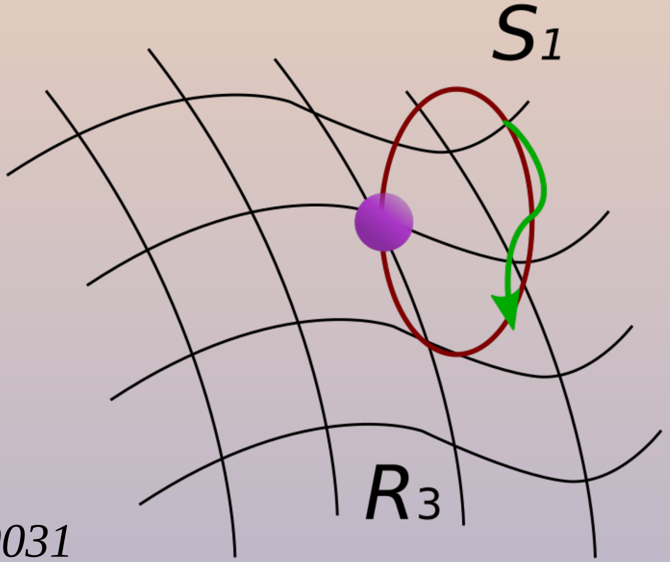
Extra dimensions in strong gravitational field

Anna Horváth

HUN-REN Wigner Research Centre for Physics
Eötvös Loránd University



In collaboration with:

Gergely Gábor Barnaföldi
Aneta Wojnar

Support:

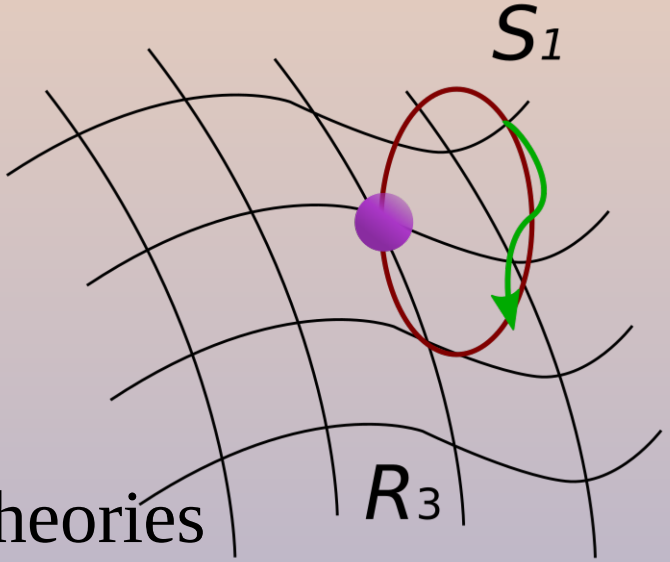
NKFIH through the DKÖP program of ELTE and OTKA K135515 and K147131
2024-1.2.5-TÉT-2024-00022, 2021-4.1.2-NEMZ_KI-2024-00031
HUN-REN's Mobility fellowship KMP-2023/101 and KMP-2024/31

Kaluza–Klein model

- 1921 Theodor Kaluza, 1926 Oskar Klein
- **Unifies gravity** and **electromagnetism** in a geometrical way

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + \kappa^2\Phi^2 A_\alpha A_\beta & \kappa\Phi^2 A_\alpha \\ \kappa\Phi^2 A_\beta & \Phi^2 \end{bmatrix}$$

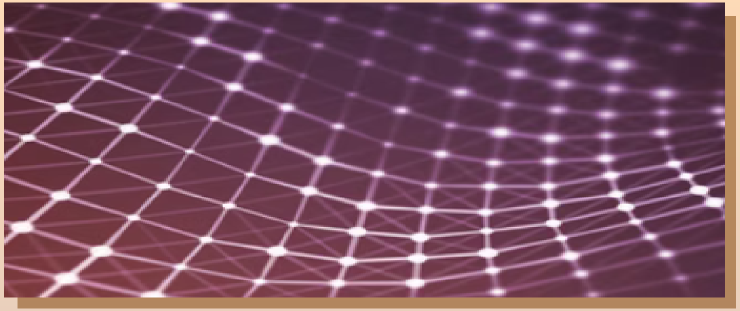
- Plus **scalar field**



- Compactified **extra** spatial **dimension**
- Base of quantum- and modified gravity theories

