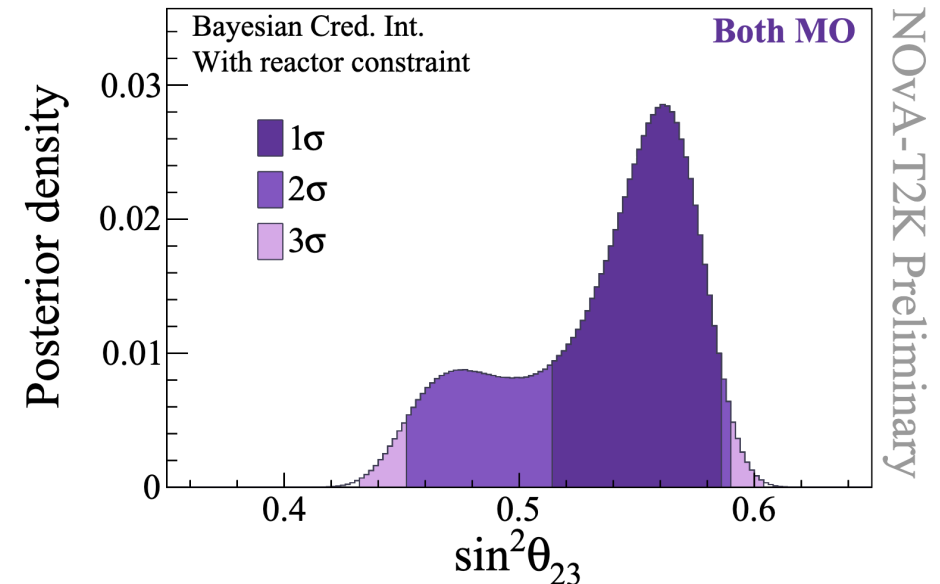
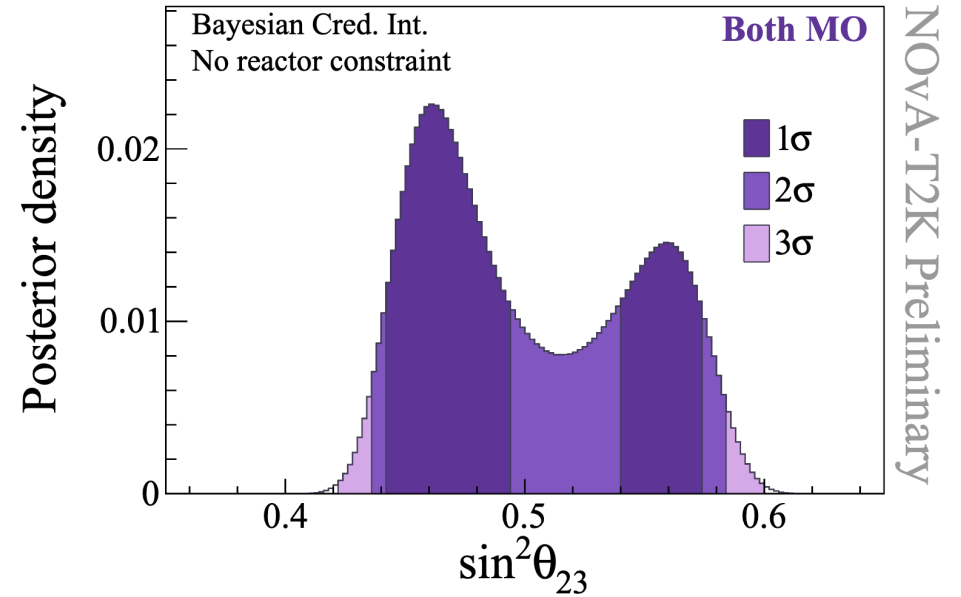
- Without any external constraint from reactor experiments, long-baseline measurements have a degeneracy in  $\sin^2 \theta_{23}$  and  $\sin^2 2\theta_{13}$  parameters.
- Using the average constraint on  $\sin^2 2\theta_{13} = 0.085 \pm 0.0027$  [PDG 2020], restricts us to a narrow posterior in  $\theta_{13}$  and lifts this degeneracy.





# Mixing angles: $\theta_{23}$ & $\theta_{13}$

- No significant preference for either octant from the joint-analysis.
- This preference shifts to a small but still insignificant preference for the upper octant when the reactor constraint on  $\theta_{13}$  is applied.



NOvA - T2K w/o reactor

NOvA - T2K - w/ reactor

Bayes factor

**1.17**

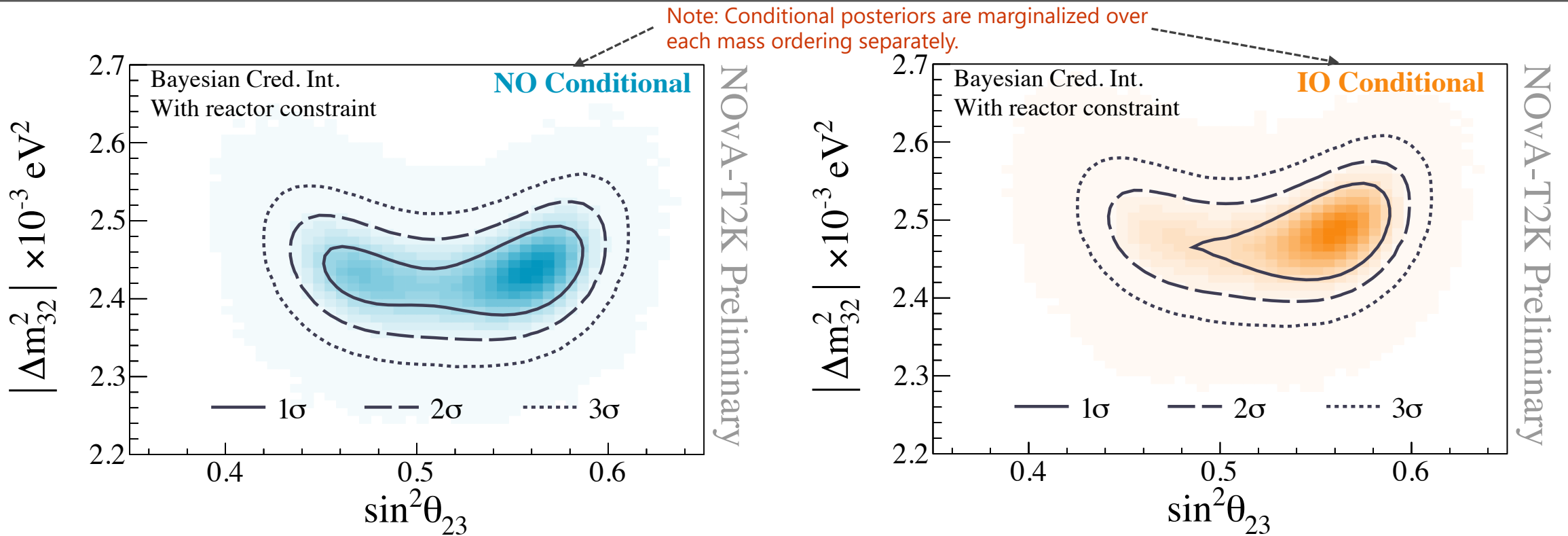
Lower Octant/Upper Octant  
~54% : ~46% posterior

**3.59**

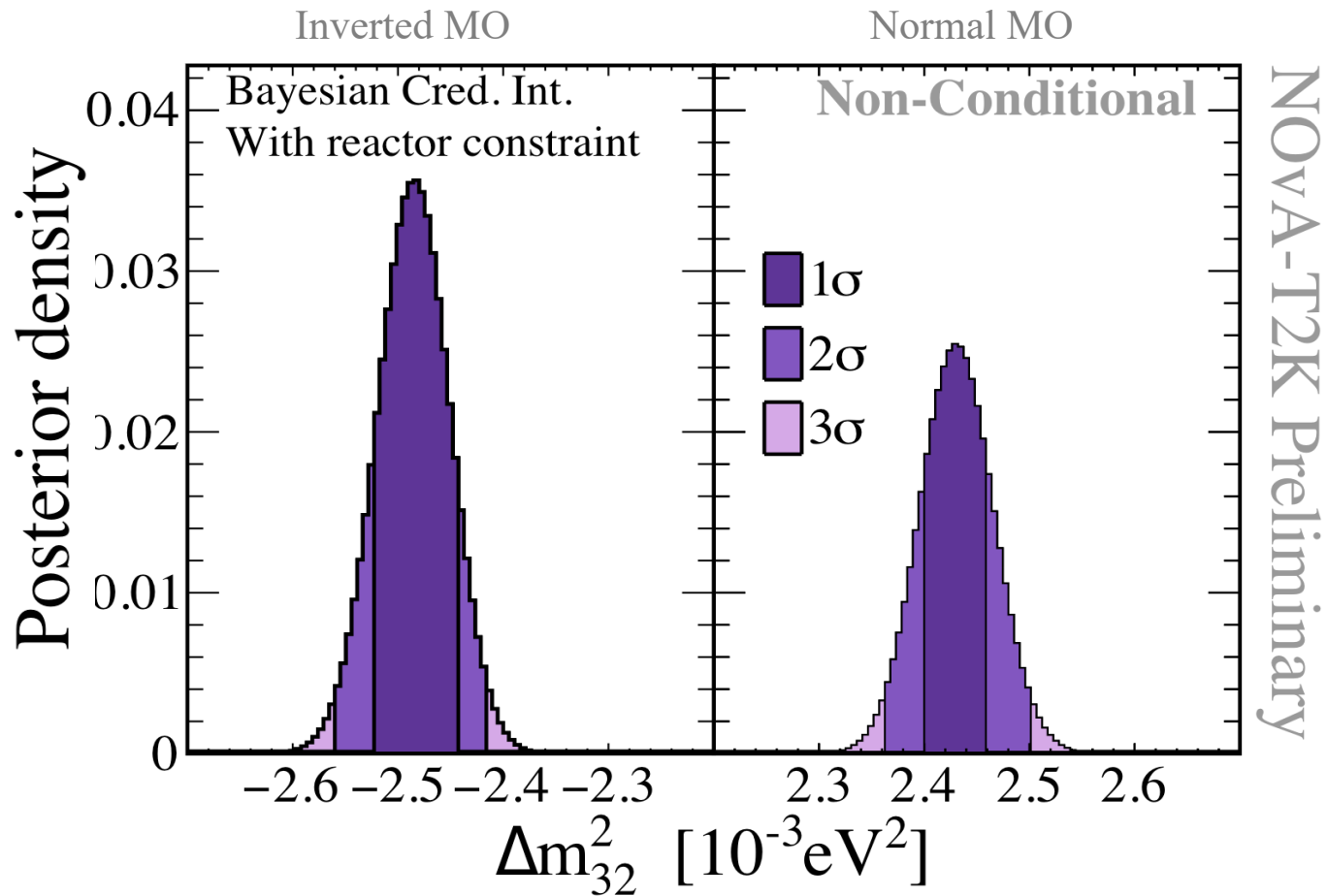
Upper Octant/Lower Octant  
~78% : 22% posterior

# $\Delta m_{32}^2$ and $\sin^2 \theta_{23}$

- Marginalizing over each mass ordering, we note a small but distinct difference in the  $\sin^2 \theta_{23}$  and  $\Delta m_{32}^2$  phase space.
- Measurements remain consistent with the maximal mixing hypothesis for  $\theta_{23}$  mixing angle.



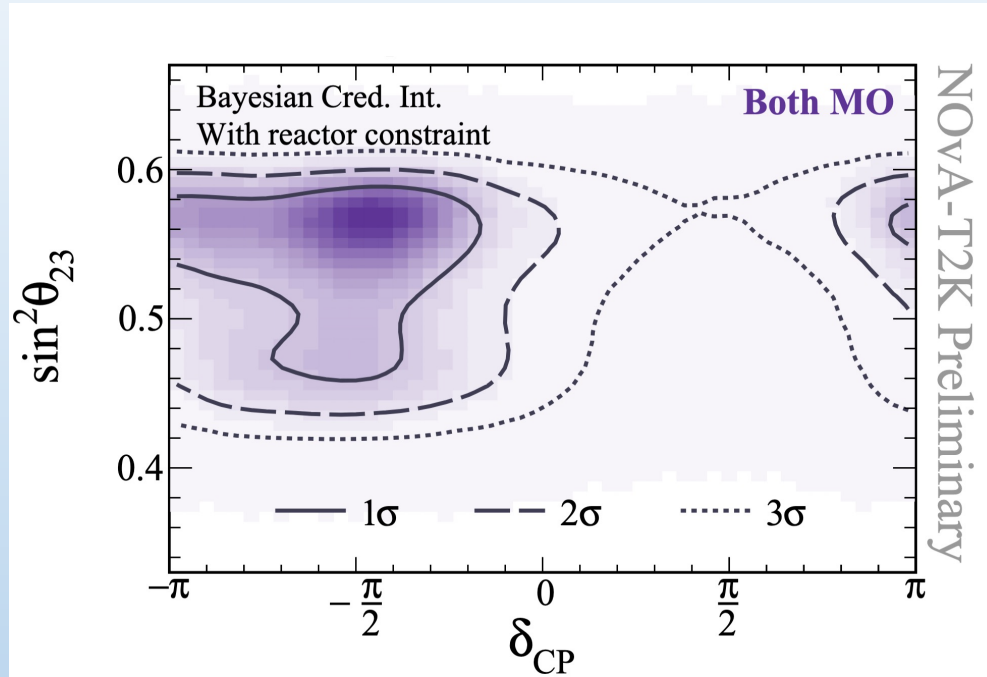
# Mass Ordering



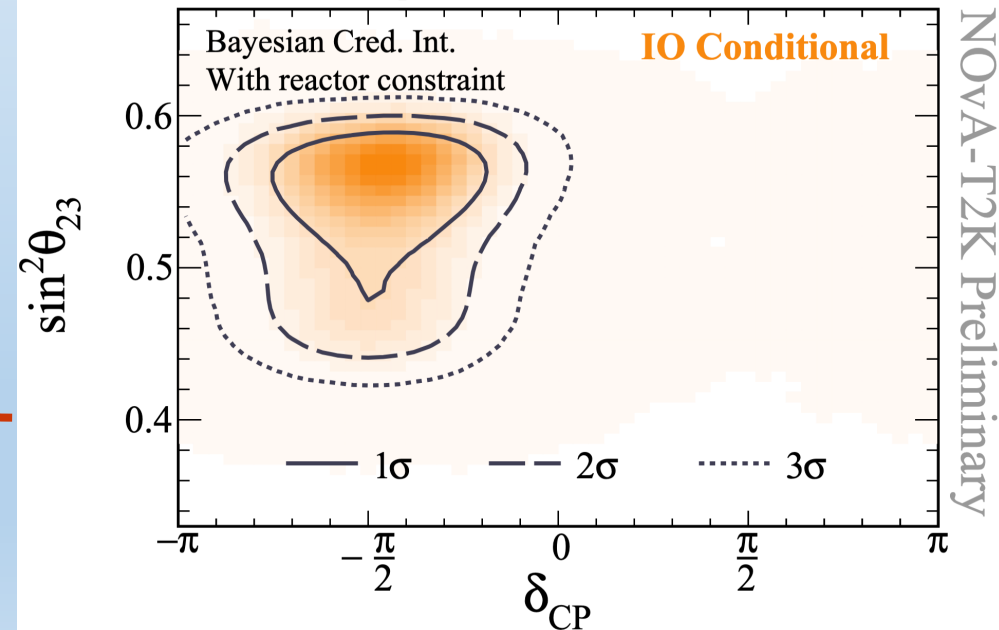
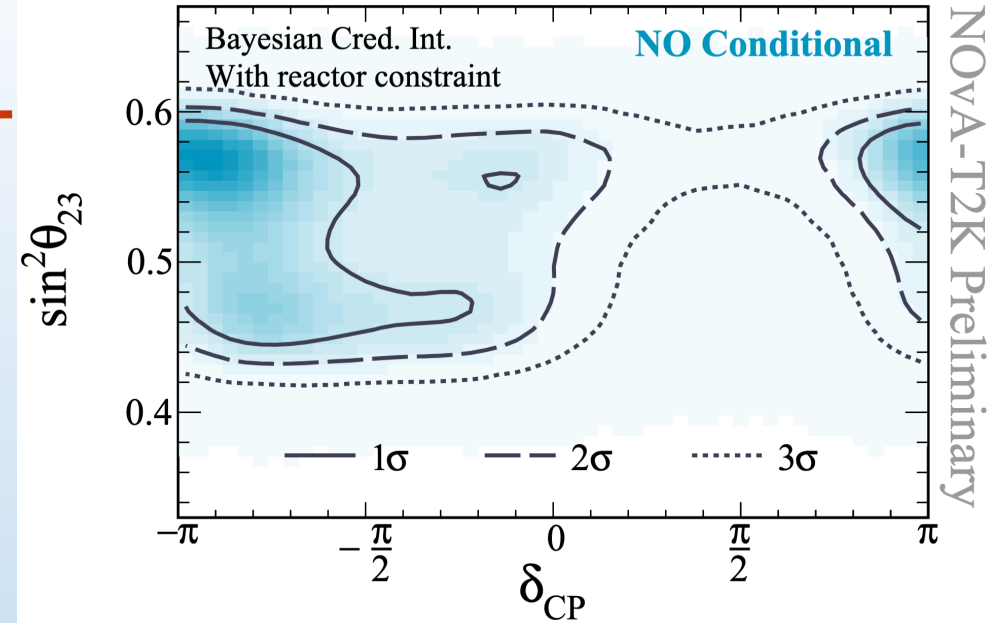
Comparing the posterior density in each mass ordering, the NOvA-T2K joint fit has a **best fit in the inverted ordering** but no significant preference

NOvA – T2K – w/ reactor	
Bayes factor	<b>1.36</b> Inverted Ordering/Normal Ordering ~58% : ~42% posterior

# CP Phase - $\delta_{CP}$

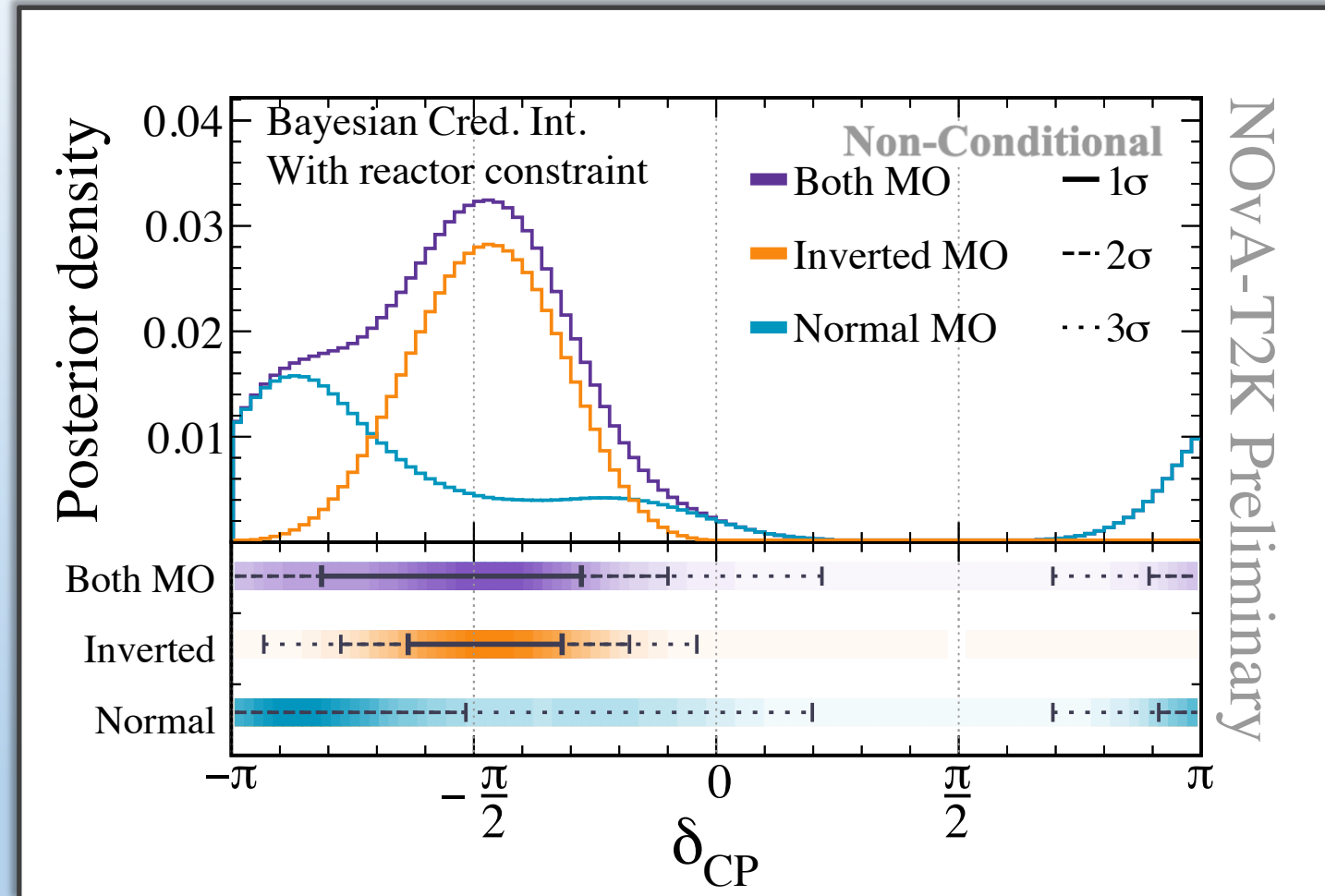


- **Normal MO:** wider range of allowed values with higher posterior density near CP conservation
- **Inverted MO:** enhanced preference for maximum CP violation and a large exclusion of  $\delta_{CP}$  phase space.



# CP Violation

- For both mass orderings,  $\delta_{CP} = +\pi/2$  lies **outside 3-sigma credible interval**.
- Normal Ordering allows for a **broad range of permissible  $\delta_{CP}$**
- For the **Inverted Ordering**, CP conserving values of  $\delta_{CP}$  ( $0, \pi$ ) lie **outside the 3-sigma** credible interval.



# CP Violation: Jarlskog

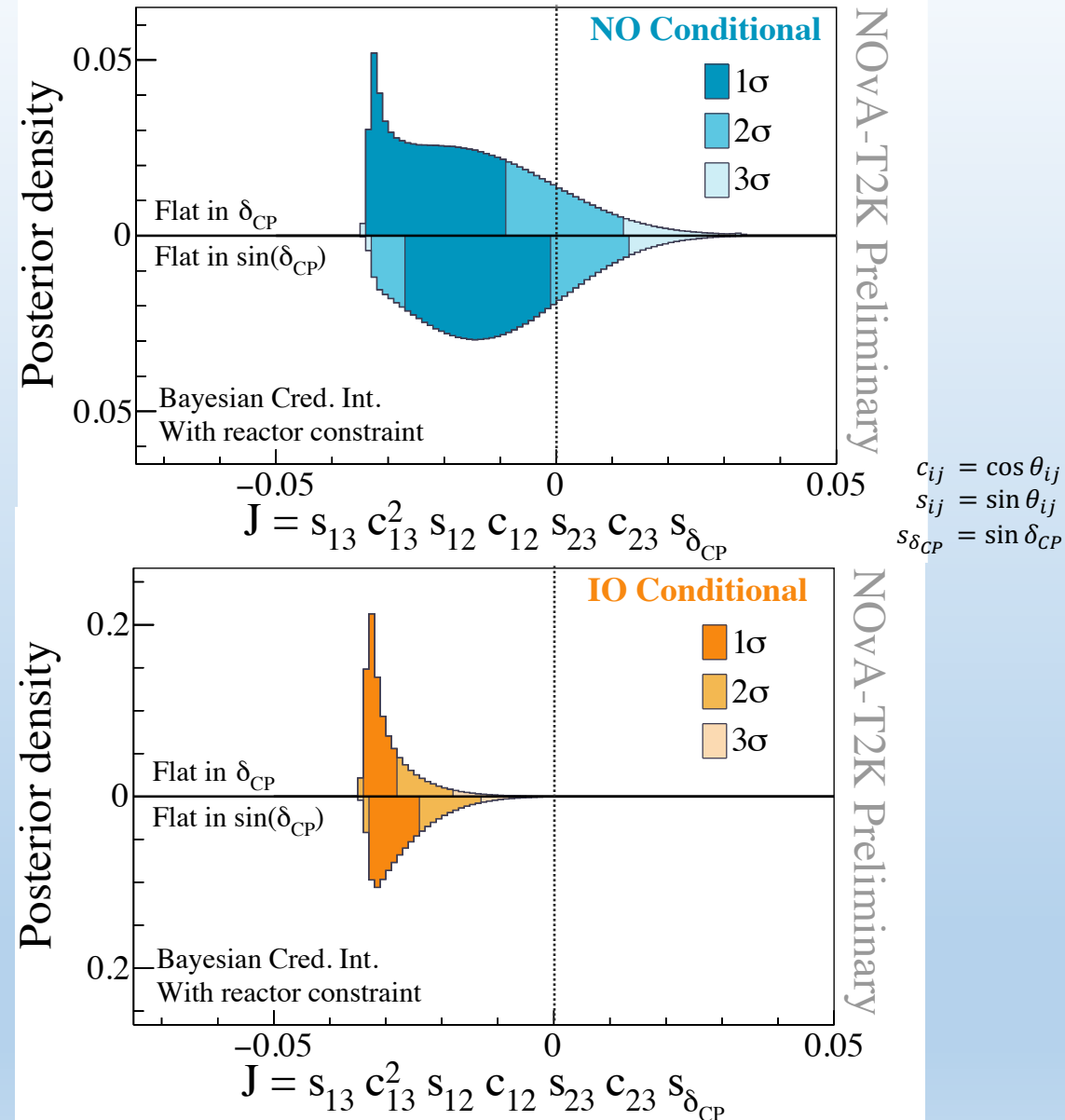
- Jarlskog-invariant is a **parameterization independent way\*** to measure CP violation.

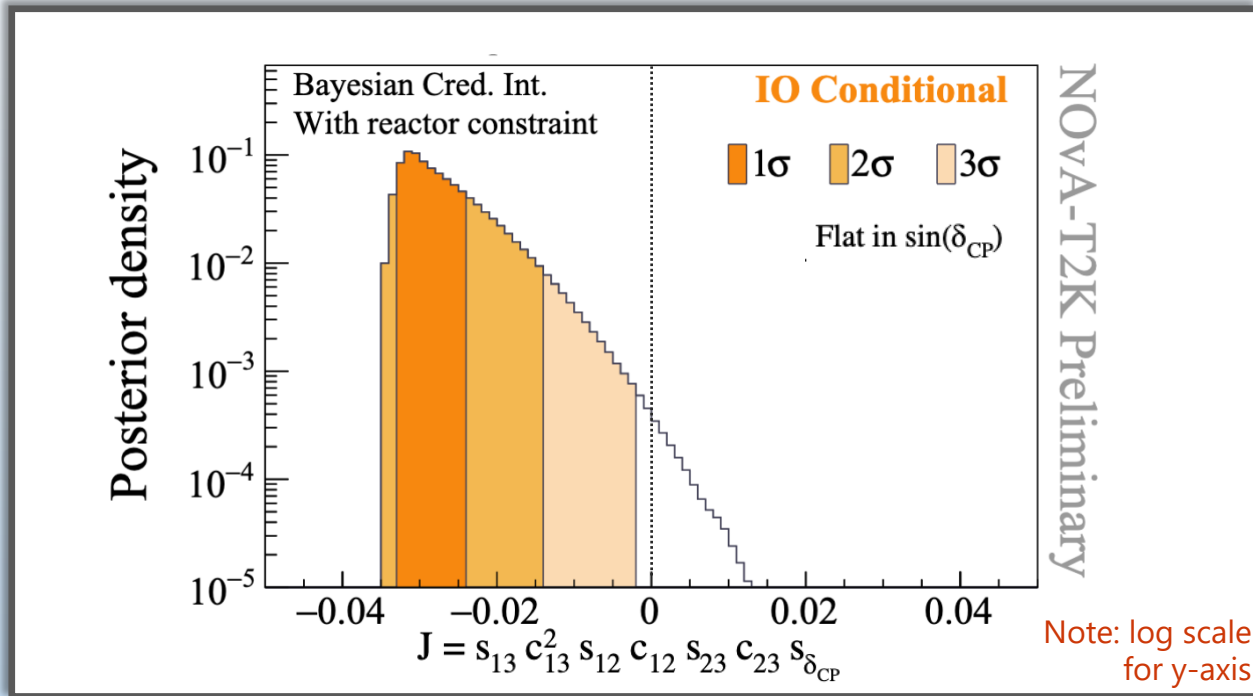
$$J = \sin \theta_{13} \cos^2 \theta_{13} \sin \theta_{12} \cos \theta_{12} \sin \theta_{23} \cos \theta_{23} \sin \delta_{CP}$$

$J=0$ : CP-Conservation  $J \neq 0$ : CP-Violation

- For **Normal Ordering**, a considerably **wider range of probable values for J**
- $J = 0$  lies outside the  $3\sigma$  interval for the Inverted Ordering**
  - for priors that are both uniform in  $\delta_{CP}$  and uniform in  $\sin \delta_{CP}$

\*Phys. Rev. D 100, 053004 (2019)

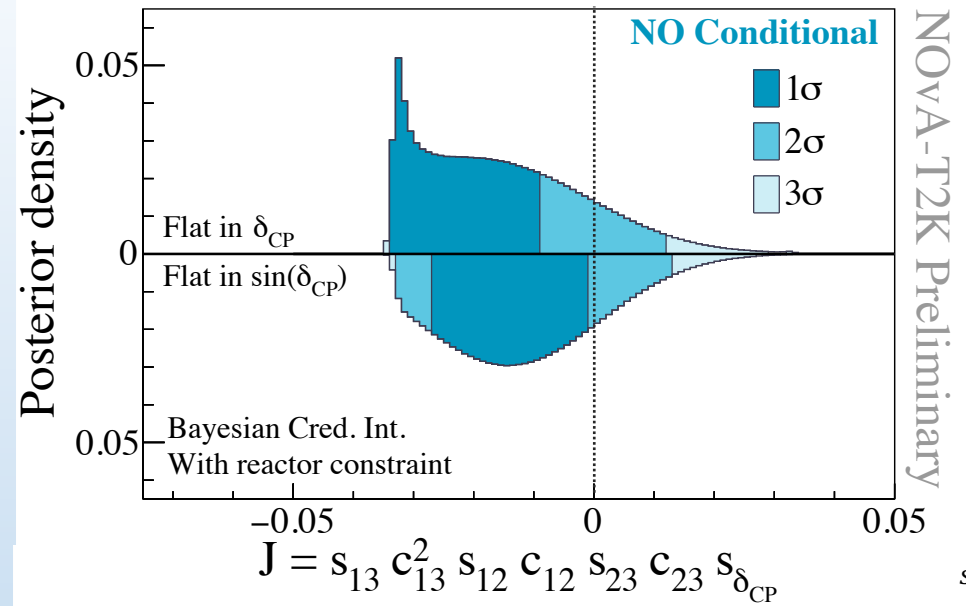




• **J = 0 lies outside the 3σ interval for the Inverted Ordering**

- for priors that are both uniform in  $\delta_{CP}$  and uniform in  $\sin \delta_{CP}$

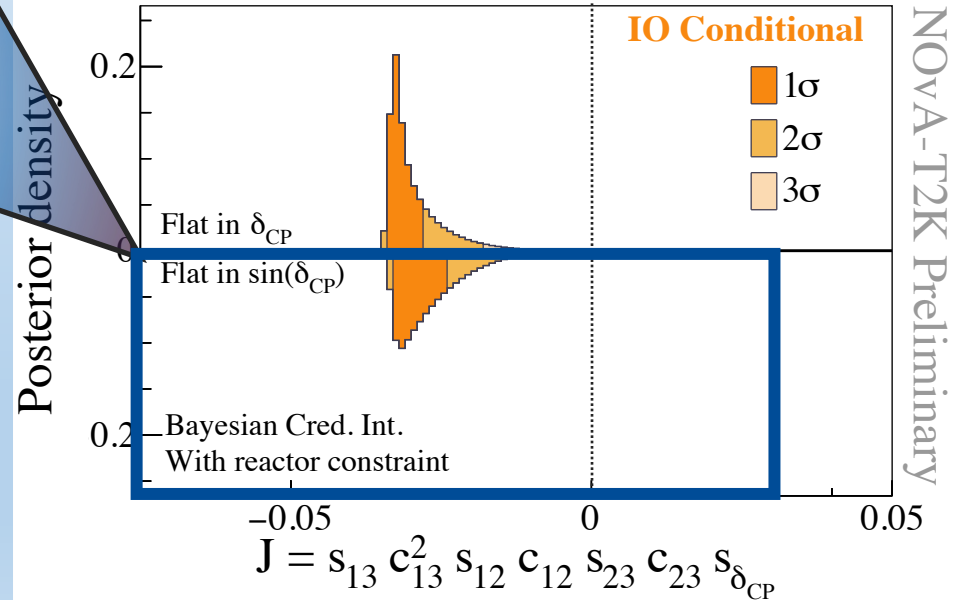
\*Phys. Rev. D 100, 053004 (2019)



$$c_{ij} = \cos \theta_{ij}$$

$$s_{ij} = \sin \theta_{ij}$$

$$s_{\delta_{CP}} = \sin \delta_{CP}$$



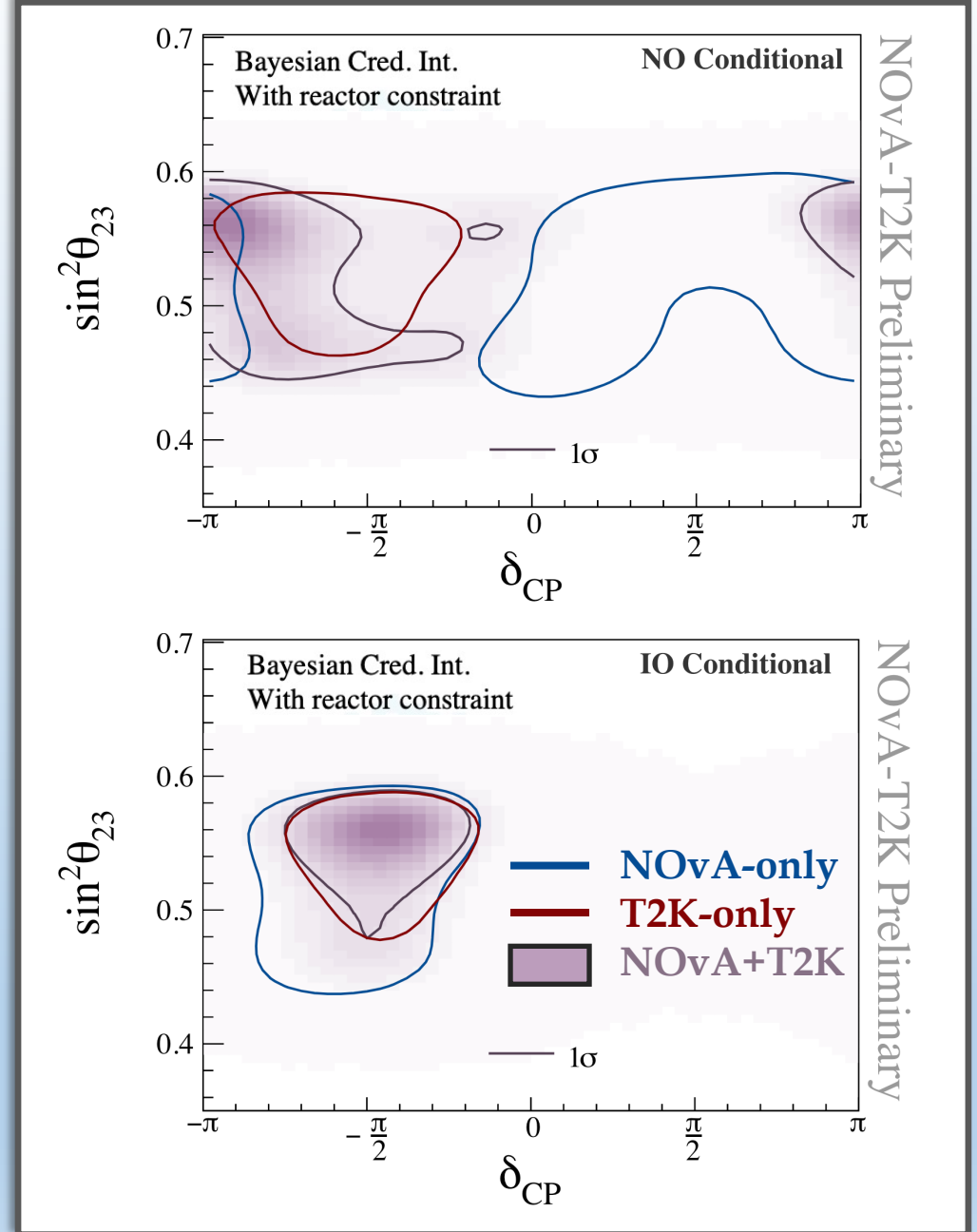
# Comparisons

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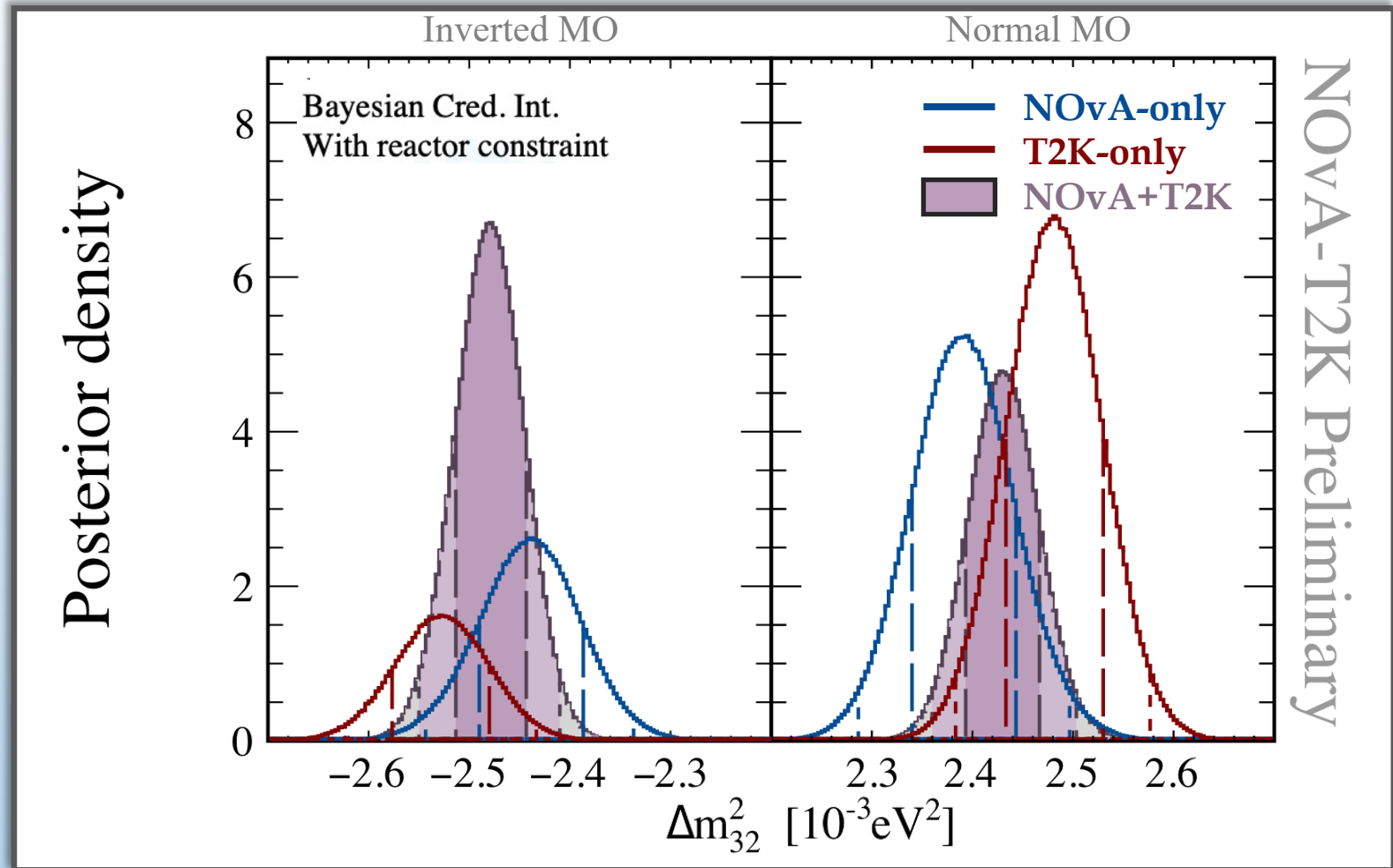
# Comparison with NOvA-only & T2K-only fits

- The **joint-fit prefers the region in the normal ordering where the two individual experiment's preference overlaps** as you'd expect.
- There is a **tighter constraint in the Inverted Ordering** where NOvA-only and T2K-only had the same best fit point.