Overduin, J. M., & Wesson, P. S. 1997, Phys. Rept., 283, 303, doi: arXiv:gr-qc/9805018

Generalized Uncertainty Principle (GUP)



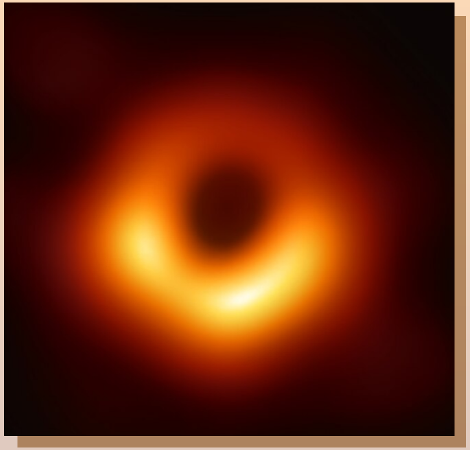
- **Minimal length** and/or **maximal momentum**
- **Schwarzschild** radius comparable to **Compton** wavelength

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \beta \Delta p^2$$

- **Modified commutators**, **dispersion relation**, speed of light
- At high energies gravity may become dominant
- Connected ideas: **noncommutative** geometry, **Lorentz** invariance **violation**

Abdel Nasser Tawfik and Abdel Magied Diab 2015 Rep. Prog. Phys. 78 126001
https://scienceexchange.caltech.edu/topics/quantum-science-explained/ask-expert-quantum/quantum-gravity-adhikari-zurek

KK, GUP and Black Holes



- **Extreme circumstances** are needed
    - **Extra dimensions** are small – large energy
    - Look for **anomalous gravitation**al effects

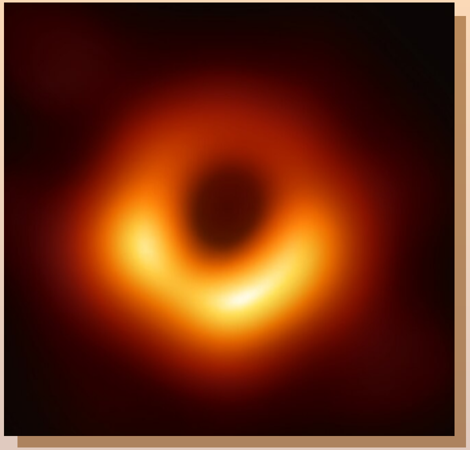
↓

- Interior of **neutron stars** – cold nuclear matter
- **White dwarfs**, core of **planets**
- **Cosmology**
- Vicinity of **black holes**

https://eventhorizontelescope.org/

Aleksander Kozak, Aneta Wojnar, “Earthquakes as probing tools for gravity theories”, 2023, arXiv:2308.01784

Solving the Einstein equations



- Kaluza – Klein metric

$$ds^2 = \gamma_{\mu\nu}dx^\mu dx^\nu + 2\gamma_{\mu 5}dx^\mu dx^5 + \gamma_{55}dx^5 dx^5$$

- **Dimensional reduction**

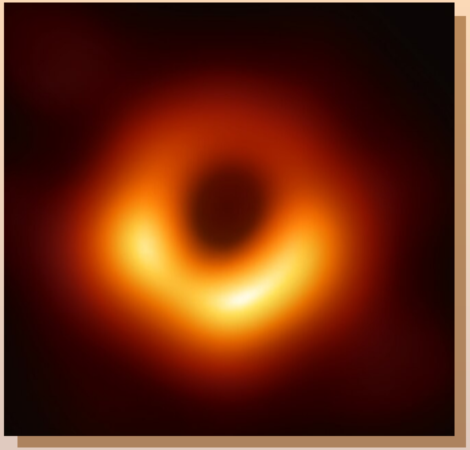
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2\sigma}\left(dx^5 + \kappa A_\mu dx^\mu\right)^2$$

- **Static, sphericall**y symmetric case, **neglecting EM**

$$ds^2 = -e^\nu dt^2 + e^{-\nu}dr^2 + e^{\lambda-\nu}d\Omega^2$$

R. Coquereaux and G. Esposito-Farese, in Annales de l'IHP Physique th´eorique, Vol. 52 (1990) pp. 113–150

Schwarzschild solution



- Usual (GR):

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

- **Generalized** (KK without EM):

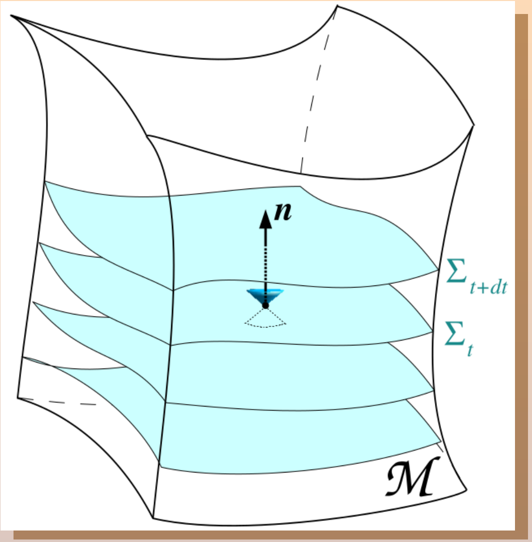
$$ds^2 = -\left(1 - \frac{a}{r}\right)^{\frac{b}{a}} dt^2 + \left(1 - \frac{a}{r}\right)^{-\frac{b}{a}} dr^2 + r^2 \left(1 - \frac{a}{r}\right)^{1-\frac{b}{a}} d\Omega^2$$

$$a^2 = b^2 + 3d^2$$

where *d* can be connected to the **scalar field** and its **derivative**

R. Coquereaux and G. Esposito-Farese, in Annales de l'IHP Physique th´eorique, Vol. 52 (1990) pp. 113–150

ADM formalism



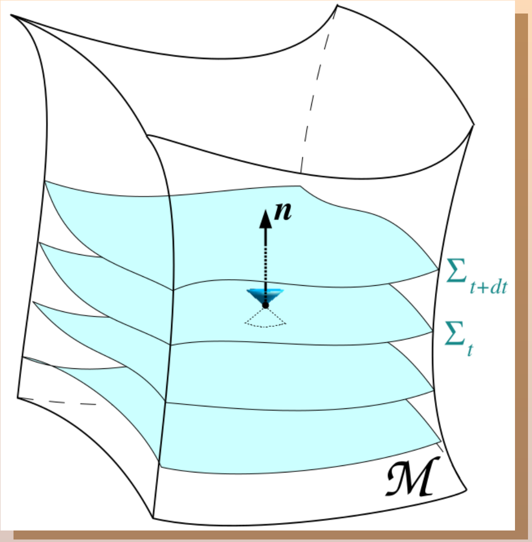
- Mathematical tool
- **Hamiltonian** formulation of GR
- Splits 4D spacetime into **3D foliations**

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- The **lapse** function N and the **shift vector** N^i define how foliations connect
- **Observer dependent** ⟶ our choice: **comoving**

Éric Gourgoulhon, “3+1 formalism and bases of numerical relativity”, 2007, arXiv:gr-qc/0703035v1

GUP and particles



- Generalized uncertainty principle:

$$\sigma_p \rho \gtrsim \pi\hbar \left[1 - \frac{\rho^2 \mathcal{R}|_{p_0}}{12\pi^2} + \xi \frac{\rho^4}{\lambda_C^2} \nabla_j N_i \nabla^j N^i |_{p_0} \right]$$

- Dispersion relation

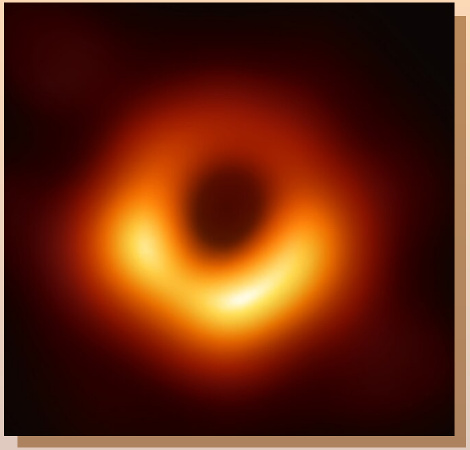
$$p^\mu p_\mu = -m^2 c^2 - \frac{\mathcal{R}}{6}$$

- Effective mass:

$$m_{\text{eff}} = \sqrt{m^2 + \frac{\mathcal{R}}{6c^2}}$$

L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).