# Comparison with NOvA-only & T2K-only fits

- The 1D posterior in  $\Delta m_{32}^2$  highlights the switch in the mass ordering preference when NOvA and T2K are combined.
- The joint-fit enhances the precision of  $\Delta m_{32}^2$  over individual experiments.

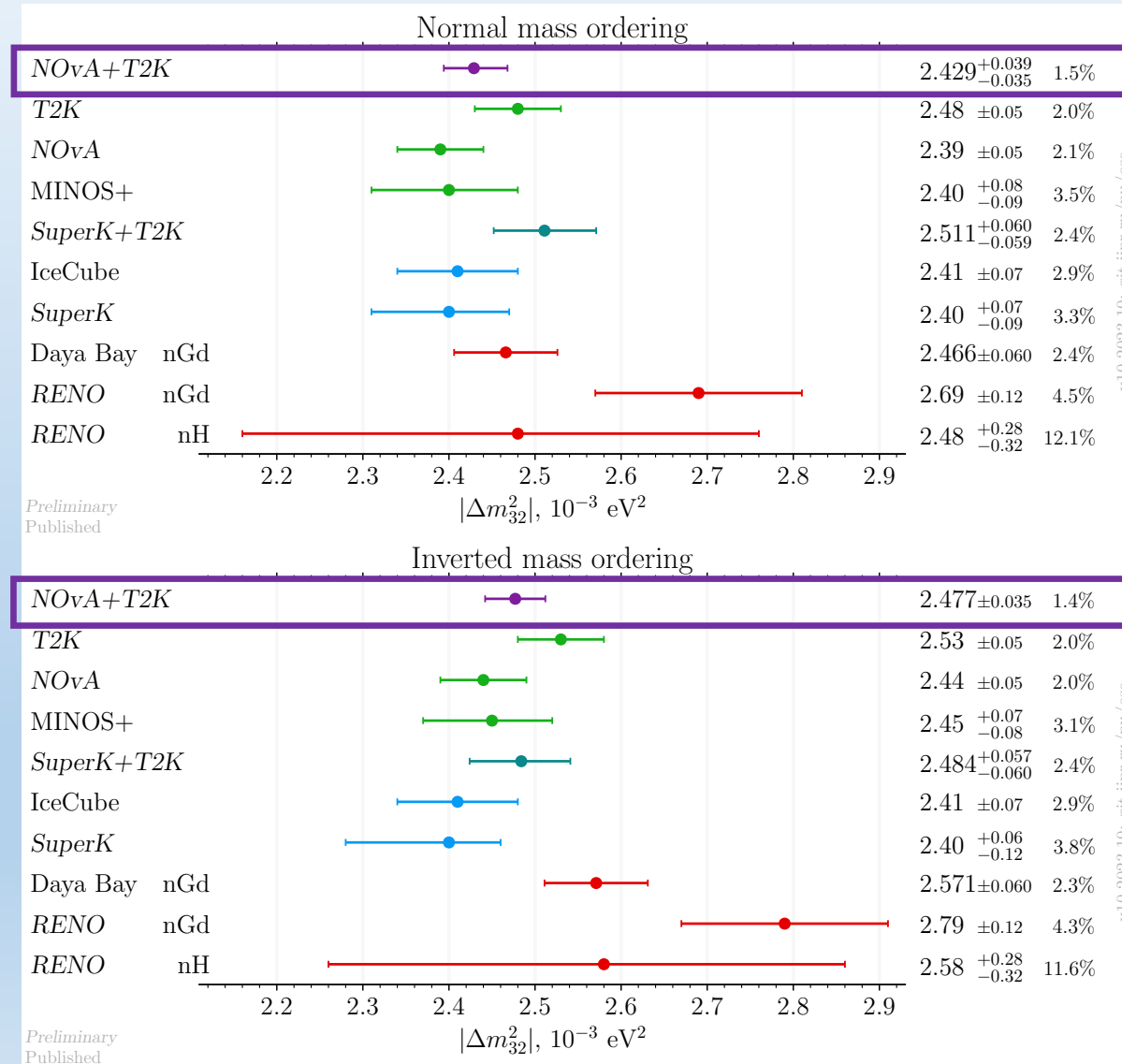


NOvA-T2K Preliminary

	NOvA only	T2K only	NOvA+T2K
Bayes factor	<p><b>2.07</b></p> <p>Normal/Inverted ~67% : ~33% posterior</p>	<p><b>4.24</b></p> <p>Normal/Inverted ~81% : ~19% posterior</p>	<p><b>1.36</b></p> <p>Inverted/Normal ~58% : ~42% posterior</p>

# Global Comparisons - $\Delta m_{32}^2$

- This analysis has the **smallest uncertainty on  $|\Delta m_{32}^2|$**  as compared to other previous measurements.

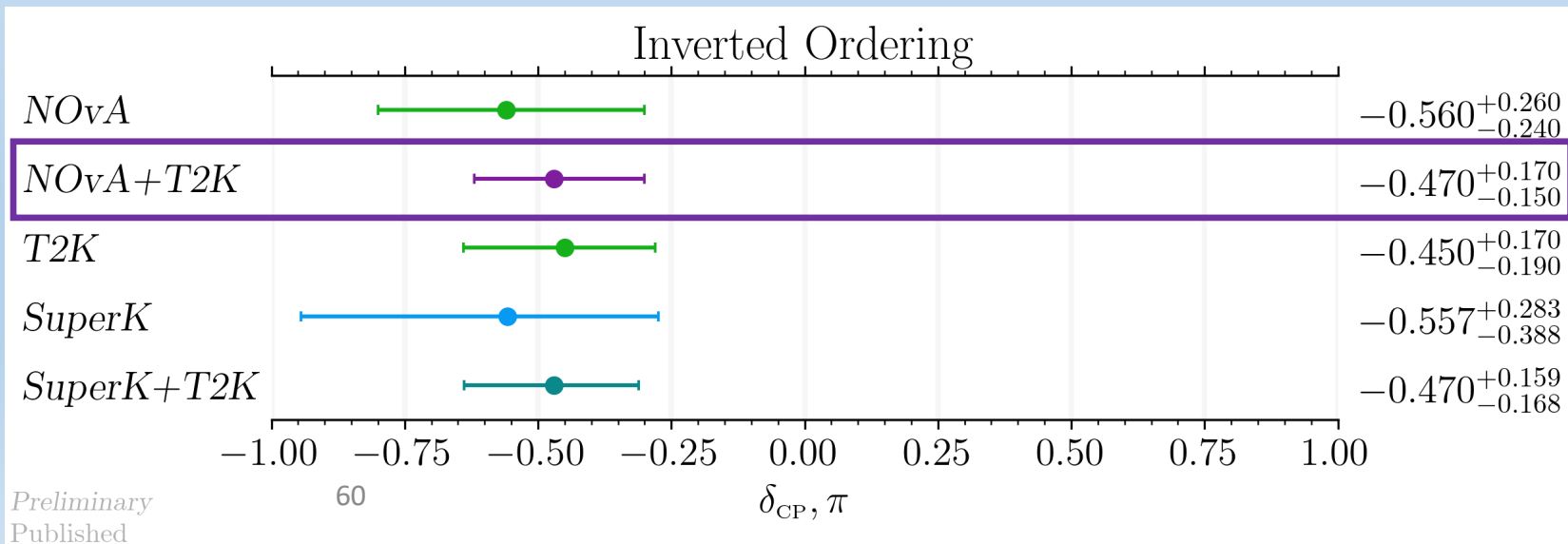
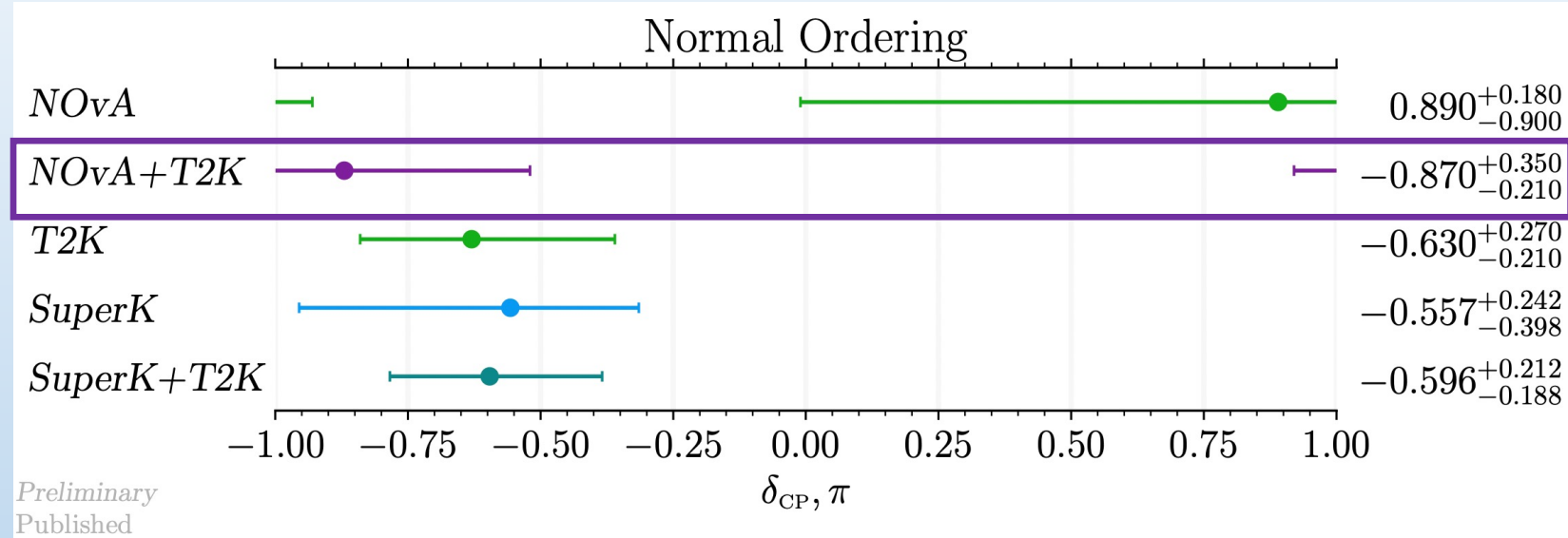


v10 2023.10: git.jinr.ru/mu/osc

v10 2023.10: git.jinr.ru/mu/osc

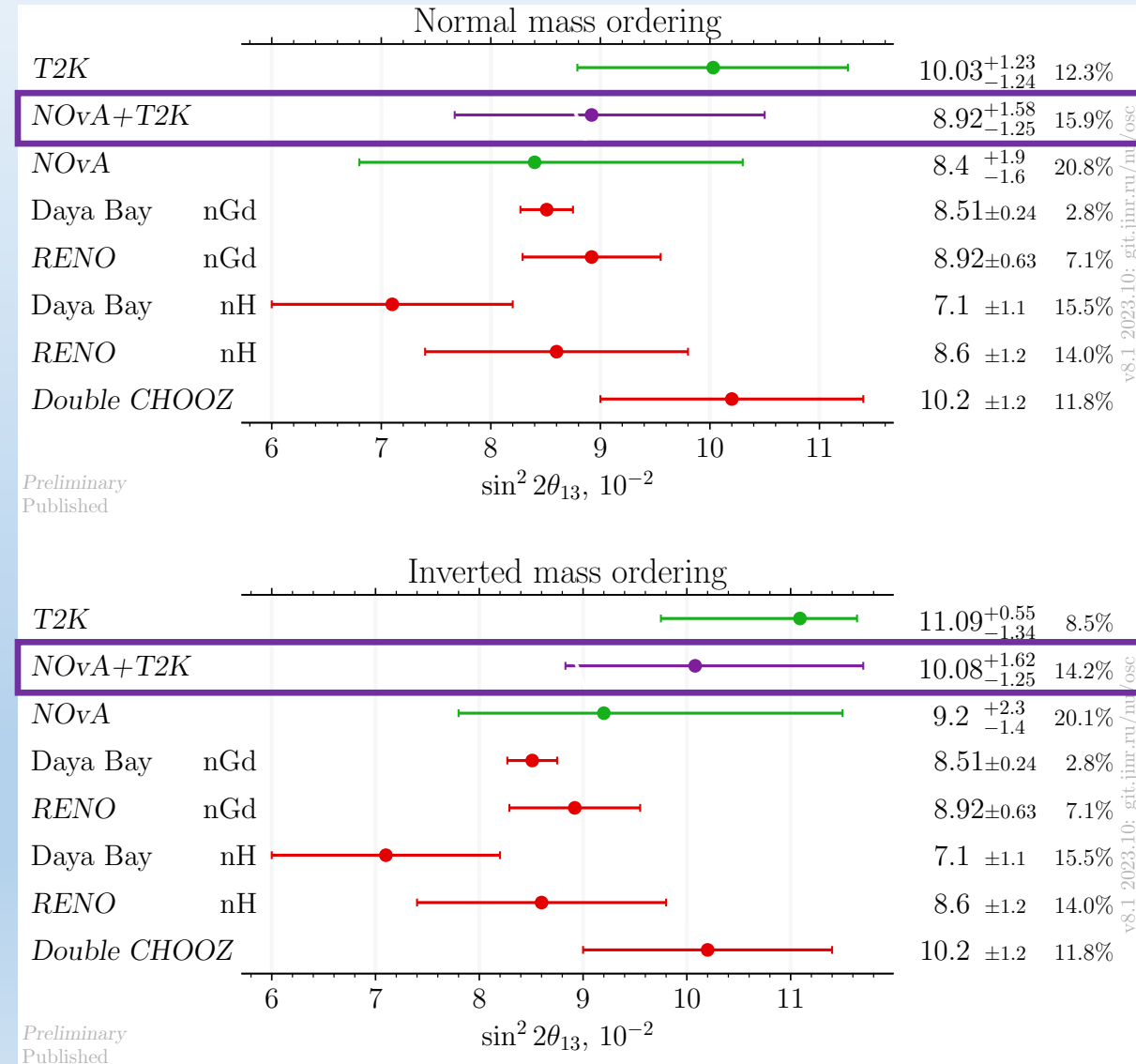
# Global Comparisons - $\delta_{CP}$

- The  $\delta_{CP}$  measurements are consistent across all experiments and their combinations.
- The uncertainty on  $\delta_{CP}$  remains large.



# Global Comparisons – $\theta_{13}$

- Daya Bay leads the precision on the measurement of  $\theta_{13}$  with 2.8% uncertainty.
- Overall, the long-baseline measurements are consistent with reactor experiments, with larger consistency in the normal ordering than the inverted ordering.

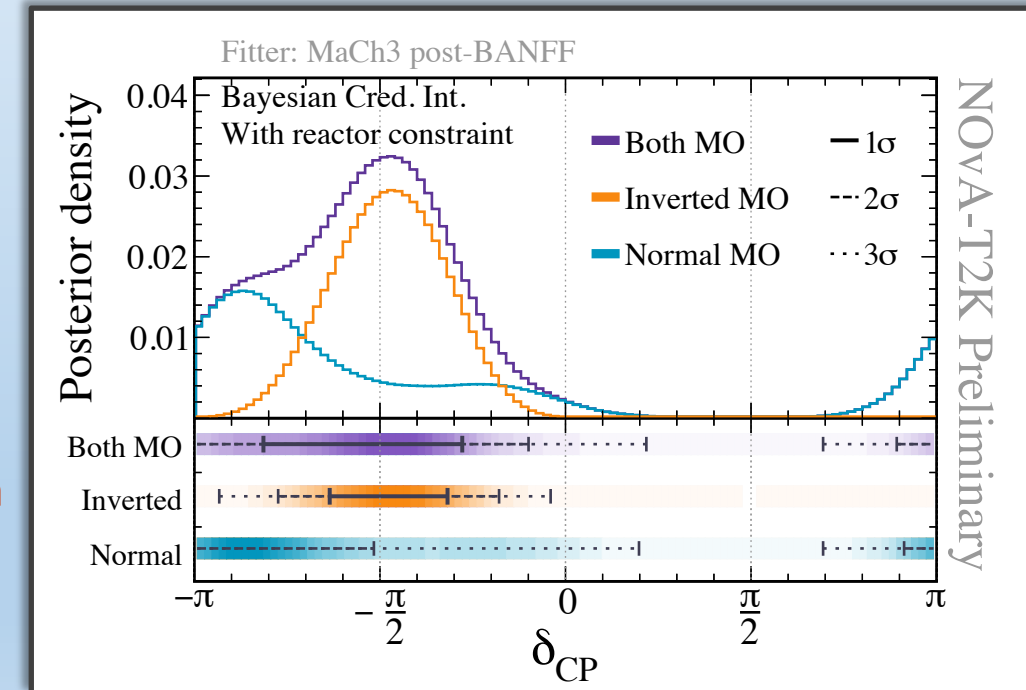
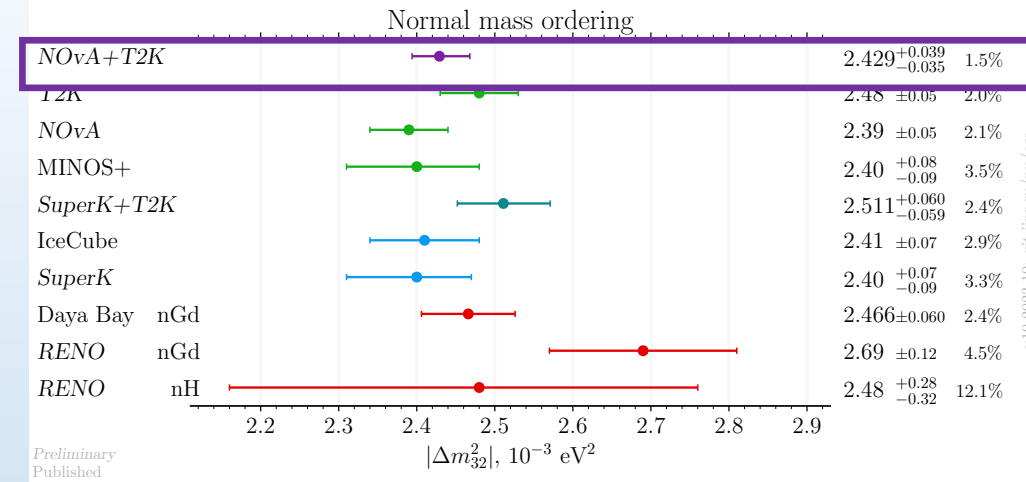


# Summary & Outlook

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# Summary

- The joint analysis of NOvA and T2K demonstrates **simultaneous compatibility** with both datasets.
- The joint analysis shows:
  - Very strong constraint on  $|\Delta m_{32}^2|$ .**
  - Mass Ordering preference remains inconclusive.
    - Small, not significant, preference for Inverted Ordering** in the joint fit whereas individual experiments prefer Normal Ordering.
  - $\delta_{CP} = +\pi/2$  lies **outside 3-sigma credible interval** for both mass orderings.
  - Normal ordering permits a wide range of permissible  $\delta_{CP}$ , while **CP conserving values for the Inverted Ordering fall outside the 3-sigma range.**
    - Similar conclusions for Jarlskog.

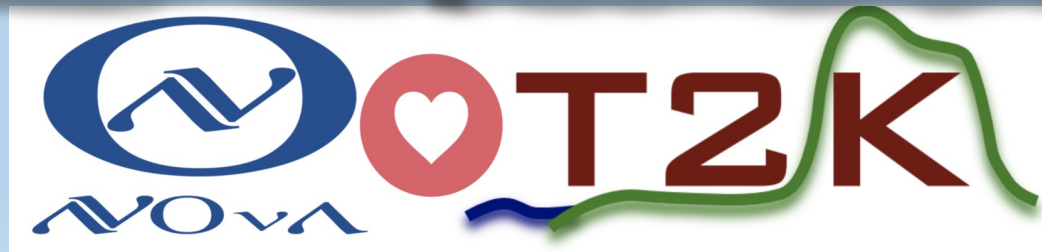


# Outlook

- Both experiments continue to collect high quality data and improve their analyses -
  - **Data is expected to double**, plus updated systematic models, detector response, and new data samples
- Collaboration and information exchange has resulted in a deeper understanding of the analyses
- We are actively exploring the scope and timeline for the next round of this work







# Backup

# NOvA+T2K+Daya Bay

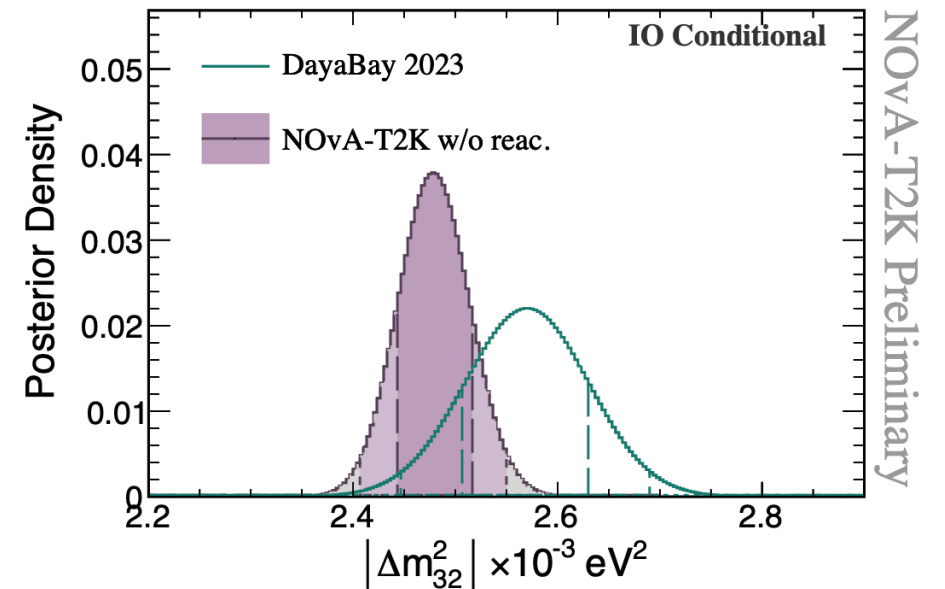
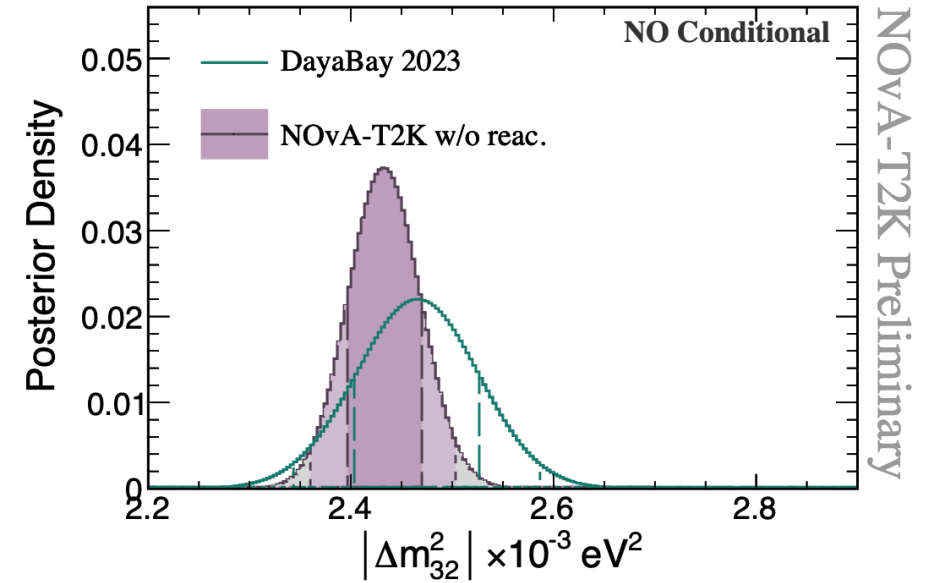
- Enhanced precision in  $\Delta m_{32}^2$  presents a “new” lever on measuring neutrino mass-ordering\*.
- In the true mass ordering, reactor and long-baseline measurements of  $\Delta m_{32}^2$  would be consistent but in the incorrect mass ordering would be wrong by different amounts.

Also see: [Stephen Parke W&C, 2023](#)

[\\*Phys. Rev. D 72: 013009, 2005](#)

## Another possible way to determine the Neutrino Mass Hierarchy

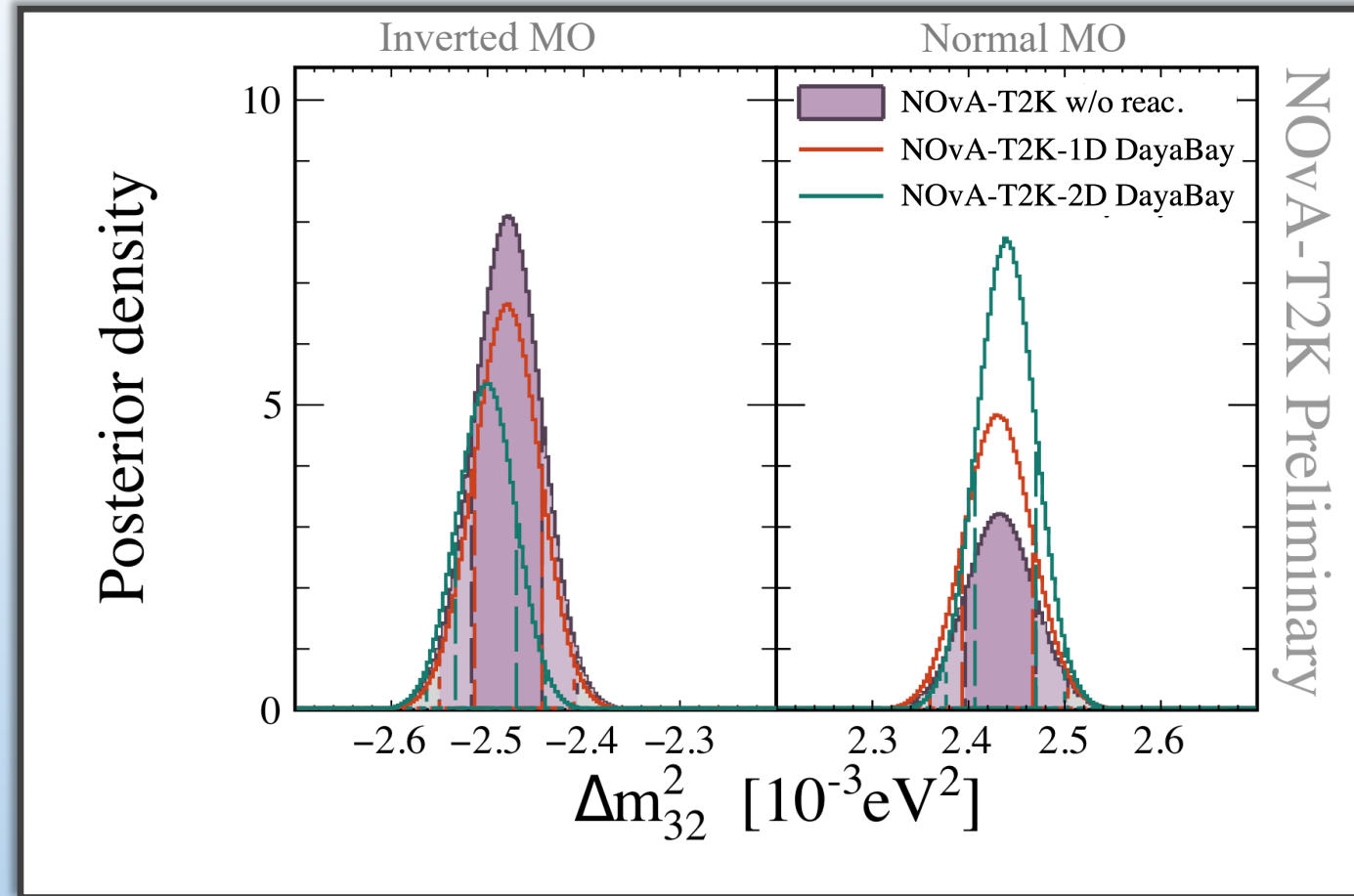
Hiroshi Nunokawa<sup>1,\*</sup>, Stephen Parke<sup>2,†</sup> and Renata Zukanovich Funchal<sup>3,‡</sup>



# NOvA+T2K+DayaBay

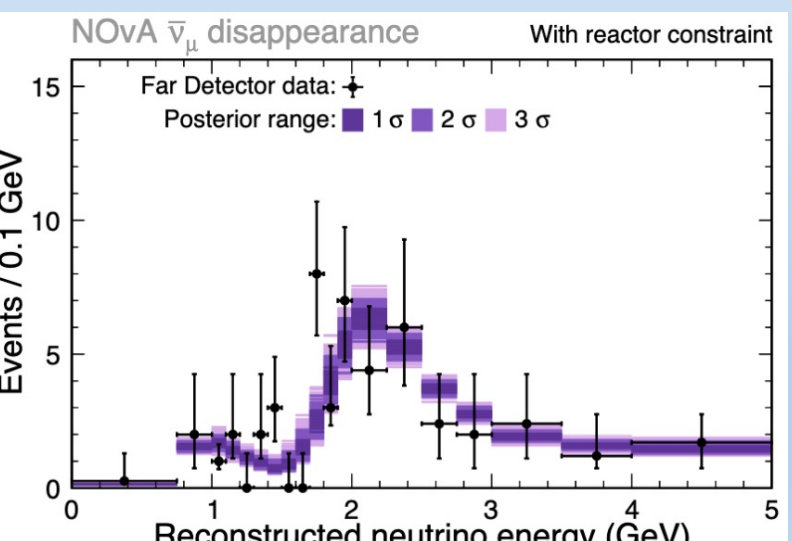
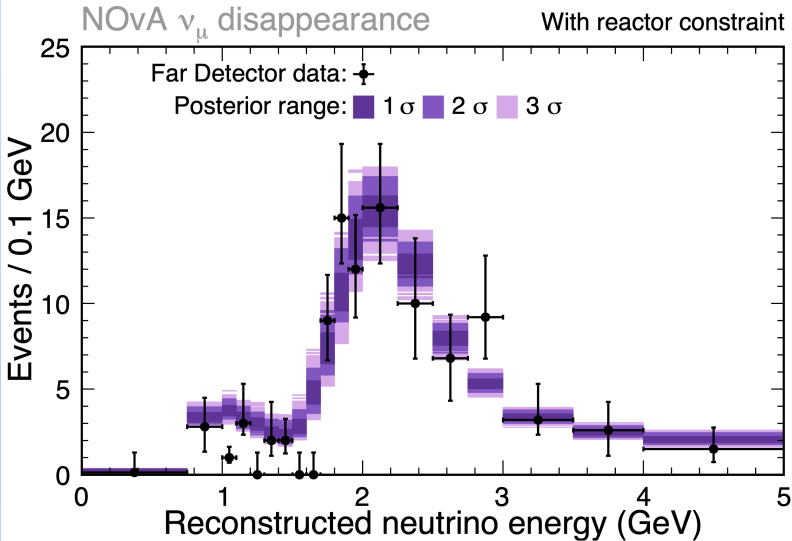
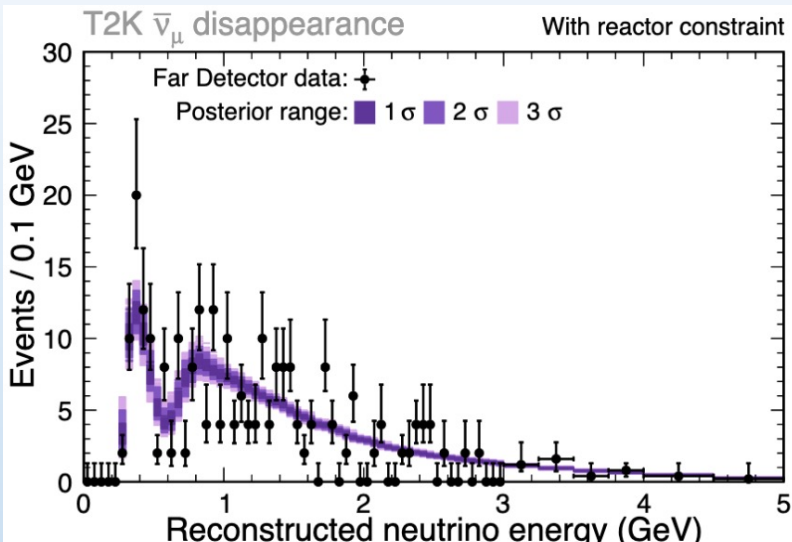
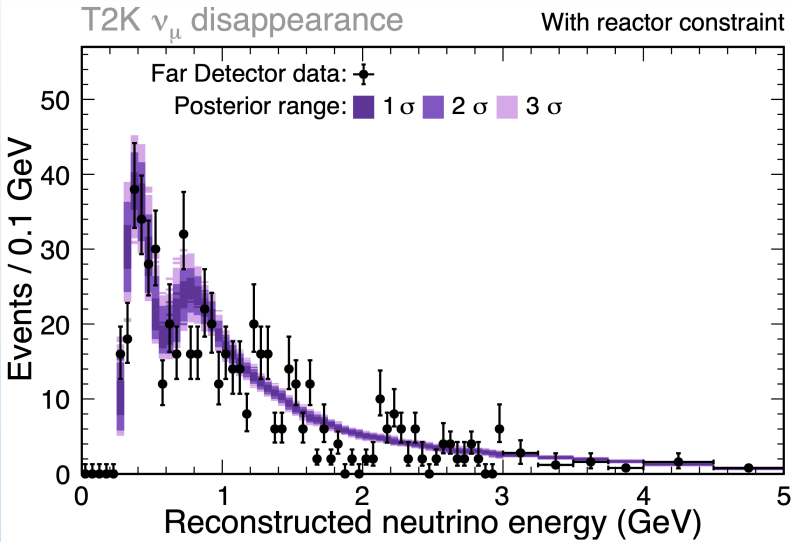
- Including the  $\Delta m_{32}^2$  constraint from the Daya Bay\*, reverse the mass ordering preference back to the Normal Ordering.
- Overall, this analysis does not show a significant preference for either mass ordering.

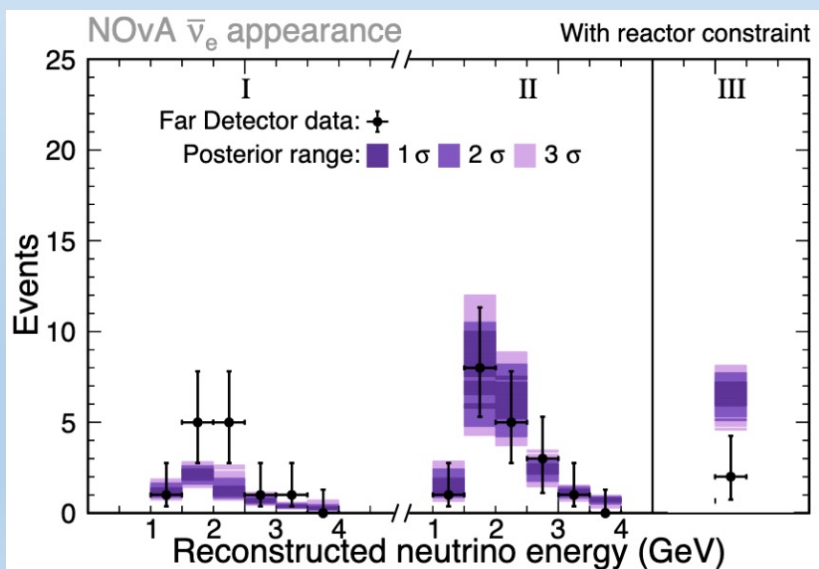
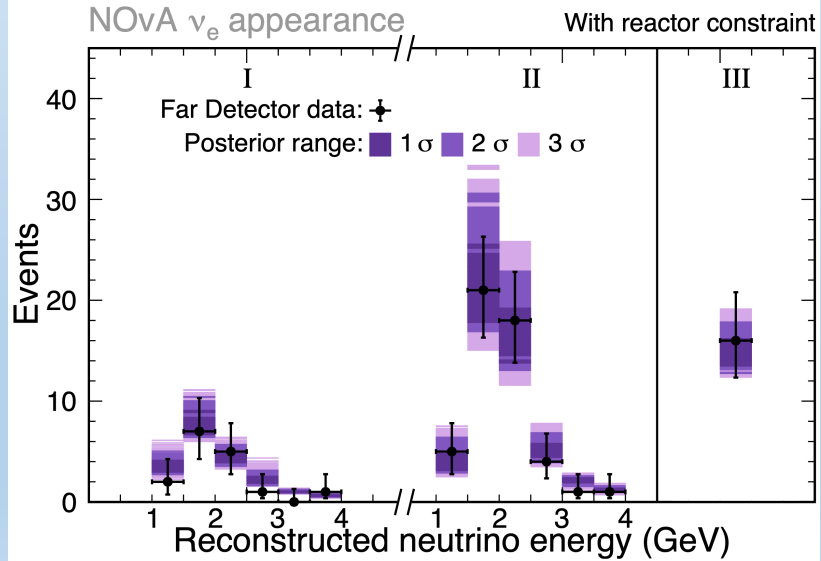
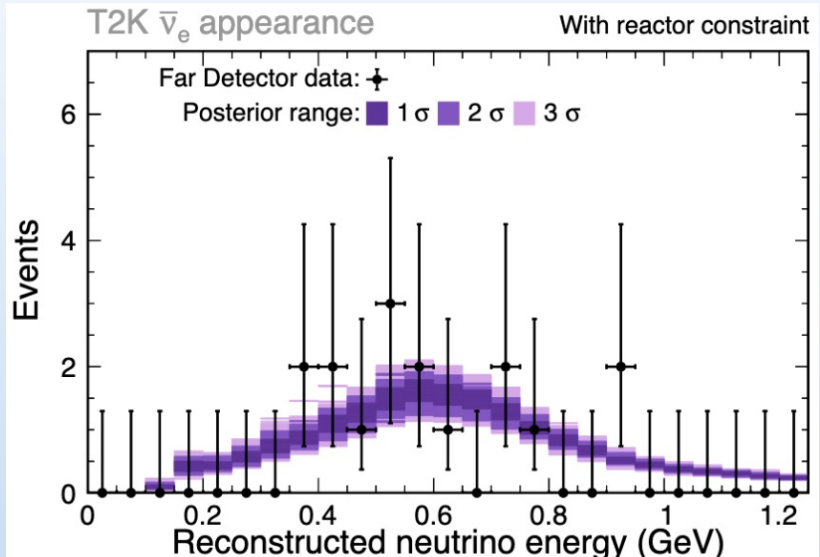
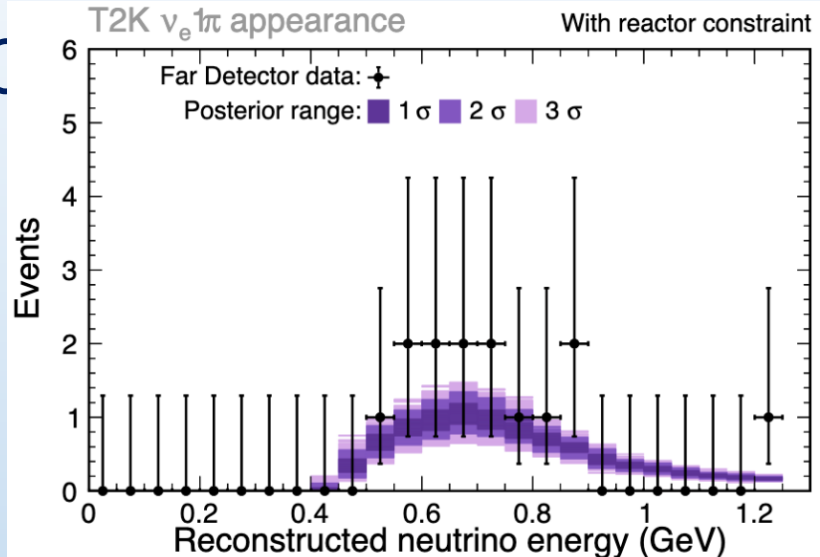
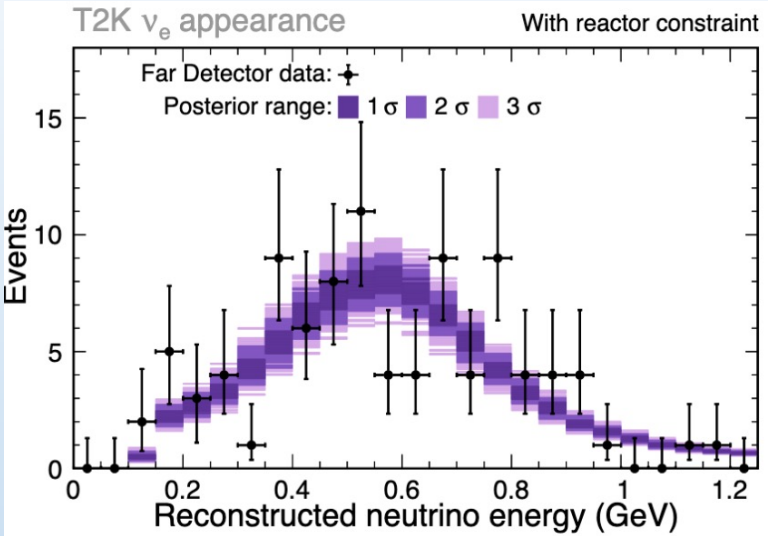
\*Phys. Rev. Lett. **130**, 161802, 2023



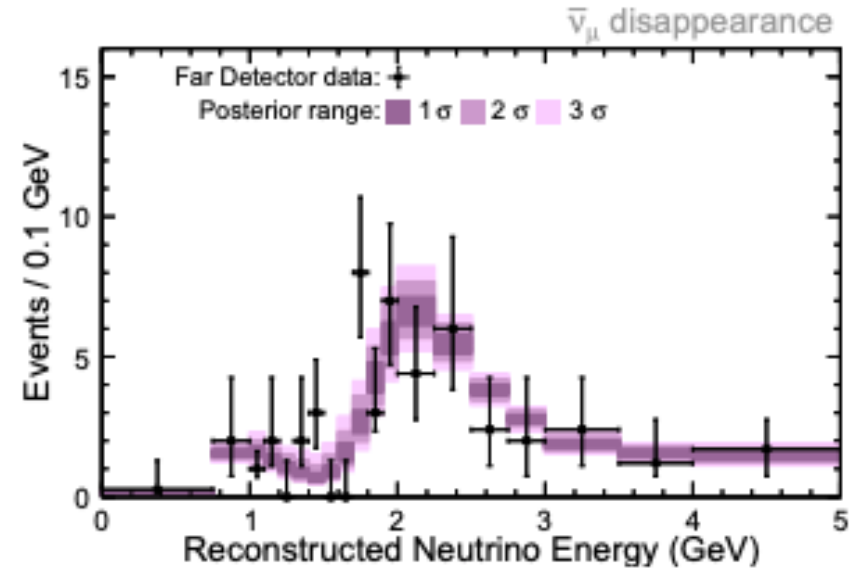
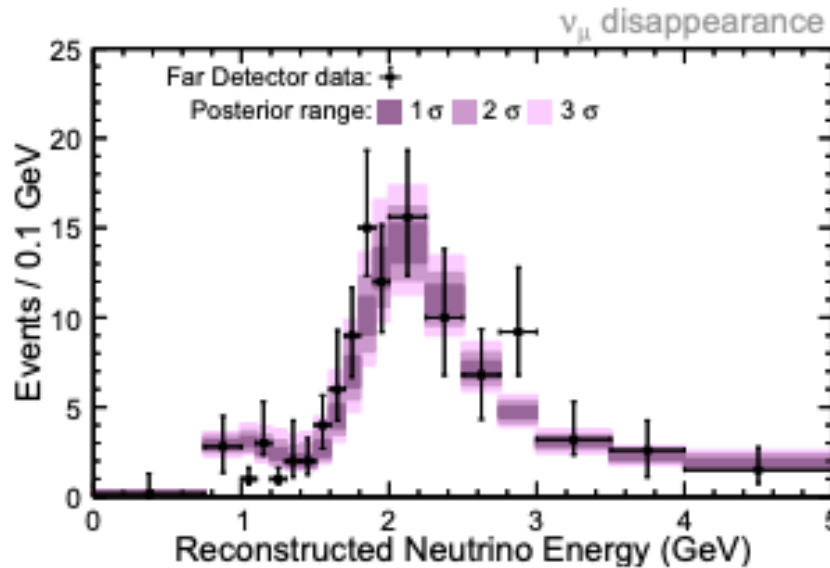
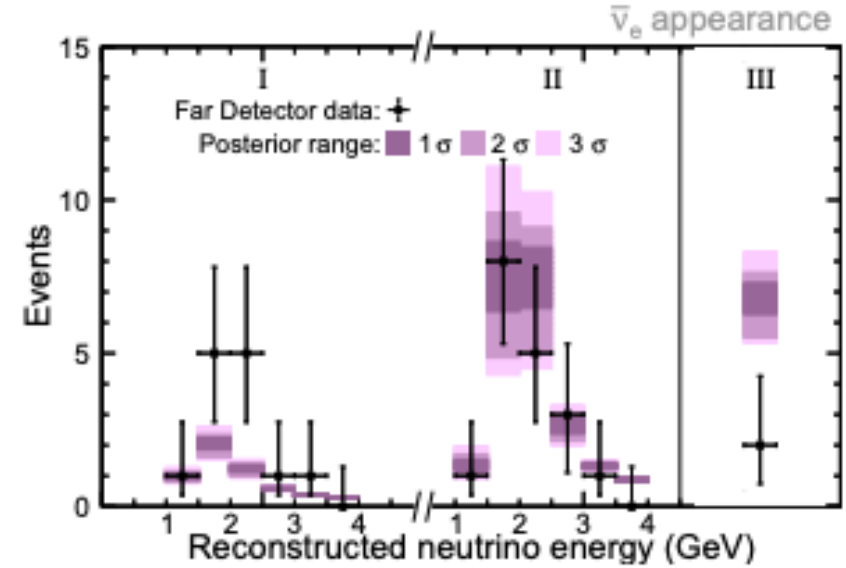
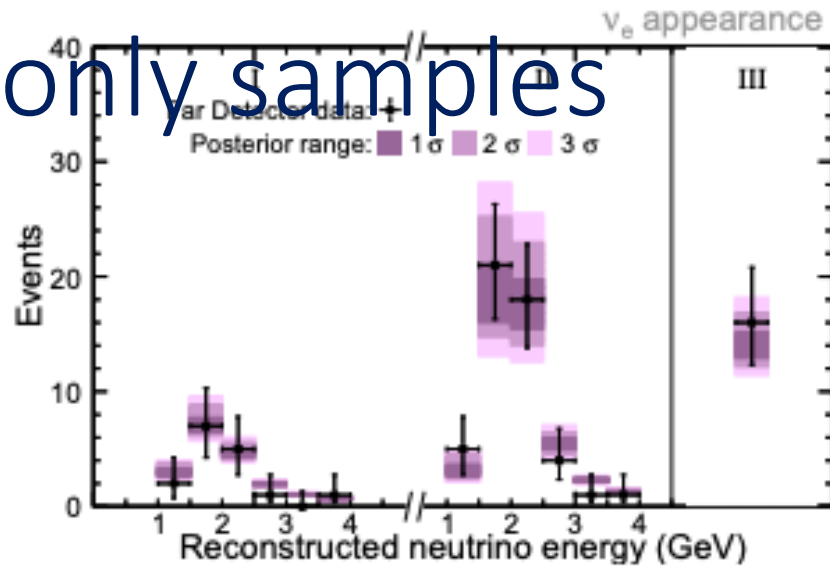
	NOvA - T2K w/o reactor	NOvA - T2K - 1D Daya Bay	NOvA - T2K - 2D Daya Bay
Bayes factor	<b>2.47</b> Inverted/Normal ~71% : ~29% posterior	<b>1.34</b> Inverted/Normal ~57% : ~43% posterior	<b>1.44</b> Normal/Inverted ~59% : ~41% posterior

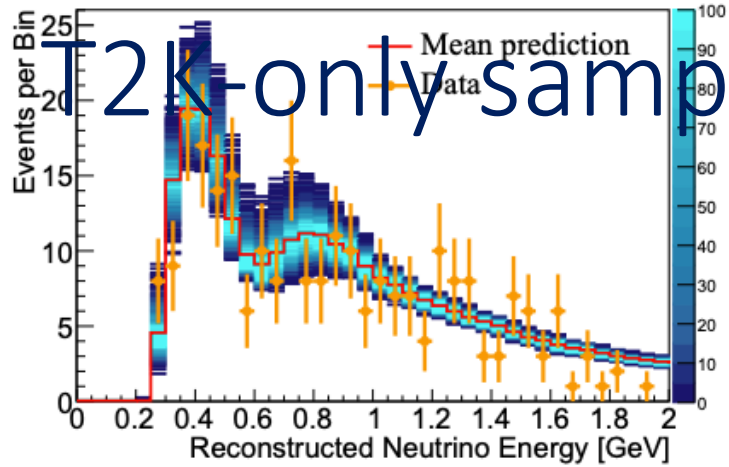
# NOvA+T2K Disappearance Data samples:



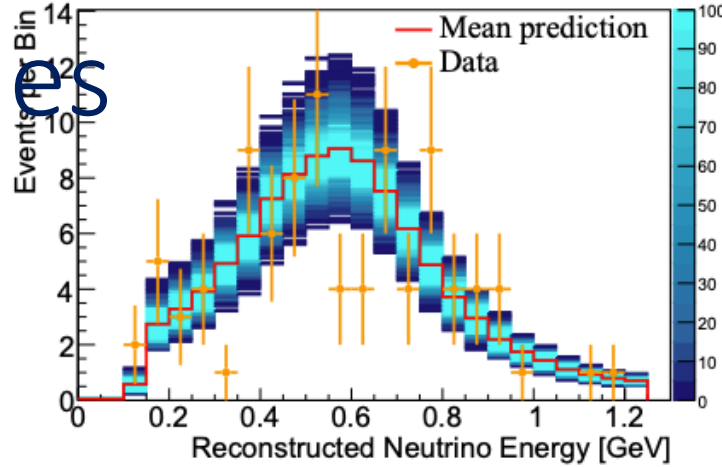


# NOvA-only samples

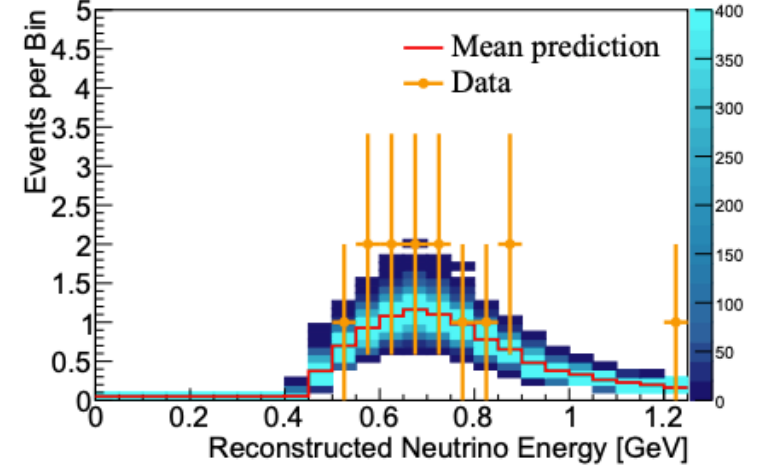




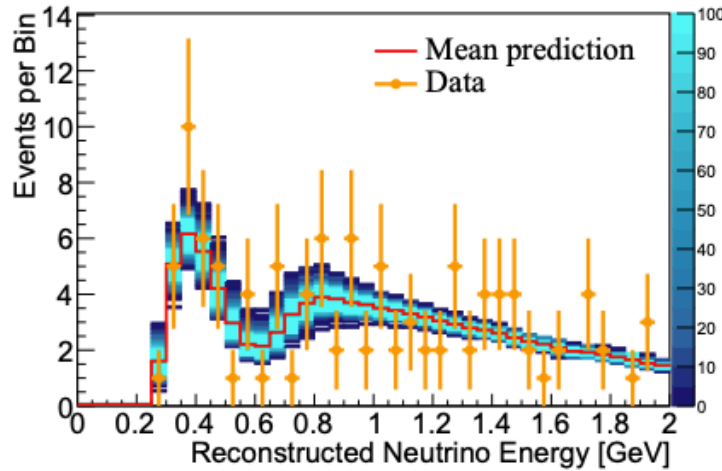
(a)  $\nu$ -mode 1R $\mu$



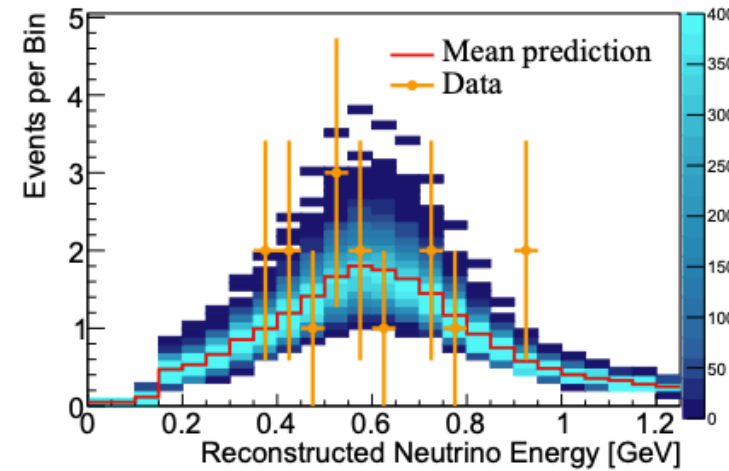
(b)  $\nu$ -mode 1Re



(c)  $\nu$ -mode 1Re1de



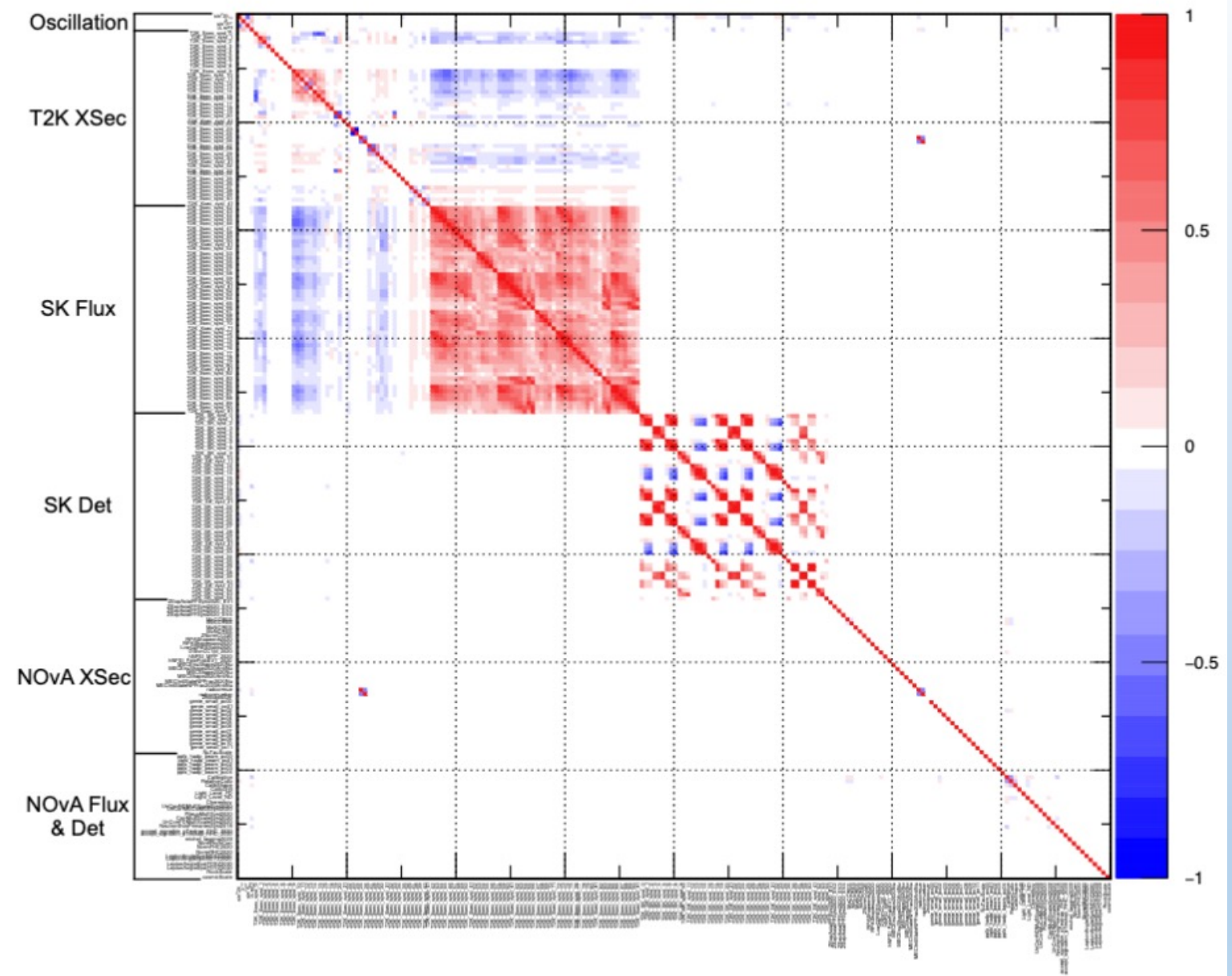
(d)  $\bar{\nu}$ -mode 1R $\mu$



(e)  $\bar{\nu}$ -mode 1Re



# Post-fit Correlation Matrix



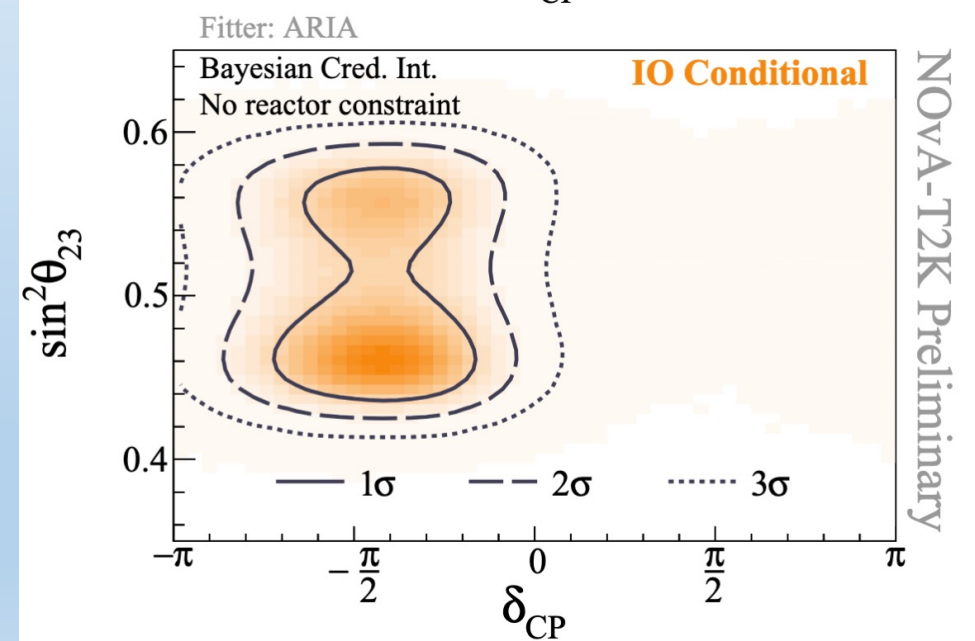
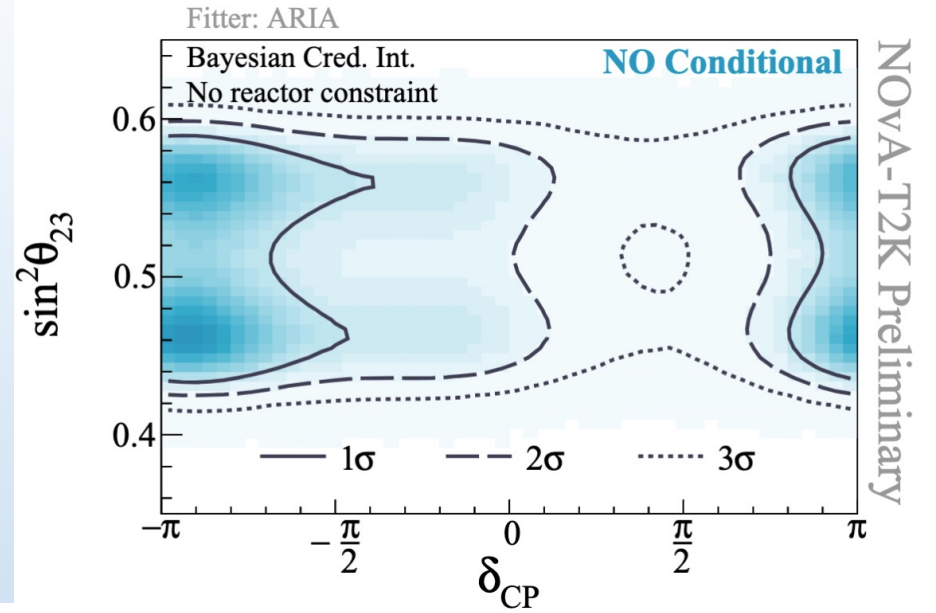
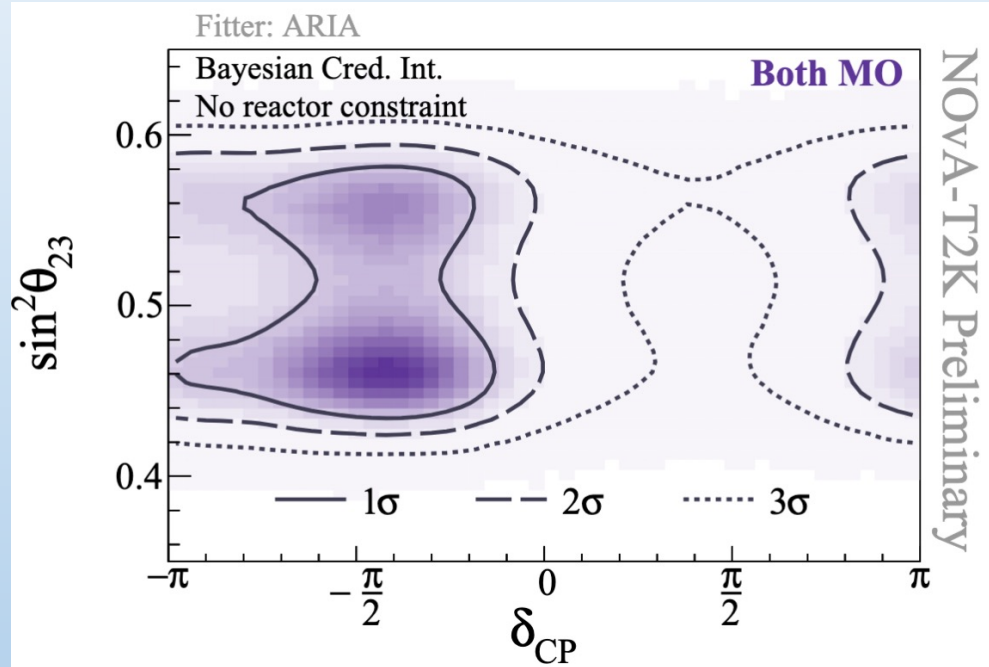
# Priors

Parameter	ARIA sampling prior	MaCh3 sampling Prior	Priors used for the analysis
$\theta_{23}$	Uniform in $\theta_{23}$	Uniform in $\sin^2\theta_{23}$	Uniform in $\sin^2\theta_{23}$
$\theta_{13}$	Uniform in $\theta_{13}$	Uniform in $\sin^2 2\theta_{13}$	Uniform in $\sin^2 2\theta_{13}$ & Gaussian reactor constraint
$ \Delta m^2_{32} $	Uniform in $ \Delta m^2_{32} $	Uniform in $ \Delta m^2_{32} $	Uniform in $ \Delta m^2_{32} $
MO	Uniform in MO with a 50% switch probability	Uniform in MO with a 50% switch probability	Uniform in MO with a 50% switch probability
$\delta_{CP}$	Uniform in $\delta_{CP}$	Uniform in $\delta_{CP}$	Uniform in $\delta_{CP}$ & Uniform in $\sin \delta_{CP}$ (for J)

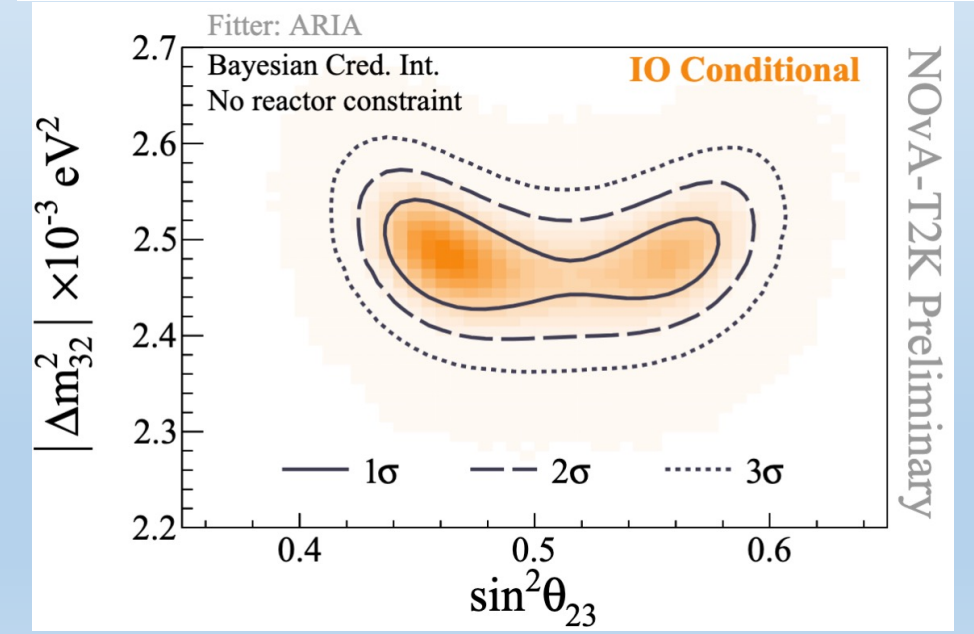
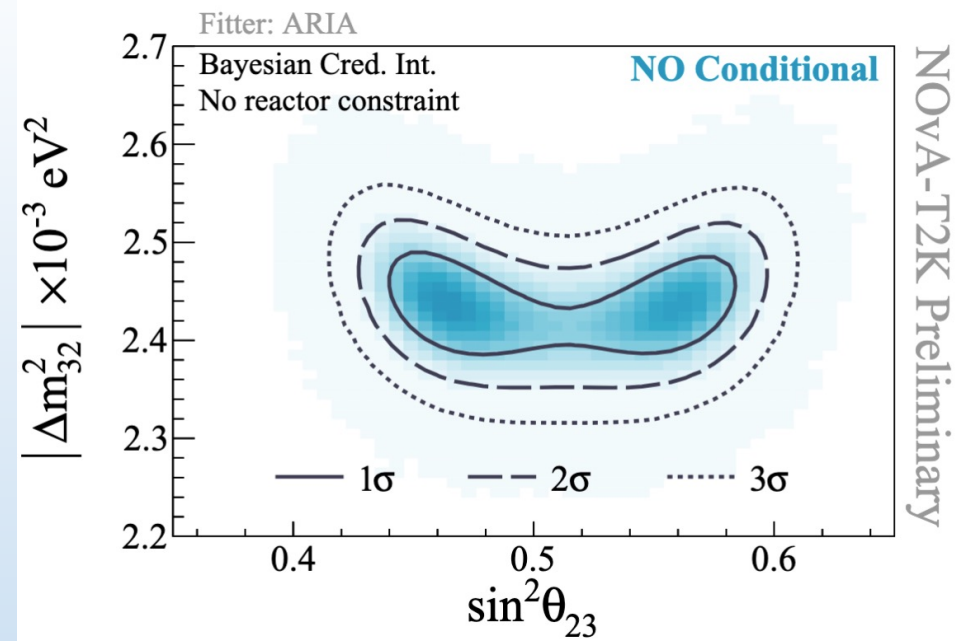
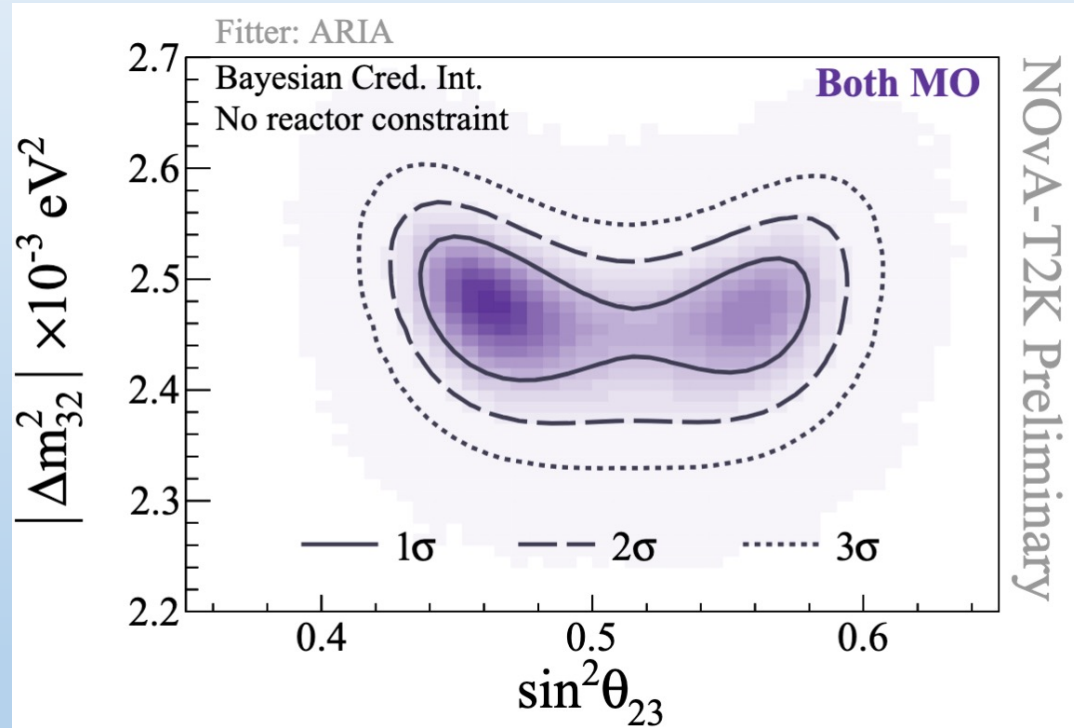
# Without reactor constraint

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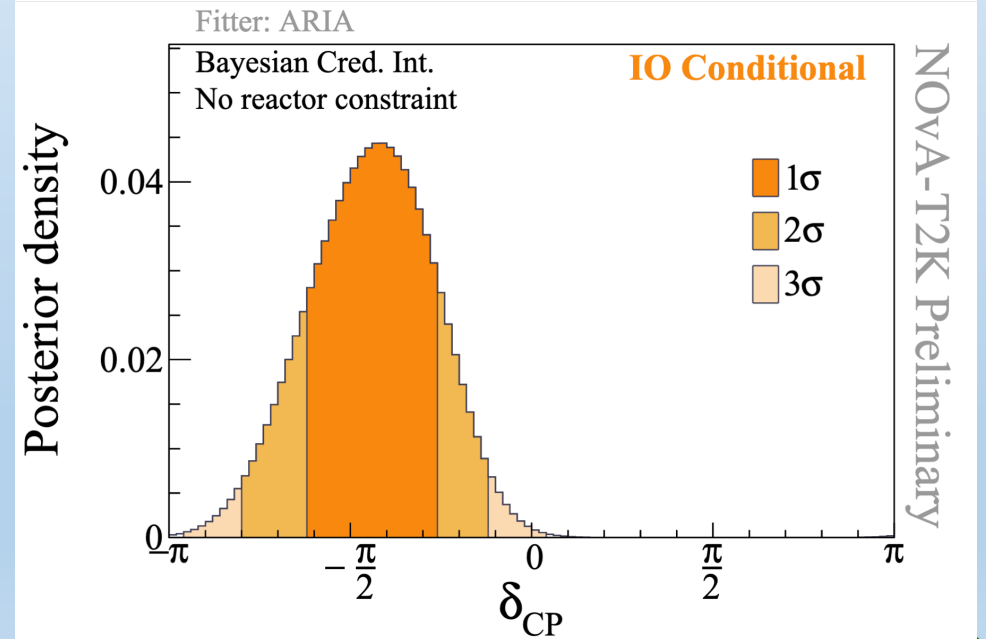
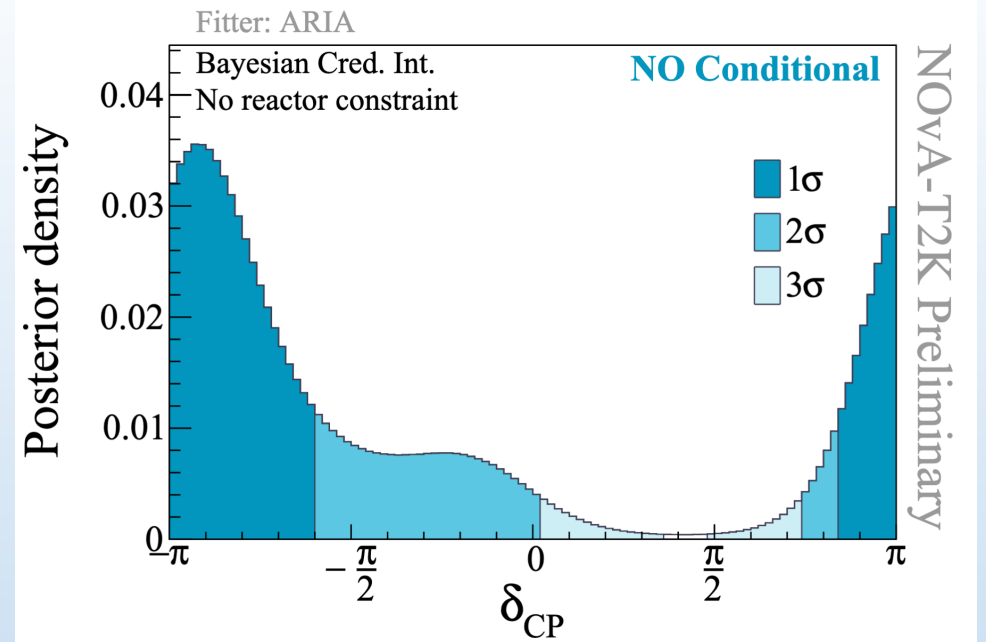
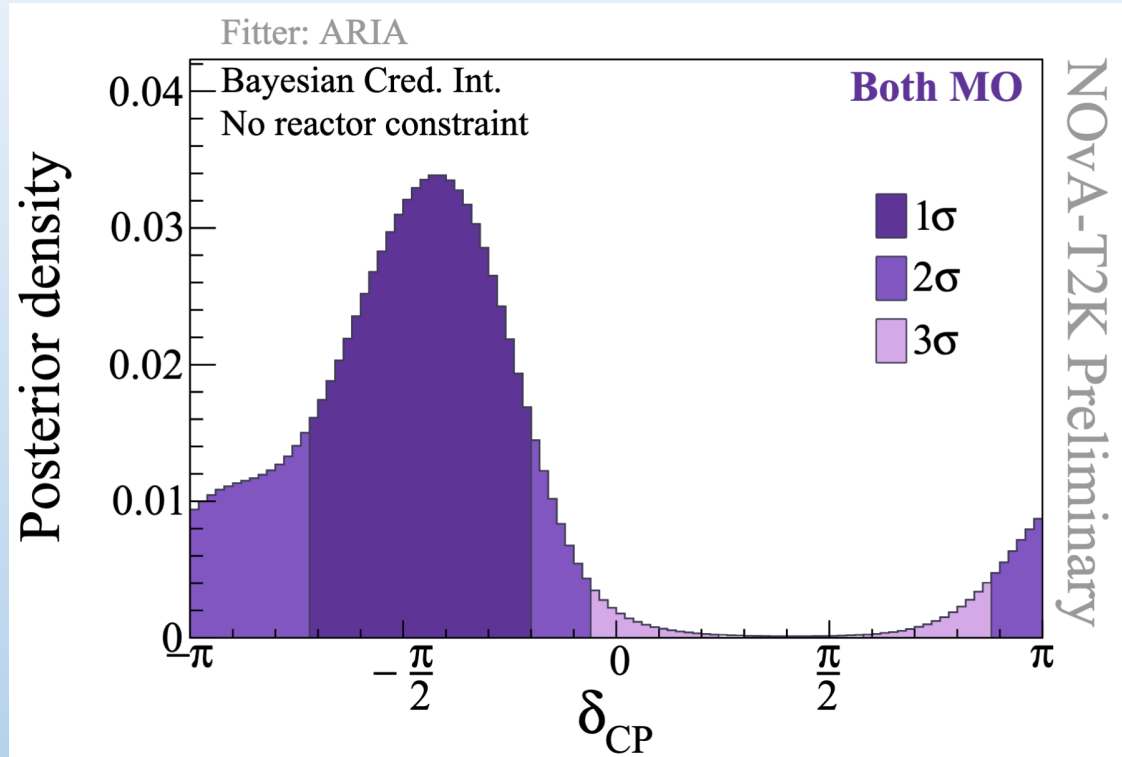
# No reactor constraint: CP Phase - $\delta_{CP}$



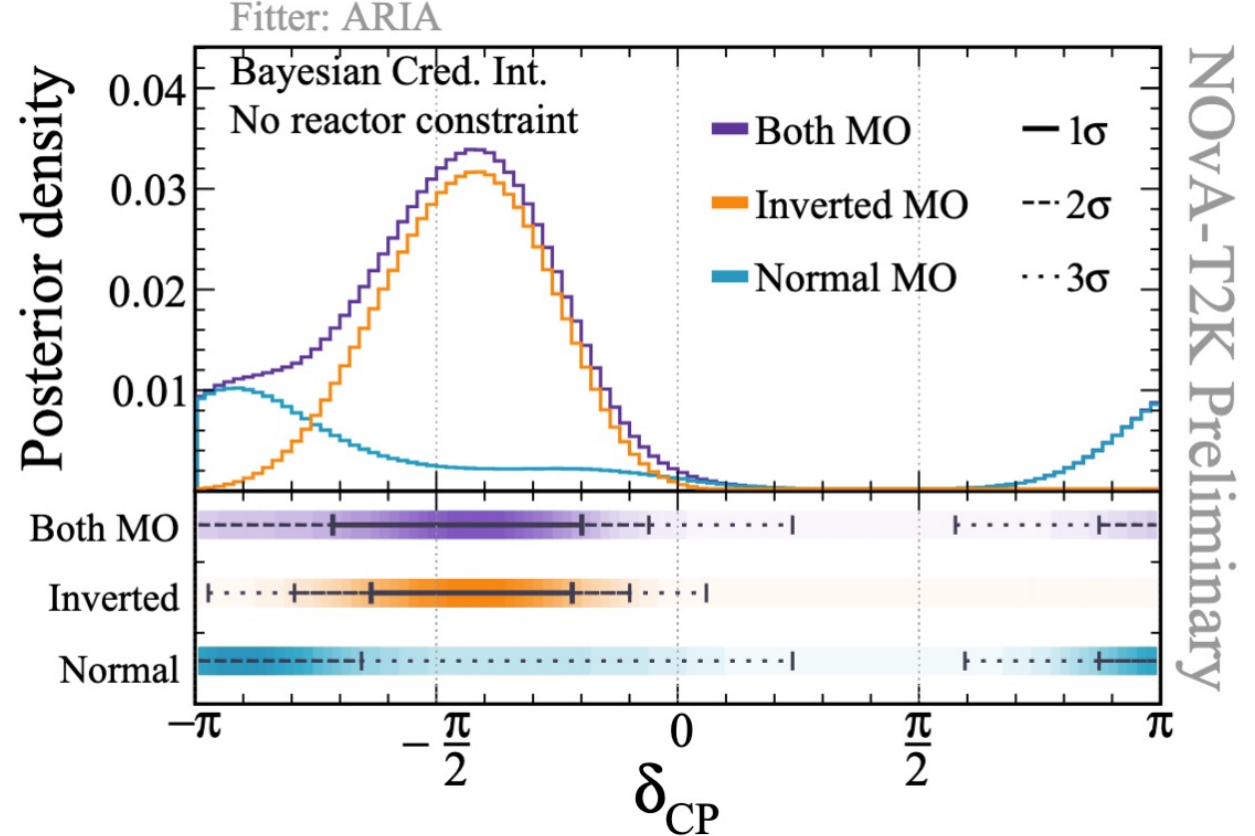
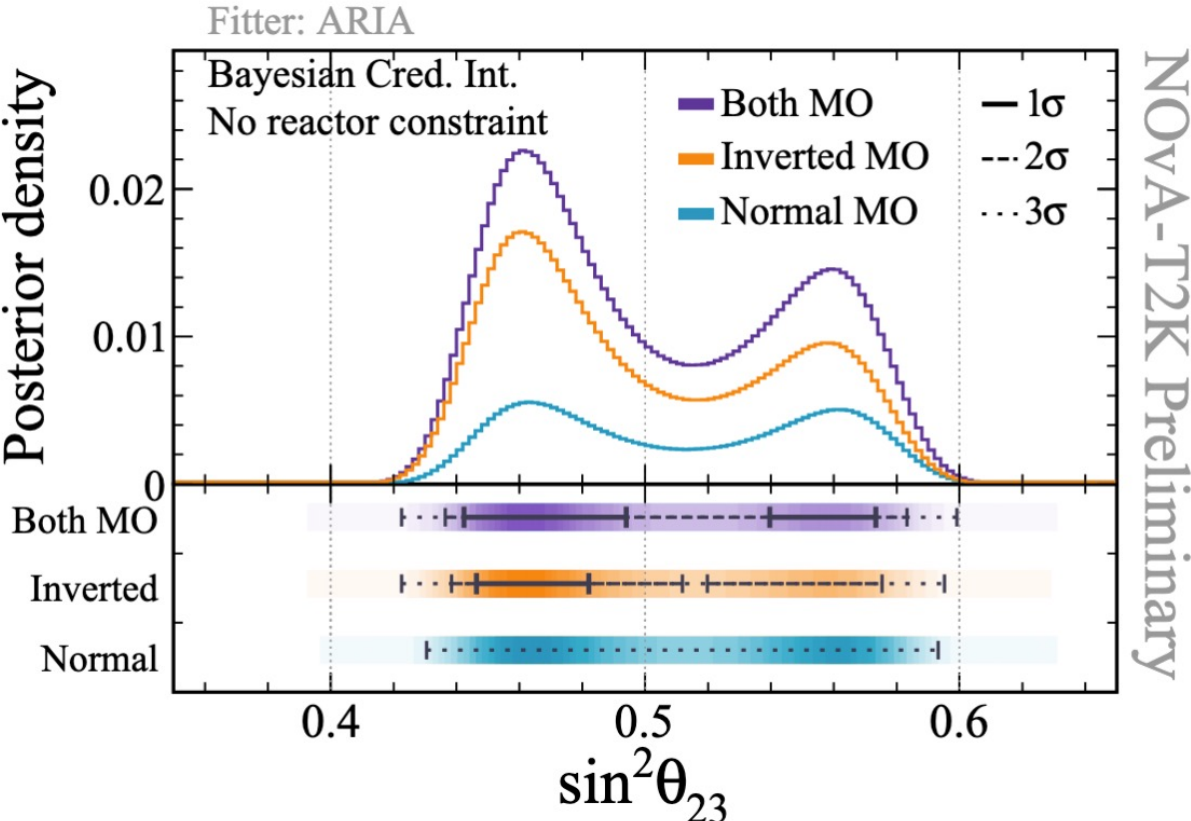
# No reactor constraint $\Delta m_{32}^2$ and $\sin^2 \theta_{23}$