Ricci scalar curvatures



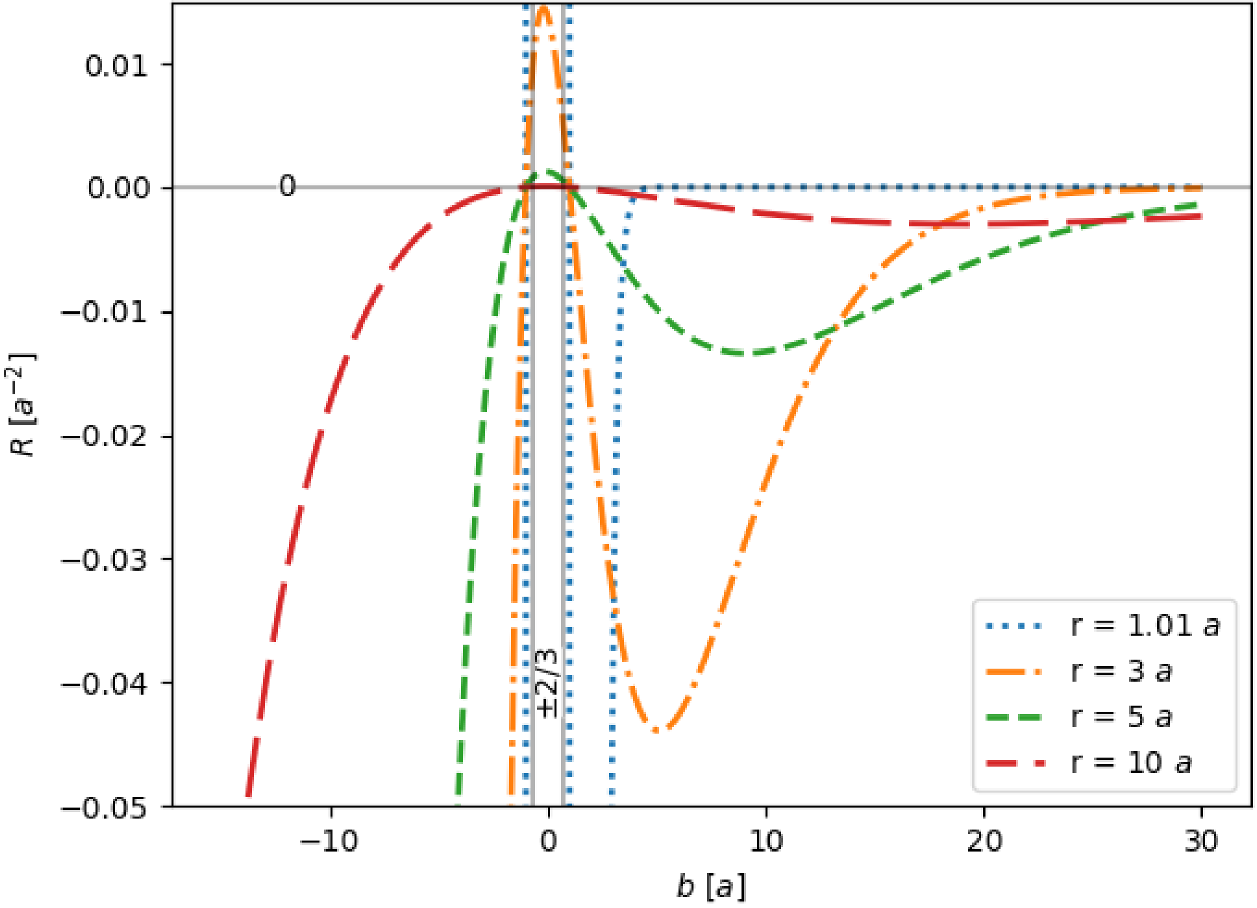
- **Spacetime** curvature (4D):

$$R = \frac{(a^2 - b^2)}{2r^4}\left(1 - \frac{a}{r}\right)^{\frac{b}{a}-2}$$

- **Phase space** curvature (3D ADM split):