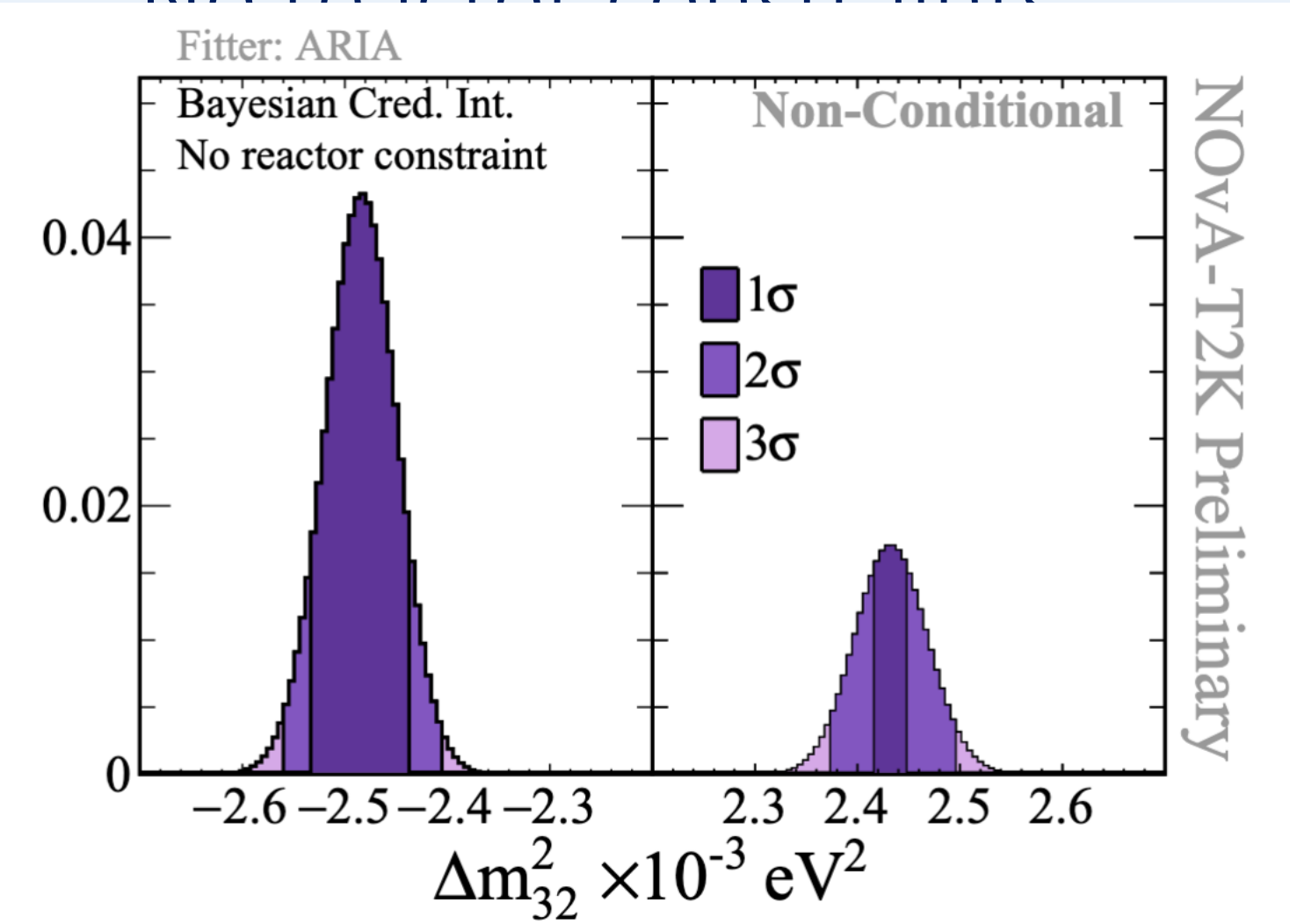
# No reactor: CP Phase - $\delta_{CP}$



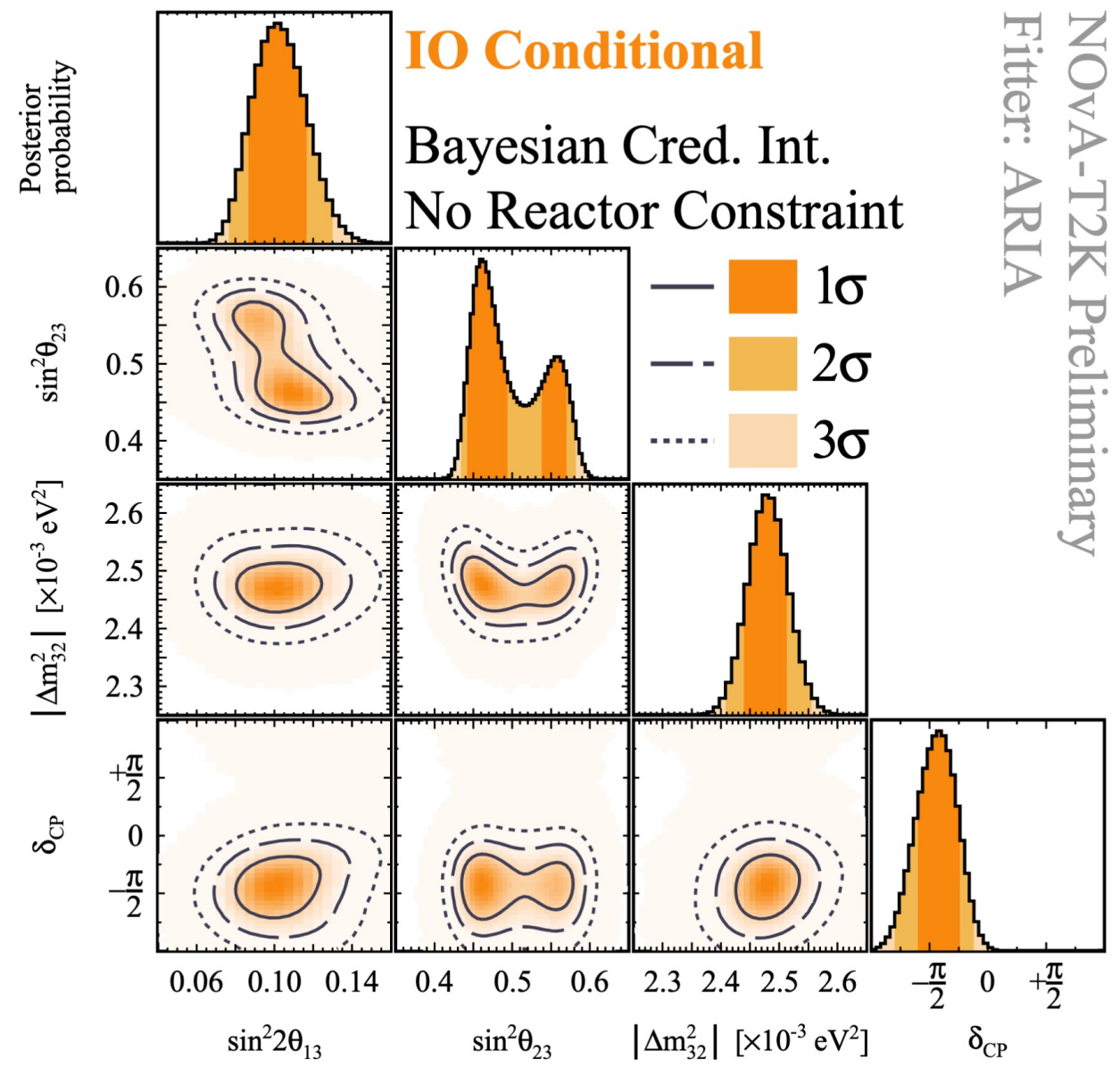
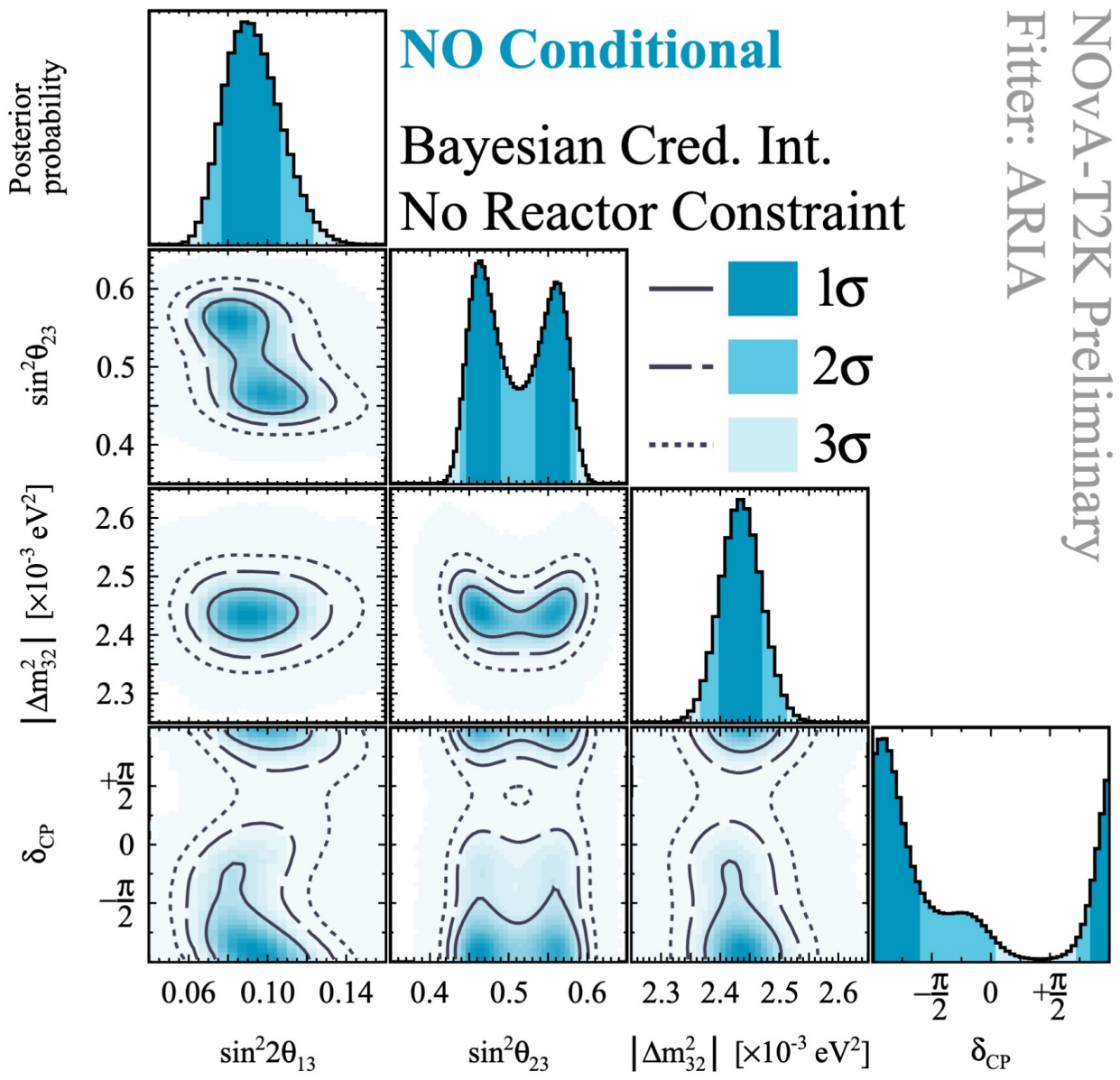
# No reactor constraint



# No reactor constraints

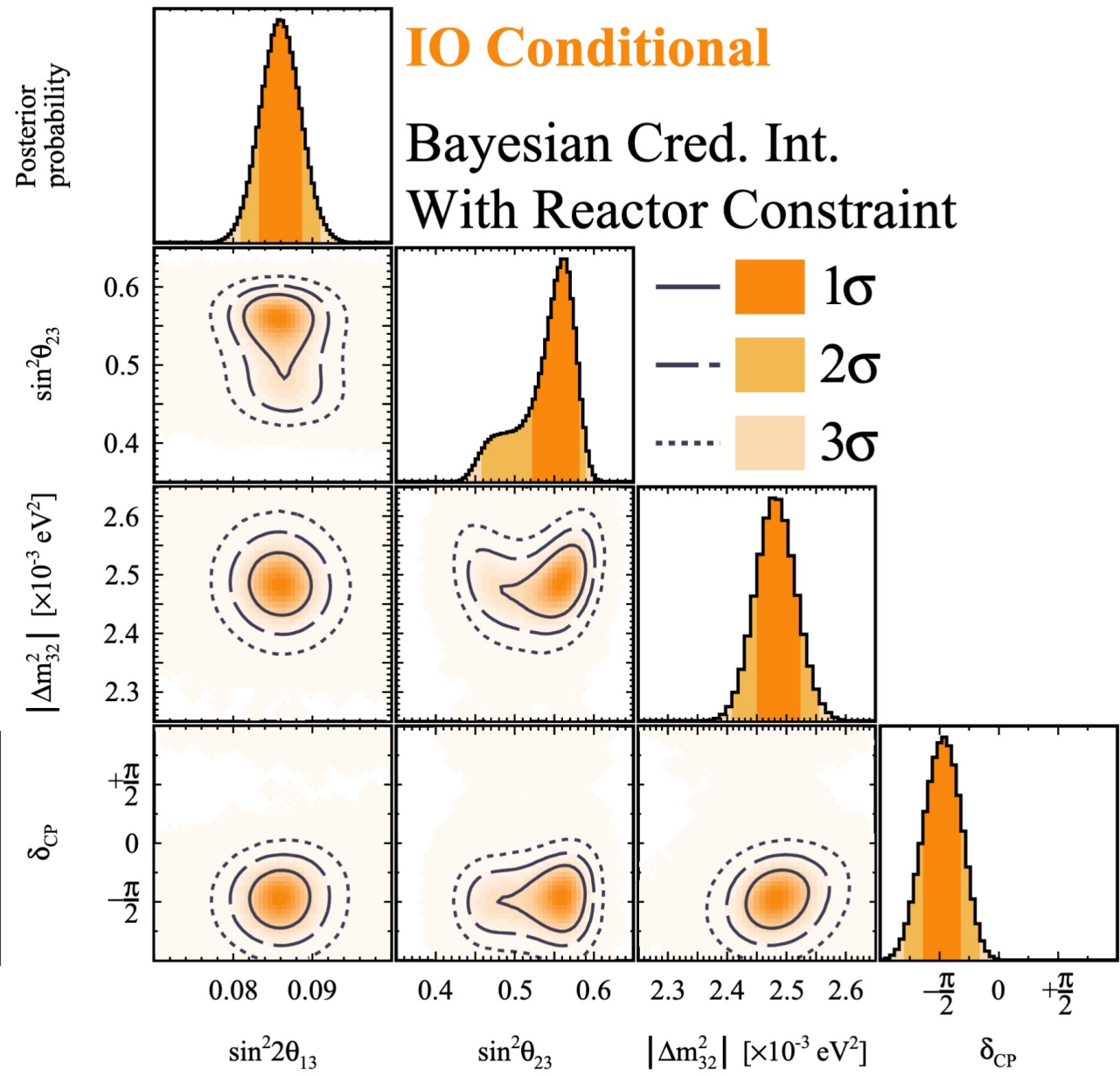
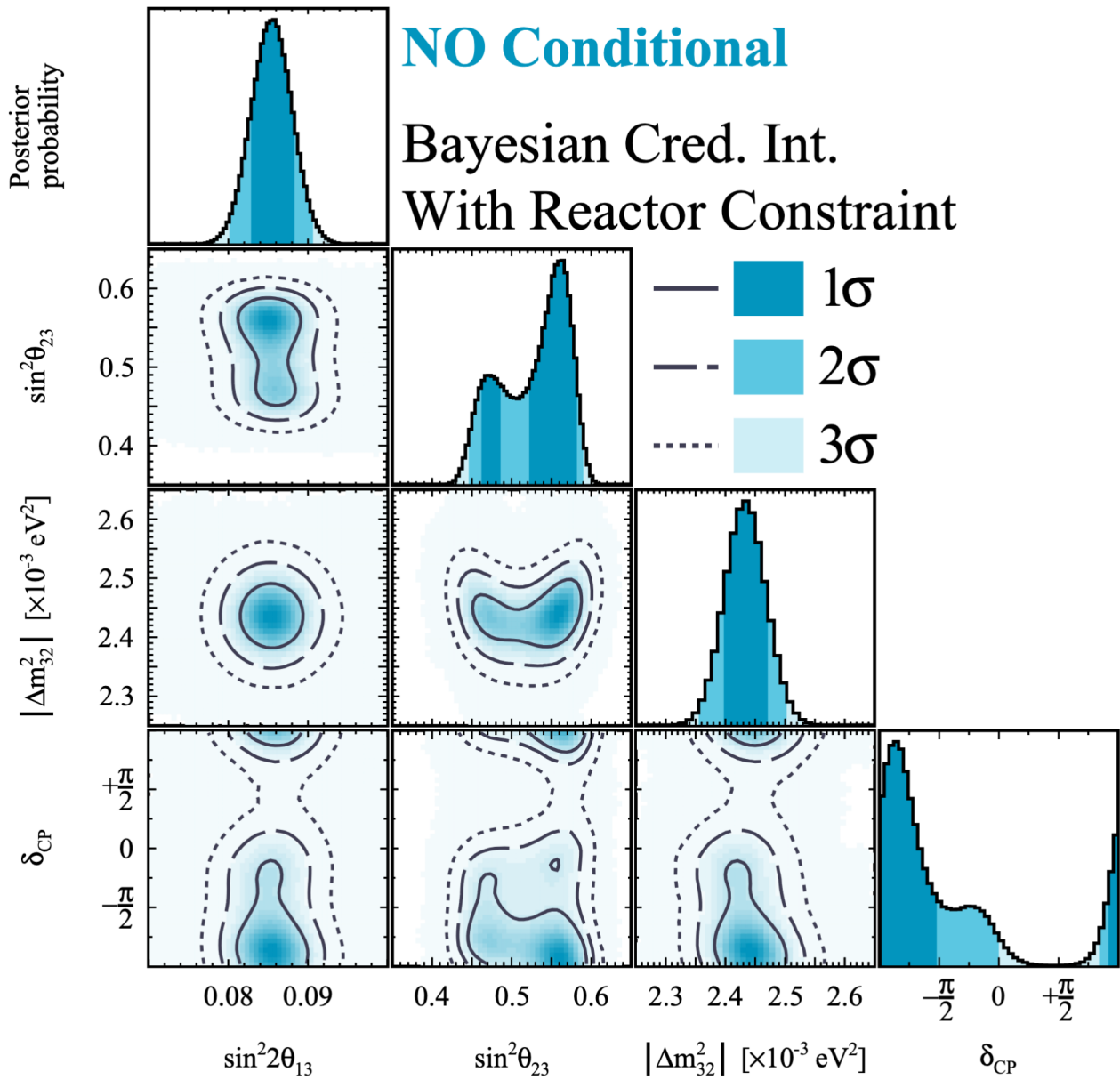






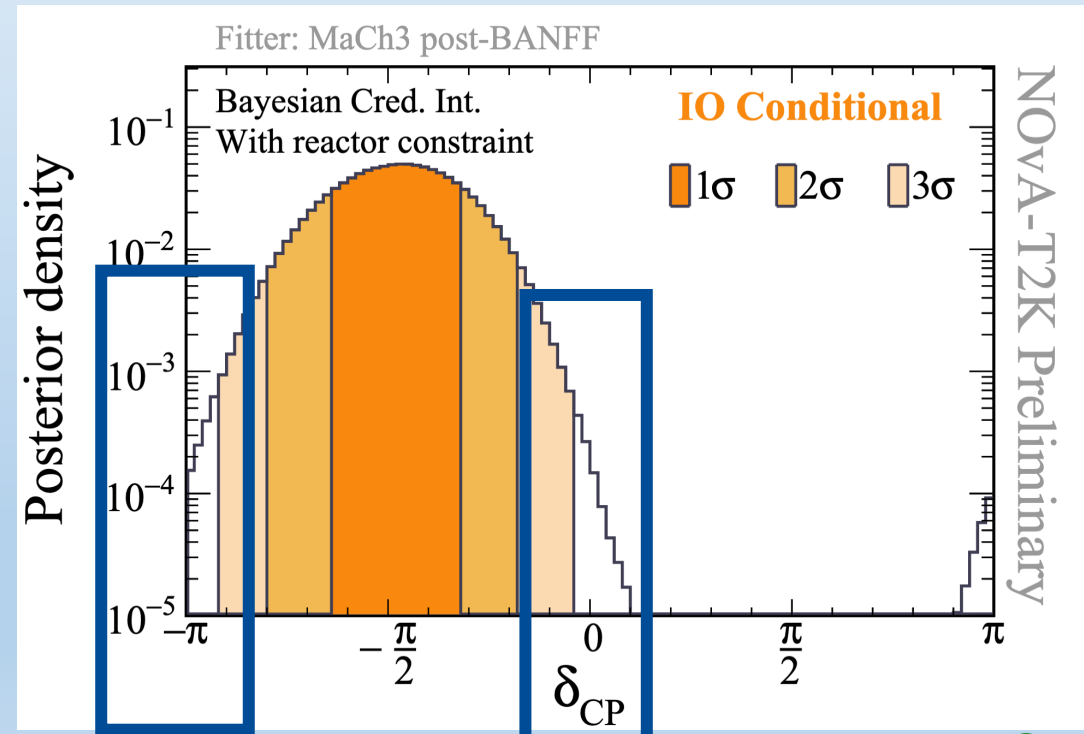
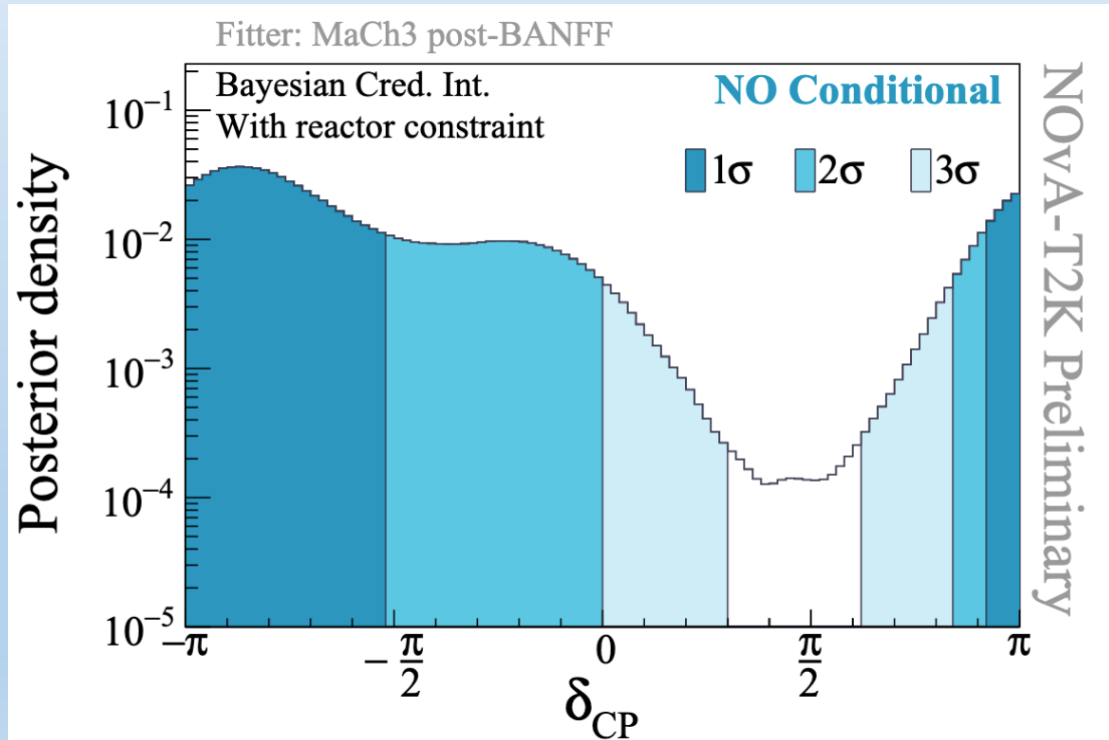
# With reactor constraint

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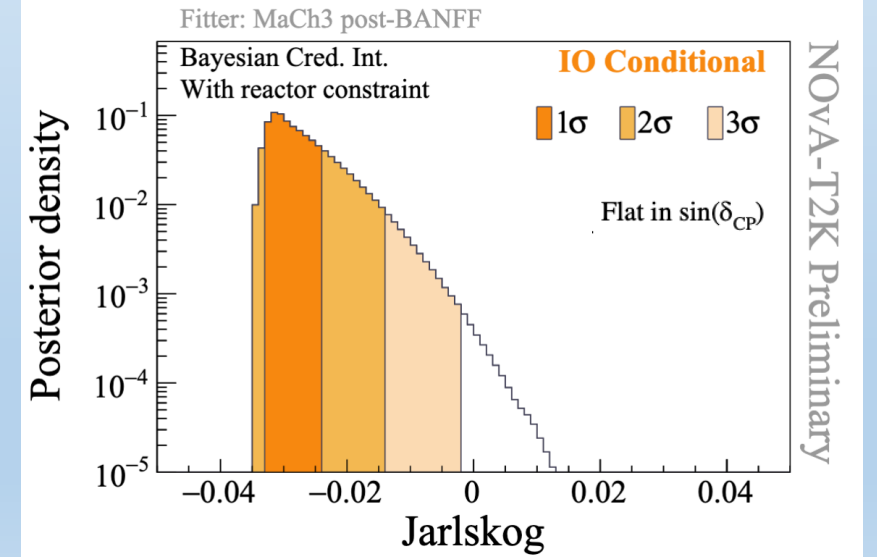
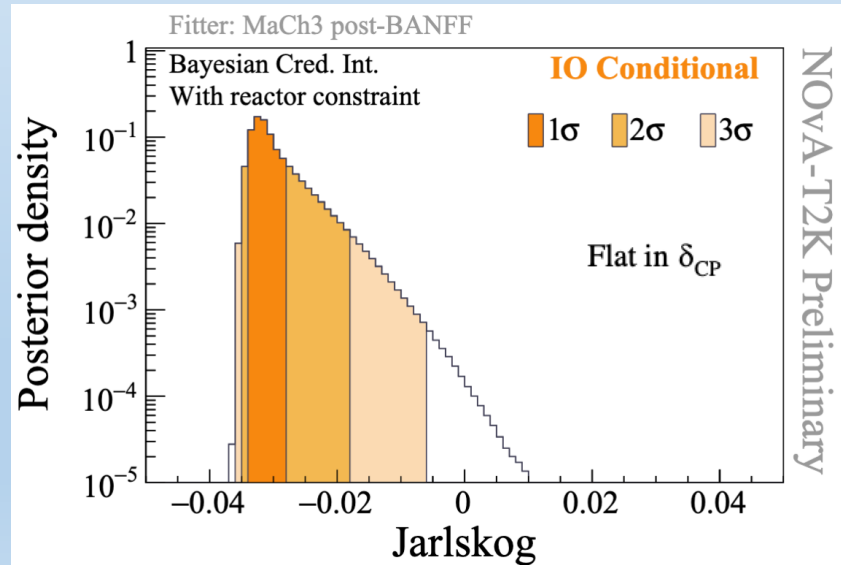
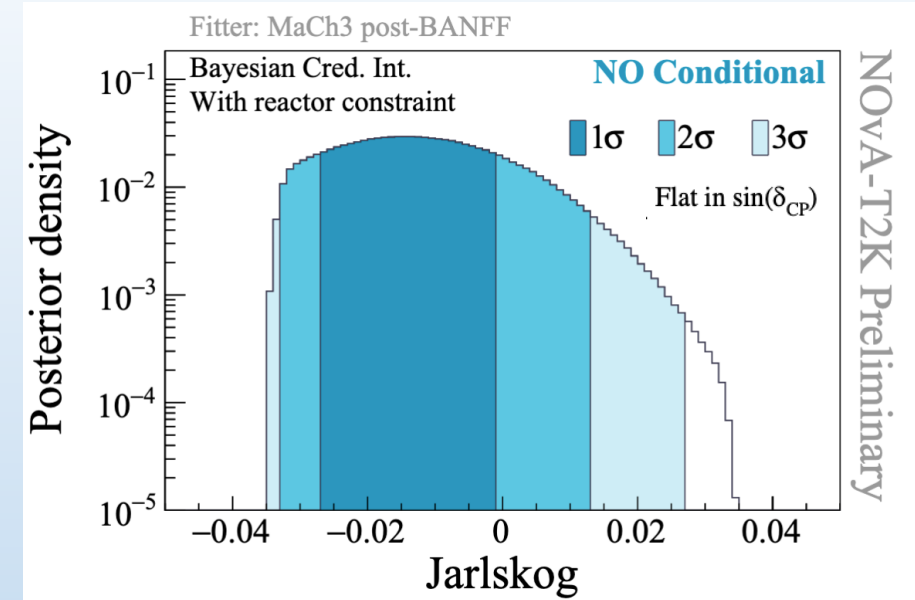
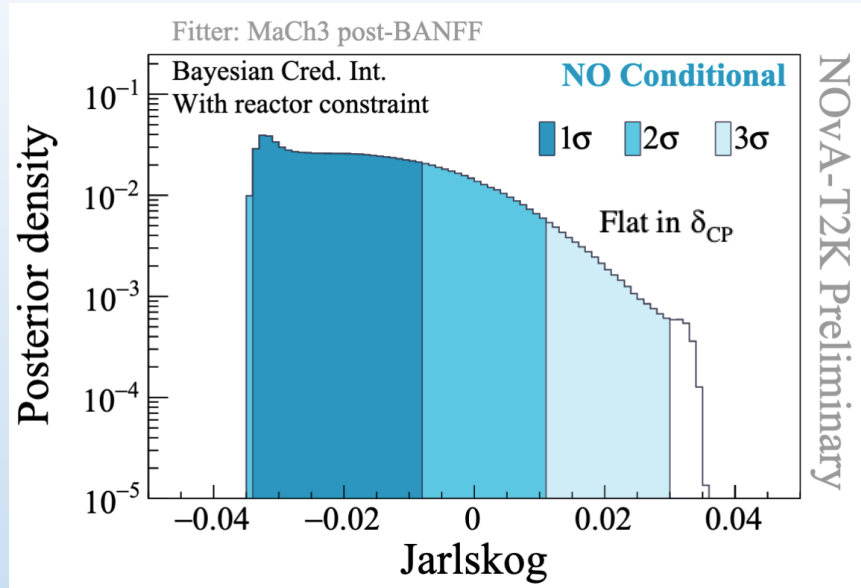


# CP Violation

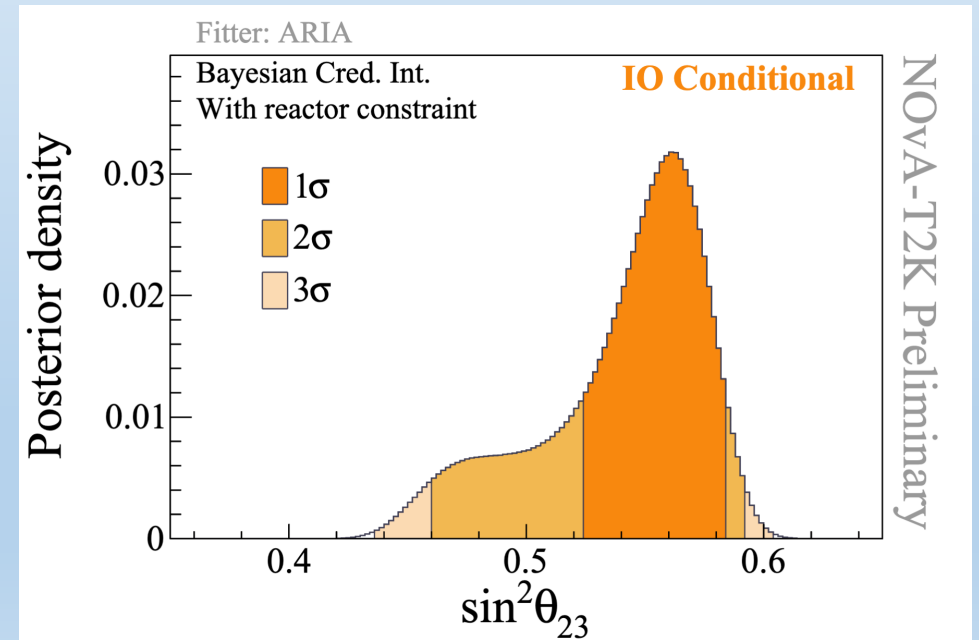
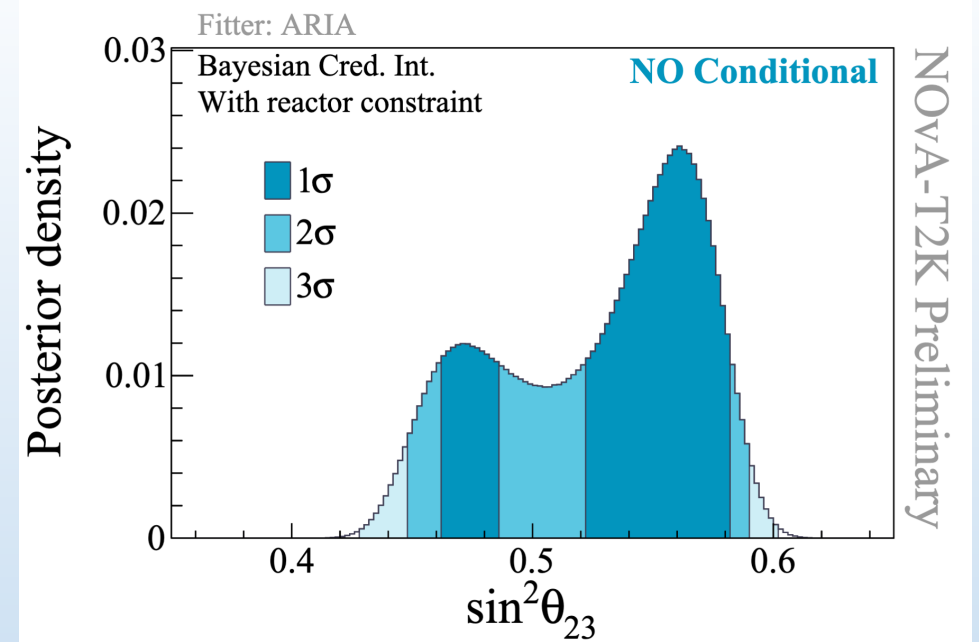
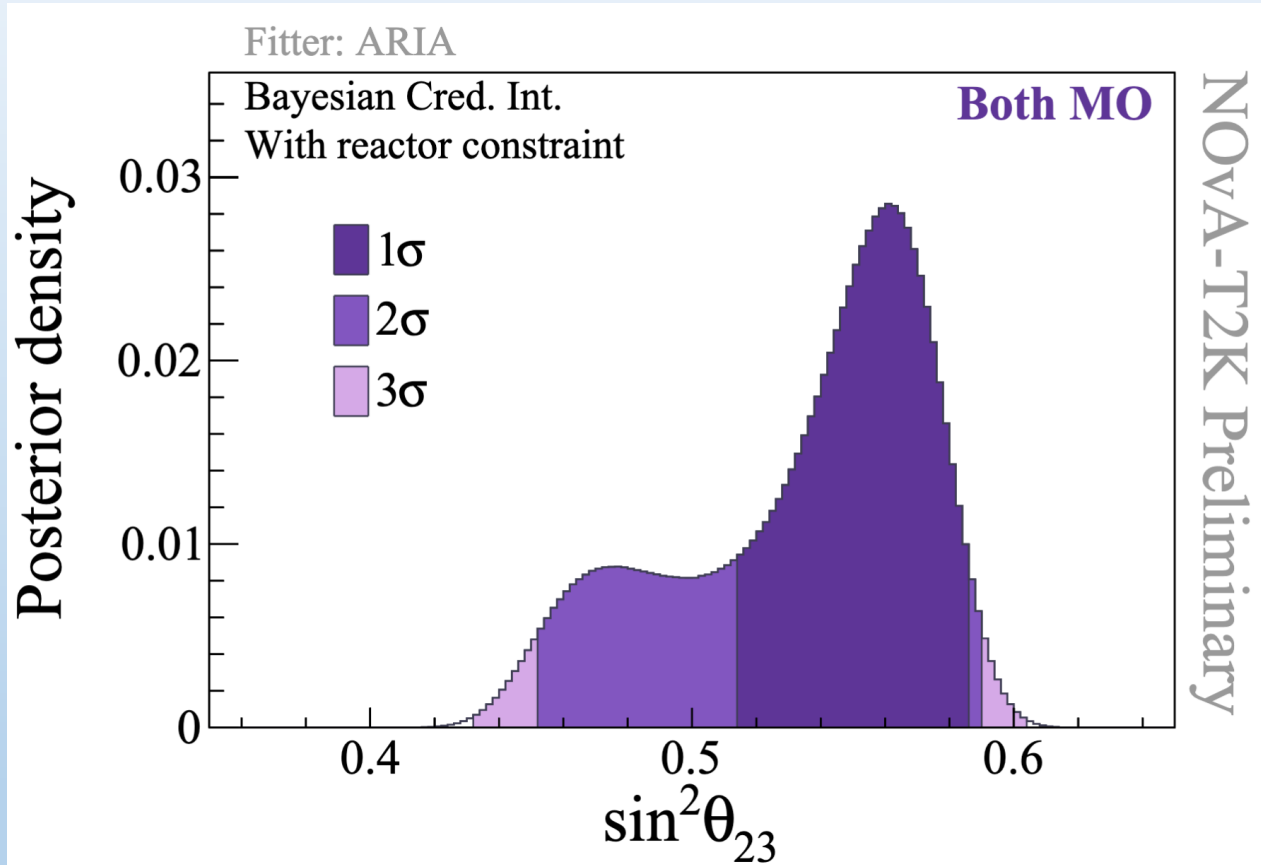
- For both mass orderings,  $\delta_{CP} = \pi/2$  lies outside 3-sigma credible interval.
- Normal Ordering allows for a broad range of permissible  $\delta_{CP}$
- For the **Inverted Ordering**, CP conserving values of  $\delta_{CP}$  ( $0, \pi$ ) lie outside the 3-sigma credible interval.



# Jarlskog



$$\Delta \sin^2 \theta_{23}$$

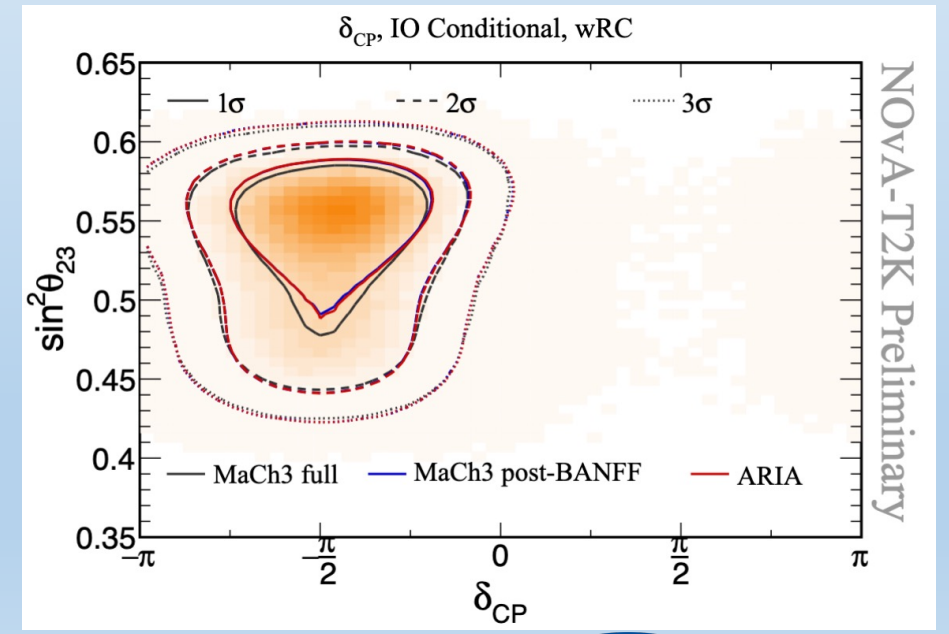
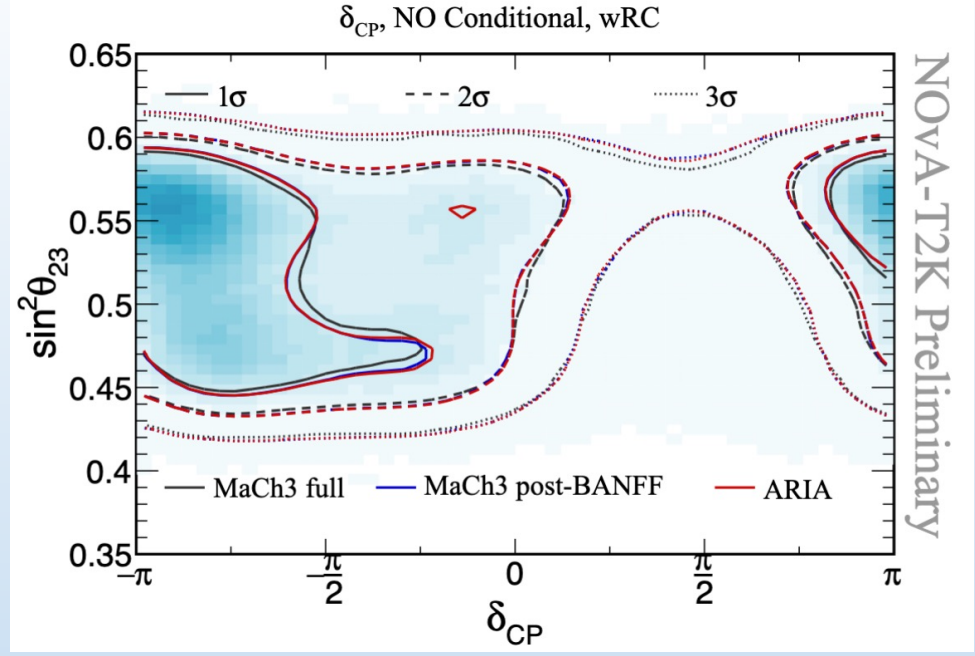
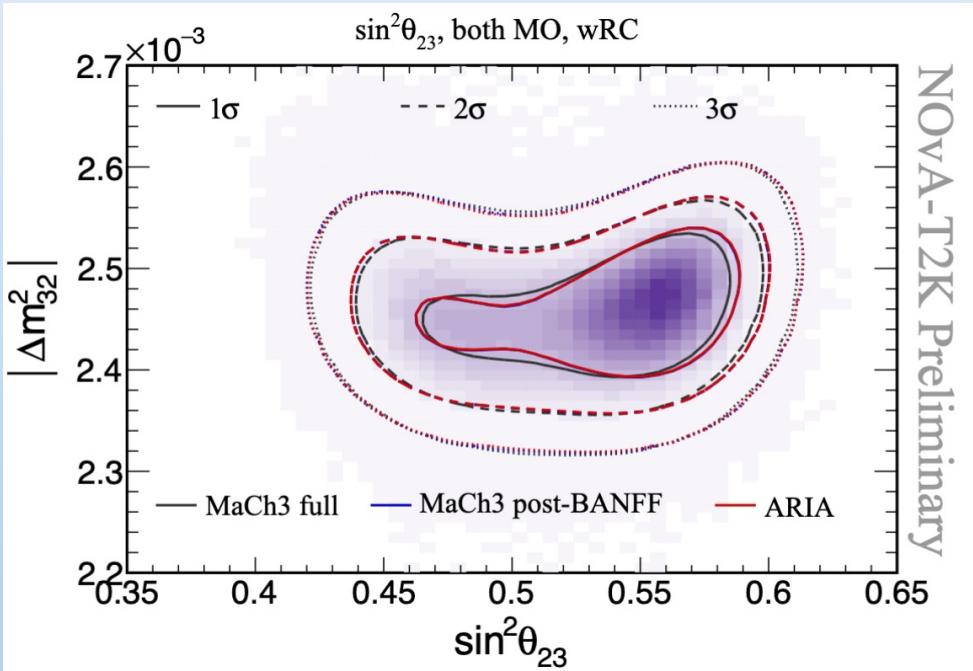


# Comparisons

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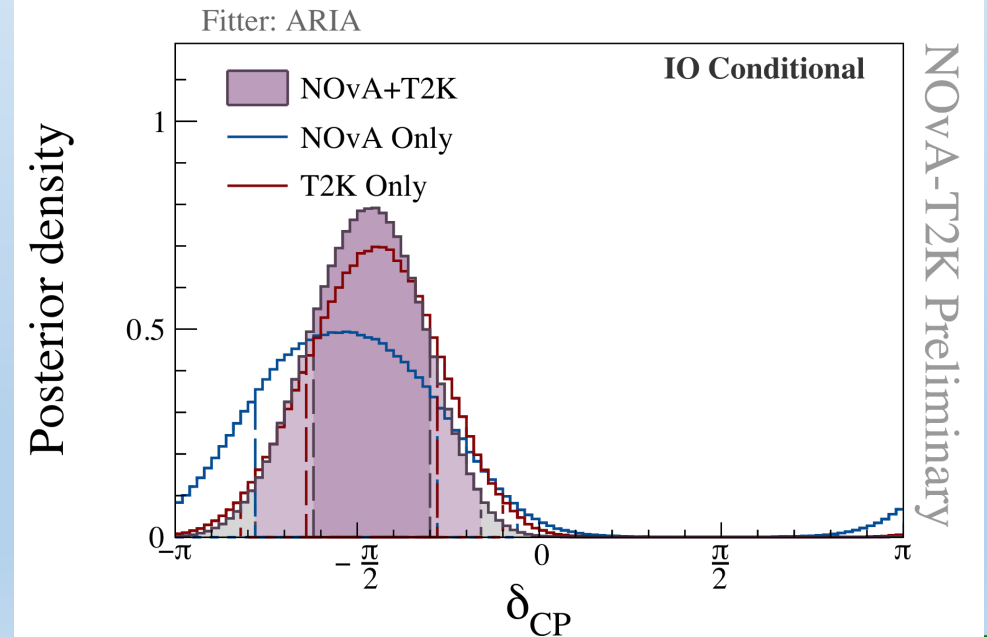
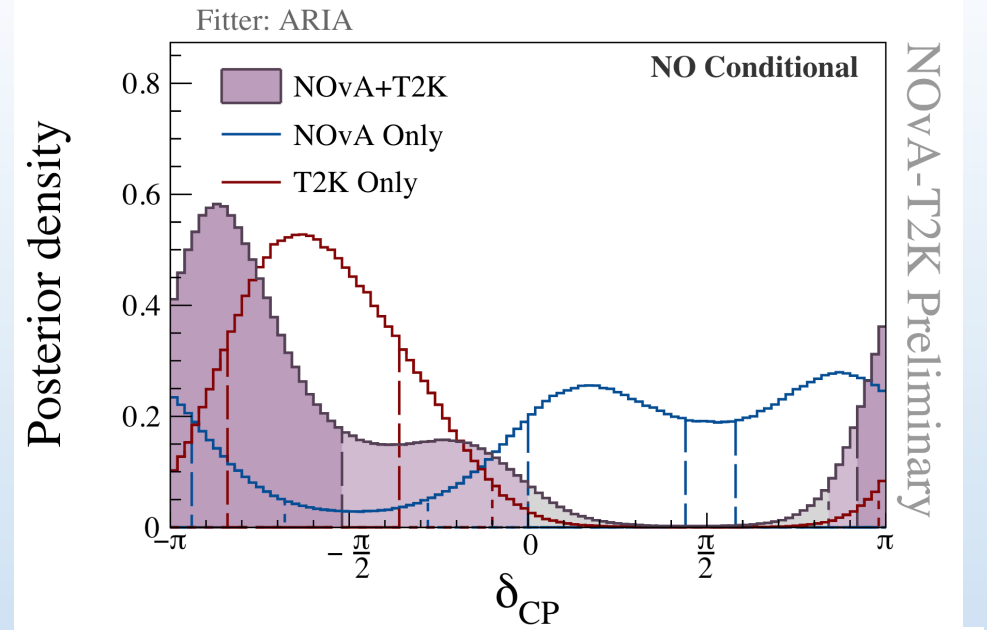
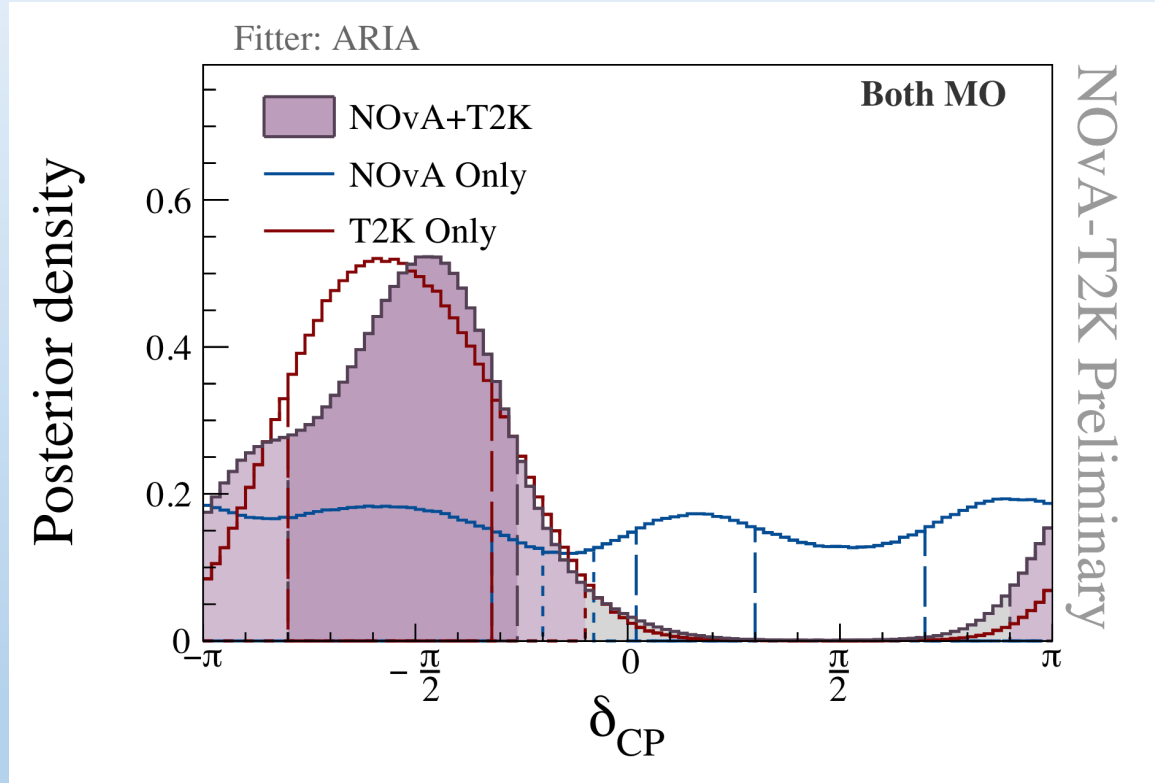
# Fitter comparisons

- All 3 sets of data-fits are consistent with each other.

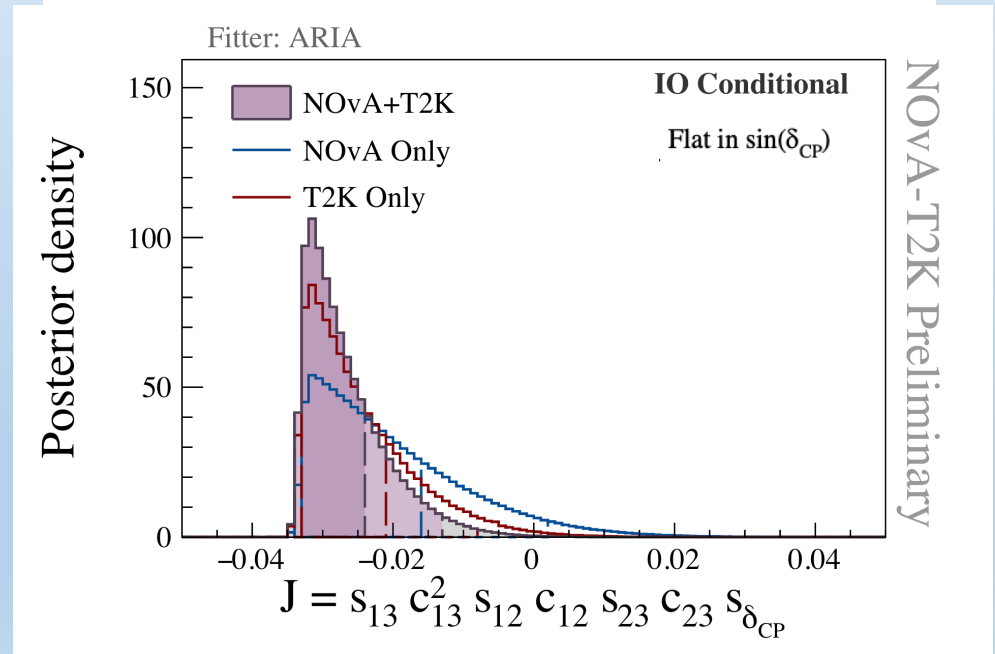
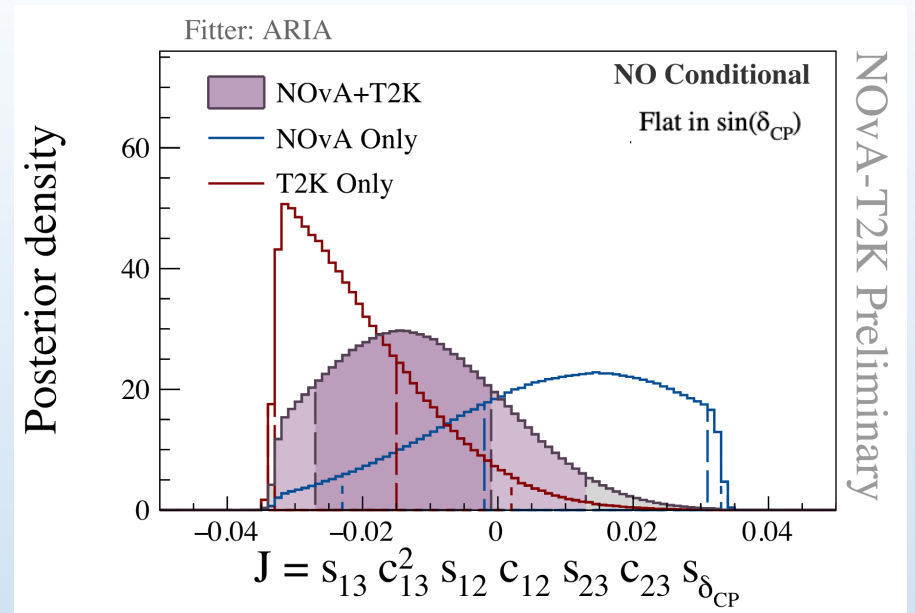
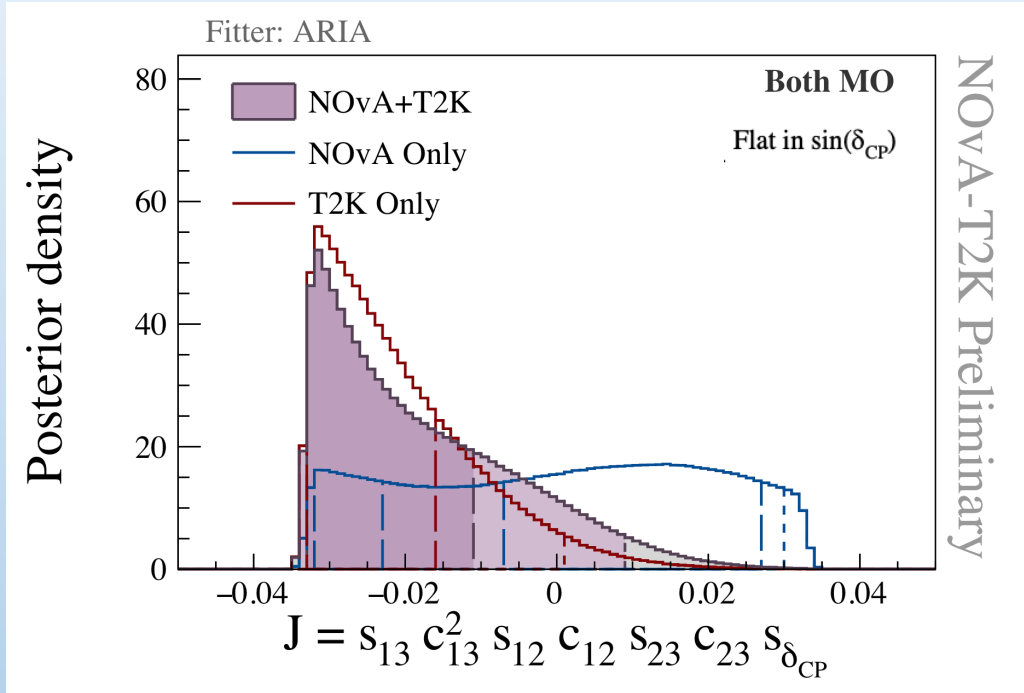




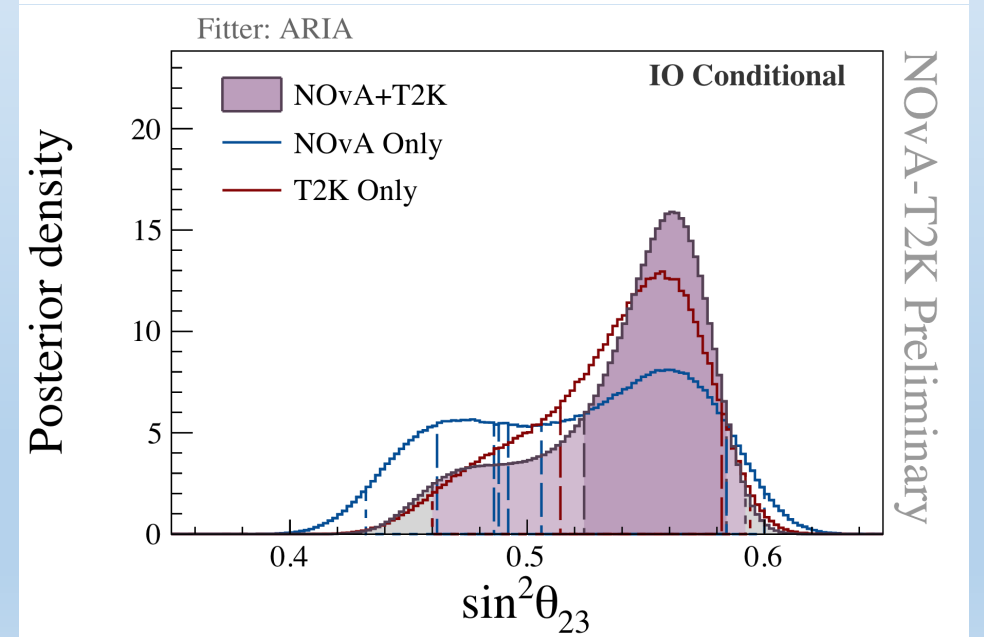
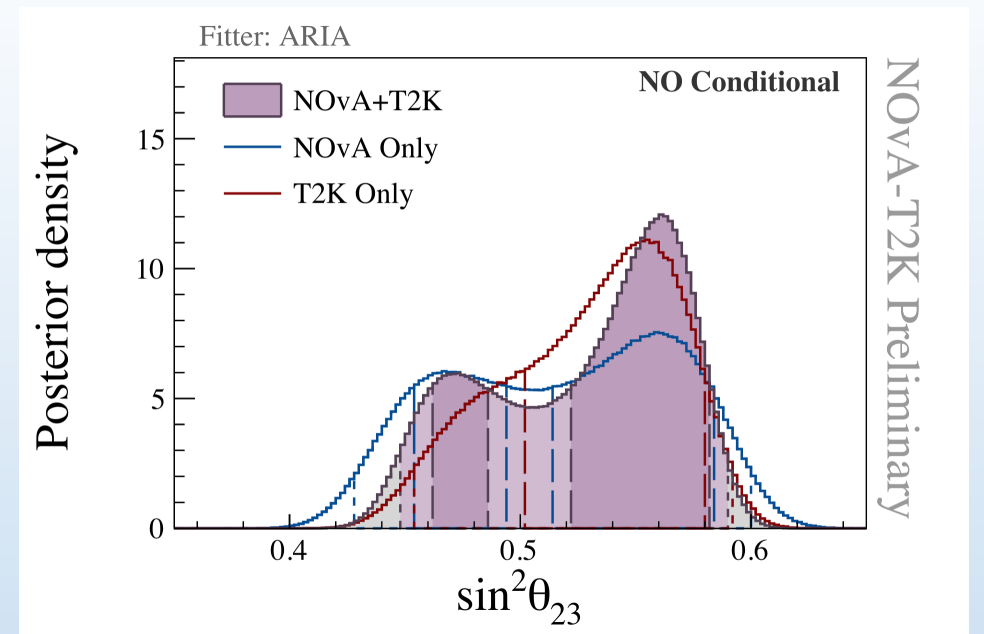
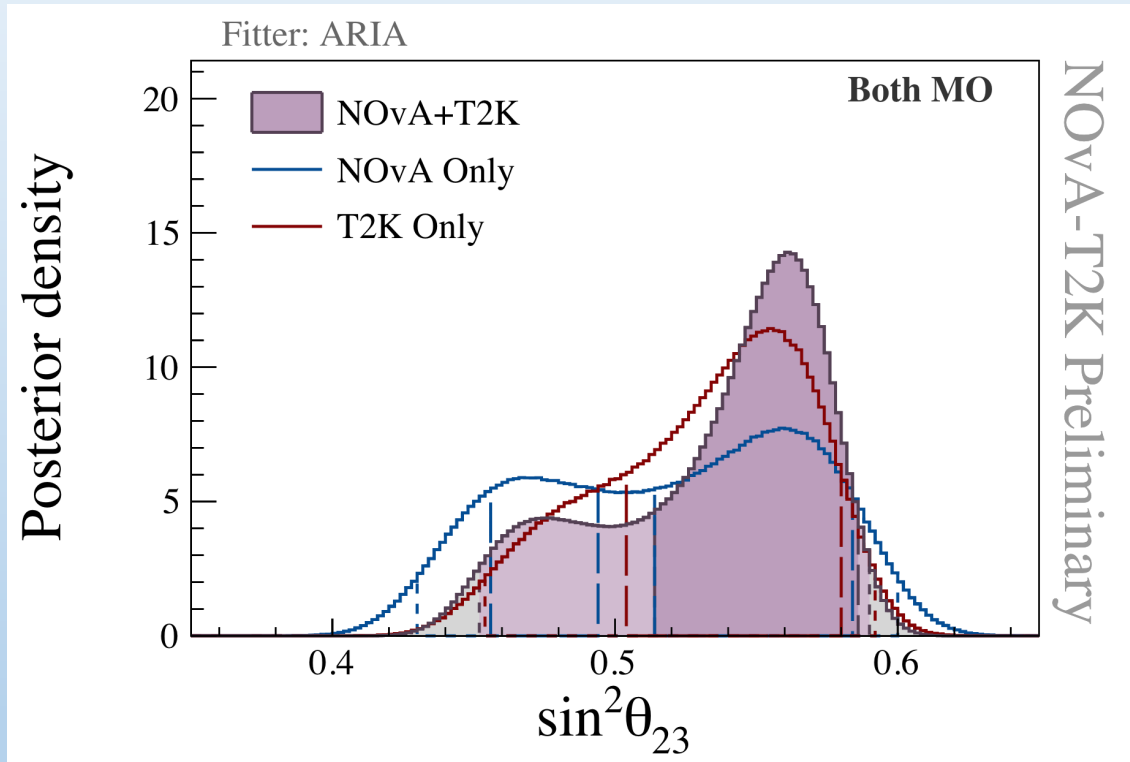
# Comparison with NOvA-only & T2K-only fits



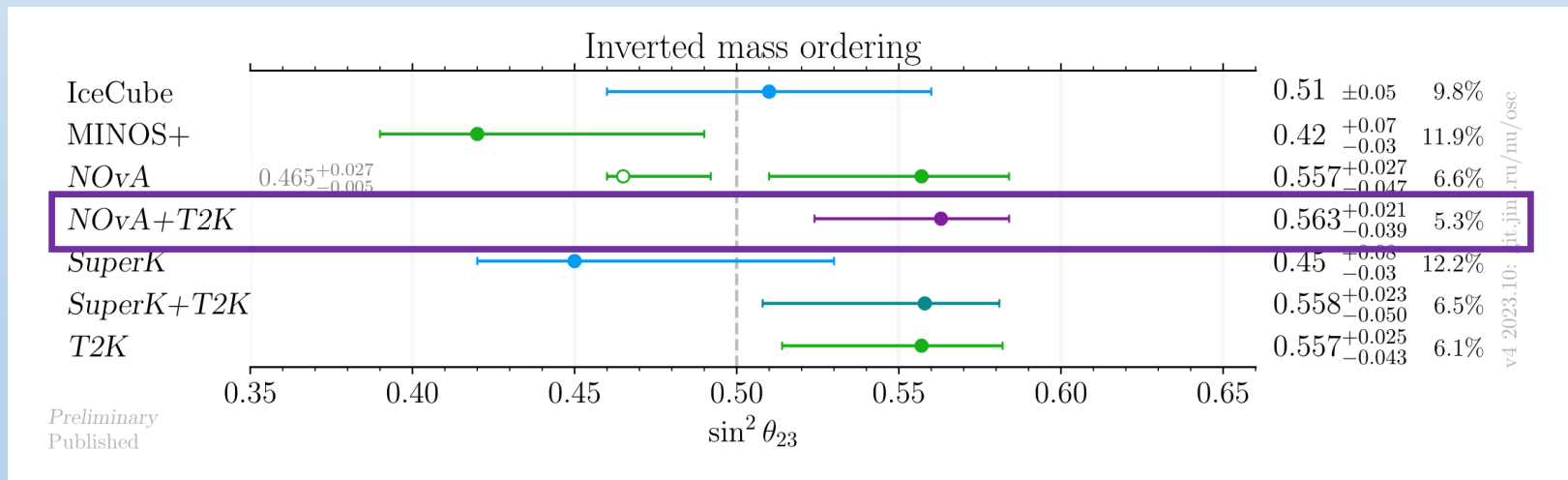
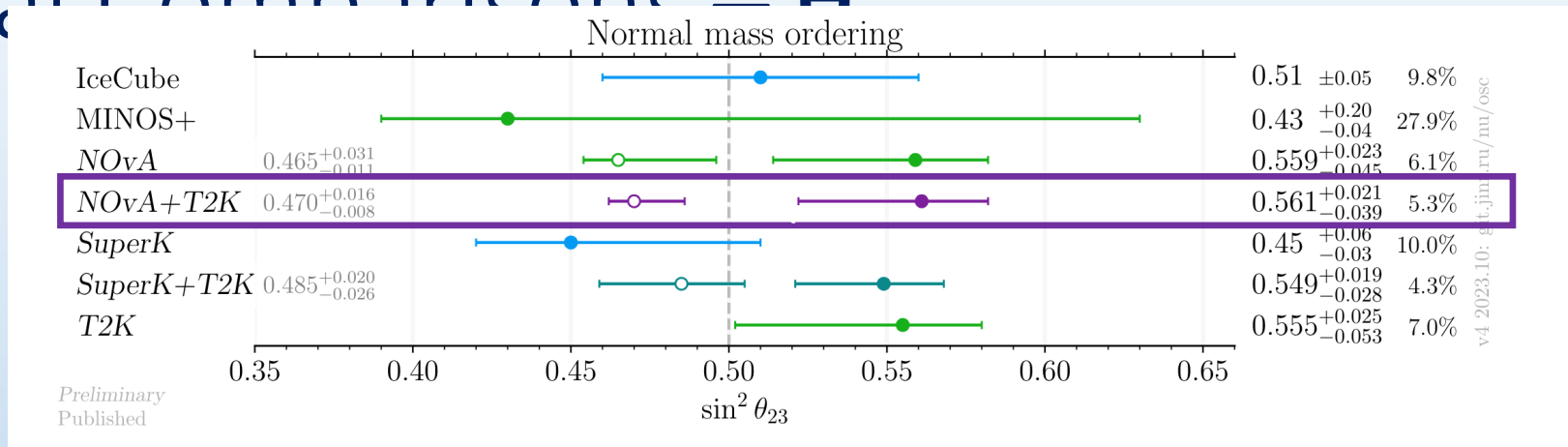
# Comparison with NOvA-only & T2K-only fits



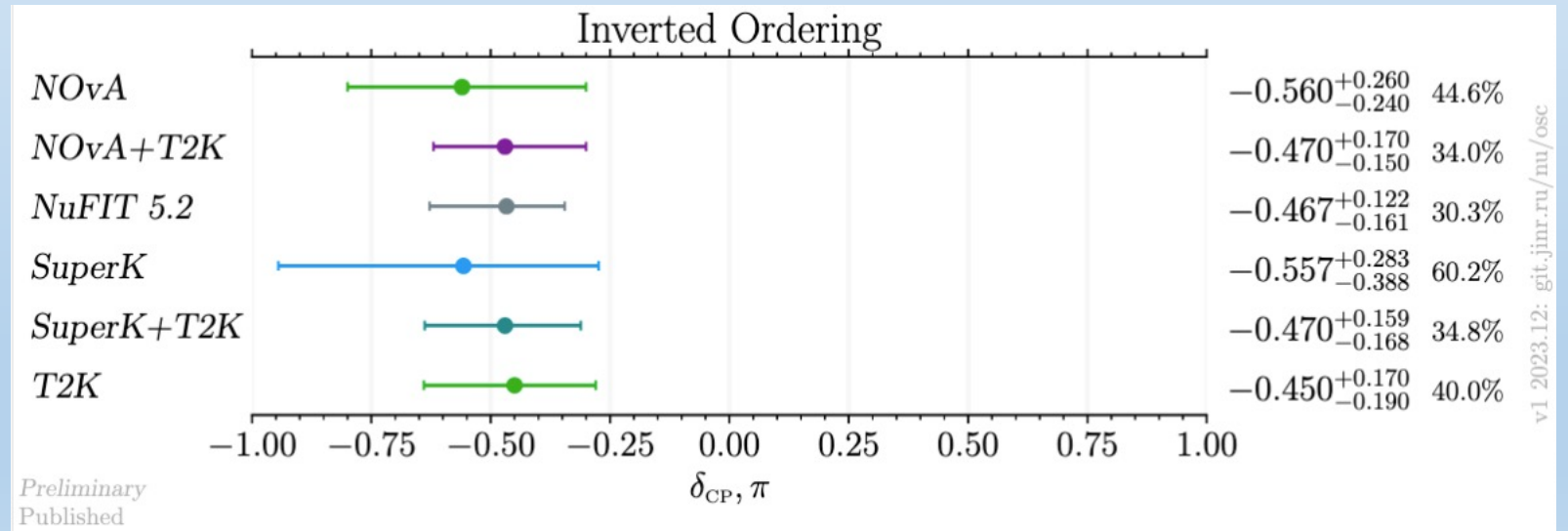
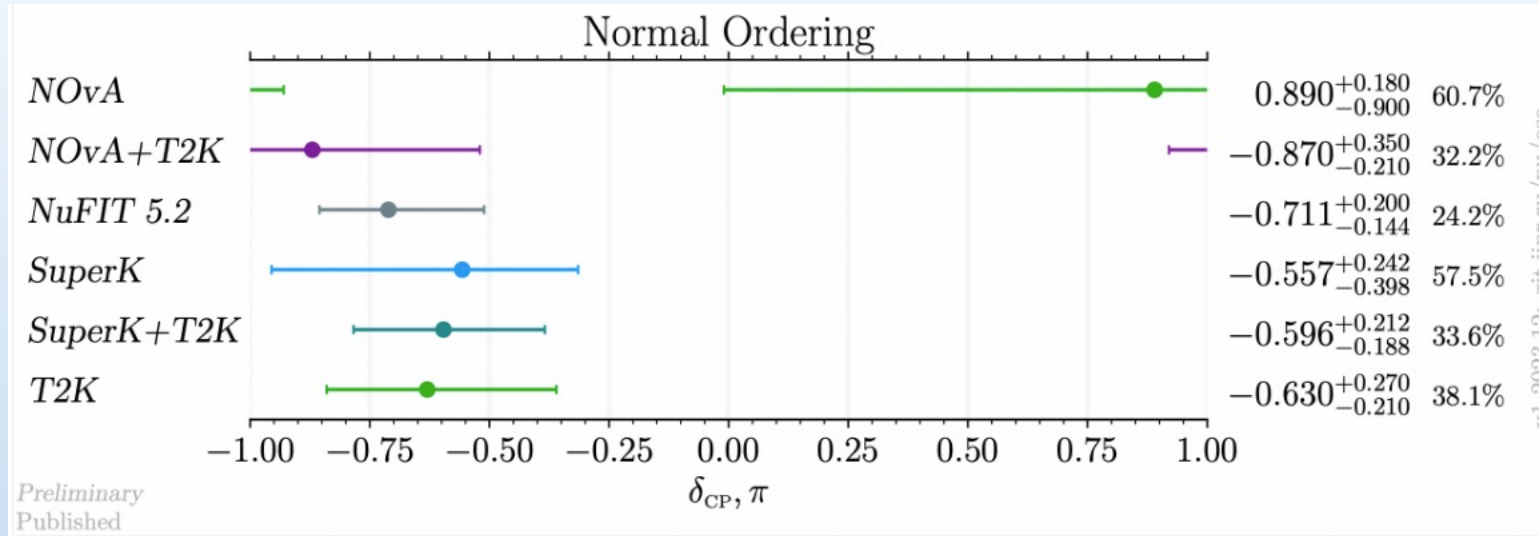
# Comparison with NOvA-only & T2K-only fits



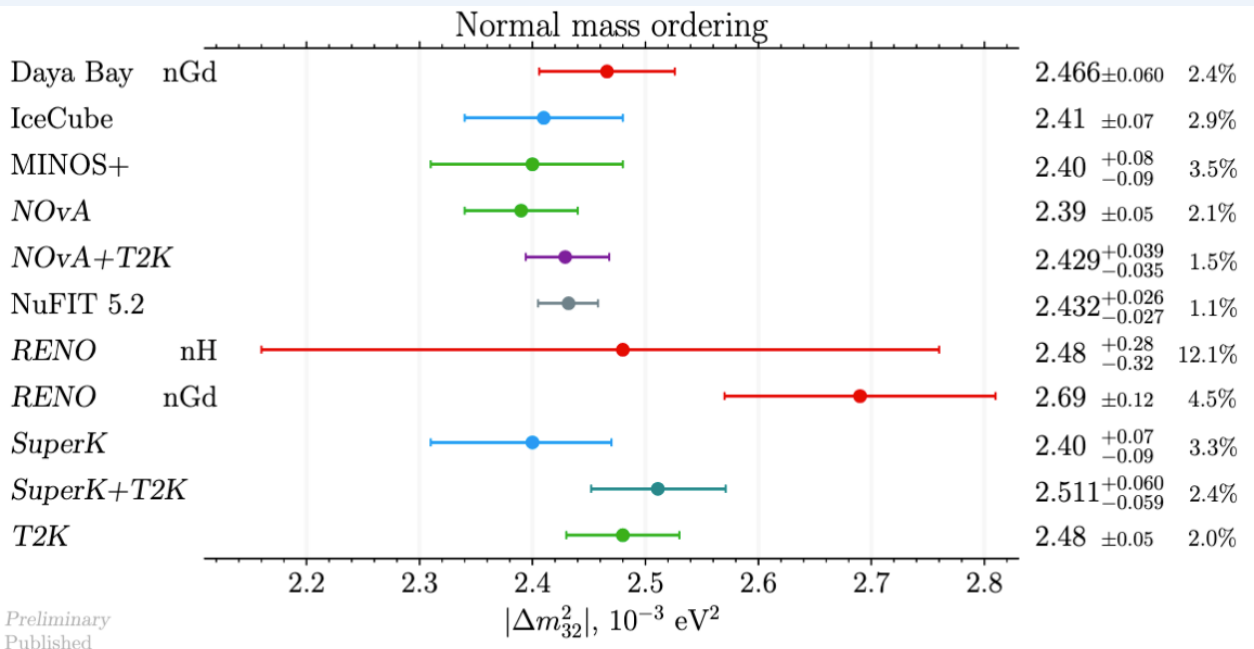
# Global Comparisons – A



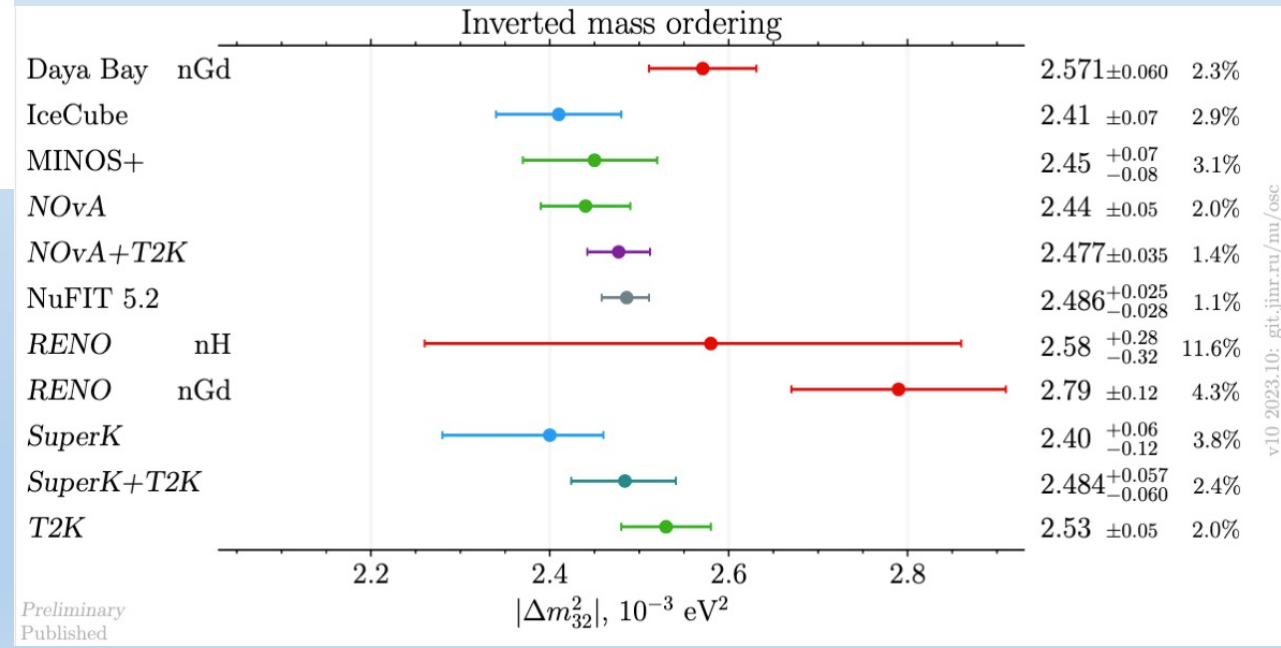
# Comparisons with global fitters



# al fitters



v10 2023.10: git.jinr.ru/mu/osc



v10 2023.10: git.jinr.ru/mu/osc

# Impact of correlations & alternate models

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# Asimov Oscillation points used for testing