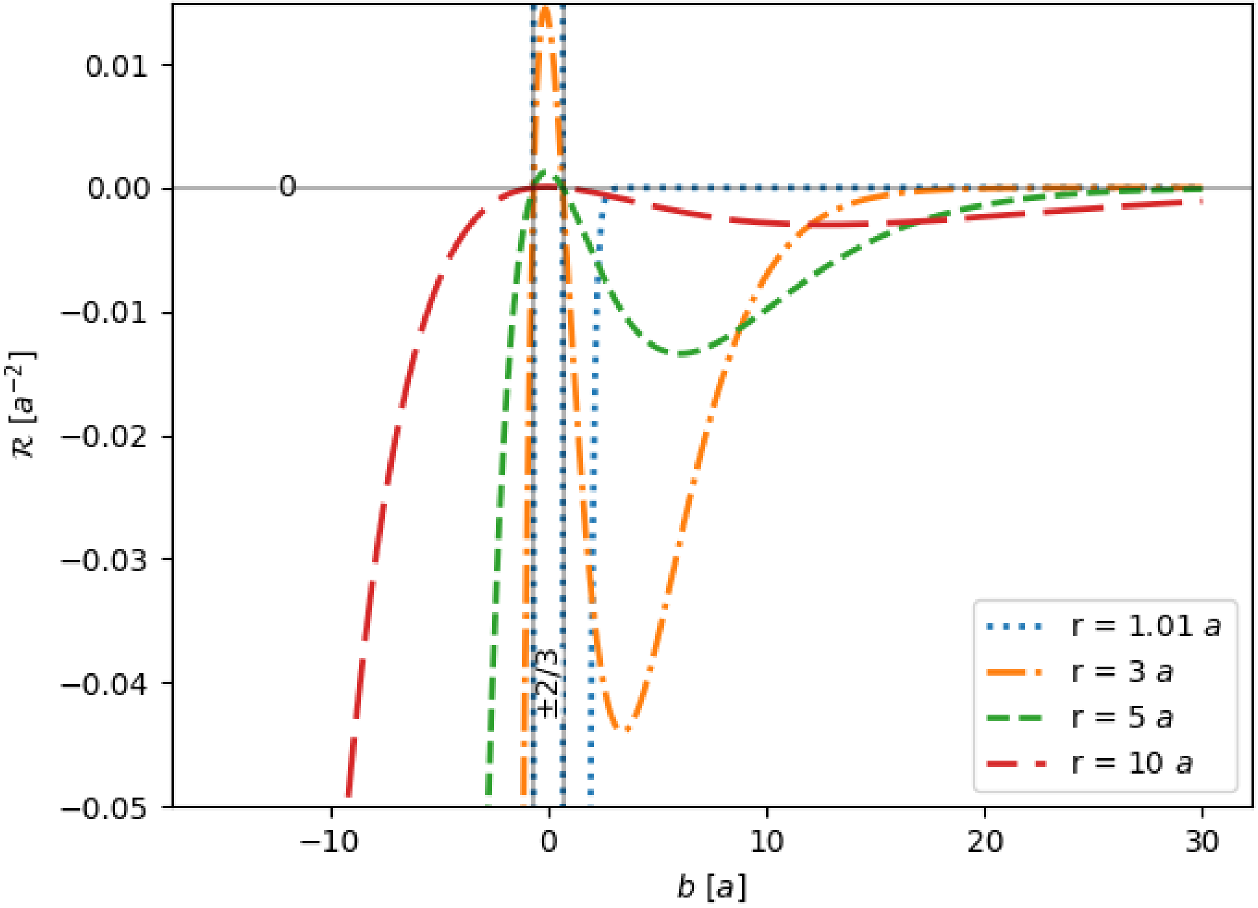
$$\mathcal{R} = \frac{(4a^2 - 9b^2)}{8r^4}\left(1 - \frac{a}{r}\right)^{\frac{3b}{2a}-2}$$

Spacetime



0.01
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
R [a⁻²]
0
±2/3
−10
0
10
20
30
b [a]
r = 1.01 a
r = 3 a
r = 5 a
r = 10 a

Phase space


0.01
0.00
−0.01
−0.02
−0.03
−0.04
−0.05
$\mathcal{R}$ [a^{-2}]
0
±2/3
−10
0
10
20
30
b [a]
r = 1.01 a
r = 3 a
r = 5 a
r = 10 a

Summary

- We considered **Kaluza–Klein** spacetime with one **extra** spatial **dimension**
- Investigated the vicinity of a **Schwarzschild black hole**
- Calculated the **spacetime** and **phase space** Ricci scalar **curvatures**
- Found **modifications** to:
    - both curvatures
    - uncertainty principle
    - dispersion relation
    - measured mass of test particles
- Possible **observation**al phenomena: position of **photon sphere**, structure and spectrum of **accretion disk**