	$\delta_{CP}$	$\Delta m_{32}^2$ [eV <sup>2</sup> ]	$\sin^2 \theta_{23}$
Asimov0 (NOvA best-fit like)	2.576	$2.41 \times 10^{-3}$	0.57
Asimov1 (T2K best-fit like)	-1.60	$2.51 \times 10^{-3}$	0.53
Asimov4 (NuFit like)	-1.60	$-2.45 \times 10^{-3}$	0.55

- Other parameters were kept constant at:

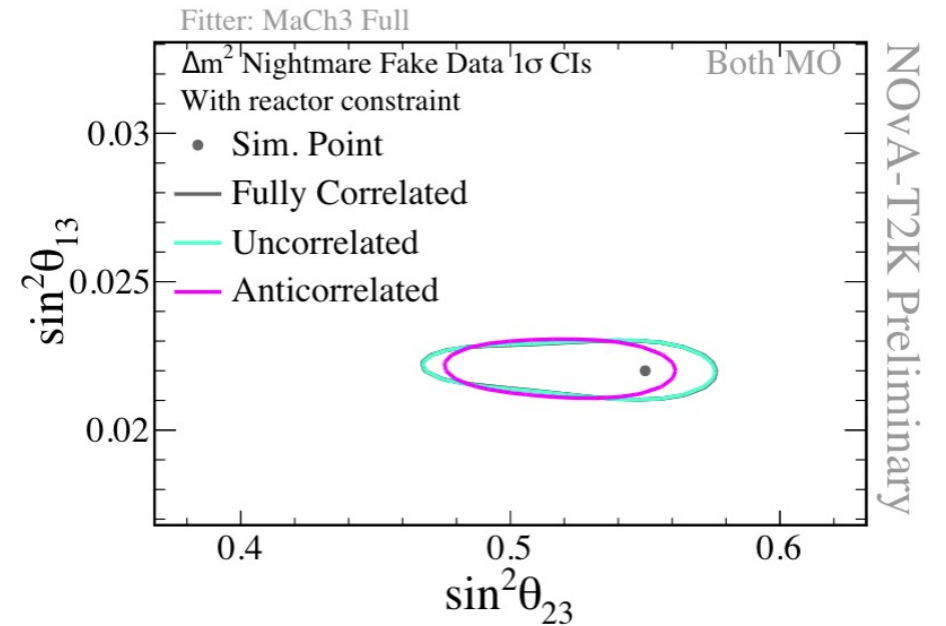
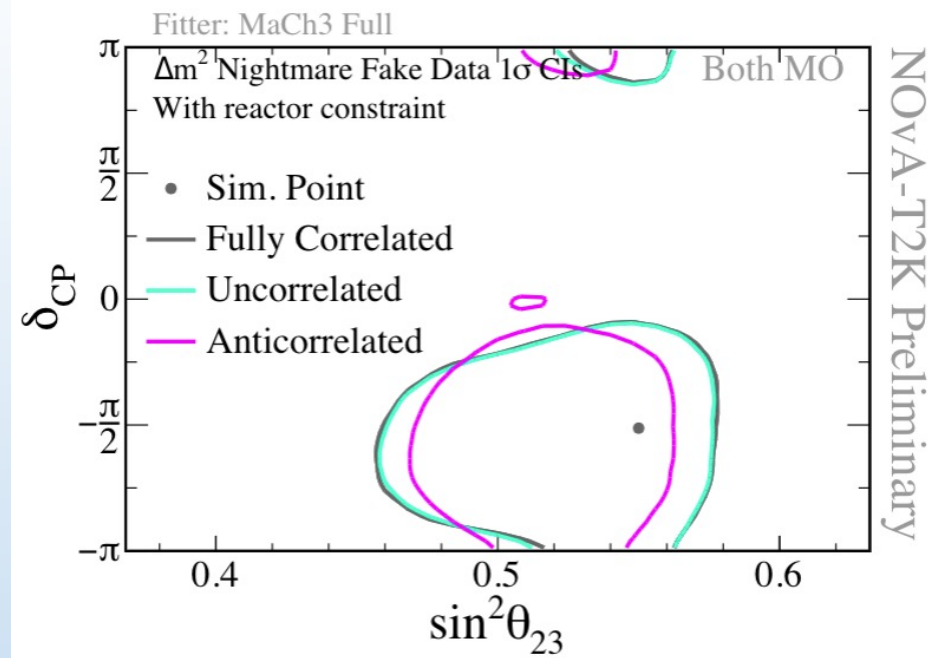
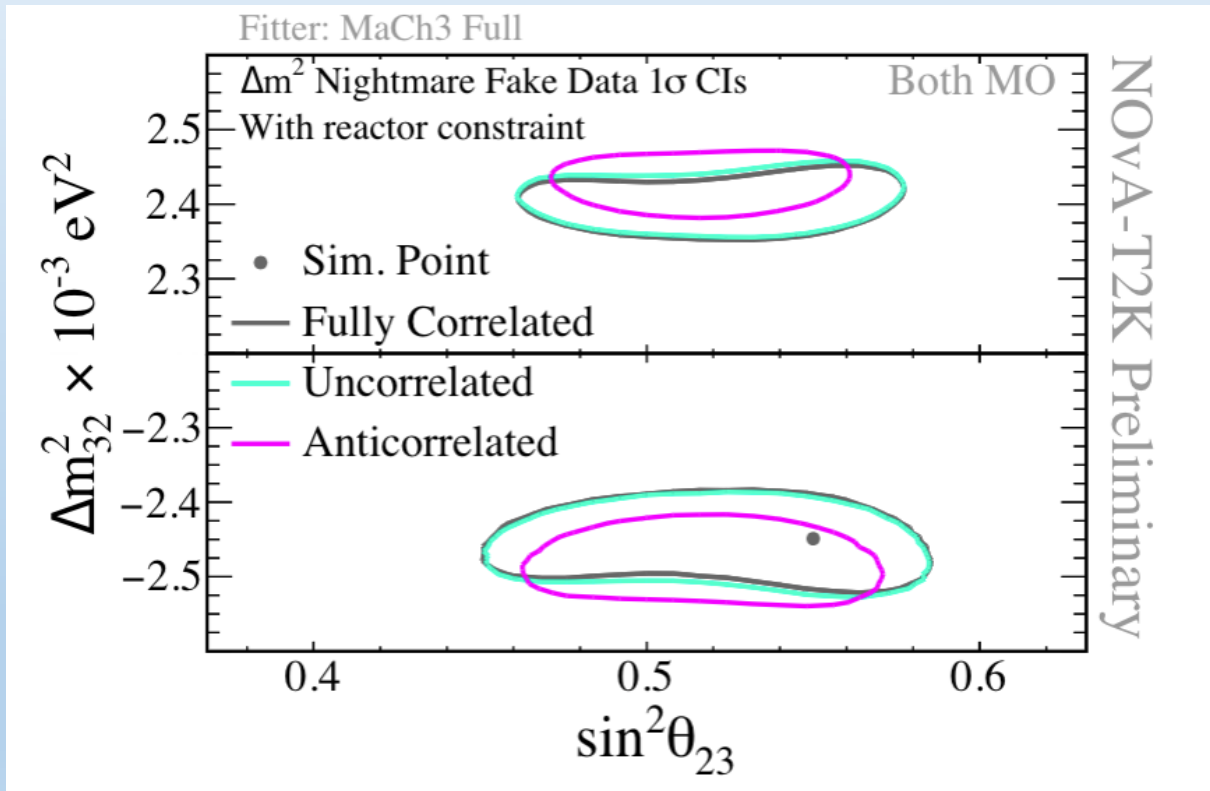
$$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{eV}^2$$

$$\sin^2 \theta_{12} = 0.307 \pm 0.013$$

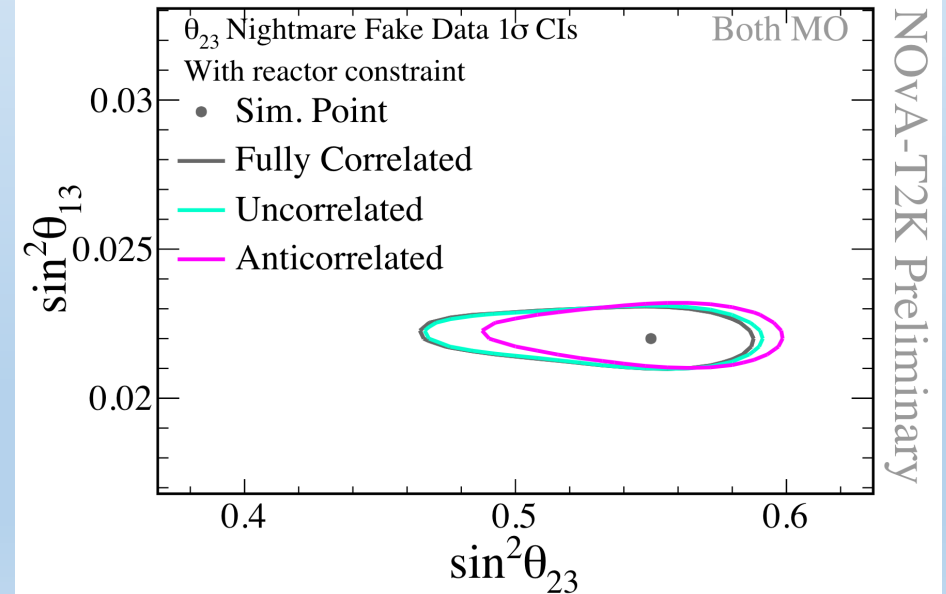
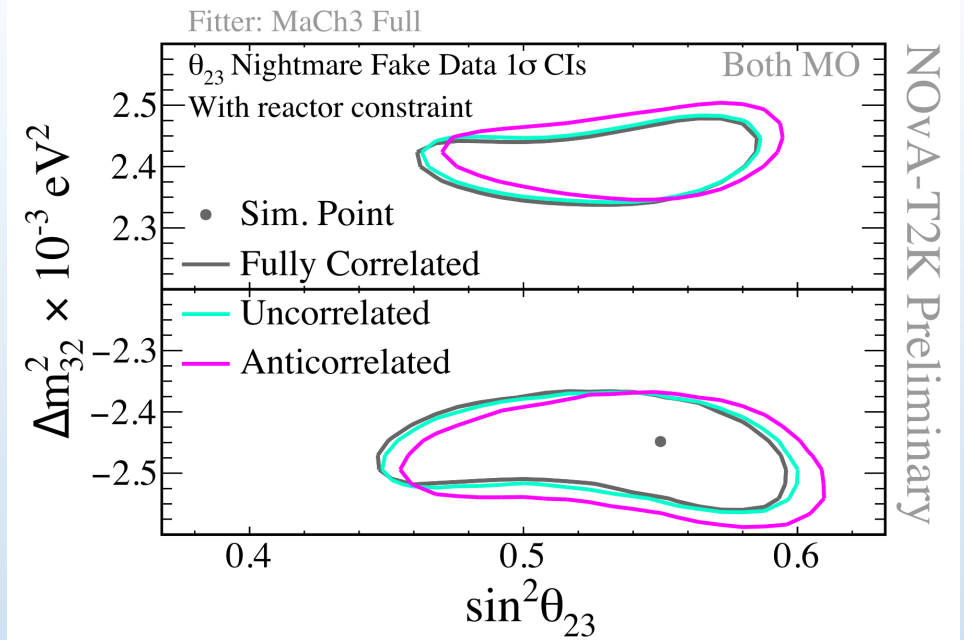
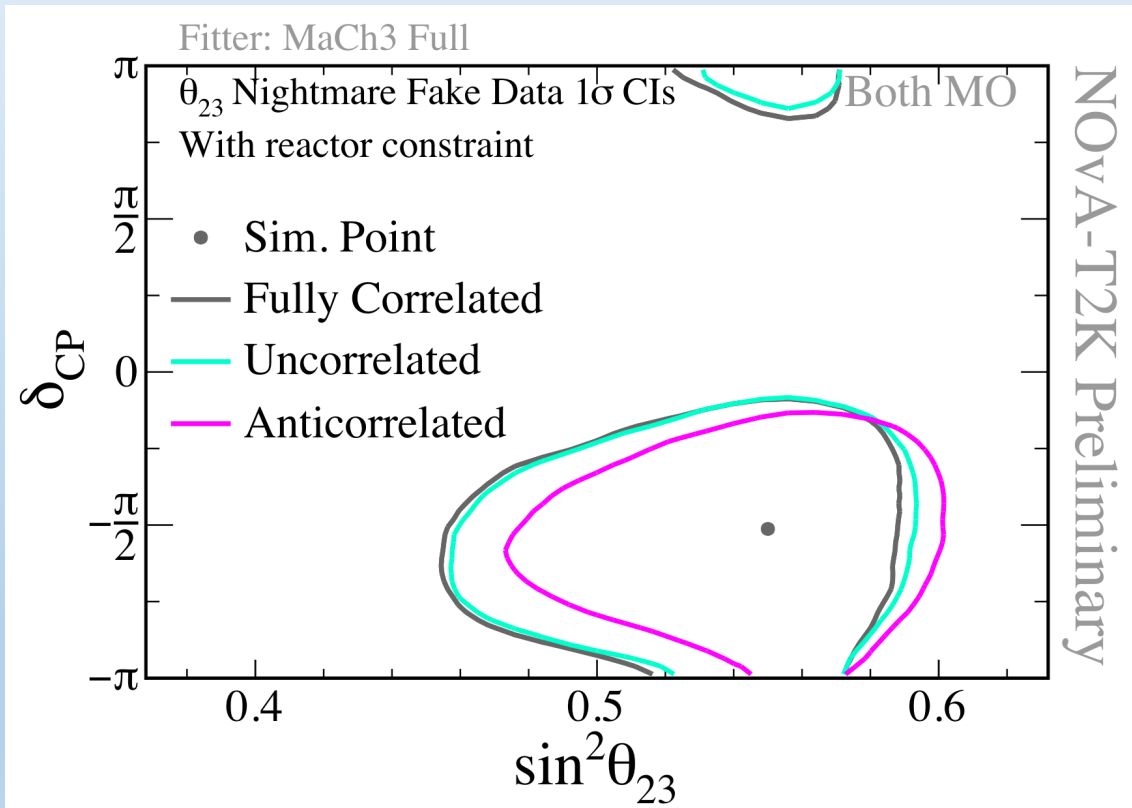
$$\sin^2 \theta_{13} = (2.18 \pm 0.07) \times 10^{-2}$$



- Impact of correlations
- $\Delta m_{32}^2$  bias mock data (nightmare) study



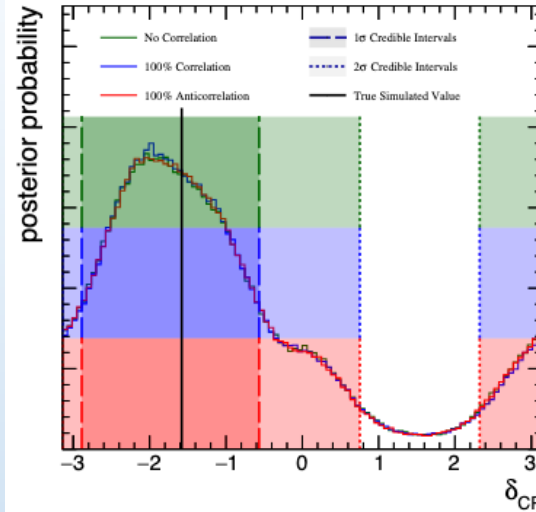
- $\theta_{23}$  bias mock data (nightmare) study



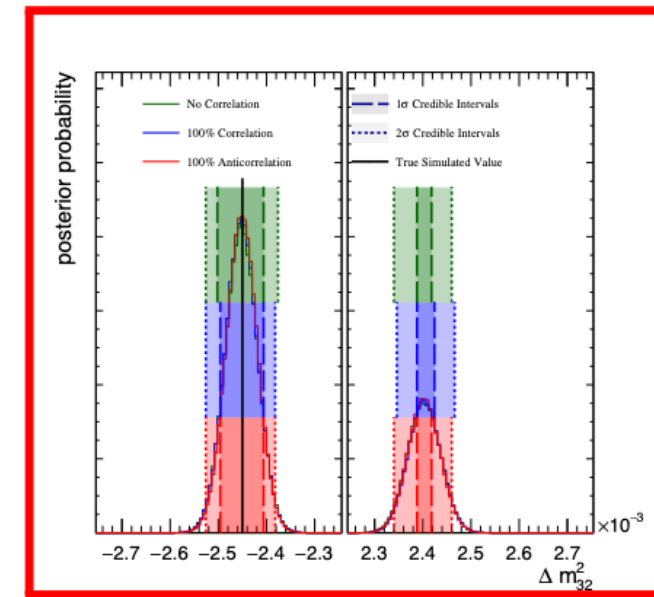
# Impact of correlations

Oscillation Parameter	Largest NOvA Systematic	Largest T2K Systematic
$\delta_{CP}$	second class currents and radiative corrections	$\sigma_{\nu_e}/\sigma_{\nu_\mu}$ cross section and antineutrino equivalents
$\sin^2 \theta_{23}$	neutron visible energy calibration	2p2h C-O scaling
$\Delta m_{32}^2$		7% SK energy-scale*

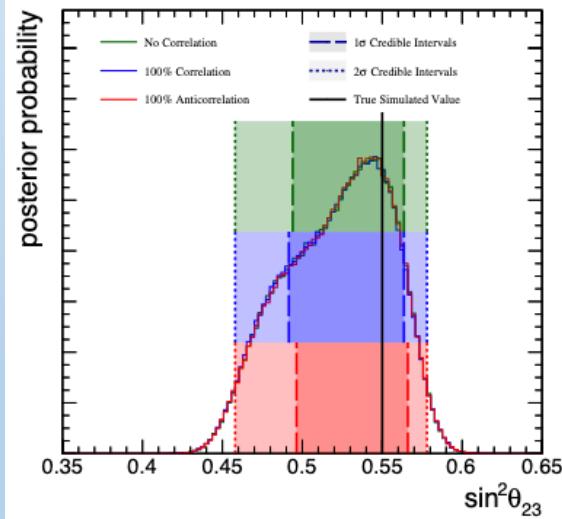
- Correlating largest systematics on  $\Delta m_{32}^2$  across both experiments.



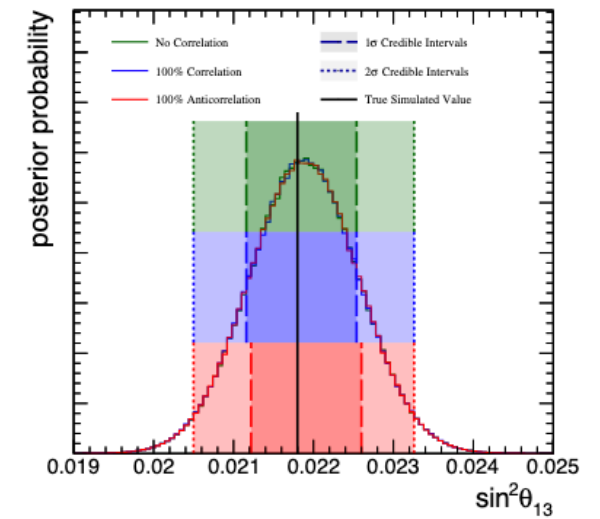
(a)  $\delta_{CP}$



(b)  $\Delta m_{32}^2$



(c)  $\sin^2 \theta_{23}$

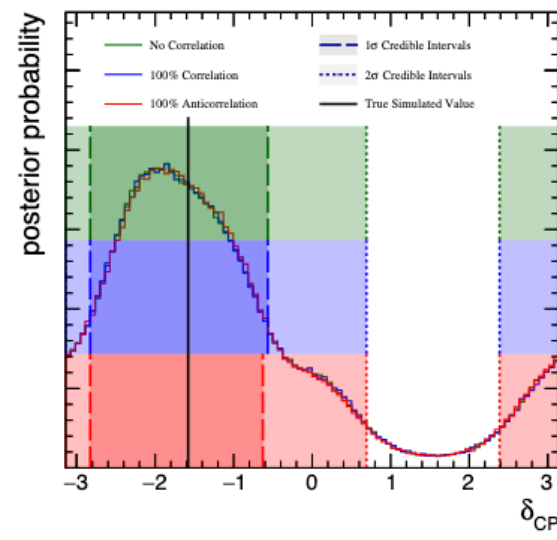


(d)  $\sin^2 \theta_{13}$

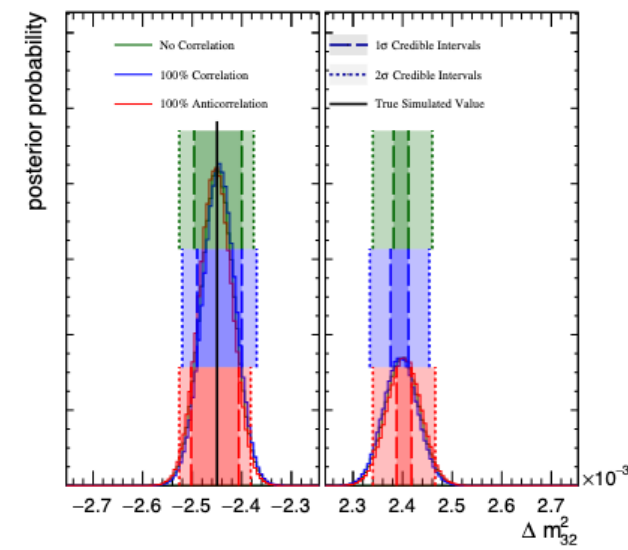
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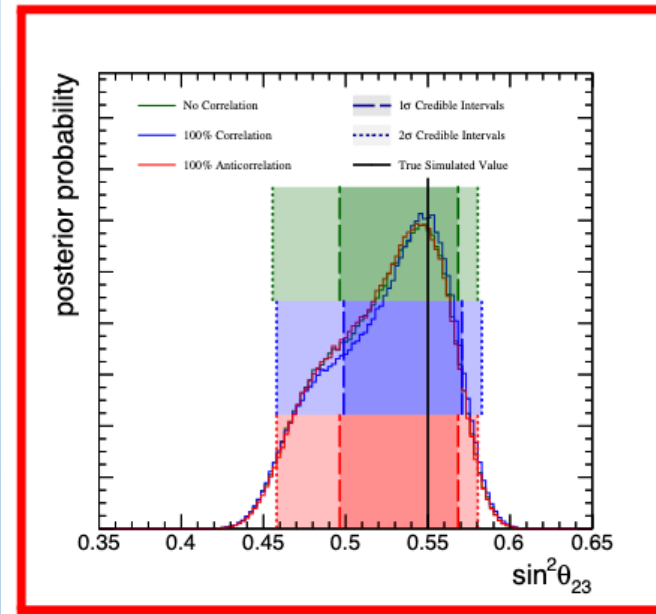
- Correlating largest systematics on  $\sin^2 \theta_{23}$  across both experiments.



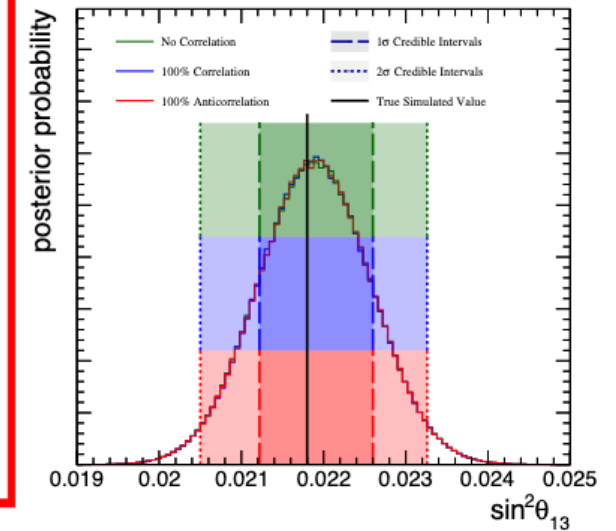
(a)  $\delta_{CP}$



(b)  $\Delta m_{32}^2$



(c)  $\sin^2 \theta_{23}$

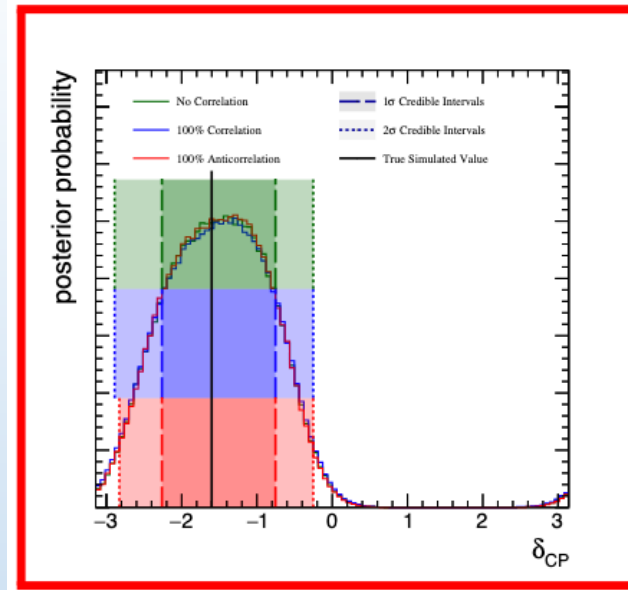


(d)  $\sin^2 \theta_{13}$

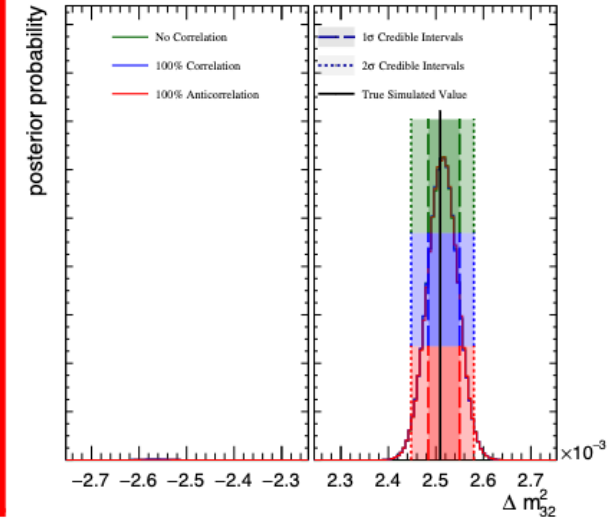
# Impact of correlations

Oscillation Parameter	Largest NOvA Systematic	Largest T2K Systematic
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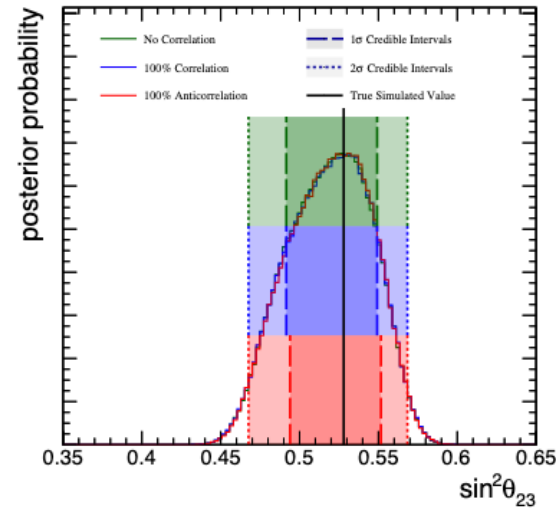
- Correlating largest systematics on  $\sin^2 \theta_{23}$  across both experiments.



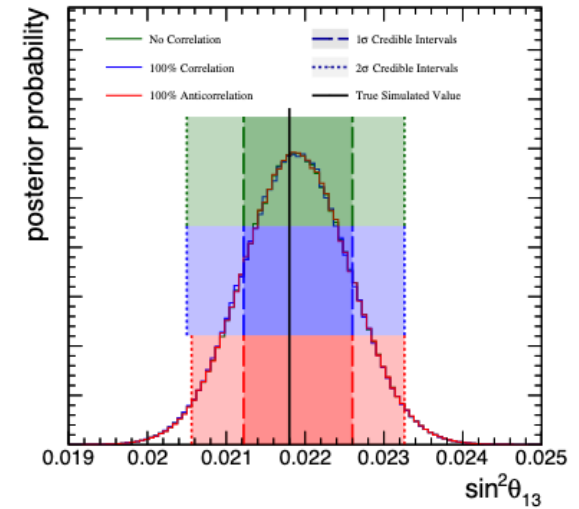
(a)  $\delta_{CP}$



(b)  $\Delta m_{32}^2$



(c)  $\sin^2 \theta_{23}$



(d)  $\sin^2 \theta_{13}$

# Impact of alternate models

- Various alternate models that had the largest impact on T2K's 2020-era fit and the two cross-experiment model checks were done for the joint analysis:
  - Non-QE: ND280 CC0 $\pi$  data are under-predicted by the T2K pre-fit prediction. This difference can be taken accounted for by the large freedom in the CCQE model. To check this large freedom does not cause bias, an alternate model where this under-prediction is attribution to only non-QE processes is produced.
  - Minerva1Pi: suppression of CC and NC resonant pion production at low- $Q^2$  to describe for GENIE v2 implementation of Rein-Seghal model to describe the data.
  - Pion SI: replaced GEANT4 model\* was replaced with NEUT's Salcedo–Oset model\*\*

\* S. Agostinelli et al., (The GEANT4 collaboration), Nucl. Instrum. Meth. A 506 (2003) 250–303 SLAC-PUB-9350

\*\* L. L. Salcedo, E. Oset, M. J. Vicente-Vacas, and C. Garcia-Recio, Nucl. Phys. A 484 (1988) 557–592 Print-87-1084 (Valencia)

# Impact of alternate models

- Test for bias 1D posteriors of  $\Delta m_{32}^2$  and  $\sin^2 \theta_{23}$ :
  - Change in the width of the 1D intervals <10%
  - Change in central value < 50% of systematic uncertainty

Alternate Model	$\Delta m_{32}^2$	$\Delta m_{32}^2$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{23}$
	Change in 1D contour < 10%	Bias in central value < 50%	Change in 1D contour < 10%	Bias in central value < 50%
Non-QE	✓	✓	✓	✓
Minerva1p	✓	✓	✓	✓
Pion-SI	✓	✓	✓	✓
NOvA-like	✓	✓	✓	✓
T2K-like	✓	✓	✓	✓

## Impact of alternate models

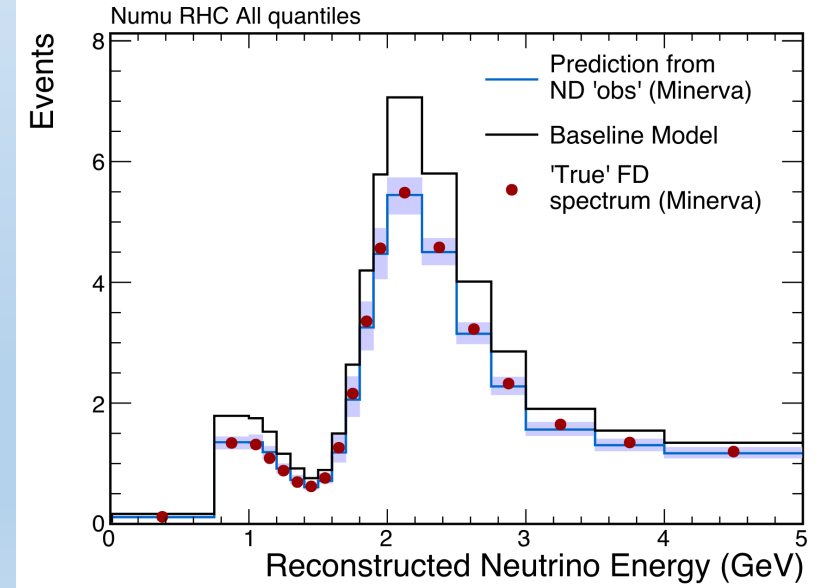
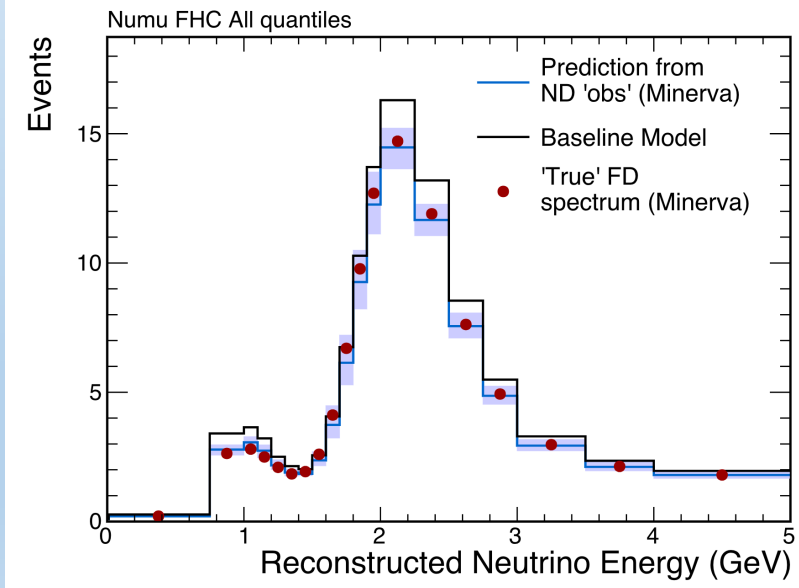
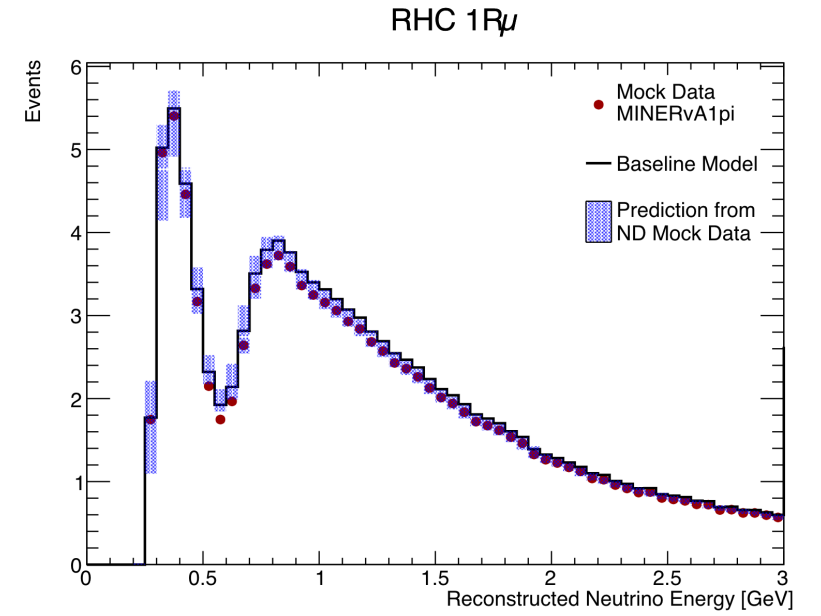
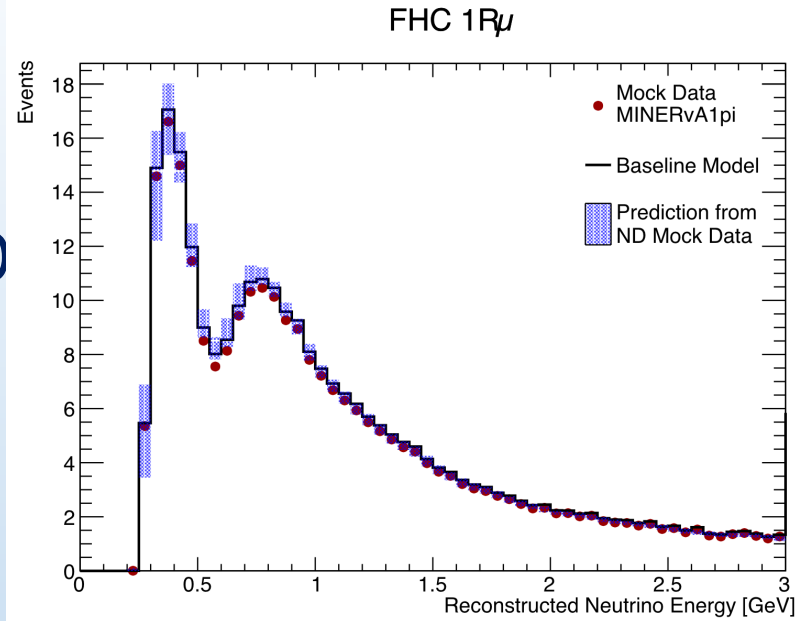
- Discrete model hypothesis:
  - Fractional change in Bayes factor for mass ordering and octant should not change any conclusions.
- Additional test on whether alternate models change our conclusion on the significance of CP violation.

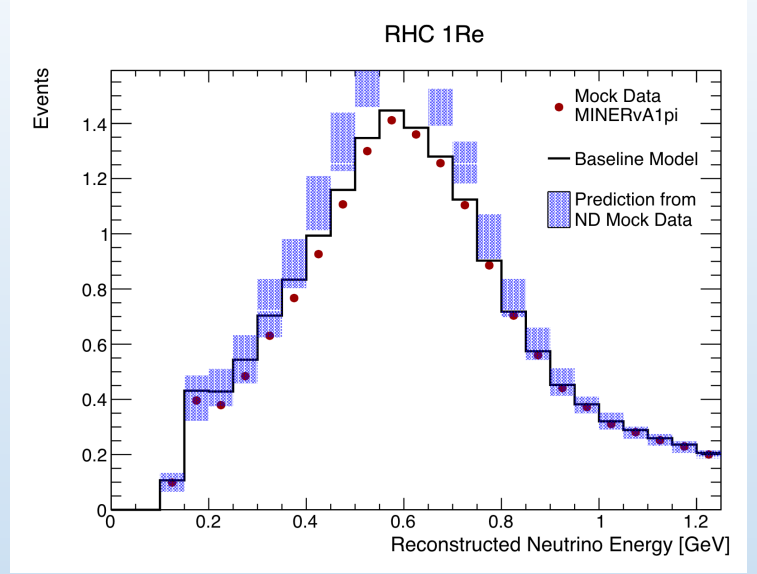
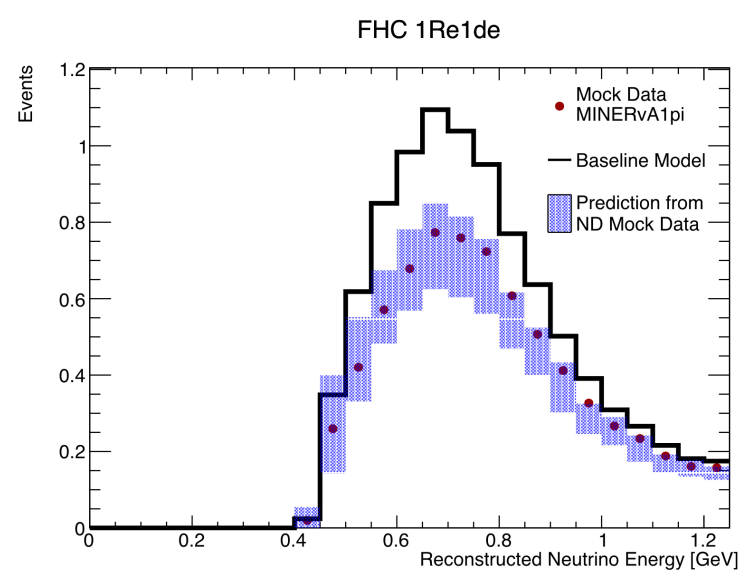
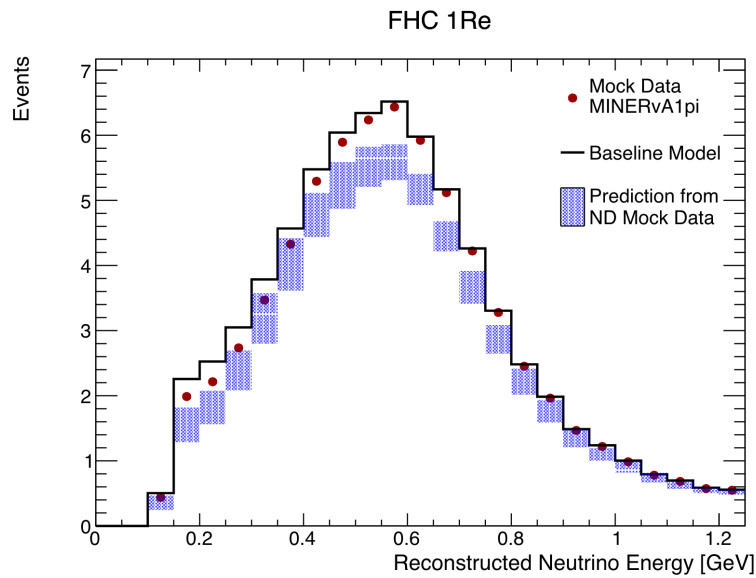
Alternate Model	Conclusion on $\delta_{CP}$	Conclusions on J	Mass Ordering Fractional change in BF	Octant Fractional change in BF
Non-QE	✓	✓	1.02	0.88
Minerva1p	✓	✓	1.03	0.92
Pion-SI	✓	✓	0.94	1.11
NOvA-like	✓	✓	1.10	1.00
T2K-like	✓	✓	1.08	1.16



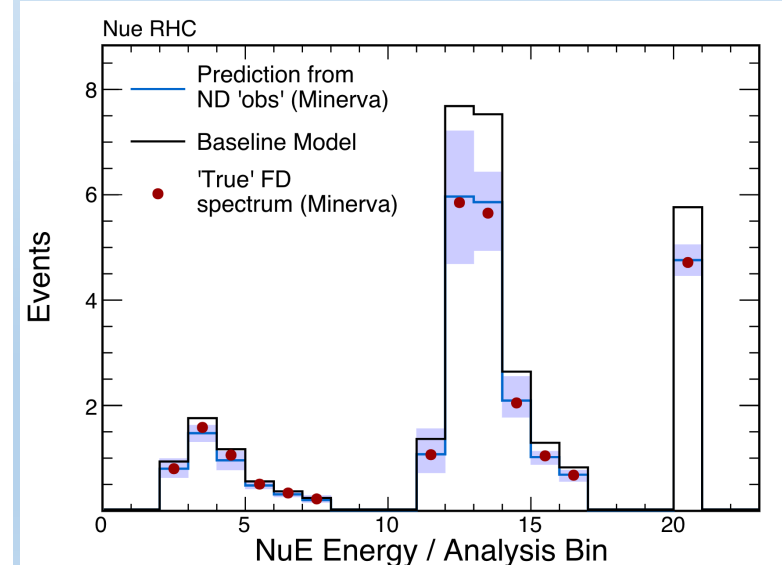
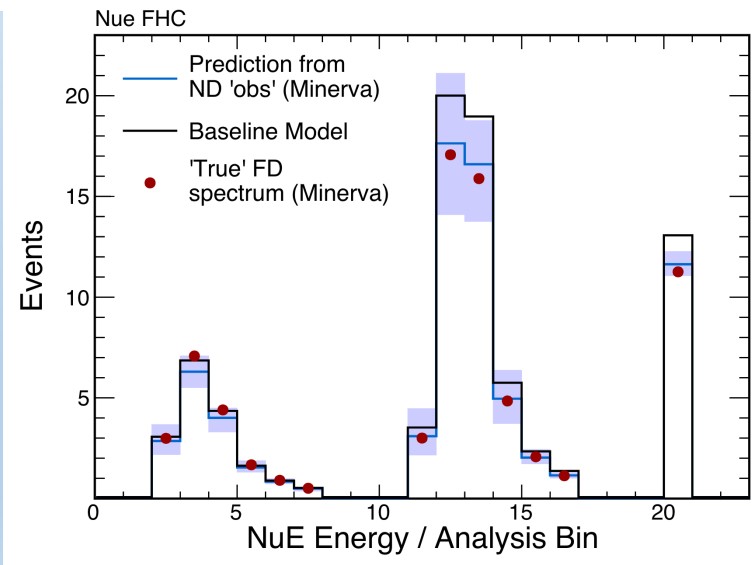
# Impact of alternate models: Minerva1p

- $\nu_{\mu}$  samples

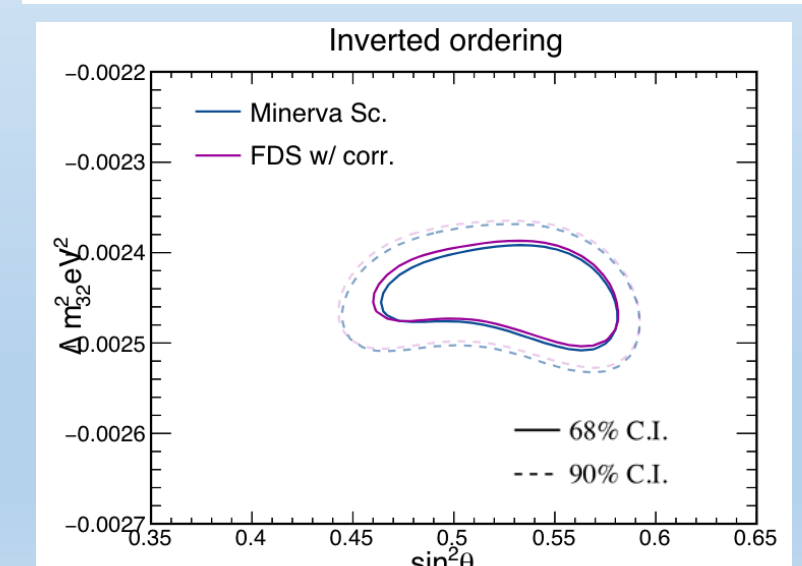
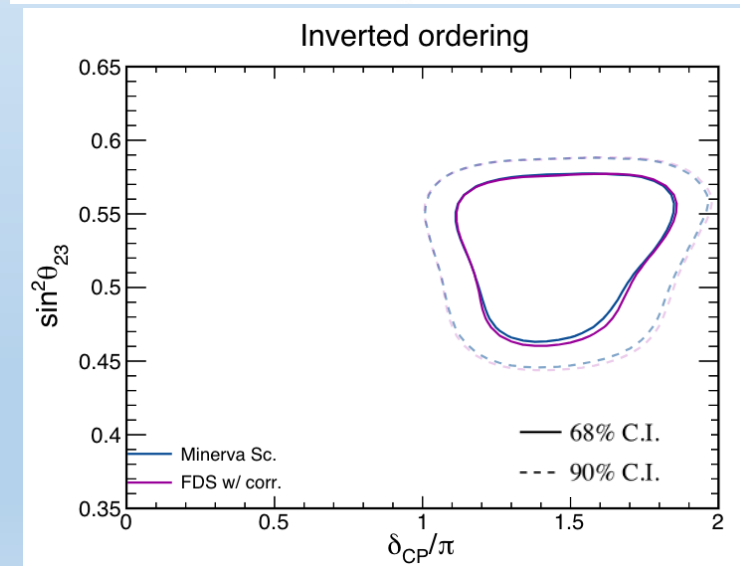
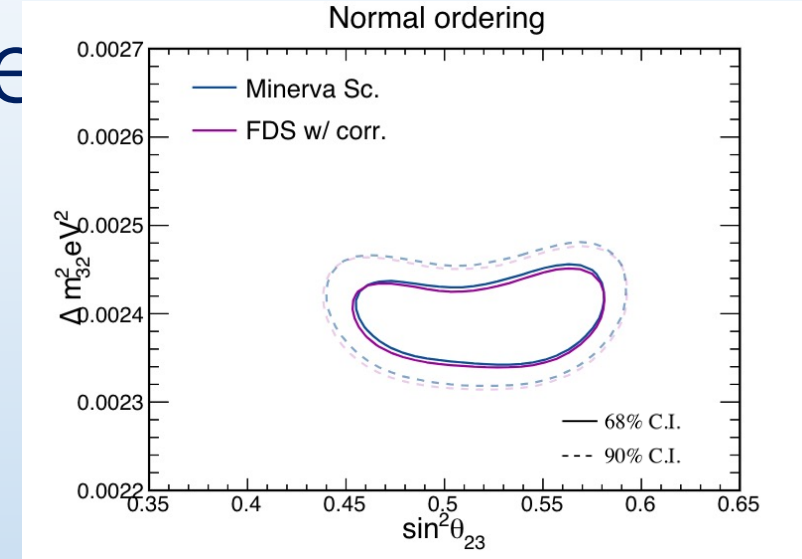
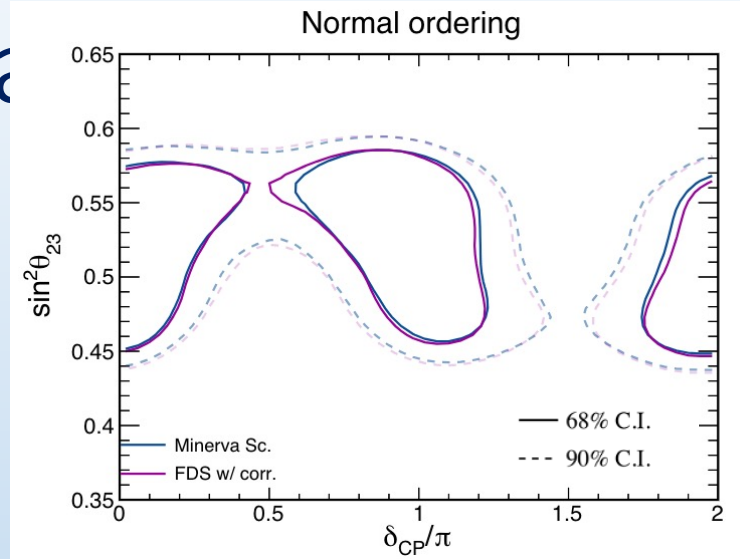


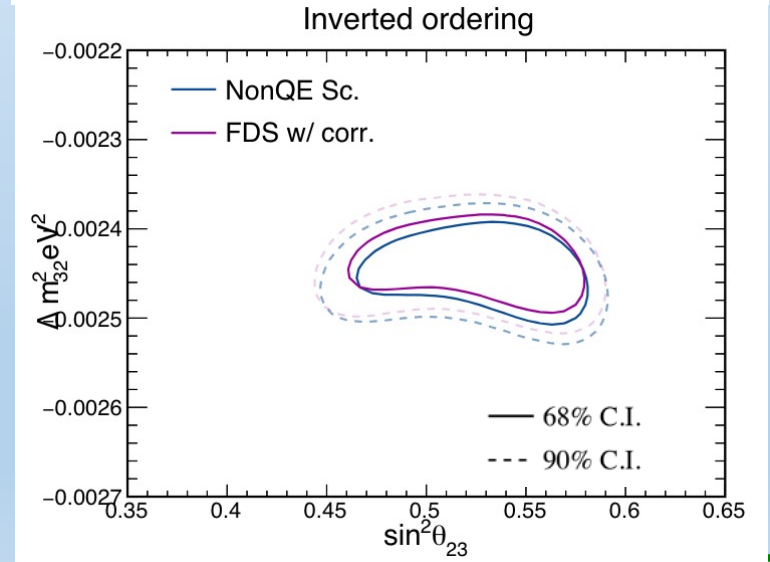
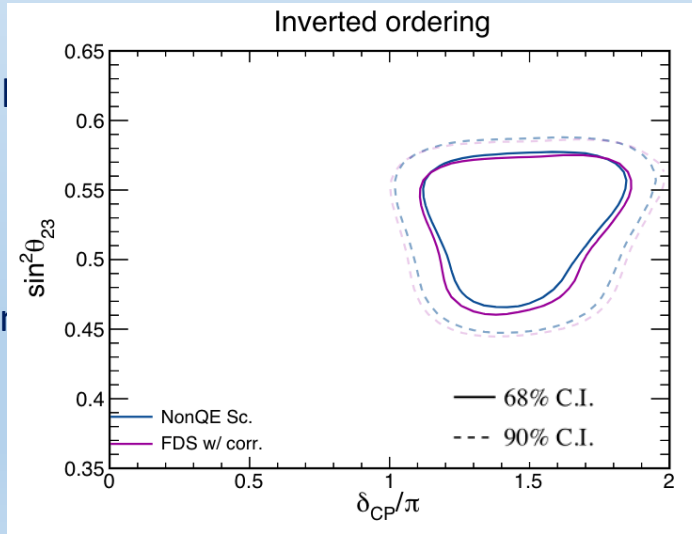
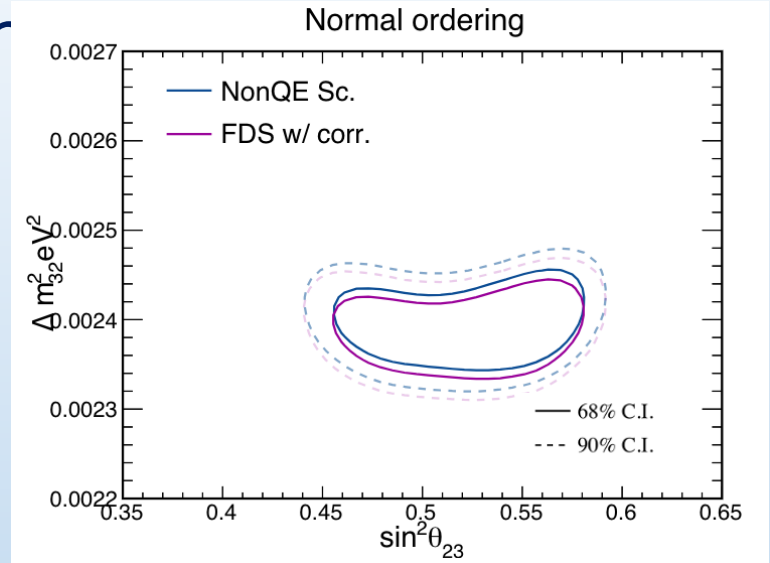
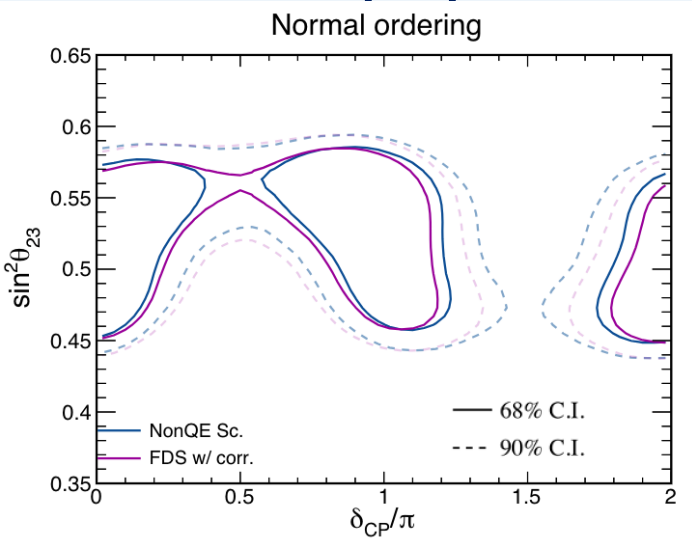
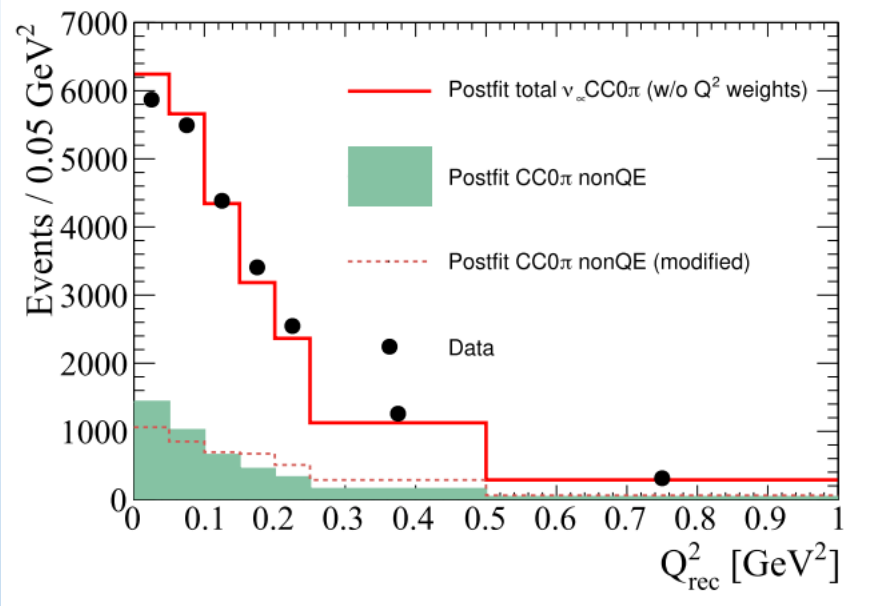


•  $\nu_e$  samples



- Impact of alternative
  - Minerva1Pi: suppression of CC and NC resonant pion production at low- $Q^2$  to describe for GENIE v2 implementation of Rein-Seghal model to describe the data.

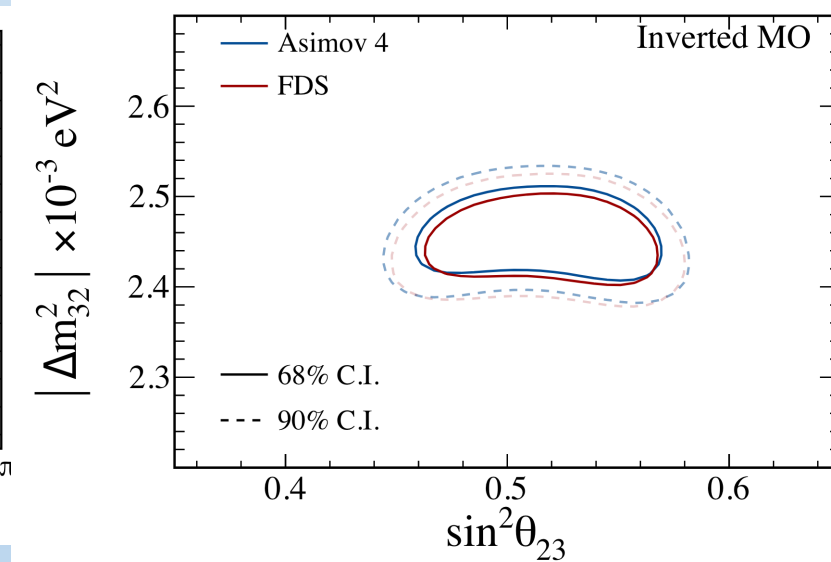
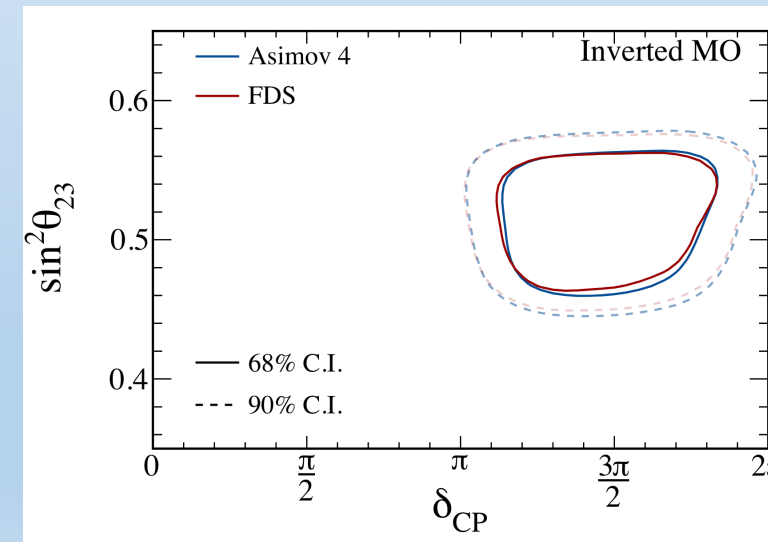
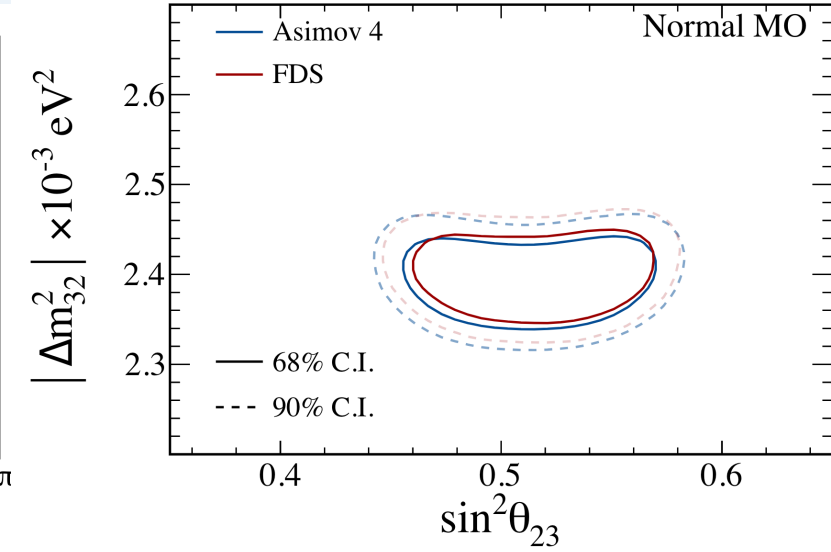
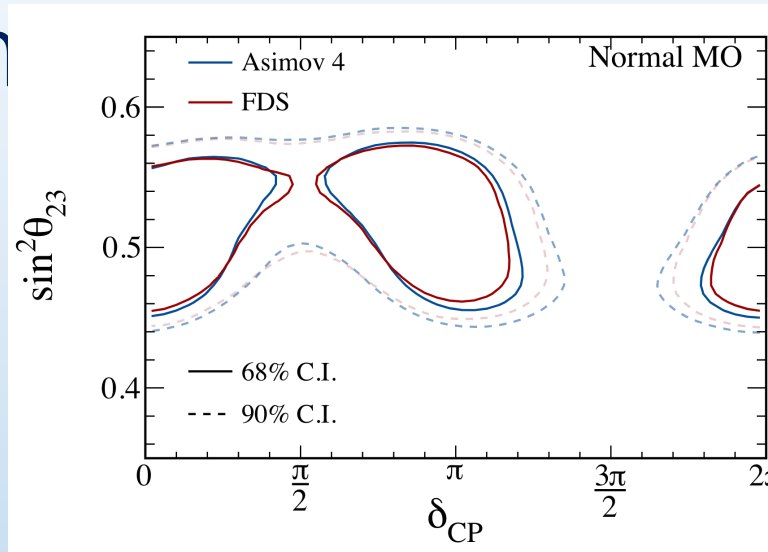




- Non-QE: ND280 CC0π data are under-predicted by the T2K pre-fit prediction. This difference can be taken into account for by the large freedom in the CCQE model. To check this large freedom does not cause bias, an alternate model where this under-prediction is attributed to only non-QE processes is produced.

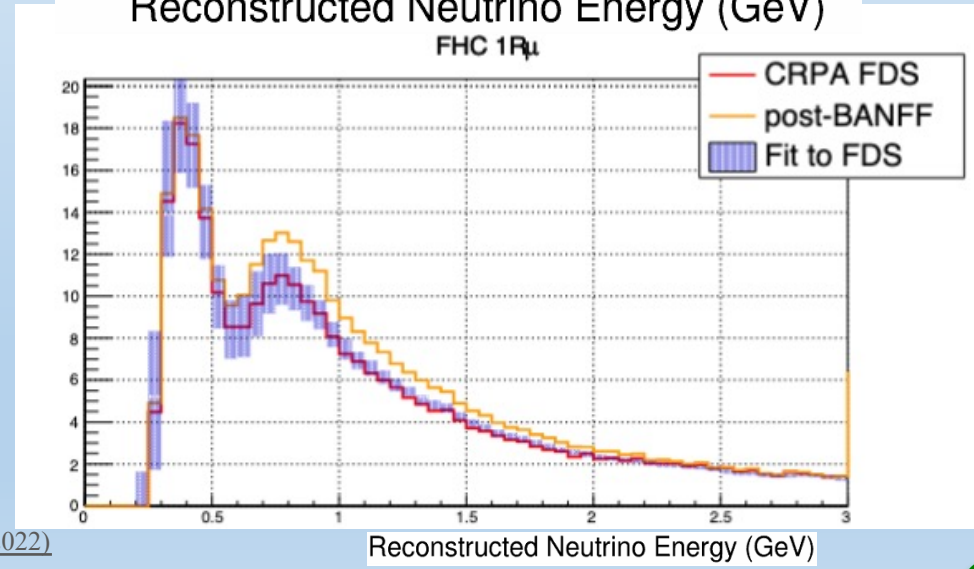
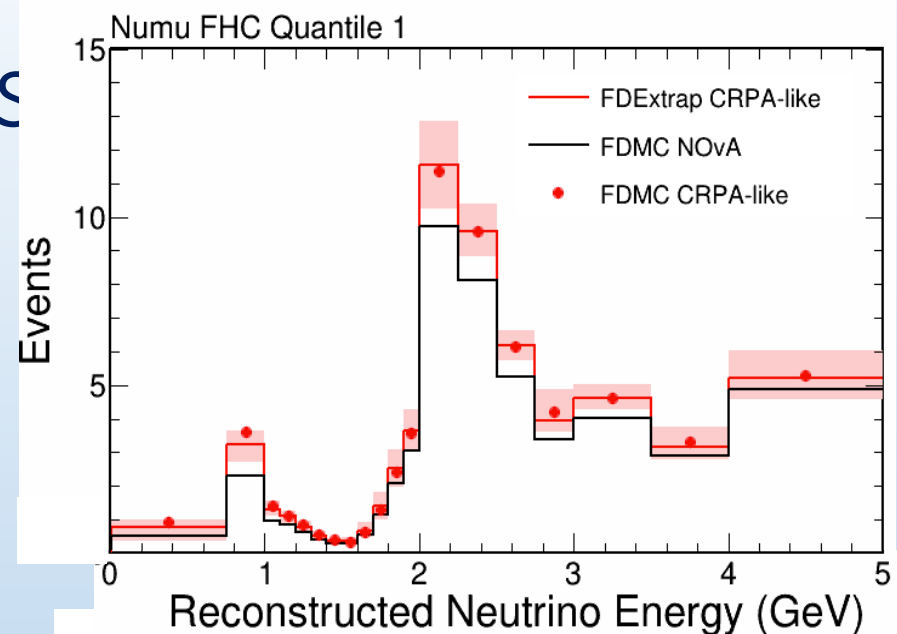
- Impact of alternative
  - Pion SI: replaced GEANT4 model\* was replaced with NEUT's Salcedo–Oset model\*\*

\* S. Agostinelli et al., (The GEANT4 collaboration), Nucl. Instrum. Meth. A 506 (2003) 250–303 SLAC-PUB-9350  
 \*\* L. L. Salcedo, E. Oset, M. J. Vicente-Vacas, and C. Garcia-Recio, Nucl. Phys. A 484 (1988) 557–592 Print-87-1084 (Valencia)



# Impact of alternate models

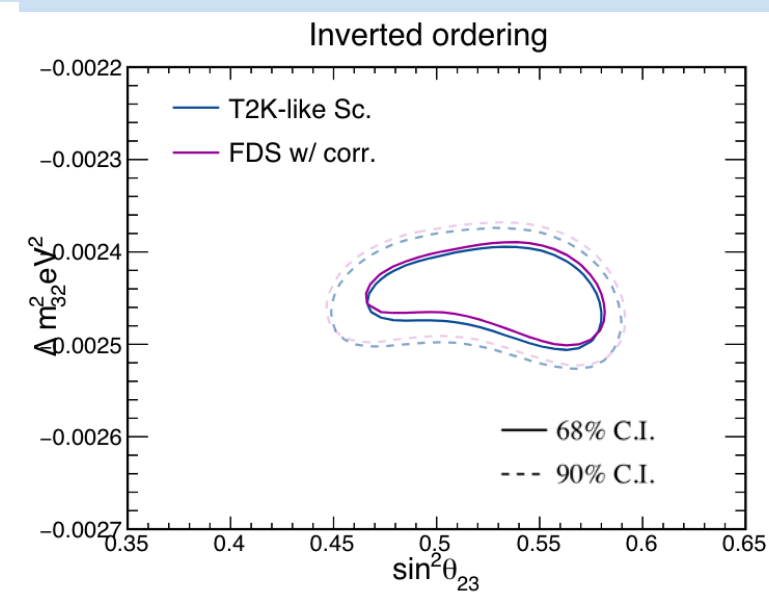
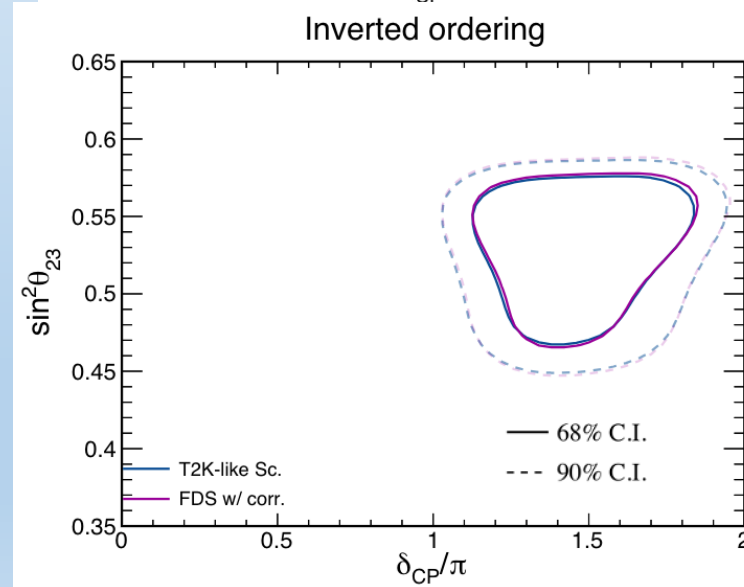
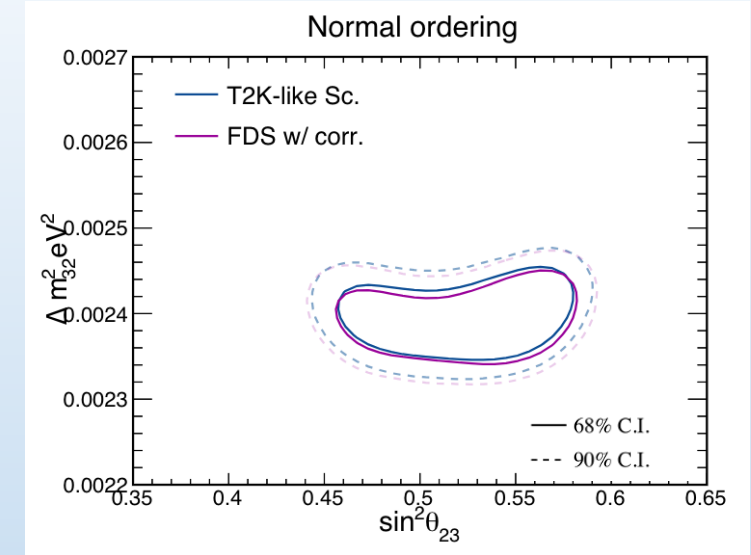
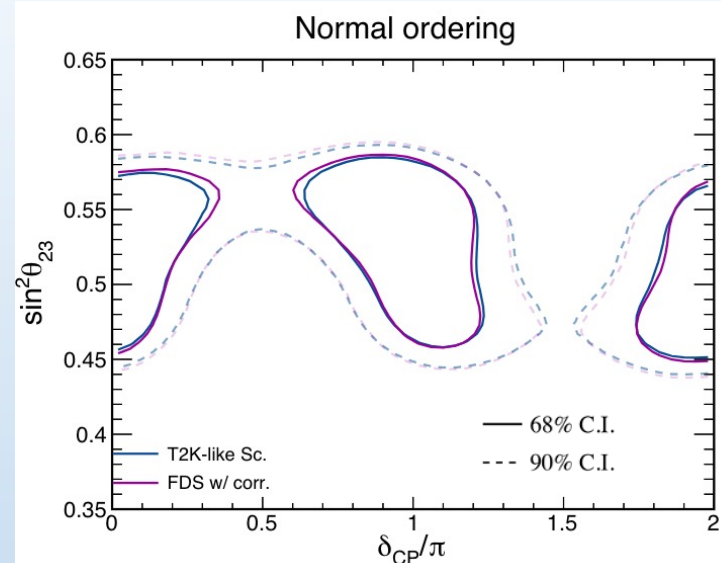
- Hartree-Fock (HF) Continuum Random Phase Approximation (CRPA)\*
- Applies modifications to the nuclear models (Spectral Function for T2K, Local Fermi Gas for NOvA)
- Recent T2K analyses have included an additional smearing on  $\Delta m_{32}^2$  based on variations seen when considering the HF-CRPA nuclear model.
  - Both NOvA and T2K independently studied the impact of this alternate nuclear model on their 2020-era analyses.
  - When taken together in the context of the joint fit, the bias is no larger than the thresholds set for any of the fake data metrics.



\* Phys. Rev. D 106, 073001 (2022)

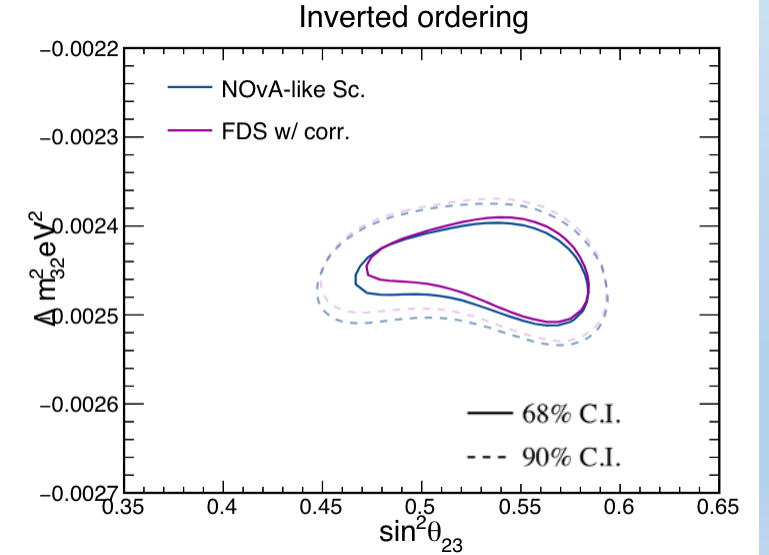
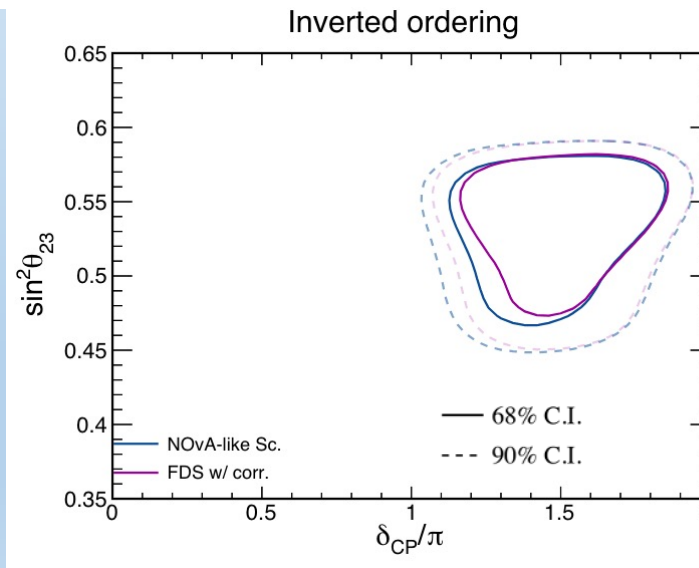
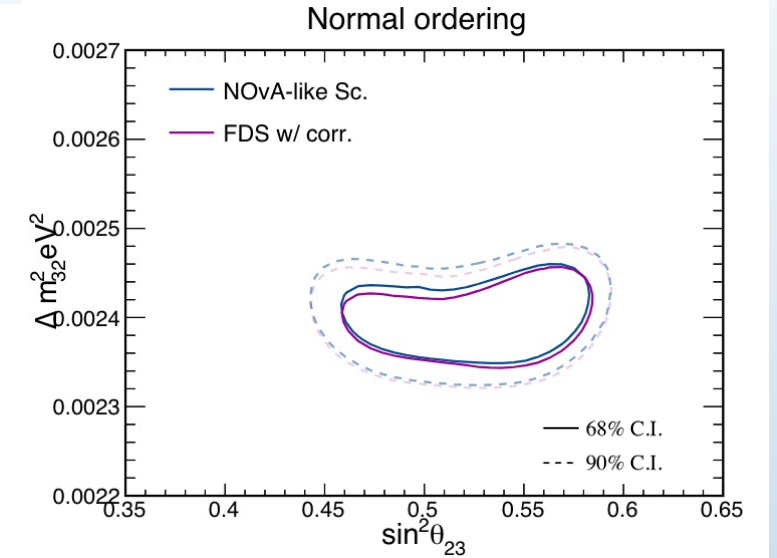
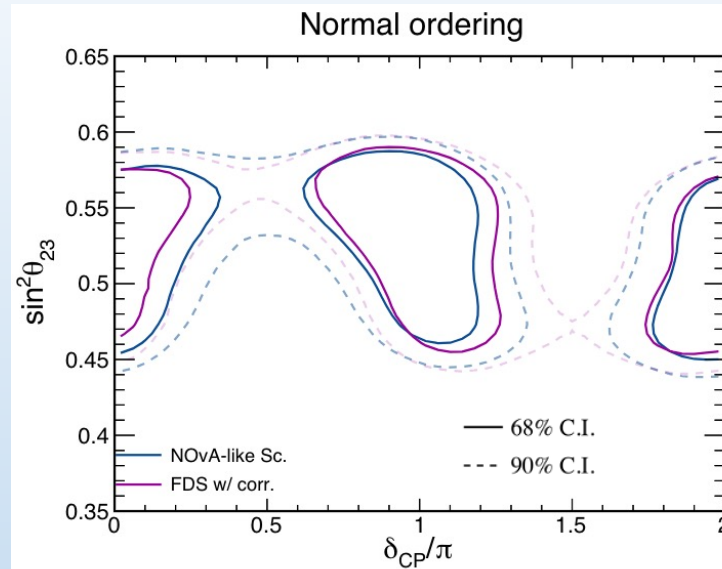
# Impact of cross-experiment model: T2K-like

- The purpose of the cross-experiment test is to verify each analysis is not broken by pseudo-data made using a representative allowed point from the other experiment's model.
- Unlike the FDS, which test a single variation, these tests also include modification of multiple processes.
- Extrapolating the individual processes consistently to the to other experiment's energy at the same time consistently is not possible.



# Impact of cross-experiment model: NOvA-like

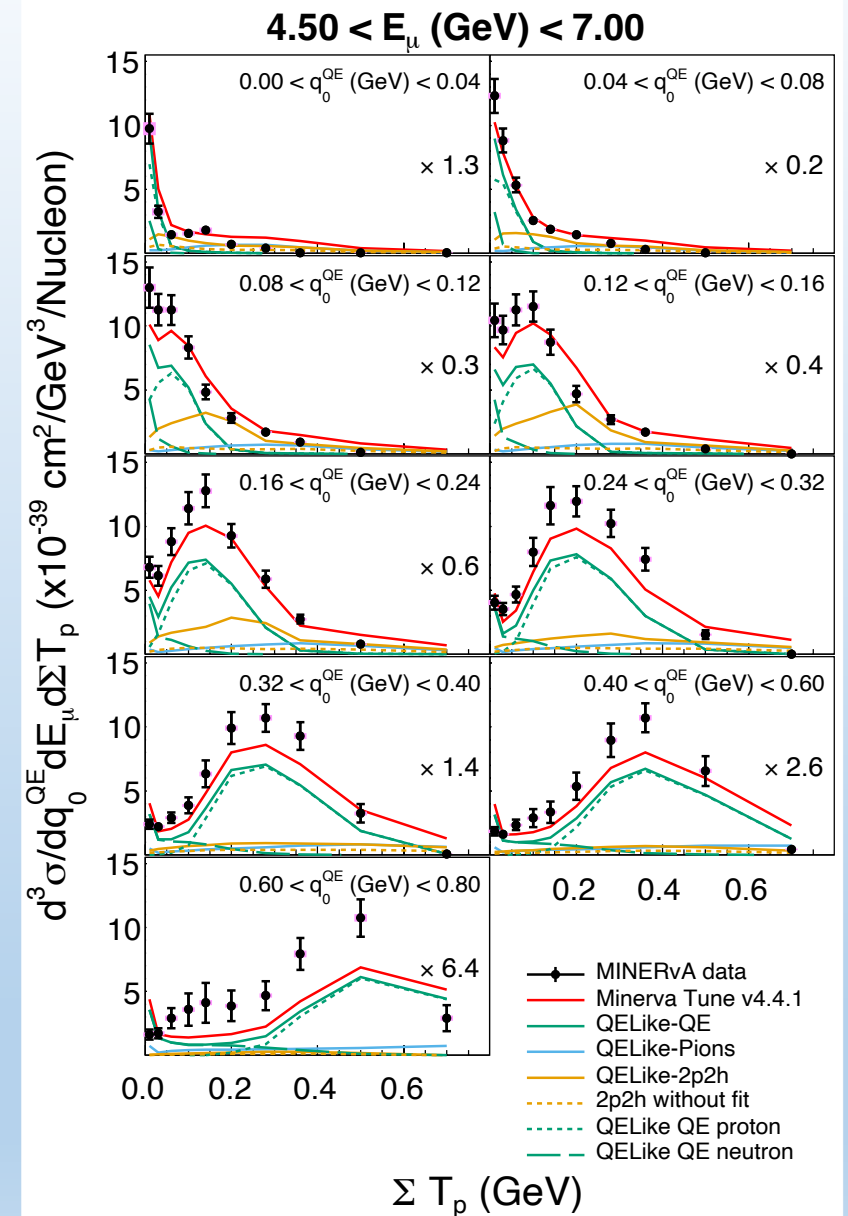
- NOvA near-to-far extrapolation method does not produce a 'post-fit' cross section model; we choose the NOvA prefit tune as the test model, but this is an arbitrary choice from a large space of valid choices.





# CC0 $\pi$ $E_{\nu, \text{reconstructed}} - E_{\text{lepton}}$ in T2K vs NOvA

- In T2K the  $p_T$  of the lepton is used to measure the recoiling energy by two body quasielastic kinematics.
- In NOvA, the visible recoil is measured.
- In this T2K-NOvA analysis, we are not relying on a single model to simultaneously describe these variables, but we may in the future
- MINERvA compares the two types of energy measures: recoil in bins of  $q_0^{\text{QE}}$  (the energy T2K adds to the muon energy)
  - Agreement with this model is poor
  - Events where the QE hypothesis says there should be lots of proton energy added, but MINERvA does not see that energy!
- T2K and NOvA naturally continue to investigate improvements in their cross section models. We appreciate the continued theoretical and experimental effort in the community



# Systematics comparison

- Models and systematics used for 2020 analysis [NOvA: [PhysRevD.106.032004](https://arxiv.org/abs/1907.04072), T2K: [arXiv:2303.03222v1](https://arxiv.org/abs/2303.03222v1)] will be used in the joint fit.
- The base-models are tuned to internal (NOvA-ND data by NOvA) and external datasets.
- The tuning modifies the underlying models drastically (eg: NOvA's 2p2h tune.)

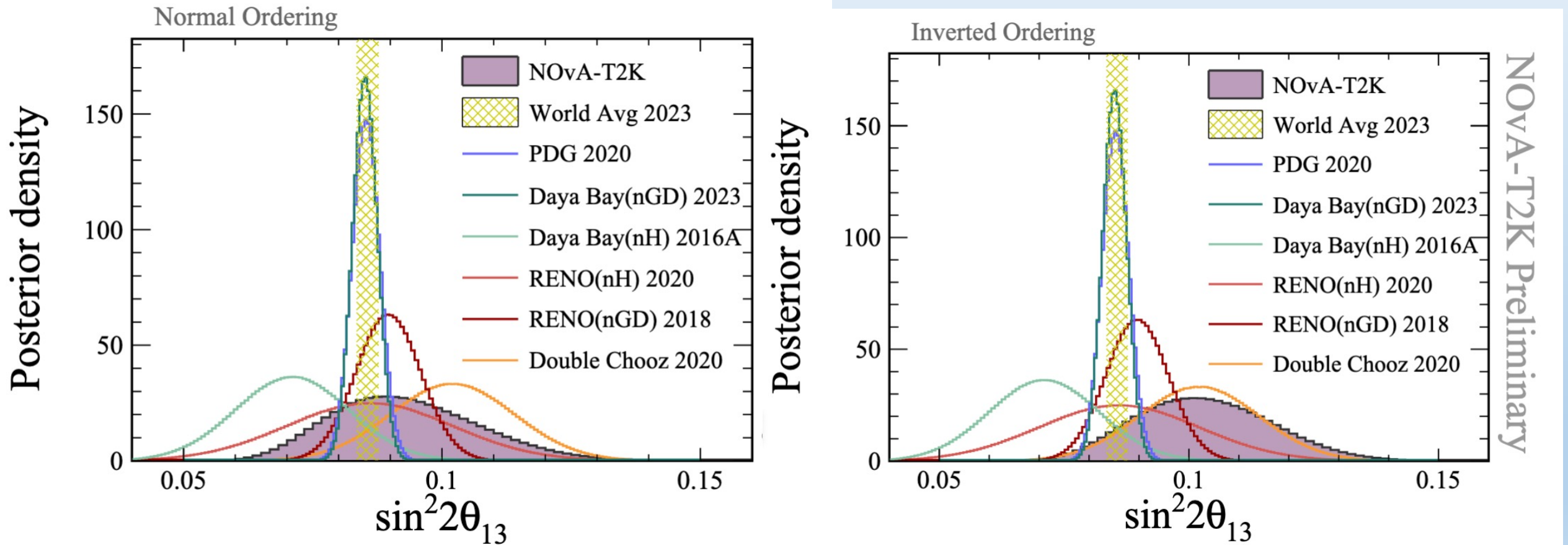
Category	NOvA Parameters	T2K Parameters
CCQE	ZNormCCQE ZExpAxialFFSyst2020_EV1 ZExpAxialFFSyst2020_EV2 ZExpAxialFFSyst2020_EV3 ZExpAxialFFSyst2020_EV4 RPAShapeenh2020 RPAShapesupp2020	$M_A^{QE}$ Q2_norm_0 Q2_norm_1 Q2_norm_2 Q2_norm_3 Q2_norm_4 Q2_norm_5 Q2_norm_6 Q2_norm_7 EB Dial C nu EB Dial C nubar EB Dial O nu EB Dial O nubar
MEC	MECEnuShape2020Nu MECEnuShape2020AntiNu MECShape2020Nu MECShape2020AntiNu MECInitStateNPFrac2020Nu MECInitStateNPFrac2020AntiNu	2p2h Norm nu 2p2h Norm nubar 2p2h C to O 2p2h Shape C 2p2h Shape O 2p2h Edep low Enu 2p2h Edep high Enu 2p2h Edep low Enubar 2p2h Edep high Enubar
RES	MaCCRES MvCCRES MaNCRES MvNCRES LowQ2RESSupp2020	CA5 MA RES ISO Bkg Low PPi ISO Bkg
FSI	hNFSI_MFP_2020 hNFSI_FateFracEV1_2020	FEFQE FEFQEH FEFINEL FEFABS FEFCX

Experiment	Generator	QE	MEC/2p2h	RES	DIS	FSI
<b>NOvA</b>	GENIE v3.0.6	Local Fermi Gas Z-expansion axial form factor	Valencia* <small>(*with NOvA 2020 tune)</small>	Berger-Sehgal	Bodek-Yang	hN Semi Classical Cascade <small>(*fit to pion scattering data)</small>
<b>T2K</b>	NEUT 5.4	Spectral Function $M_A^{QE}$ form factor	Valencia	Rein-Sehgal	Bodek-Yang	Semi- Classical Cascade

# 2D constraints from Daya Bay

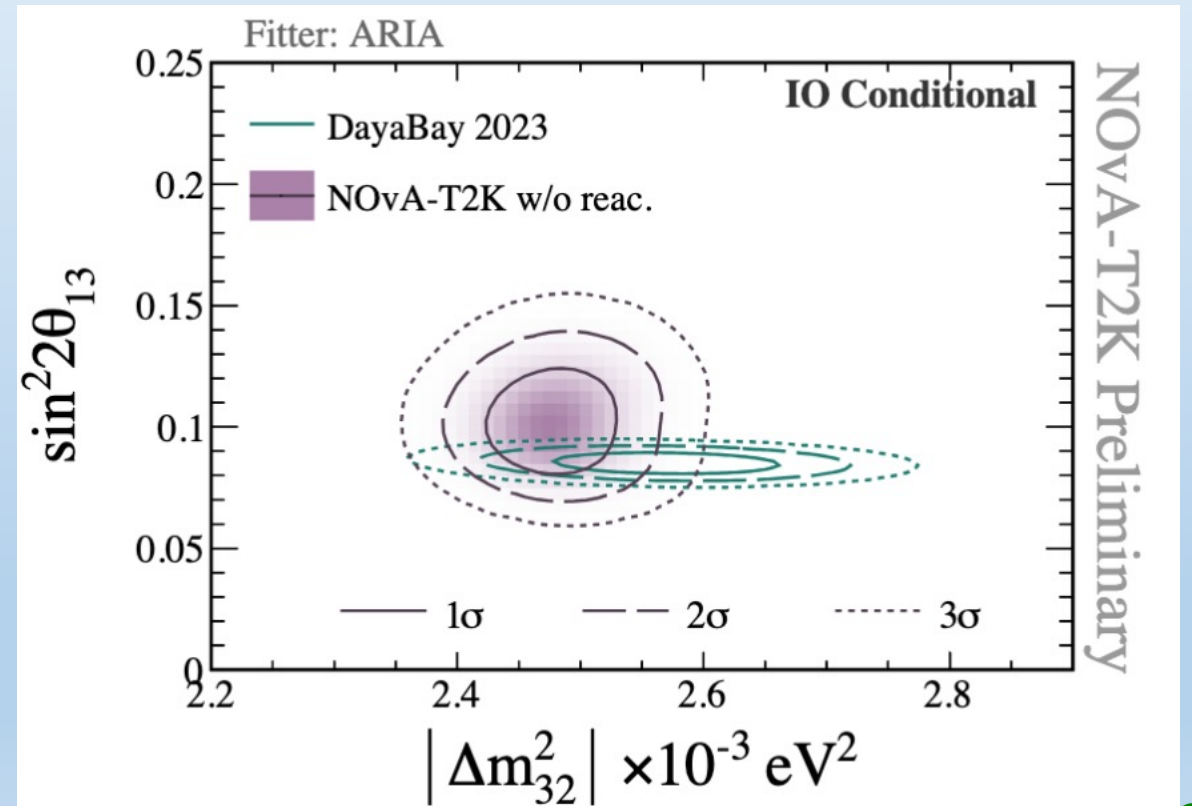
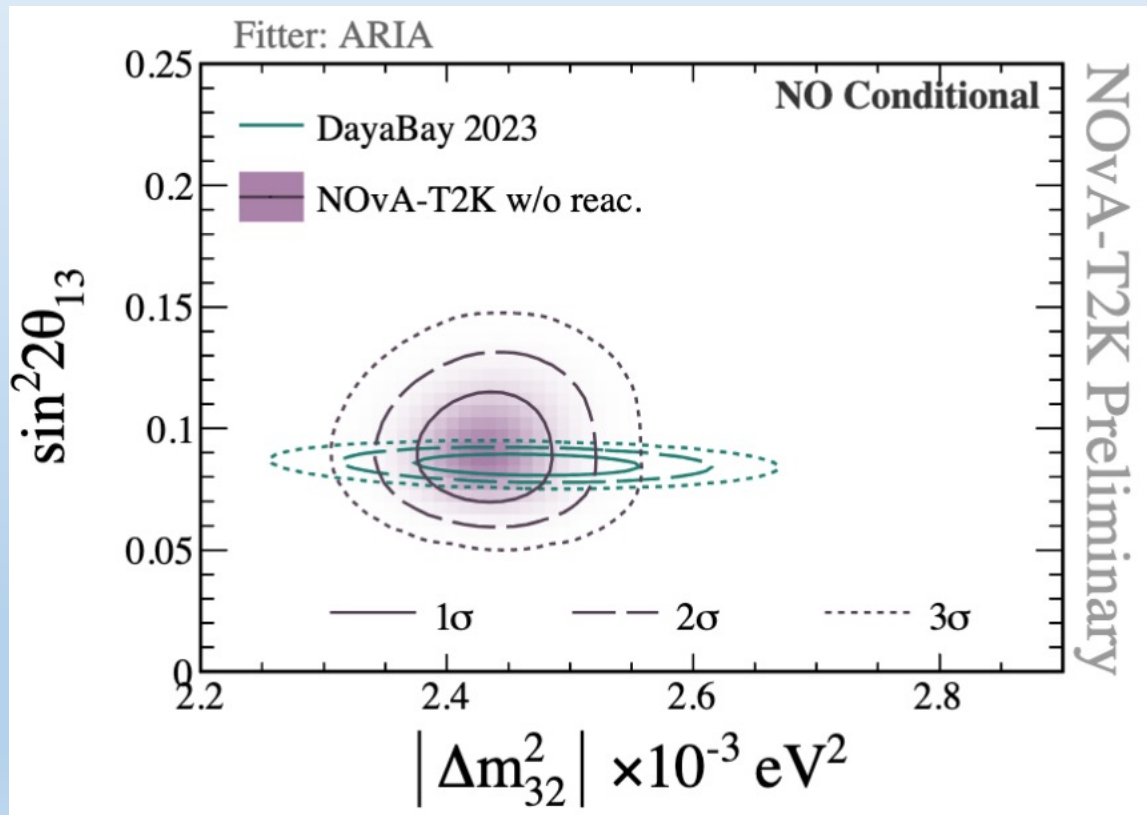
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# $\theta_{13}$ measurements



NOvA-T2K Preliminary

# NOvA+T2K+Daya Bay



- In the true mass ordering, reactor and long-baseline measurements of  $\Delta m_{32}^2$  would be consistent but the incorrect mass ordering would be wrong by different amounts

ee : reactor disappearance channel  $\rightarrow$  Daya Bay\*  
 $\mu\mu$  : long-baseline disappearance channel  $\rightarrow$  NOvA+T2K

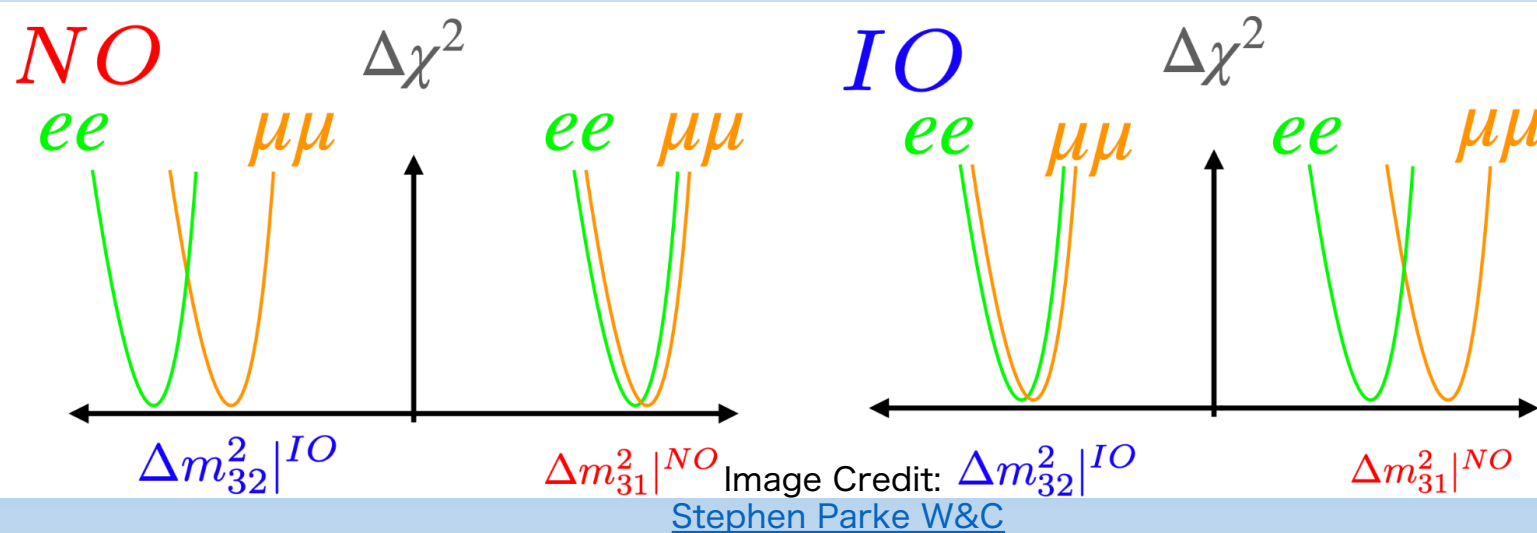
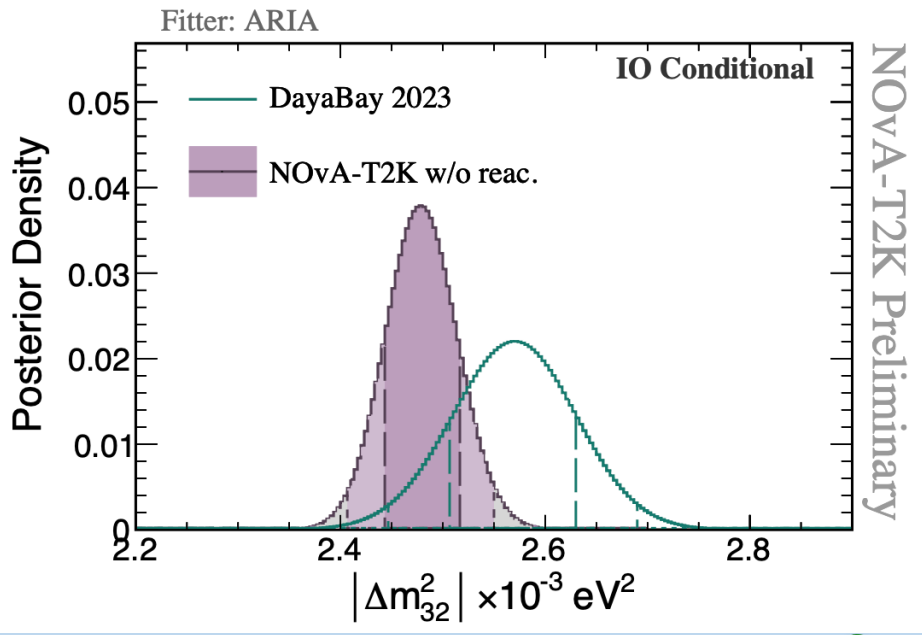
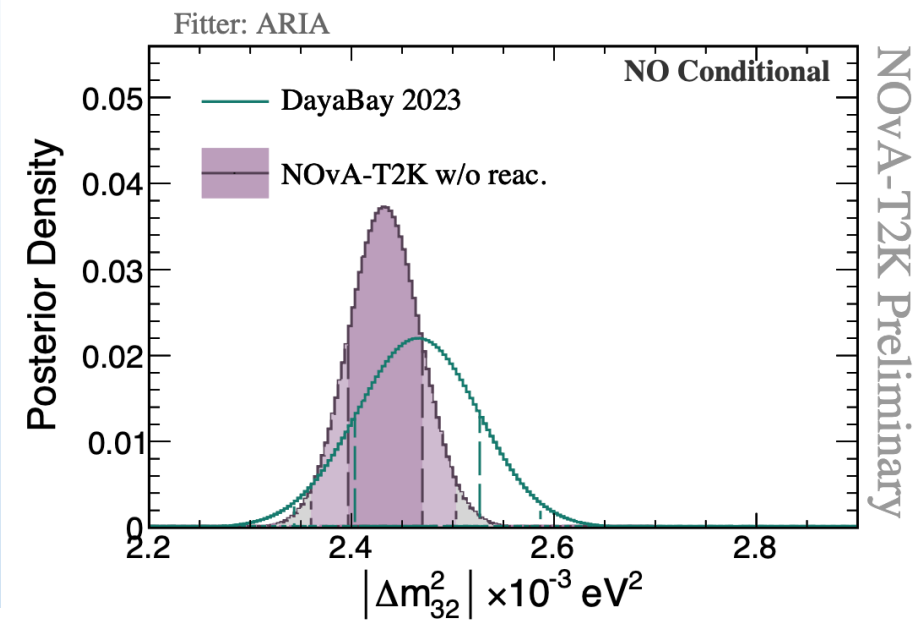


Image Credit: Stephen Parke W&C

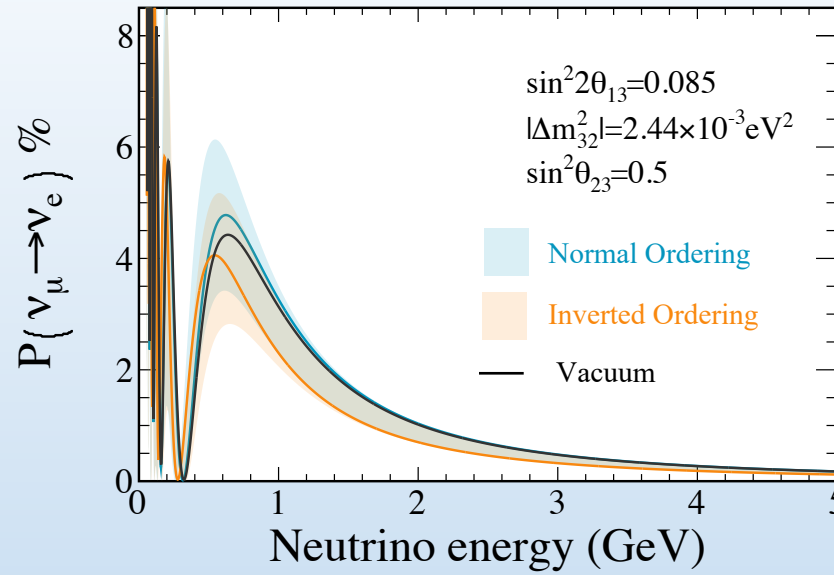


# Baselines

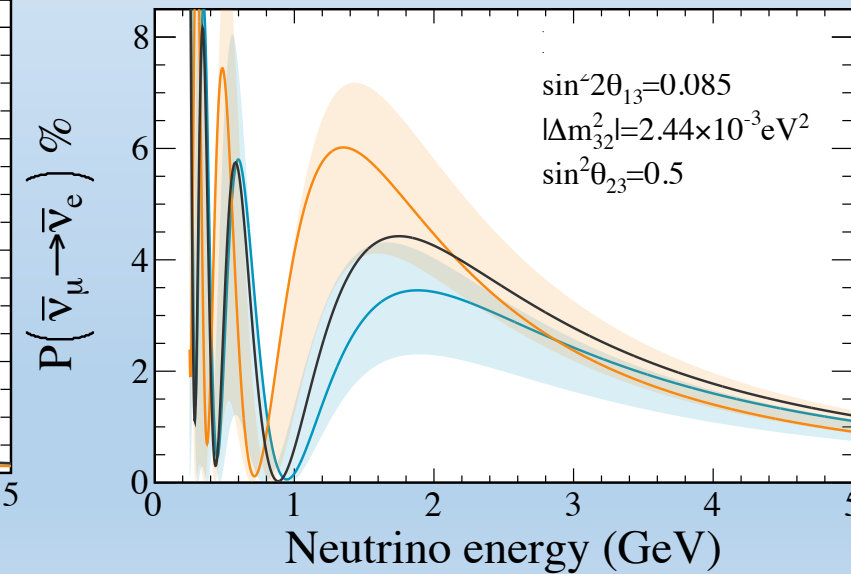
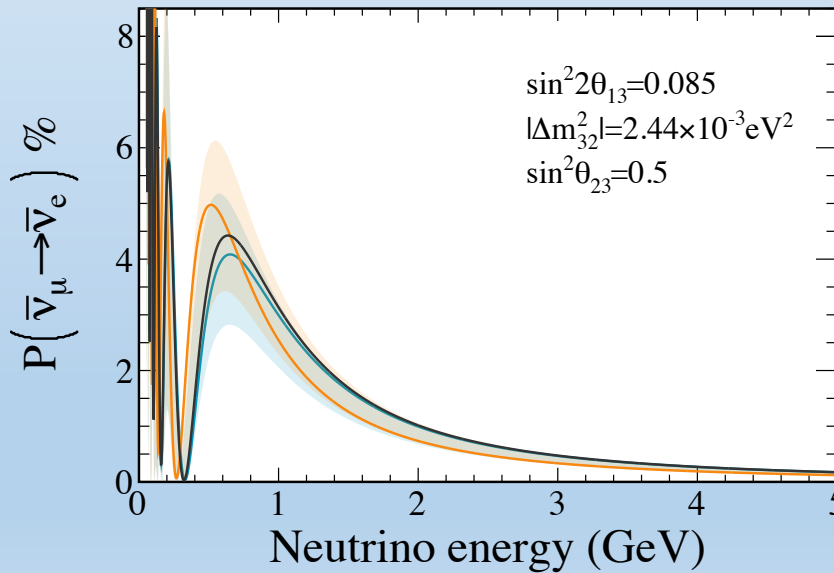
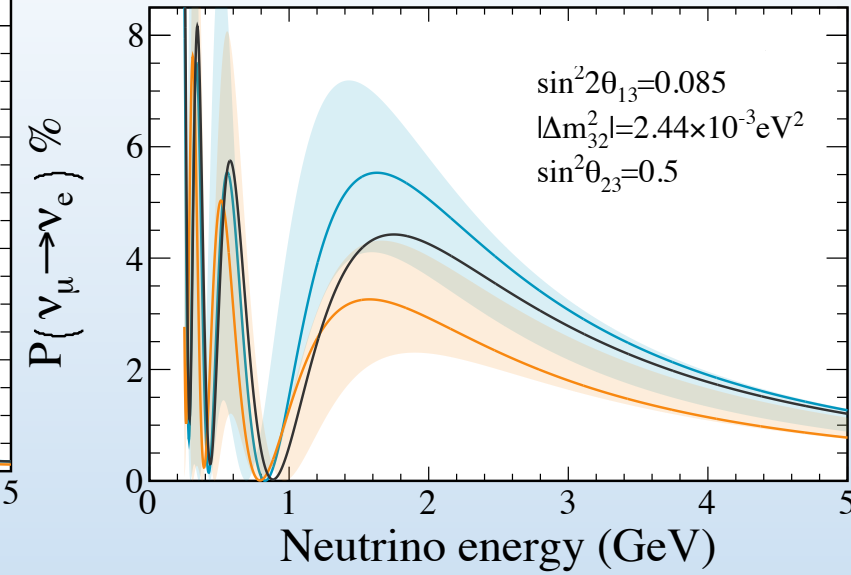
- More sensitivity to mass ordering for higher neutrino energy for the longer baseline.
- Opposite impact of matter effect and  $\delta_{CP}$  for  $\nu_e$  vs  $\bar{\nu}_e$  appearance probability.

	T2K	NOvA
L (baseline)	295 km	810 km
Energy (beam peak)	0.6 GeV	2 GeV
Matter effect	$\sim \pm 9\%$	$\sim \pm 19\%$
CP effect	$\sim \pm 30\%$	$\sim \pm 25\%$

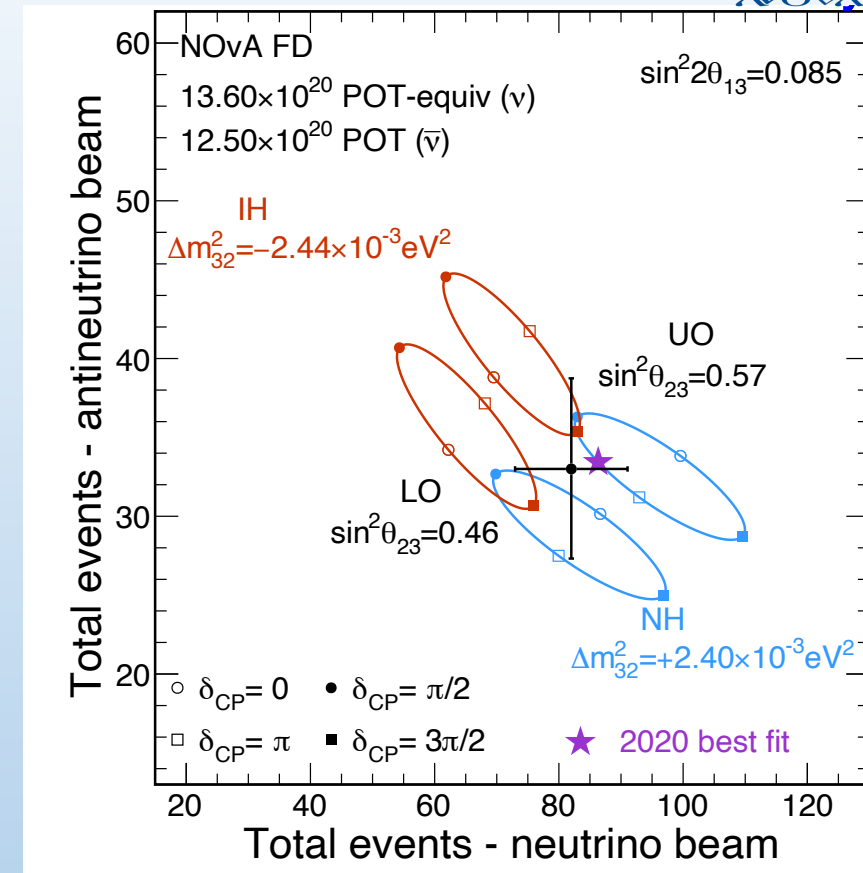
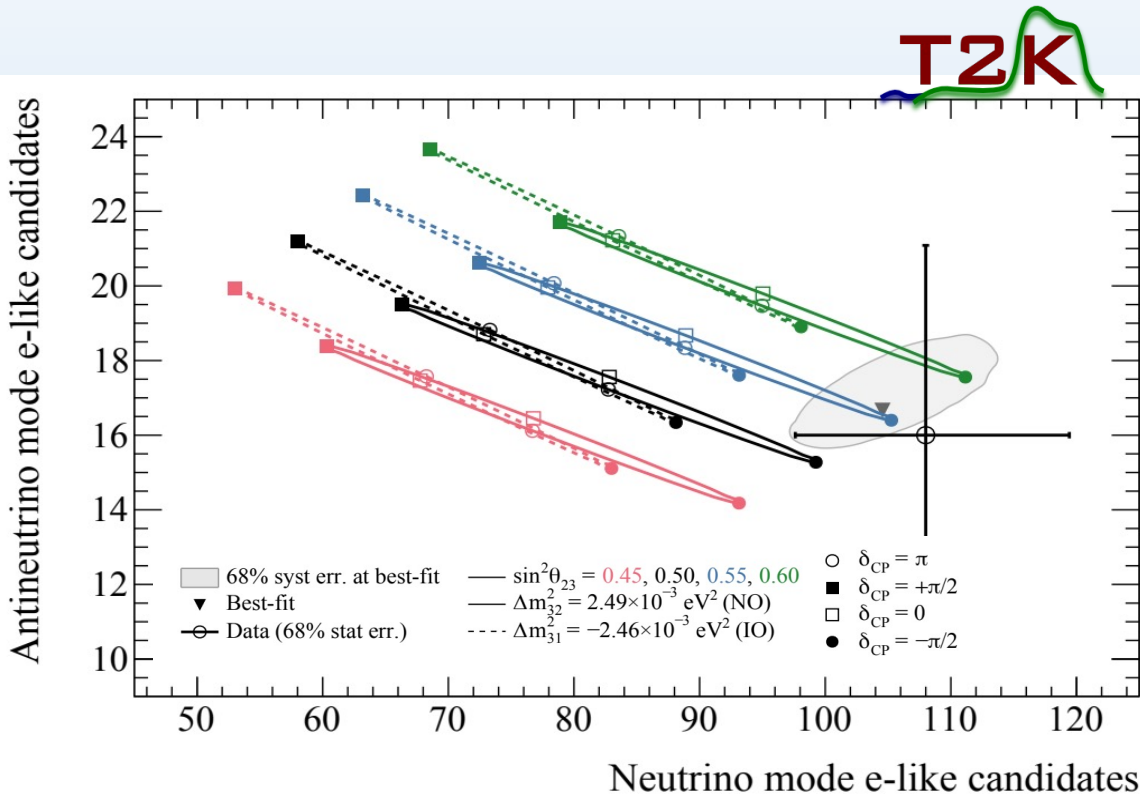
T2K: L = 295km



NOvA: L= 810 km



# Long-baseline landscape in 2020



- T2K saw an asymmetry in their  $\nu_e$  and  $\bar{\nu}_e$  appearance while NOvA did not.
- T2K's data favored large CP violation and normal mass ordering while NOvA data lies close to the degenerate  $\delta_{CP}$ -MO phase space.

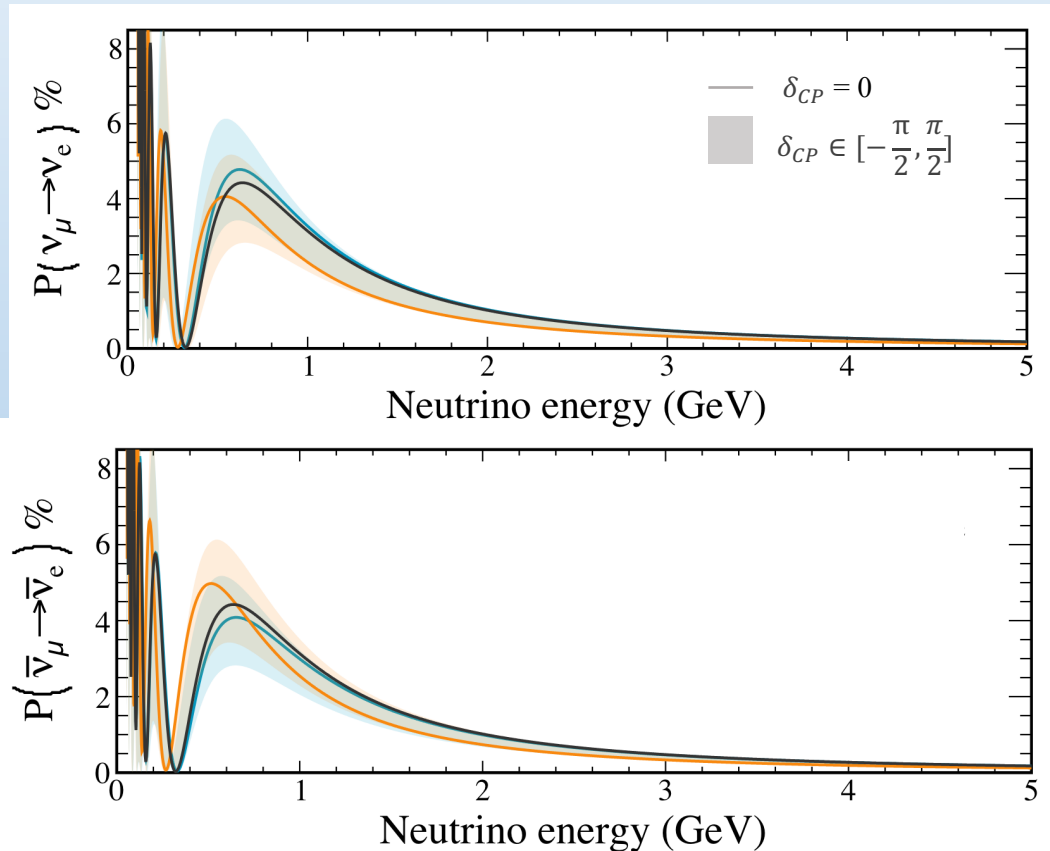




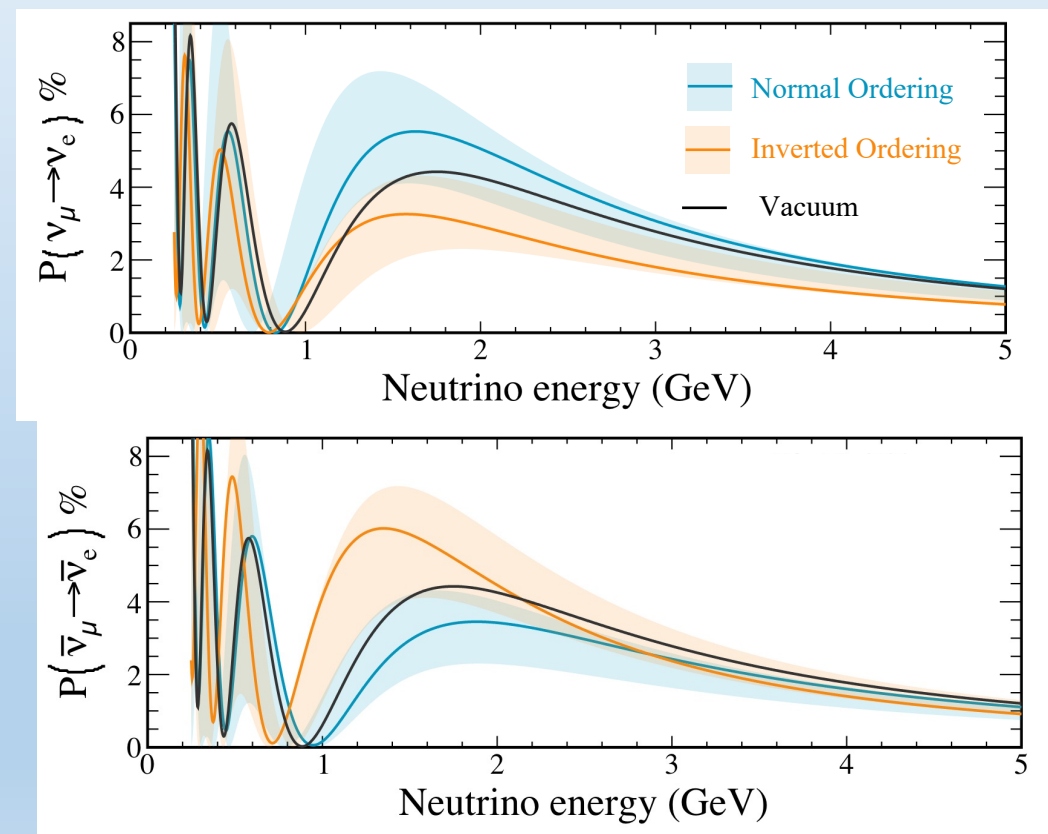
- Opposite impact of matter effect and  $\delta_{CP}$  for  $\nu_e$  vs  $\bar{\nu}_e$  appearance probability.
- Larger matter effect for higher neutrino energy  $\rightarrow$  higher sensitivity to mass ordering.
- Therefore, associated asymmetry is higher for the longer baseline.

# Baselines

**T2K: L = 295km**



**NOvA: L = 810 km**



# Long-baseline landscape in 2020

## CP-Violating Neutrino Non-Standard Interactions in Long-Baseline-Accelerator Data

Peter B. Denton,<sup>1,\*</sup> Julia Gehrlein,<sup>1,†</sup> and Rebekah Pestes<sup>1,2,‡</sup>

## Non-standard neutrino interactions as a solution to the NO $\nu$ A and T2K discrepancy

Sabya Sachi Chatterjee<sup>1,\*</sup> and Antonio Palazzo<sup>2,3,†</sup>

## Back to (Mass-)Square(d) One: The Neutrino Mass Ordering in Light of Recent Data

Kevin J. Kelly,<sup>1,\*</sup> Pedro A. N. Machado,<sup>1,†</sup> Stephen J. Parke,<sup>1,‡</sup>  
Yuber F. Perez-Gonzalez,<sup>1,2,3,§</sup> and Renata Zukanovich Funchal<sup>4,¶</sup>

## Energy-Dependent Neutrino Mixing Parameters at Oscillation Experiments

K. S. Babu,<sup>1</sup> Vedran Brdar,<sup>2,3</sup> André de Gouvêa,<sup>2</sup> and Pedro A. N. Machado<sup>3</sup>

